Probablity And Statistics Assignment 4

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-3CS3

Q1. The probability distribution of X, the number of imperfections per 10 meters of asynthetic fabric in continuous rolls of uniform width, is given as

x 0 1 2 3 4

p(x) 0.41 0.37 0.16 0.05 0.01

Find the average number of imperfections per 10 meters of this fabric.
(Try functions sum( ), weighted.mean( ), c(a %*% b) to find expected value/mean.

Solution:

x <- c(0,1,2,3,4)
fx<-c(0.41, 0.37, 0.16, 0.05, 0.01)
expected_value <-
sum(x*fx)expected_value

mean_x <-
weighted.mean(x,fx)mean_x

other <-
c(x%*%fx)other

```
> #Q1.
>
> x <- c(0,1,2,3,4)
> fx<-c(0.41, 0.37, 0.16, 0.05, 0.01)
> expected_value <- sum(x*fx)
> expected_value
[1] 0.88
>
> mean_x <- weighted.mean(x,fx)
> mean_x
[1] 0.88
>
> other <- c(x%*%fx)
> other
[1] 0.88
>
```

Q2. The time T, in days, required for the completion of a contracted project is a random variable with probability density function f(t) = 0.1 e(-0.1t) for t > 0 and 0 otherwise. Find the expectedvalue of T.

Use function integrate( ) to find the expected value of continuous random

variable T.Solution:

```
f <- function(t) {
 0.1 * exp(-0.1 * t)
}

integrand <- function(t)
{
 t * f(t)
}

result <- integrate(integrand, lower = 0, upper =
Inf)$valueresult
```

```
> #Q2.
>
> f <- function(t) {
+   0.1 * exp(-0.1 * t)
+ }
>
> integrand <- function(t)
+ {
+   t * f(t)
+ }
>
> result <- integrate(integrand, lower = 0, upper = Inf)$value
> result
[1] 10
>
```

Q3 . A bookstore purchases three copies of a book at $6.00 each and sells them for $12.00each. Unsold copies are returned for $2.00 each. Let X = {number of copies sold} and
Y = {net revenue}. If the probability mass function of X is

x 0 1 2
3
p(x) 0.1 0.2 0.2
0.5

Find the expected value of Y

Solution:

x <- c(0,1,2,3)
px <- c(0.1,0.2,0.2,0.5)

frevenue <- function(x)
{10 * x -12
}

value <- sum(frevenue(x) *
px)valuue

```
> # Q3.
>
> x <- c(0,1,2,3)
> px <- c(0.1,0.2,0.2,0.5)
>
> frevenue <- function(x) {
+   10 * x -12
+ }
>
> value <- sum(frevenue(x) * px)
> value
[1] 9
>
```

Q4. Find the first and second moments about the origin of the random variable X with probability density function f(x) = 0.5e-|x|, 1 < x < 10 and 0 otherwise. Further use the results to find Mean and Variance. (kth moment = E(Xk), Mean = first moment and Variance = secondmoment – Mean2

Solution:

```
pdf <- function(x) {
 ifelse(x > 1 & x< 10, 0.5 * exp(-x),0)
}

first_moment_integrand <-
 function(x) {x * pdf(x)
}

second_moment_integrand <-
 function(x) {x^2 * pdf(x)
}

first_moment <- integrate(first_moment_integrand, lower = 1, upper = 10)$value
second_moment <- integrate(second_moment_integrand, lower = 1, upper =

10)$valuevariance = second_moment - first_moment^2

cat("First moment (mean) is: ", first_moment,
"\n")cat("Second moment is: ",
second_moment, "\n") cat("Variance is: ",
variance, "\n")
```

```
> #Q4.
>
> pdf <- function(x) {
+   ifelse(x > 1 & x< 10, 0.5 * exp(-x),0)
+ }
>
> first_moment_integrand <- function(x) {
+   x * pdf(x)
+ }
>
> second_moment_integrand <- function(x) {
+   x^2 * pdf(x)
+ }
>
> first_moment <- integrate(first_moment_integrand, lower = 1, upper = 10)$value
> second_moment <- integrate(second_moment_integrand, lower = 1, upper = 10)$value
>
> variance = second_moment - first_moment^2
>
> cat("First moment (mean) is: ", first_moment, "\n")
First moment (mean) is:  0.3676297
> cat("Second moment is: ", second_moment, "\n")
Second moment is:  0.9169292
> cat("Variance is: ", variance, "\n")
Variance is:  0.7817776
> |
```

Q5. Let X be a geometric random variable with probability distribution f(x) =34(14)x−1, x = 1,2,3, ...
Write a function to find the probability distribution of the random variable Y = X2
and find probability of Y for X = 3. Further, use it to find the expected value and variance of Y for X =1,2,3,4,5.

Solution:

```
pmf_X <- function(x) (3/4) * (1/4)^(x - 1)
pmf_Y <- function(y) {
x <- sqrt(y)
if (x %% 1 == 0 && x >= 1) pmf_X(x) else 0
}prob_Y_9 <- pmf_Y(9)
print(paste("Probability of Y = 9 when X = 3:", prob_Y_9))
X_values <- 1:5
Y_values <- X_values^2
prob_Y_values <- sapply(Y_values, pmf_Y)
E_Y <- sum(Y_values * prob_Y_values)
E_Y2 <- sum((Y_values^2) * prob_Y_values)
Var_Y <- E_Y2 - E_Y^2
print(paste("Expected value of Y:", E_Y))
print(paste("Variance of Y:", Var_Y))
```

```
> #Q5
>
> pmf_X <- function(x) (3/4) * (1/4)^(x - 1)
>
>
> pmf_Y <- function(y) {
+   x <- sqrt(y)
+   if (x %% 1 == 0 && x >= 1) pmf_X(x) else 0
+ }
>
> prob_Y_9 <- pmf_Y(9)
> print(paste("Probability of Y = 9 when X = 3:", prob_Y_9))
[1] "Probability of Y = 9 when X = 3: 0.046875"
>
>
> X_values <- 1:5
>
>
> Y_values <- X_values^2
> prob_Y_values <- sapply(Y_values, pmf_Y)
> E_Y <- sum(Y_values * prob_Y_values)
> E_Y2 <- sum((Y_values^2) * prob_Y_values)
> Var_Y <- E_Y2 - E_Y^2
>
> print(paste("Expected value of Y:", E_Y))
[1] "Expected value of Y: 2.1826171875"
> print(paste("Variance of Y:", Var_Y))
[1] "Variance of Y: 7.61411190032959"
>
```