# CPTS 591: ELEMENTS OF NETWORK SCIENCE

## SPRING-2024

## MIDTERM EXAM

SHARATH KUMAR KARNATI
011852253

## PROBLEM-1:-

a) TRUE, (Partially)

Because the degree distribution of an ERDOS-RENYI Graph is binomial in general. But, when the network groves and is large it will be Poisson.

b) TRUE.

c) TRUE,

Because, both networks have small average path length.

d) FALSE,

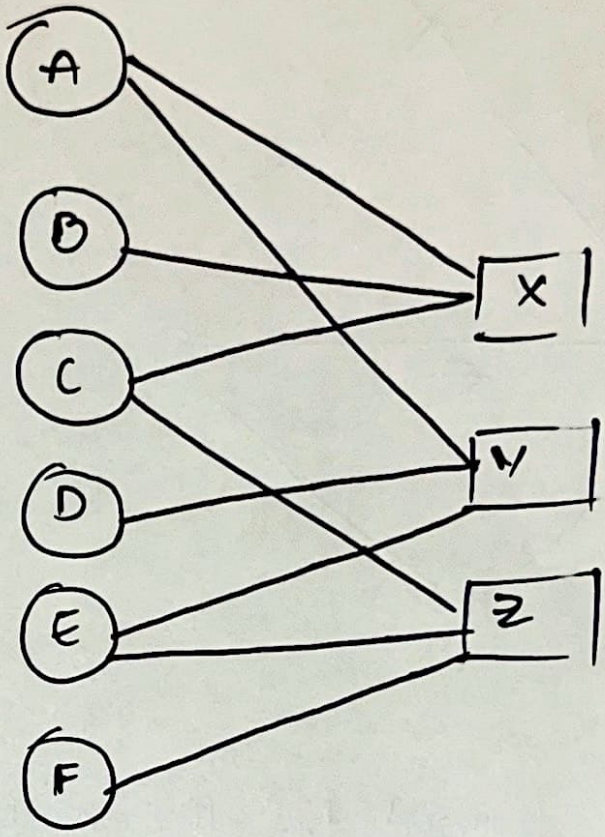
Because, they are not always equal. They varies from our graph to another graph.

e) TRUE.

PROBLEM -②:-

Given,

Graph:-




A) Derived Network:-

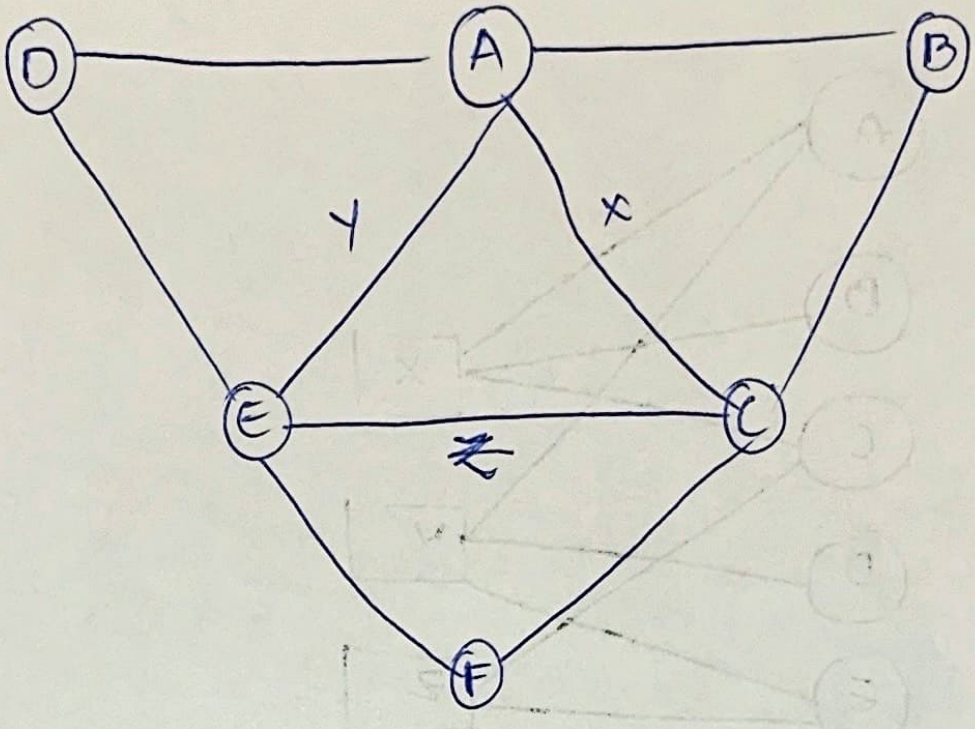
from the given graph, we have

X → A, B, C

Y → A, D, E

Z → C, E, F

then, the derived network will be:-




B) The triangle formed by the nodes A, C, E is the central triangle in which we can observe that the each node of this triangle is a part of two foci: (or) in other words they belongs to the other two triangles also.

i.e., A → has X & Y foci

C → has X & Z foci

E → has Y & Z foci

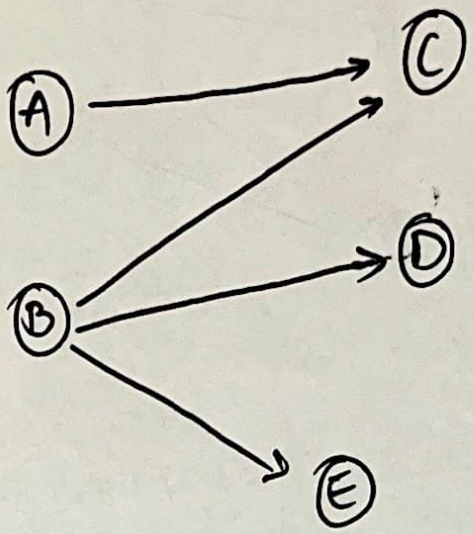
which means, A is in [ABC] & [ADE] Δles.

C is in [CFE] & [A, B, C] Δles.

E is in [E, C, F] & [A, D, E] Δles. //

PROBLEM ③ :-

Given network of five Web pages A - E.




from the given ~~graph~~ network, we can say that,

Ⓐ and Ⓑ are HUBS

Ⓒ, Ⓓ, Ⓔ are AUTHORITIES.

Now, let us consider that the weight for A and B is ①.

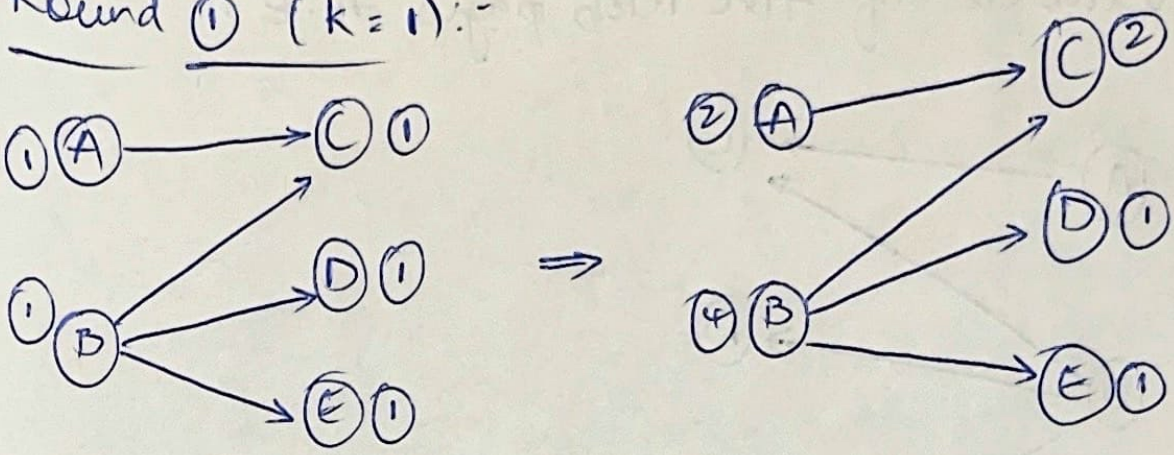
Then, for node C, the weight will be ② as there are two incoming values from A & B

for node D, the weight is ① as only one incoming value from B

for node E also, the weight is ① as only one inbound edge B.

Therefore, now the graph looks like:

Round (1) (k = 1):-




| Nodes | HUBS | AUTHORITY |
|---|---|---|
| A | 1 | 0 |
| B | 3 | 0 |
| C | 0 | 2 |
| D | 0 | 1 |
| E | 0 | 1 |

Then, the adjacency matrix will be:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 0 | 0 | 1 | 0 | 0 |
| B | 0 | 0 | 1 | 1 | 1 |
| C | 0 | 0 | 0 | 0 | 0 |
| D | 0 | 0 | 0 | 0 | 0 |
| E | 0 | 0 | 0 | 0 | 0 |

⇒

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The transpose matrix will be :-

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

⇒ Now taking k as (1) i.e., the hub weight vector as (1)

then, Authority weight vector $\boxed{V = A^T \times U}$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

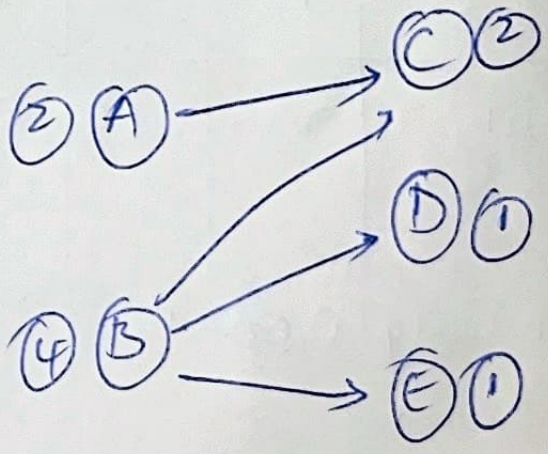
Similarly, hub weight vector $\boxed{U = A \times V}$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

then,

| k = 1 | A | B | C | D | E |
|---|---|---|---|---|---|
| Hub scores | 2 | 4 | 0 | 0 | 0 |
| Authority scores | 0 | 0 | 2 | 1 | 1 |

⇒



for $k = 2$,

Updating Hub and Authority scores,

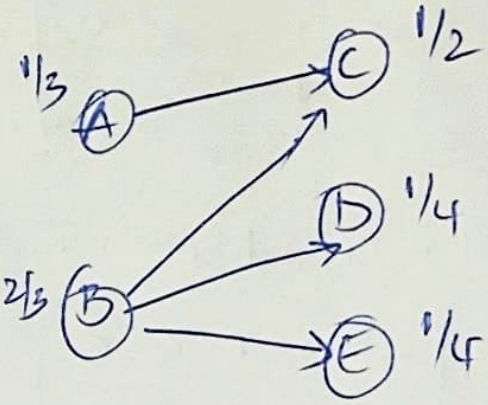
$$u = 2 + 4 = 6$$

$$\therefore u = \left( \frac{2}{6}, \frac{4}{6}, \frac{0}{6}, \frac{0}{6}, \frac{0}{6} \right)$$

Authority scores,

$$v = 2 + 1 + 1 = 4 = \left( 0, 0, \frac{2}{4}, \frac{1}{4}, \frac{1}{4} \right)$$

| $k=2$ | A | B | C | D | E |
|---|---|---|---|---|---|
| Hub scores | 1/3 | 2/3 | 0 | 0 | 0 |
| Authority scores | 0 | 0 | 1/2 | 1/4 | 1/4 |




Result:

| $k=2$ | A | B | C | D | E |
|---|---|---|---|---|---|
| HUB scores | 0.33 | 0.66 | 0 | 0 | 0 |
| Authority scores | 0 | 0 | 0.5 | 0.25 | 0.25 |

PROBLEM - 4 :-

Given,

There are three villages which has 30 people in each of it.

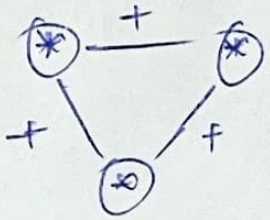
People of same village are friends, where as people of one village are enemies with the people of other village.

So, there are 90 people in total of 3 villages.

Only, the network within the people of same village is positive and it is negative outside that village.

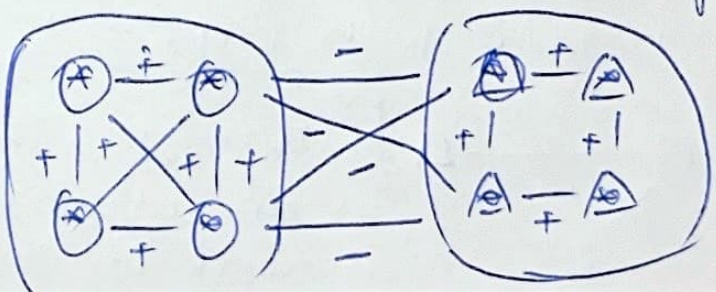
Representation of networks :-

① Within people of same village.



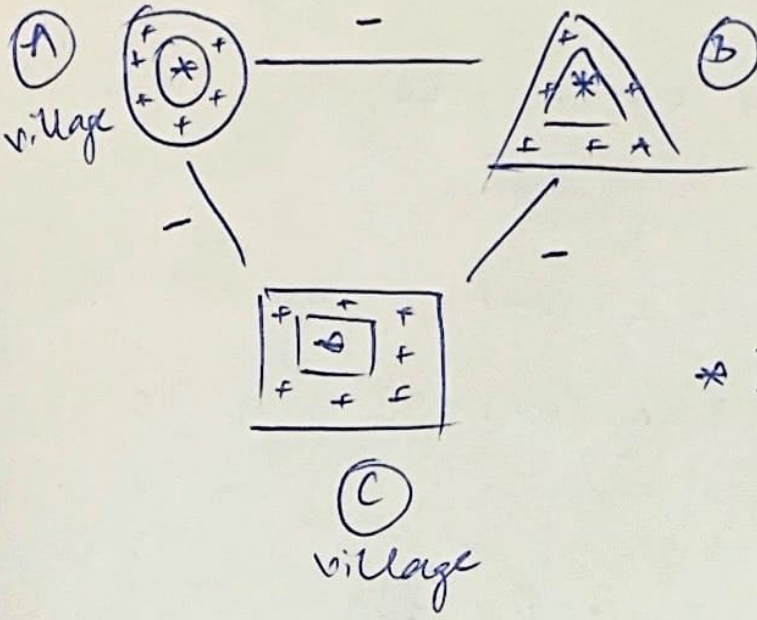
∴ It is balanced network.

② Between two different village people.



∴ It is still balanced network.

③ Between three village people



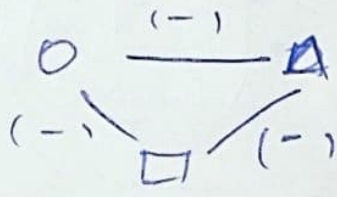

* It is now an unbalanced network.

* We can say a network is balanced when "

" If it can be partioned into two vertex subsets, so that the intra group edges are all positive and inter group edges are all negative." like in case ①

* It can be even balanced when "only if all edges are positive or when one edge is negative" like in case ②

* But when it comes to network of 3 villages. All the edges are negative with respect to other villages.




* Therefore, The given is an unbalanced network of 90 people.

## PROBLEM-5:

TRUE.

If the largest eigen value of a graph's adjacency matrix is strictly less than 2, it means that there are no eigen vectors corresponding to cycles. In the context of graphs, this indicates that the graph is cycle-free graph, as cycles would require eigenvalues equal or greater than 2.

This is because the largest eigen value is related to existence of cycles in the graph. Specifically, a graph with no cycles will have a largest eigen value equal to the square root of its maximum degree and for a graph with cycles, the largest eigen values will be greater than or equal to the square root of its maximum degree.

∴ Therefore, if the largest eigen value is strictly less than 2, the graph is cycle free.