IST-772 Quantitative Reasoning in Data science

Week3/HW-3: Sampling Distributions

Probabilities in the long run (Page-50, 51: Problems:2-7)

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1. Use R's built-in dataset ChickWeight. Capture and explain output of summary() and shape() functions

```
> cw <- ChickWeight
> summary(cw)
     weight           Time           Chick     Diet
 Min.   : 35.0   Min.   : 0.00   13     : 12   1:220
 1st Qu.: 63.0   1st Qu.: 4.00   9      : 12   2:120
 Median :103.0   Median :10.00   20     : 12   3:120
 Mean   :121.8   Mean   :10.72   10     : 12   4:118
 3rd Qu.:163.8   3rd Qu.:16.00   17     : 12
 Max.   :373.0   Max.   :21.00   19     : 12
                                 (Other):506
```

The summary function above shows the min, 1st quartile, Median, Mean, 3rd quartile and max values for each variable or column viz. weight, Time, Chick and Diet:

- Weight (in grams): Min/Max = 35/373, 1st/3rd quartile = 63/163.8, Median/Mean = 103/121.8
- Time (in days): Min/Max = 0/21, 1st/3rd quartile = 4/16, Median/Mean = 10/10.72
- Chick variable or column is type factor
- Diet variable or column is type factor comprising 4-levels viz. 1-4

```
> dim(cw)
[1] 578   4
```

The output of dim(cw) shows 578 rows and 4 columns.

2. Run the following commands. Report and explain the output of each:

summary(ChickWeight$weight)
head(ChickWeight$weight)
mean(ChickWeight$weight)
myChkWts <- ChickWeight$weight
quantile(myChkWts, 0.5)

```
> summary(cw)
     weight           Time           Chick     Diet
 Min.   : 35.0   Min.   : 0.00   13     : 12   1:220
 1st Qu.: 63.0   1st Qu.: 4.00   9      : 12   2:120
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                                 (Other):506
```

Command interpretation: Displays the min, max, central tendencies of mean/median and quartile ranges for the dataset.

```
> head(cw)
  weight Time Chick Diet
1     42    0     1    1
2     51    2     1    1
3     59    4     1    1
4     64    6     1    1
5     76    8     1    1
6     93   10     1    1
```

Command interpretation: Displays the fist few rows of the data-frame, cw (ChickWeight dataset).

```
> mean(cw$weight)
[1] 121.8183
```

Command interpretation: Calculates the mean of the weight variable or column in the ChickWeight dataset.

This value matches the Mean value for weight from the summary(cw) output.

```
> myChkWts <- cw$weight
```

Command interpretation: Using R's $, access a specific variable or column and assign those values to a variable myChkWts

```
> quantile(myChkWts, 0.5)
50%
103
```

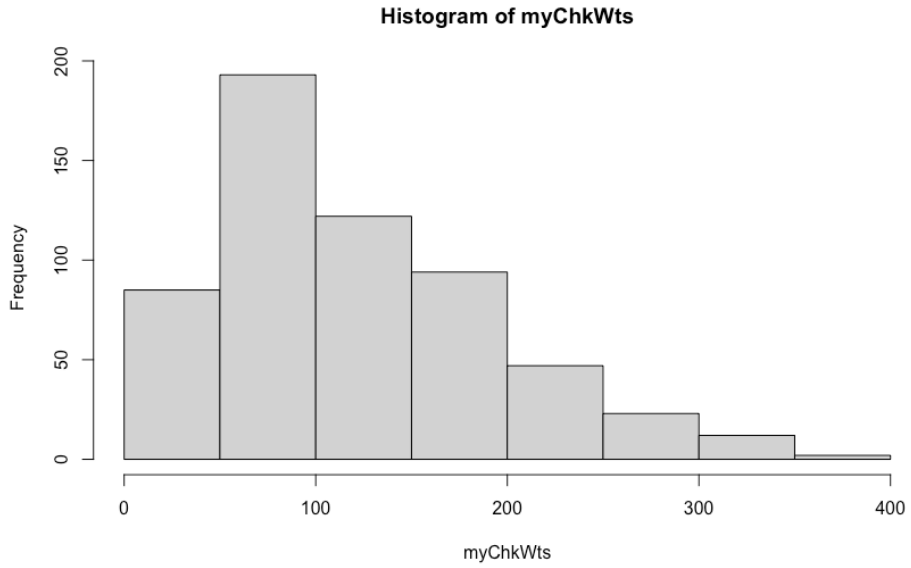
Command interpretation: The quantile function takes two arguments:

- Arg-1: Vector or data – here, chick weights
- Arg-2: Cut-point over the data

The output shows 50% (equivalent to 0.5) as 103. This value matches the median for weight from the summary(cw) output.

3. Create a histogram of the variable myChkWts and then write R code to display the 2.5% and 97.5% quantiles. Further explain the mean, median and shape of distribution. Describe clearly what the 2.5% and 97.5% quantiles signify

```
> hist(myChkWts)
```



The shape of the curve is not a regular bell-shape or normal distribution. Instead, it seems to be right-skewed or positive-skewed since there is a long tail in the positive direction of the number line.

```
> # Display quantile of 2.5% or 0.025 and 97.5% or 0.975
> quantile(myChkWts, c(0.025, 0.975))
   2.5%   97.5%
 41.000 294.575

> mean(myChkWts)
[1] 121.8183

> median(myChkWts)
[1] 103
```

The mean or average weight of chicks across 21 days is 121.82g. The median or middle point of data is 103g. Quantile output shows two values - 2.5% and 97.5% cut-points as 41g and 294.56g respectively. This means that:

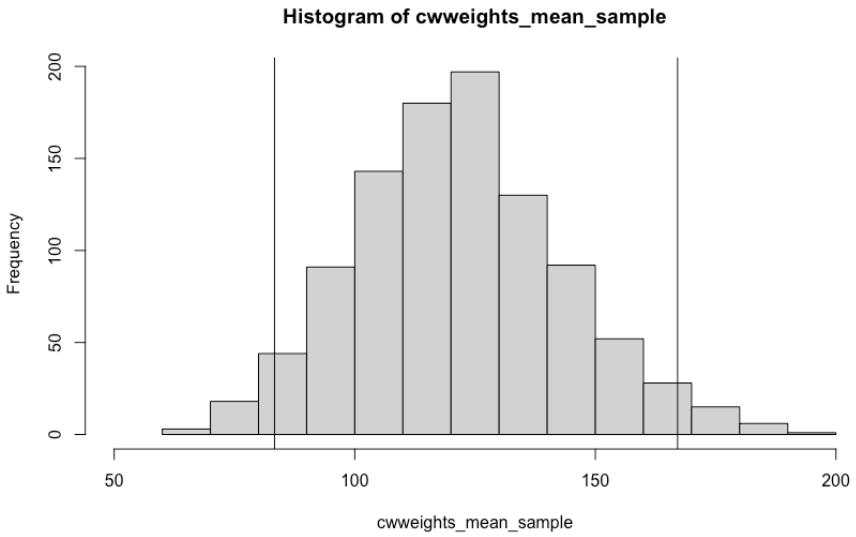
- 2.5% of all values or observations lie at or below 41
- 2.5% of all values or observations lie at or above 294.56

4. Write R code that will construct sampling distribution of means from weight data. Use a sample size of 11 with replacement. Show a histogram of the distribution of sample means. Also display the 2.5% and 97.5% quantiles of the sampling distribution on the histogram with vertical line.

We can use the following R-code to achieve this:

```
# Sample the weights
cwweights_mean_sample <- replicate(1000, mean(sample(myChkWts, size=11, replace = T)))
hist(cwweights_mean_sample, xlim=c(50, 200))
```

```
# Using abline indicate the 2.5% and 97.5% values/cut-points on the histogram
abline(v=quantile(cwweights_mean_sample, c(0.025, 0.975)))
```



5. Briefly describe what the difference between a distribution of raw data and a distribution of sample means is. Comment why the 2.5% and 97.5% quantiles are so different.

The distribution of raw data showed a right-skewed distribution whereas, the distribution of sampling means was a normal distribution.

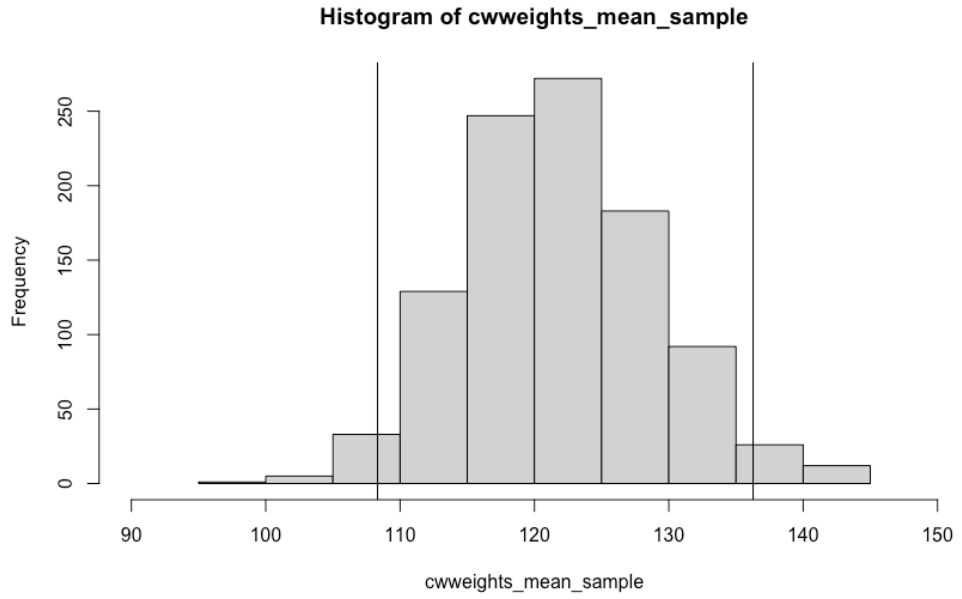
The reason distribution of sampling means tends to be a bell-shaped curve or normal distribution is because of the law of large numbers and central limit theorem:

- when you run a statistical process like sampling many times, it will generally converge on a particular result
- the distribution of sampling means starts to show a bell-shaped or normal distribution

Finally, when comparing the 2.5% and 97.5% quantiles we see 41, 294.56 respectively for raw data vs 83.33, 167.1 respectively for distribution of sampling means. Since the data tends to shift or converge towards the mean/median when sampling (with means), the quantiles tend to shift right correspondingly.

6. Redo the sampling with a larger size = 100. Explain the difference is quantiles for 2.5% and 97.5%.

Using n=100 for sampling size, we see the following histogram:



The corresponding quantiles at 2.5% and 97.5% are:

```
> quantile(cwweights_mean_sample, c(0.025, 0.975))
    2.5%    97.5%
108.3190 136.2745
```

Comparing the quantiles for 2.5% and 95.5% for sample size=11 vs 100 (83.33, 167.1 vs 108.31, 136.27) we can see a further right shift towards the central mean/median - recap that the mean/median for the weights column as initially recorded via summary() was 21/103 respectively. This aligns with our understanding of central limit theorem.