Prep for Quiz #2

1) The distribution of weekly incomes of supervisors at the ABC Company follows the normal distribution, with a mean of $1000 and a standard deviation of $100. What percent of the supervisors have a weekly income between $840 and $1200?

2) Twenty percent of the employees of ABC Company use direct deposit and have their wages sent directly to the bank. Assume we random sample five employees.
a. What is the probability that all five employees use direct deposit?
b. What is the probability that at least 2 employees use direct deposit?
c. What is the probability that 4 or more employees use direct deposit?

3) Three work shifts producing the same product sorted the finished product into 4 categories based on its quality level and displayed the results in the following table. Determine whether there is dependence between the shift and the quality of the product? (Does product quality depend on the shift that produces it?) Assume alpha= 0.05

	1^{st} Shift	**2^{nd} Shift**	**3^{rd} Shift**
Perfect product	185	175	170
Acceptable product	55	60	65
Defective product	15	15	15
Reworked product	10	15	15

4) A bullet manufacturer claims to have produced a projectile having a mean muzzle velocity of more than 3000 feet per second. From a random sample of 60 bullets he calculates a sample mean of 3012 feet per second and a sample standard deviation of 112 feet per second. Does the data from the sample support his claim?

Solutions:

Answer #1) Given:
Normal distribution
μ (mean) = 1000
σ (standard deviation) =100
x1 & x2 (salary range) = 840,1200

A) Find: **P(840≤ X ≤ 1200)** = P(X≤ 1200) – P(X≤ 840)

B) Need to first convert to standardized values

Z formula =
(for the low end of the salary range)

$$Z = \frac{x-\mu}{\sigma} = \frac{840-1000}{100} = -1.6$$

Z formula =
(for the high end of the salary range)

$$Z = \frac{x-\mu}{\sigma} = \frac{1200-1000}{100} = 2$$

C) Using the *Standard Normal Probabilities* in the back of the book (**Table C**)

For the low end of the salary range:
Looking for P(Z ≤ -1.6) so look for z at -1.60 on Table C. The probability (area under the curve) is = **0.0548** Resulting in: P(X≤ 840) = **0.0548**

or using Excel function =NORMSDIST(-1.6) (for Excel 2010 use =NORM.S.DIST(-1.6,TRUE))
or you can use =NORMDIST(840, 1000, 100, True) (for Excel 2010 use NORM.DIST)

For the high end of the salary range:
Looking for P(X≤ 1200) = P(Z ≤ 2) = **0.9772** (using Table C at z=2.0)
or using Excel function =NORMDIST(1200, 1000, 100, True)

D) **P(840≤ X ≤ 1200)** = P(X≤ 1200) – P(X≤ 840) = 0.9772 – 0.0548 = **0.9224**

This is a probability that can be stated as a percent.
Answer: **92.24%**

Answer #2a) Given:

20% of employees use direct deposit so the probability someone uses direct deposit = 0.2
p = 0.2
sample size n = 5
Employees either use direct deposit or they don't (Binomial)
Using direct deposit = success = x = 5

1) Find: **P(X = 5)**

2) Using the *Binomial Probabilities* in the back of the book (**Table B**)
Look-up: p= 0.2 (along the top of the page) and n=5 (on the left side of the table) and x=5
Or use Excel function = BINOMDIST(5, 5, 0.2, False) (for 2010 users BINOM.DIST)
Note: "Probability" is always a number between 0 and 1.

Answer: P(X = 5) = **0.0003**

Answer #2b) Given:

20% of employees use direct deposit so the probability someone uses direct deposit = 0.2
p = 0.2
sample size n = 5
Employees either use direct deposit or they don't (Binomial)
Using direct deposit = success = x >=2

1) Find: **P(X >= 2)**

2) Using the *Binomial Probabilities* in the back of the book (**Table B**)
Look-up: p= 0.2 (along the top of the page) and n=5 (on the left side of the table) and x=2, x=3, x=4, x=5 and add them up. P(X>=2) = 0.2048+0.0512+0.0064+0.0003
Or use Excel function = 1-BINOMDIST(1, 5, 0.2, True)

Answer: P(X >= 2) = **0.2627**

Answer #2c) Given:

20% of employees use direct deposit so the probability someone uses direct deposit = 0.2
p = 0.2
sample size n = 5
Employees either use direct deposit or they don't (Binomial)
Using direct deposit = success = x >=4

1) Find: **P(X >= 4)**

2) Using the *Binomial Probabilities* in the back of the book (**Table B**)
Look-up: p= 0.2 (along the top of the page) and n=5 (on the left side of the table) and x=4 and x=5 and add them up. P(X>=4) = 0.0064+0.0003
Or use Excel function = 1-BINOMDIST(3, 5, 0.2, True)

Answer: P(X >= 4) = **0.0067**

Answer #3)

	1st Shift	2nd Shift	3rd Shift	Totals
Perfect product	185	175	170	530
Acceptable product	55	60	65	180
Defective product	15	15	15	45
Reworked product	10	15	15	40
	265	265	265	795

Ho: product quality and shift **are** independent (or stated as product and shift are not dependent, In other words, the product's quality doesn't depend on a particular shift producing it.)
Ha: product quality and shift are not independent (or stated as product and shift **are** dependent)

F=(fi * fj)/N
The first F = (530 x 265)/795

Category	f	F	$(f-F)^2$ /F
perfect 1	185	176.7	0.39
perfect 2	175	176.7	0.02
perfect 3	170	176.7	0.25
accept 1	55	60.0	0.42
accept 2	60	60.0	0.00
accept 3	65	60.0	0.42
defect 1	15	15.0	0.00
defect 2	15	15.0	0.00
defect 3	15	15.0	0.00
rework 1	10	13.3	0.83
rework 2	15	13.3	0.21
rework 3	15	13.3	0.21
Totals	795		**2.74** = chi-squared

i = row
j = column
df = degrees of freedom = (i -1) x (j – 1) = (4-1) x (3-1)= 6

Use **Table E** (with df = 6) and chi-square = 2.744
along row at df = 6 the chi-square = 2.744 is closest to **p-value = 0.90**

or **use Excel function** to get the exact p-value = CHIDIST(2.744, 6)
(for Excel 2010 users =CHISQ.DIST.RT(2.744, 6))
p-value = 0.84

Or set up both **actual**

185	175	170
55	60	65
15	15	15
10	15	15

and **expected** tables

176.7	176.7	176.7

60	60	60
15	15	15
13.3	13.3	13.3

And use **Excel function** =CHITEST(actual table range, expected table range)
(for 2010 users =CHISQ.TEST(actual table range, expected table range))

("if p is low Ho must go")
If $p < \alpha$ then reject H_o
If $p \geq \alpha$ then fail to reject H_o

0.90 (or 0.84) > alpha 0.05 then fail to reject Ho
product quality and shift are independent (not dependent)
The product's quality does not depend on the shift that produced it.

Answer #4)

His claim was… mean > 3000
Ho: μ < = 3000
Ha: μ > 3000
One-sample, one-sided, upper-tail hypothesis test
n=60 (large sample size)
sample mean, x-bar = 3012
s = 112
alpha α = ? (what is the level of significance?)

Z = (x-bar – μ) / (s/ $\sqrt{n}$) = (3012 – 3000) / (112 / sqrt 60) = 12 / (112 / 7.7459)
Z = 0.8299 (rounded = 0.83)

Use Table C…Look down the Z column for 0.8 and then across to 0.03

to get a probability of 0.7967 (this is the area to the left – according to the normal curve at top of page)
This is an upper-tail test so we want the area to the **right** of the Z
1 - 0.7967 = **0.2033 = p**

Or use Excel function =1-NORM.S.DIST(0.83,TRUE)

The decision rule is:
If p > = alpha, then fail to reject Ho
If p < alpha, then reject Ho

But alpha wasn't given in this problem.

If 0.2033 > = alpha, then fail to reject Ho
If 0.2033 < alpha, then reject Ho So….

If alpha > 0.2033 then reject Ho
This seems like you need a really high alpha to reject Ho…since they are typically around 5% . We wouldn't necessarily say that the data supports the claim, unless we had a pretty large alpha > 0.2033.