Set Operators

24

- Union, Intersection and Difference are defined only for **union compatible** relations.
- Two relations are union compatible if they have the same set of attributes and the types (domains) of the attributes are the same.
- E.g., two relations that are not union compatible:
  - **Student (sNumber, sName)**
  - **Course (cNumber, cName)**

24

Relational Algebra on Bags

25

- A *bag* (or *multiset*) is like a set, but an element may appear more than once.
- Example: {1,2,1,3} is a bag.
- Example: {1,2,3} is also a bag that happens to be a set.

25

Union: ∪

26

- Consider two bags R_1 and R_2 that are union-compatible. Suppose a tuple t appears in R_1 m times, and in R_2 n times. Then in the union, t appears $m + n$ times.

R_1

A	B
1	2
3	4
1	2

R_2

A	B
1	2
3	4
5	6

$R_1 \cup R_2$

A	B
1	2
1	2
1	2
3	4
3	4
5	6

26

Intersection: ∩

27

- Consider two bags R_1 and R_2 that are union-compatible. Suppose a tuple t appears in R_1 m times, and in R_2 n times. Then in the intersection, t appears min (m, n) times.

R_1

A	B
1	2
3	4
1	2

R_2

A	B
1	2
3	4
5	6

$R_1 \cap R_2$

A	B
1	2
3	4

27

Difference: -

28

- Consider two bags R_1 and R_2 that are union-compatible. Suppose a tuple t appears in R_1 m times, and in R_2 n times. Then in $R_1 - R_2$, t appears max (0, m - n) times.

R_1

A	B
1	2
3	4
1	2

R_2

A	B
1	2
3	4
5	6

$R_1 - R_2$

A	B
1	2

28

Building Complex Expressions

29

- Combine operators with parentheses and precedence rules.
- Three notations, just as in arithmetic:
  - Sequences of assignment statements.
  - Expressions with several operators.
  - Expression trees.

Credit: Renee J. Miller

29

Sequences of Assignments

30

- Create temporary relation names.
- Renaming can be implied by giving relations a list of attributes.
  - $\pi_{A+B->C,A->A1,A->A2}$ (R)
- Example: R3 := R1 $\bowtie_C$ R2 can be written:

R4 := R1 X R2

R3 := σ_C (R4)

30

Expressions in a Single Assignment

31

Example: the theta-join R3 := R1 $\bowtie_C$ R2 can be written as

- R3 := σ_C (R1 X R2)

- Precedence of relational operators: (parentheses supercedes)
  - [σ, π, ρ] (highest).
  - [X, $\bowtie$].
  - $\cap$.
  - [$\cup$, —]

31

Expression Trees

32

- Leaves are operands --- either variables standing for relations or particular, constant relations.
- Interior nodes are operators, applied to their child or children.

32

Example: Tree for a Query

33

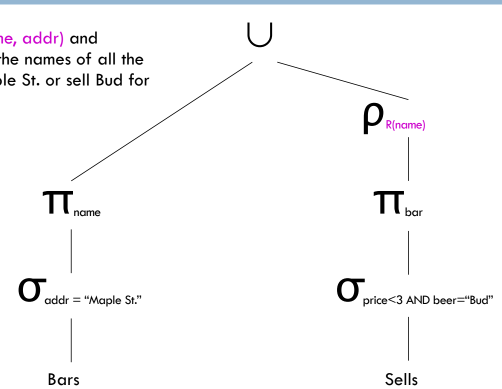
- Using the relations Bars(name, addr) and Sells(bar, beer, price), find the names of all the bars that are either on Maple St. or sell Bud for less than $3.

33

As a Tree:

34

Using the relations Bars(name, addr) and Sells(bar, beer, price), find the names of all the bars that are either on Maple St. or sell Bud for less than $3.



34

Example: Self-Join

35

- Using Sells(bar, beer, price), find the bars that sell two different beers at the same price.
- Strategy: by renaming, define a copy of Sells, called S(bar, beer1, price). The natural join of Sells and S consists of quadruples (bar, beer, beer1, price) such that the bar sells both beers at this price.

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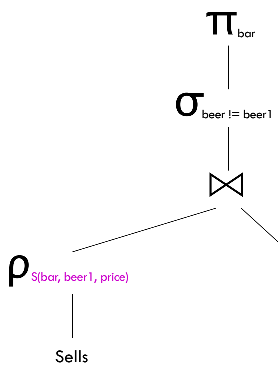
The Tree

36



Using Sells(bar, beer, price), find the bars that sell two different beers at the same price.

36

Schemas for Results

37

- Union, intersection, and difference: the schemas of the two operands must be the same, so use that schema for the result.
- Selection: schema of the result is the same as the schema of the operand.
- Projection: list of attributes tells us the schema.

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Schemas for Results

38

- Product: schema is the attributes of both relations.
  - Distinguish two attributes with the same name.
- Theta-join: same as product.
- Natural join: union of the attributes of the two relations. Keep only one copy of the equated attributes.
- Renaming: the operator tells the schema.

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Lecture Example

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The Extended Algebra

40

δ = eliminate duplicates from bags.

τ = sort tuples.

γ = grouping and aggregation.

Outerjoin : avoids "dangling tuples" = tuples that do not join with anything.

Credit: Renee J. Miller

40

Duplicate Elimination

41

- R1 := δ(R2).
- R1 consists of one copy of each tuple that appears in R2 one or more times.

41

Example: Duplicate Elimination

42

R = (

A	B)
1	2
3	4
1	2

δ(R) =

A	B
1	2
3	4

42

Sorting

43

- R1 := τ_L (R2).
  - *L* is a list of some of the attributes of R2.
- R1 is the list of tuples of R2 sorted first on the value of the first attribute on *L*, then on the second attribute of *L*, and so on.
  - Break ties arbitrarily.

43

Example: Sorting

44

R = (A B)

A	B
1	2
3	4
5	2

τ_B (R) =

(A B)

A	B
5	2
1	2
3	4

44

Aggregation Operators

45

- Aggregation operators are not formally operators of relational algebra.
- Rather, they apply to entire columns of a table and produce a single result.
- The most important examples: SUM, AVG, COUNT, MIN, and MAX.

45

Example: Aggregation

46

R = (A B)

A	B
1	3
3	4
3	2

SUM(A) = 7
COUNT(A) = 3
MAX(B) = 4
AVG(B) = 3

46

Grouping Operator

47

- R1 := γ_L (R2). *L* is a list of elements that are either:
  1. Individual (*grouping*) attributes.
  2. AGG(*A*), where AGG is one of the aggregation operators and *A* is an attribute.
     - An arrow and a new attribute name renames the component.

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Applying γ_L(R)

48

- Group *R* according to all the grouping attributes on list *L*.
  - That is: form one group for each distinct list of values for those attributes in *R*.
- Within each group, compute AGG(*A*) for each aggregation on list *L*.
- Result has one tuple for each group:
  1. The grouping attributes and
  2. The group's aggregations.

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Example: Grouping/Aggregation

49

R =

A	B	C
1	2	3
4	5	6
1	2	5

$\gamma_{A,B,AVG(C)\to X}$ (R) = ??

First, group *R* by *A* and *B* :

A	B	C
1	2	3
1	2	5
4	5	6

Then, average C within groups:

A	B	X
1	2	4
4	5	6

49

Recall: Outerjoin

50

- Suppose we join $R \bowtie_C S$.
- A tuple of *R* that has no tuple of *S* with which it joins is said to be *dangling*.
  - Similarly for a tuple of *S*.
- Outerjoin preserves dangling tuples by padding them NULL.

50

Example: Outerjoin

51

R =

A	B
1	2
4	5

S =

B	C
2	3
6	7

(1,2) joins with (2,3), but the other two tuples are dangling.

R FULL OUTERJOIN S =

A	B	C
1	2	3
4	5	NULL
NULL	6	7

51

Outer Join – Example

52

- instructor

ID	name	dept_name
10101 12121 15151	Srinivasan Wu Mozart	Comp. Sci. Finance Music

teaches

ID	course_id
10101 12121 76766	CS-101 FIN-201 BIO-101

instructor ⋈ *teaches*

ID	name	dept_name	course_id
10101 12121	Srinivasan Wu	Comp. Sci. Finance	CS-101 FIN-201

- Left Outer Join

instructor $⟕_{LEFT}$ *teaches*

ID	name	dept_name	course_id
10101 12121 15151	Srinivasan Wu Mozart	Comp. Sci. Finance Music	CS-101 FIN-201 *NULL*

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Outer Join – Example

53

- instructor

ID	name	dept_name
10101 12121 15151	Srinivasan Wu Mozart	Comp. Sci. Finance Music

teaches

ID	course_id
10101 12121 76766	CS-101 FIN-201 BIO-101

- *instructor* $⟖_{RIGHT}$ *teaches*

ID	name	dept_name	course_id
10101 12121 76766	Srinivasan Wu null	Comp. Sci. Finance null	CS-101 FIN-201 BIO-101

- *instructor* $⟗_{FULL}$ *teaches*

ID	name	dept_name	course_id
10101 12121 15151 76766	Srinivasan Wu Mozart null	Comp. Sci. Finance Music null	CS-101 FIN-201 *null* BIO-101

53

Operations on Bags

54

A **bag** = a set with repeated elements

All operations need to be defined carefully on bags

- {a,b,b,c}∪{a,b,b,b,e,f,f}={a,a,b,b,b,b,b,c,e,f,f}

- $\sigma_C(R)$: preserve the number of occurrences
- $\Pi_A(R)$: no duplicate elimination
- Cartesian product, join: no duplicate elimination

Important ! Relational Engines work on bags, not sets !

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Why Bags?

55

- SQL, the most important query language for relational databases, is actually a bag language.
- Some operations, like projection, are more efficient on bags than sets.

55

Operations on Bags

56

- Selection applies to each tuple, so its effect on bags is like its effect on sets.
- Projection also applies to each tuple, we do not eliminate duplicates.
- Products and joins are done on each pair of tuples, so duplicates in bags have no effect on how we operate.

56

Example: Bag Selection

57

R(

A,	B)
1	2
5	6
1	2

$\sigma_{A+B<5}$ (R) =

A	B
1	2
1	2

57

Example: Bag Projection

58

R(

A,	B)
1	2
5	6
1	2

π_A (R) =

A
1
5
1

58

Example: Bag Product

59

R(

A,	B)
1	2
5	6
1	2

S(

B,	C)
3	4
7	8

R X S =

A	R.B	S.B	C
1	2	3	4
1	2	7	8
5	6	3	4
5	6	7	8
1	2	3	4
1	2	7	8

59

Example: Bag Theta-Join

60

R(A, B)

A	B
1	2
5	6
1	2

S(B, C)

B	C
3	4
7	8

$R \bowtie_{R.B<S.B} S =$

A	R.B	S.B	C
1	2	3	4
1	2	7	8
5	6	7	8
1	2	3	4
1	2	7	8

60

Bag Union

61

- An element appears in the union of two bags the sum of the number of times it appears in each bag.
- Example: {1,2,1} ∪ {1,1,2,3,1} = {1,1,1,1,1,2,2,3}

61

Bag Intersection

62

- An element appears in the intersection of two bags the minimum of the number of times it appears in either bag
- Example: {1,2,1,1} ∩ {1,2,1,3} = {1,1,2}.

62

Bag Difference

63

- An element appears in the difference $A - B$ of bags as many times as it appears in *A*, minus the number of times it appears in *B*.
- Example: {1,2,1,1} – {1,2,3} = {1,1}.

63

Beware: Bag Laws != Set Laws

64

- Some, but *not all* algebraic laws that hold for sets also hold for bags.
- Example: the commutative law for union ($R \cup S = S \cup R$) *does* hold for bags.
  - Since addition is commutative, adding the number of times x appears in R and S doesn't depend on the order of R and S.

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Example: A Law That Fails

65

- Set union is *idempotent*, meaning that $S \cup S = S$.
- However, for bags, if x appears n times in S, then it appears $2n$ times in $S \cup S$.
- Thus $S \cup S$!= S in general.
  - e.g., {1} ∪ {1} = {1,1} != {1}.

What about Intersection?

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Lecture Example

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