ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

APPLIED DATA ANALYSIS

School of Computer and Communication Sciences

Applied Data Analysis Summary

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Contents

1 Introduction

1.1 General information about the course

This course covers multiple topics in the data science field such as **Data Wrangling**, **Data Management**, **Data Mining**, **Machine Learning**, **Visualization**, **Statistics** and **Story telling**. It's about **breadth**, not depth. Indeed, Data science is evolving really quickly, hence learning in depth a specific tool won't pay off.

1.2 Data Science

When we talk about Data Science, we often use the term Big Data as the enormous amount of data that exist in the world. But Big Data is not only about collecting huge amount of data. It is challenging but not enough. The real value comes from the insights. The *internet* companies (Google, Facebook, etc.) understood this many years ago.

An accurate definition of Data Analysis is given by Wikipedia:

> **Analysis of data** *is a process of* **inspecting**, **cleaning**, **transforming**, *and* **modeling data** *with the goal of* **discovering useful information**, *suggesting conclusions, and supporting decision-making. Data analysis has multiple facets and approaches, encompassing diverse techniques under a variety of names,* **in different business**, *science, and social science* **domains**. Wikipedia - Data Analysis

Therefore, a Data Scientist has to master different kinds of skills such as **Mathematics** (for the Statistics), **Programming** and the **Domain Expertise**. Drew Conway's Venn diagram, Figure 1, shows the different combination man can obtain with these three skills.

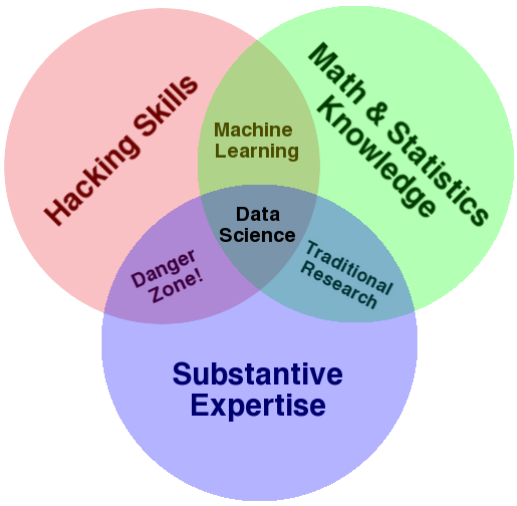


Figure 1: Venn Diagram describing the different combination of skills used by a Data Scientist (by Drew Conway)

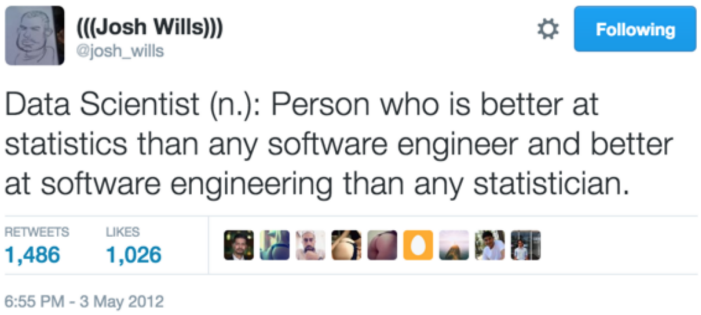
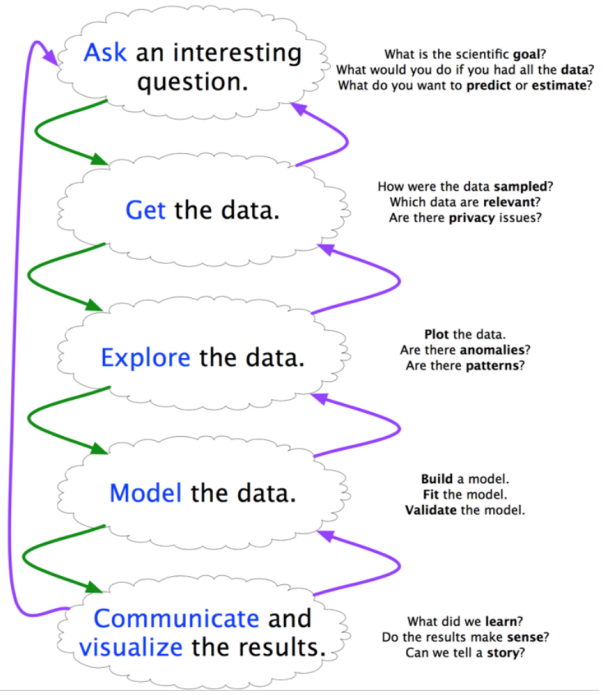


Figure 2: A tweet from Josh Wills, Data Scientist at Slack.

A practical definition of Data Science

Data Science is about the whole processing pipeline to extract information out of data. As such, a Data Scientist **understands and cares about the whole data pipeline**.



A **data pipeline** consists of 3 steps:

1. Preparing to run a model.
   Gathering, cleaning, integrating, restructuring, transforming, loading, filtering, deleting, combining, merging, verifying, extracting, shaping
2. Running the model
3. Communicating the results

A "good" Data Scientist will always go back and forth between the steps. The diagram on the left shows exactly what can happen.

In this course, you will develop the following skills:

data mining/scraping/sampling/cleaning in order to get an informative, manageable data setlength

data storage and management in order to be able to access data quickly and reliably during subsequent analysis

exploratory data analysis to generate hypotheses and intuition about the data

prediction based on statistical tools such as regression, classification, and clustering

communication of results through visualization, stories and interpretable summaries

2 Basic concepts

A data science student is attended to understand the **Grammar of Data Science**. Having some backgrounds in SQL concepts is also a good thing because, as it is very common, people love to make examples with it. Here is a brief refresh of some definitions and concepts about data science.

- **Structured data** requires two key concepts:
  - **Data model** is a collection of concepts for describing data.
  - **Schema** is a description of a particular collection of data, using a given data model.
- The **Relational model** is on of the most common approach to manage data (SQL like) and can handle most of the data. A counter example is the facebook-like data which requires **graph model**. This model is made of 2 parts:
  - The **Schema**.
    For example, `Students(sid: string, name:string, age:integer)`
  - The **Instance**, *i.e.* the data at a given time.
    Definitions:
    - **Cardinality** is the number of rows. (Number of items)
    - **Degree** or **Arity** is the number of fields. (Number of attributes)
- Definitions of some "Database" terms:
  - A **JOIN** is a mean to combine tables based on shared attributes (most of the time some **IDs**). Despite its apparent simplicity beware of the many ways to compute a JOIN and check what is the default JOIN of a language before using it. The FIG 3 summarises these possibilities.
  - **Aggregation**, **reduction**, and **groupby** are the action of reducing data with a common operation (**sum**, **count**, **average**, ...) to summarise them.
- Definitions of some "Pandas" terms:
  - **Series** are a name, ordered dictionary
    - keys are indexes
    - built on **numpy.ndarray** (so values can be any Numpy data type)
  - **DataFrame** is a table with named column
    - the columns are series
    - it is indeed a dictionary with (columnName $\rightarrow$ series)

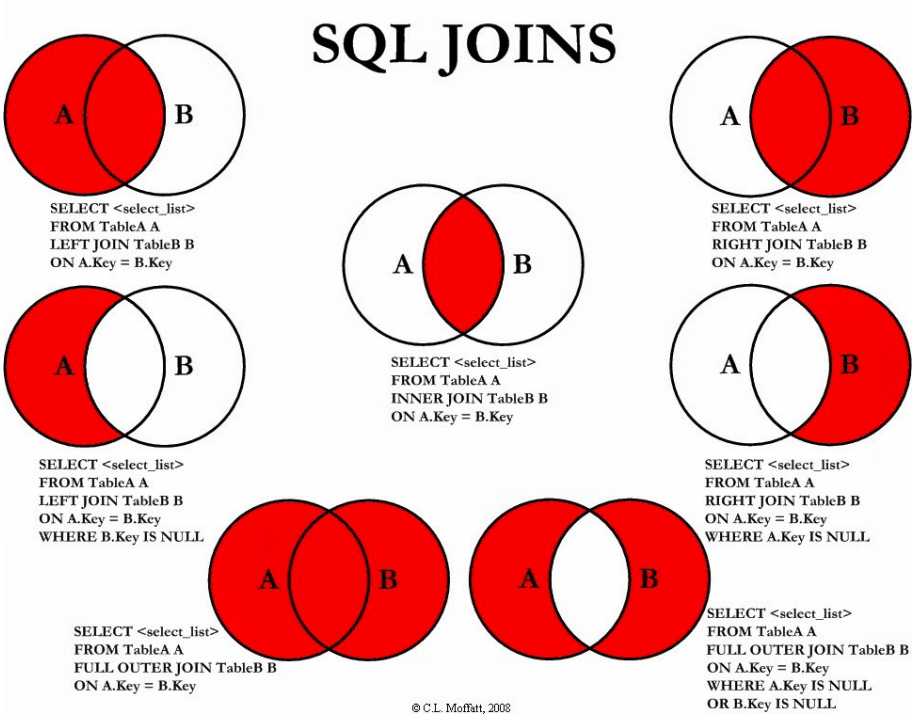


Figure 3: Different ways to join two tables and the related SQL command.

2.1 Panda vs SQL

Panda is built to allow easy and fast **data exploration** and not to be a database manager, as SQL is. Thus there are benefits and drawbacks of using it.

Pros	**Cons**
Lightweight & fast	Tables stored directly in memory
Great expressiveness (combine SQL + Python)	No post-load indexing functionality
Easy plot for data visualization (e.g. Matplotlib)	No transactions, journalings
	Large, complex joins are slower

2.2 OnLine Analytical Processing (OLAP cubes)

OLAP tools enable users to analyze multidimensional data interactively from multiple perspectives. Conceptually, it is like an n-dimensional spreadsheet (a cube) on which we can apply various operations to take decisions.

OLAP cubes are another way to see data tables and are constructed based on them, as shows FIG 4.

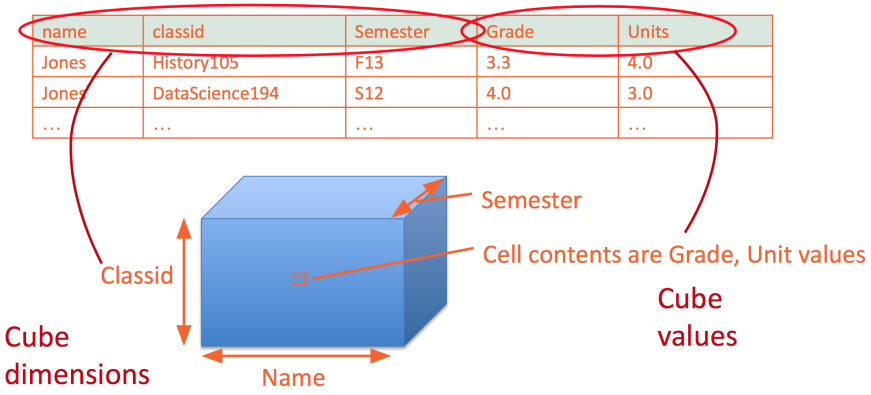
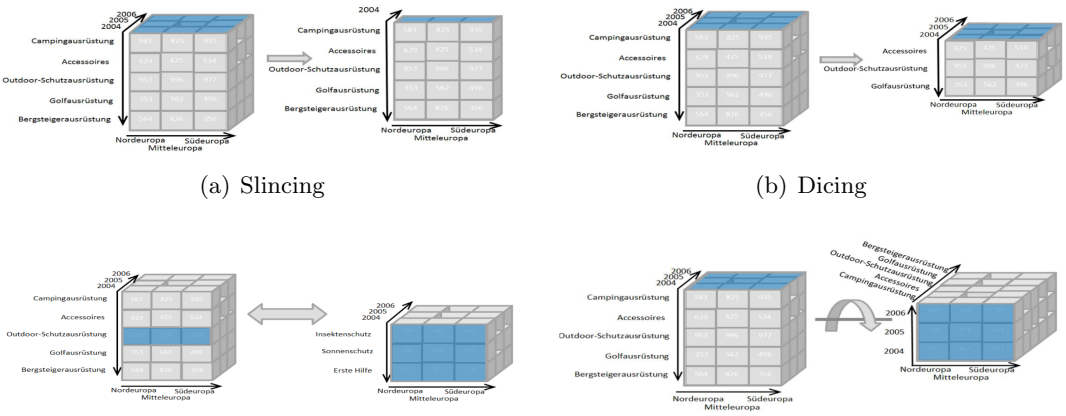


Figure 4: Construction of an OLAP cube from a table.

Operations on OLAP cubes are the following and are illustrated on FIG 5

- **Slicing** fixes one or more variable
- **Dicing** selects a range of one or more variable
- **Drilling up/down** change levels of a hierarchically indexed variable, i.e. "zoom" on a variable and see the subcategories it contains.
- **Pivoting** change the point of view of the cube. Swap an aggregated variable and a detailed one.



(a) Slincing (b) Dicing

(c) Driling up/down (d) Pivoting

Figure 5: Operations on OLAP cubes

Pros	Cons
The main advantage of OLAP cubes is that they are **conceptually simpler** to understand by a non-scientist person, e.g. a business man who has to take day-to-day decisions based on company's data. Aggregations are limited but cover the main common cases that we can encounter.	Because of the "on-line" behaviour of this approach, all types of aggregation must be pre-calculated among all combinations of axes which is very **expensive in memory and in time** (when updating the data)

3 Data Wrangling

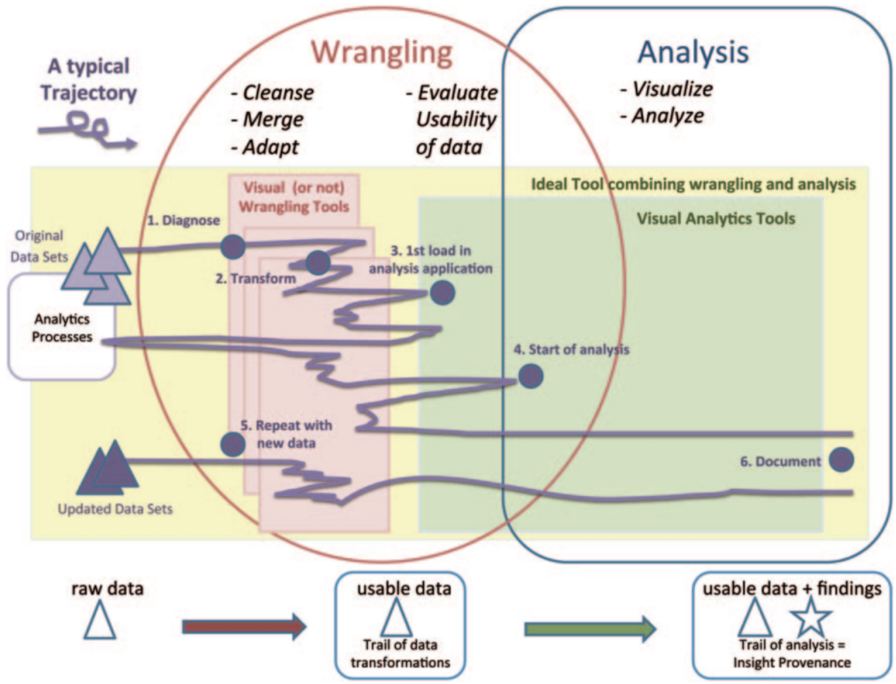


Figure 6: Things do not always happen as expected...

Before any analysis, data need to be transformed from "dirty" to clean and processable data.
Data comes from different sources (excel or SQL?), sometime collected through different methods over time, with different conventions (space or NaN?), etc ... Data wrangling goal is to **extract and standardize these raw data**. The best way to do it is to **combine automation with visualizations** in order to find outliers.
Data problems can come from (non-exhaustive):

- Missing data
- Incorrect data
- Inconsistent representations of the same data

- Non-standardized data (centimeter or inches? Fahrenheit or Celsuis ?)
- Duplicated data

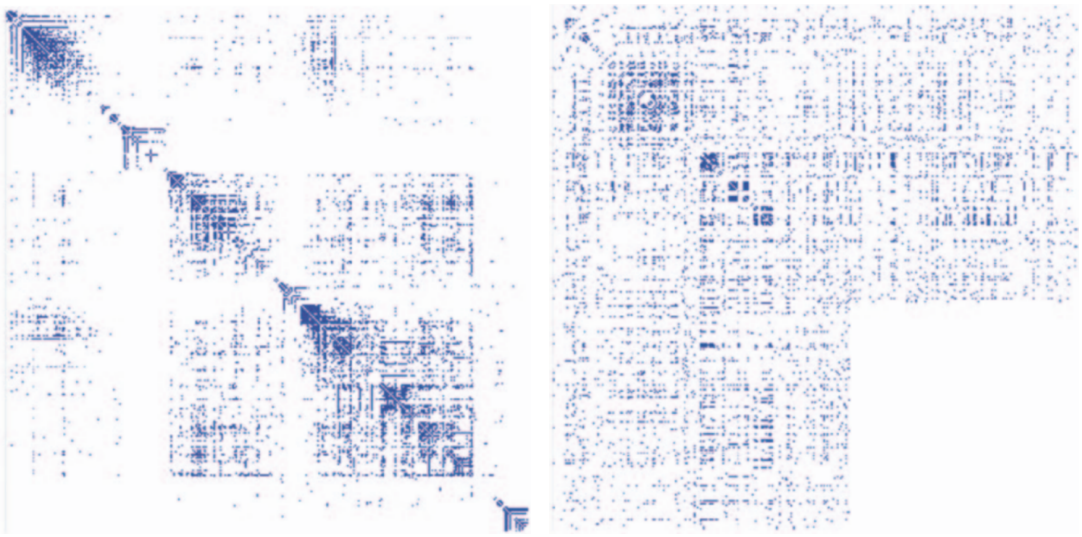
About 75% of theses problems will need **human intervention** to be corrected (by the data-scientist or by crowdsourcing).
Even if it seems really dirty, **beware not to over-sanitize the data!**. Applying what we can call "defensive programming" is not a good idea because we risk losing any interesting data, keeping only the ones that fit perfectly in our model.

3.1 Diagnosis of the data

One of the most important aspects of Data Wrangling is to **understand** the data and to **find possible problems**. In order to "diagnose" the data, two tools can be used:

- **Visualization** (A *thoughtful* visualization will always help)
- **Basic Statistics**

Matrix visualizations of the facebook graph is shown in Figure 7. The Relational visualization, Figure 7(a), does not show any particular problem in the data. But the Time dependant visualization, Figure 7(b), shows that the Facebook API reached its limit while collecting data.



(a) Relational visualization (b) Time dependant visualization

Figure 7: Matrix visualization of the facebook graph.

3.2 Dealing with missing values

Values can often miss from the data we have, because of various events (war, fire, ...). We must detect and correct these values with different methods according to the domain we are working in.

Whatever the method used, it's good to keep track of these changes to know which are original data and which are modified ones.

- Set values to zero FIG 8(a)
- Interpolate based on existing data FIG 8(b)
- Omit missing data FIG 8(c)
- Interpolation with tracks kept 8(d)

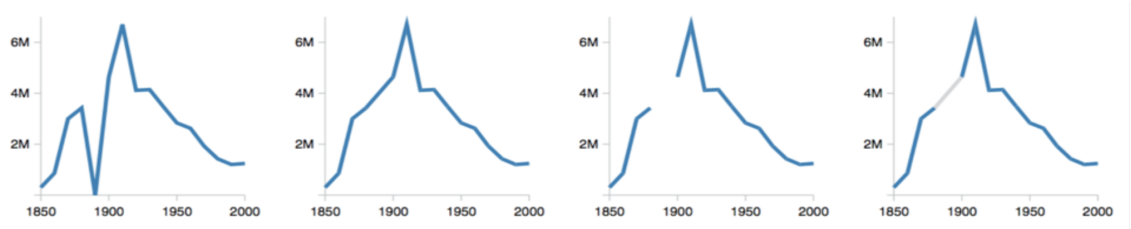


Figure 8: To deal with missing values.

3.3 General procedure

Once the data are well wrangled and before trying to analyse them, we must take care of two more steps:

1. **Deal with uncertain data** (can arise from measurement errors, wrong sampling strategies, etc.)
2. **Parse/transform data** (with aggregation and reduction techniques) to obtain meaningful records

It's always ideal to have the code and/or the documentation about the dataset you are analyzing (provenance).

4 Data Variety

The "3 Vs" of Big Data: *Volume*, *Velocity* and *Variety*. In this course, we don't address the *Volume* and *Velocity* parts (A course on Database does). Since there is a lot of variety in the data, we need to prepare the data. This flow is called **ETL**:

- **Extract** from the *source(s)*.
- **Transform** data at the source, sink, or in a *staging area*.
- **Load** data into the *sink*.

This variety of the data comes, in a first place, from the many different sources from which we extract them: *files, databases, logs, ...* Each of these sources uses (or not!) its proper convention and can contain structured (DB), semi-structured (logs) or unstructured (web page) data.

4.1 Role of Schema

The **Schema**, which specifies the *structure* and *types* of data repository, is changing. Traditional databases are **schema-on-write**, *i.e.* you cannot load data into a table without a schema. But new data stores (NoSQL for example) are **schema-on-read** or **schemaless**.

- **Schema-on-write** is typically SQL, where we must create a table before inserting data in our system. Data must scale the defined schema and this is both the strength and weakness of the system. Strength because the data is perfectly oriented and respect the constraints we establish. Weakness because schemas are always subjective in some ways and data (which are perfectly correct) may not fit with it.

- **Schema-on-read** is for instance XML, where you create the schema according to the data you read.

- Youtube and Google Cache where the first **schemaless** data system. Without schema, everything is simply stored as a string and we need a parser to return a typed data.

4.2 Examples of data

4.2.1 XML and DOM

The XML data are used mostly with HTML and specifies the data structure. An XML schema can be applied to interpret the XML data and specifies the **data types**. Figure 9 shows the XML data 9(a) and the schema 9(b) used to parse and type the data.

```
<location>
  <latitude>37.78333</latitude>
  <longitude>122.4167</longitude>
</location>
```

(a) XML data

```
<xsd:schema xmlns:xsd="http://www.w3.org/2001/XMLSchema"
elementFormDefault="unqualified">
<xsd:complexType name="location">
  <xsd:sequence>
    <xsd:element name="latitude" type="xsd:decimal"/>
    <xsd:element name="longitude" type="xsd:decimal"/>
  </xsd:sequence>
</xsd:complexType name="location">
```

(b) XML schema

Figure 9: Example of XML.

The XML is a text format that encodes **DOM** (Document-Object Models). It's a data structure often used by Web pages. The DOM is tree-structured. An example of a DOM is given in Figure 10.

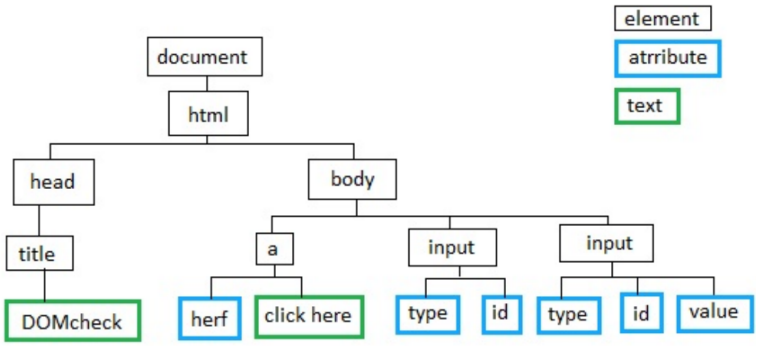


Figure 10: Example of a DOM tree for an HTML Web page.

THe XML schema allows a database to interpret the data when running queries. It can do arithmetic or range queries on numerical values, for example.

4.2.2 JSON

JSON stands for Javascript Object Notation. It's a schemaless data (schema support was added later). An example of JSON data is shown in Figure 11

```
{
  "firstName": "John",
  "lastName": "Smith",
  "age": 25,
  "address": {
       "streetAddress": "21 2nd Street",
       "city": "New York",
       "state": "NY",
       "postalCode": "10021-3100" },
  "phoneNumbers": [
      { "type": "home",
        "number": "212 555-1234" },
      { "type": "office",
        "number": "646 555-4567" } ],
  "children": [],
  "spouse": null
}
```

Figure 11: Example of a JSON data.

JSON is typically used to represent **hierarchical data structures** directly in the target language (Javascript or Java at the beginning). The transformation on the data is procedural in the target languages. It is often easier for some tasks, but it can be painful for some of them: for example schema changes.

4.2.3 Tabular data

A Tabular Data is simply data put into a table such as CSV or TSV. Definition of a table:

- A **table** is a collection of **rows** and **columns**.

- Each row has an **index**.
- Each column has a **name**.
- A **cell** is specified by an (index, name) pair.
- A cell may or may not have a **value**.

It's a very simple yet powerful data type. For example, the sensors usually output data in the form of time series, transformed into a tabular format. However, a system dealing with sensor data should:

- support both long-term (**trend**) and short-term (**real-time**) queries
- have **low latency** but also efficient. It should use **real-time indexing** for longer-term queries.
- support triggers (**alerts**) for a variety of conditions.

Therefore, the **complexity of a data format** does not determine the **complexity of the system required to properly handle it**.

4.2.4 Log files

The log files are simple text files giving information about the process. The daemons, such as `httpd`, `mysqld` or `syslogd`, usually create logs. `syslog` was developed by Eric Allman. It's a way for devices to send event messages to a server that will log all the events. Splunk is a company that built a successful business model around the syslog events.

4.2.5 Binary formats

They are often the key to performance because we **avoid expensive parsing**. The modern formats even support nested structures, various levels of schema enforcement, **compression**, etc. Some examples: Protocol Buffers (Google), Avro, Parquet, etc.

4.3 Processing the data (JSON and XML)

In order to process XML, we can use the DOM. It can also be used to process JSON data. The DOM is very easy to work with: all the data are directly accessible by links. The problem is that we **might not care about most of the data** and if the data are big, they **might not fit into the RAM**. In order to deal with these two problems, we can use a **SAX** parser which is an event-driven parser. It will find all the **open-close-tag events** in an XML document and will bf do callbacks to user code.

+ User code can respond to only a subset of events corresponding to the tag it is interested in.
+ User code can correctly compute aggregates from the data rather than create a record for each tag.

+ User code can implement flexible error recovery strategies for ill-formed XML.

– User code must implement a state machine to keep track of "where it is" in the DOM tree.

For JSON, most parsers construct the "DOM" directly. But there are a few SAX-style parsers: Jackson, JSON-simple, etc. Sometimes **SAX-style is the way to handle ill-formed datasets**, an endless array of objects, for example.

4.4 HTML and Web Services

4.4.1 HTML

Internet contains an "enormous" amount of data. Some crawlers such as Common Crawl datasets contains about 1.82 billion web pages (for 145 TB). We can use different tools to crawl data from the web. Examples for Python: Beautiful Soup, Requests, Scrapy, etc.

Most of the time, the Web pages are considered as unstructured data. But you can find some semi-structured data, *e.g.* Google WebTables. Some big "internet" companies (Google, Yahoo, Yandex and Microsoft) are sponsoring a project called **schema.org** to create structured or semi-structured Web pages. A core vocabulary for the type of fields is given. schema.org is more and more used. It's also used by knowledge bases such as Google Knowledge Graph. **WikiData** is a community project to create an open database of structured data taken from Wikipedia.

4.4.2 Web Services

Screen-scraping the content of a large website was possible before, but become more and more difficult nowadays. This is mainly due to the content "hidden" behind a form or an authentification. Take for example facebook without account, or the IS-academia course page if you do not select a semester. Therefore big companies are providing Web Service APIs[1] to retrieve data from their website. There are two kinds of Web Services:

- The old way: XML-based RPC-style messages: SOAP
- The new way: REST-style stateless interactions, URLs encode state

4.4.2.1 RPC

The SOAP RPC [2] messages typically encode arguments that are presented to the calling program as parameters and return values. HTTP POST/GET are used to communicate.

[1] Application Program Interface: Set of subroutine definitions, protocols, and tools for building software and applications. In this particular case, the APIs are used to retrieve the data from the Web page, *e.g.* Facebook API to retrieve the contacts.

[2] SOAP = Simple Object Access Protocol, RPC = Remote Procedure Call

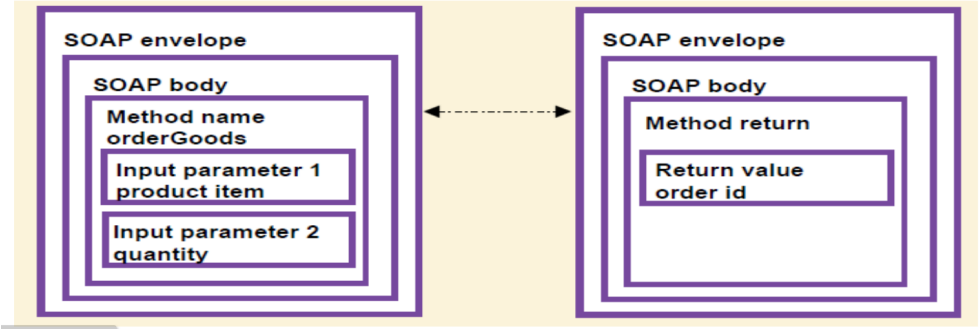


Figure 12: Example of a SOAP RPC exchange.

This kind of procedure (same for XML-RPC) requires a request-response cycle. This often leads to longer "conversations". The RPC-style is being quickly superseded by newer and more user-friendly technologies.

In **RPC systems**, the design emphasis is on **verbs**. It uses functions such as *getUser()*, *addUser()*, etc.

4.4.2.2 REST

REST[3] is a **stateless** client/server protocol. The principles are:

1. Each message in the protocol contains all the information needed by the receiver to understand and/or process it. This constraint attempts to *"keep things simple"* and avoids needless complexity.

2. Set of Uniquely Addressable Resources

   - *"Everything is a Resource"* in a RESTful system
   - Requires universal syntax for resource identification, *e.g.* URI.

3. Set of Well-Defined Operations that can be applied to all resources

   - In the context of HTTP (REST APIs), the primary methods are:
     POST, **GET**, **PUT**, and **DELETE**
     These are similar (but not exactly) to the database notion of CRUD (Create, Read, Update, and Delete)

4. The use of Hypermedia both for Application Information and State Transitions

   - Resources are typically stored in a structured data format that supports hypermedia links, such as XHTML or JSON.

In **REST systems**, the design emphasis is on **nouns**. It uses the HTTP Protocols (POST, GET, PUT, and DELETE) a *User*, a *Location*, etc.

[3] REpresentation State Transfer

5 Statistics on the Data

When we explore and analyze data, it would be great if we only had to look at some statistics numbers and make automatically a conclusion about them. Sadly, it's not the case. At all.

5.1 Examples of famous mistakes due to statistics

5.1.1 Anscombe's quartet: Sensivity of outliers & Robust statistics

The FIG 13 show four different data distribution that present, despite all of that, the same means on x and y, the same variance on x and y, and, thus, the same linear regression function. This is due to the statistics used to define them.

- Min, Max, Mean, Standard Deviation (Std) and Range are sensitive to outliers and then are **not robust statistics**.
- Median, quartils, (and others) are not sensitive and then are said to be **robust statistics**.

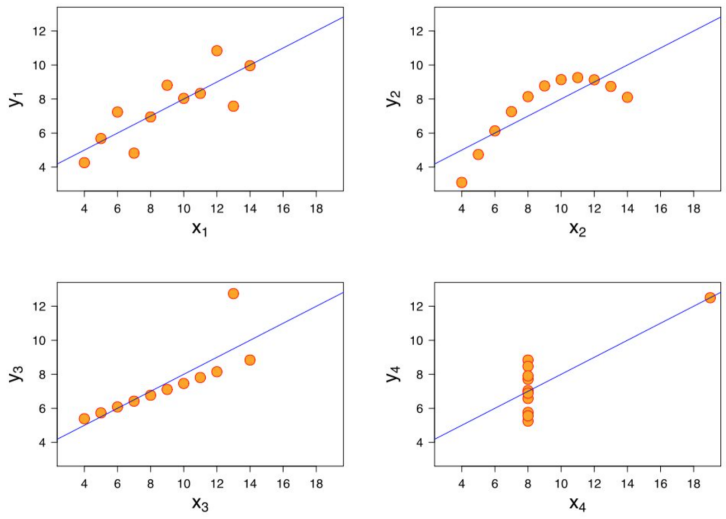


Figure 13: Anscombe's quartet

5.1.2 Simpson's paradox: aggregation of data

Certain tendencies can appear, disappear or even reverse themselves when aggregating the data! This was the case when media started blaming Berkey of being unfair with women applications (looking at left table of FIG 14). After further investigation (and de-aggregation of the data), it appeared that at the opposite... Berkeley was unfair with men! (Right table of the same FIG).

This paradox comes from the fact that women (according to these tables) tended to apply for more competitive departments, with lower rates of admission. **When aggregating the data, we lost this subtlety and then draw a wrong conclusion.**
Simpson's paradox can appear in a lot of cases and can be very hard to detect. The wikipedia page of Simpson's paradox describes a lot of examples and, for the ones interested, a great book relates lots of statistical errors that drove to miscarriages of justice: Leila Schneps and Coralie Colmez, Math on Trial: How Numbers Get Used and Abused in the Courtroom

	Applicants	Admitted
Men	8442	**44%**
Women	4321	35%

Department	Men		Women	
	Applicants	Admitted	Applicants	Admitted
A	825	62%	108	**82%**
B	560	63%	25	**68%**
C	325	**37%**	593	34%
D	417	33%	375	**35%**
E	191	**28%**	393	24%
F	373	6%	341	**7%**

Figure 14: Berkley admission tables of 1973

5.2 Refresh of basic statistics concept

- **Probabilities**: mathematical theory that describes uncertainty.

- **Statistics**: Set of techniques for extracting useful info from data

> **Probability and Statistics** *are related areas of mathematics which concern themselves with analyzing the relative frequency of events. Still, there are fundamental differences in the way they see the world:*
> ***Probability deals with predicting the likelihood of future events**, while **statistics involve the analysis of the frequency of past events**.*
> *Probability is primarily a theoretical branch of mathematics, which studies the consequences of mathematical definitions. Statistics is primarily an applied branch of mathematics, which tries to make sense of observations in the real world.*
> Steven S. Skiena, "Calculated Bets", Cambridge University Press, 2001

5.2.1 Bayes Theorem

The theorem express the very intuitive statement that:
The probability of observing event A and B is the probability of observing B multiplied by the probability of observing A knowing that B occurred.
Mathematically it's expressed as:

$$P(A|B)P(B) = P(A \cap B) = P(B|A)P(A) \tag{1}$$

Or equivalently

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{2}$$

More about Bayes Theorem on wikipedia.

5.2.2 Random Variables

A **random variable** is a quantity that can take various values, each one associated with a probability of apparitions. The some of these probabilities will always be 1.

$$X : \Omega \to E \tag{3}$$

Ω being a probability space and E a measurable space (usually $E = \mathbb{R}$).
Any random variable can be described by its cumulative distribution function, which describes the probability that the random variable will be less than or equal to a certain value.
Two **independent variables** are defined as

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B). \tag{4}$$

or equivalently (by Bayes Theorem)

$$\mathbb{P}(A \mid B) = \mathbb{P}(A) \tag{5}$$

More about Bayes Theorem on wikipedia.

5.2.3 Law of Large Numbers

The Law of Large Numbers links, in some way, the probability to the statistics. It's, again, a very intuitive statement, even if not so easy to prove (as always in mathematics).

> In probability theory, the **law of large numbers** (LLN) is a theorem that describes the result of performing the same experiment a large number of times. According to the law, **the average of the results obtained from a large number of trials should be close to the expected value**, and will tend to become closer as more trials are performed. Wikipedia

A common mistake is to deduce that, in the case of playing heads or tails for example, observing a lot of time **heads** increase the probability of observing **tails**. This is absolutely wrong. The variables are perfectly independent and, according to Eq 5, the probability stays exactly 50%. This is the perfect example of confusing probabilities with statistics.

5.2.4 Central Limit Theorem

Central Limit Theorem states that the mean of independent and identically-distributed random variables will converge to **Gaussian Distribution** (Normal Distribution).

5.3 Most common distributions

- **Gaussian Distribution** (fig 15) results from independent and identically-distributed variables $f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ More on wikipedia

- **Poisson Distribution** (fig 16) describe the observation of events happening in a delimited time-laps. E.g: an event happens in average 4 times each 10 minutes ($\lambda = 4$). What's the probability that it appears after only 3 times in this same interval ($k = 3$)? $p(k) = \frac{\lambda^k}{k!} e^{-\lambda}$ More on wikipedia

- **Exponential Distribution** (fig 17) describes the time between two events in a Poisson process. $P(x) = \lambda e^{-\lambda x}$ More on wikipedia

- **Binomial Distribution** (fig 18) describes the discrete distribution on success in a yes/no experiment. (E.g. coin tossing or any win/lose game). $f(k; n, p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ More on wikipedia

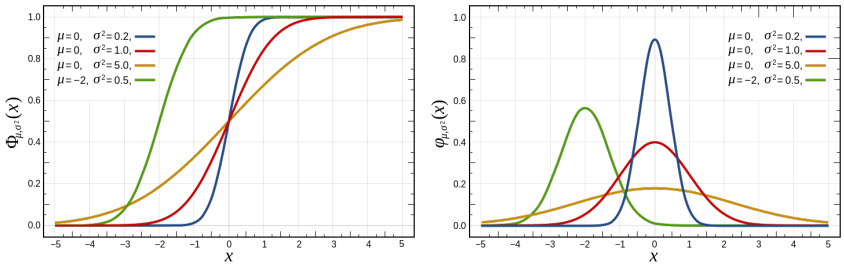
- **Multinomial Distribution** generalizes Binomial law. More on wikipedia

- **Zipf Distribution** is an empirical discret description of word frequency in a text. More on wikipedia

- **Pareto Distribution** is the equivalent of Zipf in a continuous space. It allows, amongst other things, to give a theoretical base of the "80-20 principle" (20% of the causes produce 80% of the effects). More on wikipedia

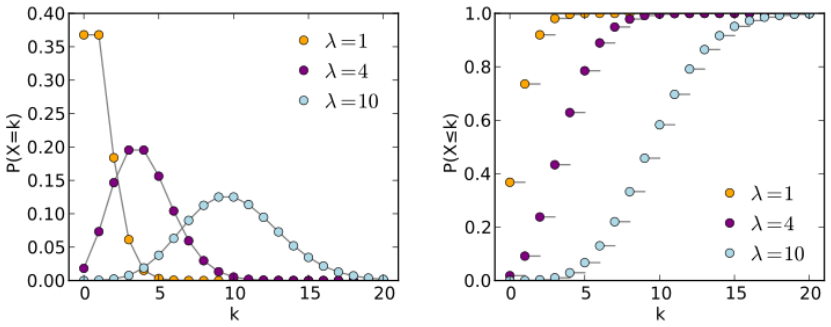
- **Yule-Simon distribution** describes discret frequencies of term too. More on wikipedia

You should understand the distribution of your data before applying a model!



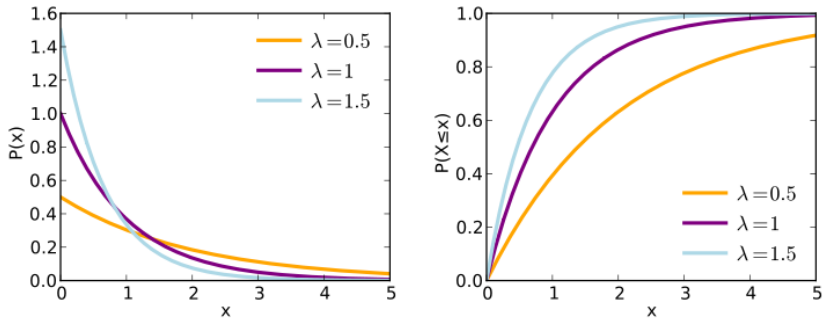
(a) Probability density function (b) Cumulative distribution function

Figure 15: Gaussian distribution



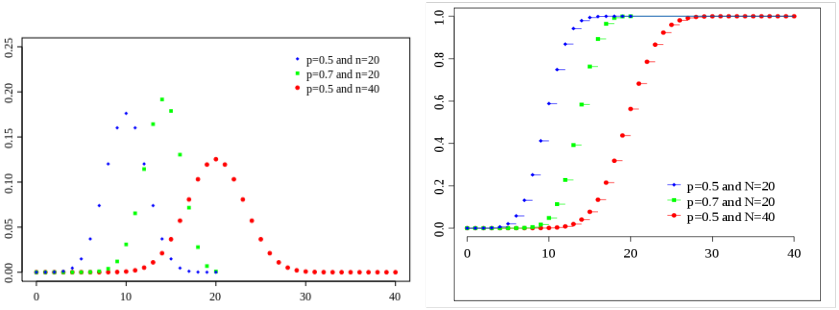
(a) Probability density function (b) Cumulative distribution function

Figure 16: Poisson distribution



(a) Probability density function (b) Cumulative distribution function

Figure 17: Exponential distribution



(a) Probability density function (b) Cumulative distribution function

Figure 18: Binomial distribution

5.4 Measurement on Samples

In practice, we (almost) never analyse the entire population. We always work on a subset of it called **sample**. The **variance** is the variation between elements of our sample, that we hope to be the same as the population. The **biases** is the systematic variation between the entire population and the sample we chose.
When randomly select elements of the population to be part of the sample, we have a great chance that the bias is small (i.e. that the distribution of the sample corresponds to the distribution of the population). But do not forget that there is a probability (even if a small one) to select elements that **biased our measures**! This probability can even increase when you clean the data, if you don't do it wisely.
A stupid example could be a study on population education in which, during the cleaning, you remove the answers containing misspelling. Uneducated people are more likely to commit misspelling so, removing them, you artificially biais the sample.

5.5 Test Statistic

(The only good and easy-to-understand explanation about test statistics I ever found is available on hamelg.blogspot)
The idea behind test statistics is **instead of proving that our assumption is true, let's calculate the probability that our observations occur by chance (null hypothesis or H_0). If this probability is very low, then there is a good chance that our hypothesis is true!**
The probability that this happens by chance is called *pvalue* and we usually consider that if $pvalue \leq 0.05$ our hypothesis is true ($pvalue \leq 0.01$ in some strict cases).

5.5.1 Example with t-test

Let's take the example on FIG 19. The **Null Hypothesis** H_0 represents the distribution of observation we can do, assuming there is no correlation between the values we measured. The **Alternative** is H_A, our hypothesis which states that, at the opposite, there is correlations.

- If the observation we test is $x = 3$, there is **less than 5% probabilities that it was produced by** H_0 ($pvalue \leq 0.05$). Our hypothesis H_A is considered true.
- If the observation we test is $x = 0$, there are **more than 5% probabilities that it was produced by** H_0 (more or less 40%). Our hypothesis H_A is considered false.

An important thing to notice is that **it does not provide the truth on statistic**, it only provides information about how likely is the null hypothesis! Let's look back to the example

- The $x = 3$ observation **could have been produced by the red area**, meaning that it's part of the small 5% chance of being produced by the H_0. The test will say that our hypothesis is true, which will be a **false positive**.

- The $x = 0$ observation **could have been produced by the blue area**, meaning that even it H_0 have great chance to produce it, it was in fact produced by H_A. The test will say that our hypothesis is false, which will be a **false negative**.

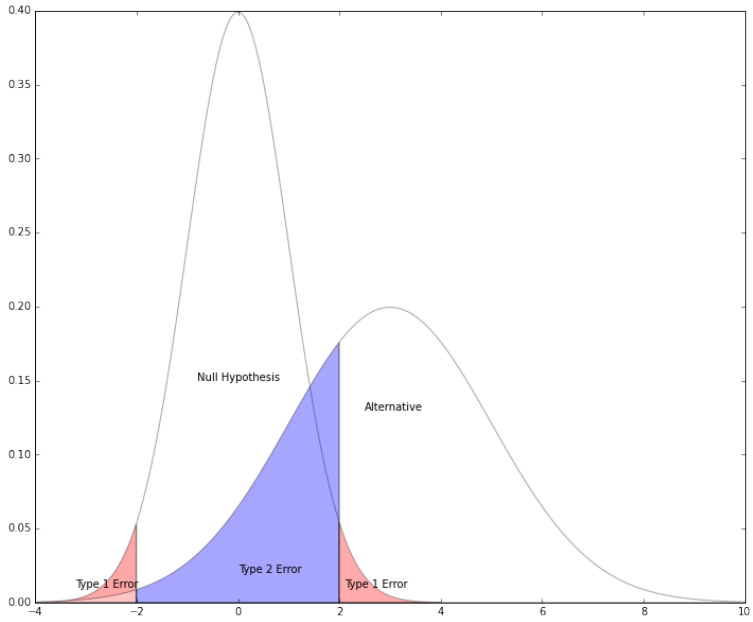


Figure 19: T-Test example

5.5.2 Choose the right test

A lot of tests exist and we must choose wisely which one to use, according to data and hypothesis characteristics. FIG 20 show a decision tree helping to choose the test which suits best our situation.

- Question ?
- Data type ?
- Sample size
- Variance known?
- Variance of several groups equals?
- ...

5.5.3 Family-wise Error

Following this simple math equation, we can figure out that the more experiment we do to test the hypothesis, the more likely we are to find that one of them are spuriously right!

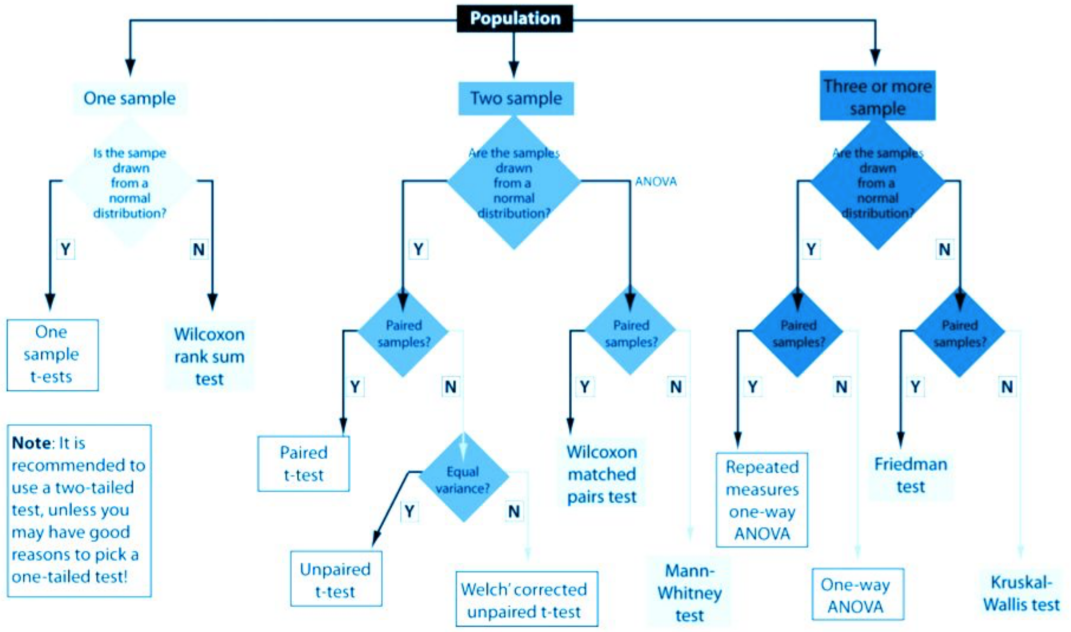


Figure 20: Statistical test decision tree

This is because the reverse point of view of the test. We are testing the fact that H_0 is unlikely, not directly that H_A is likely.

$$\begin{aligned} P(false\ positive) &= \alpha = 0.05 \\ P(true\ positive) &= 1 - \alpha = 0 - 95 \\ P(true\ positive\ on\ each\ experiment) &= (1-\alpha)^k \\ P(at\ least\ on\ false\ positive\ on\ one\ experiment) &= 1 - (1-\alpha)^k \end{aligned} \tag{6}$$

To counter that, two possible correction exists

- Bonferroni correction : $\alpha_c = \frac{\alpha}{k}$
- Sidak correction: $\alpha_c = 1 - (1-\alpha)^{1/k}$

5.5.4 Non-Parametric tests

All the tests so fare assume that the data are **normally distributed** and that the samples are **independent of each other and all have the same distribution.** (IID) They may be inaccurate if those assumptions are not met. Therefore, make sure the data satisfy the assumptions of the test we're using. Watch out for:

- **Outliers** will corrupt many tests that use variance estimates.
- **Correlated values as samples**, e.g. if you repeated measurements on the same subjective

- **Skewed (bias) distributions** give invalid results.

Some tests make no assumptions and thus can be used on very general cases: **K-S test**, **Permutation test** and **Bootstrap confidence interval**.

5.5.5 K-S test

K-S (Kolmogorov-Smirnov) test is a very useful test for checking whether two (continuous or discrete) distributions are the same.

- In the **one-sided test**, an observed distribution (e.g. some observed values or a histogram) is compared against a reference distribution (e.g., power-law).
- In the **two-sided test**, two observed distributions are compared.
- The K-S statistic is just the **max distance between the CDFs** (Cumulative Distribution Function) of the two distributions.
- The K-S test can be used to test **whether a data sample has a normal distribution** or not.
- Thus it can be used as a sanity check for any common parametric test (which assumes normally distributed data).
- It can also be used to compared distributions of data values in large data pipeline: **Most errors will distort the distribution of a data parameter and a K-S test can detect this.**

This test is expensive! Check for more information on the wikipedia page.