Silver Market Timing–
A Macroeconomic Perspective

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1 Introduction

The silver commodity is a long-established constituent of the precious metals market. It typically provides investors with greater volatility than what is commonly observed in the gold market. In this article, I investigate the determinants of the spot silver price and provide a valuable tool for industry practitioners.

In the following, I first present a simple, theoretical pricing model. Secondly, I am empirically investigating key properties of this model by using recent market data. From these findings, I finally construct a statistical fair-value indicator for the silver spot price. This indicator conveniently absorbs statistically significant macroeconomic information that is relevant to silver spot pricing.

All results are 100% reproducible. All data and code are available on my company's GitHub repo.

2 Silver Spot Pricing Hypothesis

The silver commodity (spot symbol: `^XAGUSD`) can be thought of in terms of a zero-coupon paying bond with an infinite time to maturity. Assuming cashing-out within finite time, we are interested in the asset's spot price at the end of the arbitrary holding period, T. The future selling price, P_T, is a function of the spot price at the beginning of the investment period, P_0, and USD inflation, i.

$$P_T = f(P_0, i)$$

Assuming continuous-time, we can discount the future spot price, P_T, to the current spot price, P_t, using the risk-free USD interest rate r.[1]

$$P_t = \frac{f(P_0, i)}{e^{r(T-t)}} \tag{1}$$

For illustration purposes, let's assume that $f(P_0, i) = P_0 e^{i(T-t)}$

$$P_t = \frac{P_0 e^{i(T-t)}}{e^{r(T-t)}}$$

The partial derivatives with respect to inflation, i, and the risk-free USD interest rate, r, yield:

$$\frac{\partial P_t}{\partial i} = \frac{(T-t)P_0 e^{i(T-t)}}{e^{r(T-t)}}$$

[1] This implicitly assumes risk-neutrality, which further assumes an arbitrage-free market.

$$\frac{\partial P_t}{\partial r} = \frac{(t-T)P_0 e^{i(T-t)}}{e^{r(T-t)}}$$

Note that $P_0, t, T > 0$ and $T > t$. Thus,

$$\begin{aligned} &\textbf{1.} \quad \frac{\partial P_t}{\partial i} > 0 \\ &\textbf{2.} \quad \frac{\partial P_t}{\partial r} < 0 \end{aligned}$$

Hence, we have established that,

1. The silver spot price, P_t, *increases* with rising USD inflation, i
2. The silver spot price, P_t, *decreases* with rising risk-free USD rates, r.

A Word of Caution

Let's denote s the *spread* between USD inflation and the USD risk-free rate, $s = i - r$. It is *tempting* to say, whenever the *change* in the inflation-rate spread, s, is *positive*, P_t is increasing. In other words, whenever the change in inflation is greater than the change in the interest rate, the silver price, P_t is rising:

$$\Delta s > 0 \implies \Delta P_t > 0$$

However, it is important to understand that we cannot make such claim. Remember that the exact form of $f(P_0, i)$ in equation (1) is *unknown*. Hence, we cannot infer that Δi and Δr have an *equal* impact on P_t. However, the exact impact can be estimated using data, which is done in the next section.

3 Empirical Validation

In the following, I am empirically investigating the silver spot pricing hypthesis from the previous section. Hence, it is necessary to find real-world proxies for USD inflation and USD risk-free rate.

When it comes to inflation, I am using the 10-year Breakeven Inflation Rate by the Federal Reserve Bank of St. Louis.[2] At each point in time, this rate represents the *expected inflation* over the next 10 years. Hence, a *change* in the Breakeven Inflation Rate can be interpreted as a change in expected inflation. This rate is derived from 10-year US Treasury bonds with and without inflation protection.

[2]Federal Reserve Bank of St. Louis, 10-Year Breakeven Inflation Rate [T5YIE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/T5YIE, April 9, 2021.

When it comes to the risk-free rate, I am using 10-year US treasury bond rates, which are not indexed for inflation.[3]

This selection implies a finite-time holding horizon of 10-years for this commodity.

For daily data of the last 3 years (from 1 April 2018 to 31 March 2021), I am fitting the following linear regression model:

$$\Delta P_t = \hat{\beta}^i \Delta i_t + \hat{\beta}^r \Delta r_t + \epsilon_t \tag{2}$$

where:

P_t	Daily XAGUSD exchange rate
ΔP_t	Continuously compounded return of P_t, $ln\left(\frac{P_t}{P_{t-1}}\right)$
Δi_t	Change in the 10-year Breakeven Inflation rate, $i_t - i_{t-1}$
Δr_t	Change in the 10-year Treasury rate, $r_t - r_{t-1}$
ϵ_t	Error term, $E[\epsilon_t] = 0$

Table 1: Linear Regression Results

	$\hat{\beta}$	SE($\hat{\beta}$)	t-value	p-value
Δi_t	0.1999	0.0235	8.5240	<0.001
Δr_t	-0.1017	0.0165	-6.1752	<0.001

Adj Rsq: 0.0961

The linear regression model in (2) explains roughly 10% of the total variation in the Silver spot price return. In terms of financial econometrics, this is a remarkably convincing result. Using the linear regression output, we can use statistical theory to (in-)validate the hypotheses made in the previous section. For the *true* value of β, we are interested whether $\beta^i > 0$ and $\beta^r < 0$. These one-sided Frequentist hypothesis tests are carried out below.

Hypothesis Test for Inflation-Beta

$$\mathbf{H_0}: \quad \beta^i \leq 0$$
$$\mathbf{H_1}: \quad \beta^i > 0$$

$$P[\text{data} | \beta^i \leq 0] = P[T_{\text{df=752}} > 8.52] < \mathbf{0.001}$$

[3] Board of Governors of the Federal Reserve System (US), 10-Year Treasury Constant Maturity Rate [DGS10], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/DGS10, April 9, 2021.

Given H_0 is true, the probability of observing this data (or more extreme) is < 0.001. Hence, we reject H_0 for *all common levels of statistical significance* and accept that $\beta^i > 0$.

Hypothesis Test for Risk-Free Rate-Beta

$$\begin{aligned} \mathbf{H_0}: &\quad \beta^r \geq 0 \\ \mathbf{H_1}: &\quad \beta^r < 0 \end{aligned}$$

$$P[\text{data}|\beta^r \geq 0] = P[T_{\text{df}=752} < -6.18] < \mathbf{0.001}$$

Given H_0 is true, the probability of observing this data (or more extreme) is < 0.001. Hence, we reject H_0 for *all common levels of statistical significance* and accept that $\beta^r < 0$.

The results of the hypothesis tests provide strong statistical evidence that the two statements derived from the partial derivatives in the previous section are true.

In summary, we found that;

1. The silver price *increases* with rising expected inflation. This is backed by strong statistical evidence.
2. The silver price *decreases* with rising 10Y US treasury rates. This is backed by strong statistical evidence.
3. The silver price is estimated to be *twice as sensitive* to changes in inflation than it is to changes in US treasury rates.

4 Construction of a Fair-Value Silver Indicator

Using the regression results in the previous part, we can try to construct a silver fair value indicator. According to the linear regression model above,

$$\widehat{\Delta P_t} = 0.2\Delta i_t - 0.1\Delta r_t$$

Hence,

$$\mathbb{E}[0.2\Delta i - 0.1\Delta r - \Delta P_t] = 0$$

This equation within the expectation can be used to construct a fundamental Silver spot price indicator. The rolling-time period, Δ, can be arbitrarily chosen, e.g. 15 trading days (resembling 3 weeks). The daily indicator value is then computed as follows:[4]

[4]Note that continuously compounded returns are additive

$$0.2[i_t - i_{t-15}] - 0.1[r_t - r_{t-15}] - ln\left(\frac{P_t}{P_{t-15}}\right)$$

It might be convenient to think of the indicator's values in terms of simple compounding return. Hence the indicator can be transformed to reflect that:

$$\mathrm{I}_t^S = 100\left(\exp\left[0.2(i_t - i_{t-15}) - 0.1(r_t - r_{t-15}) - ln\left(\frac{P_t}{P_{t-15}}\right)\right] - 1\right)$$

The Silver Indicator, I_t^S, can be interpreted as the *fundamentally* expected percentage return over the next 3 weeks:

- $I_t^S > 0$: The current silver spot price is *fundamentally undervalued*
- $I_t^S < 0$: The current silver spot price is *fundamentally overvalued*
- $I_t^S = 0$: The current silver spot price trades at fair value

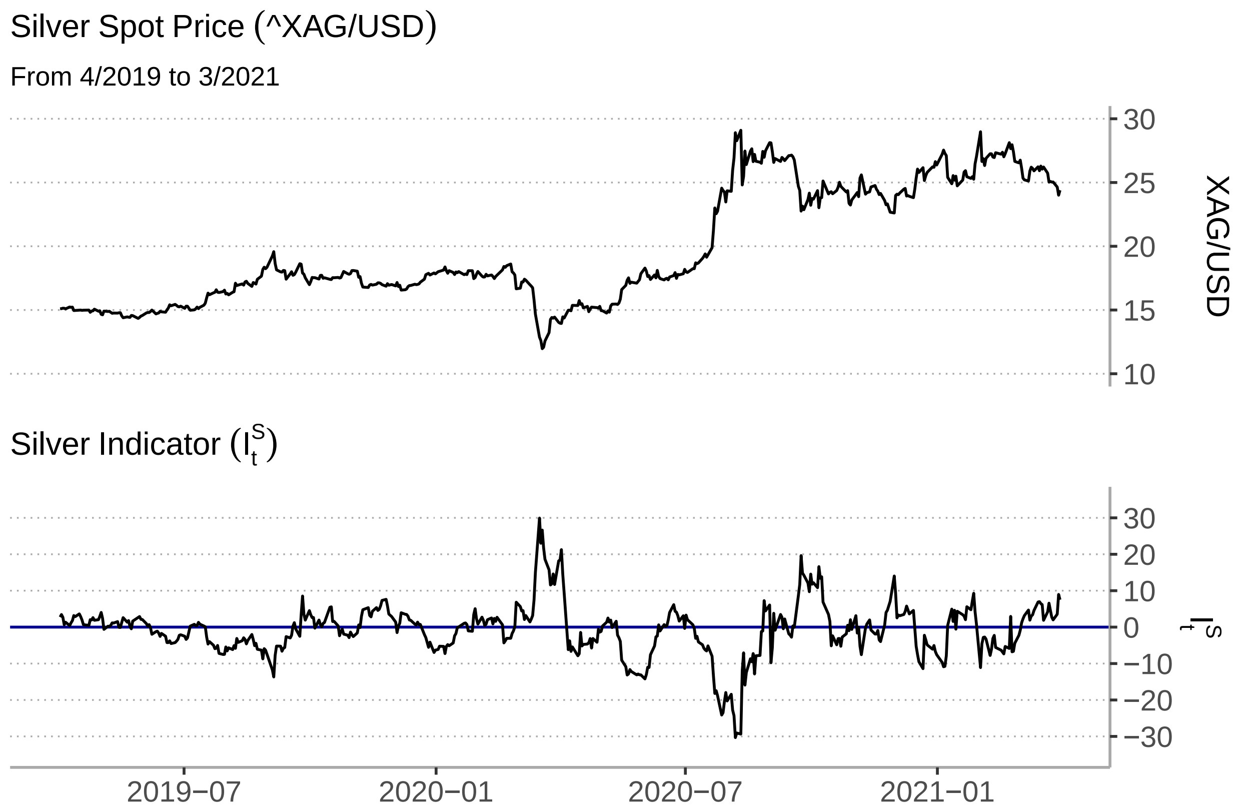
Limitations

Acknowledge that the I_t^S is *only* based on developments of the USD risk-free rate and the expected USD inflation. There is reasonable statistical evidence to use these two macroeconomic variables. Any other potential factors (e.g. sentiment, other macroeconomic variables, storage cost, etc) are neglected. This is due to a lack of data availability and/or a lack of sensible theoretical considerations to include such additional variables.

Note that whenever $I_t^S \neq 0$, I_t^S *must* eventually revert to 0. This happens because of any of the following reasons:

a. The silver spot price will move such that $I_t^S \to 0$, yielding a statistical arbitrage opportunity in the silver market
b. The *future* expected inflation will move such that $I_t^S \to 0$
c. The *future* risk-free USD rate will move such that $I_t^S \to 0$
d. The market fails to properly reflect macroeconomic changes within the 3-week rolling indicator window
e. A combination of any of the above

Regardless of these limitations, the indicator performs well in explaining the silver spot price movements from 2019–2021. Below is a plot of the silver spot price (`^XAGUSD`) against I_t^S.



Observe that,

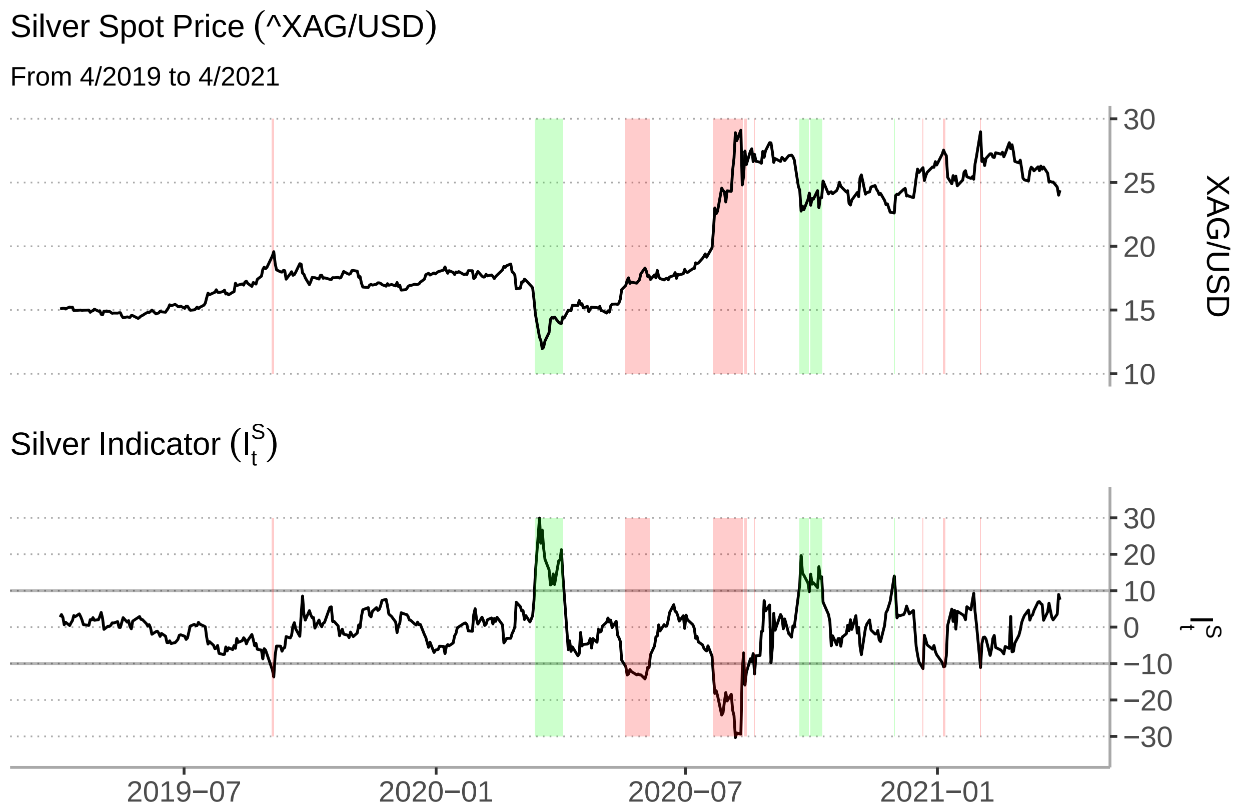
1. I_t^S is mean-reverting around zero
2. Whenever I_t^S diverges significantly away from zero, this indicates strong buy/sell signals, which in part materialize in subsequent `^XAG/USD` movements

Hence, the I_t^S indicator can be used to absorb and compare the information given by the most crucial macroeconomic variables versus the actual price changes.

4.1 Silver Market Timing Using I_t^S

To avoid signals in the I_t^S that are due to noise, let's arbitrarily define a fundamental buy/sell signal whenever $|I_t^S| \geq 10$.

In the plot below, all values $I_t^S \leq -10$ are highlighted in red (i.e. *sell* signal), and values $I_t^S \geq 10$ are highlighted in green (i.e. *buy* signal).



Remember that this indicator signals diversions from the estimated *macroeconomically justifiable* fair-value price. For greater detail, in the appendix I am providing zoomed-in plots over specific time periods.

5 A Note on Crypto-Currency

Below is a plot depicting the 10-year US expected inflation[5] (top), the Bitcoin spot price, `BTC/USD` (middle), and the silver spot price, `XAG/USD` (bottom) from January 2020 to December 2020.

[5]Federal Reserve Bank of St. Louis, 10-Year Breakeven Inflation Rate [T5YIE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/T5YIE, April 9, 2021.

From January 2020 to November 2020, `BTC/USD` and `XAG/USD` shared similar price developments. Both assets seemed to be driven by a rise in expected inflation. It can be inferred that, similar to silver, `BTC/USD` provides inflation protection. On these terms, `BTC/USD` can be considered a *low transaction cost* and *zero storage cost* competitor to the silver commodity.

However, after October 2020, `BTC/USD` appreciated greatly without macroeconomic justification. More specifically, `BTC/USD` rose from USD 11,736 on 19 October 2020 to roughly USD 30,000 by the end of the year. On 12 April 2021, it reached more than USD 60,000. Simultaneously, the overall crypto currency market had an increase in market capitalization proportionally to the `BTC/USD` appreciation.

Opinion: The risk for the silver commodity is that some of it's potential market capitalization for inflation-protection might be substituted for investment in cryptocurrency. Hence, further gain in acceptance and trust in cryptocurrency for the purpose of preserving value might, in part, drive away demand from the silver commodity market.

6 Conclusion

In this article, I validate how long-term risk-free rates and inflation are determinants of the silver spot price. More specifically, empirical analysis suggests that the silver spot price is roughly *twice* as sensitive to changes in USD expected inflation than it is to changes in the 10-year U.S. Treasury rate.

When it comes to silver market-timing, it can be concluded the following:

A Strong Bullish `XAG/USD` Case

- A silver indicator value $I_t^S \geq 10$
- An *unexpected* rise in inflation[6]
- A *future* decrease in long-term risk-free rate[7]
- An *increase* in demand for silver as a resource for production
- A *decrease* in trust in cryptocurrency

A Strong Bearish `XAG/USD` Case

- A silver indicator value $I_t^S \leq -10$
- An *unexpected* decrease in inflation[8]
- A *future* increase in long-term risk-free rate[9]
- A *decrease* in demand for silver as a resource for production
- A continued *increase* in trust and acceptance in cryptocurrency

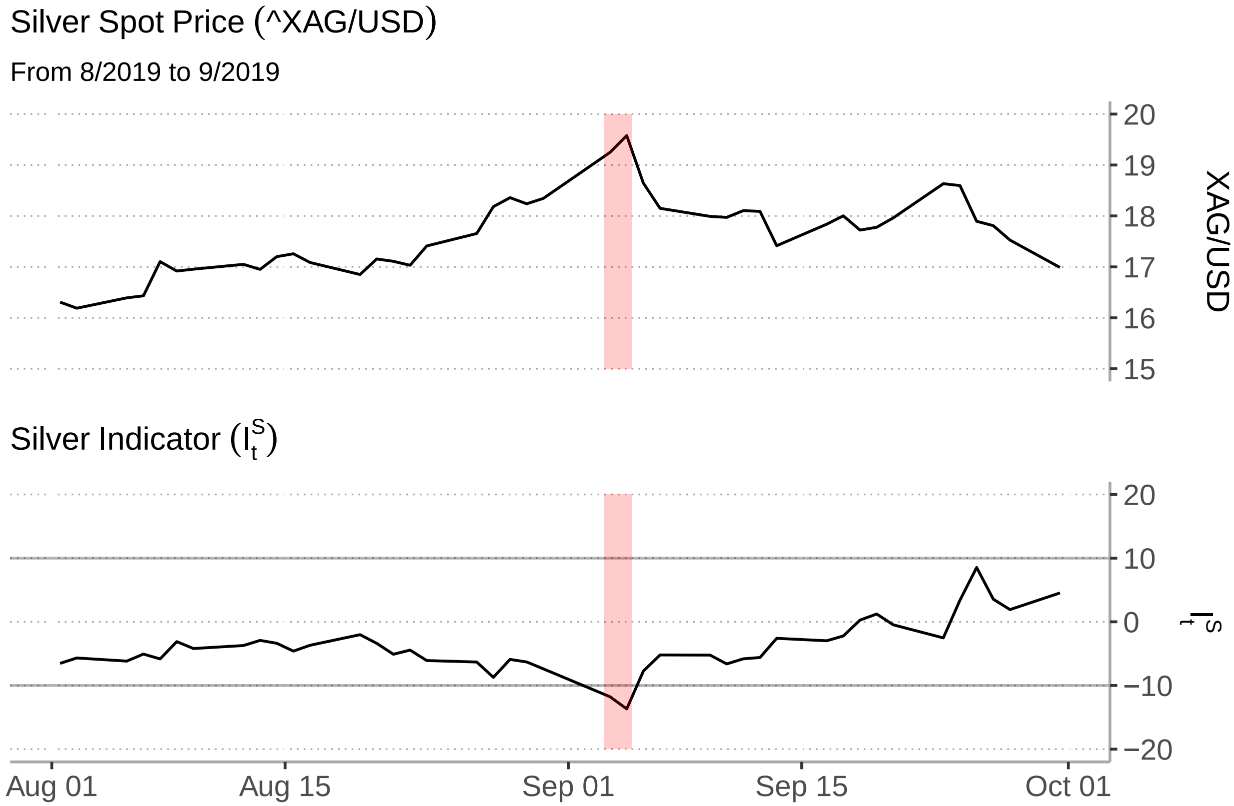
[6]Note that a rise in *expected inflation* is incorporated in the silver indicator I_t^S

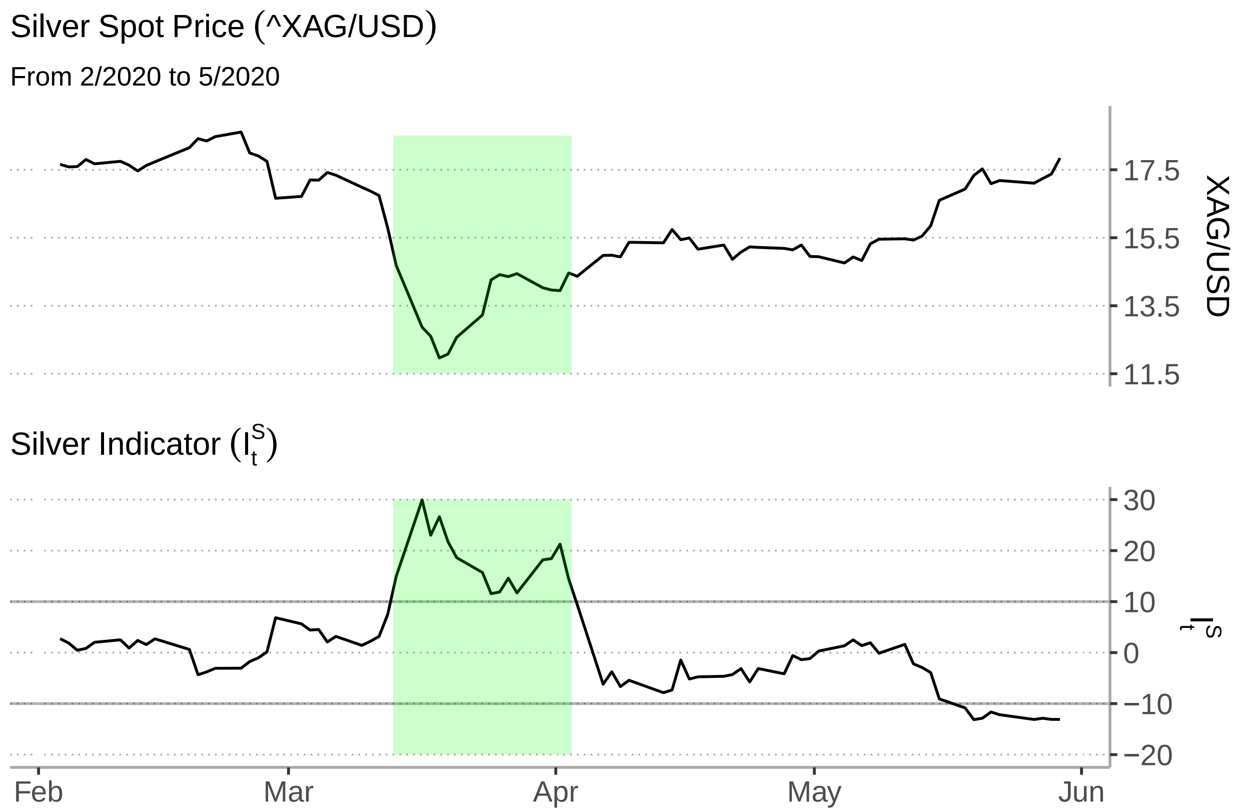
[7]Note that a decrease in the long-term risk-free rate is incorporated in the silver indicator I_t^S

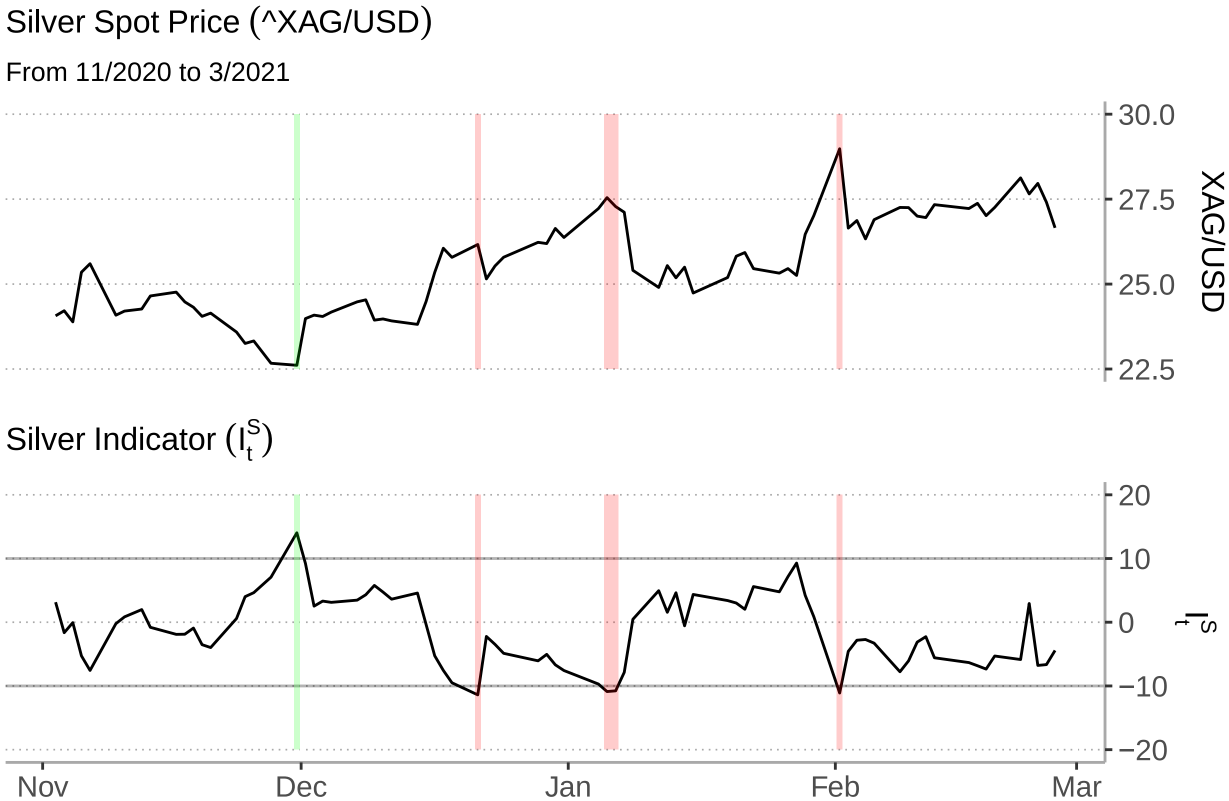
[8]Note that a decrease in *expected inflation* is incorporated in the silver indicator I_t^S

[9]Note that a rise in the long-term risk-free rate is incorporated in the silver indicator I_t^S

7 Appendix: Zoomed Plots




Silver Spot Price (^XAG/USD)
From 2/2020 to 5/2020
17.5
15.5
13.5
11.5
XAG/USD
Silver Indicator (I_t^S)
30
20
10
0
−10
−20
I_t^S
Feb
Mar
Apr
May
Jun


Silver Spot Price (^XAG/USD)
From 11/2020 to 3/2021
30.0
27.5
25.0
22.5
XAG/USD
Silver Indicator (I_t^S)
20
10
0
−10
−20
I_t^S
Nov
Dec
Jan
Feb
Mar

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