

CSE 402 - BIOMETRICS AND PATTERN  
RECOGNITION

PROF. ARUN ROSS

MICHIGAN STATE UNIVERSITY

THANKS TO: THOMAS SWEARINGEN

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**FACE RECOGNITION**

**PART 2**

## TERMINOLOGY

- ▶ **Label**: a specific class of a data sample (e.g., an image of a face is labeled with an ID of the person)
- ▶ **Training Data**: labeled examples used by a classifier to learn a task
- ▶ **Test Data**: data (*separate from training data*) used to evaluate classifier performance; a test label is compared with the classifier predicted label to evaluate classifier performance
- ▶ **Supervised Learning Method**: a method that uses class labels of the training data to learn
- ▶ **Unsupervised Learning Method**: a method that does not use the class labels of the training data to learn

## SOME APPROACHES TO FACE MATCHING

- ▶ Appearance-Based
  - ▶ **Principal Component Analysis**
  - ▶ **Linear Discriminant Analysis**
- ▶ Model-Based
  - ▶ Elastic Bunch Graph Matching
- ▶ Texture-Based
  - ▶ Local Binary Pattern (LBP)
- ▶ Deep Learning
  - ▶ Convolutional Neural Network



FACE RECOGNITION

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**PRINCIPAL COMPONENT  
ANALYSIS (PCA)**

## PRINCIPAL COMPONENT ANALYSIS (PCA)

- ▶ Early automated face recognition method
- ▶ **Goal:** learn a subspace that accounts for as much variability in the training data as possible
- ▶ Does not use identity information during training (*Unsupervised Learning*)

## PCA DATA SETUP

1. Let  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N$   $\swarrow$   $d \times 1$  column vector denote each training sample
2. Compute average of the training set

$$\swarrow \text{ } d \times 1 \text{ column vector} \quad \boldsymbol{\mu} = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i$$

3. Define the data matrix

$$\swarrow \text{ } d \times N \text{ matrix} \quad \mathbf{X} = \left[ (\mathbf{x}_1 - \boldsymbol{\mu}) \ (\mathbf{x}_2 - \boldsymbol{\mu}) \cdots (\mathbf{x}_N - \boldsymbol{\mu}) \right]$$

## PCA SUBSPACE LEARNING

4. Calculate the covariance matrix

$$\begin{array}{c} d \times d \text{ matrix} \swarrow \\ \mathbf{C} = \mathbf{X}\mathbf{X}^T \\ \swarrow \quad \searrow \\ d \times N \text{ matrix} \quad N \times d \text{ matrix} \end{array}$$

5. Compute the Eigenvectors of the covariance matrix by solving

$$\begin{array}{c} d \times 1 \text{ column vector} \quad \mathbf{C}\mathbf{E} = \lambda\mathbf{E} \\ \swarrow \\ \text{where } \mathbf{E} = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_d] \text{ are the Eigenvectors} \\ \swarrow \\ d \times d \text{ matrix} \end{array}$$

## PCA COMPARING FACES

Instead of taking all "d" eigen-vectors,  $E_k$  takes the top "k" eigen-vectors (eigen-vectors are arranged in columns)

1. Represent two unseen images,  $\mathbf{x}_i$  and  $\mathbf{x}_j$ , as a weighted sum of Eigenvectors

$$k \times 1 \text{ vector} \longrightarrow \boldsymbol{\omega}_i = \mathbf{E}_k^T (\mathbf{x}_i - \boldsymbol{\mu}) \quad \boldsymbol{\omega}_j = \mathbf{E}_k^T (\mathbf{x}_j - \boldsymbol{\mu})$$

2. Compare difference of weighted sums to threshold (t)

$$\| \boldsymbol{\omega}_i - \boldsymbol{\omega}_j \|^2 \leq t$$

Faces are considered a match if the difference is less than the threshold, otherwise they are not a match



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**Algorithm 1** Principal Component Analysis for Faces
 

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$\mathbf{X}_{\text{train}} \leftarrow \text{LoadTrainData}()$ $\mathbf{X}_{\text{test}} \leftarrow \text{LoadTestData}()$ $\mathbf{V}, \boldsymbol{\mu} \leftarrow \text{Eigenfaces}(\mathbf{X}_{\text{train}})$ $\mathbf{D} \leftarrow \text{CompareFaces}(\mathbf{X}_{\text{test}}, \mathbf{V}, \boldsymbol{\mu})$	$\triangleright \mathbf{X}_{\text{train}} = [\mathbf{x}_1^{\text{train}}, \dots, \mathbf{x}_N^{\text{train}}]$ $\triangleright \mathbf{X}_{\text{test}} = [\mathbf{x}_1^{\text{test}}, \dots, \mathbf{x}_M^{\text{test}}]$ $\triangleright \text{Learn Subspace}$ $\triangleright \text{Obtain distances scores for test data}$
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**function** EIGENFACES( $\mathbf{X}$ )

 $\boldsymbol{\mu} \leftarrow \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i$ 
 $\triangleright$  calculate mean

 $\mathbf{X}_c \leftarrow \mathbf{X} - [\boldsymbol{\mu}, \dots, \boldsymbol{\mu}]$ 
 $\triangleright$  center data

 $\mathbf{C} \leftarrow \mathbf{X}_c \mathbf{X}_c^T$ 
 $\triangleright$  compute covariance matrix

 $\mathbf{V} \leftarrow \text{Eigenvectors}(\mathbf{C})$ 
 $\triangleright$  Find Eigenvectors

**return**  $\mathbf{V}, \boldsymbol{\mu}$ 
**end function**
**function** COMPAREFACES( $\mathbf{X}, \mathbf{V}, \boldsymbol{\mu}$ )

 $\boldsymbol{\Omega} = \mathbf{V}^T (\mathbf{X} - [\boldsymbol{\mu}, \dots, \boldsymbol{\mu}])$ 
 $\triangleright \boldsymbol{\Omega} = [\boldsymbol{\omega}_1, \dots, \boldsymbol{\omega}_M]$ 
**for each**  $\boldsymbol{\omega}_i, \boldsymbol{\omega}_j$  pair in  $\boldsymbol{\Omega}$  **do**
 $D_{i,j} = \|\boldsymbol{\omega}_i - \boldsymbol{\omega}_j\|^2$ 
 $\triangleright$  Compare test samples

**end for**
**return**  $\mathbf{D}$ 
**end function**


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## VISUALIZING PCA EIGENVECTORS

- ▶ Train on **4,000 aligned images** from the *Oxford VGG Face dataset*
- ▶ Obtain the Eigenvectors corresponding to the 4 largest Eigenvalues
  - ▶ Called *Eigenfaces*
- ▶ Reshape from 1D vector to 2D image



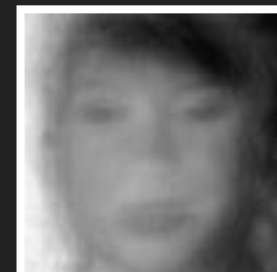
Mean



Largest Eigenvalue



2nd Largest Eigenvalue



3rd Largest Eigenvalue



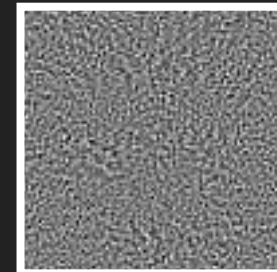
4th Largest Eigenvalue

## COMPRESSED REPRESENTATION

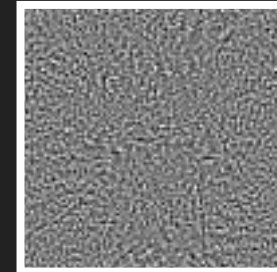
- ▶ **Not all Eigen vectors needed to adequately represent the data**
- ▶ Eigenvectors with small Eigenvalues **mostly model noise** rather than discriminant info relating to identity
- ▶ Represent data using only  $k$  Eigenvectors ( $k < d$ )

$$k \times 1 \text{ vector} \longrightarrow \omega_i = E_k^T x$$

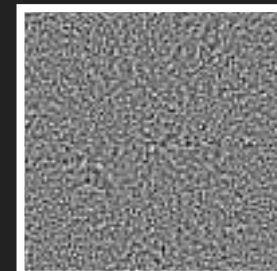
$\uparrow$   
 $E_k \text{ is } d \times k$



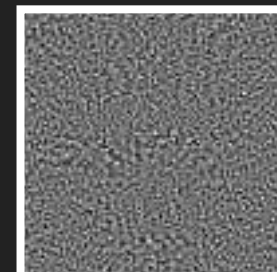
5th Smallest Eigenvalue



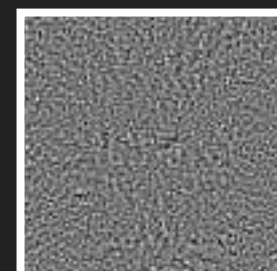
4th Smallest Eigenvalue



3rd Smallest Eigenvalue



2nd Smallest Eigenvalue



Smallest Eigenvalue

The background of the slide is a dark gray field filled with a dense, repeating pattern of stylized human faces. The faces are rendered in a minimalist, geometric style with various features like eyes, noses, and mouths. Some faces are light gray, while others are a darker shade, creating a textured, crowd-like effect.

FACE RECOGNITION

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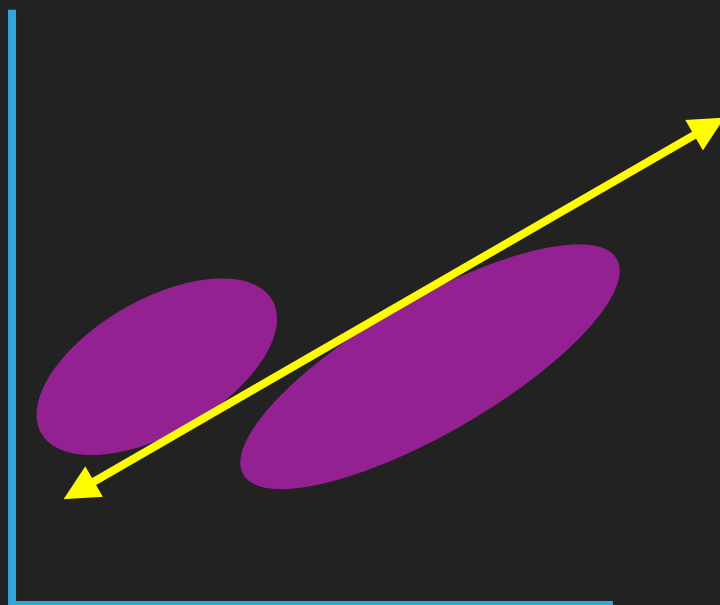
**LINEAR DISCRIMINANT  
ANALYSIS (LDA)**

## LINEAR DISCRIMINANT ANALYSIS (LDA)

- ▶ Unlike PCA, LDA makes use of identity information (*Supervised Learning*)
- ▶ **Goal:** learn a subspace that *minimizes intra-class variation* and *maximizes inter-class variation*
  - ▶ Images from the same subject are closer together
  - ▶ Images from different subjects further apart

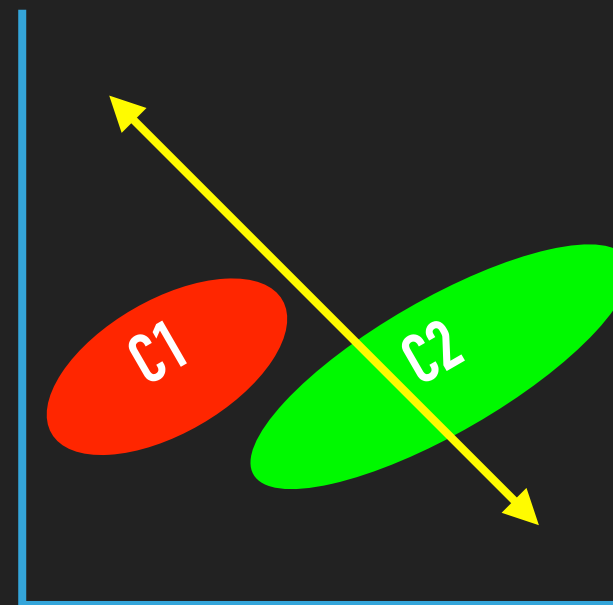
## PCA VS LDA

↔ = Principal Axis



**PCA**

Principal axis follows the direction with the most variation for all data.




**LDA**

Principal axis follows the direction with the most *inter-class* variation.

## LDA INITIALIZATION

1. Let  $(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \dots, (\mathbf{x}_N, y_N)$  be the training set where  $\mathbf{x}_i$  is the data and  $y_i = \{1, \dots, C\}$  is the class label
2. Compute mean for each class (identity)

$d \times 1$  column vector


$$\mu_c = \frac{1}{N_c} \sum_{\{i \mid y_i=c\}} \mathbf{x}_i$$

where  $N_c$  is the number of samples with class  $c$

## LDA SCATTER MATRICES

3. Use training data to calculate between class and within class scatter matrices

within class  $\longrightarrow$   $S_w = \sum_{c=1}^C \sum_{\{i \mid y_i=c\}} (\mathbf{x}_i - \boldsymbol{\mu}_c) (\mathbf{x}_i - \boldsymbol{\mu}_c)^T$   
 $d \times d$  matrix

between class  $\longrightarrow$   $S_b = \sum_{c=1}^C N_c (\boldsymbol{\mu}_c - \boldsymbol{\mu}) (\boldsymbol{\mu}_c - \boldsymbol{\mu})^T$   
 $d \times d$  matrix

where  $\boldsymbol{\mu} = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i$  (mean of all data)



## LDA SUBSPACE

4. Calculate Eigenvectors by solving

minimized  $\longrightarrow S_w^{-1} S_b E = \lambda E$   
maximized

5. Represent any data vector  $\mathbf{x}$  by

$d \times 1$  vector  $\longrightarrow \omega_i = E^T (\mathbf{x} - \mu)$

6. Compare representations like in PCA (norm of difference)

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**Algorithm 1** Linear Discriminant Analysis for Faces
 

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$\mathbf{X}_{\text{train}}, \mathbf{y}_{\text{train}} \leftarrow \text{LoadTrainData}()$   $\triangleright \mathbf{y}_{\text{train}} = [y_1^{\text{train}}, \dots, y_N^{\text{train}}]$   
 $\mathbf{X}_{\text{test}} \leftarrow \text{LoadTestData}()$   
 $\mathbf{V}, \boldsymbol{\mu} \leftarrow \text{Fisherfaces}(\mathbf{X}_{\text{train}}, \mathbf{y}_{\text{train}})$   $\triangleright$  learn subspace  
 $\mathbf{D} \leftarrow \text{CompareFaces}(\mathbf{X}_{\text{test}}, \mathbf{V}, \boldsymbol{\mu})$   $\triangleright$  obtain distances scores for test data

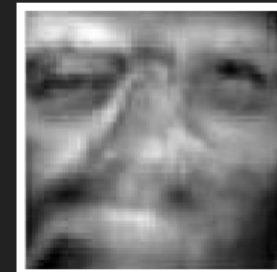
**function** FISHERFACES( $\mathbf{X}, \mathbf{y}$ )  
   **for** each  $c \in [1, \dots, C]$  **do**  
      $N_c \leftarrow |\{i | y_i = c\}|$   $\triangleright$  num. samples with class  $c$   
      $\boldsymbol{\mu}_c \leftarrow \frac{1}{N_c} \sum_{\{i | y_i = c\}} \mathbf{x}_i$   $\triangleright$  calculate class mean  
   **end for**  
    $\boldsymbol{\mu} \leftarrow \frac{1}{N} \sum_{i=1}^N \mathbf{x}_i$   $\triangleright$  calculate overall mean  
    $\mathbf{S}_w = \sum_{c=1}^C \sum_{\{i | y_i = c\}} (\mathbf{x}_i - \boldsymbol{\mu}_c) (\mathbf{x}_i - \boldsymbol{\mu}_c)^T$   
    $\mathbf{S}_b = \sum_{c=1}^C N_c (\boldsymbol{\mu}_c - \boldsymbol{\mu}) (\boldsymbol{\mu}_c - \boldsymbol{\mu})^T$   
    $\mathbf{V} \leftarrow \text{Eigenvectors}(\mathbf{S}_w^{-1} \mathbf{S}_b)$   $\triangleright$  find Eigenvectors  
   **return**  $\mathbf{V}, \boldsymbol{\mu}$   
**end function**

**function** COMPAREFACES( $\mathbf{X}, \mathbf{V}, \boldsymbol{\mu}$ )  
    $\boldsymbol{\Omega} = \mathbf{V}^T (\mathbf{X} - [\boldsymbol{\mu}, \dots, \boldsymbol{\mu}])$   $\triangleright \boldsymbol{\Omega} = [\boldsymbol{\omega}_1, \dots, \boldsymbol{\omega}_M]$   
   **for** each  $\boldsymbol{\omega}_i, \boldsymbol{\omega}_j$  pair in  $\boldsymbol{\Omega}$  **do**  
      $D_{i,j} = \|\boldsymbol{\omega}_i - \boldsymbol{\omega}_j\|^2$   $\triangleright$  compare test samples  
   **end for**  
   **return**  $\mathbf{D}$   
**end function**

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## VISUALIZING LDA EIGENVECTORS

- ▶ Train on images from the *ORL Face dataset*
- ▶ Obtain the Eigenvectors corresponding to the 5 largest Eigenvalues
  - ▶ Called *Fisherfaces*
- ▶ Reshape from 1D vector to 2D image



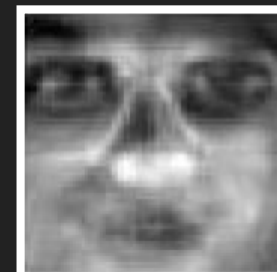
Largest Eigenvalue



2nd Largest Eigenvalue



3rd Largest Eigenvalue



4th Largest Eigenvalue



5th Largest Eigenvalue



FACE RECOGNITION

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**ELASTIC BUNCH GRAPH  
MATCHING (EBGM)**

## ELASTIC BUNCH GRAPH MATCHING (EBGM)

- ▶ Model-based method
- ▶ Robust to *occlusions* or *pose*
- ▶ Uses ***Gabor filters at specific face landmarks***
- ▶ Organizes filter responses in a **graph**

## FACE LANDMARKS

- ▶ Point of interest on the face
  - ▶ corners of eyes
  - ▶ tip of the nose
  - ▶ corners of the mouth
  - ▶ etc.
- ▶ Also known as fiducial points

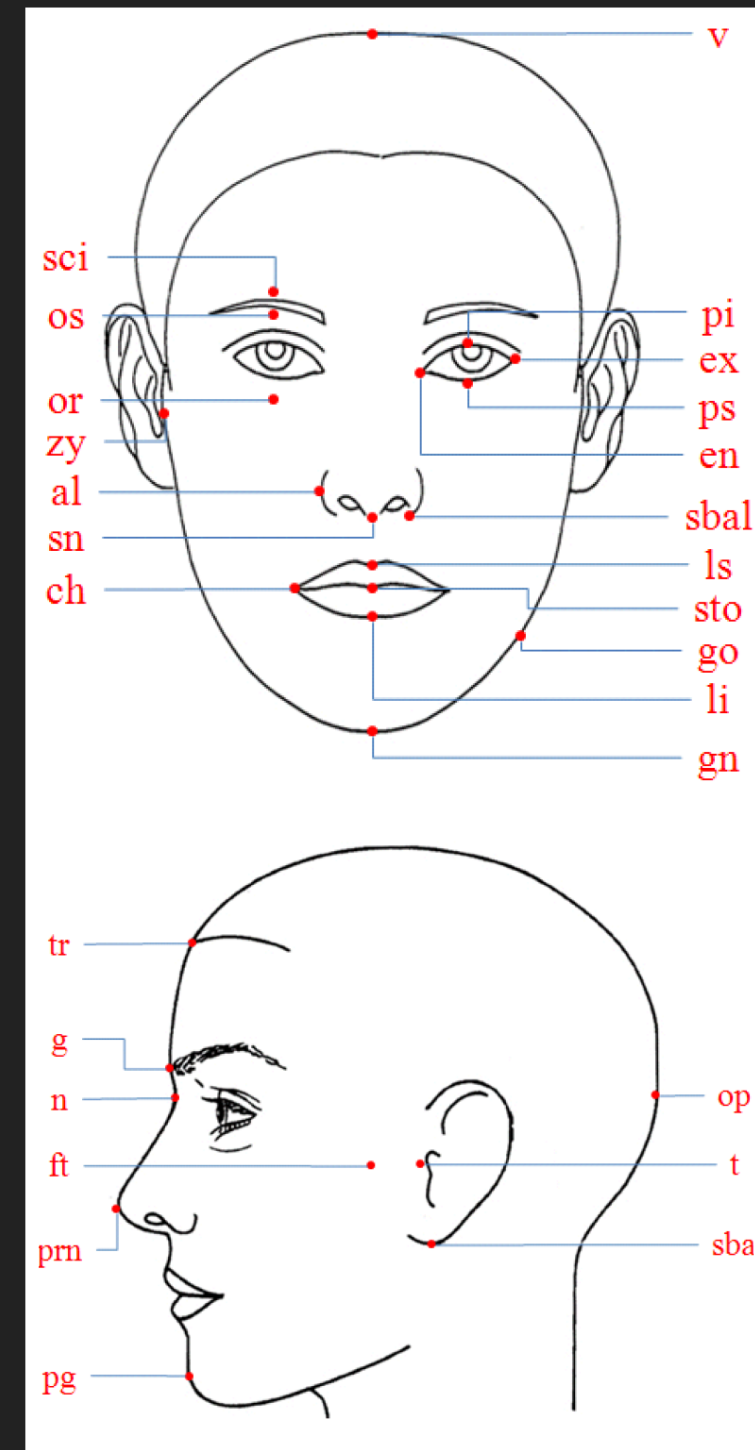
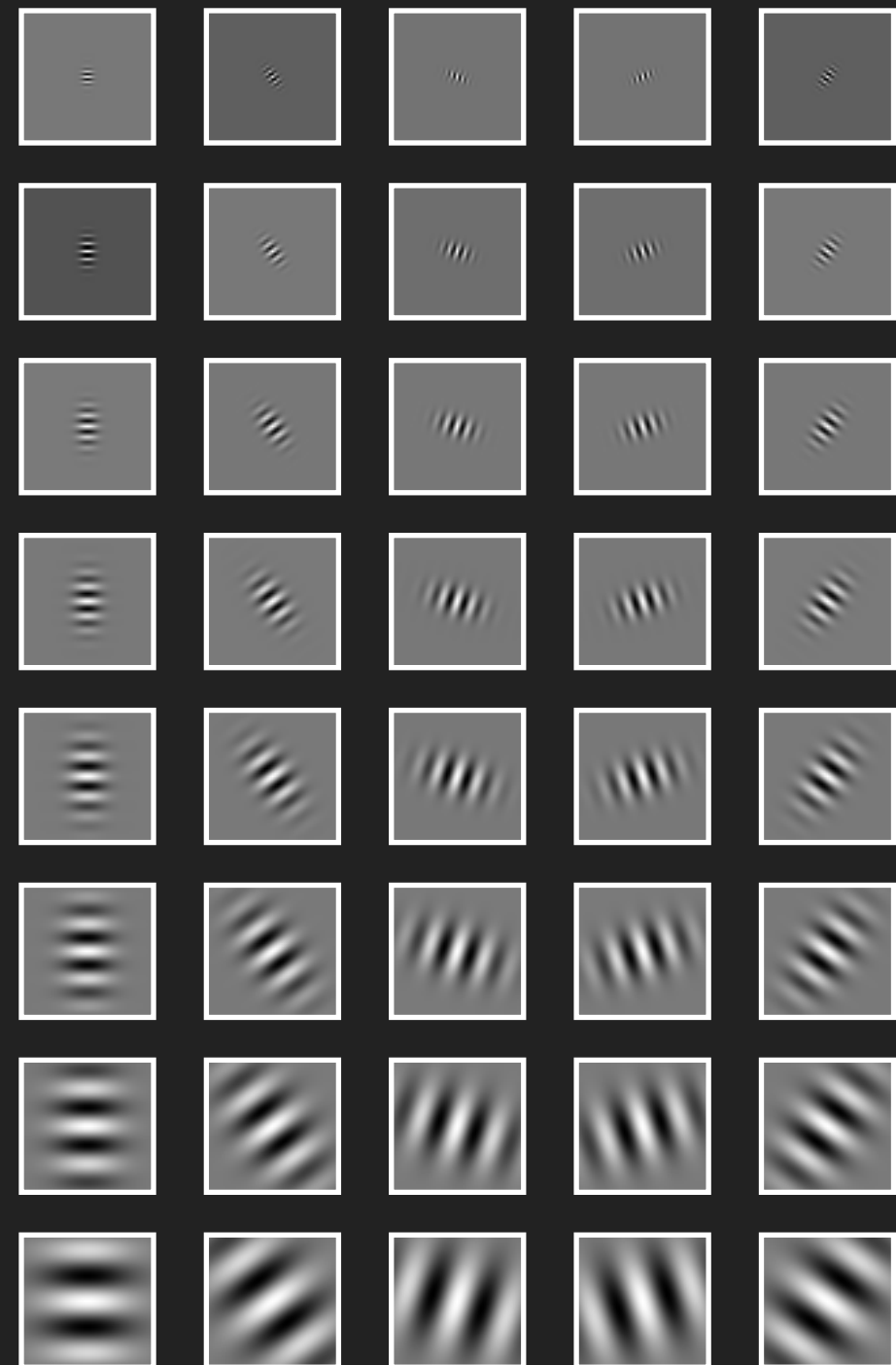


Image from *Introduction to Biometrics*, 2011.  
(adapted from *Anthropometry of the Head and Face*, 1994)

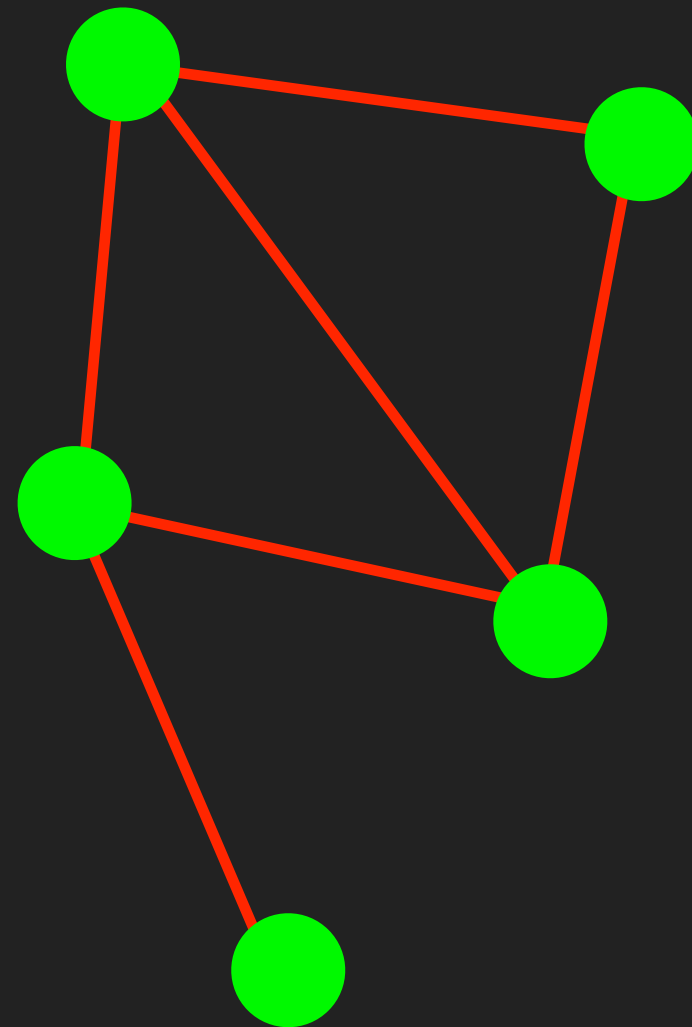
## GABOR FILTERS

- ▶ Apply a set of filters to a *small area* centered around each landmark
- ▶ Total of 40 responses for each landmark
  - ▶ 8 different scales
  - ▶ 5 different directions
- ▶ A set of filter responses is called a **jet**



## GRAPHS

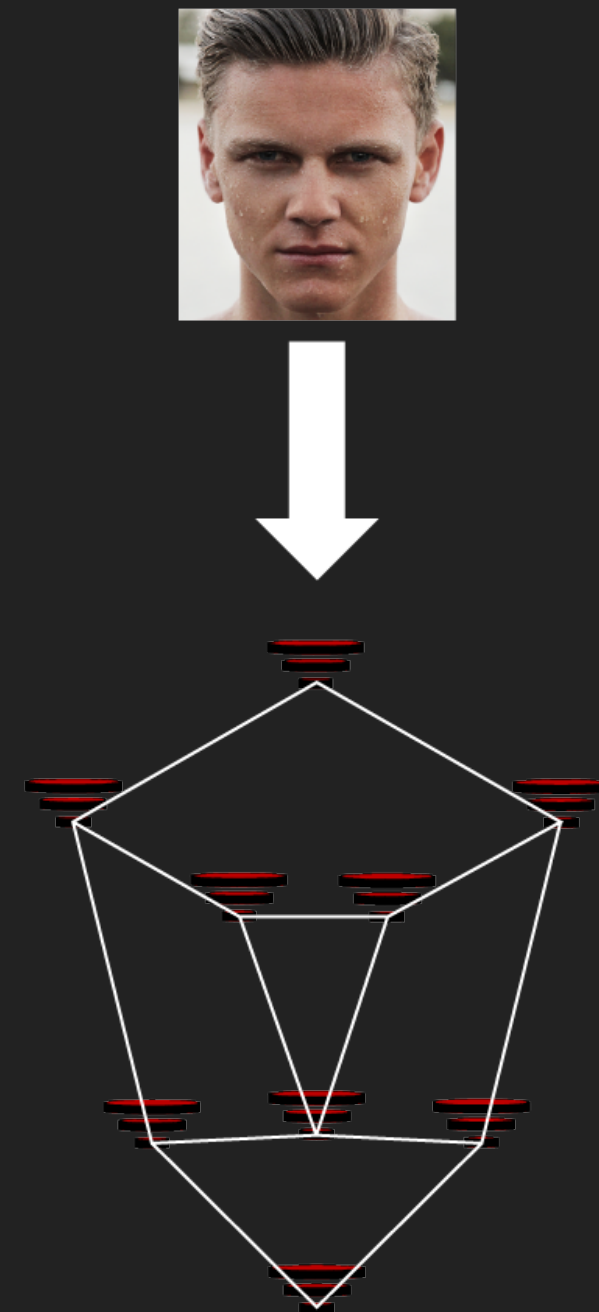
- ▶ Structure consisting of nodes and edges
- ▶ Edges relate nodes to one another
- ▶ Edges may have an associated weight that defines a relationship between 2 nodes





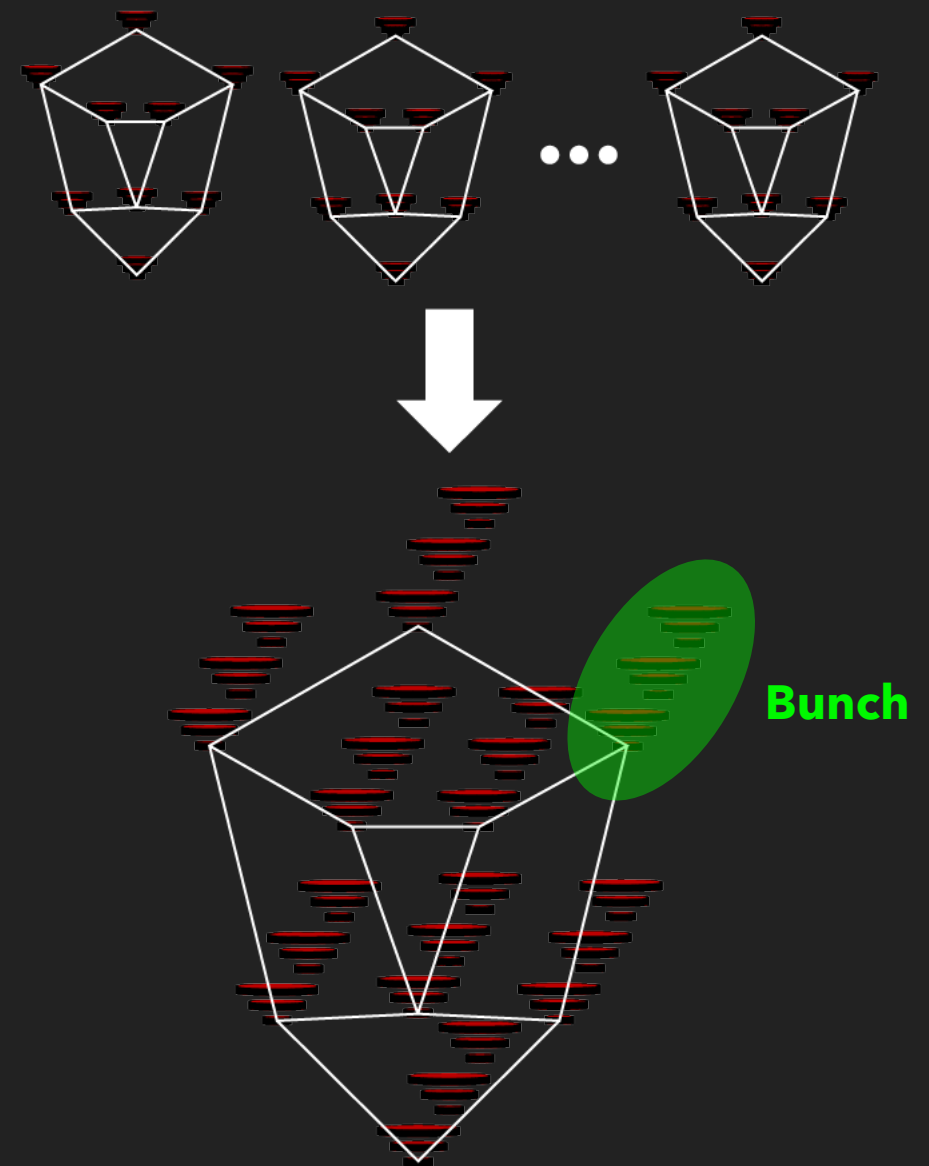
## TRAINING - 1<sup>ST</sup> STAGE

- ▶ Landmark points are identified manually for the first few images
- ▶ Landmarks for subsequent images are discovered automatically by comparing the Gabor jets
- ▶ Manual correction of mislabeled landmarks may be required
- ▶ Edge weights denote the distance between face landmarks



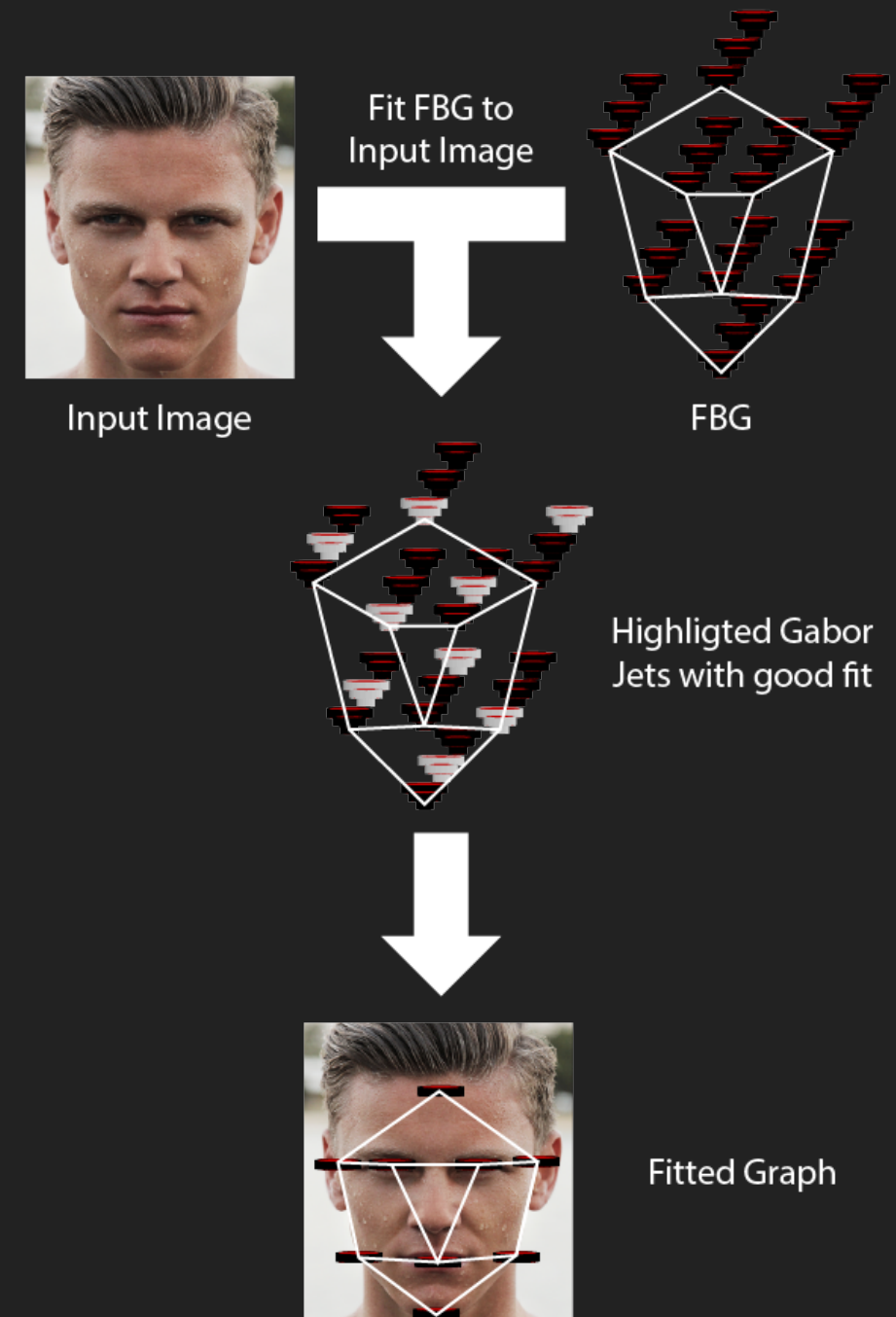
## TRAINING - 2<sup>ND</sup> STAGE

- ▶ Image graphs are combined into a stack-like structure called a Face Bunch Graph (FBG)
- ▶ The combined jets for a particular landmark is called a **bunch**
- ▶ Several FBGs may be created for different poses



## MATCHING USING FBG

- ▶ Obtain approximate face position for an image using a condensed FBG (each bunch is averaged)
- ▶ Refine face position using the full FBG
- ▶ Result is a fitted graph for an image
- ▶ Fitted graphs for two face images are compared by taking the **average similarity between the jets at each corresponding landmark**



## REVIEW

- ▶ Face Images are typically acquired using a VIS spectrum camera like a camera on your phone
- ▶ Viola-Jones uses a cascade classifier to detect faces
- ▶ PCA learns a subspace from the training data
- ▶ LDA learns a subspace similar to PCA but takes into account the class label (i.e., the subject ID)