Blockchain Principles and Applications

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Most slides have been adapted from ECE/COS 470, Pramod Viswanath, Princeton 2024 and CS251, Dan Boneh, Stanford 2023.

Recap

Bitcoin Security

- **Safety**: A transaction/block confirmed by one user is soon confirmed by all other users and remains confirmed forever after.

- **Liveness**: all (honest) transactions get included into blocks, and further that the blocks feature in the longest chain.

Protocol level attacks

√ Create valid blocks

x Mine on the tip of the longest chain

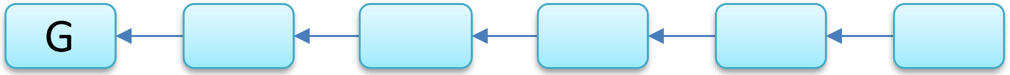
x Publish the blocks once mined

We looked at one strategy called private attack

Longest Chain Protocol

Where should the mined block hash-point to?

Latest block?



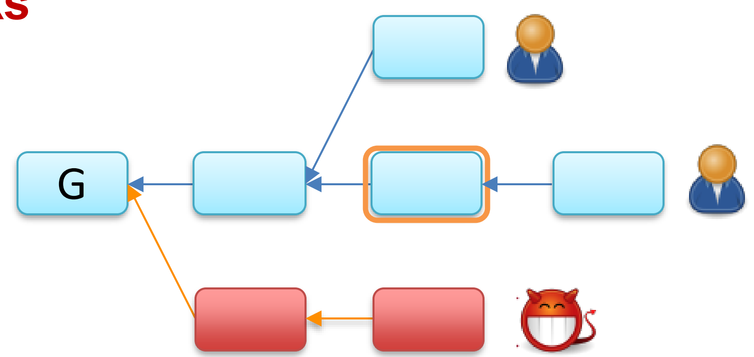
Longest Chain Protocol

Where should the mined block hash-point to?

However, blockchain may have **forks**

because of network delays

because of adversarial action



Longest Chain Protocol

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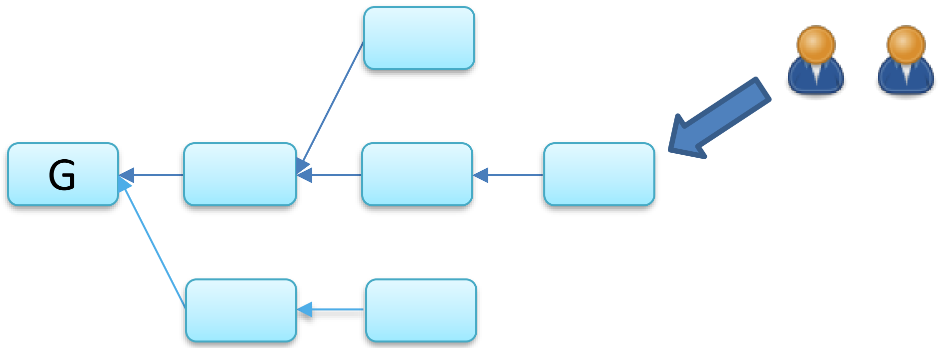
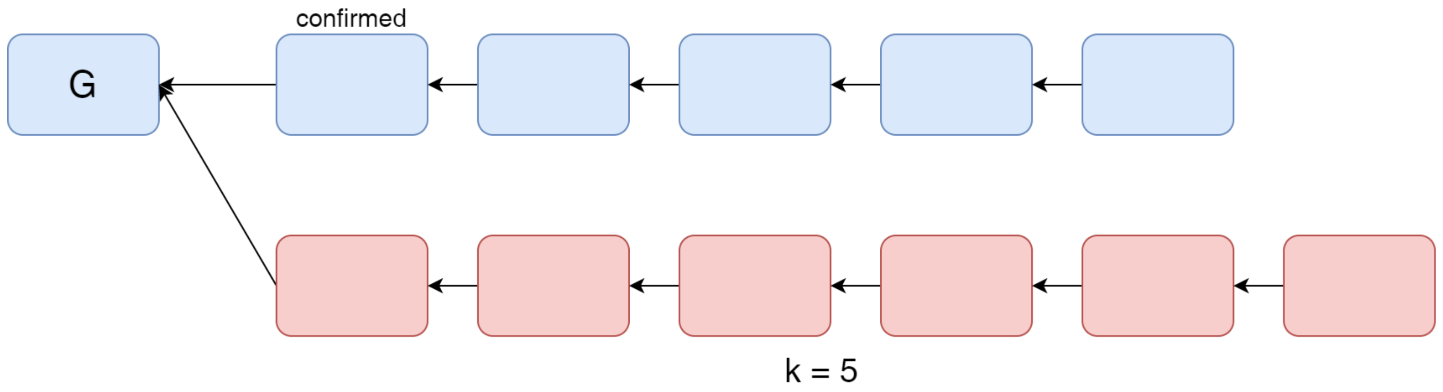


Longest chain protocol

attach the block to the leaf of the longest chain in the block tree

k Deep Confirmation Rule

- A block is **confirmed** if it is **buried k-deep in the longest chain**
- An attacker would need more than k blocks to double spend



Double Spend Attack

Adversary can point its block to an older part of the chain

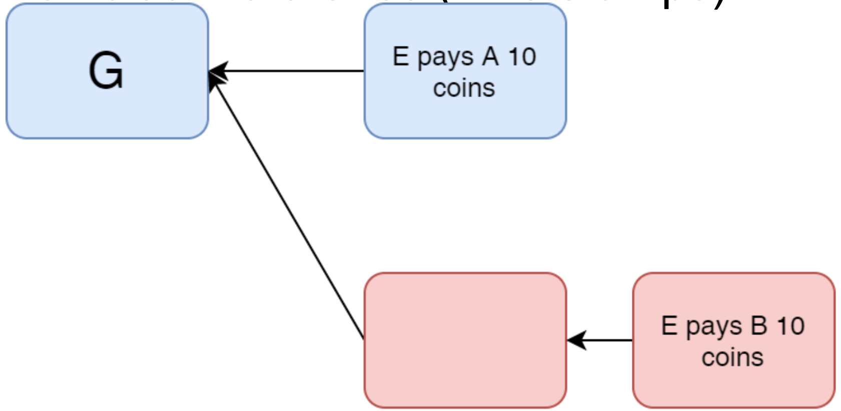
Duplicate transaction inserted

Plausible Deniability

network latency

an offline user will not know which block came earlier

blocks have no wall clock reference (time stamps).



Stochastic Process

A **stochastic process** is a mathematical model that describes a collection of random variables indexed by time (or another parameter), representing how a system evolves under uncertainty. It is widely used in probability theory, statistics, and applications like finance, physics, and machine learning.

Definition

A stochastic process is a family of random variables $\{X(t) : t \in T\}$, where:

- T is the index set (often representing time, either discrete or continuous).
- $X(t)$ is a random variable representing the system state at time t.
- The randomness means that each observation of the process may yield different outcomes.

Poisson process

A Poisson process $N(t)$ with rate (or intensity) λ satisfies the following properties:

1. **Independent increments**: The number of events occurring in disjoint time intervals are independent.

2. **Stationary increments**: The number of events occurring in any interval of length t follows a Poisson distribution with mean λt:

$$P(N(t) = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0, 1, 2, \ldots$$

3. **No simultaneous events**: The probability of more than one event occurring in an infinitesimally small interval $[t, t + \Delta t]$ is negligible:

$$P(N(t + \Delta t) - N(t) = 1) \approx \lambda \Delta t, \quad P(N(t + \Delta t) - N(t) \geq 2) \approx 0.$$

Poisson process

Interpretation of λ

- λ represents the average number of events occurring per unit time.
- The **interarrival times** between consecutive events are exponentially distributed with mean $1/\lambda$, meaning the waiting time between events follows:

$$P(T > t) = e^{-\lambda t}, \quad t \geq 0.$$

Applications

- **Network traffic**: Packet arrivals in a network often follow a Poisson process.
- **Reliability analysis**: Failures of a system might be modeled as a Poisson process.
- **Queueing theory**: Customers arriving at a service counter can be modeled by a Poisson process.
- **Finance**: Modeling the arrival of trades or price changes in high-frequency trading.

Exponential distribution

Probability density function

The probability density function (pdf) of an exponential distribution is

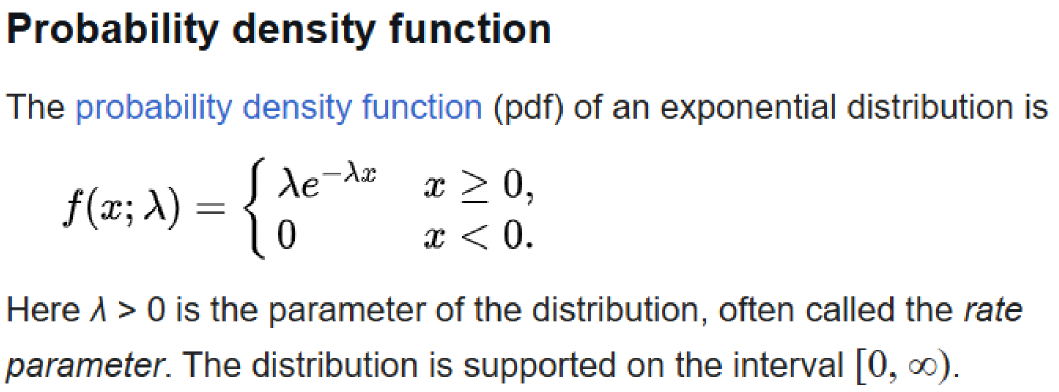
$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0, \\ 0 & x < 0. \end{cases}$$

Here $\lambda > 0$ is the parameter of the distribution, often called the *rate parameter*. The distribution is supported on the interval $[0, \infty)$. If a

The **cumulative distribution function (CDF)** of an **exponential distribution** with rate parameter $\lambda > 0$ is given by:

$$F(t) = P(T \le t) = \begin{cases} 1 - e^{-\lambda t}, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

Exponential distribution



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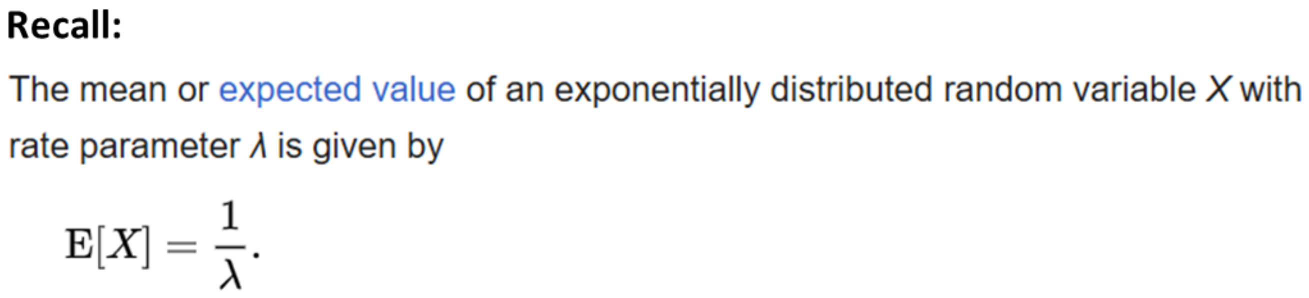
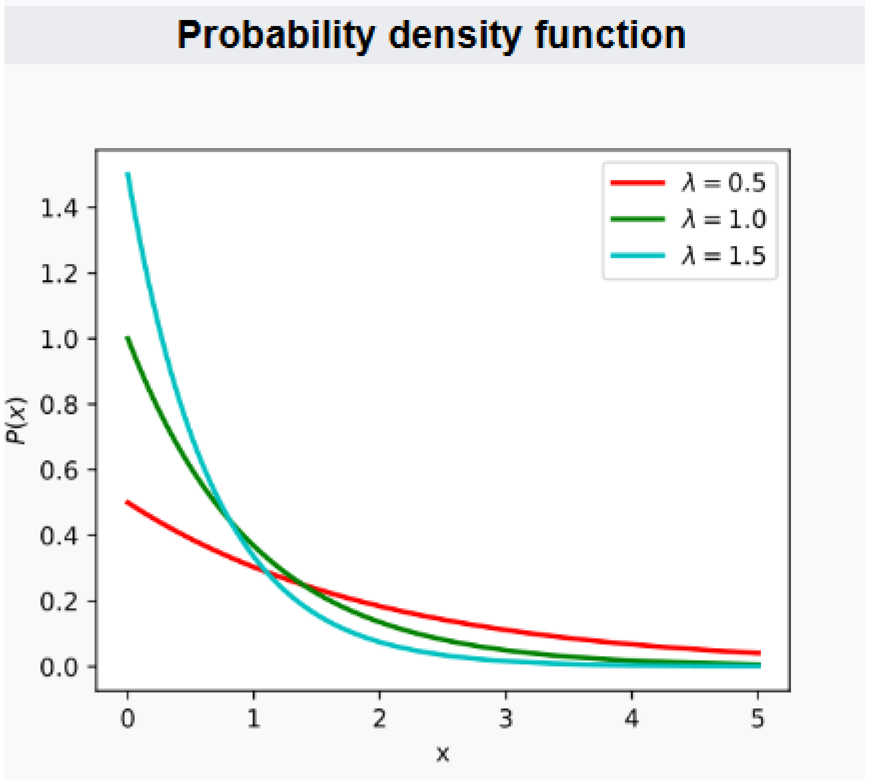


Recall:

The mean or expected value of an exponentially distributed random variable X with rate parameter λ is given by

$$\mathrm{E}[X] = \frac{1}{\lambda}.$$

Exponential distribution



Mining as a Poisson process

The times at which a new block is mined is modeled as a Poisson process with rate λ. Here the average inter-block time, $\frac{1}{\lambda}$, is set based on the target difficulty in the PoW mining operation; for Bitcoin $\frac{1}{\lambda} = 10$ minutes. A Poisson process is one in which new events (or arrivals) occur at random intervals following the exponential distribution. Moreover, the intervals between any two events are independent of, and statistically identical to, each other. Recall that an exponential random variable X with parameter λ has distribution

$$\mathbb{P}(X \geq t) = \exp(-\lambda t)\ \forall t \geq 0.$$

Memorylessness property of exponential random variable

An exponentially distributed random variable *T* obeys the relation

$$\Pr\left(T > s + t \mid T > s\right) = \Pr(T > t), \qquad \forall s, t \geq 0.$$

This can be seen by considering the complementary cumulative distribution function:

$$\begin{aligned}
\Pr\left(T > s + t \mid T > s\right) &= \frac{\Pr\left(T > s + t \cap T > s\right)}{\Pr\left(T > s\right)} \\
&= \frac{\Pr\left(T > s + t\right)}{\Pr\left(T > s\right)} \\
&= \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} \\
&= e^{-\lambda t} \\
&= \Pr(T > t).
\end{aligned}$$

Mining as a Poisson process

Also recall that a Poisson random variable Y with parameter λ has distribution

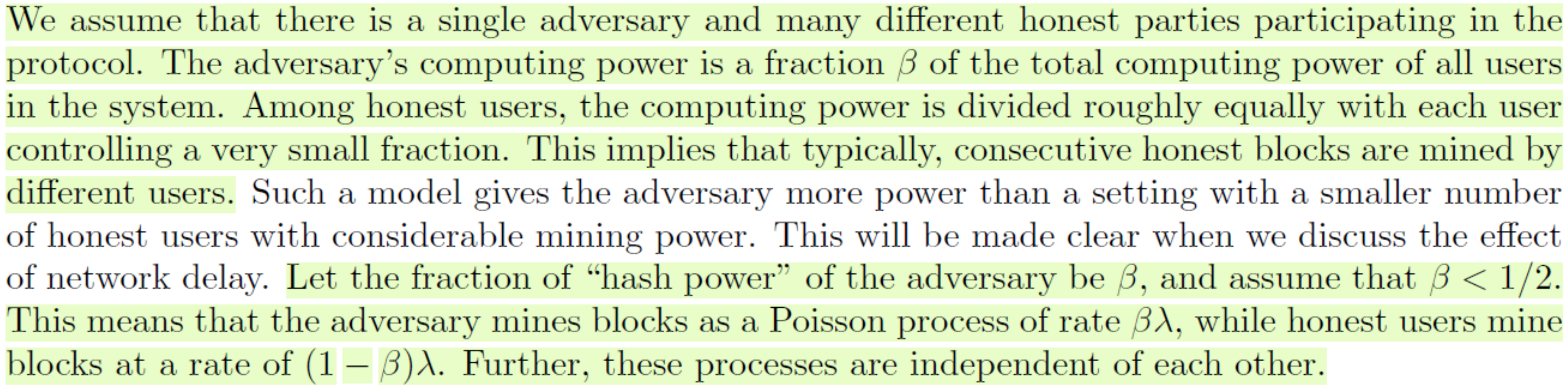
$$\mathbb{P}(Y = k) = \exp(-\lambda)\frac{\lambda^k}{k!} \ \forall k \geq 0.$$

In a Poisson process, the number of events in an interval of length T is a Poisson random variable with parameter λT. Moreover, the number of events in disjoint intervals of time are independent. If we consider small intervals ($\lambda T \ll 1$), there is one event in the interval with probability λT and none otherwise. Thus, a Poisson process can be emulated by counting the occurrence of heads in a sequence of (independent) coin tosses, with the probability of heads being very small. We now see why the mining process has such a property.

Mining as a Poisson process

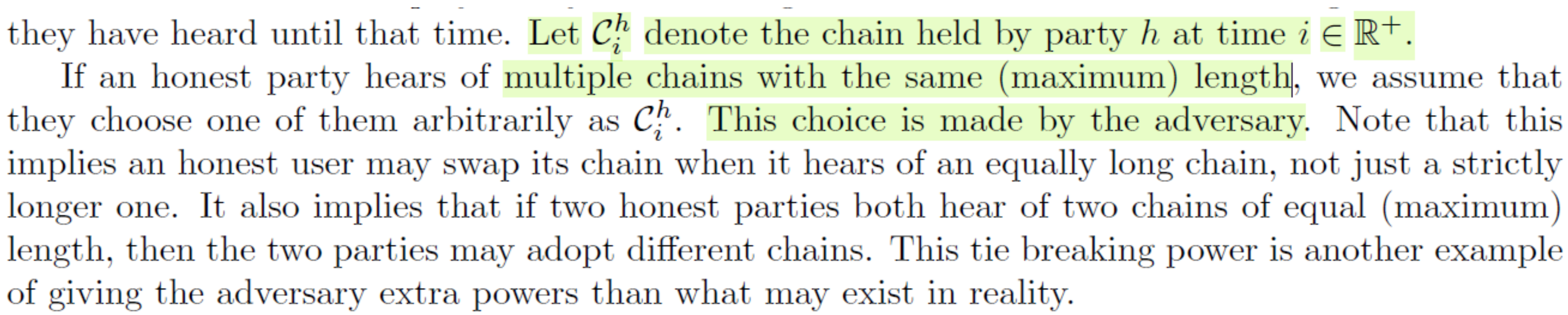
In modeling the mining process as a Poisson process, we focus only at the times at which new, valid blocks are created. The Poisson process model holds irrespective of the number of miners, their individual computation power, whether different miners are working on the same block or different blocks, and when different users receive newly mined blocks. The parameter λ of the mining process, called the **mining rate**, is equal to the average number of blocks mined per unit time. In Bitcoin, λ is 1/(600s), i.e., one block every 600 seconds (ten minutes). In Ethereum (which also has the same Nakamoto consensus protocol), the rate is much faster: λ is 1/(13s).

Mining as a Poisson process



We assume that there is a single adversary and many different honest parties participating in the protocol. The adversary's computing power is a fraction β of the total computing power of all users in the system. Among honest users, the computing power is divided roughly equally with each user controlling a very small fraction. This implies that typically, consecutive honest blocks are mined by different users. Such a model gives the adversary more power than a setting with a smaller number of honest users with considerable mining power. This will be made clear when we discuss the effect of network delay. Let the fraction of "hash power" of the adversary be β, and assume that $\beta < 1/2$. This means that the adversary mines blocks as a Poisson process of rate $\beta\lambda$, while honest users mine blocks at a rate of $(1 - \beta)\lambda$. Further, these processes are independent of each other.

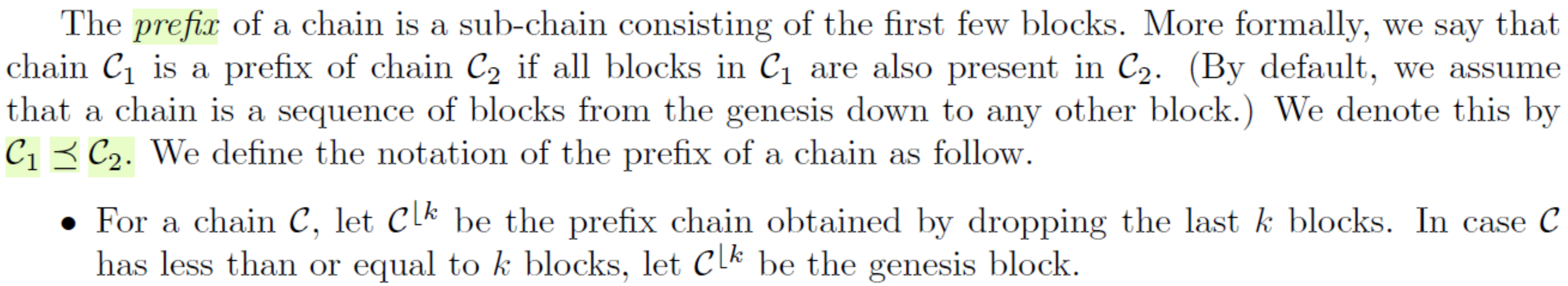
Nakamoto consensus protocol model



they have heard until that time. Let $\mathcal{C}_i^h$ denote the chain held by party h at time $i \in \mathbb{R}^+$.

If an honest party hears of multiple chains with the same (maximum) length, we assume that they choose one of them arbitrarily as $\mathcal{C}_i^h$. This choice is made by the adversary. Note that this implies an honest user may swap its chain when it hears of an equally long chain, not just a strictly longer one. It also implies that if two honest parties both hear of two chains of equal (maximum) length, then the two parties may adopt different chains. This tie breaking power is another example of giving the adversary extra powers than what may exist in reality.

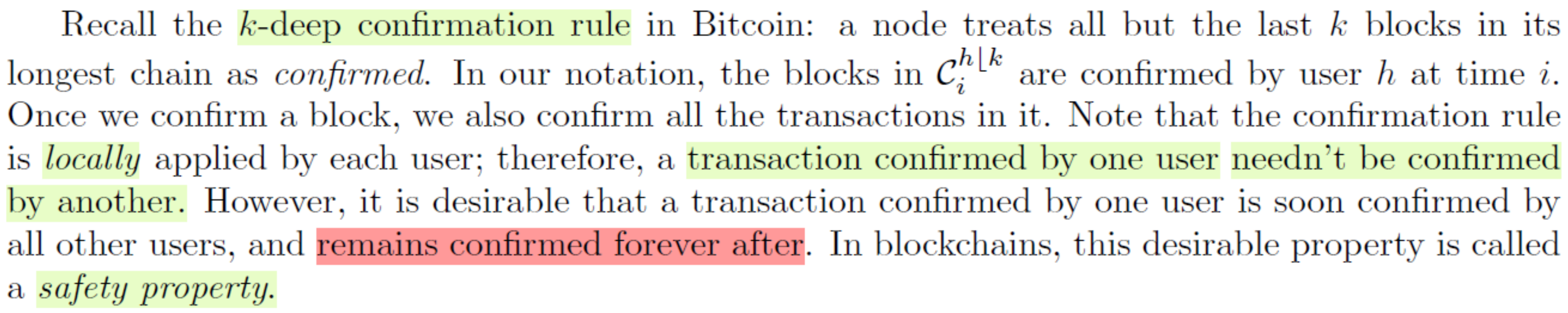
Nakamoto consensus protocol model



The *prefix* of a chain is a sub-chain consisting of the first few blocks. More formally, we say that chain $\mathcal{C}_1$ is a prefix of chain $\mathcal{C}_2$ if all blocks in $\mathcal{C}_1$ are also present in $\mathcal{C}_2$. (By default, we assume that a chain is a sequence of blocks from the genesis down to any other block.) We denote this by $\mathcal{C}_1 \preceq \mathcal{C}_2$. We define the notation of the prefix of a chain as follow.

- For a chain $\mathcal{C}$, let $\mathcal{C}^{\lfloor k}$ be the prefix chain obtained by dropping the last k blocks. In case $\mathcal{C}$ has less than or equal to k blocks, let $\mathcal{C}^{\lfloor k}$ be the genesis block.

Nakamoto consensus protocol model



Recall the *k*-deep confirmation rule in Bitcoin: a node treats all but the last k blocks in its longest chain as *confirmed*. In our notation, the blocks in $\mathcal{C}_i^{h\lfloor k}$ are confirmed by user h at time i. Once we confirm a block, we also confirm all the transactions in it. Note that the confirmation rule is *locally* applied by each user; therefore, a transaction confirmed by one user needn't be confirmed by another. However, it is desirable that a transaction confirmed by one user is soon confirmed by all other users, and remains confirmed forever after. In blockchains, this desirable property is called a *safety property.*

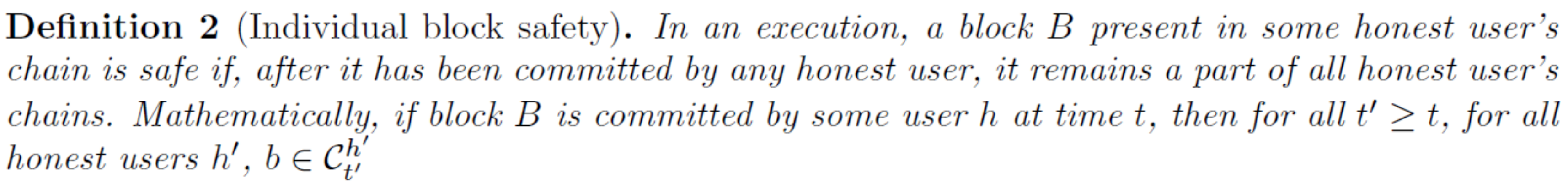
Formal definitions of safety

Definition 1 (Common Prefix Property). *For a blockchain protocol, the k-common prefix property holds during an execution of the protocol if any block that is committed by one honest user appears in every honest user's chain thereafter. Mathematically, for all pairs of times $i_1 \leq i_2$, for all pairs of honest users h_1, h_2,*

$$\mathcal{C}_1^{\lfloor k} \preceq \mathcal{C}_2$$

where $\mathcal{C}_1 \equiv \mathcal{C}_{i_1}^{h_1}$, $\mathcal{C}_2 \equiv \mathcal{C}_{i_2}^{h_2}$.

Formal definitions of safety



Definition 2 (Individual block safety). *In an execution, a block B present in some honest user's chain is safe if, after it has been committed by any honest user, it remains a part of all honest user's chains. Mathematically, if block B is committed by some user h at time t, then for all $t' \geq t$, for all honest users h', $b \in \mathcal{C}_{t'}^{h'}$*

Mining as a Poisson process

Time to a successful mining event is an **exponential** random variable

$$T \sim exp(\lambda) \;\; if \;\; \Pr(T \geq t) = e^{-\lambda t}$$

Memoryless:

$$\Pr(T \geq t + t_0 | T \geq t_0) = \frac{\Pr(T \geq t + t_0)}{\Pr(T \geq t_0)} = \frac{e^{-\lambda(t+t_0)}}{e^{-\lambda t_0}} = e^{-\lambda t} = \Pr(T \geq t)$$

Number of mined blocks in time T is a **Poisson** random variable

$$X \sim Poi(\lambda T) \;\; if \;\; \Pr(X = k) = \frac{(\lambda T)^k e^{-\lambda T}}{k!}$$

The mining process is a Poisson process with rate λ, proportional to hash power

Mining as a Poisson Process

Mathematical fact: The sum of multiple independent Poisson processes is still a Poisson process

Consequence: the honest/adversarial mining processes are independent Poisson processes with constant mining rate

Honest mining: Poisson process with rate $(1-\beta)\lambda$

Adversarial mining: Poisson process with rate $\beta\lambda$

Sum of Two Independent Poisson Processes

Let:

- $N_1(t)$ be a Poisson process with rate λ_1.
- $N_2(t)$ be a Poisson process with rate λ_2.

If $N_1(t)$ and $N_2(t)$ are independent, then their sum:

$$N(t) = N_1(t) + N_2(t)$$

is also a **Poisson process** with rate:

$$\lambda = \lambda_1 + \lambda_2.$$

Private attack

Consider a simple model of the blockchain system with all users split into two groups: many honest users and a singe adversarial user. The adversarial user is trying to re-write the last k blocks in the ledger by performing a private attack. Assume that the total mining rate is fixed at λ. We suppose that the adversarial fraction of hash power is β (for "bad"); the honest fraction of hash power is $1 - \beta$.

Suppose the adversary wishes to violate the safety of a block B. To do so, the adversary must ensure that block B is first confirmed by some (or all) honest users, and then, at some time in the future, must dislodge B from the longest chain, i.e., it must create a fork from a block preceding B, and must eventually produce a chain of length equal to or longer than the longest chain containing B, after B has been confirmed.

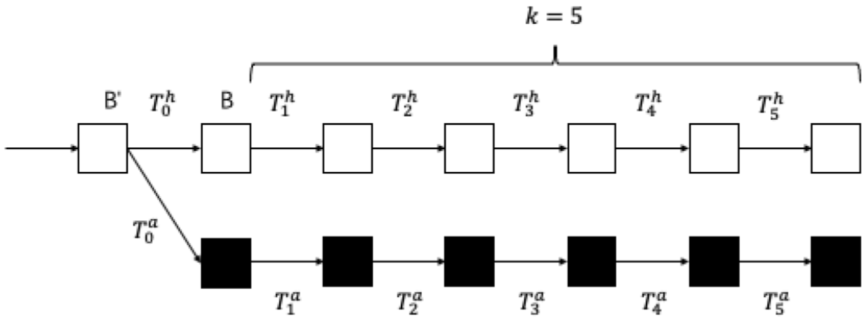
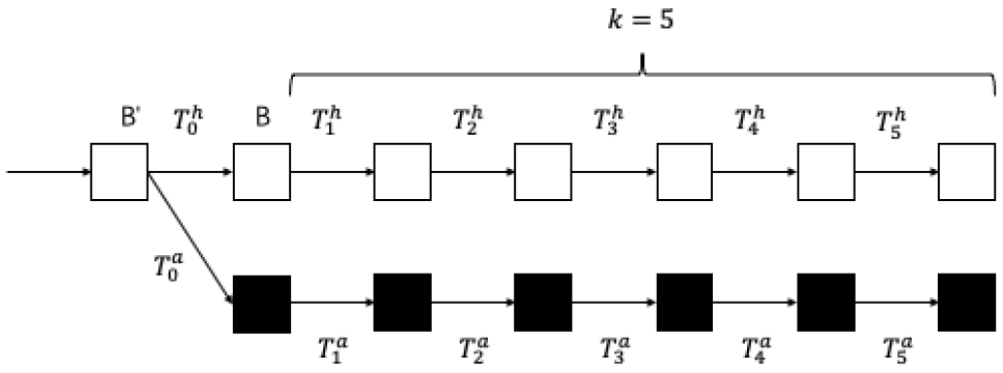


Figure 1: Private attack on block B with $k = 5$.

Private attack

One possible attack is the private attack, introduced in Lecture 3, which we investigate here in more detail. Let B' be the parent of block B. One option is for the adversary to mine a conflicting block on B' immediately after block B' is mined. It keeps mining in private, creating an ever-increasing chain. The honest users, unaware of the private chain, continue to mine following the longest chain rule, below B. In the private attack, the adversary does not contribute to the chain the honest nodes are mining on. Note that the honest and adversarial chains are independent of each other and distributed as Poisson processes with rate $(1-\beta)\lambda$ and $\beta\lambda$, respectively.



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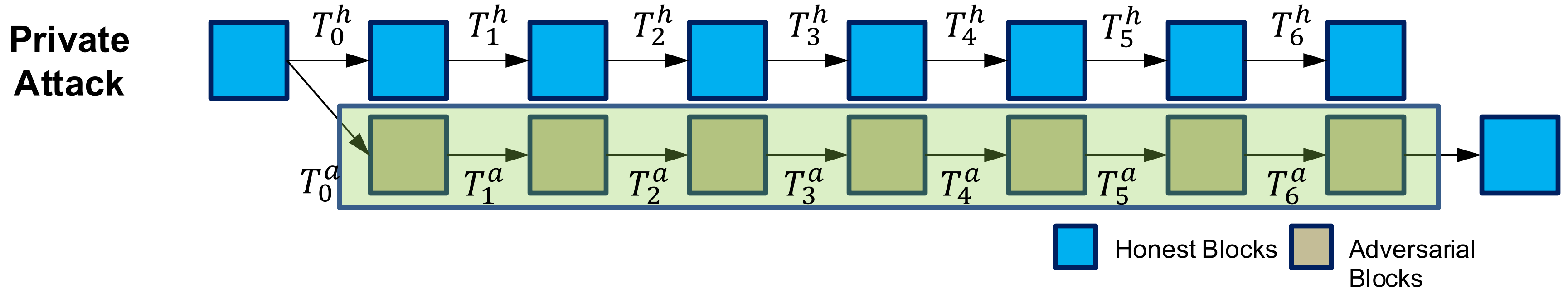
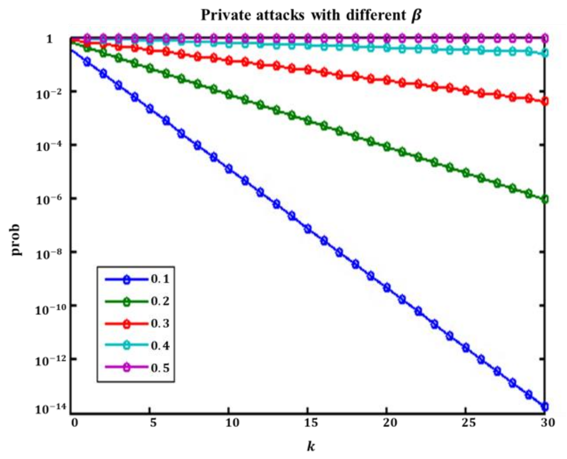
Private attack



Attack Success

$k \to \infty$: $\sum_{i=0}^{\infty} T_i^h > \sum_{i=0}^{\infty} T_i^a \to \beta\lambda > (1-\beta)\lambda \to \beta > \frac{1}{2}$

$k < \infty$: $p_a = e^{-c(k+1)}$



Private attack ($k \to \infty$)

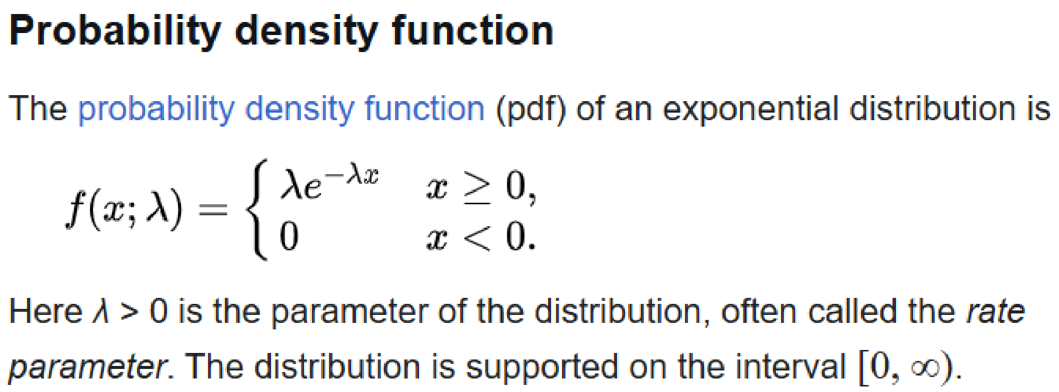
Mathematically, the attack is successful if:

$$\sum_{i=0}^{k} T_i^h > \sum_{i=0}^{k} T_i^a$$

where:

- T_i^h are the times taken by honest miners to find their blocks.
- T_i^a are the times taken by the adversary to find their blocks.

Private attack ($k \to \infty$)



Probability density function

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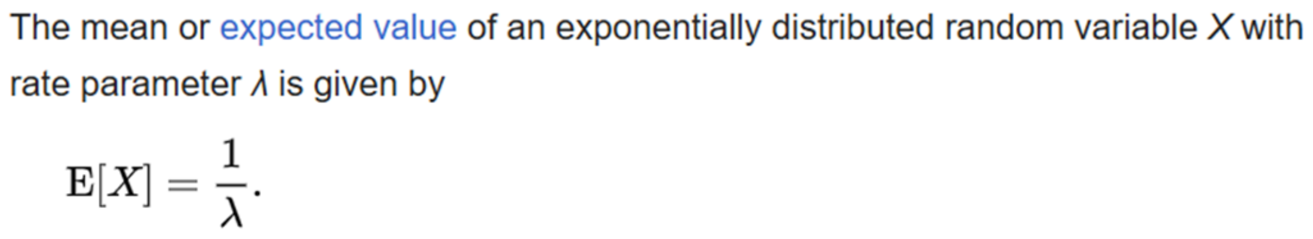
Recall:



The mean or expected value of an exponentially distributed random variable X with rate parameter λ is given by

$$\mathrm{E}[X] = \frac{1}{\lambda}.$$

Private attack ($k \rightarrow \infty$)

- Block mining times are modeled as *exponential random variables*:
  - Honest miners find a block in an *exponential* time distribution with mean $\frac{1}{(1-\beta)\lambda}$, where β is the fraction of total hash power controlled by the adversary.
  - The adversary finds a block in an *exponential* time distribution with mean $\frac{1}{\beta\lambda}$.

Private attack ($k \to \infty$)

By the *Strong Law of Large Numbers (LLN)*, the sample mean of i.i.d. random variables converges almost surely to the expected value:

$$\frac{1}{k+1}\sum_{i=0}^{k} T_i^h \to \mathbb{E}[T_i^h] = \frac{1}{(1-\beta)\lambda} \quad \text{as } k \to \infty.$$

$$\frac{1}{k+1}\sum_{i=0}^{k} T_i^a \to \mathbb{E}[T_i^a] = \frac{1}{\beta\lambda} \quad \text{as } k \to \infty.$$

Thus, taking the difference:

$$\frac{1}{k+1}\sum_{i=0}^{k} (T_i^h - T_i^a) \to \frac{1}{(1-\beta)\lambda} - \frac{1}{\beta\lambda}$$

Private attack ($k \to \infty$)

- The attack is successful if this limit is **positive**, meaning that the adversary is mining faster than the honest miners on average.
- This translates to the condition:

$$\frac{1}{(1-\beta)\lambda} > \frac{1}{\beta\lambda}$$

- Simplifying:

$$(1-\beta)^{-1} > \beta^{-1}$$

$$\beta > \frac{1}{2}$$

Private attack (k>0 and finite)

We define the probability of deconfirmation as:

$$P\left(\sum_{i=0}^{k}(T_i^h - T_i^a) > 0\right)$$

This means the sum of the mining times for honest miners is greater than that of the adversary, implying that the adversary successfully replaces the longest chain.

Private attack (k>0 and finite)

Instead of directly working with the probability, we use an exponential bound:

$$P\left(\sum_{i=0}^{k}(T_i^h - T_i^a) > 0\right) = P\left(e^{s\sum_{i=0}^{k}(T_i^h - T_i^a)} > 1\right).$$

By **Markov's inequality**, for any positive function X:

$$P(X > 1) \leq \mathbb{E}[X].$$

Applying this to our case:

$$P\left(e^{s\sum_{i=0}^{k}(T_i^h - T_i^a)} > 1\right) \leq \mathbb{E}\left[e^{s\sum_{i=0}^{k}(T_i^h - T_i^a)}\right].$$

Thus,

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Private attack (k>0 and finite)

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Recall:

Statement of Markov's Inequality

Let X be a non-negative random variable (i.e., $X \geq 0$) with **expected value** $E[X]$. Then, for any $a > 0$:

$$P(X \geq a) \leq \frac{E[X]}{a}.$$

Thus,

$$P\left(\sum_{i=0}^{k}(T_i^h - T_i^a) > 0\right) \leq \mathbb{E}\left[e^{s\sum_{i=0}^{k}(T_i^h - T_i^a)}\right].$$

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Thus,

$$P\left(\sum_{i=0}^{k}(T_i^h - T_i^a) > 0\right) \leq \mathbb{E}\left[e^{s\sum_{i=0}^{k}(T_i^h - T_i^a)}\right].$$

Private attack (k>0 and finite)

Since the mining times T_i^h and T_i^a are independent **exponential random variables**, we factorize the expectation:

$$\mathbb{E}\left[e^{s\sum_{i=0}^{k}(T_i^h - T_i^a)}\right] = \mathbb{E}\left[\prod_{i=0}^{k} e^{s(T_i^h - T_i^a)}\right].$$

Using the **independence property**:

$$= \prod_{i=0}^{k} \mathbb{E}\left[e^{s(T_0^h - T_0^a)}\right].$$

Thus, the bound simplifies to:

$$P\left(\sum_{i=0}^{k}(T_i^h - T_i^a) > 0\right) \leq \left(\mathbb{E}\left[e^{s(T_0^h - T_0^a)}\right]\right)^{k+1}.$$

Private attack (k>0 and finite)

For an **exponential random variable** $X \sim \text{Exp}(\lambda)$, the moment generating function is:

$$\mathbb{E}[e^{sX}] = \frac{\lambda}{\lambda - s}, \quad \text{for } s < \lambda.$$

Applying this for $T_0^h \sim \text{Exp}((1-\beta)\lambda)$ and $T_0^a \sim \text{Exp}(\beta\lambda)$:

$$\mathbb{E}\left[e^{s(T_0^h - T_0^a)}\right] = \frac{(1-\beta)\lambda}{(1-\beta)\lambda - s} \times \frac{\beta\lambda - s}{\beta\lambda}.$$

Simplifying:

$$\mathbb{E}\left[e^{s(T_0^h - T_0^a)}\right] = \frac{\beta\lambda}{\beta\lambda + s} \times \frac{(1-\beta)\lambda + s}{(1-\beta)\lambda}.$$

$$= \left(\frac{\beta\lambda}{\beta\lambda + s} \times \frac{(1-\beta)\lambda + s}{(1-\beta)\lambda}\right)^{k+1}.$$

Private attack (k>0 and finite)

To minimize the probability bound, we choose the optimal s. The best choice is:

$$s = \frac{(1 - 2\beta)\lambda}{2}.$$

Substituting this minimizes the exponent, giving the bound:

$$P\left(\sum_{i=0}^{k}(T_i^h - T_i^a) > 0\right) \leq e^{-c(k+1)},$$

where:

$$c = -\log_e\left(4\beta(1-\beta)\right) > 0.$$

Private attack (k>0 and finite)

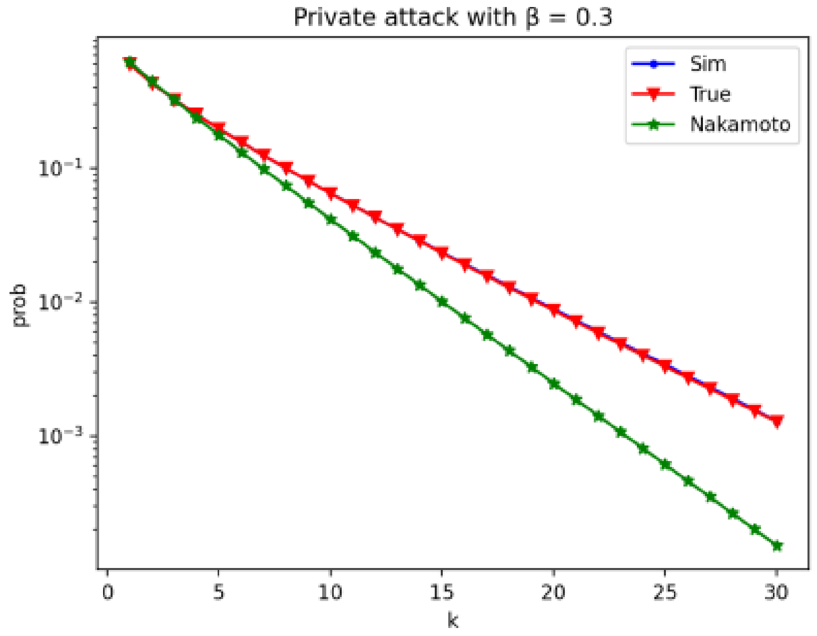
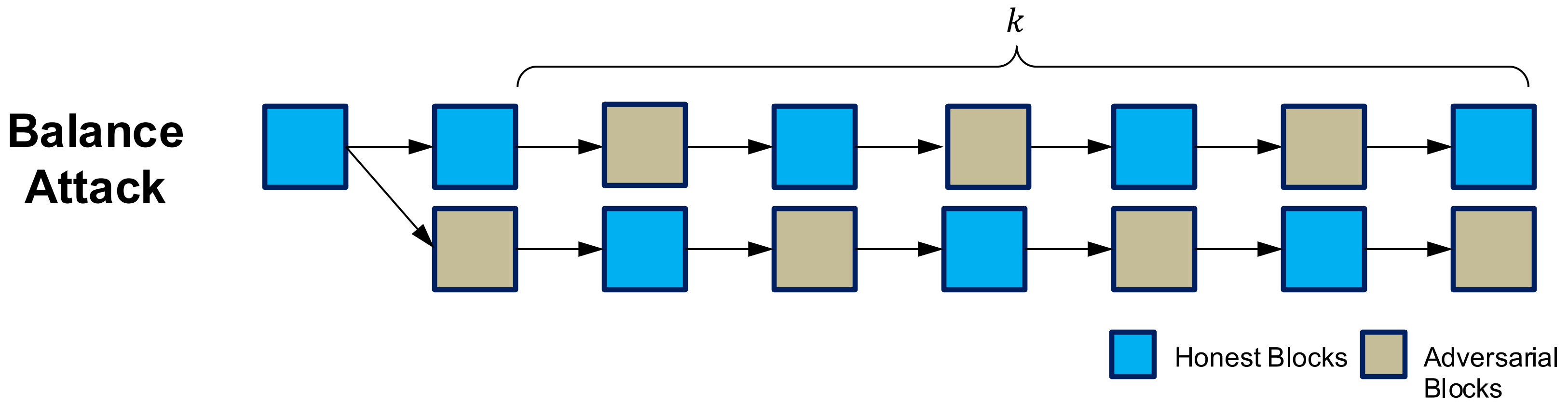


Figure 2: Private attack with $\beta = 0.3$.

Private attack is the worst-case attack

- Private attacks constitute a particularly serious threat.
  - However, they are not the only possible attack on the safety of the blockchain.

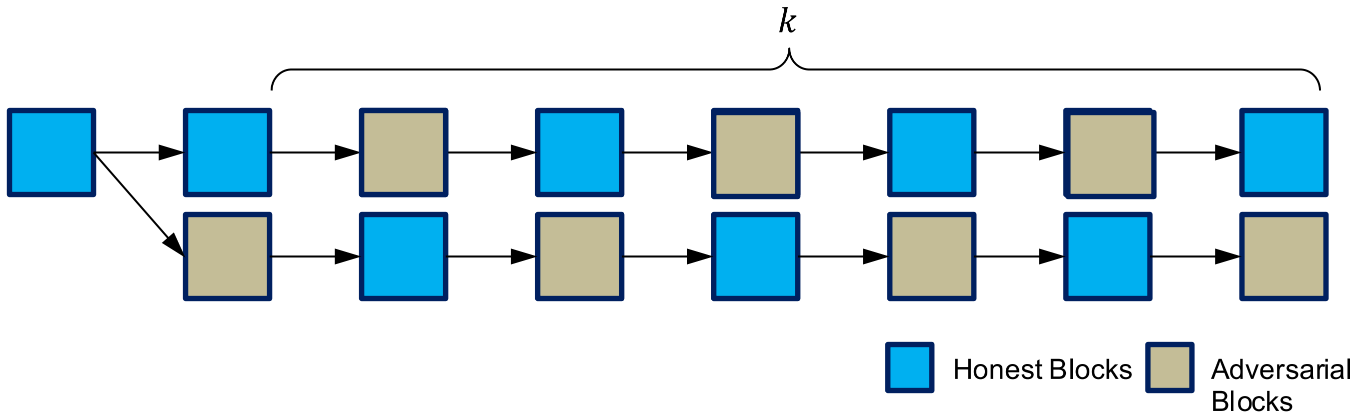
Balance attack



Balance attack

- An attacker could
  - Launch a balance attack to split the honest mining power among the two chains
    - Thus reducing the rate of growth of each one
  - Then launching a fatal private attack

Private attack is the worst-case attack



A_k = # adv blocks, H_k = # of honest blocks

$A_k + H_k \geq 2k + 2$

$A_k \geq H_k$

$\Longrightarrow$ $A_k \geq max(k, H_k) + 1$ $\Longrightarrow$ **Number of adversarial blocks is enough to launch a private attack**

Private attack is the worst-case attack

- In the special case when the network delay is zero,
  - It turns out that whenever any attack on safety is successful, the private attack is also successful.

Safety analysis under bounded network delay

- Block propagation delay in the Internet is of the order of a few seconds on average.

- **Natural Forking**
  - Honest miners may occasionally **fork** naturally due to network delay — they may not see each other’s blocks in time.

- **Honest Forking (Top row)**
  - In this model:
    - Network delay causes multiple honest miners to unknowingly mine on different tips.
    - This creates forks among honest blocks.
    - This reduces the **effective chain growth rate** of honest miners, because only one of the forked chains can eventually be accepted.

Safety analysis under bounded network delay

No Honest Forking (Middle row)

If there's no delay or perfect coordination, honest miners **do not fork**.

- The honest chain grows at a rate of $(1-\beta)\lambda$.

Reduced Honest Chain Growth

In the presence of forking:

- Effective chain growth of honest miners is reduced to:

$$\frac{(1-\beta)\lambda}{1+(1-\beta)\lambda\Delta}$$

This reflects that the more frequently honest miners fork (due to Δ), the **slower** their longest chain grows.

Safety analysis under bounded network delay

Expected Number of Blocks Mined per Growth

Let's walk through what happens **between two honest chain growth events**.

1. Suppose a block B is mined at time t, and it eventually extends the chain.
2. During the next Δ seconds, miners don't know about B, so they mine more blocks.
   - These are **"wasted"** blocks that do **not** help grow the chain.
   - On average, number of these blocks is:

$$(1-\beta)\lambda\Delta$$

3. Then comes **1 block** (after the delay) that successfully builds on top of B and grows the chain.

Safety analysis under bounded network delay

Total Blocks per Chain Growth

So, in each "cycle" (from one real growth to the next), there are:

- $(1-\beta)\lambda\Delta$ **wasted** blocks (forks), and
- **1 useful block** that grows the chain.

So, **total number of blocks** mined in this cycle is:

$$1 + (1-\beta)\lambda\Delta$$

Safety analysis under bounded network delay

Now the Probability

So, what's the **chance** that **a randomly picked honest block** is the one that actually grew the chain?

Only **1 out of** $1 + (1 - \beta)\lambda\Delta$ blocks grows the chain.

Therefore:

$$\boxed{\text{Probability that a block grows the chain} = \frac{1}{1 + (1 - \beta)\lambda\Delta}}$$

Safety analysis under bounded network delay

In each round like this:

- You get $(1-\beta)\lambda\Delta$ **"wasted" blocks** (forks at level ℓ)
- Then, finally **1 good block** gets added at level $\ell + 1$

So, you need:

$$1 + (1-\beta)\lambda\Delta \text{ blocks}$$

(on average) to **grow the chain by 1 level**.

This gives us the **probability** that any block grows the chain:

$$\frac{1}{1 + (1-\beta)\lambda\Delta}$$

And then the **honest chain growth rate** becomes:

$$(1-\beta)\lambda \times \frac{1}{1 + (1-\beta)\lambda\Delta} = \frac{(1-\beta)\lambda}{1 + (1-\beta)\lambda\Delta}$$

Safety analysis under bounded network delay

Attack Success Condition

For the private attack to succeed, the adversary's chain must grow faster than the *effective* honest chain.

So the success condition becomes:

$$\beta\lambda > \frac{(1-\beta)\lambda}{1+(1-\beta)\lambda\Delta}$$

You can interpret this as:

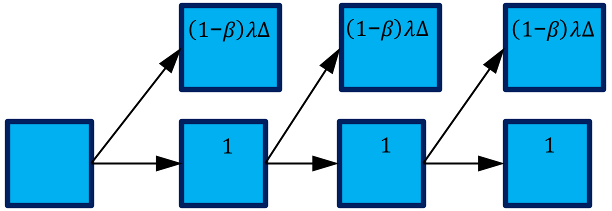
> The adversary wins if their block creation rate is higher than the effective growth rate of the honest chain (which is slowed down by forking and delay).

Private Attack (With Honest Forking)

Δ - synchronous network model: network delays bounded by Δ

Natural forking:

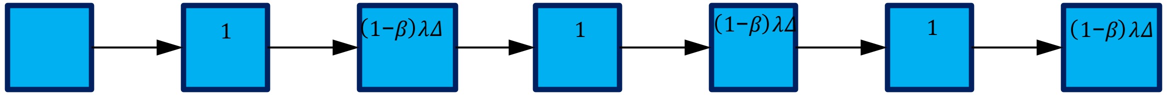
Honest forking:



Reduced honest chain growing rate

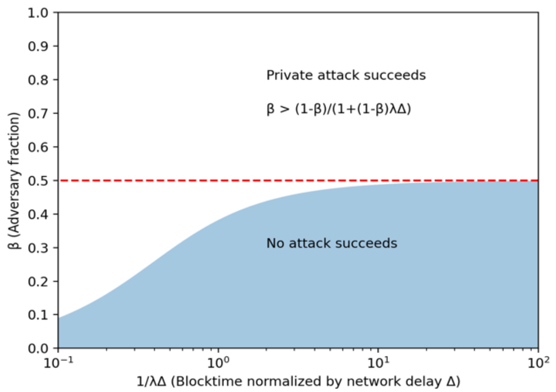
$$\Longrightarrow \frac{(1-\beta)\lambda}{1+(1-\beta)\lambda\Delta}$$

No honest forking:



Attack Success

$$\xrightarrow{\Delta \text{ network delay}} \beta\lambda > \frac{(1-\beta)\lambda}{1+(1-\beta)\lambda\Delta} \Longrightarrow$$



Summary

- Model Bitcoin mining as Poisson processes
- Analysis against the private attack
- Safety analysis beyond the private attack – all possible protocol attacks

Resources

- ECE/COS 470, Pramod Viswanath, Princeton 2024
- CS251, Dan Boneh, Stanford 2023