Day. M T W T F S

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Date: __/__/__

NAME:
Rimsha Imran

Class:
BSCS - 1D

Roll NO:
P20-0046

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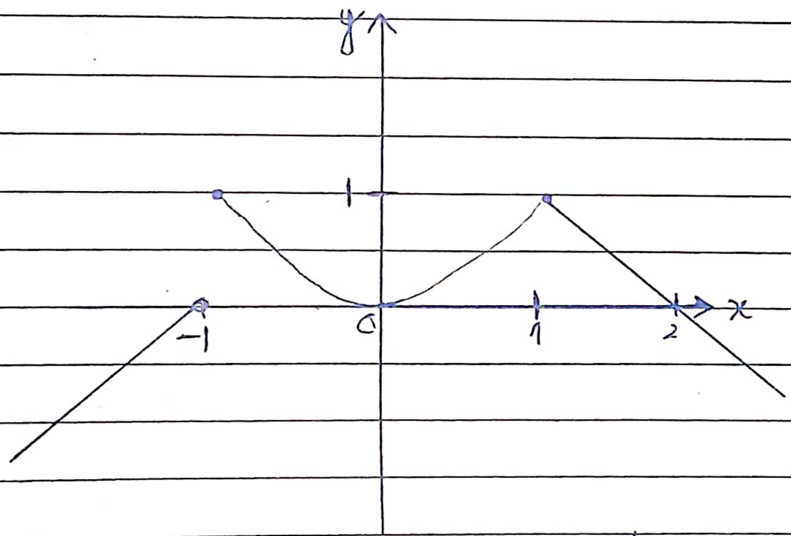
Q1 Sketch the graph of the function and use it to determine the value of a for which $\lim_{x \to a} f(x)$ exist.

$$f(x) = \begin{cases} 1+x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ 2-x & \text{if } x > 1 \end{cases}$$

Sol:



From the graph of $f(x)$ we see that $\lim_{x \to a} f(x)$ exist for all a except $a = -1$. Notice that the right and left limits are different at $a = -1$

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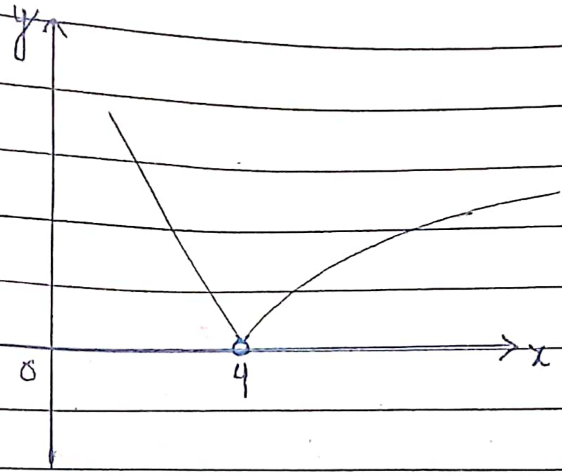
ii $f(x) = \begin{cases} \sqrt{x+4} & \text{if } x>4 \\ \sqrt{8-2x} & \text{if } x<4 \end{cases}$



(I) $f(4) = 0$

(II) Since $f(x) = \lim \sqrt{x-4}$ for $x>4$, we have

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4} \sqrt{x-4}$$

$$= \sqrt{4-4}$$

$$= 0$$

Since $f(x) = 8-2x$ for $x<4$, we have

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (8-2x)$$

$$= 8-2(4)$$

$$= 8-8$$

$$= 0$$

The right and left hand limits are equal. Thus the limit exist at $x=4$ and

$$\lim_{x\to 4} f(x) = 0$$

(III)

So the fun $f(x)$ is continous at $x=4$.

③

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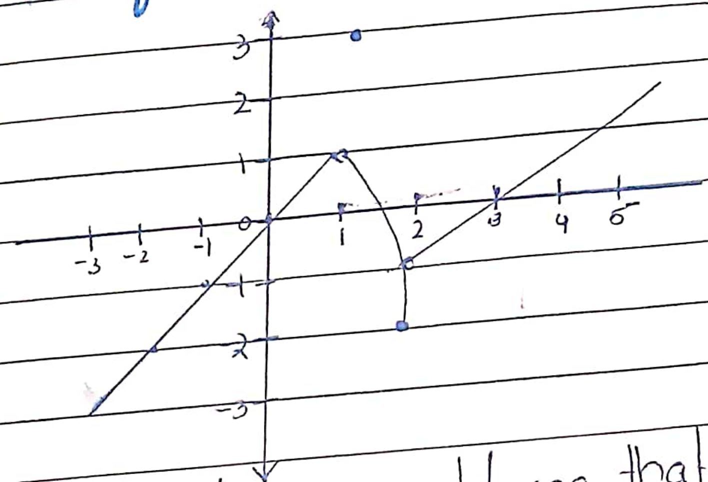
iii $\begin{cases} x & \text{if } x<1 \\ 3 & \text{if } x=1 \\ -x^2+2 & \text{if } 1<x\leq 2 \\ x-3 & \text{if } x>2 \end{cases}$

Sol



From the graph we would see that the limit of function exist at $x=1$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x$$

$$= 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2-x^2)$$

$$= 2-1^2$$

$$= 1$$

So the $\lim_{x\to 1} f(x) = 1$

iv Does $\lim_{x\to 2} g(x)$ exist for $g(x) = \dfrac{x^2+x-6}{|x-2|}$

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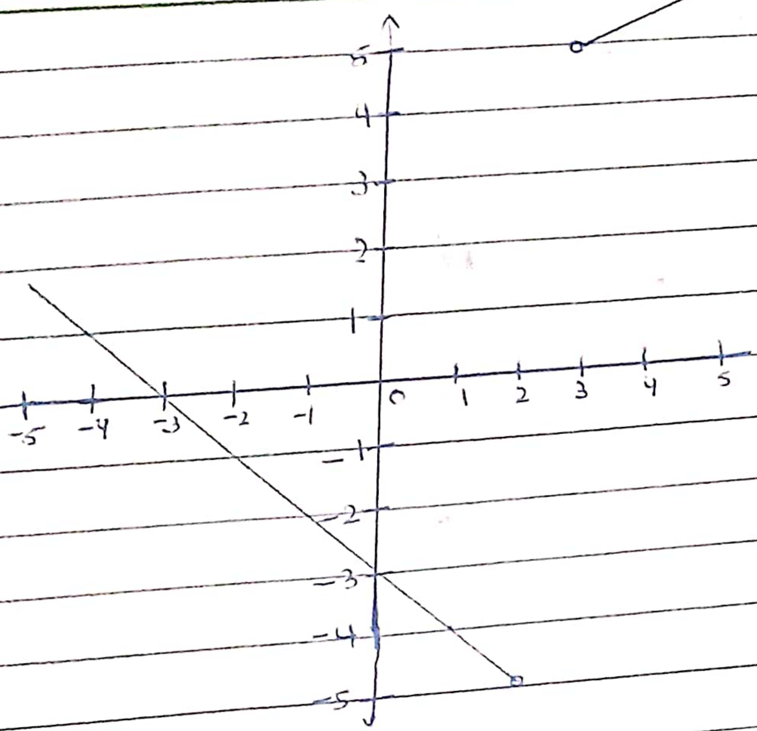


$$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} \frac{x^2+x-6}{|x-2|}$$

$$= \lim_{x\to 2^+} \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}} \rightarrow \text{Since } x-2>0 \text{ if } x\to 2^+$$

$$= \lim_{x\to 2^+} (x+3)$$

$$= 5$$

$$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} \frac{x^2+x-6}{|x-2|}$$

$$= \lim_{x\to 2^-} \frac{(x+3)(x-2)}{(2-x)} \rightarrow \text{Since } x-2<0 \text{ if } x\to 2^-$$

$$= \lim_{x\to 2^-} -(x+3)$$

$$= -(2+3)$$

$$= -5$$

Since right hand and left hand limits of g at $x=2$ are not equal, $\lim_{x\to 2} g(x)$

does not exist.

v $B(t) = \begin{cases} 4 - \frac{1}{2}t & \text{if } t < 2 \\ \sqrt{t + c} & \text{if } t \geq 2 \end{cases}$

Sol: First we have to find the value of c

So, if $\lim_{t \to 2} B(t)$ exist then put L.H.S = R.H.S

L.H.S.

$$\lim_{x \to 2^-} B(t) = \lim_{t \to 2^-} 4 - \frac{1}{2}(t)$$

$$= \lim_{t \to 2^-} 4 - \frac{1}{2}(2)$$

$$= 4 - 1$$

$$= 3$$

R.H.L

$$\lim_{t \to 2^+} B(t) = \lim_{t \to 2^+} \sqrt{t + c}$$

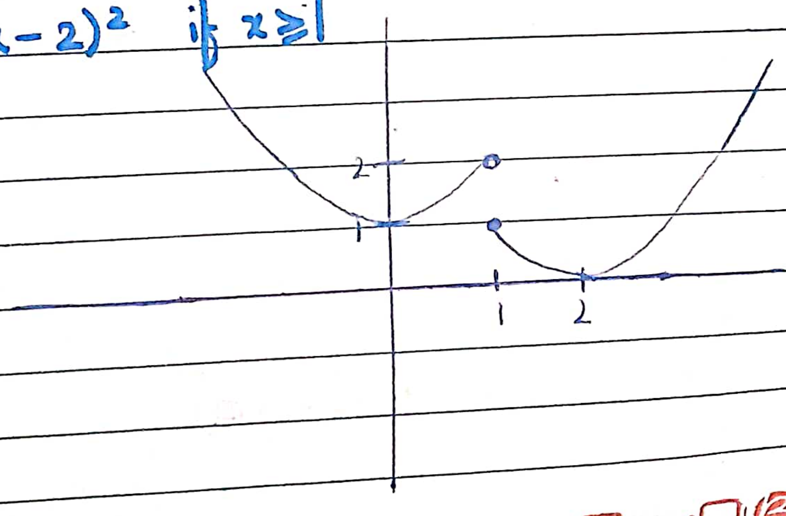
$$= \sqrt{2 + c}$$

Putting L.H.S = R.H.S

$3 = \sqrt{2 + c}$

$9 = 2 + c$, $c = 7$

vi $f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ (x-2)^2 & \text{if } x \geq 1 \end{cases}$



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from the graph of $f(x)$ we would see that that $\lim_{x \to a} f(x)$ exist for all a except at $x = 1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2 + 1)$$

$$= 1 + 1$$

$$= 2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x - 2)^2$$

$$= (1 - 2)^2$$

$$= 1$$

R.H.S Limit $\neq$ L.H.S limit

Q2 Evalute the limit if exist:

ii $\lim_{u \to 2} \frac{\sqrt{4u+1} - 3}{u - 2}$

$\left(\frac{0}{0}\right)$ by directly apply the ...

Sol

$$= \lim_{u \to 2} \frac{\sqrt{4u+1} - 3}{u - 2}$$

$$= \lim_{u \to 2} \frac{\sqrt{4u+1} - 3}{u - 2} \times \frac{\sqrt{4u+1} + 3}{\sqrt{4u+1} + 3}$$

$$= \lim_{u \to 2} \frac{(\sqrt{4u+1})^2 - (3)^2}{(u - 2)(\sqrt{4u+1} + 3)}$$

$$= \lim_{u \to 2} \frac{4u + 1 - 9}{(u - 2)(\sqrt{4u+1} + 3)}$$

$$= \lim_{u\to 2} \frac{4u-8}{(u-2)(\sqrt{4u+1}+3)}$$

$$= \lim_{u\to 2} \frac{4(\cancel{u-2})}{(\cancel{u-2})(\sqrt{4u+1}+3)}$$

$$= \lim_{u\to 2} \frac{4}{\sqrt{4u+1}+3}$$

$$\Rightarrow \frac{4}{\sqrt{4(2)+1}+3} = \frac{\cancel{4}\,2}{\cancel{6}\,3}$$

$$= \frac{2}{3}$$

iii $\lim_{x\to 0} \frac{|x|}{x}$

Sol:

$$= \lim_{x\to 0^+} \frac{|x|}{x}$$

$$= \lim_{x\to 0^+} \frac{x}{x} \rightsquigarrow \text{when } x>0 \; |x| = x$$

$$= \lim_{x\to 0^+} 1$$

$$= 1$$

$$= \lim_{x\to 0^-} \frac{|x|}{x}$$

$$= \lim_{x\to 0^-} \frac{-x}{x} \rightsquigarrow \text{when } x<0 \; |x| = -x$$

$$= \lim_{x\to 0^-} -1$$

$= -1$

Since the left and right hand limits are not equal so $\lim_{x\to 0} \frac{|x|}{x}$ does not exist.

iv $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right]$

Sol

$= \lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right)$

$= \lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{x}\right)$ → Since for $x>0$ $|x| = x$

$= \lim_{x\to 0^+} 0$

$= 0$

v $f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

prove that $\lim_{x\to 0} f(x) = 0$

Sol

As $0 \le f(x) \le x^2$ for all x

and $\lim_{x\to 0} 0 = 0$, $\lim_{x\to} x^2 = 0$

So by Squeeze Theorem, $\lim_{x\to 0} f(x)=0$

i $\lim_{x\to\infty}\left(\sqrt{4x^2+2x}-\sqrt{4x^2-2x}\right)$ (∞)

Sol.

$$\lim_{x\to\infty}\left(\sqrt{4x^2+2x}-\sqrt{4x^2-2x}\right)$$

Divide & multiply by $\sqrt{4x^2+2x}+\sqrt{4x^2-2x}$

$$\lim_{x\to\infty}\left(\sqrt{4x^2+2x}-\sqrt{4x^2-2x}\right)\times\frac{\left(\sqrt{4x^2+2x}+\sqrt{4x^2-2x}\right)}{\sqrt{4x^2+2x}+\sqrt{4x^2-2x}}$$

$$\lim_{x\to\infty}\frac{\left(\sqrt{4x^2+2x}\right)^2-\left(\sqrt{4x^2-2x}\right)^2}{\sqrt{4x^2+2x}+\sqrt{4x^2-2x}}$$

$$\lim_{x\to\infty}\frac{\cancel{4x^2}+2x-\cancel{4x^2}+2x}{\sqrt{4x^2+2x}+\sqrt{4x^2-2x}}$$

$$\lim_{x\to\infty}\frac{4x}{\sqrt{4x^2+2x}+\sqrt{4x^2-2x}}$$

$$\lim_{x\to\infty}\frac{\frac{4\cancel{x}}{\cancel{x}}}{\frac{\sqrt{4x^2+2x}+\sqrt{4x^2-2x}}{x}}$$

$$\lim_{x\to\infty}\frac{4}{\frac{\sqrt{4x^2+2x}}{\sqrt{x^2}}+\frac{\sqrt{4x^2-2x}}{\sqrt{x^2}}}$$

$$\lim_{x\to\infty}\frac{4}{\sqrt{\frac{\cancel{x^2}(4+2/x)}{\cancel{x^2}}}+\sqrt{\frac{\cancel{x^2}(4-2/x)}{\cancel{x^2}}}}$$

$$\lim_{x\to\infty}\frac{4}{\left(\sqrt{4+\frac{2}{x}}\right)+\left(\sqrt{4-\frac{2}{x}}\right)}$$

$$\Rightarrow \frac{4}{\sqrt{4+2/\infty}+\sqrt{4-2/\infty}} = \frac{4}{\sqrt{4}+\sqrt{4}}$$

$$\Rightarrow \frac{4}{2+2} = \frac{4}{4}$$

$$= 1$$

vi $\lim_{x\to\infty} \frac{2x^{5/3}-x^{1/3}+7}{x^{8/5}+3x+\sqrt{x}}$

Sol:

$$\lim_{x\to\infty} \frac{\frac{2x^{5/3}}{x^{8/5}} - \frac{x^{1/3}}{x^{8/5}} + \frac{7}{x^{8/5}}}{\frac{x^{8/5}}{x^{8/5}} + \frac{3x}{x^{8/5}} + \frac{\sqrt{x}}{x^{8/5}}}$$

$$\lim_{x\to\infty} \frac{2x^{\frac{5}{3}-\frac{8}{5}} - x^{\frac{1}{3}-\frac{8}{5}} + \frac{7}{x^{8/5}}}{1 + 3x^{1-8/5} + x^{\frac{1}{2}-8/5}}$$

$$\lim_{x\to\infty} \frac{2x^{1/15} - \frac{1}{x^{19/15}} + 7/x^{8/5}}{1 + 3/x^{3/5} + 1/x^{11/10}}$$

$$\lim_{x\to\infty} = \frac{2(\infty)^{1/15} - \frac{1}{(\infty)^{19/15}} + \frac{7}{(\infty)^{8/5}}}{1 + 3/(\infty)^{3/5} + 1/(\infty)^{11/10}}$$

$$\lim = \frac{2(\infty)^{1/15}}{1}$$

$$\lim_{x\to\infty} = \infty$$

Soln:

$$= \frac{2x^{5/3} - x^{1/3} + 7}{x^{5/3}\left[x^{1} + \frac{3x}{x^{5/3}} + \frac{x^{1/2}}{x^{5/3}}\right]}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^{5/3}}\left[2 - \frac{x^{1/3}}{x^{5/3}} + \frac{7}{x^{5/3}}\right]}{\cancel{x^{5/3}}\left[x + \frac{3}{x^{2/3}} + \frac{1}{x^{2/3}}\right]}$$

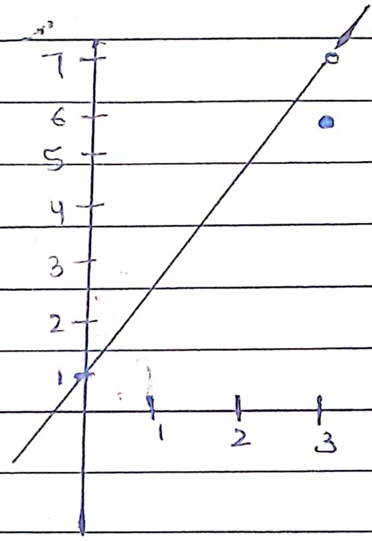
$$= \lim_{x\to\infty} \frac{2}{\infty} = 0$$

Q3 Where are each of the following function discontinuous.

ii $f(x) = \begin{cases} \dfrac{2x^2-5x-3}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

<u>Sol</u>

The function is discontinuous at $x = 3$



$\lim\limits_{x \to 3} f(x) = \lim\limits_{x \to 3} \dfrac{2x^2-5x-3}{x-3}$

$= \lim\limits_{x \to 3} \dfrac{(2x+1)\cancel{(x-3)}}{\cancel{(x-3)}}$

$= \lim\limits_{x \to 3} (2x+1)$

$= 2(3)+1$

$= 7$

but $f(3) = 6$

So the function is discontinuous at $x = 3$

iii $f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1-x^2 & \text{if } x > 0 \end{cases}$

Note: Follow the 3-steps of continuity

(12)

You have to check the left and Right hand limit

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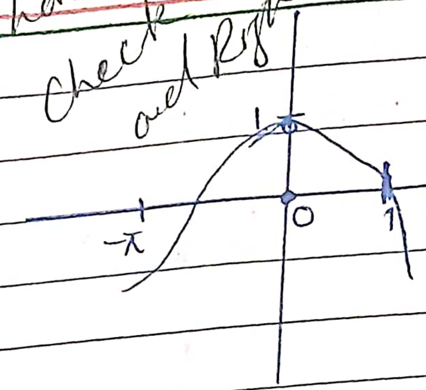

The function is discontinuous at $x=0$

because $\lim_{x\to 0} f(x) \neq f(0)$

$\lim_{x\to 0^+} f(x) =$ ___

$\lim_{x\to 0^-} f(x) =$ ___

$\lim_{x\to 0} f(x) - \lim_{x\to 0} f(x)$

$= 1$

but

$f(0) = 0$

v The gravitational force exerted by the planet Earth on a unit mass at a distance r from the center of the planet is

$$F(r) = \begin{cases} \dfrac{GMr}{R^3} & \text{if } r < R \\ \dfrac{GM}{r^2} & \text{if } r \geq R \end{cases}$$

where M is the mass of Earth, R is its radius and G is the gravitational constant. Is F a continuous function of r?

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Each piece of F is continuous on its domain.

We have to check continuity at $r = R$

(1) $F(R) = \frac{GM}{R^2}$

(2) $\lim_{r \to R^-} F(r) = \lim_{x \to R^-} \frac{GMr}{R^3}$

$= \frac{GMR}{R^3}$

$= \frac{GM}{R^2}$

$\lim_{x \to R^+} F(r) = \lim_{r \to R^+} \frac{GM}{r^2}$

$= \frac{GM}{R^2}$

So $\lim_{r \to R} = \frac{GM}{R^2}$

(3) $\lim_{r \to R} f(r) = f(R)$

Since $F(R) = \frac{GM}{R^2}$

Therefore F is continuous at R.

vi $f(x) = \begin{cases} 2^x & \text{if } x \leq 1 \\ 3-x & \text{if } 1 < x \leq 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$

Discuss the continuity at $x=1$ and $x=4$

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Checking continuity at $x=1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2^x$

$= 2^1$

$= 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 3-x$

$= 3-1$

$= 2$

B/L LHL = RHL

So $\lim_{x\to 1} f(x) = 2$ Therefore

Now

$f(x) = x^2+1$ when $x=1$

$f(1) = 1^2+1$

$= 2$

So the function is continuous at $x=1$

because $\lim_{x\to 1} f(x) = f(1)$

Checking continuity at $x=4$

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (3-x)$

$= 3-4$

$= -1$

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \sqrt{x}$$

$$= \sqrt{4}$$

$$= 2$$

$\lim_{x\to 4} f(x)$ does not exist

So function is discontinuous at $x=4$

iii $f(x) = \dfrac{x^2-x-2}{1+\cos x}$

Sol.

To find the discontinuity of function

Denominator $1+\cos x = 0$

$$\cos x = -1$$

$$x = \cos^{-1}(1)$$

So the function is discontinuous at

$$x = (2n+1)\pi$$

iv $y = \dfrac{1}{1+e^{-x\mp 1}}$

Sol: Note that funtion $f(x)$ is continuous at the point $x=a$ if and only if $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$

Now, $\lim_{x\to 0^-} \dfrac{1}{1+e^{-\frac{1}{x}}} = \dfrac{1}{1+e^{\infty}} = \dfrac{1}{1+\cancel{\infty}} = \dfrac{1}{\infty} = 0$

$$= 0$$

$\lim_{x\to 0^+} \dfrac{1}{1+e^{-\frac{1}{x}}} = \dfrac{1}{1+e^{-\infty}} = \dfrac{1}{1+\frac{1}{e^{\infty}}} = \dfrac{1}{1+\frac{1}{\infty}} = \dfrac{1}{1+0} = 1$

$$= 1$$

$L.H.S \neq R.H.S$. So the given function is discontinuous at $x=0$

But it continuous everywhere.

Q4 Find the number at which f(x) is continuous at which of these number is f(x) is continuous from left, from the right or neither.

iii $f(x) = \begin{cases} x^4 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

This is by Theorem But here we have to check the continuity at $x=0$

Sol

$f(x) = x^4 . \sin\left(\frac{1}{x}\right)$ is continuous on $(-\infty, 0) \cup (0, \infty)$ since it is a product of a polynomial and a composite of a trignometric function and a rational function.

(ii) Now since $-1 \leq \sin(1/x) \leq 1$, we have $-x^4 \leq x^4 \sin(1/x) \leq x^4$. Because $\lim_{x \to 0} (-x^4) = 0$ and $\lim_{x \to 0} (x^4) = 0$

The Squeeze Theorem gives us

$\lim_{x \to 0} (x^4 \sin(1/x)) = 0$

And

(I) $\rightarrow$ $f(0) = 0$

Thus f is continuous at 0. ~~and~~, hence ~~on $(-\infty, \infty)$~~

(III) $\lim_{x \to 0} f(x) = f(0)$.

Thus $f(x)$ is continous

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i $f(x) = \begin{cases} x+1 & \text{if } x \le 1 \\ 1/x & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \ge 3 \end{cases}$

$f(1) = 2$

~~$f(3) = 0$~~

Sol:

L.H.S at 1

$= \lim_{x \to 1^-} x+1$

$\Rightarrow 1+1 = 2$

R.H.S at 1

$= \lim_{x \to 1^+} \frac{1}{x}$

$\Rightarrow \frac{1}{1} = 1$

L.H.S $\neq$ R.H.S hence the function at $x=1$ is discontinuous.

L.H.S at $x=3$

$= \lim_{x \to 3^-} \frac{1}{x}$

$= \frac{1}{3}$

R.H.S at 3

$= \lim_{x \to 3^+} \sqrt{x-3}$

$\Rightarrow \sqrt{3-3} = 0$

and $f(3) = 0$

L.H.S $\neq$ R.H.S hence the function is discontinuous at $x=3$

Since $f(1) = 2$ and $\lim_{x \to 1^-} f(x) = 2$ therefore the function continuous from left at $x=1$

$f(3) = 0$ and $\lim_{x \to 3^+} f(x) = 0$ therefore function is continuous from right at $x=3$.

ii $f(x) = \begin{cases} x+2 & \text{if } x<0 \\ e^x & 0 \le x \le 1 \\ 2-x & x>1 \end{cases}$

L.H.S at 0

$= \lim_{x \to 0^-} x+2$

$\Rightarrow 0+2 = 2$

R.H.S at 0

$= \lim_{x \to 0^+} e^x$

$\Rightarrow e^{(0)} = 1$

First check that $f(0) = 1$ then go for the 2nd condition.

L.H.S $\neq$ R.H.S hence function is discontinuous at $x=0$

L.H.S at 1

$= \lim_{x \to 1^-} e^x$

$\Rightarrow e^1 = e$

R.H.S at 1

$= \lim_{x \to 1^+} 2-x$

$\Rightarrow 2-1 = 1$

L.H.S $\neq$ R.H.S hence function is discontinuous at $x=1$

Since $\lim_{x \to 0^+} f(x) = f(0)$ therefore function is continuous from right at $x=0$

$\lim_{x \to 1^-} f(x) = f(1)$ therefore function is continuous from left at $x=0$

~~iv $g(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ x & \text{if } x \text{ is irrational} \end{cases}$~~

Skip this Question

Sol

let $x = a$

there are infinite numbers to reach till 0

$\lim_{x \to a} \neq f(a)$

there are infinite number to reach till 0 is irrational

discontinuous function.

Skip this Question

v continuous at $(-\infty, \infty)$

$$f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 1 \\ \ln & \text{if } x > 1 \end{cases}$$

Note

$\boxed{f(1) = 0}$

L.H.S at 1

$\lim_{x \to 1^-} 1 - x^2$

$= 1 - (1)^2$

$= 0$

R.H.S at 1

$= \text{limit}_{x \to 1^+} \ln$

$= \ln(1)$

$= 0$

L.H.S = R.H.S

So function is continuous.

vi Using continuity to evaluate limit

$$\lim_{x \to 2} \frac{x^3}{\sqrt{x^2 + x - 2}}$$

Sol

$$= \lim_{x \to 2} \frac{x^3}{\sqrt{x^2 + x - 2}}$$

Since the fn is continuous therefore the limit is exist

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$$= \frac{(2)^3}{\sqrt{(2)^2+(2)-2}}$$

$$= \frac{8}{\sqrt{4+2-2}}$$

$$= \frac{8}{\sqrt{4}}$$

$$= \frac{8}{2}$$

$$= 4$$

Continuous at $x = 2$

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Q5 Find the value of a and b that make f continuous everywhere.

i $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x<2 \\ ax^2-bx+3 & \text{if } 2 \le x < 3 \\ 2x-a+b & \text{if } x \ge 3 \end{cases}$

Sol

At $x=2$:

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2-4}{x-2}$

$= \lim_{x\to 2^-} \frac{(x+2)(x-2)}{(x-2)}$

$= \lim_{x\to 2^-} (x+2)$

$= 4$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (ax^2-bx+3)$

$= 4a-2b+3$

We must have $4a-2b+3=4$

$4a-2b=1 \rightarrow (i)$

At $x=3$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (ax^2-bx+3)$

$= 9a-3b+3$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (2x-a+b)$$

$$= 6-a+b$$

We must have $9a-3b+3 = 6a+b$

$$10a-4b = 3 \quad (2)$$

By solving (i) and (2)

$$-8a+4b = -2$$
$$10a-4b = 3$$
$$2a = 1$$

So $a = \frac{1}{2}$, $b = \frac{1}{2}$

Thus for f to be continuous on $(-\infty, \infty)$

$a = b = \frac{1}{2}$

ii Let $g(x) = \begin{cases} 2x-x^2 & \text{if } 0 \le x \le 2 \\ 2-x & \text{if } 2 < x \le 3 \\ x-4 & \text{if } 3 < x < 4 \\ \pi & \text{if } x \ge 4 \end{cases}$

For each of the number 2, 3 and 4 discover whether g is continuous from left, continuous from right, or continuous at a number.

Sol.

- Checking continuity from left at $x=2$

$$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} (2x-x^2)$$

$$= 2(2)-(2)^2$$

$= 0$

Also $g(x) = 2x - x^2$ at $x = 2$

$g(2) = 2(2) - (2)^2$

$= 0$

So the $g(x)$ is continuous from left at $x=2$

- Checking continuity from right at $x=2$

$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} 2 - x$

$= 2 - 2$

$= 0$

Also $g(x) = 2x - x^2$ at $x = 2$

$= 2(2) - (2)^2$

$= 0$

As $\lim_{x \to 2^+} g(x) = f(2)$

So $g(x)$ is continuous from right at $x=2$

$g(x)$ is also continuous at point $x=2$

because $\lim_{x \to 2} f(x) = g(2)$

- Checking continuity from left at $x = 3$

$\lim_{x \to 3^-} g(x) = \lim_{x \to 3^-} (2 - x)$

$= 2 - 3$

$= -1$

Also $g(x) = 2 - x$ at $x = 3$

$g(3) = 2 - 3$

$= -1$

As $\lim_{x\to 3^-} g(x) = g(3)$ So the function is continuous from left at $x=3$

- Checking continuity at $x=3$ from right

$\lim_{x\to 3^+} g(x) = \lim_{x\to 3^+} (x-4)$

$= 3-4$

$= -1$

Also $g(x) = 2-x$ at $x=3$

$g(3) = 2-3$

$= -1$

As $\lim_{x\to 3^+} g(x) = g(3)$ So the function is continuous from right at $x=3$

Also $g(x)$ is continuous at point $x=3$ because $\lim_{x\to 3} g(x) = g(3)$

- Checking Continuity from left at $x=4$

$\lim_{x\to 4^-} g(x) = \lim_{x\to 4^-} (x-4)$

$\Rightarrow 4-4 = 0$

Also $g(x) = \pi$ at $x=4$

$g(4) = \pi$

Aso $\lim_{x\to 4^-} g(x) \neq g(4)$ so the $g(x)$ is discontinuous from left at $x=4$.

- Checking Continuity from right at $x=4$.

$$\lim_{x\to 4^+} g(x) = \lim_{x\to 4^+} \pi$$

$$= \pi$$

Also at $x=4$ $g(x)=\pi$

$$g(4)=\pi$$

As $\lim_{x\to 4^+} g(x) = g(4)$, so the $g(x)$ is continuous from right at $x=4$

As $\lim_{x\to 4} g(x)$ does not exist so the function is not continuous at point $x=4$.

iii Find the horizontal asymptotes of

$$f(x) = \frac{x^3-2}{|x|^3+1}$$

Sol

We can find horizontal asymptote when $x-\infty$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^3-2}{|x|^3+1}$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Now when $x<0$, it gets to $-\infty$

Now $\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{x^3-2}{-x^3+1} \rightsquigarrow \div$ by $x^3 \rightsquigarrow \lim_{x\to -\infty} \frac{\frac{x^3}{x^3} - \frac{2}{x^3}}{-\frac{x^3}{x^3} + \frac{1}{x^3}}$

$$\Rightarrow \frac{1-\frac{2}{\infty^3}}{-1+\frac{1}{\infty^3}} = -1.$$

Now when $x \geq 0$, it tends towards $+\infty$

$\lim_{x\to +\infty} f(x) = \lim_{x\to\infty} \frac{x^3-2}{x^3+1} \rightsquigarrow \div$ by $x^3 \rightsquigarrow \frac{\frac{x^3}{x^3} - \frac{2}{x^3}}{\frac{x^3}{x^3} + 1/x^3} \Rightarrow \frac{1-2/x^3}{1+1/x^3}$

$$\Rightarrow \frac{1-2/\infty^3}{1+1/\infty^3} \Rightarrow 1$$

So its horizontal asymtotes are 1 and -1

iv Find horizontal asymtots of $y = 2 + \frac{\sin x}{x}$ using sandwich theorem

Sol.

$$\lim_{x\to\infty} 2 + \frac{\sin x}{x}$$

$$\lim_{x\to\infty} 2 + \lim_{x\to\infty} \frac{\sin x}{x}$$

$$-1 \le \sin x \le 1$$

$$\lim_{x\to\infty} \frac{1}{-x} \le \frac{\sin x}{x} \le \frac{1}{x}$$

according to squeeze theorem $\lim_{x\to\infty} \frac{\sin x}{x} = 0$ $\lim_{x\to\infty} \frac{1}{x} = 0$

So, $\lim_{x\to\infty} = \frac{\sin x}{x} = 0$

$\lim_{x\to\infty} \frac{1}{-x} = 0$

$y = 2 + \cancel{\frac{\sin x}{x}}^{\,0}$

$\lim_{x\to\infty} f(y) = 0$ $\boxed{y=2}$ → vertical

~~f(y) has horizontal asymptote at 0~~

Use limit to determine the equation for all vertical and horizontal asymptots. $y = \frac{x^2+x-6}{x^2+2x-8}$

For vertical

~~$x^2 + 2x - 8$~~

~~$x = -1 \pm \sqrt{-7}$~~

~~it does not exist~~

~~at real number~~

For horizontal

$$y = \frac{x^2+x-6}{x^2+2x-8}$$

For vertical

x^2+2x-8

$(x+4)(x-2)$

So y has vertical asymptot at $x=-4$ and $x=2$

$$= \frac{\frac{x^2}{x^2} + \frac{x}{x^2} - \frac{6}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{8}{x^2}}$$

$$= \frac{1 + \frac{1}{x} + \frac{6}{x^2}}{1 + \frac{2}{x} - \frac{8}{x^2}}$$

$$\lim_{x\to\infty} = \frac{1 + \frac{1}{\infty} + \frac{6}{\infty^2}}{1 + \frac{2}{\infty} - \frac{8}{\infty^2}}$$

$$= \frac{1}{1}$$

$$= 1$$

So f has horizonal asymptote at $\underline{1}$

$\boxed{y=1}$

Vertical and horizontal asymptotes

$$y = \frac{\sqrt{ax^2+5}}{x-b}$$

Sol for vertical

$x-b=0$

$x=b$

Vertical asymptote at $x=b$

For horizontal

$$y = \frac{\sqrt{ax^2+5}}{x-b}$$

$$= \frac{\frac{\sqrt{ax^2+5}}{x}}{\frac{x-b}{x}}$$

$$= \frac{\frac{\sqrt{ax^2+5}}{\sqrt{x^2}}}{\frac{x}{x} - \frac{b}{x}}$$

$$= \frac{\sqrt{\frac{ax^2}{x^2} + \frac{5}{x^2}}}{1 - \frac{b}{x}}$$

$$= \frac{\sqrt{a + \frac{5}{x^2}}}{1 - \frac{b}{x}}$$

$$\lim_{x \to \infty} = \frac{\sqrt{a + \frac{5}{\infty^2}}}{1 - \frac{b}{\infty}}$$

$$= \sqrt{a}$$

horizontal asymplote at $y = \sqrt{a}$