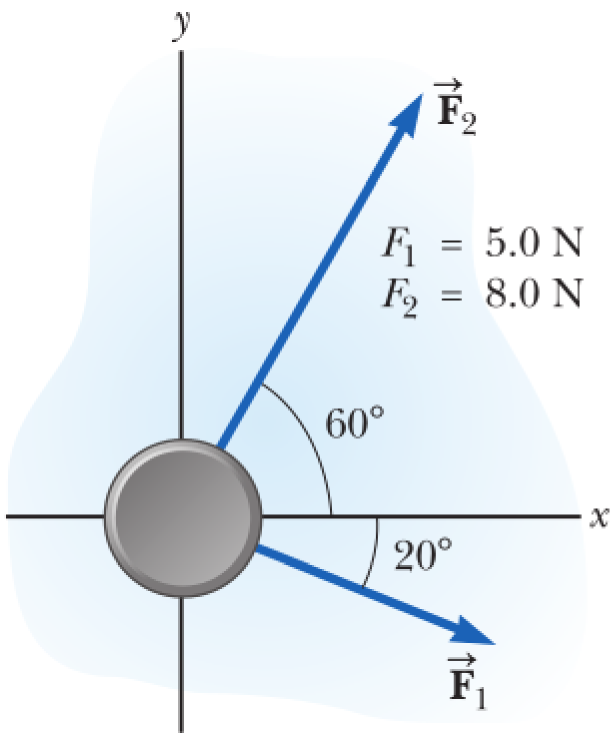
Example 5.1 An Accelerating Hockey Puck

A hockey puck having a mass of 0.30 kg slides on the frictionless, horizontal surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in the figure. The force $\boldsymbol{F}_1$ has a magnitude of 5.0 N, and the force $\boldsymbol{F}_2$ has a magnitude of 8.0 N. Determine both the magnitude and the direction of the puck's acceleration.



$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos(-20°) + F_2 \cos 60°$$

$$= (5.0\text{ N})(0.940) + (8.0\text{ N})(0.500) = 8.7\text{ N}$$

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin(-20°) + F_2 \sin 60°$$

$$= (5.0\text{ N})(-0.342) + (8.0\text{ N})(0.866) = 5.2\text{ N}$$

$$a_x = \frac{\sum F_x}{m} = \frac{8.7\text{ N}}{0.30\text{ kg}} = 29\text{ m/s}^2$$

$$a_y = \frac{\sum F_y}{m} = \frac{5.2\text{ N}}{0.30\text{ kg}} = 17\text{ m/s}^2$$

$$a = \sqrt{(29\text{ m/s}^2)^2 + (17\text{ m/s}^2)^2} = \boxed{34\text{ m/s}^2}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = \boxed{31°}$$

Conceptual Example 5.2 How Much Do You Weigh in an Elevator?

You have most likely been in an elevator that accelerates upward as it moves toward a higher floor. In this case, you feel heavier. In fact, if you are standing on a bathroom scale at the time, the scale measures a force having a magnitude that is greater than your weight. Therefore, you have tactile and measured evidence that leads you to believe you are heavier in this situation.

Are you heavier?

Conceptual Example 5.3 You Push Me and I'll Push You

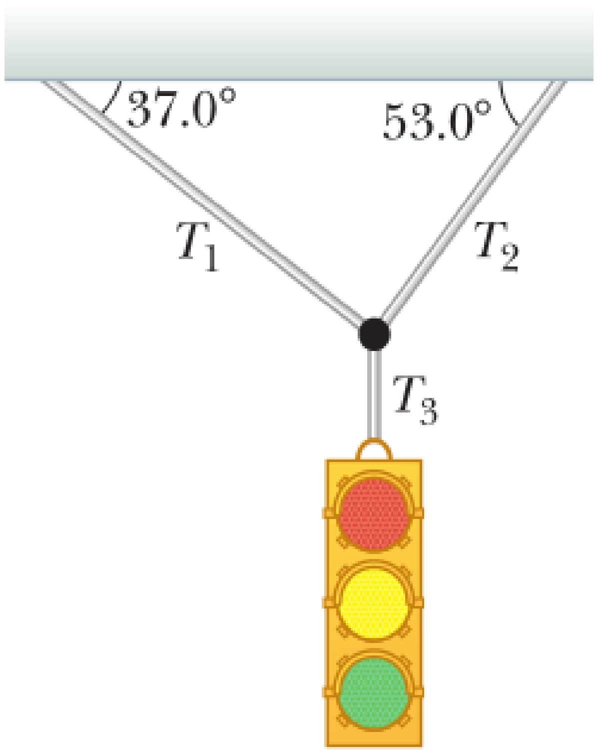
A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart.

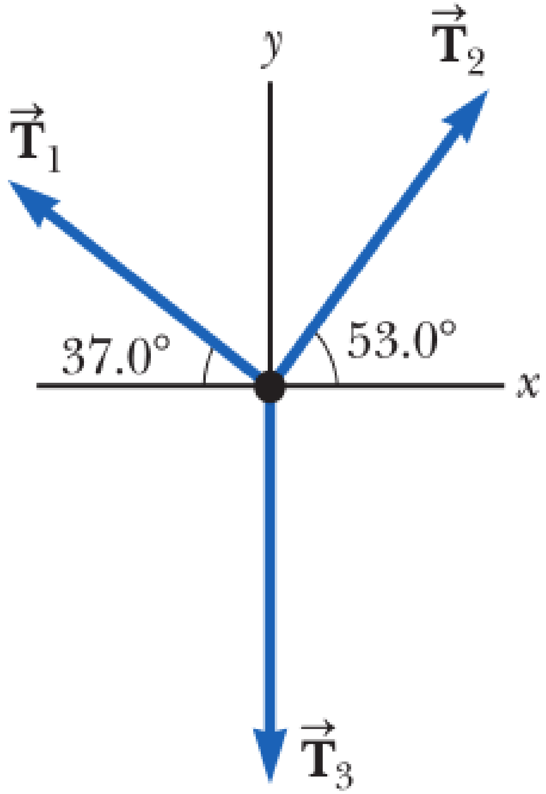
(A) Who moves away with the higher speed?

(B) Who moves farther while their hands are in contact?

Example 5.4 A Traffic Light at Rest

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?





$$\sum F_y = 0 \quad \rightarrow \quad T_3 - F_g = 0$$

$$T_3 = F_g = 122\text{ N}$$

Force	x Component	y Component
$\vec{\mathbf{T}}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\vec{\mathbf{T}}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\vec{\mathbf{T}}_3$	0	-122 N

$$(1)\ \sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

$$(2)\ \sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122\text{ N}) = 0$$

$$T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33 T_1$$

$$T_1 \sin 37.0° + (1.33 T_1)(\sin 53.0°) - 122\text{ N} = 0$$

$$T_1 = 73.4\text{ N}$$

$$T_2 = 1.33 T_1 = 97.4\text{ N}$$

the cables will not break.

Conceptual Example 5.5 Forces Between Cars in a Train

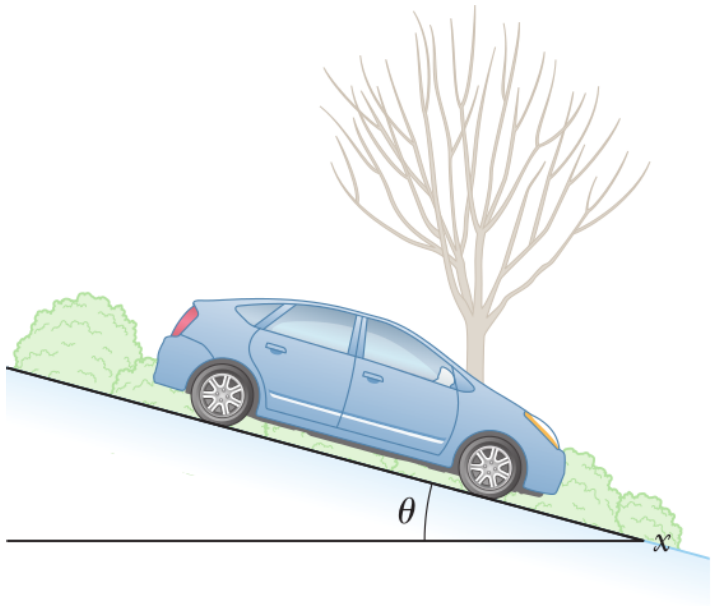
Train cars are connected by *couplers*, which are under tension as the locomotive pulls the train. Imagine you are on a train speeding up with a constant acceleration. As you move through the train from the locomotive to the last car, measuring the tension in each set of couplers, does the tension increase, decrease, or stay the same? When the engineer applies the brakes, the couplers are under compression. How does this compression force vary from the locomotive to the last car? (Assume only the brakes on the wheels of the engine are applied.)

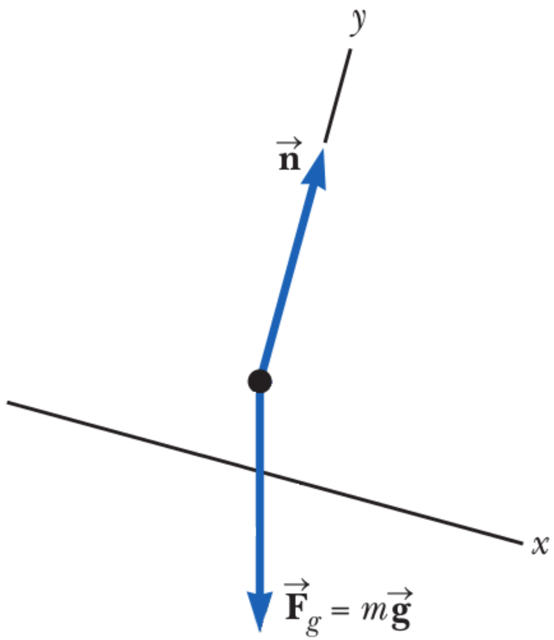
Example 5.6 The Runaway Car

A car of mass m is on an icy driveway inclined at an angle θ.

(A) Find the acceleration of the car, assuming the driveway is frictionless.

(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is d.
How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?





Apply these models to the car:

$$(1) \quad \sum F_x = mg\sin\theta = ma_x$$

$$(2) \quad \sum F_y = n - mg\cos\theta = 0$$

Solve Equation (1) for a_x:

$$(3) \quad a_x = \boxed{g\sin\theta}$$

Analyze Defining the initial position of the front bumper as $x_i = 0$ and its final position as $x_f = d$, and recognizing that $v_{xi} = 0$, choose Equation 2.16 from the particle under constant acceleration model, $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$d = \tfrac{1}{2}a_x t^2$$

Solve for t:

$$(4) \quad t = \sqrt{\frac{2d}{a_x}} = \boxed{\sqrt{\frac{2d}{g\sin\theta}}}$$

Use Equation 2.17, with $v_{xi} = 0$, to find the final velocity of the car:

$$v_{xf}^2 = 2a_x d$$

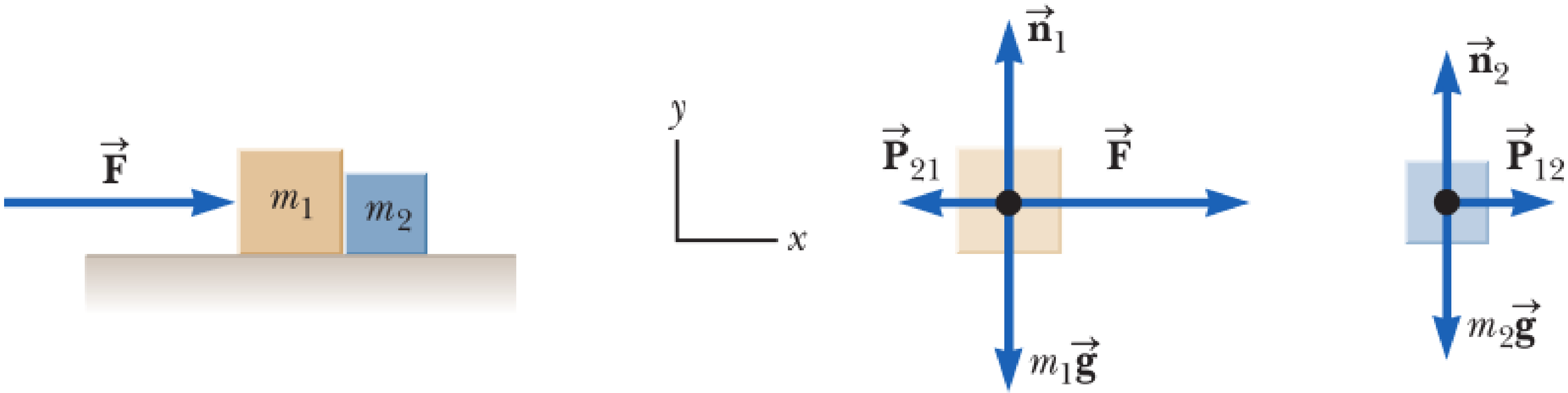
$$(5) \quad v_{xf} = \sqrt{2a_x d} = \boxed{\sqrt{2gd\sin\theta}}$$

Example 5.7 One Block Pushes Another

Two blocks of masses m_1 and m_2, with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface. A constant horizontal force $\boldsymbol{F}$ is applied to m_1.

(A) Find the magnitude of the acceleration of the system.

(B) Determine the magnitude of the contact force between the two blocks.



$$\sum F_x = F = (m_1 + m_2)a_x$$

$$a_x = \boxed{\frac{F}{m_1 + m_2}}$$

$$\sum F_x = P_{12} = m_2 a_x$$

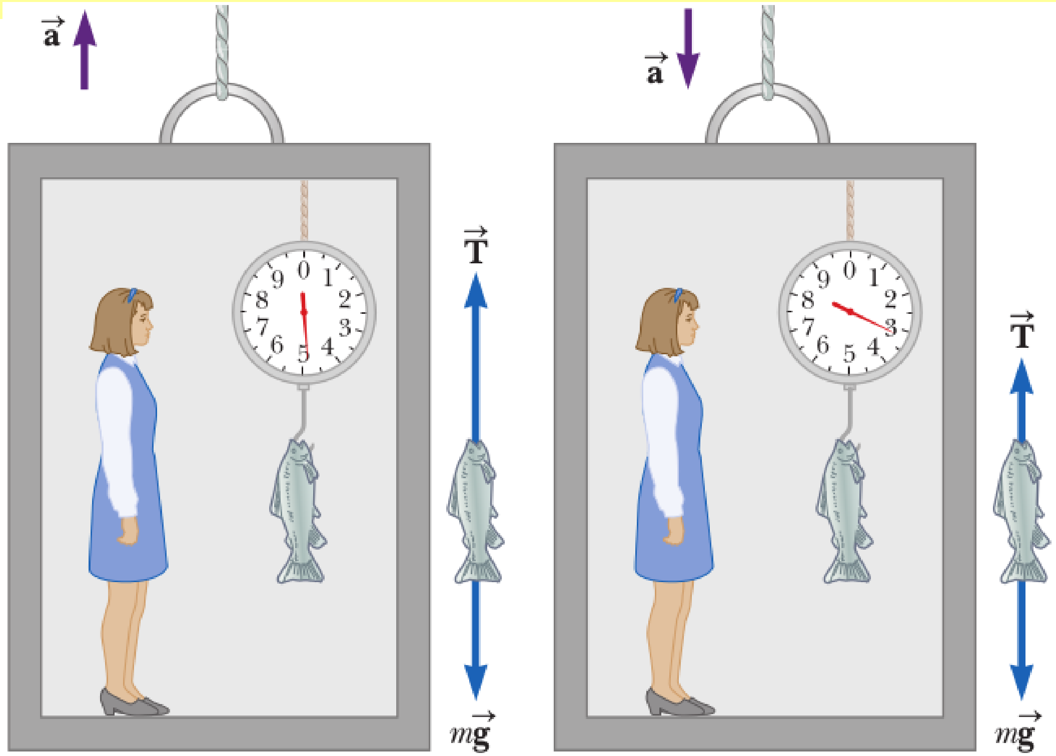
$$P_{12} = m_2 a_x = \boxed{\left(\frac{m_2}{m_1 + m_2}\right)F}$$

Example 5.8 Weighing a Fish in an Elevator

A person weighs a fish of mass *m* on a spring scale attached to the ceiling of an elevator.

(A) Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

(B) Evaluate the scale readings for a 40.0-N fish if the elevator moves with an acceleration $a_y = \pm 2.00$ m/s².



Apply Newton's second law to the fish:

$$\sum F_y = T - mg = ma_y$$

Solve for T:

$$(1)\ T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right) = F_g\left(\frac{a_y}{g} + 1\right)$$

Evaluate the scale reading from Equation (1) if $\vec{\mathbf{a}}$ is upward:

$$T = (40.0\ \mathrm{N})\left(\frac{2.00\ \mathrm{m/s^2}}{9.80\ \mathrm{m/s^2}} + 1\right) = \boxed{48.2\ \mathrm{N}}$$

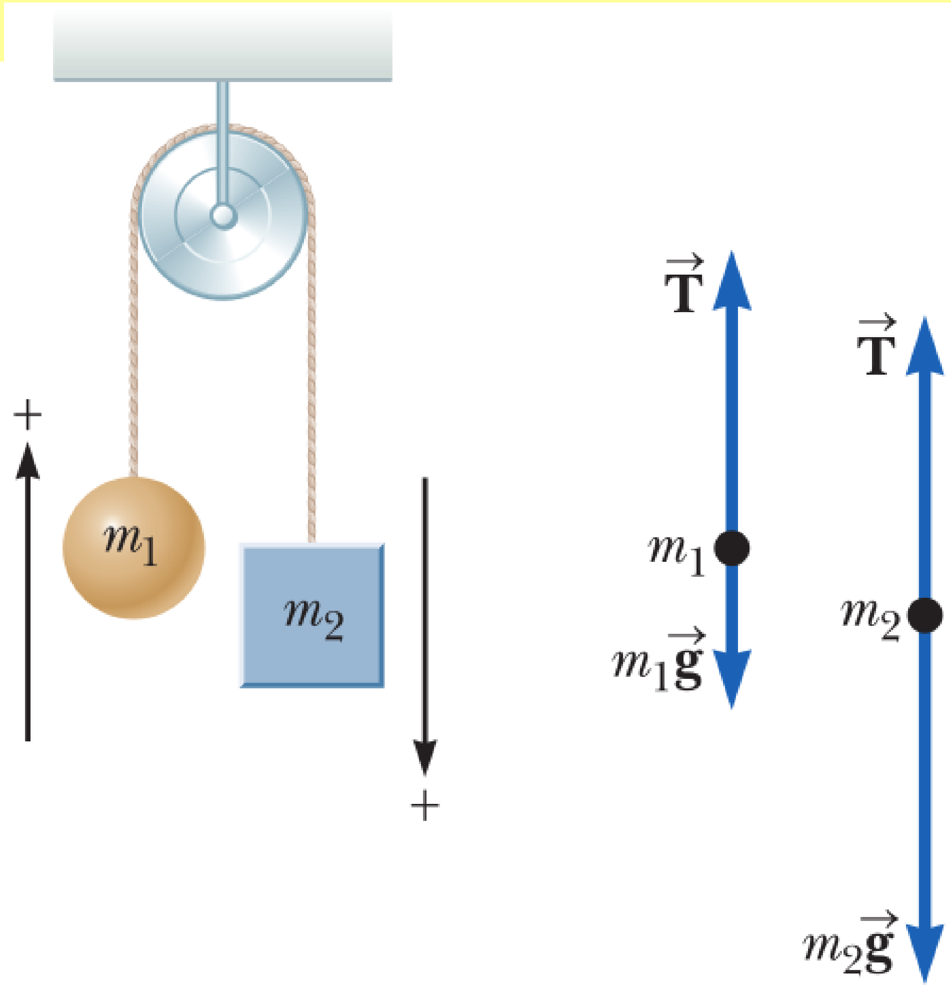
Evaluate the scale reading from Equation (1) if $\vec{\mathbf{a}}$ is downward:

$$T = (40.0\ \mathrm{N})\left(\frac{-2.00\ \mathrm{m/s^2}}{9.80\ \mathrm{m/s^2}} + 1\right) = \boxed{31.8\ \mathrm{N}}$$

Example 5.9 The Atwood Machine

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, the arrangement is called an Atwood machine. The device is sometimes used in the laboratory to determine the value of g.

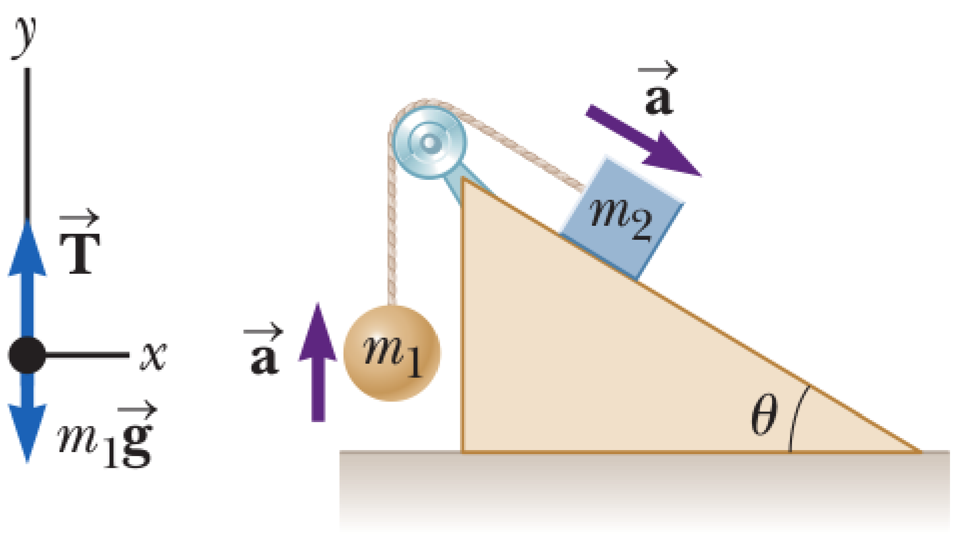
Determine the magnitude of the acceleration of the two objects and the tension in the lightweight string.

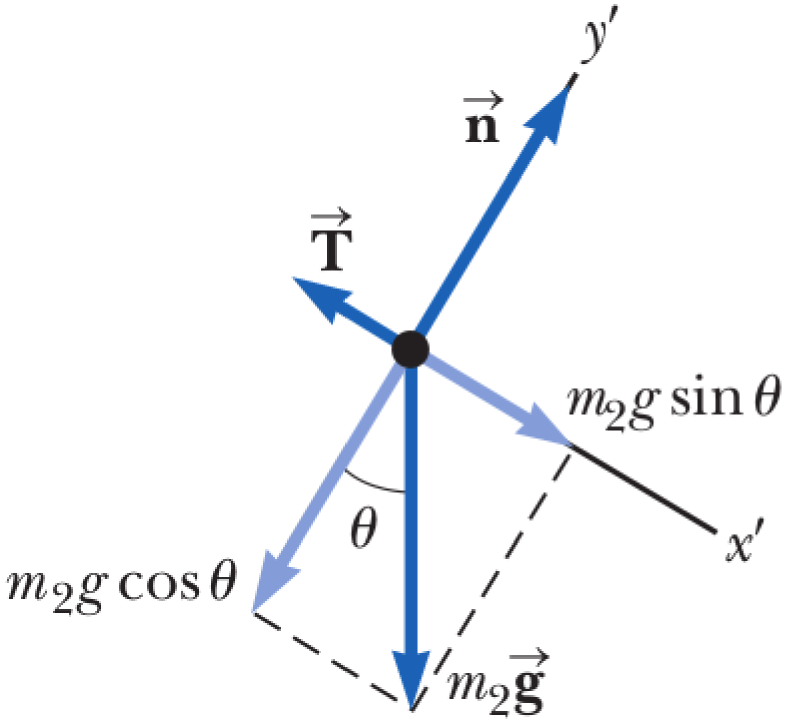


Apply Newton's second law to object 1:	(1) $\sum F_y = T - m_1 g = m_1 a_y$
Apply Newton's second law to object 2:	(2) $\sum F_y = m_2 g - T = m_2 a_y$
Add Equation (2) to Equation (1), noticing that T cancels:	$-m_1 g + m_2 g = m_1 a_y + m_2 a_y$
Solve for the acceleration:	(3) $a_y = \boxed{\left(\dfrac{m_2 - m_1}{m_1 + m_2}\right) g}$
Substitute Equation (3) into Equation (1) to find T:	(4) $T = m_1 (g + a_y) = \boxed{\left(\dfrac{2 m_1 m_2}{m_1 + m_2}\right) g}$

Example 5.10 Acceleration of Two Objects Connected by a Cord

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in the figure. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.





Apply Newton's second law in the y direction to the ball, choosing the upward direction as positive:

$$(1) \quad \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

Apply the particle under a net force model to the block in the x' direction and the particle in equilibrium model in the y' direction:

$$(2) \quad \sum F_{x'} = m_2 g \sin\theta - T = m_2 a_{x'} = m_2 a$$

$$(3) \quad \sum F_{y'} = n - m_2 g \cos\theta = 0$$

In Equation (2), we replaced $a_{x'}$ with a because the two objects have accelerations of equal magnitude a.

Solve Equation (1) for T:

$$(4) \quad T = m_1(g + a)$$

Substitute this expression for T into Equation (2):

$$m_2 g \sin\theta - m_1(g + a) = m_2 a$$

Solve for a:

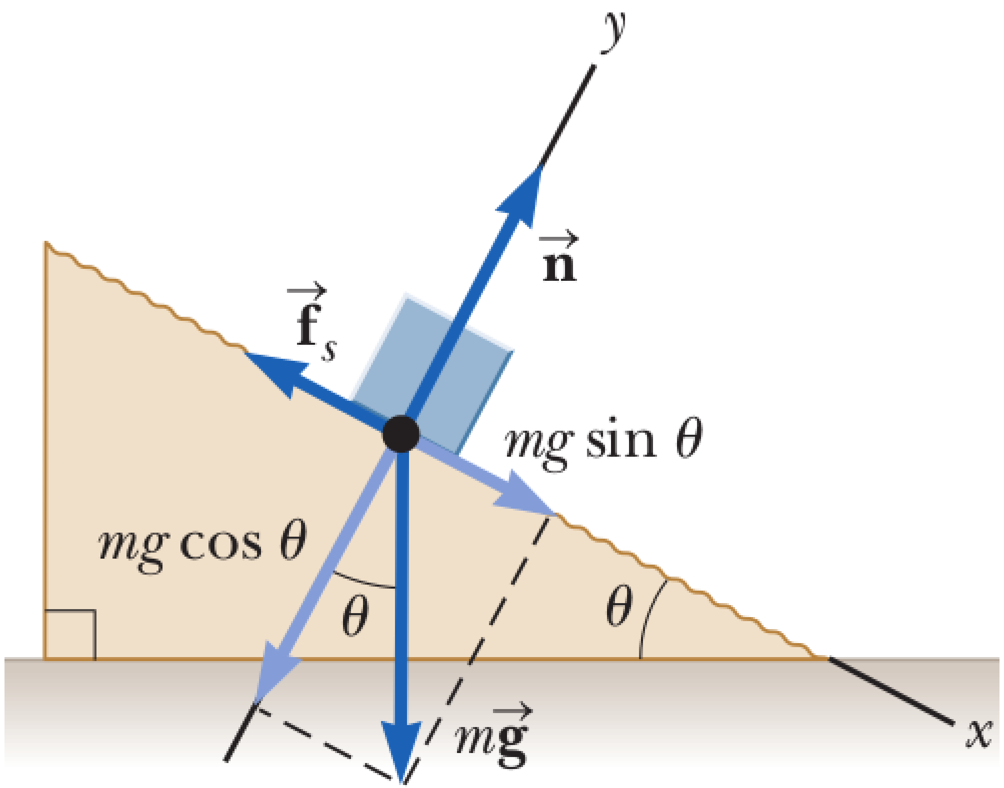
$$(5) \quad a = \boxed{\left(\frac{m_2 \sin\theta - m_1}{m_1 + m_2}\right) g}$$

Substitute this expression for a into Equation (4) to find T:

$$(6) \quad T = \boxed{\left(\frac{m_1 m_2(\sin\theta + 1)}{m_1 + m_2}\right) g}$$

Example 5.11 Experimental Determination of μ_s and μ_k

The following is a simple method of measuring coefficients of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal. The incline angle is increased until the block starts to move. Show that you can obtain μ_s by measuring the critical angle θ_c at which this slipping just occurs.



$$(1) \quad \sum F_x = mg \sin \theta - f_s = 0$$

$$(2) \quad \sum F_y = n - mg \cos \theta = 0$$

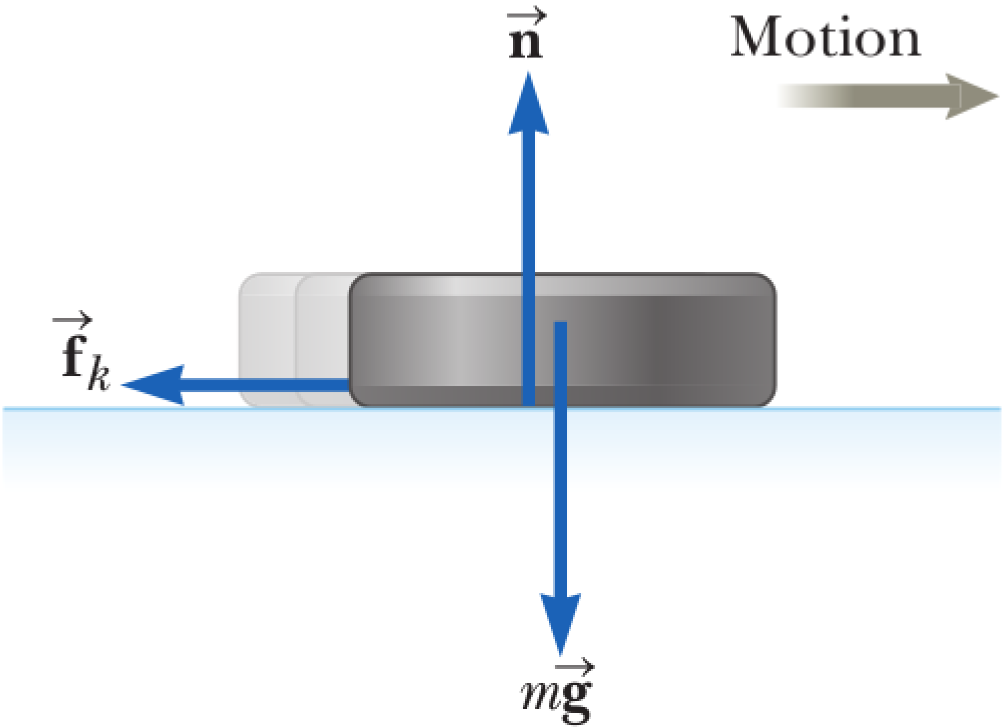
$$(3) \quad f_s = mg \sin \theta = \left(\frac{n}{\cos \theta}\right) \sin \theta = n \tan \theta$$

$$\mu_s n = n \tan \theta_c$$

$$\mu_s = \tan \theta_c$$

Example 5.12 The Sliding Hockey Puck

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.



$$(1)\quad \sum F_x = -f_k = ma_x$$

$$(2)\quad \sum F_y = n - mg = 0$$

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

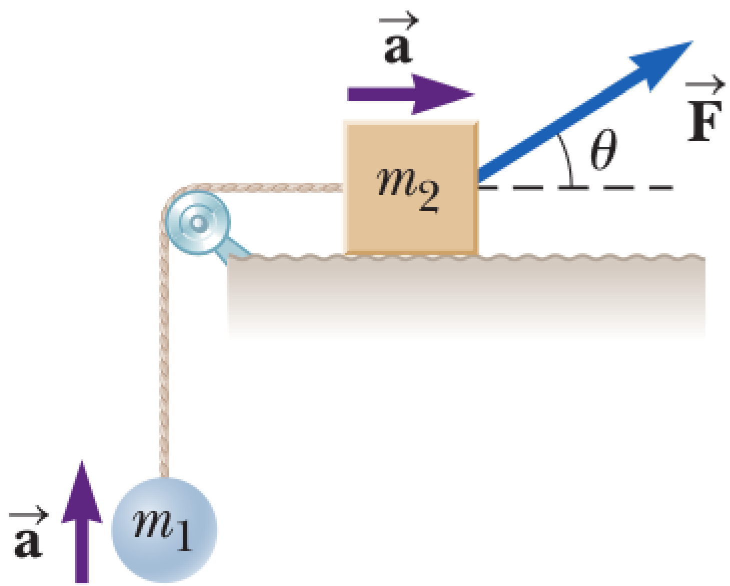
$$0 = v_{xi}^{\ 2} + 2a_x x_f = v_{xi}^{\ 2} - 2\mu_k g x_f$$

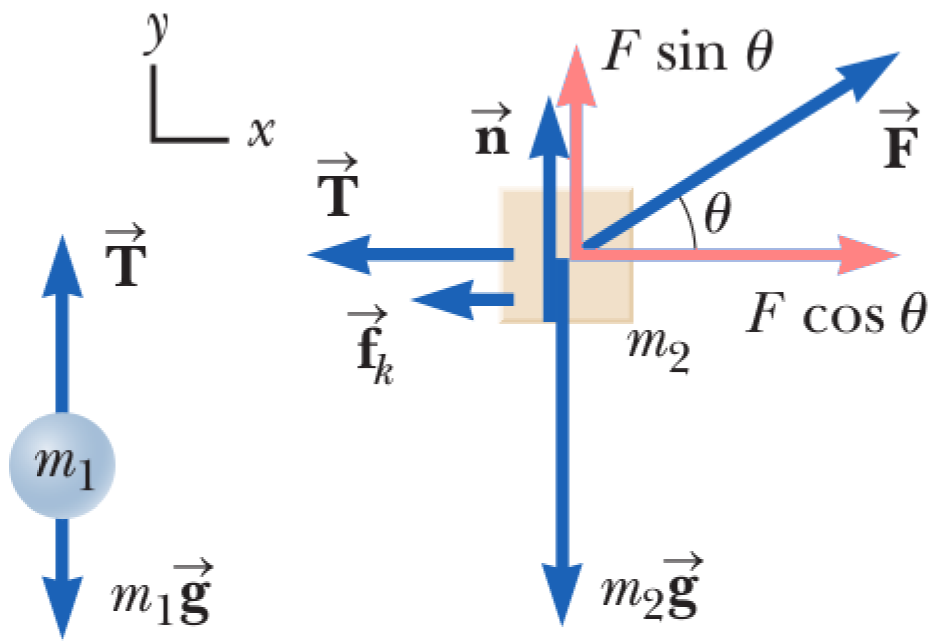
$$\mu_k = \frac{v_{xi}^{\ 2}}{2gx_f}$$

$$\mu_k = \frac{(20.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(115\ \text{m})} = \boxed{0.177}$$

Example 5.13 Acceleration of Two Connected Objects When Friction Is Present

A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley. A force of magnitude F at an angle θ with the horizontal is applied to the block, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.





$$(1) \quad \sum F_x = F\cos\theta - f_k - T = m_2 a_x = m_2 a$$

$$(2) \quad \sum F_y = n + F\sin\theta - m_2 g = 0$$

$$(3) \quad \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

$$n = m_2 g - F\sin\theta$$

$$(4) \quad f_k = \mu_k(m_2 g - F\sin\theta)$$

$$F\cos\theta - \mu_k(m_2 g - F\sin\theta) - m_1(a + g) = m_2 a$$

$$(5) \quad a = \boxed{\frac{F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}}$$