Exercises on Oscillations and Waves

Exercise 1.1
You find a spring in the laboratory. When you hang 100 grams at the end of the spring it stretches 10 cm. You pull the 100 gram mass 6 cm from its equilibrium position and let it go at $t = 0$. Find an equation for the position of the mass as a function of time t.

Lets first find the period of the oscillations, then we can obtain an equation for the motion. The period $T = 2\pi\sqrt{m/k}$. The mass m is 0.1 Kg. To find k, we use the fact that 100 grams causes the spring to stretch an additional 10 cm. Since $F = k\Delta x$, we have

$$
\begin{aligned}
mg &= k\Delta x \\
0.1(9.8) &= k(0.1) \\
k &= 9.8 \; N/m
\end{aligned}
$$

The period of the motion is therefore $T = 2\pi\sqrt{0.1/9.8} \approx 0.635 \; sec$. At $t = 0$ the mass is at its maximum distance from the origin. Thus, $x(t) = 0.6 \; cos(2\pi t/T)$. Using $T = 0.635$ sec gives

$$
\begin{aligned}
x(t) &= 0.6 \; cos(2\pi t/0.635) \\
x(t) &\approx 0.6 \; cos(9.9t)
\end{aligned}
$$

The cosine function is the appropriate one, since at $t = 0$ the mass is at its maximum distance from equilibrium.

Exercise 1.2
Joan has two pendula, one has a length of 1 meter, and the other one is longer. She sets them both swinging at the same time. After the 1 meter pendulum has completed 12 oscillations, the longer one has only completed 11. How long is the longer pendulum?

One can solve this problem by taking the ratio of the equation for the periods of the two pendula. Since the period of a pendulum (for small amplitudes) is approximately $T = 2\pi\sqrt{l/g}$, we have

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{l_2}{g}}$$

for each of the pendula. Dividing these two equations we have

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} \tag{1}$$

Solving for l_2 we have

$$l_2 = l_1\left(\frac{T_2}{T_1}\right)^2 \tag{2}$$

Using $l_1 = 1\ meter$, and $T_2/T_1 = 12/11$ yields

$$l_2 = 1(12/11)^2 \approx 1.19\ meters \tag{3}$$

So the longer pendulum is 1.19 meters long.

Exercise 1.3
A spring is hanging freely from the ceiling. You attach an object to the end of the spring and let the object go. It falls down a distance 49 cm and comes back up to where it started. It continues to oscillate in simple harmonic motion going up and down a total distance of 49 cm from top to bottom. What is the period of the simple harmonic motion?

You might think that there is not enough information to solve the problem, but lets try and see if we need to know more about the system. Let the mass of the object be m and the spring constant be labeled k. The period of simple harmonic motion for an ideal spring is given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{4}$$

To solve for the period T, we need to know the ratio of m/k. What does the 49 cm tell us? This is the total distance from the top to the bottom of the simple harmonic motion. Let $d = 49\ cm$. This means that the equilibrium position lies $d/2$ from the

top. This is the distance ($d/2$ cm) that the mass m would stretch the spring due to its weight mg. In terms of the spring constant we have

$$mg = k(\frac{d}{2})$$
$$\frac{m}{k} = \frac{d}{2g}$$

Substituting into the equation for the period we have

$$T = 2\pi\sqrt{\frac{d}{2g}} \approx \frac{\pi}{\sqrt{10}} \approx 0.99 \; sec \tag{5}$$

since $d = 49$ cm, and we have used $g = 980 \; cm/s^2$. Looks like we had enough information after all.

Exercise 1.4
Consider the system shown in the figure, which consists of a rod of length L and mass M that can spin around its center. There is a little hole in the center of the rod, which allows the rod to rotate, without friction, about its center. There is a spring, with spring constant k, attached to one end. The rod is pulled to one side and set free to oscillate. Is the motion one of simple harmonic motion? What is the period of oscillation. Express your answer in terms of M, L, and k.

To determine if the motion is simple harmonic, we need to see if the restoring force from the equilibrium position is *proportional* to the displacement from equilibrium. Our system is a rod that can rotate about its center. The appropriate equation for rotation about a fixed axis is

$$I\alpha = \tau_{net} \tag{6}$$

where I is the rotational inertial about the axis of rotation, $\alpha = d\theta/dt$ is the angular acceleration about the axis, and τ_{net} is the net torque about the axis. The rotational inertial for a thin rod of mass M, length L, that rotates about its center is $ML^2/12$. The torque about an axis equals the force applied times the moment arm. In our case, if the angular displacements are small, the force is perpendicular to the rod. The torque is therefore given by $\tau = rF = (L/2)F$, where F is the force that the spring exerts on the end of the rod.

3

The force the spring exerts on the rod equals k times the extension of the spring. If $\theta = 0$ is the equilibrium position, then the spring exerts a force equal to $k(L/2)\theta$, where θ is the angle the rod is rotated from its equilibrium position. Putting the pieces together, the equation of motion for the rod is

$$
\begin{aligned}
I\alpha &= \tau_{net} \\
(\frac{ML^2}{12})\frac{d^2\theta}{dt^2} &= -rF \\
(\frac{ML^2}{12})\frac{d^2\theta}{dt^2} &= -\frac{L}{2}\,k(L\,\frac{\theta}{2})
\end{aligned}
$$

The minus sign is needed since the force is a restoring force. If θ is positive, the force (and the torque) is in the negative direction. If θ is negative, the force is in the positive direction. The spring always pushes or pulls the rod back to its equilibrium position. Simplifying the equation above gives

$$
\frac{d^2\theta}{dt^2} = -\frac{3k}{M}\,\theta \tag{7}
$$

since the L's cancel. The solution to this equation is $\theta(t) = A\,sin(\omega t)$, where $\omega = \sqrt{3k/M}$. So the period of the motion is $T = 2\pi/\omega$ or

$$
T = 2\pi\sqrt{\frac{M}{3k}} \tag{8}
$$

Exercise 1.5
Consider a spring that is standing on end in the vertical position. You place 100 grams on the spring and it compresses a distance of 9.8 cm.

a) If an additional 200 grams are placed on top of the 100 gram mass, how much will the spring compress?

This is an easy question if we assume that the spring obeys Hooke's Law. If 100 grams compresses the spring 9.8 cm, then 200 grams will compress the spring $2(9.8) = 19.6$ cm.

b) What is the spring constant?

The spring constant is defined in the equation $F_x = -kx$. Consider the 100 gram mass. It exerts a force of $F = mg = (0.1 \ kg)(9.8 \ m/s^2) = 0.98 \ N$ on the spring. The compression is $0.098 \ m$, so the spring constant is

$$k = \frac{F}{x} = \frac{0.98}{0.098} = 10 \ N/m \tag{9}$$

You would get the same result if you considered the 200 gram mass and its compression.

Exercise 1.6
A massless spring attached to a wall lies on a frictionless table. It has a block of mass 2 kg attached to one end. Initially the block is at rest. Another block, also of mass 2 kg is sliding along on the tabletop with a speed of 8 m/s. At time $t = 0$, the moving block collides with the block on the spring. The two stick together and oscillate back and forth. If the spring constant is $16 \ N/m$, find an expression $x(t)$ which describes the motion of the two blocks that are stuck together.

Since there are no external forces during the collision, linear momentum is conserved. Let v_f be the final speed after the collision. then

$$
\begin{aligned}
final \ total \ momentum &= initial \ total \ momentum \\
(m_1 + m_2)v_f &= m_1 v_0 \\
(2 + 2)v_f &= 2(8) \\
v_f &= 4 \ m/s
\end{aligned}
$$

This speed of 4 m/s is the initial speed for the oscillatory motion. Since the spring obeys Hooke's law, the motion is one of simple harmonic (i.e. sinusoidal) with $\omega = \sqrt{k/m} = \sqrt{16/4} = 2 \ s^{-1}$. The general expression for simple harmonic motion is:

$$x(t) = x_0 cos(\omega t) + \frac{v_0}{\omega} sin(\omega t) \tag{10}$$

For our example, $x_0 = 0$ since the blocks are at $x = 0$ at $t = 0$. Thus only the sin term is present:

$$x(t) = 2sin(2t) \ meters \tag{11}$$
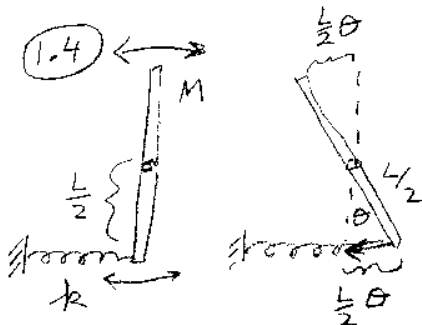
where t is in seconds.

(1.3)
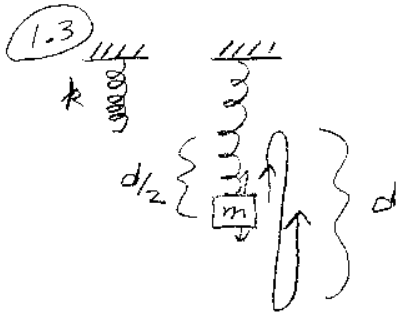
$$T = 2\pi \sqrt{\frac{m}{k}}$$

but $mg = k\left(\frac{d}{2}\right)$

$$\frac{m}{k} = \frac{d}{2g}$$

So $\quad T = 2\pi \sqrt{\frac{d}{2g}} = 2\pi \sqrt{\frac{49}{2(980)}} = 2\pi \sqrt{\frac{1}{40}} = \boxed{\frac{\pi}{\sqrt{10}}}$ sec
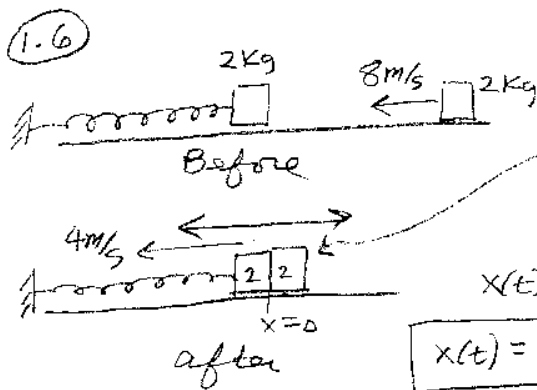
(1.4)

$$I \frac{d^2\theta}{dt^2} = \tau$$

$$I \frac{d^2\theta}{dt^2} = \frac{L}{2} F = -\frac{L}{2}\left(k \frac{L}{2}\theta\right)$$

$$\frac{ML^2}{12} \frac{d^2\theta}{dt^2} = -\frac{L^2}{4} k \theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{3k}{M}\theta \quad \Rightarrow \quad \boxed{T = 2\pi \sqrt{\frac{M}{3k}}}$$

(1.6)

2Kg   8m/s   2Kg

Before

4m/s

after $x=0$

$$2(8) = 4(v_0)$$

$$v_0 = 4 \ M/s$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{16}{4\,Kg}} = 2 \ (1/sec)$$

$$x(t) = \frac{v_0}{\omega} \sin \omega t$$

$$\boxed{x(t) = 2 \sin 2t}$$

Exercise 1.7

An object of mass m is oscillating back and forth in simple harmonic motion. The maximum distance from equilibrium is A, and the period of oscillation is T. At $t = 0$ the object is at the origin, $x = 0$, and moving in the $-x$ direction. Find the following in terms of m, T, and A:

a) The equation of motion of the object.

The general form for the equation of motion is $x(t) = x_0 cos(\omega t) + (v_0/\omega) sin\omega t$. In our case, $x_0 = 0$. Since the maximum amplitude is A, we have $|v_0/\omega| = A$. Since the object is initially moving in the "-" direction, $v_0 < 0$ so $v_0/\omega = -A$. Since $\omega = (2\pi)/T$, the equation of motion is therefore:

$$x(t) = -Asin(\frac{2\pi}{T}t) \tag{12}$$

b) The maximum speed of the object.

To find the velocity of the object for any time t, we can just differentiate $x(t)$. Thus,

$$v(t) = \frac{dx}{dt}$$

$$v(t) = -\frac{A2\pi}{T}cos(\frac{2\pi}{T}t)$$

using the chain rule. Since the cos function varies between -1 and $+1$, the maximum speed is $(2\pi A)/T$.

c) The maximum acceleration of the object.

To find the acceleration of the object for any time t, we can just differentiate $v(t)$:

$$a(t) = \frac{dv}{dt}$$

$$a(t) = (\frac{A2\pi}{T})^2 sin(\frac{2\pi}{T}t)$$

using the chain rule again. Since the sin function varies between -1 and $+1$, the maximum acceleration is $((2\pi A)/T)^2$.

d) The total energy of the object.

The total energy of an object moving in simple harmonic motion equals its kinetic energy as it passes through the equilibrium position. This is true since at $x = 0$ the object has no potential energy (i.e. it is at a minimum) and all of its energy is in the form of kinetic energy. Thus, we have

$$
\begin{aligned}
E_{tot} &= \frac{m}{2} v_{max}^2 \\
&= \frac{m}{2} \left(\frac{2\pi A}{T} \right)^2 \\
E_{tot} &= \frac{2\pi^2 m A^2}{T^2}
\end{aligned}
$$

Note: all our answers for this problem apply to any type of simple harmonic motion. The answers don't only apply to a mass on a spring.

Exercise 1.8
Consider the U-shaped tube containing a fluid shown in the figure. The cross sectional area of the tube is A, and the total length of the tube is l. The fluid is pushed up on one side and released. The fluid sloshes back and forth in periodic motion (assuming there are no frictional forces). Is the motion simple harmonic? If so, what is the period of the motion?

To determine what kind of motion will occur, we displace the fluid a distance y from equilibrium and determine what the net force is. Suppose the surface on the right side of the tube is raised a distance y. Then there is a total height difference of $2y$ above the surface of the left side of the tube. The net force on the fluid will be the weight of the fluid contained in a height of $2y$, which we call Δm. Since the cross-sectional area of the tube is A, the weight unbalance or net force equals $F_{net} = (\Delta m)g = (2y)A\rho g$, where ρ is the density of the fluid. Since the force acts in the opposite direction to y we have:
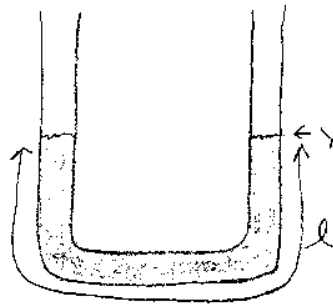
$$
F_{net} = -2A\rho g y \tag{13}
$$

The net force equals the total mass of the fluid times its acceleration:

$$
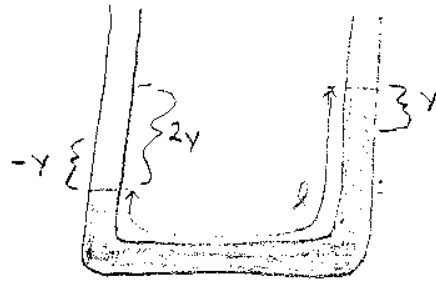m \frac{d^2 y}{dt^2} = -2A\rho g y \tag{14}
$$

8

(1.8)

EQUILIBRIUM

$$\left| F_{NET} \right| = (\Delta m)g = (2yA)\rho \, g$$

$$m \frac{d^2y}{dt^2} = -2yA\rho g$$

$$(\ell A \rho) \frac{d^2y}{dt^2} = -2yA\rho g$$

$$\frac{d^2y}{dt^2} = -\left(\frac{2g}{\ell}\right) y$$

$$\underset{\omega^2}{\uparrow}$$

$$\omega = \sqrt{\frac{2g}{\ell}} \qquad \text{or} \qquad \boxed{T = 2\pi \sqrt{\frac{\ell}{2g}}}$$

where m is the total mass of the fluid. Since the total length of the fluid is l, $m = Al\rho$. Subsituting this expression for m into the equation above yields:

$$Al\rho\frac{d^2y}{dt^2} = -2A\rho gy$$

$$\frac{d^2y}{dt^2} = -\frac{2g}{l}y$$

Thus, we see that the restoring force (i.e. or acceleration) is proportional to the displacement from equilibrium (y). The solution to the differential equation above is a perfect sinusoidal $y(t) = Asin(\omega t)$ where $\omega = \sqrt{2g/l}$. The force is a linear restoring force and **will produce simple harmonic motion**. The period T equals $2\pi/\omega$:

$$T = 2\pi\sqrt{\frac{l}{2g}} \tag{15}$$

Exercise 2.1
Eric has a semi-infinite string and is holding it at one end. He moves the end of the string up and down in a sinusoidal motion in time at a rate of 50 times per second. The distance from equilibrium that he moves the end is 0.05 meters. The speed of the wave on the string is 200; m/s. Find an equation for $y(x,t)$ that describes the displacement of the string from equilibrium in terms of x (the distance from the end) and t time.

Since he is moving the end in a perfect sinusoidal motion, the shape of the wave will be a sine wave. The wavelength λ of the traveling sine wave equals $\lambda = v/f = 200/50 = 4$ meters. So an equation for the wave can be

$$y(x,t) = Asin(\frac{2\pi}{\lambda}x - 2\pi ft)$$

$$= 0.05sin(\frac{\pi}{2}x - 100\pi t) \; meters$$

This is a sine wave that travels in the $+x$ direction. If the wave were to travel in the $-x$ direction, the function would be $y(x,t) = 0.05sin(\frac{\pi}{2}x + 100\pi t)$ meters. We could also add a phase in the sin argument.
Note: if he were to move the end up and down in a motion other than sinusoidal,

then the shape would not be a perfect sine wave.

Exercise 2.2
Eric takes the same string with the same tension as in Exercise 2.1, but cuts it so its length is 1 meters. He fixes both ends, and forms a standing wave with an amplitude of 0.05 meters. The standing wave produced is the fundamental. Find an equation for $y(x,t)$ that describes the displacement of the string from equilibrium in terms of x and t.

Since the tension is the same as before, the speed of the wave on the string is 200 m/s. The standing wave is the fundamental, which means that there is one anti-mode. Therefore, the wavelength is equal to twice the length of the string, $\lambda = 2L = 2\ meters$. The frequency of the oscillations is $f = v/\lambda = 200/2 = 100\ Hz$. For a standing wave, the space and time functions separate. Since the frequency is 100 Hz, the time function is $sin(2\pi100t) = sin(200\pi t)$. Since the wavelength is 2 meters, the space function is $sin(2\pi x/\lambda) = sin(\pi x)$. Therefore, the function $y(x,t)$ becomes:

$$y(x,t) = 0.05\ sin(\pi x)\ sin(200\pi t) \tag{16}$$

for

$$0 \leq x \leq 1\ meter \tag{17}$$

The equation is only valid for values of x between 0 and 1 meter, which is the length of the string. We could add a phase in the time part: $sin(200\pi t + \phi)$ which would depend on when we choose $t = 0$. However, we cannot add a phase to the position part, since $x = 0$ and $x = L = 1\ meter$ are nodes. $y(0,t) = y(L,t) = 0$ for all times. Also, note the difference between the traveling wave solution of Exercise 2.1 and the standing wave of this exercise.

Exercise 2.3
Gisela is standing on the side of the autobahn (freeway). Hans has just passed her traveling 44 m/s (100 mph) and is traveling north. Wolfram is traveling towards her from the south at a speed of 60 m/s (136 mph). He is also traveling towards the north. Wolfram honks his horn, which has a frequency of 400 Hz. Take the speed of sound in air to be 340 m/s. Find:

a) The frequency of the horn that Gisela hears.

Gisela is at rest in the medium, and Wolfram is traveling towards her. Thus, she will hear the note with a frequency higher than 400 Hz. We can use the Doppler formula with the observer at rest with respect to the medium and the source moving in the medium:

$$
\begin{aligned}
f &= f_0 \left(\frac{1 \pm v_o/V}{1 \pm v_s/V}\right) \\
&= 400 \, \frac{1}{1 - (60/340)} \\
f &\approx 486 \; Hz
\end{aligned}
$$

Since the observer is at rest in the medium, $v_o = 0$. The sign in the denominator is negative since Gisela hears a higher frequency (i.e. the source is moving towards her). If Wolfram were moving away from her, there would be a + sign in the denominator.

b) The frequency of the horn that Hans hears.

Once again we can use the Doppler formula:

$$
\begin{aligned}
f &= f_0 \left(\frac{1 \pm v_o/V}{1 \pm v_s/V}\right) \\
&= 400 \frac{1 - (44/340)}{1 - (60/340)} \\
f &\approx 423 \; Hz
\end{aligned}
$$

Hans the observer is moving in the medium. Since Hans is moving away from the source, his motion will lower the frequency resulting in a negative sign in the numerator. Since the source (Wolfram) is moving towards the observer, the sources motion will tend to increase the frequency Hence, the minus sign in the denominator. If they were to both travel at the same speed, then Hans would hear the note unshifted in frequency. Remember that when using the Doppler formula for sound, the speeds are with respect to the medium.

Exercise 2.4
Flipper is driving her car along the road. To determine her speed, she drives her car

towards a big wall and honks the horn. The horn has a frequency of 400 Hz. She hears the sound that is reflected off the wall, and notices that the two notes, the horn and reflected wave, result in a beat frequency of 4 beats/sec. How fast is she traveling?

At the wall, the note will be heard with a higher frequency than 400 Hz. Since she is traveling towards the wall, she will hear a still higher note. Since the beat frequency is 4 Hz, the note that she hears after the sound has bounced off the wall is $(400 + 4) = 404\ Hz$.

It is probably easiest to break the problem up into two parts. First find the frequency that a person standing next to the wall would hear the sound as. Then, take this frequency as the source that Flipper is traveling towards. Since the wall is stationary, $v_o = 0$. Let flippers speed be v. This gives

$$f_{at\ wall} = 400\left(\frac{1}{1 - v/340}\right) \tag{18}$$

In this case, Flipper's horn is the source of the sound. The wall is the observer and is stationary. There is a minus sign in the denominator since she is moving towards the wall (i.e. the frequency at the wall is greater than 400 Hz.) The frequency $f_{at\ wall}$ is the frequency in air reflected back from the wall. Flipper is driving towards the wall, so she will hear the higher frequency of 404. Thus, we have
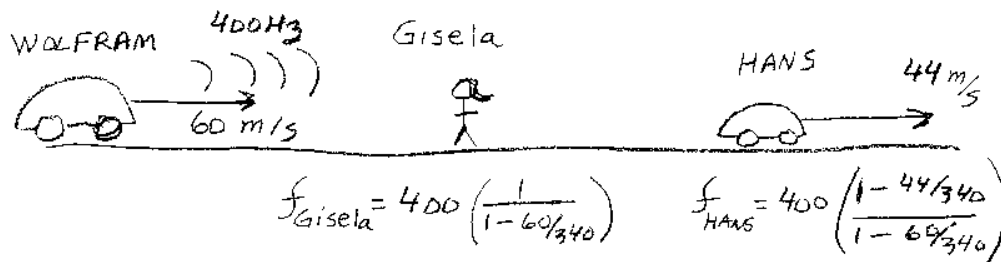
$$
\begin{aligned}
404 &= f_{at\ wall}(1 + v/340) \\
404 &= 400\left(\frac{1}{1 - v/340}\right)(1 + v/340) \\
404 &= 400\,\frac{1 + v/340}{1 - v/340}
\end{aligned}
$$

We can solve the last equation for v, which gives $v \approx 1.69$ m/s. This is the method that dolphins use to determine if they are closing in on their prey.
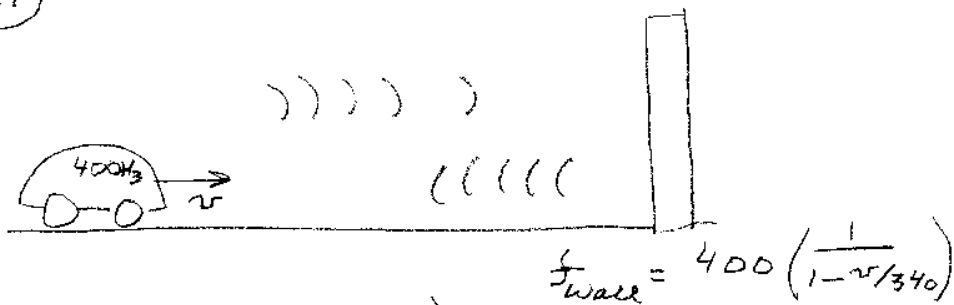
Exercise 2.5
John has two identical guitar strings of the same length. One has a tension F_1. When he plucks this string it produces a sound of 440 Hz for its fundamental note. The second string has a tension that is 1 % higher than the first, $F_2 = 1.01F_1$. If both string are plucked at the same time, what is the beat frequency?

(2.3)

WOLFRAM   400Hz   Gisela   HANS   44 m/s

60 m/s

$$f_{Gisela} = 400\left(\dfrac{1}{1 - \frac{60}{340}}\right) \qquad f_{HANS} = 400\left(\dfrac{1 - \frac{44}{340}}{1 - \frac{60}{340}}\right)$$

(2.4)

400Hz v

$$f_{wall} = 400\left(\dfrac{1}{1 - v/340}\right)$$

$$404 = 400\left(\dfrac{1 + v/340}{1 - v/340}\right)$$

$$404\left(1 - \frac{v}{340}\right) = 400\left(1 + \frac{v}{340}\right)$$

$$404\left(340 - v\right) = 400\left(340 + v\right)$$

$$340(4) = v(804)$$

$$v = \frac{340(4)}{804} \simeq 1.69 \, m/s$$

14

Lets first find the frequency that the second string will produce when plucked. Since the string is vibrating in its fundamental mode, $f = v/(2L)$. In terms of the tension F, $v = \sqrt{F/\mu}$, and therefore $f = \sqrt{F/\mu}/(2L)$. This equation is valid for each string:

$$f_2 = \sqrt{\frac{F_2}{\mu}}/(2L)$$

$$f_1 = \sqrt{\frac{F_1}{\mu}}/(2L)$$

Dividing the two equations yields:

$$\frac{f_2}{f_1} = \sqrt{\frac{F_2}{F_1}}$$

$$f_2 = f_1\sqrt{\frac{F_2}{F_1}}$$

$$f_2 = 440\sqrt{1.01}$$

$$f+2 \approx 442\ Hz$$

So John will hear a beat frequency of $f_{beat} = 442 - 440 = 2$ beats/sec.

Exercise 2.6
Hector is attending a rock concert and is sitting 2 meters from the speaker. The music is loud, and he measures the intensity level to be 120 db. He knows that this is bad for his ears, so he wants to move to where the intensity level is only 90 db. How far away from the speaker should he be?

Lets first change intensity level to intensity in unit of W/m^2. 120 db is a sound intensity of 1 W/m^2, and 90 db is a sound intensity of $10^{-3}\ W/m^2$. Sound intensity, I, decreases as the inverse square law:

$$I = \frac{P}{4\pi r^2} \tag{19}$$

where P is the power output of the point source (i.e. speaker). This can be re-written as

15

$$Ir^2 = \frac{P}{4\pi} \qquad (20)$$

Since the power is the same no matter where Hector sits, we must have

$$I_2 r_2^2 = I_1 r_1^2 \qquad (21)$$

where position 1 is at 2 meters, and position 2 is far away. Substuting in the appropriate quantities, we have

$$
\begin{aligned}
10^{-3} r_2^2 &= (1)2^2 \\
r_2 &= 2\sqrt{1000}
\end{aligned}
$$

$$r_2 \approx 63 \ meters$$

So Hector should move 63 meters away from the speaker. The actual power P that the speaker is putting out is $P = I(4\pi r^2) = 1(4\pi 2^2) \approx 50$ Watts.

noindent **Exercise 2.7**

Lorraine has two organ pipes. Both pipes are open at both ends. One pipe is 60 cm long, and the other is 61 cm long. If both are played in their fundamental mode at the same time, what is the beat frequency? Take the speed of sound in air to be 340 m/s.

The beat frequency is the difference in frequency of the two pipes. Let's find the fundamental frequency of each pipe. Since the pipes are open at both ends, the fundamental frequency will have the length equal to 1/2 the wavelength. Thus, the fundamental frequency f is related to the length L by

$$f = \frac{v}{2L} \qquad (22)$$

where v is the speed of sound. The first pipe has a frequency of $f_1 = 340/(2(0.6)) \approx$ 283.3 Hz. The frequency of the second pipe is $f_2 = 340/(2(0.61)) \approx 278.7$ Hz. So the beat frequency is $f_{beat} = f_1 - f_2 \approx 283.3 - 278.7 \approx 4.6$ Hz. This means that the sound will have around 4.6 loud-soft oscillations per second.

Exercise 2.8

Eva notices that the "A" string on her violin is a little flat. She measures the frequency to be 436 Hz. The "A" note should be 440 Hz. The tension in the string is F_1. To obtain the proper note, she needs to increase the tension to F_2. What is the proper tension F_2 in terms of F_1?

We need to determine how the fundamental frequency depends on tension F. If L is the length of the string, then $f = v/(2L)$. The speed v depends on the tension according to $v = \sqrt{F/\mu}$. Substituting into the equation for f, we have

$$f = \frac{1}{2L}\sqrt{\frac{F}{\mu}} \tag{23}$$

Since the length of the string L and the mass per length μ are not changed, we see that the fundamental frequency is proportional to the square-root of F. Taking the ratio of f_2/f_1, L and μ cancel giving:

$$\frac{f_2}{f_1} = \sqrt{\frac{F_2}{F_1}} \tag{24}$$

Solving for F_2, we have

$$
\begin{aligned}
F_2 &= F_1(\frac{f_2}{f_1})^2 \\
&= F_1(\frac{440}{436})^2 \\
F_2 &\approx 1.02 F_1
\end{aligned}
$$

So Eva needs to increase the tension in the string by around 2%.

Exercise 2.9

My friend Soumya has given me an exotic wind instrument from the villages of Bengal. I am not sure if it will behave as an open-open pipe, or an open-closed pipe. I make overtone notes, and produce successive resonant frequencies of 510, 850, and 1190 Hz. Is the instrument an open-open pipe or an open-closed pipe? What is the fundamental frequency?

If the instrument is an open-open pipe, the resonant frequencies are

$$f_n = \frac{nv}{2L} \qquad (25)$$

where $n = 1,\ 2,\ 3,\ \cdots$, an integer. If the instrument is an open-closed pipe, the resonant frequencies are

$$f_n = \frac{nv}{4L} \qquad (26)$$

where $n = 1,\ 3,\ 5,\ \cdots$, an odd integer. In either case, the difference of successive resonant frequencies is $v/(2L)$. For our instrument, the difference of successive resonant frequencies is $(850 - 510) = 340$ Hz, which means $v/(2L) = 340$ Hz. If the instrument were open-open, then all overtones would be multiples of 340 Hz. Since 510, 850, and 1190 are not multiples of 340 the instrument does not behave like an open-open pipe. It must be an open-closed pipe. Let's see if it checks out. The fundamental frequency is $f_1 = v/(4L) = (1/2)v/(2L) = (1/2)340 = 170$ Hz. The frequency 510 is 3 times f_1, 850 is 5 times f_1, and 1190 is 7 times f_1. So the measured frequencies are successive resonances for an open-closed pipe that has a fundamental frequency of 170 Hz.

Exercise 2.10
Two speakers are located 3 meters apart and are emitting sine wave tones **coherently**. George is standing at the point **A**, which is 6 meters away from the line joining the two speakers. At the point **A** the sound he hears from the speakers is very loud, i.e. constructive interference, since he is equal distance from the speakers. He now walks "north" as shown in the figure. At first the sound becomes soft, then when he reaches the point labeled **B** the sound from the speakers is loud, i.e. constructive interference. If the distance from **A** to **B** is 1.5 meters, what is the frequency of the sine wave tone? Take the speed of sound in air to be 340 m/s.

Since there is constructive interference when George is at point **B**, the distance from **B** to one speaker "1" minus the distance from **B** to speaker "2" must equal a mulitple of the wave length. This is the first maximum that George hears, so $d_1 - d_2 = \lambda$, where d_1 is the distance to speaker "1" and d_2 is the distance to speaker "2". Using the distances in the figure, we have

$$
\begin{aligned}
\lambda &= d_1 - d_2 \\
&= \sqrt{3^2 + 6^2} - 6 \\
\lambda &= 0.708 \ meters
\end{aligned}
$$

So the frequency of the sine wave tone is $f = 340/0.708 \approx 480$ Hz.

Exercise 2.11
Sally's roommate likes to listen to pure sine wave tones. She sets up two speakers at each end of the room. The speakers are 10 meters apart and are each producing the exact same wave **coherently** that is a single sine wave of 170 Hz. Sally likes to study in the middle of the room, which is 5 meters from each speaker. However, the constructive interference at this point results in a very loud sound and Sally cannot study well. How far towards one of the speakers should she move so that the waves from the speakers cancel, allowing her to study better? Take the speed of sound in air to be 340 m/s.

To have the sound waves produce destructive interference, the distance from Sally to one speaker minus her distance to the other speaker must be one-half a wavelength. The wavelength of the note is $\lambda = 340/170 = 2$ meters. Suppose Sally moves a distance x to the right as shown in the figure. Then the distance to speaker "1" is $d_1 = 5+x$, and the distance to speaker "2" is $d_2 = 5-x$. For destructive interference, $d_1 - d_2$ must equal $\lambda/2$:
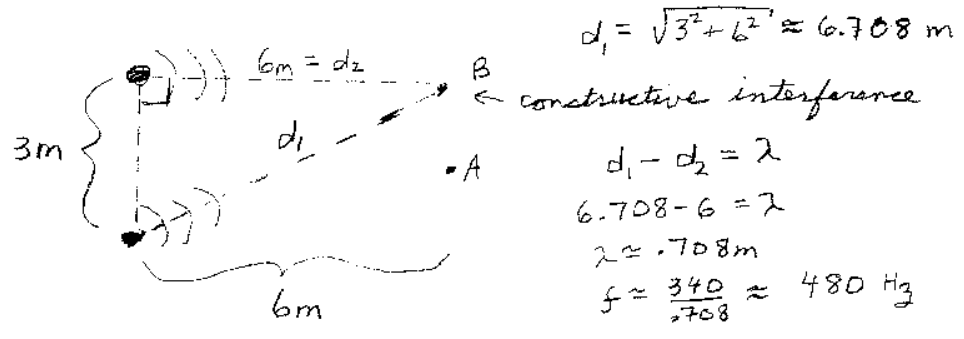
$$
\begin{aligned}
\frac{\lambda}{2} &= d_1 - d_2 \\
\frac{2}{2} &= (5+x) - (5-x) \\
1 &= 2x \\
x &= \frac{1}{2} \; meter
\end{aligned}
$$

So if she moves 0.5 meters to the right, she will be 5.5 meters from the speaker on the left, and 4.5 meters from the speaker on the right. The difference in the distance from the speakers is $(5.5 - 4.5)$ or 1 meter. This difference is one half a wavelength, which results in the cancelation of the two waves. Now she can study her physics better.
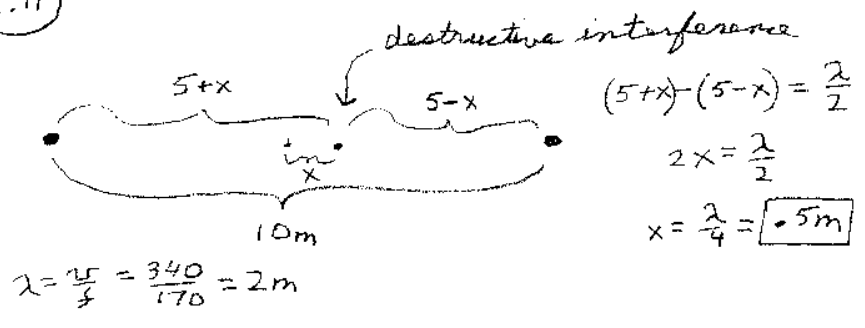
Exercise 2.12
Consider the set-up shown in the figure. There are two sources of sound waves that are producing the exact same sound wave (i.e. **coherent**)that is a single sine function of frequency 340 Hz. The speakers are separated by a distance of 20 cm. The
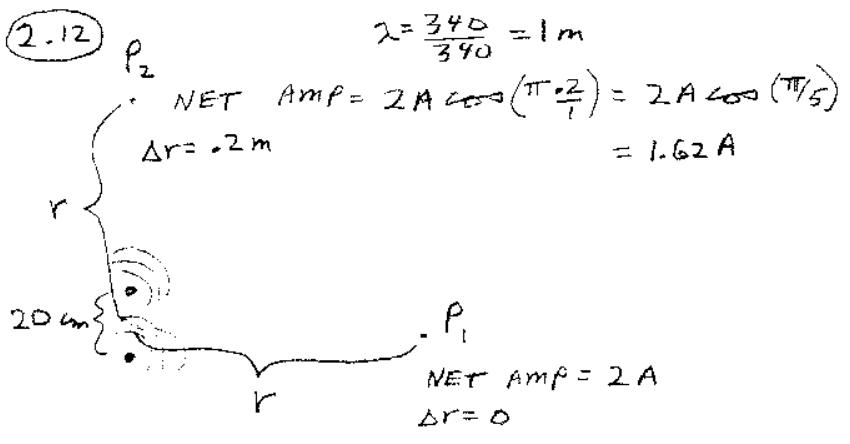
19

(2.10)

$d_1 = \sqrt{3^2 + 6^2} \approx 6.708\ m$

6m = d_2 B   ← constructive interference

3m

d_1

• A

6m

$d_1 - d_2 = \lambda$

$6.708 - 6 = \lambda$

$\lambda \approx .708m$

$f \approx \dfrac{340}{.708} \approx 480\ Hz$

(2.11)

destructive interference

5+x   5−x

10m

$\lambda = \dfrac{v}{f} = \dfrac{340}{170} = 2m$

$(5+x) - (5-x) = \dfrac{\lambda}{2}$

$2x = \dfrac{\lambda}{2}$

$x = \dfrac{\lambda}{4} = \boxed{.5m}$

(2.12) P_2

$\lambda = \dfrac{340}{340} = 1m$

NET AMP $= 2A \cos\left(\pi \cdot \dfrac{.2}{1}\right) = 2A \cos(\pi/5)$

$\Delta r = .2\ m$

$= 1.62\ A$

r

20 cm

P_1

r

NET AMP $= 2A$

$\Delta r = 0$

point P_1 is located a distance r from the center of the speakers perpendicular to the line joining them. The point P_2 is located the same distance r from the center of the speakers along the line joining them. Harvey stands at the point P_1, and Ellen stands at the point P_2. How does the amplitude of the sound that Ellen hears compare to the amplitude that Harvey hears? Take the speed of sound to be 340 m/s.

As discussed in lecture (see the lecture notes), the net amplitude from two coherent identical sinusoidal sources equals *Net Amplitude* $\approx 2A\cos(\pi\Delta r/\lambda)$. In this expression, Δr is the difference in distance from the observer to the sources. We also assume that the amplitude at the observer, A, is the same from each source. In our case, Ellen (point P_2) is closer to one speaker than the other. However, we will assume that $r \gg 20\ cm$, so at Ellen's location the amplitude is essentially the same for each source of sound. Since $f = 340$ Hz, we have $\lambda = 340/340 = 1$ meter. So, the net amplitude that Harvey experiences is

$$\begin{aligned}
(Net\ Amplitude)_1 &= 2A\cos(\pi(0)/1) \\
&= 2A
\end{aligned}$$

At Harvey's location (P_1) $\Delta r = 0$, since each speaker is the same distance away from him. The net amplitude that Ellen experiences is

$$\begin{aligned}
(Net\ Amplitude)_2 &= 2A\cos(\pi(0.2)/1) \\
&= 2A\cos(\pi/5) \\
&\approx 1.62A
\end{aligned}$$

At Ellen's location (P_2) one speaker is 20 cm closer than the other to her, so $\Delta r = 0.2$ meters. So we see that the amplitude at point P_2 is $(1.62/2) = 0.81$ or 81% the amplitude at point P_1.

We note that if the two sources were not coherent, the interference effects would average to zero, and the average amplitude at point P_1 would be the same as that at point P_2.