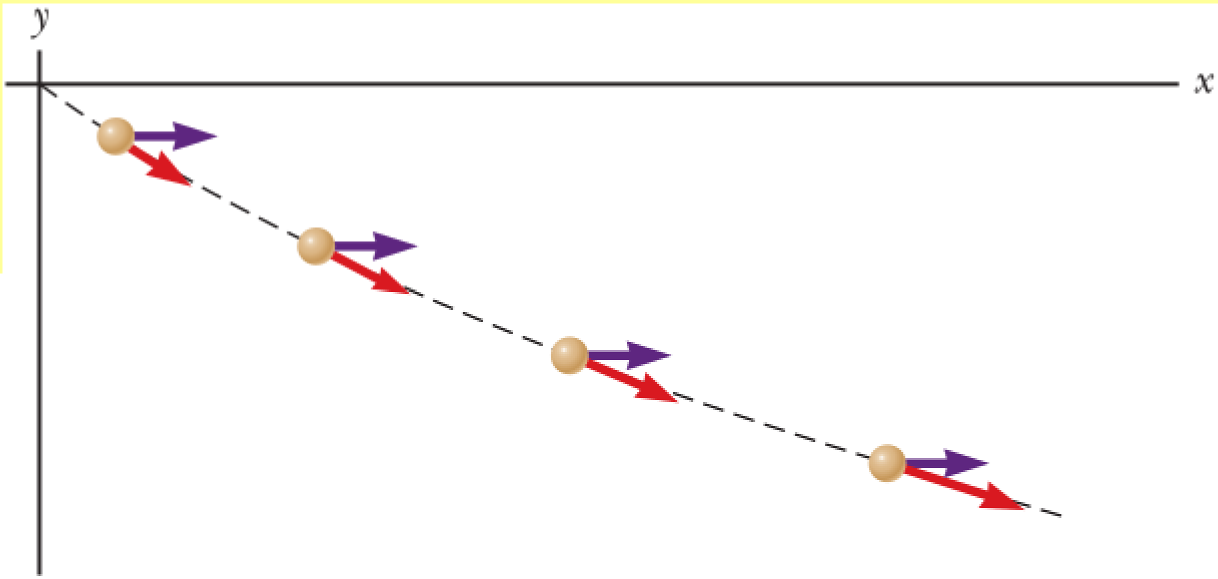
# Example 4.1 Motion in a Plane

A particle moves in the *xy* plane, starting from the origin at $t = 0$ with an initial velocity having an $x$ component of 20 m/s and a $y$ component of –15 m/s. The particle experiences an acceleration in the x direction, given by $a_x = 4.0\ \text{m/s}^2$.

(A) Determine the total velocity vector at any time.

(B) Calculate the velocity and speed of the particle at $t = 5.0$ s and the angle the velocity vector makes with the $x$ axis.

(C) Determine the $x$ and $y$ coordinates of the particle at any time $t$ and its position vector at this time.



$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = (v_{xi} + a_x t)\hat{\mathbf{i}} + (v_{yi} + a_y t)\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}_f = [20 + (4.0)t]\hat{\mathbf{i}} + [-15 + (0)t]\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}_f = \boxed{[(20 + 4.0t)\hat{\mathbf{i}} - 15\hat{\mathbf{j}}]}$$

$$\vec{\mathbf{v}}_f = [(20 + 4.0(5.0))\hat{\mathbf{i}} - 15\hat{\mathbf{j}}] = \boxed{(40\hat{\mathbf{i}} - 15\hat{\mathbf{j}})\text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15\text{ m/s}}{40\text{ m/s}}\right) = \boxed{-21^\circ}$$

$$v_f = |\vec{\mathbf{v}}_f| = \sqrt{{v_{xf}}^2 + {v_{yf}}^2} = \sqrt{(40)^2 + (-15)^2}\text{ m/s} = \boxed{43\text{ m/s}}$$

$$x_f = v_{xi}t + \tfrac{1}{2}a_x t^2 = \boxed{20t + 2.0t^2}$$

$$y_f = v_{yi}t = \boxed{-15t}$$

$$\vec{\mathbf{r}}_f = x_f\hat{\mathbf{i}} + y_f\hat{\mathbf{j}} = \boxed{(20t + 2.0t^2)\hat{\mathbf{i}} - 15t\hat{\mathbf{j}}}$$

## Example 4.2 The Long Jump

A long jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(A) How far does he jump in the horizontal direction?

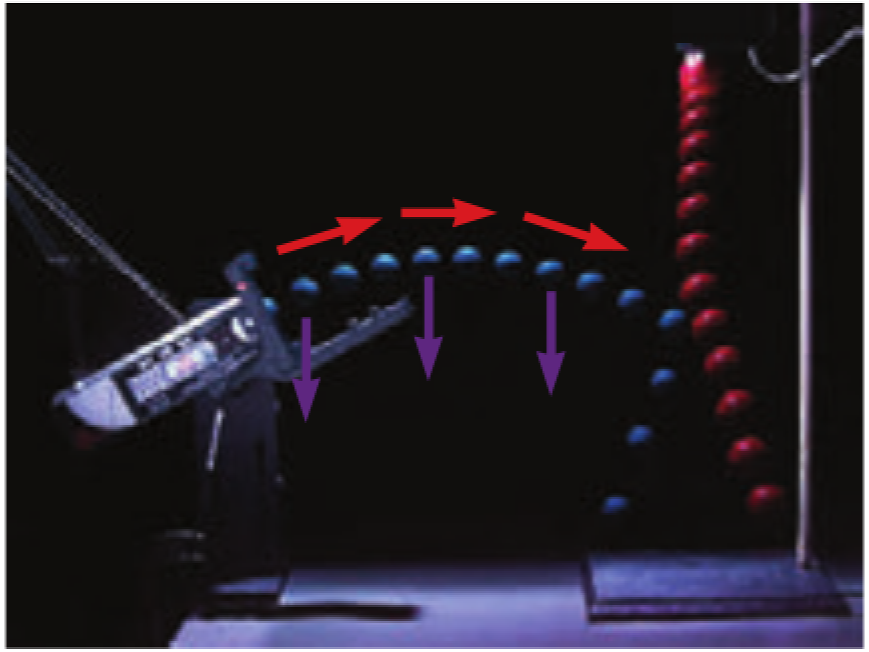
(B) What is the maximum height reached?

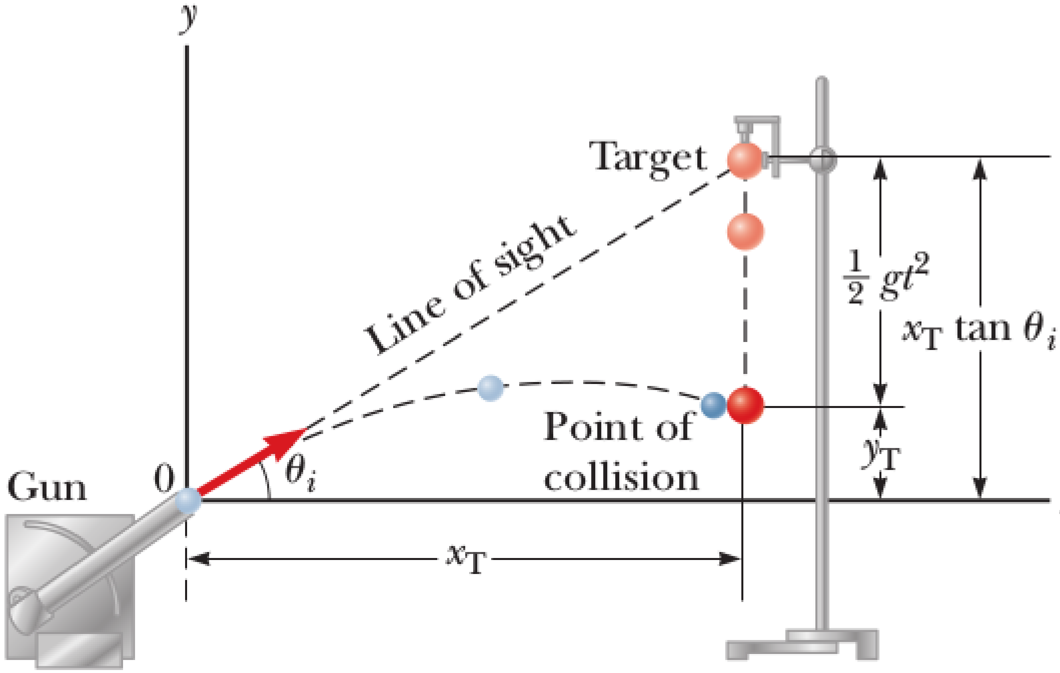
$$R = \frac{v_i^{\,2} \sin 2\theta_i}{g} = \frac{(11.0 \text{ m/s})^2 \sin 2(20.0°)}{9.80 \text{ m/s}^2} = \boxed{7.94 \text{ m}}$$

$$h = \frac{v_i^{\,2} \sin^2\theta_i}{2g} = \frac{(11.0 \text{ m/s})^2 (\sin 20.0°)^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.722 \text{ m}}$$

## Example 4.3 A Bull's-Eye Every Time

In a popular lecture demonstration, a projectile is fired at a target in such a way that the projectile leaves the gun at the same time the target is dropped from rest. Show that if the gun is initially aimed at the stationary target, the projectile hits the falling target as shown in the figure.





(1) $y_T = y_{iT} + (0)t - \frac{1}{2}gt^2 = x_T \tan\theta_i - \frac{1}{2}gt^2$

(2) $y_P = y_{iP} + v_{yiP}t - \frac{1}{2}gt^2 = 0 + (v_{iP}\sin\theta_i)t - \frac{1}{2}gt^2 = (v_{iP}\sin\theta_i)t - \frac{1}{2}gt^2$

$x_P = x_{iP} + v_{xiP}t = 0 + (v_{iP}\cos\theta_i)t = (v_{iP}\cos\theta_i)t$

$$t = \frac{x_P}{v_{iP}\cos\theta_i}$$

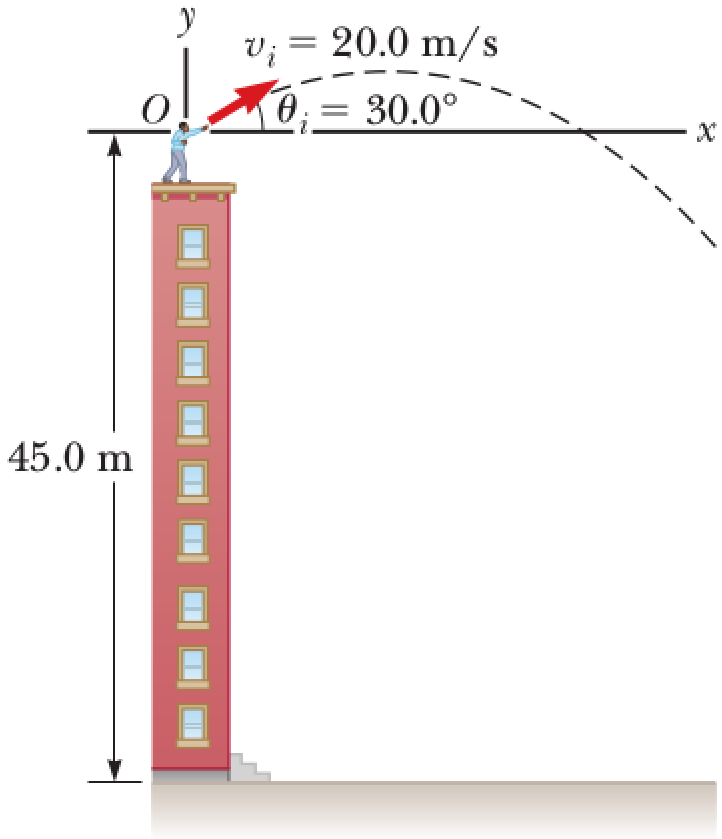
(3) $y_P = (v_{iP}\sin\theta_i)\left(\dfrac{x_P}{v_{iP}\cos\theta_i}\right) - \frac{1}{2}gt^2 = x_P\tan\theta_i - \frac{1}{2}gt^2$

## Example 4.4 That's Quite an Arm!

A stone is thrown from the top of a building upward at an angle of 30.0º to the horizontal with an initial speed of 20.0 m/s as shown in the figure. The height from which the stone is thrown is 45.0 m above the ground.

(A) How long does it take the stone to reach the ground?

(B) What is the speed of the stone just before it strikes the ground?



$$v_{xi} = v_i \cos\theta_i = (20.0\text{ m/s})\cos 30.0° = 17.3\text{ m/s}$$

$$v_{yi} = v_i \sin\theta_i = (20.0\text{ m/s})\sin 30.0° = 10.0\text{ m/s}$$

$$y_f = y_i + v_{yi}t - \tfrac{1}{2}gt^2$$

$$-45.0\text{ m} = 0 + (10.0\text{ m/s})t + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$t = \boxed{4.22\text{ s}}$$

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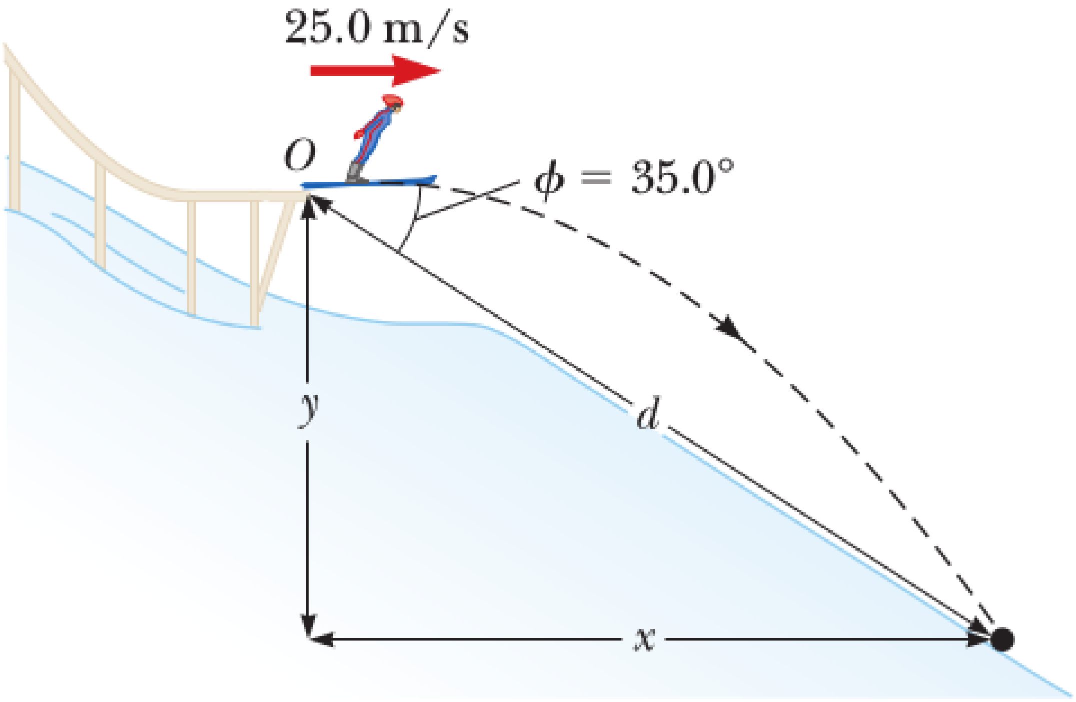
$$v_{yf} = v_{yi} - gt$$

$$v_{yf} = 10.0\text{ m/s} + (-9.80\text{ m/s}^2)(4.22\text{ s}) = -31.3\text{ m/s}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(17.3\text{ m/s})^2 + (-31.3\text{ m/s})^2} = \boxed{35.8\text{ m/s}}$$

## Example 4.5 The End of the Ski Jump

A ski jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s as shown in the figure. The landing incline below her falls off with a slope of 35.0º. Where does she land on the incline?



$$(1) \quad x_f = v_{xi} t$$

$$(2) \quad y_f = v_{yi} t - \tfrac{1}{2} g t^2$$

$$(3) \quad d \cos \phi = v_{xi} t$$

$$(4) \quad -d \sin \phi = -\tfrac{1}{2} g t^2$$

$$-d \sin \phi = -\tfrac{1}{2} g \left( \frac{d \cos \phi}{v_{xi}} \right)^2$$

$$d = \frac{2 v_{xi}{}^2 \sin \phi}{g \cos^2 \phi} = \frac{2(25.0 \text{ m/s})^2 \sin 35.0^\circ}{(9.80 \text{ m/s}^2) \cos^2 35.0^\circ} = 109 \text{ m}$$

$$x_f = d \cos \phi = (109 \text{ m}) \cos 35.0^\circ = \boxed{89.3 \text{ m}}$$

$$y_f = -d \sin \phi = -(109 \text{ m}) \sin 35.0^\circ = \boxed{-62.5 \text{ m}}$$

## Example 4.6 The Centripetal Acceleration of the Earth

(A) What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

(B) What is the angular speed of the Earth in its orbit around the Sun?

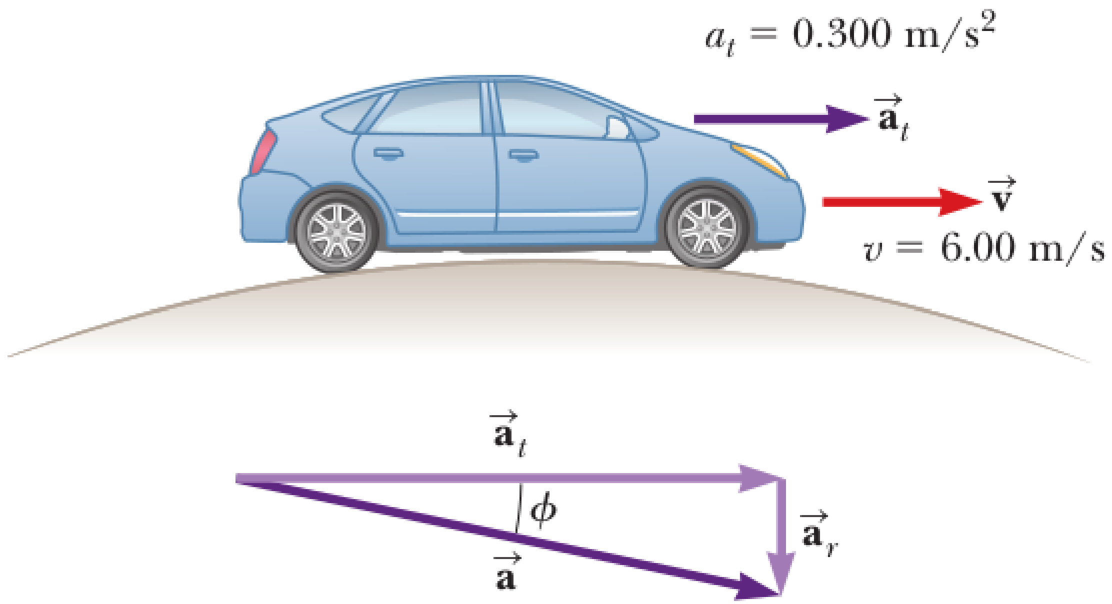
$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$a_c = \frac{4\pi^2(1.496 \times 10^{11}\ \text{m})}{(1\ \text{yr})^2}\left(\frac{1\ \text{yr}}{3.156 \times 10^7\ \text{s}}\right)^2 = \boxed{5.93 \times 10^{-3}\ \text{m/s}^2}$$

$$\omega = \frac{2\pi}{1\ \text{yr}}\left(\frac{1\ \text{yr}}{3.156 \times 10^7\ \text{s}}\right) = \boxed{1.99 \times 10^{-7}\ \text{s}^{-1}}$$

# Example 4.7 Over the Rise

A car leaves a stop sign and exhibits a constant acceleration of 0.300 m/s² parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like an arc of a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What are the magnitude and direction of the total acceleration vector for the car at this instant?



$$a_r = -\frac{v^2}{r} = -\frac{(6.00\ \text{m/s})^2}{500\ \text{m}} = -0.072\,0\ \text{m/s}^2$$

$$\sqrt{a_r^2 + a_t^2} = \sqrt{(-0.072\,0\ \text{m/s}^2)^2 + (0.300\ \text{m/s}^2)^2}$$

$$= \boxed{0.309\ \text{m/s}^2}$$

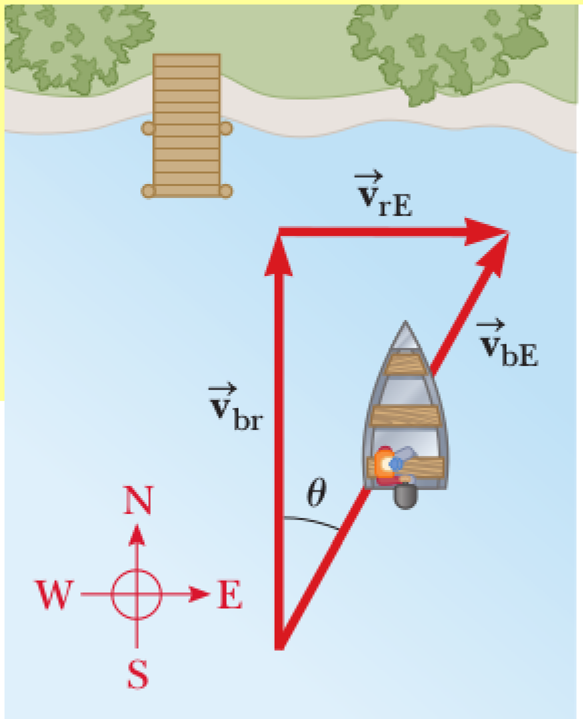
$$\phi = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\left(\frac{-0.072\,0\ \text{m/s}^2}{0.300\ \text{m/s}^2}\right) = \boxed{-13.5°}$$

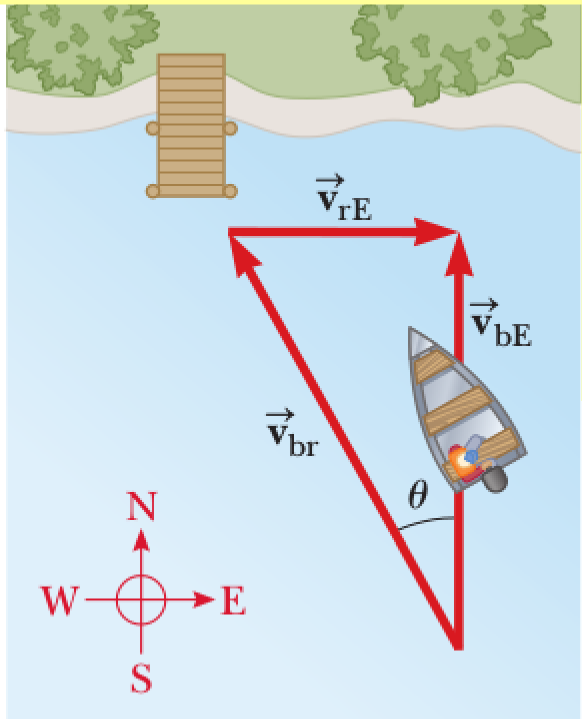
# Example 4.8 A Boat Crossing a River

A boat crossing a wide river moves with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

(A) If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.

(B) If the boat travels with the same speed of 10.0 km/h relative to the river and is to travel due north, what should its heading be?



$$v_{bE} = \sqrt{{v_{br}}^2 + {v_{rE}}^2} = \sqrt{(10.0\ \mathrm{km/h})^2 + (5.00\ \mathrm{km/h})^2}$$

$$\boxed{= 11.2\ \mathrm{km/h}}$$

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = \boxed{26.6^\circ}$$

$$v_{bE} = \sqrt{{v_{br}}^2 - {v_{rE}}^2} = \sqrt{(10.0\ \mathrm{km/h})^2 - (5.00\ \mathrm{km/h})^2} = 8.66\ \mathrm{km/h}$$

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{bE}}\right) = \tan^{-1}\left(\frac{5.00}{8.66}\right) = \boxed{30.0^\circ}$$