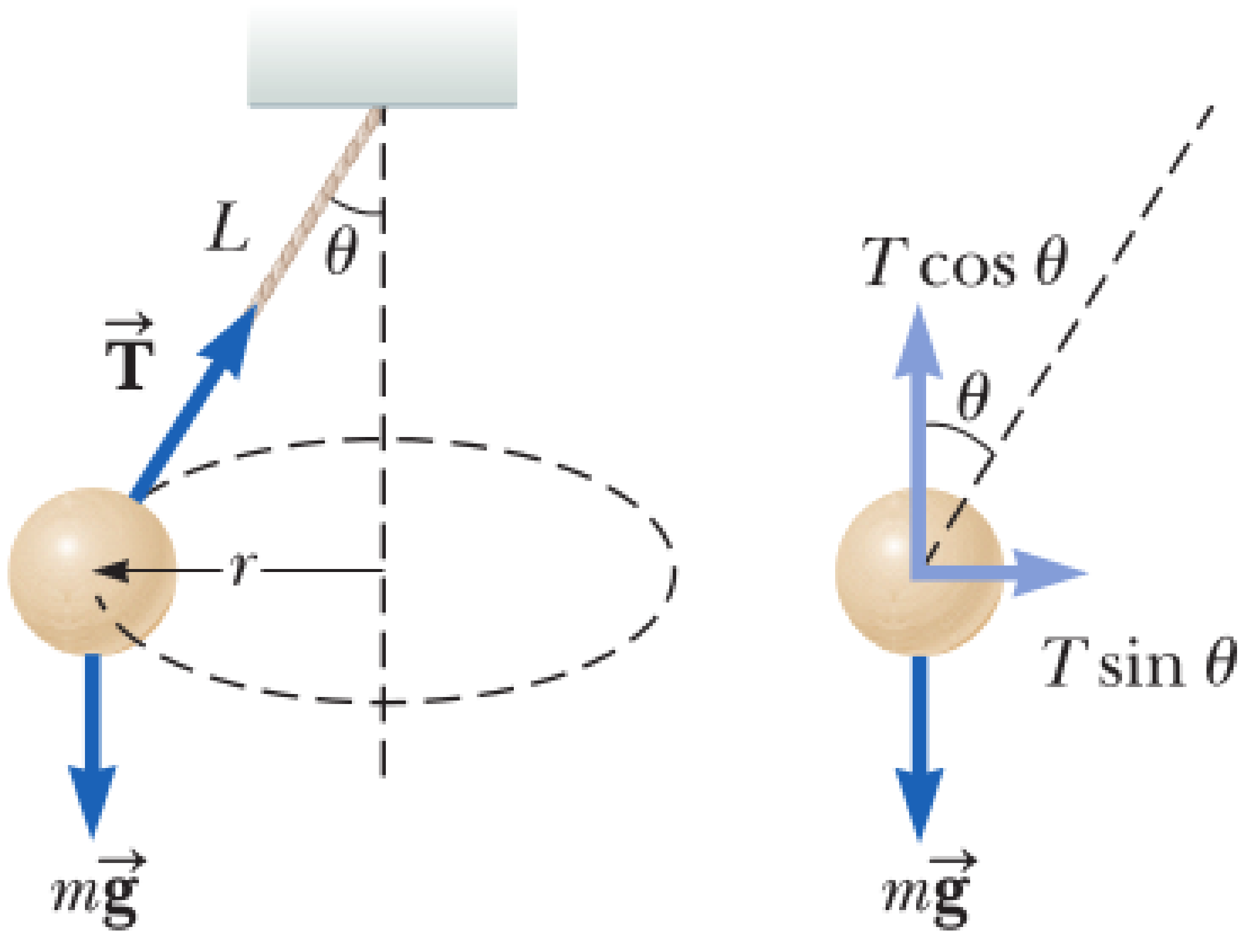
Example 6.1 The Conical Pendulum

A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r as shown in the figure. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum*.) Find an expression for v.



$$\sum F_y = T\cos\theta - mg = 0$$

$$(1)\ T\cos\theta = mg$$

$$(2)\ \sum F_x = T\sin\theta = ma_c = \frac{mv^2}{r}$$

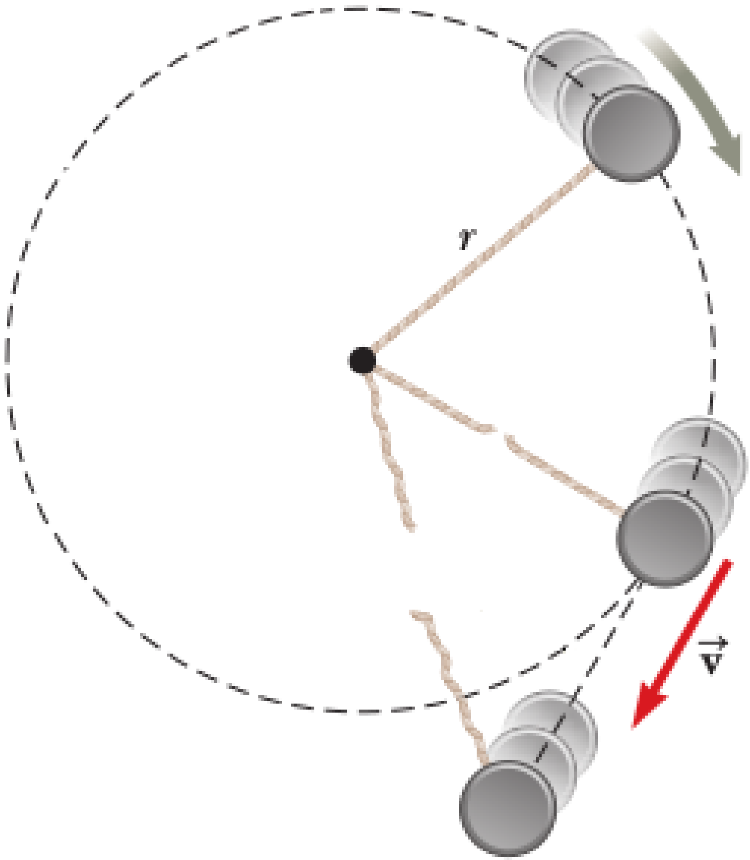
$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

$$v = \boxed{\sqrt{Lg\sin\theta\tan\theta}}$$

Example 6.2 How Fast Can It Spin?

A puck of mass 0.500 kg is attached to the end of a cord 1.50 m long. The puck moves in a horizontal circle as shown in the figure. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the puck can move before the cord breaks?



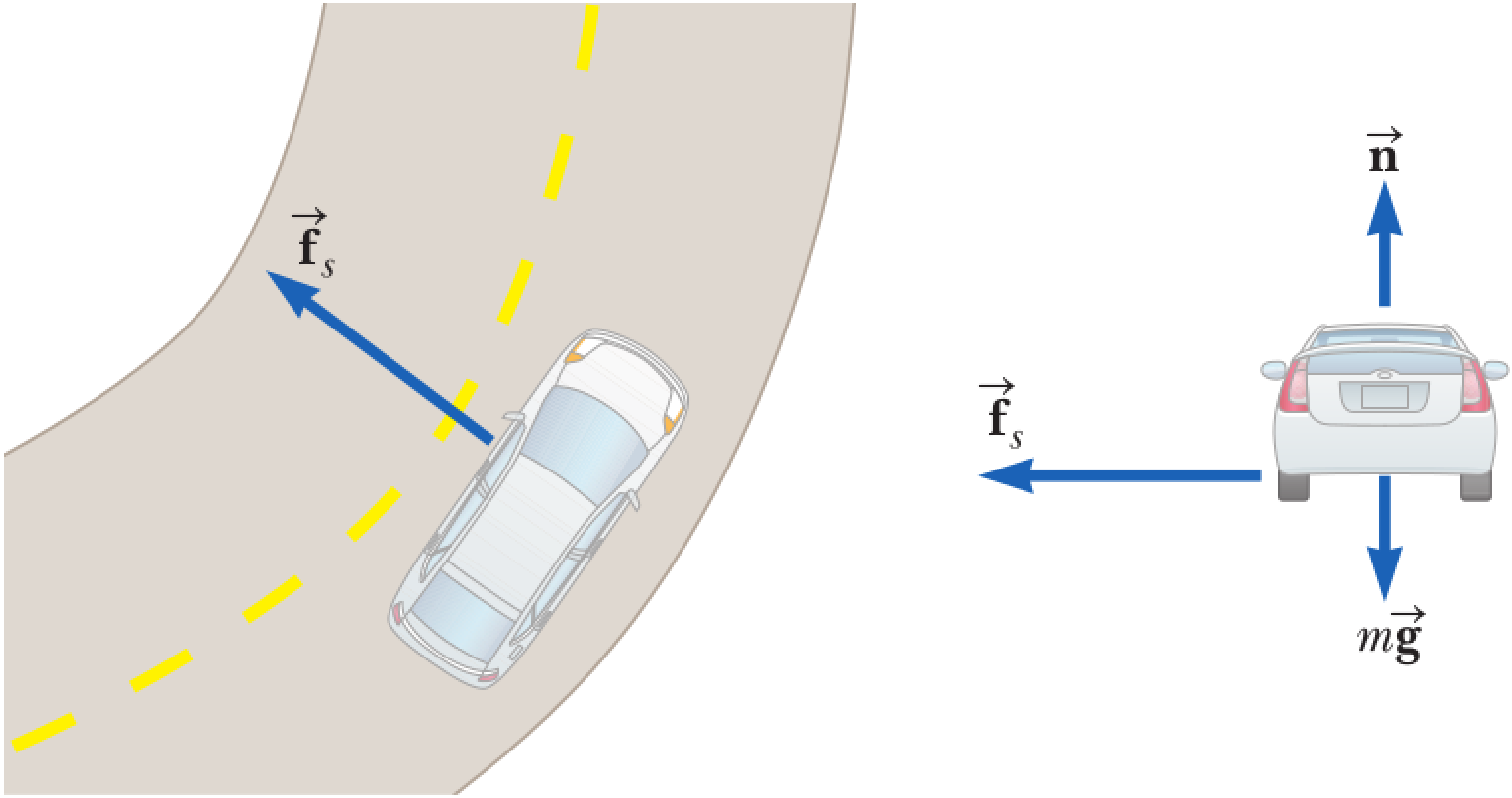
$$T = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{Tr}{m}}$$

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0\ \mathrm{N})(1.50\ \mathrm{m})}{0.500\ \mathrm{kg}}} = \boxed{12.2\ \mathrm{m/s}}$$

Example 6.3 What Is the Maximum Speed of the Car?

A 1500-kg car moving on a flat, horizontal road negotiates a curve as shown in the figure. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.



$$(1) \quad f_{s,\max} = \mu_s n = m\,\frac{v_{\max}^2}{r}$$

$$\sum F_y = 0 \;\rightarrow\; n - mg = 0 \;\rightarrow\; n = mg$$

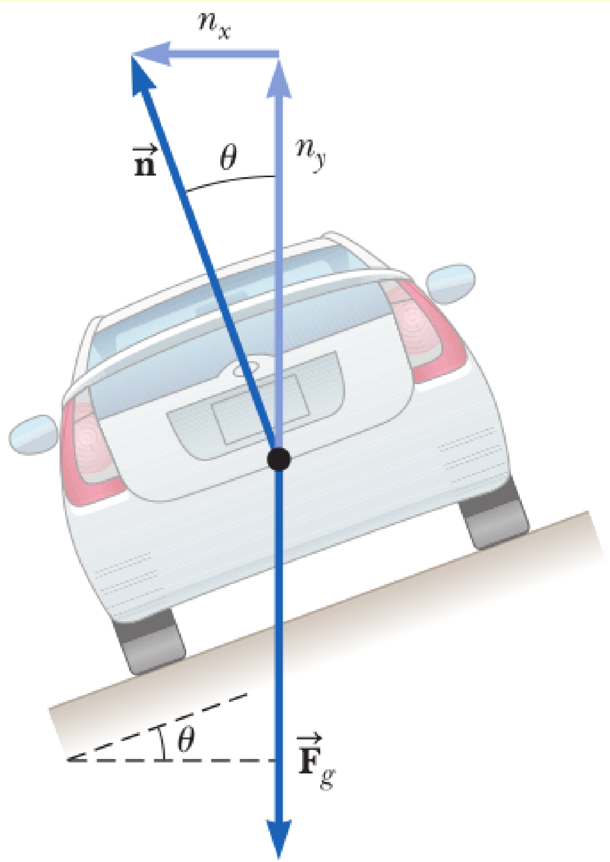
$$(2) \quad v_{\max} = \sqrt{\frac{\mu_s n r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

$$v_{\max} = \sqrt{(0.523)(9.80\ \mathrm{m/s^2})(35.0\ \mathrm{m})} = \boxed{13.4\ \mathrm{m/s}}$$

Example 6.4 The Banked Roadway

A civil engineer wishes to redesign the curved roadway in Example 6.3 in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a road is usually *banked*, which means that the roadway is tilted toward the inside of the curve as seen in the figure. Suppose the designated speed for the ramp is to be 13.4 m/s and the radius of the curve is 35.0 m.

At what angle should the curve be banked?



$$(1)\quad \sum F_r = n \sin\theta = \frac{mv^2}{r}$$

$$\sum F_y = n\cos\theta - mg = 0$$

$$(2)\quad n\cos\theta = mg$$

$$(3)\quad \tan\theta = \frac{v^2}{rg}$$

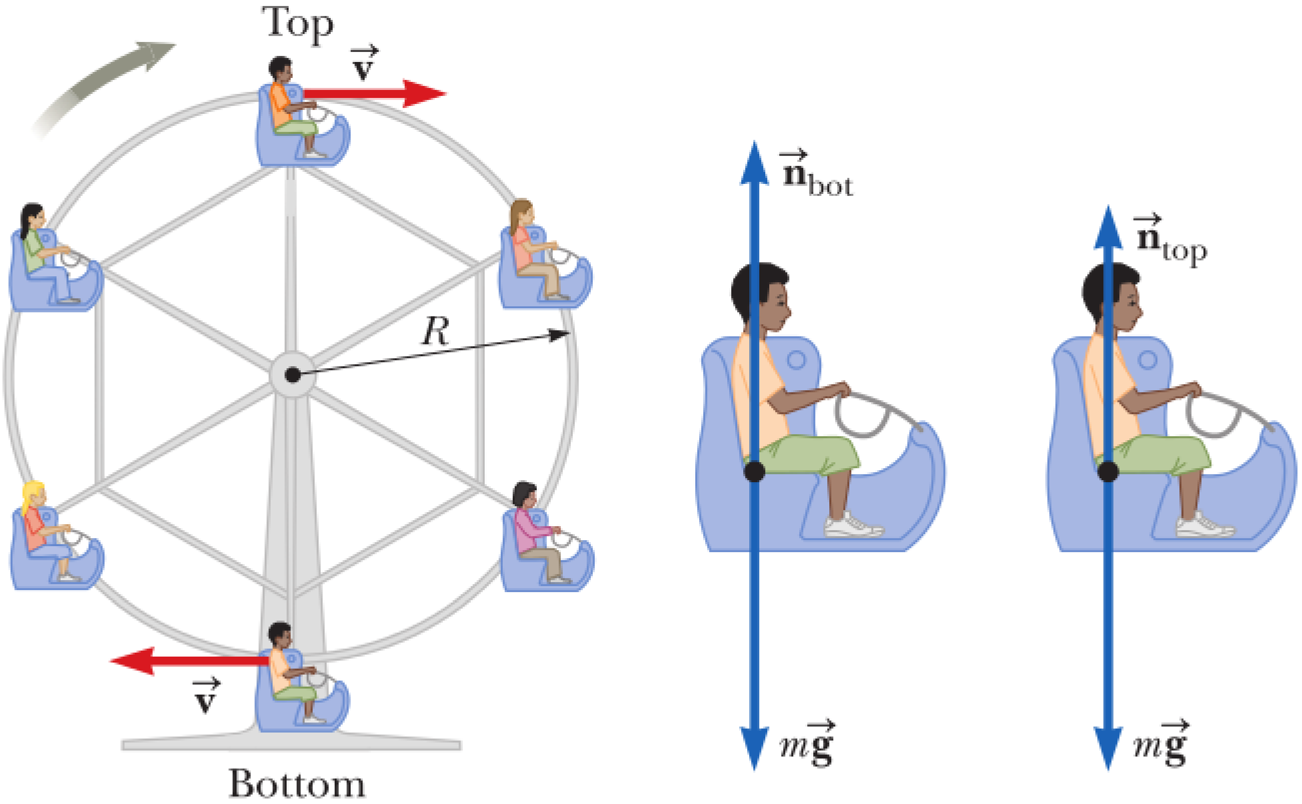
$$\theta = \tan^{-1}\left[\frac{(13.4\ \text{m/s})^2}{(35.0\ \text{m})(9.80\ \text{m/s}^2)}\right] = \boxed{27.6^\circ}$$

Example 6.5 Riding the Ferris Wheel

A child of mass m rides on a Ferris wheel as shown in the figure. The child moves in a vertical circle of radius 10.0 m at a constant speed of 3.00 m/s.

(A) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child, mg.

(B) Determine the force exerted by the seat on the child at the top of the ride.



$$\sum F = n_{\text{bot}} - mg = m\frac{v^2}{r}$$

$$n_{\text{bot}} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

$$n_{\text{bot}} = mg\left[1 + \frac{(3.00\text{ m/s})^2}{(10.0\text{ m})(9.80\text{ m/s}^2)}\right]$$

$$= \boxed{1.09\,mg}$$

$$\sum F = mg - n_{\text{top}} = m\frac{v^2}{r}$$

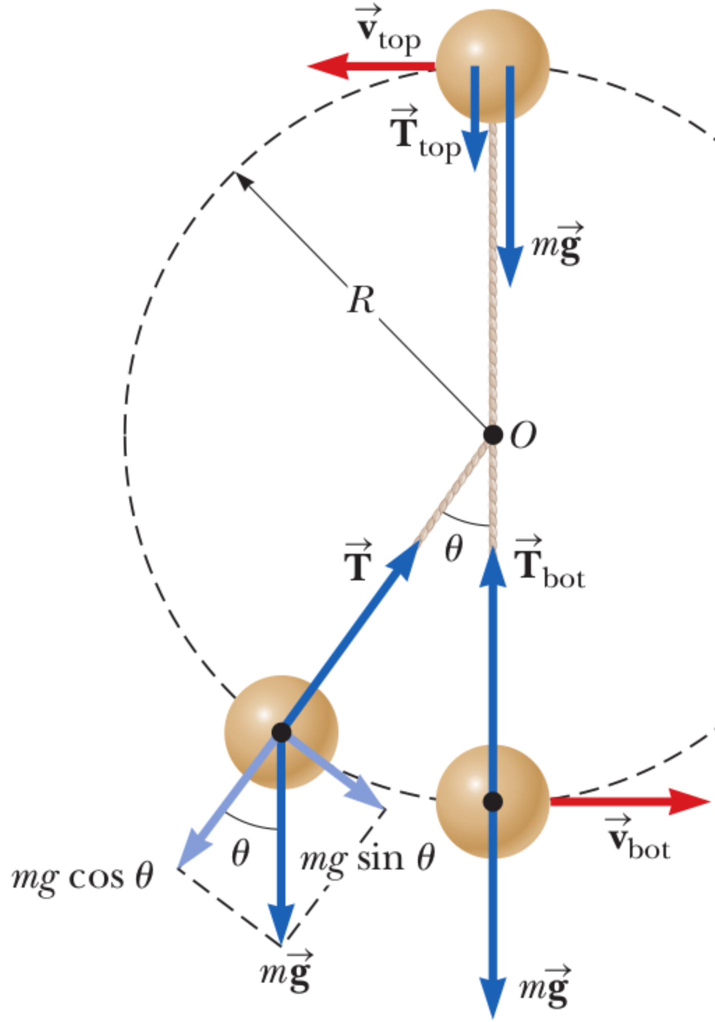
$$n_{\text{top}} = mg - m\frac{v^2}{r} = mg\left(1 - \frac{v^2}{rg}\right)$$

$$n_{\text{top}} = mg\left[1 - \frac{(3.00\text{ m/s})^2}{(10.0\text{ m})(9.80\text{ m/s}^2)}\right]$$

$$= \boxed{0.908\,mg}$$

~~Example 6.6~~ ~~Keep Your Eye on the Ball~~

A small sphere of mass m is attached to the end of a cord of length R and set into motion in a vertical circle about a fixed point O. Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.



$$\sum F_t = mg \sin\theta = ma_t$$

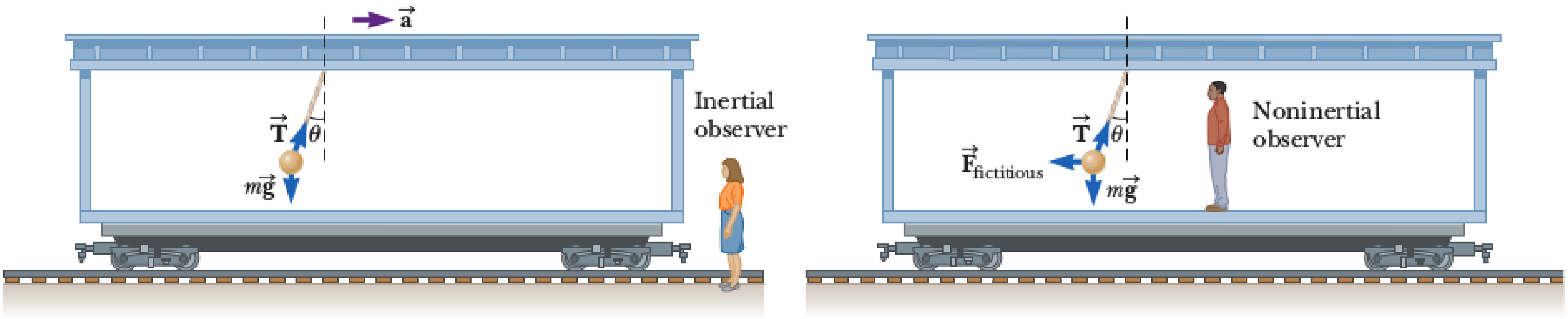
$$a_t = \boxed{g \sin\theta}$$

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = \boxed{mg\left(\frac{v^2}{Rg} + \cos\theta\right)}$$

Example 6.7 Fictitious Forces in Linear Motion

A small sphere of mass *m* hangs by a cord from the ceiling of a boxcar that is accelerating to the right as shown in the figure. Both the inertial observer on the ground and the noninertial observer on the train agree that the cord makes an angle θ with respect to the vertical. The noninertial observer claims that a force, which we know to be fictitious, causes the observed deviation of the cord from the vertical. How is the magnitude of this force related to the boxcar's acceleration measured by the inertial observer.



$$
\text{Inertial observer} \begin{cases} (1) \quad \sum F_x = T\sin\theta = ma \\ (2) \quad \sum F_y = T\cos\theta - mg = 0 \end{cases}
$$

$$
\text{Noninertial observer} \begin{cases} \sum F'_x = T\sin\theta - F_{\text{fictitious}} = 0 \\ \sum F'_y = T\cos\theta - mg = 0 \end{cases}
$$

Example 6.8 Sphere Falling in Oil

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time at which the sphere reaches 90.0% of its terminal speed.

$$b = \frac{mg}{v_T} = \frac{(2.00\ \text{g})\,(980\ \text{cm/s}^2)}{5.00\ \text{cm/s}} = 392\ \text{g/s}$$

$$\tau = \frac{m}{b} = \frac{2.00\ \text{g}}{392\ \text{g/s}} = \boxed{5.10 \times 10^{-3}\ \text{s}}$$

$$0.900\,v_T = v_T(1 - e^{-t/\tau})$$

$$1 - e^{-t/\tau} = 0.900$$

$$e^{-t/\tau} = 0.100$$

$$-\frac{t}{\tau} = \ln\ (0.100) = -2.30$$

$$t = 2.30\tau = 2.30(5.10 \times 10^{-3}\ \text{s}) = 11.7 \times 10^{-3}\ \text{s}$$

$$= \boxed{11.7\ \text{ms}}$$

Conceptual Example 6.9 The Skysurfer

Consider a skysurfer who jumps from a plane with his feet attached firmly to his surfboard, does some tricks, and then opens his parachute.
Describe the forces acting on him during these maneuvers.

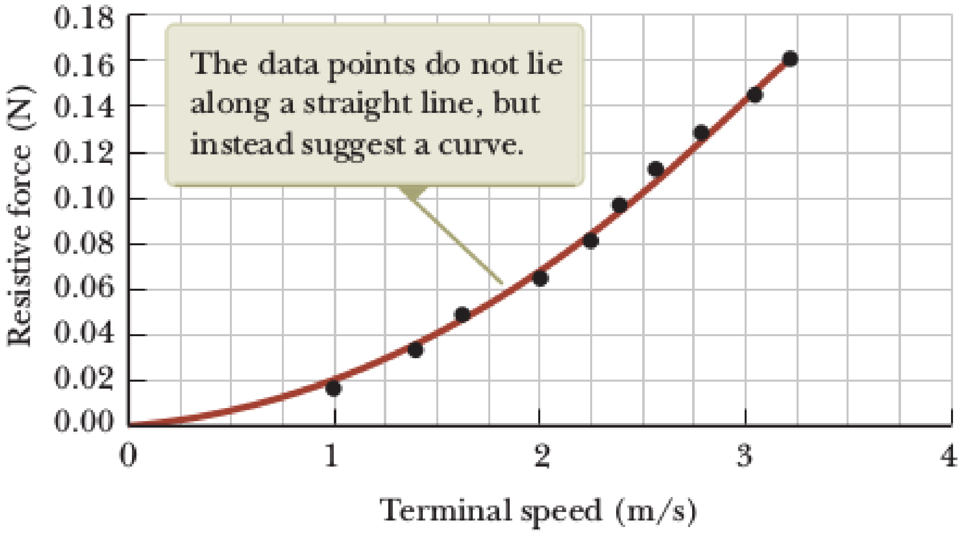
Example 6.10 Falling Coffee Filters

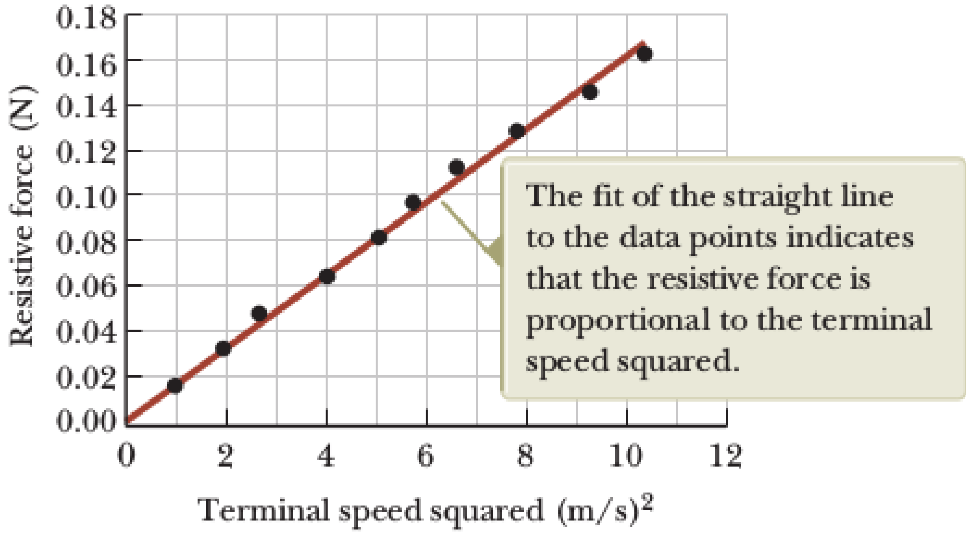
The dependence of resistive force on the square of the speed is a simplification model. Let's test the model for a specific situation. Imagine an experiment in which we drop a series of bowl-shaped, pleated coffee filters and measure their terminal speeds, see table for typical terminal speed data from a real experiment using these coffee filters as they fall through the air. The time constant τ is small, so a dropped filter quickly reaches terminal speed. Each filter has a mass of 1.64 g. When the filters are nested together, they combine in such a way that the front-facing surface area does not increase. Determine the relationship between the resistive force exerted by the air and the speed of the falling filters.

Terminal Speed and Resistive Force for Nested Coffee Filters

Number of Filters	v_T (m/s)[a]	R (N)
1	1.01	0.016 1
2	1.40	0.032 2
3	1.63	0.048 3
4	2.00	0.064 4
5	2.25	0.080 5
6	2.40	0.096 6
7	2.57	0.112 7
8	2.80	0.128 8
9	3.05	0.144 9
10	3.22	0.161 0

[a]All values of v_T are approximate.


0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Resistive force (N)
The data points do not lie along a straight line, but instead suggest a curve.
0
1
2
3
4
Terminal speed (m/s)


0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Resistive force (N)
The fit of the straight line to the data points indicates that the resistive force is proportional to the terminal speed squared.
0
2
4
6
8
10
12
Terminal speed squared (m/s)²

Example 6.11 Resistive Force Exerted on a Baseball

A pitcher hurls a 0.145-kg baseball past a batter at 40.2 m/s. Find the resistive force acting on the ball at this speed.

$$D = \frac{2mg}{v_T^2 \rho A}$$

$$R = \tfrac{1}{2} D\rho A v^2 = \frac{1}{2}\left(\frac{2mg}{v_\tau^2 \rho A}\right)\rho A v^2 = mg\left(\frac{v}{v_\tau}\right)^2$$

$$R = (0.145\ \mathrm{kg})(9.80\ \mathrm{m/s^2})\left(\frac{40.2\ \mathrm{m/s}}{43\ \mathrm{m/s}}\right)^2 = 1.2\ \mathrm{N}$$