assn1_3

March 3, 2025

1 3 Numerical Application: Generation of Efficient Frontier

2 Environment

I am using Python here because only Python, R, and Matlab are allowed.

2.1 Imports

[44]:
```
import yfinance as yf
import polars as pl
import numpy as np
import scipy.optimize as opt
import matplotlib.pyplot as plt
```

3 1.

Download data.

[19]:
```
tickers = ["META", "AMZN", "AAPL", "NFLX", "GOOG"]
start = "2022-01-01"
end = "2024-12-31"

data = yf.download(tickers, start=start, end=end)["Close"]
data
```

```
[*********************100%***********************]  5 of 5 completed
```

[19]:

Ticker	AAPL	AMZN	GOOG	META	NFLX
Date					
2022-01-03	178.879898	170.404495	144.555084	337.251709	597.369995
2022-01-04	176.609650	167.522003	143.899460	335.249359	591.150024
2022-01-05	171.911865	164.356995	137.160675	322.936432	567.520020
2022-01-06	169.042068	163.253998	137.058533	331.194855	553.289978
2022-01-07	169.209137	162.554001	136.513992	330.527435	541.059998
…	…	…	…	…	…
2024-12-23	254.989655	225.059998	195.990005	599.849976	911.450012
2024-12-24	257.916443	229.050003	197.570007	607.750000	932.119995
2024-12-26	258.735504	227.050003	197.100006	603.349976	924.140015

```
2024-12-27  255.309296  223.750000  194.039993  599.809998  907.549988
2024-12-30  251.923019  221.300003  192.690002  591.239990  900.429993

[752 rows x 5 columns]
```

Convert to Polars DataFrame.

[20]:
```python
df = pl.from_pandas(data.reset_index())
df = df.rename({"Date": "date"})
df
```

[20]: shape: (752, 6)

date	AAPL	AMZN	GOOG	META	NFLX
---	---	---	---	---	---
datetime[ns]	f64	f64	f64	f64	f64
2022-01-03 00:00:00	178.879898	170.404495	144.555084	337.251709	597.369995
2022-01-04 00:00:00	176.60965	167.522003	143.89946	335.249359	591.150024
2022-01-05 00:00:00	171.911865	164.356995	137.160675	322.936432	567.52002
2022-01-06 00:00:00	169.042068	163.253998	137.058533	331.194855	553.289978
2022-01-07 00:00:00	169.209137	162.554001	136.513992	330.527435	541.059998
…	…	…	…	…	…
2024-12-23 00:00:00	254.989655	225.059998	195.990005	599.849976	911.450012
2024-12-24 00:00:00	257.916443	229.050003	197.570007	607.75	932.119995
2024-12-26 00:00:00	258.735504	227.050003	197.100006	603.349976	924.140015
2024-12-27 00:00:00	255.309296	223.75	194.039993	599.809998	907.549988
2024-12-30 00:00:00	251.923019	221.300003	192.690002	591.23999	900.429993

4 2.

Compute daily and annualized return.

[25]:
```python
returns_df = (
    df.with_columns(
        [(pl.col(ticker) / pl.col(ticker).shift(1) - 1).
  ↪alias(f"{ticker}_daily_return") for ticker in tickers]
    )
    .drop_nulls()
    .with_columns(
        [(pl.col(f"{ticker}_daily_return") * 252).
  ↪alias(f"{ticker}_annualized_return") for ticker in tickers]
    )

)
returns_df
```

[25]:
```
shape: (751, 16)


  date        AAPL        AMZN        GOOG        …   AMZN_annu   AAPL_annu
NFLX_annu   GOOG_ann
  ---         ---         ---         ---             alized_re   alized_re
alized_re   ualized_
  datetime[   f64         f64         f64             turn        turn
turn        return
  ns]                                                 ---         ---
---         ---
                                                      f64         f64
f64         f64


  2022-01-0   176.60965   167.52200   143.89946   …   -4.262728   -3.19825
-2.623889   -1.14293
  4                       3
  7
  00:00:00

  2022-01-0   171.91186   164.35699   137.16067   …   -4.761059   -6.703154
-10.07318   -11.8011
  5           5           5           5                                     1
  13
  00:00:00

  2022-01-0   169.04206   163.25399   137.05853   …   -1.691167   -4.206742
-6.318668   -0.18766
```

6 8 8 3
2
00:00:00

2022-01-0 169.20913 162.55400 136.51399 … -1.08052 0.249058
-5.570235 -1.00120
7 7 1 2
9
00:00:00

2022-01-1 169.22880 161.48599 138.07786 … -1.655684 0.029292
-0.563571 2.886855
0 6 2 6

00:00:00

… … … … … … … …
…
2024-12-2 254.98965 225.05999 195.99000 … 0.156855 0.772369
0.665317 3.957088
3 5 8 5

00:00:00

2024-12-2 257.91644 229.05000 197.57000 … 4.467615 2.892473
5.714889 2.031535
4 3 3 7

00:00:00

2024-12-2 258.73550 227.05000 197.10000 … -2.200393 0.800273
-2.157399 -0.59948
6 4 3 6
5
00:00:00

2024-12-2 255.30929 223.75 194.03999 … -3.662633 -3.337016
-4.523867 -3.91234
7 6 3
5
00:00:00

2024-12-3 251.92301 221.30000 192.69000 … -2.759326 -3.342384
-1.977014 -1.75323
0 9 3 2
5
00:00:00

5 3.

Compute expected returns vector μ.

[26]:
```
mu = (
    returns_df.select([pl.col(f"{ticker}_annualized_return").mean().
    ↪alias(f"{ticker}_mean") for ticker in tickers])
    .to_numpy()
    .flatten()
)
mu
```

[26]:
```
array([0.30920058, 0.16117727, 0.15153147, 0.26472372, 0.15015158])
```

Compute covariance matrix Σ.

[27]:
```
annualized_returns_np = returns_df.select([pl.
    ↪col(f"{ticker}_annualized_return") for ticker in tickers]).to_numpy()
Sigma = np.cov(annualized_returns_np, rowvar=False)
Sigma
```

[27]:
```
array([[59.89081909, 28.2131141 , 16.73612824, 28.90034123, 24.09146925],
       [28.2131141 , 37.06979297, 14.75868059, 24.01321127, 20.68442646],
       [16.73612824, 14.75868059, 18.51798169, 14.96888643, 13.90722188],
       [28.90034123, 24.01321127, 14.96888643, 60.69335964, 17.16245177],
       [24.09146925, 20.68442646, 13.90722188, 17.16245177, 27.07180831]])
```

6 4.

Create Λ from 0 to 0.5.

[56]:
```
Lambda = np.arange(0.001, 0.501, 0.001)
Lambda[:5]
```

[56]:
```
array([0.001, 0.002, 0.003, 0.004, 0.005])
```

Optimize the minimum of the function $-(\mu^T w - \frac{\lambda_t}{2} \cdot w^T \Sigma w)$ i.e. the maximum of $\mu^T w - \frac{\lambda_t}{2} \cdot w^T \Sigma w$

[47]:
```
def portfolio_optimization(lambda_t: float):
    def objective(w):
        return -(mu @ w - (lambda_t / 2) * w.T @ Sigma @ w)

    constraints = ({'type': 'eq', 'fun': lambda w: np.sum(w) - 1})
    bounds = [(None, None)] * len(tickers)
```

```
    w0 = np.ones(len(tickers)) / len(tickers)

    result = opt.minimize(objective, w0, constraints=constraints, bounds=bounds)
    return result.x if result.success else None
```

Compute the optimal portfolios for each λ_t.

[54]:
```
optimal_portfolios = {lambda_t: portfolio_optimization(lambda_t) for lambda_t␣
↪in Lambda}

optimal_portfolios[0.001]
```

[54]:
```
array([ 4.13257389, -1.93778343, -0.8873206 ,  1.8536281 , -2.16109796])
```

Compute portfolio statistics for each λ_t.

[49]:
```
portfolio_stats = []

for lambda_t in Lambda:
    w_opt = portfolio_optimization(lambda_t)
    if w_opt is not None:
        mu_p = mu @ w_opt
        sigma_p = np.sqrt(w_opt.T @ Sigma @ w_opt)
        portfolio_stats.append((lambda_t, mu_p, sigma_p))
stats_df = pl.DataFrame(
    {
        "Lambda": [x[0] for x in portfolio_stats],
        "Expected Return": [x[1] for x in portfolio_stats],
        "Expected Volatility": [x[2] for x in portfolio_stats],
        # "Portfolio Weights": [x[3].tolist() for x in portfolio_stats],
    }
)
stats_df
```

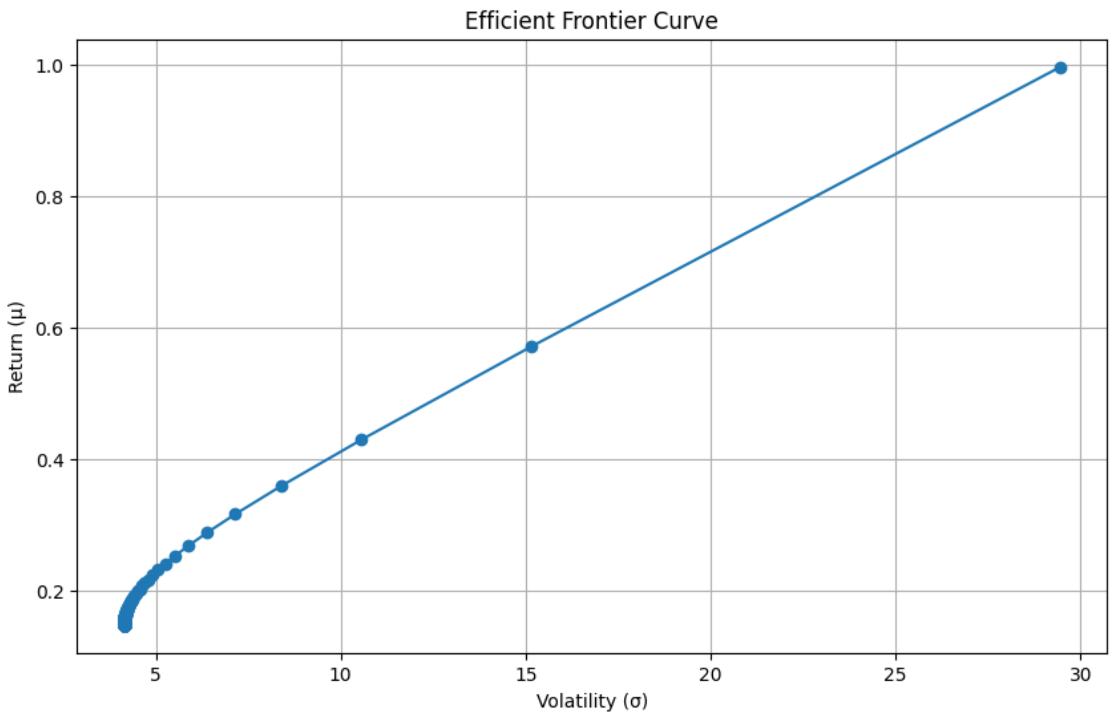
[49]: shape: (500, 3)

Lambda	Expected Return	Expected Volatility
---	---	---
f64	f64	f64
0.001	0.997218	29.453461
0.002	0.572003	15.155693
0.003	0.430282	10.563914
0.004	0.359428	8.382332
0.005	0.316968	7.15209
…	…	…
0.496	0.148526	4.13637
0.497	0.148523	4.136368

0.498	0.148519	4.136367
0.499	0.148516	4.136365
0.5	0.148512	4.136363

Plot efficient frontier.

[53]:
```
plt.figure(figsize=(10, 6))
plt.plot(stats_df["Expected Volatility"], stats_df["Expected Return"],␣
↪marker='o', linestyle='-', label='Efficient Frontier')
plt.xlabel("Volatility ( )")
plt.ylabel("Return ( )")
plt.title("Efficient Frontier Curve")
plt.grid(True)
plt.show()
```



The shape is similar to that of the square root curve, which then levels out into a linear function.