Inference from Scientific Data, 2020 - Worksheet 1

Canvas submission deadline: Wednesday, 28th October, at 16:00
Marks out of a total of 20 are shown in brackets

Question 1: It is estimated that approximately 75% of the galaxies in the universe at large (known as "the field") are spirals. However, observations of 14 galaxies at the core of a large galaxy cluster found that only 3 of them are spirals. What is the probability of finding three or fewer spiral galaxies in this sample if it had been taken from the field? What would you conclude from this result? **[4]**

Question 2: A large Cherenkov detector for solar neutrinos records detections at a mean rate of one per hour. If the detector runs for 6 hours:

a) Plot the probability distribution for the number of counts recorded. **[2]**

b) What is the probability of detecting exactly 6 neutrinos? **[1]**

c) What is the probability of detecting 10 or more neutrinos? **[2]**

d) Compare the expected number of detections with the most likely number of detections, and comment briefly on this. **[2]**

e) More counts than expected are recorded during the run, and the experimenters suspect they have witnessed some kind of unusual event. What is the minimum number of counts that would be required for this to be a one-in-a-hundred occurrence (i.e. for there to be a 1% or less probability of happening by chance on a normal observing run of 6 hours)? **[1]**

Question 3: N people measure the length of a pen, and make measurements $\{x_i\}$ with corresponding (different) Gaussian measurement uncertainties $\{\sigma_i\}$ for $i = 1, 2, \ldots, N$.

a) Using a flat, uninformative prior, perform a Bayesian parameter estimation to find the the most likely value (i.e. maximum *a posteriori* value) for the length of the pen, X. **[3]**

b) Given the following set of measurements and uncertainties (x_i, σ_i), what is the most likely value for X? All measurements are in PU (pen units):
(1.1, 0.05); (1.01, 0.01); (0.99, 0.01); (0.98, 0.01); (1, 0.02); (1.3, 0.4). **[2]**

c) Estimate the uncertainty on the estimate of X? **[3]**