Group Studies - Asteroseismology

Sharif

March 2020

Contents

1 Oscillations

1.1 Convection

The stratification of a star depends on hydrostatic equilibrium and any perturbation from this balance will damped and restored to equilibrium. With this condition in place, instabilities arise as a result of the delicate balance between temperature and pressure. For example by considering the ideal gas law, equation 5, it can be seen that if the temperature drops too quickly with increasing r, the density may have to increase in order to compensate for this, fan maintain the correct pressure gradient for hydrostatic equilibrium. However, A layer of higher density material sitting above a layer of lower density material is unstable and leads to what is called convective instability. A region of the star is determined to be convectively unstable or not by applying the Schwarzschild criterion in which a the motion of a perturbed bubble of gas is analysed by considering the buoyant forces acting on it.
Consider a bubble of stellar plasma which is randomly perturbed such that it begins to rise. Its buoyant forces per unit volume at any point along the path can be found by considering the difference between its density ρ_b and the density of the surrounding stellar material ρ_s.

$$f = -g(\rho_b - \rho_s) = -g\Delta\rho \tag{1}$$

where g is the gravitational field strength. If $f < 0$, the bubble will sink back to its equilibrium position and the region can be deemed convectively stable, however if $f > 0$, the bubble will rise and if it continues to be less dense than its surroundings the region is known as being convectively unstable. Whether it remains less dense than its surroundings as it rises can be derived by assuming that the stellar plasma is an ideal gas and that the bubble behaves adiabaticlly with respect to the surroundings. Adiabatic assumprions are valid because the rising of a bubble occurs on the local dynamical timescale which is much shorter than the timescale for hear exchange Pols (2011). Considering the convectively unstable situation in which $f > 0$ therefore, $\Delta\rho < 0$.

$$\Delta\rho = \left[\frac{\mathrm{d}\rho_\mathrm{b}}{dr} - \frac{\mathrm{d}\rho_\mathrm{s}}{dr}\right]\mathrm{dr} \simeq \left[\frac{\mathrm{d}\rho}{dr}_{ad} - \frac{\mathrm{d}\rho_\mathrm{s}}{dr}\right]\Delta r \tag{2}$$

$$\frac{\mathrm{d}\rho}{dr}_{ad} < \frac{\mathrm{d}\rho}{dr} \tag{3}$$

where the subscript b and s have been dropped and the derivative representing the change within the bubble is indicated with the subscript "ad" since we assumed that the bubble was adiabatic. This is then linked to a temperature gradient by making the following substitutions.
The adiabatic term on the LHS of equation 3 will be analysed using the adiabatic exponent, Γ_1

$$\Gamma_1 = \left(\frac{\mathrm{d}\ln P_b}{\mathrm{d}\ln\rho_b}\right) \rightarrow \frac{\mathrm{d}\rho}{dr}_{ad} = \frac{\rho}{P}\frac{1}{\Gamma_1}\frac{\mathrm{dP}}{dr} \tag{4}$$

where P is the pressure. The term on the RHS of equation 3 will be analysed by assuming that the plasma acts as an ideal gas and therefore ρ is a function of P and T only.

$$\rho = \frac{\mu m_u P}{k_b T} \tag{5}$$

where μ is the mean molecular weight, m_u is the atomic mass unit k_b is the Boltzmann constant and T is the temperature. This can be rewritten as the derivatives of quantities which scale with radius.

$$\frac{1}{\rho}\frac{d\rho}{dr} = \frac{1}{P}\frac{dP}{dr} - \frac{1}{T}\frac{dT}{dr} \tag{6}$$

Substituting these equations into equation 3 gives

$$\frac{\rho}{P}\frac{1}{\Gamma_1}\frac{\mathrm{dP}}{dr} < \frac{\rho}{P}\frac{dP}{dr} - \frac{\rho}{T}\frac{dT}{dr} \tag{7}$$

which rearranges to

$$\frac{\Gamma_1 - 1}{\Gamma_1}\frac{\rho}{P}\frac{dP}{dr} < \frac{\rho}{T}\frac{dT}{dr}. \tag{8}$$

Due to the partial ionisation and subsequent departure from ideal conditions, Γ_1 must be replaced by Γ_2 where

$$\frac{\Gamma_2}{\Gamma_2 - 1} = \frac{d\ln P}{d\ln T}_{ad} \tag{9}$$

giving

$$-\frac{d\ln T}{d\ln P}_{ad}\frac{\rho}{P}\frac{dP}{dr} < \frac{\rho}{T}\frac{dT}{dr} \tag{10}$$

By expanding the logarithmic term, a series of cancellations leads to

$$\frac{dT}{dr}_{ad} > \frac{dT}{dr} \tag{11}$$

or, more compactly,

$$\nabla > \nabla_{ad} \tag{12}$$

This equation describes a region in which a rising bubble of gas looses energy and cools as a slower rate (less negative gradient) than the cooling of the surrounding plasma. Consequently, it maintains a greater temperature and therefore smaller density than its surroundings and continues to rise unhindered until new instabilities set in resulting in turbulent flow and the bubble dissolving. The substitution of Γ_1 for Γ_2 represents the fact that the bubble is not truly adiabatic with respect to its surroundings but still looses energy at a relatively slow rate. See figure 2 for a graphical representation of this situation.

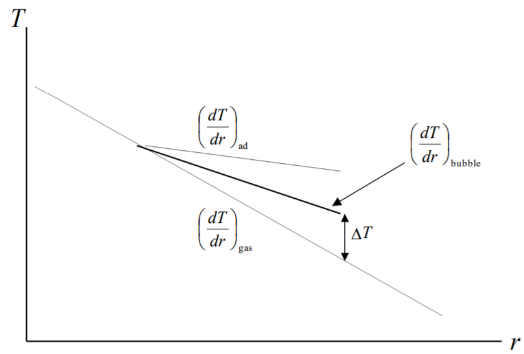

Figure 1: Temperature gradients showing how a bubble of gas cools at a slower rate than the surrounding material as it rises through it. This results in the bubble having a temperature excess ΔT causing it to continue to rise. Chaplin (2019)

More realistic models include a term for the result of the changing composition of the stellar plasma, μ with increasing radius leading to

$$\nabla > \nabla_{ad} + \nabla_{\mu} \tag{13}$$

However, this effect is small and since TRILEGAL does not provide variables which scale with the radius of the star, this will be omitted.
To consider the regime in which convection will take place, we will assume that energy is being transported completely by radiation, therefore the temperature gradient is equal to the radiative temperature gradient, $\nabla = \nabla_R$.

$$\nabla = \frac{d\ln T}{d\ln P} = \frac{P}{T}\frac{dT}{dP} = \frac{P}{T}\frac{dT}{dr}\frac{dr}{dP} \tag{14}$$

where the middle term on the RHS is found by considering the radiative flux through and area of stellar material

$$F_R = -\frac{\lambda\tilde{c}}{3}\frac{du_R}{dr} \tag{15}$$

where λ is the mean free path of the photons and is given by $\lambda = (\kappa\rho)^-1$ where κ is the opacity, $\tilde{c}$ is the speed of sound and u_R is the energy density of radiation and is given by

$$u_R = aT^4 \tag{16}$$

where a is the radiation density constant. Substituting this in gives the following equation

$$F_R = -\frac{-4a\tilde{c}T^3}{3\kappa\rho}\frac{dT}{dr} \tag{17}$$

By substituting the inverse square law for flux and rearranging for $\frac{dT}{dr}$, we arrive at

$$\frac{dT}{dr} = \frac{3\kappa\rho L}{16\pi a\tilde{c}r^2T^3} \tag{18}$$

The third term on the RHS is given by hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \rightarrow \frac{dr}{dP} = -\frac{r^2}{Gm\rho} = \frac{r^2k_bT}{Gm\mu m_uP} \tag{19}$$

Combining these terms in equation 14 gives the radiative temperature gradient in a star

$$\nabla_R = \frac{3k_b}{16\pi\tilde{c}Gm_u}\frac{\kappa}{\mu}\frac{L}{m}\frac{\rho}{T^3} \tag{20}$$

This equation suggests that for convection to take place when $\nabla_R > \nabla_{ad}, therefore, \nabla_R$ must be large. This occurs in K-Dwarfs and M-Dwarfs when the κ is large. Kramers law, $\kappa = \kappa_0\rho T^{-7/2}$ suggests that this occurs in stars with a low surface temperature. It will also occur when $\frac{\rho}{T^3}$ is large which is satisfied in the outer region of cool stars.
The assumption that the stellar material can be modelled as an ideal gas allows for the adiabatic temperature gradient to be found fairly easily. $\Gamma_1 = \frac{c_p}{c_v} = 5/3$ for a fully ionised gas giving $\nabla_{ad} = \frac{\Gamma_1 - 1}{\Gamma_1} = 0.4$. The radius dependent variables in ∇_R were accessed online Christensen-Dalsgaard (1996) allowing for a model to be plotted for the Sun, showing where the consecutively unstable region lies.

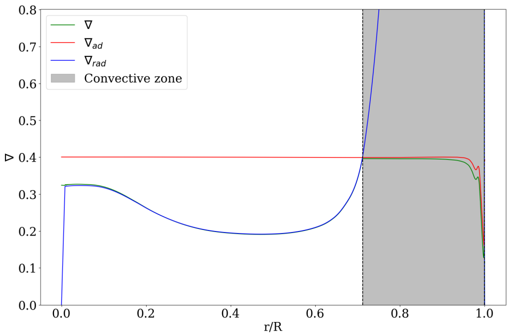

Figure 2: Model of the Sun showing three temperature gradients, ∇, ∇_{ad} and ∇_{rad}. It can be seen in the outer $\simeq 30\%$ of the star $\nabla_{rad} > \nabla_{ad}$ creating the convective envelope.

The stars that we are considering however have greater opacities and lower effective temperatures in the outer layers than the Sun, increasing the radiative temperature gradient resulting in the convective envelope extending much further towards the core of the star as per equation 20. The effective temperature is dependednt on the mass of the star and stars with masses less than $0.35 M_\odot$ are completely convective Pols (2011). The depth of the convective envelope as a function of total mass is represented in figure 3

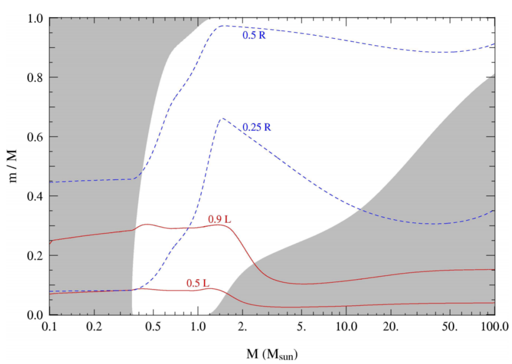

Figure 3: Diagram showing the depth of the convective envelope in grey as a function of the star's total mass. Depth is represented in this plot by the proportion of stellar mass from the surface. The solid red line represents the the shells in which 50% and 90% of the total luminosity are produced and the dashed blue lines show where the shells which lie at 25% and 50% of the total radius. Pols (2011)

The stars involved in this project have masses between $0.80 M_\odot$ and $0.50 M_\odot$ and can therefore be expected to have convective zones in the outer $\simeq 40\%$ of their mass. This is fairly similar to the case for the Sun. Their cores however, remain radiative due to the pp-chain dominating by means of energy production which is distributed over a larger area causing ∇_{rad} to be small in the core.
By scaling the opacity by

$$\nabla_{rad} = \nabla_{rad\odot} \frac{T_{eff}}{T_{eff\odot}}^{-\frac{7}{2}} \tag{21}$$

the following model are obtained for a K-Dwarf of effective temperature 4976.2K. This scale has assumed that the only scaling variable that changes between the Sun and the K-Dwarf is the opacity. More accurate models would include the proper ∇_μ, $\rho(r)$ and other quantities that scales with stellar radius, however the TRILEGAL database does not contain data which makes this calculation possible and therefore figure 4 should be treated as a illustrative guide.

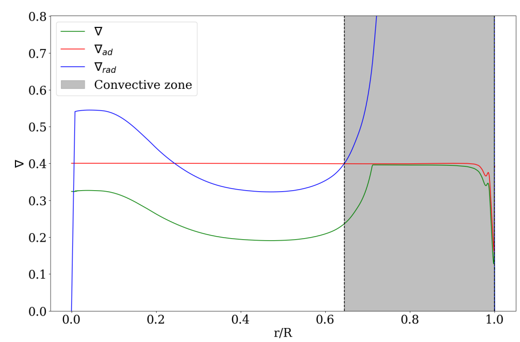

Figure 4: Stellar model for a K-Dwarf with $T_{eff} = 4976.2K$. It can be seen by comparing to figure 2 that the depth of the convective zone has increased as a result of the increase in opacity.

1.2 Granulation

At the upper boundary of the convective zone, $\nabla_{rad} = \nabla_{ad}$ Pols (2011). This results in the buoyant force acting on a bubble of hot gas vanishing. However, the inertia acquired by a bubble of gas (assuming that it has not already dissolved) causes it to overshoot the convective boundary where it is strongly braked, especially if it is entering an area of low density such as the photosphere. After overshooting into the photosphere, the bubble experiences a sharp drop in opacity allowing it to radiate away its energy excess causing it to cool enough such that it is now denser than the the surrounding material. The sign of the buoyant force in equation 1 changes and the bubble falls back into the depths of the star. Each filament can be compared to the water from a water fountain rushing upwards and then spilling over where it is hot and bright as it rises, and cool and dark when it sinks. This overshooting and subsequent sinking results in oscillatory motion at the surface of the star across many adjacent convective filaments causing a granulation pattern at the surface of the star. Due to granulation being a direct effect of convection, any star which has a convective envelope must have some degree of granulation.

1.3 P-Modes

P-modes are the standing waves that are set up because of coherent excitations from convective cells at the surface of the star. They are projected into the stellar interior at a range of angles and experience refraction back towards the surface as a result of the temperature gradient within the star. As a result, only the modes with no angular component (ones that are projected tangential to the stellar surface) reach the core of the star. The "p" refers to the fact that the restoring force is the pressure gradient within the star. Due to the damping from the granulation, their power spectra resemble a Lorentzian. There is a range of modes that oscillate through the star's interior depending on the internal properties of the star. Each mode is assigned a radial order, n, and angular degree, l, as a result of the way the mode distorts the shape of the star where radial modes have $l = 0$. Because of geometric cancellation and technical limitations, it is difficult to observe angular degrees greater than $l = 2$. Geometric cancellation refers to the symmetry of modes on either side of a star causing difficulty in measuring changes in its observed flux.

2 Modelling

2.1 The Granulation

The granulation spectrum is modelled as a lorentzian centered at a frequency of zero Hz, it is given by the equation 24.

$$P(\nu) = \frac{4\sqrt{2}\sigma_c^2}{1 + (2\pi\nu\tau_c)^2} \tag{22}$$

Where τ is the ..... and σ is the .... Their scaling relations from the Sun are given according to equations 25

$$\sigma_c = \sigma_{c,\odot}\left[\left(\frac{T_{eff}}{T_{eff\odot}}\right)\left(\frac{R}{R_\odot}\right)\left(\frac{M_\odot}{M}\right)\right] \tag{23a}$$

$$\tau_c = \tau_{c,\odot}\left[\left(\frac{T_{eff}}{T_{eff,\odot}}\right)^{\frac{1}{2}}\left(\frac{g_\odot}{g}\right)\right] \tag{23b}$$

3 Results

3.1 Array creation

3.2 Granulation

3.3 P-Modes

P-modes are the standing waves that are set up because of coherent excitations from convective cells at the surface of the star. They are projected into the stellar interior at a range of angles and experience refraction back towards the surface as a result of the temperature gradient within the star. As a result, only the modes with no angular component (ones that are projected tangential to the stellar surface) reach the core of the star. The "p" refers to the fact that the restoring force is the pressure gradient within the star. Due to the damping from the granulation, their power spectra resemble a Lorentzian. There is a range of modes that oscillate through the star's interior depending on the internal properties of the star. Each mode is assigned a radial order, n, and angular degree, l, as a result of the way the mode distorts the shape of the star where radial modes have $l = 0$. Because of geometric cancellation and technical limitations, it is difficult to observe angular degrees greater than $l = 2$. Geometric cancellation refers to the symmetry of modes on either side of a star causing difficulty in measuring changes in its observed flux.

4 Modelling

4.1 The Granulation

The granulation spectrum is modelled as a lorentzian centered at a frequency of zero Hz, it is given by the equation 24.

$$P(\nu) = \frac{4\sqrt{2}\sigma_c^2}{1 + (2\pi\nu\tau_c)^2} \tag{24}$$

Where τ is the ..... and σ is the .... Their scaling relations from the Sun are given according to equations 25

$$\sigma_c = \sigma_{c,\odot}\left[\left(\frac{T_{eff}}{T_{eff\odot}}\right)\left(\frac{R}{R_\odot}\right)\left(\frac{M_\odot}{M}\right)\right] \tag{25a}$$

$$\tau_c = \tau_{c,\odot}\left[\left(\frac{T_{eff}}{T_{eff,\odot}}\right)^{\frac{1}{2}}\left(\frac{g_\odot}{g}\right)\right] \tag{25b}$$

5 Results

5.1 Array creation

References

William Chaplin. 3. observational signatures of convection, 2019. URL https://canvas.bham.ac.uk/courses/34222/pages/AEXPart1:Asteroseismology.

J. Christensen-Dalsgaard. Solar models, 1996. URL https://users-phys.au.dk/~jcd/solar_models/.

Onno Pols. Stellar structure and evolution, Sep 2011. URL https://www.astro.ru.nl/~onnop/education/stev_utrecht_notes/.