MDNPN:

Initialize:

$$
\begin{aligned}
h_1^\mu &\equiv \nu_1 \equiv 0 \\
\tilde{x}_1 &\equiv 0 \\
\mu_1 &\equiv x_1 \\
\beta_1^\mu &\equiv \log\left(\eta_1^\mu/(1-\eta_0^\mu)\right), \qquad \text{(inverse of sigmoid)} \\
\beta_1^\nu &\equiv \log\left(\eta_1^\nu/(1-\eta_0^\nu)\right), \qquad \text{(inverse of sigmoid)} \\
\eta_0^{\mu,\text{prod}} &\equiv 1-\eta_1^\mu \\
\eta_0^{\nu,\text{prod}} &\equiv 1
\end{aligned}
$$

For all $t \geq 2$:

$$\eta_t^\mu \equiv \frac{1}{1+\exp(-\beta_{t-1}^\mu)} \tag{1}$$

$$\eta_t^\nu \equiv \frac{1}{1+\exp(-\beta_{t-1}^\nu)} \tag{2}$$

$$\eta_t^{\mu,\text{prod}} \equiv \eta_{t-1}^{\mu,\text{prod}} \cdot (1-\eta_t^\mu) \tag{3}$$

$$\eta_t^{\nu,\text{prod}} \equiv \eta_{t-1}^{\nu,\text{prod}} \cdot (1-\eta_t^\nu) \tag{4}$$

$$\nu_t = \nu_{t-1} + \eta_t^\nu \left(\left(\tilde{x}_t^2 - 1\right)^2 - \nu_{t-1}\right)/(1-\eta_t^{\nu,\text{prod}}) \tag{5}$$

$$\tilde{x}_t \equiv \frac{x_t - \mu_{t-1}}{\max\left(\sqrt{\nu_t}, 1e^{-8}\right)}, \tag{6}$$

$$\mu_t \equiv \mu_{t-1} + \eta_t^\mu (x_t - \mu_{t-1})/(1-\eta_t^{\mu,\text{prod}}) \tag{7}$$

$$v_t \equiv v_{t-1} + \eta_t^\nu \left(\left(\tilde{x}_t^2 - 1\right)^2 - v_{t-1}\right)/(1-\eta_t^{\nu,\text{prod}}) \tag{8}$$

$$\beta_t^\mu \equiv \beta_{t-1}^\mu + \theta\, \tilde{x}_t\, h_{t-1}^\mu \sqrt{2\eta_t^\mu - (\eta_t^\mu)^2} \tag{9}$$

$$\beta_t^\nu \equiv \beta_{t-1}^\nu + \theta \frac{\tilde{x}_t^2 - 1}{\max\left(v_t, 1e^{-8}\right)} h_{t-1}^\nu \sqrt{2\eta_t^\nu - (\eta_t^\nu)^2} \tag{10}$$

$$h_t^\mu \equiv (1-\eta_t^\mu) h_{t-1}^\mu + \tilde{x}_t \tag{11}$$

$$h_t^\nu \equiv (1-\eta_t^\nu) h_{t-1}^\nu + \tilde{x}_t^2 - 1 \tag{12}$$