S#	Samples	$\bar{y}$	s^2	$T = s/\sqrt{\bar{y}} \times 100$

Lab # 03

① let continuous variable x has the pdf

$$f(x) = 2x \quad , \quad 0 \leq x \leq 1$$

ⓐ let x_1 & x_2 are random sample of size from the distn

Find $\left(\frac{x_1}{x_2} < 2\right)$ in three ways

i) Analytically using joint distn of x_1 & x_2 directly

α ii) By first finding the distn of $Y = x_1/x_2$ using cdf & then using distance of y evaluate $P(Y<2)$

iii) using simulation of 100 replication & comment on the accuracy of simulated probability.

ⓑ let x_1, x_2, x_3 are random sample of size 3, let y is the largest observation

i) Find pdf of Y

ii) Find $P(Y < 0.8)$ in two ways

a) Analytically using pdf

b) using simulation of 100 replicants & comment

S.No	X_1	X_2	X_3	X_1/X_2	$Y>2$	Y=Largest	$y>0.8$
1	0.912	0.940	0.735	0.959	1	0.940	0
2	0.987	0.879	0.728	1.122	1	0.987	0
3	0.501	0.964	0.628	0.519	1	0.964	0
4	0.486	0.739	0.178	0.657	1	0.739	1
5	0.594	0.606	0.660	0.980	1	0.666	1
6	0.926	0.651	0.624	1.422	1	0.926	0
7	0.600	0.173	0.528	3.468	0	0.660	1
8	0.881	0.946	0.850	0.931	1	0.946	0
9	0.462	0.740	0.961	0.624	1	0.961	0
10	0.480	0.212	0.770	2.264	0	0.770	1
11	0.394	0.951	0.948	0.414	1	0.951	0
12	0.994	0.918	0.819	1.082	1	0.994	0
13	0.161	0.920	0.743	0.175	1	0.920	0
14	0.459	0.951	0.697	0.482	1	0.951	0
15	0.669	0.655	0.151	1.021	1	0.669	1
16	0.881	0.605	0.909	1.456	1	0.909	0
17	0.563	0.989	0.597	0.569	1	0.989	0
18	0.463	0.919	0.722	0.503	1	0.919	0
19	0.889	0.972	0.653	0.914	1	0.972	0
20	0.965	0.641	0.126	1.505	1	0.965	0
21	0.956	0.889	0.626	1.075	1	0.956	0
22	0.846	0.391	0.793	2.291	0	0.846	0
23	0.834	0.714	0.987	1.168	1	0.987	0
24	0.925	0.966	0.169	0.957	1	0.925	0
25	0.962	0.827	0.870	1.163	1	0.962	0
26	0.991	0.386	0.700	2.567	0	0.991	0
27	0.874	0.947	0.711	0.922	1	0.947	0
28	0.788	0.054	0.651	14.542	0	0.788	1
29	0.491	0.844	0.522	0.581	1	0.844	0
30	0.575	0.453	0.417	1.269	1	0.575	1
31	0.698	0.480	0.924	1.454	1	0.924	0
32	0.506	0.363	0.641	1.393	1	0.641	1
33	0.511	0.667	0.914	0.766	1	0.914	0
34	0.686	0.786	0.844	0.871	1	0.844	0
35	0.901	0.484	0.262	1.861	1	0.901	0
36	0.735	0.505	0.732	1.455	1	0.735	1
37	0.877	0.988	0.582	0.887	1	0.988	0
38	0.258	0.719	0.757	0.358	1	0.757	1
39	0.579	0.437	0.176	1.324	1	0.579	1
40	0.830	0.957	0.562	0.867	1	0.957	0
41	0.279	0.502	0.949	0.555	1	0.949	0
42	0.141	0.507	0.760	0.278	1	0.760	1
43	0.793	0.706	0.034	1.123	1	0.793	1
44	0.849	0.933	0.938	0.909	1	0.938	0
45	0.956	0.758	0.881	1.261	1	0.956	0
46	0.785	0.170	0.937	4.617	0	0.937	0
47	0.805	0.293	0.893	2.747	0	0.893	0
48	0.556	0.571	0.822	0.973	1	0.822	0
49	0.731	0.836	0.813	0.874	1	0.836	0
50	0.723	0.469	0.346	1.541	1	0.723	1

S#	Samples	$\bar{y}$	S^2	$T = s/\bar{y} \times 100$

S.No	X_1	X_2	X_3	X_1/X_2	$Y>2$	Y = Largest	$y>0.8$
51	0.723	0.464	0.346	1.085	1	0.924	0
52	0.238	0.992	0.542	0.239	1	0.992	0
53	0.417	0.750	0.979	0.556	1	0.979	0
54	0.472	0.536	0.782	0.880	1	0.782	1
55	0.394	0.890	0.690	0.442	1	0.890	1
56	0.642	0.519	0.692	1.236	1	0.682	1
57	0.891	0.736	0.723	1.210	1	0.891	1
58	0.774	0.275	0.428	2.814	0	0.774	1
59	0.482	0.170	0.899	2.835	0	0.899	1
60	0.331	0.934	0.788	0.354	1	0.934	0
61	0.308	0.393	0.662	0.783	1	0.662	1
62	0.289	0.912	0.641	0.316	1	0.912	0
63	0.070	0.676	0.927	0.103	1	0.927	0
64	0.849	0.679	0.682	1.250	1	0.849	1
65	0.977	0.971	0.861	1.006	1	0.977	0
66	0.513	0.648	0.698	0.791	1	0.698	1
67	0.988	0.461	0.270	2.143	0	0.988	0
68	0.077	0.770	0.570	0.1	1	0.770	1
69	0.928	0.508	0.294	1.826	1	0.928	0
70	0.648	0.593	0.943	1.092	1	0.943	0
71	0.870	0.270	0.610	3.222	0	0.870	1
72	0.402	0.306	0.614	1.313	1	0.614	1
73	0.873	0.578	0.913	1.510	1	0.913	0
74	0.368	0.228	0.742	1.614	1	0.742	1
75	0.881	0.944	0.951	0.933	1	0.951	0
76	0.768	0.272	0.741	2.823	0	0.768	1
77	0.294	0.854	0.731	0.344	1	0.854	1
78	0.275	0.558	0.952	0.492	1	0.952	0
79	0.636	0.506	0.667	1.256	1	0.667	1
80	0.458	0.813	0.796	0.563	1	0.813	1
81	0.225	0.662	0.913	0.339	1	0.913	0
82	0.536	0.949	0.715	0.564	1	0.949	0
83	0.522	0.992	0.844	0.526	1	0.992	0
84	0.173	0.973	0.856	0.177	1	0.973	0
85	0.585	0.929	0.612	0.629	1	0.929	0
86	0.712	0.921	0.787	0.773	1	0.921	0
87	0.636	0.861	0.526	0.738	1	0.861	1
88	0.9	0.674	0.770	1.335	1	0.900	0
89	0.766	0.993	0.964	0.771	1	0.993	0
90	0.536	0.690	0.824	0.776	1	0.824	1
91	0.876	0.917	0.834	0.955	1	0.917	0
92	0.667	0.700	0.941	0.952	1	0.941	0
93	0.686	0.814	0.083	0.842	1	0.814	1
94	0.603	0.800	0.712	0.753	1	0.800	1
95	0.982	0.746	0.777	1.316	1	0.982	0
96	0.825	0.954	0.728	0.864	1	0.954	0
97	0.952	0.927	0.378	1.026	1	0.952	0
98	0.464	0.577	0.384	0.804	1	0.577	1
99	0.976	0.705	0.780	1.384	1	0.976	0
100	0.519	0.844	0.861	0.697	1	0.861	1

$$(P<2) = \frac{\Sigma Y\ (\#\text{ of 1's})}{\text{Total obs}} = \frac{88}{100} = 0.88$$

S# Samples $\bar{y}$ S^2 $T = S/\bar{y}$ ...

(b) ...

(i) Analytically:

For joint $4x_1x_2$

For cdf of Y:

$$F_Y(y) = P(Y \le y)$$

$$= P(x_1/x_2 \le 2) = P(x_2 \ge x_1/2)$$

$$= P(x_1/x_2 < 2) = 1 - P(x_2 \ge x_1/2)$$

$$= 1 - \int_{x_1=0}^{1}\int_{x_2=0}^{x_1/2} 4x_1x_2 \, dx_1\, dx_2$$

$$= 1 - \int_{x_1=0}^{1} \left| 4x_1 \frac{x_2^2}{2} \right|_0^{x_1/2} dx_1$$

$$= 1 - \int_{x_1=0}^{1} \left[2x_1 x_2^2 \right]_0^{x_1/2} dx_1$$

$$= 1 - \int_{x_1=0}^{1} \left[2x_1 \left(\frac{x_1}{2}\right)^2 - 0 \right] dx_1$$

$$= 1 - \int_{x_1=0}^{1} \left[2x_1 \cdot \frac{x_1^2}{4} \right] dx_1$$

$$= 1 - \int_{x_1=0}^{1} \frac{x_1^3}{2} \, dx_1$$

$$= 1 - \frac{1}{2} \left| \frac{x_1^4}{4} \right|_0^1$$

$$= 1 - \frac{1}{8} \left((1)^4 - 0 \right)$$

$$= 1 - \frac{1}{8} \Rightarrow \frac{8-1}{8}$$

$$= \frac{7}{8}$$

$$\boxed{P(x_1/x_2 > 2) \Rightarrow 0.875}$$

Analytically		By simulation
0.875	$\cong$	0.88

S#	Samples	$\bar{y}$	S^2	$T = S/\bar{y} \times 100$

(b) Here, y - Largest

$f(x_1, x_2, x_3) = 8 x_1 x_2 x_3$

For Cdf of y

$$F_y(y) = 8 \int_{x_1=0}^{x_2} \int_{x_2=0}^{x_3} \int_{x_3=0}^{y} x_1 x_2 x_3 \, dx_1 \, dx_2 \, dx_3$$

$$= 8 \int_{x_2=0}^{x_3} \int_{x_3=0}^{y} \left| \frac{x_1^2}{2} \right|_0^{x_2} x_2 x_3 \, dx_2 \, dx_3$$

$$= 4 \int_{x_2=0}^{x_3} \int_{x_3=0}^{y} [x_2^2 - 0] \, x_2 x_3 \, dx_2 \, dx_3$$

$$\Rightarrow 4 \int_{x_2=0}^{x_3} \int_{x_3=0}^{y} x_2^3 x_3 \, dx_2 \, dx_3$$

$$\Rightarrow 4 \int_{x_3=0}^{y} \left| \frac{x_2^4}{4} \right|_0^{x_3} x_3 \, dx_3$$

$$\Rightarrow \int_{x_3=0}^{y} [x_3^4 - 0] \, x_3 \, dx_3$$

$$\Rightarrow \int_{x_3=0}^{y} x_3^5 \, dx_3$$

$$\Rightarrow \left| \frac{x^6}{6} \right|_0^{y} \Rightarrow \frac{1}{6}(y^6 - 0)$$

$$\boxed{F_y(y) = \frac{y^6}{6}}$$

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Pdf:

$$f_y(y) = \frac{d}{dy} F_y(y)$$

$$= \frac{d}{dy}\left(\frac{y^6}{6}\right) \Rightarrow \frac{6y^5}{6}$$

$$\boxed{f_y(y) = y^5}$$

Pdf:

$$= \int_0^{0.8} y^5 \, dy \Rightarrow \left. \frac{y^6}{6} \right|_0^{0.8}$$

$$= 0.0436$$

Analytically	Simulation
0.0436	0.43

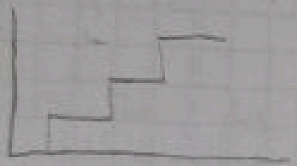
(a) (b)

	sample	R# aa	y_i	$\bar{y}_i = \frac{\Sigma y}{10}$	S^2	$\hat{V}(\bar{y})$	$B.E = 2\sqrt{\hat{V}(\bar{y})}$	$\hat{P}$	$\hat{\tau} = N\bar{y}$
1	1 2 3 ⋮ 10	05 — — — —						$2/10$	
2									

Q#1

x	$f(x)$	$F(x)$	Range
1	$1/15$	$1/15$	0 — 0.066
2		$3/15$	0.067 — 0.2
3			0.0201 — 0.4
4			0.401 — 0.66
5	$5/15$	$15/15$	0.661 — 1

(a)

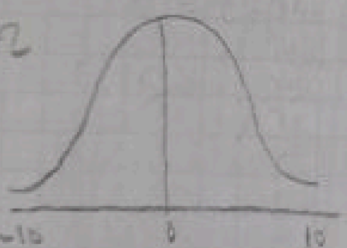


(b)

S#	Rand #	x
1		
2		
3		
⋮		
30		

x	T.m	f	$e.f = b/\Sigma b$
1			
2			
3			
4			
5			

Q#2



(b) $F(x) = \int_{-\infty}^{x} \frac{e^{-t}}{(1+e^{-t})^2} dt$

$y = 1 + e^{-t} \Rightarrow \boxed{dt = \frac{-dy}{e^{-t}}}$

when $t = x$ then $y = 1 + e^{-x}$

when $t = -\infty$ then $y = \infty$

(d)

$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$

$F(x) = -\int_{\infty}^{1+e^{-x}} \frac{1}{y^2} dy$

$\Rightarrow -\int_{a}^{b} f(x)\, dx = \int_{b}^{a} f(x)\, dx$

$\Rightarrow \int_{1+e^{-x}}^{\infty} 1/y^2\, dy$

$\boxed{F(x) = \frac{1}{1+e^{-x}}}$

(c) $y = e^{-x}$

$\ln y = -x \ln x$

$1/y\, dy = \ln y$

Lab # 04

Q) Let X_1 & X_2 be two Binomial r.v each with $n_1 = n_2 = n$ & $p = 0.6$. Let Y be the sum of the r.v. What is the distⁿ of Y? Analytically find the probability that Y exceeds 8.

* Now suppose you do not know the distⁿ of sum & still want to estimate the probability (i) conduct a simulation from Binomial ($n=5$, $p=0.60$) with 100 replications to estimate the desired probability. Comment on the accuracy of the estimated probability.

Sol:

x	$f(x) = {}^nC_x p^x q^{n-x}$	$F(x)$	C.B
0	0.01024	0.01024	0 – 0.01
1	0.0768	0.08704	0.01 – 0.08
2	0.2304	0.31744	0.08 – 0.31
3	0.3456	0.66304	0.31 – 0.66
4	0.2592	0.92224	0.66 – 0.92
5	0.07776	1	0.92 – 1
	$\Sigma f(x) = 1$		

S.No	U_1	U_2	X_1	X_2	$Y = X_1 + X_2$	$Y > 8$
1	0.68	0.197	4	2	6	0
2	0.498	0.18	3	4	7	0
3	0.079	0.258	1	2	3	0
4	0.357	0.703	3	4	7	0
5	0.914	0.948	4	5	9	1
6	0.825	0.251	4	2	7	0
7	0.448	0.11	3	2	5	0
8	0.018	0.855	1	4	5	0
9	0.514	0.497	3	3	6	0
10	0.522	0.581	3	3	6	0
11	0.123	0.793	2	4	6	0
12	0.413	0.952	3	5	8	0
13	0.18	0.02	2	1	3	0
14	0.629	0.562	3	3	6	0
15	0.07	0.448	1	3	4	0
16	0.933	0.587	5	3	8	0
17	0.848	0.939	4	5	9	1
18	0.472	0.609	3	3	6	0
19	0.468	0.673	3	4	7	0
20	0.021	0.305	1	2	3	0
21	0.186	0.752	2	4	6	0
22	0.802	0.115	4	2	6	0

S.No	U_1	U_2	X_1	X_2	$Y = X_1 + X_2$	$Y > 8$
23	0.177	0.944	2	5	7	0
24	0.3	0.079	2	1	3	0
25	0.944	0.388	5	3	8	0
26	0.4	0.205	3	2	5	0
27	0.201	0.289	2	2	4	0
28	0.275	0.069	2	1	3	0
29	0.613	0.458	1	3	4	0
30	0.284	0.359	2	3	5	0
31	0.887	0.273	4	2	6	0
32	0.708	0.888	4	4	8	0
33	0.63	0.243	4	2	6	0
34	0.727	0.754	4	4	8	0
35	0.716	0.566	4	3	7	0
36	0.451	0.85	3	4	7	0
37	0.135	0.061	2	1	3	0
38	0.044	0.429	1	3	4	0
39	0.169	0.669	2	4	6	0
40	0.681	0.964	4	5	9	1
41	0.59	0.456	3	3	6	0
42	0.493	0.647	3	3	6	0
43	0.765	0.577	4	3	7	0
44	0.771	0.625	4	3	7	0
45	0.678	0.479	4	3	7	0
46	0.003	0.049	0	1	1	0
47	0.569	0.436	3	3	6	0
48	0.28	0.256	2	2	4	0
49	0.209	0.773	2	4	6	0
50	0.781	0.466	4	3	7	0
51	0.604	0.476	3	3	6	0
52	0.118	0.384	2	3	5	0
53	0.85	0.816	4	4	8	0
54	0.125	0.5	2	3	5	0
55	0.32	0.79	3	4	7	0
56	0.175	0.877	2	4	6	0
57	0.491	0.985	3	5	8	0
58	0.375	0.23	3	2	5	0
59	0.178	0.731	2	4	6	0
60	0.396	0.331	3	3	6	0
61	0.039	0.369	1	3	4	0
62	0.657	0.65	3	3	6	0
63	0.978	0.998	5	5	10	1
64	0.775	0.842	4	4	8	0
65	0.911	0.503	4	3	7	0
66	0.339	0.708	3	4	7	0
67	0.212	0.683	2	4	6	0
68	0.686	0.647	4	3	7	0
69	0.343	0.443	3	3	6	0
70	0.651	0.043	3	1	4	0
71	0.361	0.846	3	4	7	0
72	0.02	0.362	1	3	4	0
73	0.285	0.844	2	4	6	0
74	0.123	0.79	2	4	6	0

S.No	U_1	U_2	X_1	X_2	$Y = X_1 + X_2$	$Y > 8$
75	0.153	0.408	2	3	7	0
76	0.998	0.816	5	4	9	1
77	0.409	0.997	3	5	8	0
78	0.497	0.056	3	1	4	0
79	0.276	0.402	2	3	5	0
80	0.555	0.698	3	4	7	0
81	0.257	0.904	2	4	6	0
82	0.228	0.926	2	5	7	0
83	0.086	0.894	1	4	5	0
84	0.839	0.515	4	3	7	0
85	0.257	0.034	2	1	3	0
86	0.14	0.116	2	2	4	0
87	0.265	0.103	2	2	4	0
88	0.123	0.585	2	3	5	0
89	0.088	0.072	2	1	3	0
90	0.159	0.62	2	3	5	0
91	0.011	0.534	1	3	4	0
92	0.67	0.062	4	1	5	0
93	0.692	0.957	2	5	7	0
94	0.036	0.976	1	5	6	0
95	0.91	0.154	4	2	6	0
96	0.036	0.211	1	2	3	0
97	0.268	0.131	2	2	4	0
98	0.577	0.993	3	5	8	0
99	0.593	0.928	3	5	8	0
100	0.666	0.433	4	3	7	0

$\Rightarrow \frac{5}{100}$

$\Rightarrow 0.05$

Samples | $\bar{y}$ | S^2 | $T = S/\bar{y} \times 100$

Analytically:

$$\Rightarrow {}^{10}C_9 \times p^9 \times q^{10-9} + {}^{10}C_{10} \times p^{10} \times q^{10-10}$$

$$\Rightarrow {}^{10}C_9 \times (0.6)^9 \times (0.4)^1 + {}^{10}C_{10} \times (0.6)^{10} \times (0.4)^0$$

$$\Rightarrow 0.04 + 0.006$$

$$\Rightarrow 0.046$$

Comment:- The probability of 100 repletion. The eshmation by simulation is 0.05 & analytically is 0.046 which is approximately equal means eshmation of probability is very precise & authentic

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Lab # 05

Q let x_1 & x_2 be two poission r.v each with $\lambda_1 = \lambda_2 = 1$. let Y be the sum of the r.v. What is the distⁿ of Y? Analytically find the prob of Y exceeds 5. Now suppose you don't know the distⁿ of sum & still want to estimate the probability (i) Conduct a simulation from poission (with $\lambda = 1$) with 100 replications to estimate the deserved probability comment on the accuracy of the estimated probability

Sol:

x_1	$f(x_1) = \frac{e^{-\lambda}\lambda^{x}}{x_1!}$	$F(x_1)$	C.B
0	0.3678	0.3678	0 – 0.36
1	0.3678	0.7356	0.36 – 0.73
2	0.1839	0.9195	0.73 – 0.91
3	0.0613	0.9808	0.91 – 0.98
4	0.0153	0.9961	0.98 – 0.996
5	0.0030	0.999	0.996 – 0.999
6	0.0005	1	0.999 – 1
	$\sum f(x) = 1$		

x_2	$f(x_2) = \frac{e^{-\lambda}\lambda^{x_2}}{x_2!}$	$F(x_2)$	C.B
0	0.13533528	0.13533528	0 – 0.13
1	0.27067056	0.40600584	0.13 – 0.40
2	0.27067056	0.67667640	0.40 – 0.67
3	0.18044704	0.85712344	0.67 – 0.85
4	0.09022352	0.94734696	0.85 – 0.94
5	0.03608940	0.98343636	0.94 – 0.98
6	0.01202980	0.99546616	0.98 – 0.99
7	0.00343708	1	0.99 – 1
	$\sum f(x_2) = 1$		

S.NO	U_1	U_2	X_1	X_2	$Y = X_1 + X_2$	$Y > 5$
1	0.078	0.429	0	2	2	0
2	0.669	0.798	1	3	4	0
3	0.669	0.713	1	3	4	0
4	0.145	0.92	0	4	4	0
5	0.077	0.668	0	2	2	0
6	0.494	0.59	1	2	3	0
7	0.274	0.321	0	1	1	0
8	0.18	0.637	0	2	2	0
9	0.41	0.722	1	3	4	0
10	0.472	0.654	1	2	3	0
11	0.363	0.86	1	4	5	0
12	0.8	0.646	2	2	4	0
13	0.304	0.481	0	2	2	0
14	0.505	0.784	1	3	4	0
15	0.281	0.008	0	0	0	0
16	0.904	0.918	2	4	6	1
17	0.14	0.709	0	3	3	0
18	0.926	0.348	3	1	4	0
19	0.852	0.298	2	1	3	0
20	0.4	0.582	1	2	3	0
21	0.358	0.375	0	1	1	0
22	0.404	0.79	1	3	4	0
23	0.772	0.997	2	6	8	1
24	0.516	0.192	1	1	2	0
25	0.866	0.356	2	1	3	0
26	0.37	0.049	1	0	1	0
27	0.612	0.576	1	2	3	0
28	0.212	0.965	0	5	5	0
29	0.005	0.016	0	0	0	0
30	0.199	0.324	0	1	1	0
31	0.621	0.892	1	4	5	0
32	0.572	0.546	1	2	3	0
33	0.358	0.386	0	1	1	0
34	0.643	0.84	1	3	4	0
35	0.746	0.425	2	2	4	0
36	0.266	0.08	0	0	0	0
37	0.957	0.931	3	4	7	1
38	0.237	0.561	0	2	2	0
39	0.539	0.193	1	1	2	0
40	0.459	0.663	1	2	3	0
41	0.68	0.136	1	1	2	0
42	0.664	0.002	1	0	1	0
43	0.868	0.198	2	1	3	0
44	0.761	0.438	2	2	4	0
45	0.891	0.404	2	2	4	0
46	0.321	0.57	0	2	2	0
47	0.618	0.513	1	2	3	0
48	0.086	0.91	0	4	4	0
49	0.612	0.981	1	5	6	1
50	0.565	0.112	1	0	1	0

S# Samples y

S.NO	U_1	U_2	X_1	X_2	$Y=X_1+X_2$	$Y>5$
51	0.791	0.368	2	1	3	0
52	0.53	0.264	1	1	2	0
53	0.782	0.924	2	4	6	1
54	0.371	0.574	1	2	3	0
55	0.678	0.853	1	4	5	0
56	0.624	0.731	1	3	4	0
57	0.392	0.31	1	1	2	0
58	0.394	0.963	1	5	6	1
59	0.037	0.083	0	0	0	0
60	0.571	0.305	1	1	2	0
61	0.551	0.018	1	0	1	0
62	0.661	0.948	1	5	6	1
63	0.43	0.358	1	1	2	0
64	0.771	0.075	2	0	2	0
65	0.381	0.278	1	1	2	0
66	0.156	0.795	0	3	3	0
67	0.637	0.94	1	4	5	0
68	0.025	0.735	0	3	3	0
69	0.612	0.448	1	2	3	0
70	0.448	0.669	1	2	3	0
71	0.391	0.831	1	3	4	0
72	0.373	0.897	1	4	5	0
73	0.761	0.257	2	1	3	0
74	0.851	0.944	2	5	7	1
75	0.891	0.612	2	2	4	0
76	0.204	0.988	0	5	5	0
77	0.154	0.208	0	1	1	0
78	0.95	0.491	3	2	5	0
79	0.908	0.334	2	1	3	0
80	0.38	0.739	1	3	4	0
81	0.166	0.918	0	4	4	0
82	0.93	0.704	3	3	6	1
83	0.626	0.563	1	2	3	0
84	0.852	0.973	2	5	7	1
85	0.638	0.942	1	5	6	1
86	0.446	0.023	1	0	1	0
87	0.845	0.841	2	3	5	0
88	0.516	0.132	1	1	2	0
89	0.571	0.268	1	1	2	0
90	0.054	0.795	0	3	3	0
91	0.037	0.891	0	4	4	0
92	0.386	0.72	1	3	4	0
93	0.325	0.913	0	4	4	0
94	0.048	0.596	0	2	2	0
95	0.119	0.073	0	0	0	0
96	0.352	0.977	0	5	5	0
97	0.814	0.268	2	1	3	0
98	0.31	0.397	0	1	1	0
99	0.069	0.916	0	4	4	0
100	0.091	0.909	0	4	4	0
						11

$\Rightarrow \frac{11}{100}$

$= 0.11$

S#	Samples	$\bar{Y}$	S^2	T-S/-

<u>Analytically:</u>

$\lambda = 3 \quad , \lambda = 0, 1, 2, 3, 4$

$P(Y>5) = 1 - P(Y<5)$

$= 1 - \{P(Y=0) + P(Y=1) + P(Y=2) + P(Y=3) + P(Y=4)\}$

$= 1 - \{0.0497 + 0.14936 + 0.22404 + 0.22404 + 0.168031\}$

$= 1 - 0.81517$

$\boxed{P(Y>5) = 0.18483}$

$\text{Error} = |P_A - P_S|$

$= |0.11 - 0.18|$

$\boxed{\text{Error} = 0.07}$

$\text{Accuracy} = (1 - \text{Error}) \times 100$

$= (1 - 0.07) \times 100$

$= 93\%$

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Comment: The simulated prob is 93% accurate to the analytical prob.

Lab # 06

Q (a) Draw a pair of random variables (U_1, U_2) from $U(0,1)$

(b) Obtain a pair of standard Normal random variables (Z_1, Z_2) using box Muller Transformation

$$Z_1 = \sqrt{-2\ln U_1}\ \cos(2\pi U_2)$$

$$Z_2 = \sqrt{-2\ln U_1}\ \sin(2\pi U_2)$$

(c) Based on (a) & (b) simulate two normal r.v X_1 & X_2 from $N(\mu = 5, \sigma^2 = 2)$ using $x_i = \mu + \sigma z_i$

(d) Now use simulated sampling dist^n of $Y_1 = X_1 + X_2$ & $Y_2 = X_1 - X_2$ To show that Y_1 & Y_2 are independent.

Sol:

S#	U_1	U_2	Z_1	Z_2	X_1	X_2	Y_1	Y_2
1	0.136	0.613	-1.51	-1.30	2.86	3.161	6.021	
2	0.923	0.525	-0.385	-0.062	4.441	4.912	9.353	
3	0.77	0.902	0.590	-0.417	5.834	4.410	10.244	
4	0.897	0.349	-0.271	0.378	4.616	5.534	10.15	
5	0.627	0.664	-0.497	-0.818	4.297	3.829	8.126	
6	0.884	0.098	0.405	0.276	5.572	5.404	10.976	
7	0.854	0.738	0.166	-0.536	5.234	4.241	9.475	
8	0.35	0.226	0.216	1.432	5.305	7.025	12.33	
9	0.372	0.256	-0.066	1.504	4.910	7.126	12.046	
10	0.81	0.25	0	0.649	5	5.917	10.917	
11	0.277	0.297	-0.466	1.522	4.34	7.166	11.506	
12	0.159	0.217	0.394	1.836	5.597	2.653	13.21	
13	0.4	0.739	-0.092	-1.390	4.868	3.090	7.958	
14	0.223	0.524	-1.712	-0.260	2.578	4.632	7.21	
15	0.808	0.024	0.645	0.098	5.912	5.138	11.05	
16	0.919	0.019	0.408	0.048	5.576	5.067	10.643	
17	0.628	0.437	-0.889	0.371	3.742	5.524	9.266	
18	0.94	0.73	-0.044	-0.349	4.937	4.506	9.443	
19	0.441	0.019	1.270	0.152	6.786	5.214	12	
20	0.16	0.746	-0.048	-1.913	4.932	2.294	7.226	
21	0.390	0.52	-1.344	-0.169	3.099	4.760	7.859	
22	0.633	0.704	-0.272	-0.916	4.615	3.704	8.319	
23	0.073	0.337	-1.189	1.954	3.318	7.763	11.081	
24	0.073	0.577	-2.025	-1.064	2.136	3.495	5.631	
25	0.233	0.164	1.355	1.037	6.916	6.466	13.382	

S#	U_1	U_2	Z_1	Z_2	X_1	X_2	Y_1	Y_2
26	0.713	0.179	0.354	0.742	5.500	6.049	11.549	
27	0.815	0.547	-0.611	-0.186	4.135	4.736	8.871	
28	0.464	0.35	-0.728	1.002	3.970	6.417	10.387	
29	0.419	0.124	0.938	0.926	6.326	6.309	12.635	
30	0.91	0.515	-0.432	-0.040	4.389	4.943	9.332	
31	0.751	0.737	-0.061	-0.754	4.913	3.933	8.846	
32	0.919	0.479	-0.407	0.054	4.424	5.076	9.6	
33	0.713	0.768	0.092	-0.817	5.130	3.844	8.974	
34	0.147	0.258	-0.098	1.955	4.861	7.764	12.625	
35	0.185	0.707	-0.490	-1.770	4.307	2.496	6.863	
36	0.416	0.787	0.305	-1.288	5.431	3.178	8.609	
37	0.273	0.531	-1.580	-0.311	2.765	4.560	7.325	
38	0.572	0.777	0.178	-1.041	5.251	3.527	8.778	
39	0.676	0.765	0.083	-0.881	5.117	3.754	8.871	
40	0.863	0.126	0.381	0.386	5.538	5.545	11.083	
41	0.238	0.18	0.721	1.533	6.019	7.16		
42	0.853	0.457	-0.543	0.150	4.232	5.212		
43	0.402	0.61	-1.046	-0.860	3.529	3.783		
44	0.31	0.892	1.128	-1.033	6.595	3.539		
45	0.643	0.17	0.452	0.823	5.634	6.163		
46	0.166	0.174	0.973	1.881	6.376	7.660		
47	0.656	0.601	-0.739	-0.544	3.954	4.230		
48	0.802	0.74	-0.041	-0.662	4.942	4.063		
49	0.574	0.325	-0.478	0.938	4.324	6.326		
50	0.054	0.054	2.278	0.804	8.221	6.137		
51	0.525	0.076	1.008	0.521	6.425	5.736		
52	0.121	0.639	-1.319	-1.575	3.134	2.772		
53	0.744	0.376	-0.547	0.540	4.226	5.763		
54	0.281	0.7	-0.492	-1.515	4.304	2.857		
55	0.028	0.748	-0.033	-2.673	4.953	1.219		
56	0.892	0.216	0.101	0.467	5.142	5.660		
57	0.908	0.186	0.171	0.404	5.341	5.57		
58	0.687	0.662	-0.455	-0.737	4.356	3.957		
59	0.288	0.503	-1.577	-0.029	2.764	4.998		
60	0.799	0.654	-0.380	-0.551	4.462	4.220		
61	0.705	0.358	-0.425	0.527	4.398	5.745		
62	0.517	0.765	0.108	-1.143	5.152	3.383		
63	0.517	0.92	1.006	-0.553	6.422	4.217		
64	0.129	0.746	-0.050	-2.023	4.929	2.139		
65	0.308	0.33	-0.739	1.344	3.954	3.099		
66	0.324	0.59	-1.267	-0.804	3.208	3.862		
67	0.42	0.904	1.084	-0.747	6.533	3.943		
68	0.964	0.012	0.270	0.020	5.381	5.028		
69	0.985	0.695	-0.058	-0.163	4.917	4.769		
70	0.517	0.052	1.087	0.368	6.537	5.520		
71	0.949	0.723	-0.654	-0.318	4.923	4.550		
72	0.149	0.273	-0.281	1.930	4.602	7.729		
73	0.187	0.927	1.641	-0.810	7.320	3.854		
74	0.8	0.37	-0.457	0.486	4.353	5.687		
75	0.095	0.736	-0.190	-2.161	4.731	1.943		
76	0.299	0.641	-0.982	-1.203	3.611	3.298		

S#	U_1	U_2	Z_1	Z_2	X_1	X_2	Y_1	Y_2
77	0.922	0.21	0.100	0.390	5.141	5.551		
78	0.83	0.606	-0.479	-0.377	4.322	4.46		
79	0.168	0.966	1.845	-0.400	7.609	4.43		
80	0.132	0.79	0.500	-1.949	5.707	2.243		
81	0.642	0.684	-0.379	0.474	4.464	5.670		
82	0.19	0.264	-0.160	1.815	4.773	7.566		
83	0.76	0.964	0.610	-0.420	5.862	4.406		
84	0.955	0.627	-0.211	-0.217	4.701	4.693		
85	0.568	0.645	-0.651	-0.840	4.079	3.812		
86	0.426	0.676	-0.585	-1.167	4.172	3.349		
87	0.771	0.614	-0.543	-0.473	4.232	4.331		
88	0.134	0.703	-0.583	-1.918	4.175	2.287		
89	0.911	0.507	-0.431	-0.018	4.390	4.974		
90	0.508	0.162	0.611	0.990	5.864	6.400		

Lab # 07

Q) Let x_1 & x_2 be the sample of size 2 from the dist^n, $f(x) = e^{-x}$; $x > 0$

a) Using 100 replications, construct the sampling dist^n of $Y = \frac{x_1}{x_2}$ (on freq dist^n & Histogram using classes 0-3, 3-6, ..., 21-24 & >)

b) The exact sampling dist^n of y is $f(y) = \frac{1}{(1+y)^2}$; $y>0$, which is an $F(2,2)$. Show that cdf is given by. $F(y) = 1 - \frac{1}{1+y}$

c) Using cdf compute the prob & hence expected freq of each class

e) Comment on the accuracy of simulated sampling dist^n.

d) Superimpose the expected freq curve on the histogram

Sol:

$$f(x) = e^{-x} \quad ; x > 0$$

$$\cancel{F(x) = \int_0^x e^{-x} \Rightarrow F(x) = 1 - e^{-x}}$$

For sampling

$$x = -\ln[1 - F(x)]$$

From cdf of exact S.D:

$$F(y) = \frac{1}{(1+y)^2} \quad ; y > 0$$

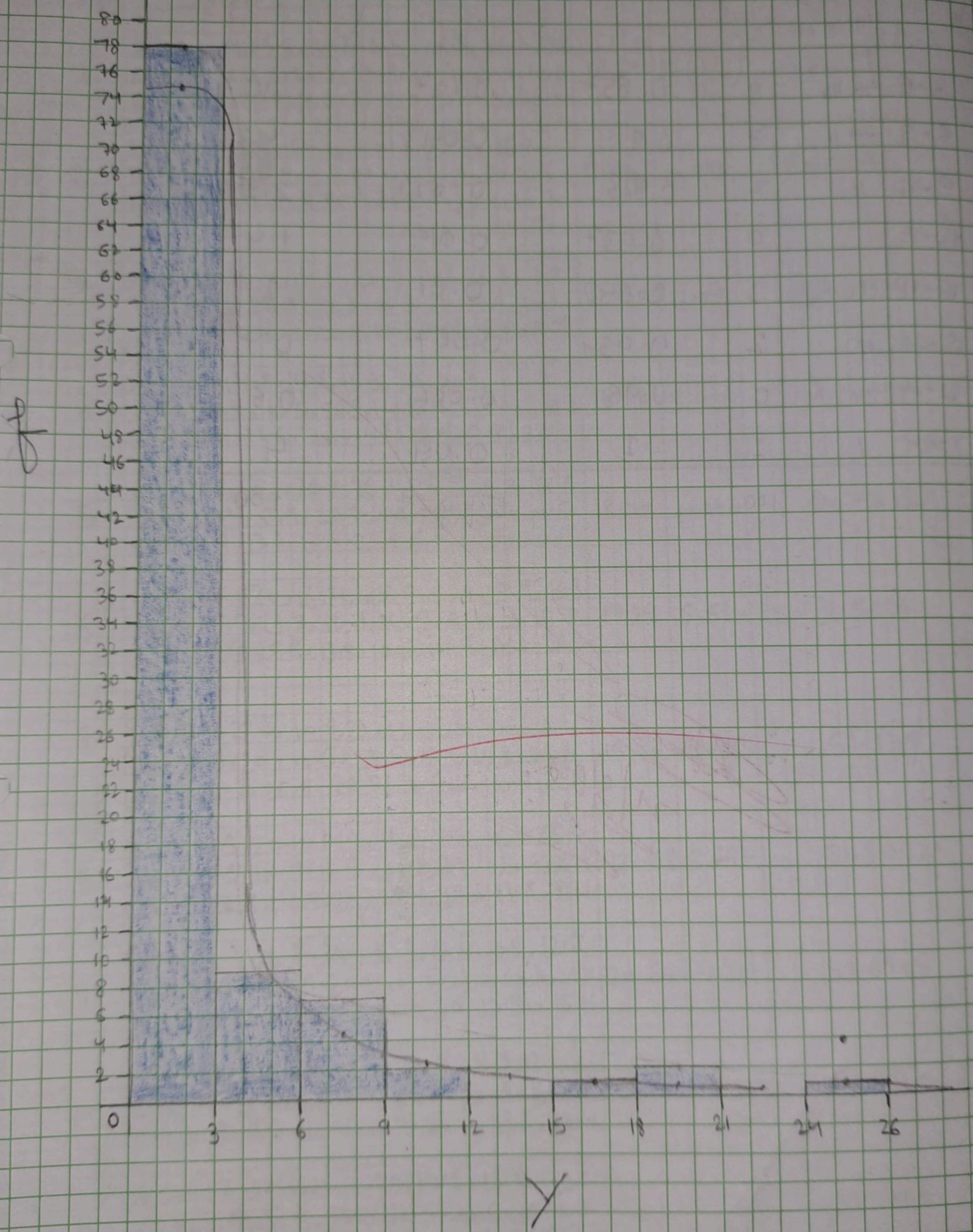
$$F(y) = \int_0^y \frac{1}{(1+y)^2}\, dy \Rightarrow F(y) = 1 - \frac{1}{1+y}$$

$$x_1 = -\ln[1 - F(x)]$$

S#	X_1	X_2	$Y = X_1/X_2$
1	1.68	1.29	1.302
2	5.11	1.59	3.213
3	0.80	1.09	0.733
4	0.22	0.54	0.407
5	0.22	1.09	0.201
6	2.26	0.35	6.457
7	1.65	0.56	2.946
8	0.10	3.07	0.032
9	0.42	0.49	0.857
10	0.79	2.12	0.372
11	0.72	1.17	0.615
12	1.94	1.77	1.096
13	0.07	0.01	7
14	1.89	3.12	0.605
15	0.12	2.28	0.052
16	0.37	0.54	0.685
17	1.80	2.64	0.681
18	1.47	0.29	5.068
19	1.04	0.05	20.8
20	1.06	0.90	1.177
21	0.466	1.13	0.412
22	1.76	0.28	6.285
23	0	1.39	0
24	2.51	0.21	11.952
25	2.95	0.12	24.583
26	1.64	0.86	1.906
27	0.58	0.03	19.333
28	0.13	2.51	0.051
29	3.14	1.62	1.938
30	0.14	0.51	0.274
31	0.39	0.18	2.166
32	0.07	0.34	0.205
33	1.48	1.15	1.286
34	0.63	3.54	0.177
35	2.64	0.33	8
36	0.03	0.72	0.041
37	0.01	0.25	0.04
38	0.51	2.00	0.255
39	1.59	0.74	2.148
40	1.44	0.44	3.272
41	0.99	1.13	0.876
42	0.42	1.09	0.385
43	0.10	0.20	0.5
44	1.16	0.47	2.468
45	0.78	0.27	2.888
46	5.52	5.80	0.951
47	1.68	2.38	0.705
48	0.12	0.22	0.545
49	1.16	0.84	1.380
50	3.14	0.27	11.629

S#	X_1	X_2	$Y = X_1/X_2$
51	0.05	0.48	0.104
52	0.35	1.87	0.187
53	0.89	0.12	7.416
54	3.54	1.33	2.661
55	0.01	0.54	0.018
56	1.10	0.97	1.134
57	1.23	0.17	7.235
58	0.08	0.14	0.571
59	0.28	0.38	0.736
60	1.61	0.30	5.366
61	2.00	0.81	2.469
62	0.70	1.35	0.518
63	0.73	4.13	0.176
64	0.50	0.23	2.173
65	4.34	1.46	2.972
66	1.16	0.27	4.296
67	0.48	1.14	0.421
68	0.05	0.13	0.384
69	2.64	2.95	0.894
70	0.31	0.58	0.534
71	1.20	2.14	0.560
72	0.21	0.12	1.75
73	0.27	0.39	0.692
74	0.29	0.17	1.705
75	0.87	1.64	0.530
76	0.14	0.32	0.437
77	0.01	0.56	0.017
78	0.89	0.12	7.416
79	0.51	0.67	0.761
80	0.25	2.98	0.086
81	0.55	0.92	0.597
82	0.57	0.54	1.055
83	0.55	0.71	0.775
84	0.49	0.12	4.083
85	1.10	0.81	1.358
86	0.81	0.26	3.115
87	1.02	1.13	0.902
88	1.37	2.27	0.603
89	0.46	3.10	0.148
90	0.34	0.90	0.109
91	0.61	0.47	1.297
92	1.48	6.21	0.238
93	1.69	0.30	5.633
94	0.69	0.39	1.769
95	0.66	0.45	1.466
96	0.23	0.83	0.277
97	1.73	0.39	4.435
98	1.85	0.22	8.409
99	1.95	0.11	17.727
100	0.12	2.83	0.042

Scale:
on x-axis: 3 boxes = 3 units
on y-axis: 1 box = 2 units



26.3, 23.5, 20, 10.5, 16.4, 26.8, 5.9, 22.3

Y	T.M	f	$F(y) = 1 - \frac{1}{1+y}$	$Prob = \Delta F(y)$	$E.F = N \times P$
0-3	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 ||||	78	0.75	0.75	75
3-6	卌 ||||	9	0.857	0.107	10.7
6-9	卌 ||	7	0.9	0.043	4.3
9-12	||	2	0.923	0.023	2.3
12-15		0	0.937	0.014	1.4
15-18	|	1	0.947	0.01	1
18-21	||	2	0.954	0.007	0.7
21-24		0	0.96	0.006	0.6
Y>24	|	1	1	0.04	4
		100		$\Sigma f(y) = 1$	100

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Lab # 08

S#	Samples	$\bar{y}$	S^2	$T = S/\bar{y} \times 100$
1	25.9, 16.4, 23.5, 14.1, 10.5, 26.8, 26.3, 12.8 26.8, 13.8, 26.8, 26.3, 25.9, 23.5, 25.9, 10.5 22.3, 23.5, 25.9, 16.3, 26.8, 5.9, 26.3, 26.3	19.24	10.061	58.2812
2	20.4, 20.4, 26.8, 16.4, 16.4, 14.1, 22.3, 16.3 16.3, 10.9, 25.9, 16.3, 14.1, 16.3, 26.8, 26.8 16.4, 26.8, 16.4, 5.9, 13.8, 5.9, 20.4, 16.3	19.96	10.040	56.28044
3	14.8, 20, 16.4, 14.8, 26.3, 14.8, 26.8, 10.9 14.8, 22.3, 26.8, 26.8, 16.3, 20, 5.9, 10.9 20.4, 16.4, 16.4, 20.4, 22.3, 14.1, 16.4, 10.5	20.91	12.108	59.01735
4	16.4, 20, 14.8, 14.8, 16.3, 20, 16.3, 25.9 13.8, 26.3, 10.5, 56.3, 14.8, 16.3, 14.8, 10.5 14.8, 20.4, 12.6, 12.6, 14.8, 25.9, 5.9, 20	15.95	5.431	34.04588
5	14.1, 26.8, 56.3, 26.8, 26.3, 26.8, 10.9, 14.8 26.3, 22.3, 5.9, 25.9, 14.1, 14.1, 5.9, 10.5 26.8, 26.3, 13.8, 12.6, 25.9, 5.9, 10.5, 20	18.46	10.047	54.4022
6	10.9, 14.1, 20, 16.4, 25.9, 14.1, 25.9, 16.3 26.3, 25.9, 23.5, 14.1, 20, 5.9, 10.5, 14.8 26.8, 25.9, 25.9, 13.8, 10.5, 14.8, 14.1, 26.3	17.89	5.564	31.5936
7	12.8, 5.9, 22.3, 10.9, 20, 16.5, 16.4, 16.3 20, 22.3, 20, 10.5, 23.5, 5.9, 26.3, 26.3 16.5, 26.8, 25.9, 25.8, 10.9, 22, 10.9, 25.9	19.52	9.983	51.1319
8	14.1, 16.3, 20.4, 14.8, 14.8, 20, [illegible], 25.9 16.5, 23.5, 16.9, 16.3, 14.8, [illegible], 16.4, 26.8 20.4, 26.8, 14.9, 20, [illegible], 23.6, 13.8, 14.1	18.058	5.692	31.50321
9	10.5, 13.8, 26.8, 20, 16.3, 16.4, 10.5, 12.8 12.6, 16.4, 25.9, 20.4, 13.8, 22.5, 26.3, 20.4 26.8, 26.8, 20, 22.3, 16.4, 20, 10.5, 16.4	20.812	9.301	49.6905
10	16.4, 14.8, 23.5, 16.3, 26.8, 16.3, 12.8, 10.5 16.4, 25.9, 12.6, 16.4, 10.5, 10.9, 16.4, 16.4 25.9, 14.8, 16.3, 20.4, 10.9, 26.3, 13.8, 16.4	20.72	12.327	59.4917
11	10.5, 14.8, 16.4, 26.4, 20, 10.5, 13.8, 12.6 12.6, 16.4, 5.9, 16.4, 16.4, 10.9, 20.4, 25.9 10.5, 20.4, 13.8, 25.9, 16.4, 12.6, 10.9, 56.3	16.80	6.045	32.1405
12	25.9, 12.8, 22.3, 20.4, 16.3, 12.8, 23.5, 14.8 56.3, 26.3, 23.5, 26.8, 20, 12.6, 20.4, 14.1 26.3, 23.5, 20, 10.5, 16.4, 26.8, 5.9, 22.3	18.71	10.072	53.68242

S#	Samples	$\bar{y}$	S^2	$T = S/\bar{y} \times 100$
13	26.3, 22.3, 5.9, 20, 22.3, 23.5, 10.5, 25.9, 10.5, 25.9, 16.4, 56.3, 16.4, 10.5, 23.5, 10.9, 5.9, 20.4, 16.4, 16.3, 56.3, 13.8, 26.8, 16.4, 16.4	21.66	11.896	54.9113
14	56.3, 20, 14.8, 12.8, 12.6, 22.3, 26.8, 14.8, 20, 14.1, 16.3, 12.6, 10.9, 14.8, 56.3, 14.8, 14.1, 23.5, 12.6, 23.5, 14.8, 10.5, 14.8, 10.9, 13.8	19.14	11.928	62.60706
15	25.9, 10.9, 14.8, 16.4, 10.5, 16.3, 12.8, 16.3, 12.8, 56.3, 14.8, 10.9, 26.3, 16.4, 20.4, 25.9, 13.8, 14.1, 20, 13.8, 23.5, 5.9, 16.4, 14.8, 16.4	17.85	9.475	53.06339
16	23.5, 16.3, 26.8, 15.3, 22.3, 5.9, 26.3, 26.8, 26.3, 25.9, 10.9, 10.5, 14.8, 10.5, 16.4, 12.8, 20.4, 26.8, 13.8, 5.9, 10.9, 16.4, 5.9, 16.4, 25.9	17.38	7.248	41.68391
17	14.8, 22.3, 13.8, 26.3, 20.4, 25.9, 26.3, 23.5, 26.8, 20.4, 25.9, 12.6, 16.3, 25.9, 20, 56.3, 56.3, 14.1, 14.1, 25.9, 16.4, 26.3, 14.1, 26.8, 12.8	23.37	11.222	48.0147
18	16.3, 5.9, 13.8, 12.8, 10.9, 25.9, 20.4, 16.4, 25.9, 12.6, 16.5, 25.9, 12.8, 16.4, 16.4, 23.5, 10.5, 14.1, 23.5, 13.8, 20.4, 22.3, 5.9, 16.5, 26.8	16.56	6.325	38.1760
19	20, 26.8, 22.3, 14.8, 12.6, 10.5, 20, 22.3, 12.8, 16.3, 56.3, 12.6, 16.4, 12.6, 14.1, 26.3, 20, 14.1, 16.3, 26.8, 26.8, 16.4, 26.3, 16.3, 26.3	20.24	9.252	45.7114
20	5.9, 26.8, 56.3, 16.4, 20, 16.9, 26.3, 14.8, 25.9, 26.3, 5.9, 14.8, 13.8, 26.3, 23.5, 56.3, 20, 25.9, 14.1, 14.1, 5.9, 12.6, 20, 12.8, 23.5	20.73	12.665	61.07735
21	20, 16.4, 26.3, 16.4, 16.3, 12.8, 14.1, 13.9, 14.3, 12.6, 12.8, 14.9, 5.9, 20, 16.4, 16.3, 25.9, 13.8, 26.3, 26.8, 20.4, 25.9, 20, 25.9, 13.9	17.76	5.805	32.671
22	25.9, 26.3, 14.8, 26.8, 22.3, 12.6, 56.3, 12.8, 26.8, 14.8, 16.3, 26.3, 16.4, 20, 14.1, 22.3, 13.8, 14.1, 16.9, 12.8, 16.4, 23.5, 20.4, 5.9, 10.9	19.34	9.706	50.186
23	13.8, 25.9, 16.3, 22.3, 26.8, 25.9, 26.8, 5.9, 25.9, 23.5, 25.9, 56.3, 12.8, 14.8, 25.9, 16.3, 26.3, 26.8, 10.9, 25.9, 12.8, 25.9, 13.8, 26.3, 20.4	22.16	9.566	43.152
24	56.3, 20, 20.4, 16.3, 25.9, 25.9, 20.4, 10.5, 14.8, 26.3, 14.8, 20.4, 12.8, 16.4, 20, 16.3, 14.1, 14.1, 12.8, 16.9, 16.3, 25.9, 22.3, 16.3, 16.3	19.46	8.995	46.223
25	16.9, 14.1, 20, 16.3, 25.9, 10.5, 14.8, 13.8, 20.4, 14.1, 56.3, 16.3, 22.3, 10.5, 23.5, 26.3, 26.3, 22.3, 25.9, 16.3, 10.5, 26.3, 26.8, 25.9, 12.6	20.35	9.556	46.944

S#	Samples	$\bar{y}$	S^2	$T = S/\bar{y} \times 100$
26	10.5, 14.8, 22.3, 5.9, 25.9, 26.3, 16.4, 10.5, 26.8 14.8, 25.9, 12.6, 16.3, 12.8, 14.1, 13.8, 26.8 14.1, 20, 12.8, 22.3, 25.9, 25.9, 20, 16.3	18.15	6.247	34.414
27	16.3, 14.8, 10.5, 22.3, 16.3, 56.3, 25.9, 14.1, 26.8 22.3, 25.9, 16.3, 12.8, 14.1, 13.8, 10.5, 23.5 20.4, 56.3, 16.3, 13.8, 26.3, 5.9, 25.9, 10.5	20.71	12.222	58.997
28	14.1, 22.3, 14.8, 10.5, 14.1, 56.3, 10.5, 26.3 14.8, 22.3, 26.3, 5.9, 10.5, 26.8, 26.3, 26.8, 16.4 26.8, 16.3, 20.4, 14.1, 22.3, 26.3, 26.8, 25.9	20.95	9.882	47.155
29	25.9, 26.3, 23.5, 14.1, 25.9, 14.1, 26.8, 23.5, 26.8 16.4, 14.1, 26.3, 10.9, 16.4, 13.8, 12.8, 25.9 12.6, 25.9, 14.1, 23.5, 16.3, 10.9, 14.1, 16.4	19.09	6.001	31.432
30	14.1, 20.4, 16.3, 25.9, 26.8, 16.3, 14.8, 26.3 23.5, 10.5, 22.3, 16.3, 12.6, 25.9, 14.8, 5.9, 23.5 13.8, 14.8, 22.3, 16.3, 14.8, 26.3, 16.4, 14.8	18.22	5.671	31.111
31	10.9, 25.9, 16.3, 56.3, 25.9, 16.4, 22.3, 14.1, 5.9 25.9, 16.3, 14.8, 20, 25.9, 12.6, 26.3, 20 56.3, 56.3, 56.3, 16.4, 10.5, 10.5, 10.5, 14.8	20.8	12.10	58.17
32	26.3, 10.5, 12.6, 25.9, 23.5, 23.5, 13.8, 14.8 25.9, 22.3, 13.8, 26.8, 16.4, 16.9, 16.4, 16.3, 16.9 16.4, 26.3, 25.9, 23.5, 5.9, 14.8, 25.9, 56.3	20.6	9.5	46.11
33	56.3, 5.9, 10.9, 13.8, 5.9, 23.5, 25.9, 26.3 25.9, 20, 23.5, 14.8, 10.9, 13.8, 16.3, 26.8, 14.8 26.8, 16.3, 26.8, 26.3, 22.3, 25.9, 26.8, 14.1	20.4	12.03	58.97
34	12.6, 5.9, 12.6, 16.4, 10.5, 12.8, 16.3, 23.5, 26.3 14.8, 26.3, 26.3, 12.8, 25.9, 23.3, 12.8, 13.9 14.8, 14.9, 14.8, 25.9, 26.8, 22.3, 26.3, 12.8	17.4	10.14	58.27
35	26.8, 12.8, 20.4, 12.6, 16.3, 16.4, 20.4, 26.3, 26.8 26.8, 13.8, 26.3, 22.3, 26.3, 24.3, 14.8, 23.5 16.4, 20.4, 26.8, 16.3, 26.8, 12.8, 25.9, 26.3	20.6	10.06	48.8
36	16.4, 5.9, 10.9, 12.6, 16.3, 14.1, 12.6, 10.9, 20.4 56.3, 25.9, 22.3, 16.4, 16.3, 16.3, 20, 20 26.8, 16.3, 10.5, 12.6, 25.9, 25.9, 12.8, 12.6	21.17	12.14	57.34
37	10.9, 25.9, 12.8, 12.8, 10.5, 5.9, 5.9, 14.8, 23.5 22.3, 22.3, 16.3, 20.4, 20, 16.4, 16.3, 22.3 10.9, 14.1, 26.8, 23.5, 13.8, 10.9, 10.5, 16.3	16.97	7.15	42.13
38	26.8, 16.4, 12.6, 10.9, 14.1, 20, 14.1, 16.4, 26.8 16.3, 16.3, 10.5, 12.8, 20, 13.8, 5.9, 14.8 26.3, 14.1, 10.5, 26.8, 12.8, 16.4, 16.4, 13.8	16.04	6.342	39.85

S#	Samples	$\bar{y}$	S^2	$T = S/\bar{y} \times 100$
39	25.9, 16.5, 26.8, 20, 14.1, 26.8, 5.9, 16.4, 25.9 56.3, 22.3, 23.5, 10.5, 12.6, 16.4, 22.3, 25.9 5.9, 14.1, 23.5, 20, 14.1, 16.3, 26.3, 16.3	19.26	9.883	51.313
40	16.3, 16.4, 16.3, 14.1, 23.5, 16.4, 26.8, 16.4, 26.3 20, 26.3, 12.6, 10.9, 14.8, 20, 26.8, 14.8 56.3, 10.9, 26.8, 10.5, 25.9, 16.4, 10.5, 26.8	18.47	6.273	33.99
41	22.3, 20, 16.3, 10.5, 16.3, 16.4, 13.8, 5.9 23.5, 16.9, 14.1, 14.8, 56.3, 14.8, 26.8, 25.9, 26.3 25.9, 14.8, 26.8, 20, 10.9, 20, 16.4, 16.3	19.44	9.58	49.27
42	20, 5.9, 10.9, 14.1, 20, 14.8, 25.9, 26.3, 14.8 12.6, 20.4, 16.3, 25.9, 26.8, 14.8, 20, 5.9 25.9, 16.4, 14.1, 25.9, 26.8, 14.8, 5.9, 16.3	17.66	6.729	38.10
43	10.9, 22.3, 14.8, 56.3, 12.8, 12.6, 20.4, 20, 5.9 14.1, 10.9, 5.9, 16.4, 16.3, 10.5, 26.3, 25.9 26.8, 14.8, 13.8, 20, 16.4, 16.3, 10.5, 23.5	17.8	9.935	55.81
44	14.1, 10.9, 23.5, 5.9, 13.8, 26.8, 14.8, 16.4, 16.3 10.5, 25.9, 26.3, 25.9, 14.8, 22.3, 12.8, 16.3 20.4, 14.1, 10.9, 5.9, 13.8, 20, 22.3, 10.5	16.6	6.216	37.4
45	10.9, 25.9, 14.8, 16.3, 13.8, 16.3, 10.5, 14.1, 20 16.3, 10.5, 5.9, 26.8, 25.9, 12.8, 16.3, 16.4 16.3, 10.5, 10.9, 16.4, 20, 5.9, 20.4, 12.8	15.46	5.33	34.47
46	10.9, 20, 16.4, 16.3, 12.8, 12.6, 20.4, 16.4, 16.3 26.3, 25.9, 14.8, 56.3, 5.9, 23.5, 10.9, 16.4 16.3, 10.5, 14.1, 10.9, 13.8, 20, 10.5, 22.3	17.22	7.085	41.14
47	20, 25.9, 26.3, 5.9, 13.8, 23.5, 10.9, 16.5, 16.4 13.8, 23.5, 10.9, 14.1, 16.3, 20, 5.9, 23.5 10.9, 12.6, 20.4, 22.3, 26.8, 12.6, 20.4, 10.5	16.70	6.345	37.99
48	14.8, 12.6, 26.4, 10.5, 16.4, 13.8, 23.5, 56.3, 12.8 12.6, 16.3, 22.3, 20.5, 5.9, 20.4, 10.9, 16.3 10.5, 5.9, 23.5, 16.9, 16.3, 10.5, 23.5, 10.9	16.69	9.75	58.414
49	25.9, 56.3, 16.3, 10.5, 26.8, 14.8, 25.9, 56.3, 12.8 10.5, 12.8, 10.5, 20, 23.5, 12.8, 10.5, 20.5 5.9, 26.8, 22.3, 20.4, 25.9, 26.3, 10.5, 16.3	20.8	12.46	59.90
50	56.3, 22.3, 14.8, 20, 16.3, 10.5, 26.8, 12.6, 10.5 14.1, 22.3, 12.6, 20.4, 10.5, 20, 23.5, 10.9 14.1, 12.6, 20.4, 13.8, 23.5, 12.8, 20.4, 20	18.48	9.264	50.129
	$\bar{T} = \frac{2345.49101}{50} = 46.909$			2345.49101

Mean of popn = 19.464
SD of popn = 9.829

Q2 Sol:

Jackknife Method:

S#	Samples	$\bar{y}$	s^2	$T = s/\bar{y} \times 100$
1	10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3, 10.5 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.68	9.97	50.8
2	14.1, 23.5, 5.9, 13.8, 20, 16.4, 16.3, 10.5 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.82	9.87	49.8
3	14.1, 10.9, 5.9, 13.8, 20, 16.4, 16.3, 10.5 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.29	10.00	52.08
4	14.1, 10.9, 23.5, 13.8, 20, 16.4, 16.3, 10.5 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	20.02	9.64	48
5	14.1, 10.9, 23.5, 5.9, 20, 16.4, 16.3, 10.5 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.7	9.96	51.3
6	14.1, 10.9, 23.5, 5.9, 13.8, 16.4, 16.3, 10.5 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.44	10.04	51.7

S#	Samples	$\bar{y}$	s^2	$T = s/\bar{y} \times 100$
7	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.3, 10.5 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.59	10.02	51.3
8	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 10.5 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.59	10.01	51.33
9	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.83	9.85	48.74
10	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.17	9.93	51.98
11	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 26.8, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.19	9.94	51.78
12	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 14.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.15	9.91	51.69
13	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 22.3, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.6	9.99	50.96
14	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 56.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.3	10.02	51.91
15	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 12.8, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	17.9	6.27	35.02
16	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.6 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.2	10.42	54.27
17	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8, 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.9	9.93	50.40
18	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8 12.6, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.4	10.03	51.70
19	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8 12.6, 20.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8	19.5	10.02	51.38

S#	Samples	$\bar{y}$	s^2	$T = s/\bar{y} \times 100$
20	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8 12.6, 20.4, 16.4, 10.5, 26.3, 25.9, 26.8, 14.8	19.5	10.0	51.28
21	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8 12.6, 20.4, 16.4, 16.3, 26.3, 25.9, 26.8, 14.8	19.3	9.8	50.77
22	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8 12.6, 20.4, 16.4, 16.3, 10.5, 25.9, 26.8, 14.8	19.1	9.93	51.98
23	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8 12.6, 20.4, 16.4, 16.3, 10.5, 26.3, 26.8, 14.8	19.1	9.94	52.04
24	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8 12.6, 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 14.8	19.1	9.91	51.88
25	14.1, 10.9, 23.5, 5.9, 13.8, 20, 16.4, 16.3 10.5, 26.3, 25.9, 26.8, 14.8, 22.3, 56.3, 12.8 12.6, 20.4, 16.4, 16.3, 10.5, 26.3, 25.9, 26.8	19.6	9.99	50.96
				2472.1

$$\bar{\hat{T}} = \frac{2472.1}{25}$$

$$\bar{\hat{T}} = 50.602$$

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Mean of popⁿ = 19.464
SD of popⁿ = 9.829

$$\hat{T} = \frac{s}{\bar{y}} \times 100$$

$$= \frac{9.829}{19.464} \times 100$$

$$\boxed{\hat{T} = 50.49}$$

$$= \bar{\hat{T}}^{*} - \hat{T}$$

$$= 50.602 - 50.49$$

$$= 0.112$$

Q-1: Using the Bootstrap to construct sampling distribution: You are given return (%) on equity (a measure of rate of return on investment of a company) for a random sample of 25 companies as follows:

01 14.1	02 10.9	03 23.5	04 5.9	05 13.8	06 20	07 16.4	08 16.3	09 10.5
10 26.3	11 25.9	12 26.8	13 14.8	14 22.3	15 56.3	16 12.8	17 12.6	18 20.4
19 16.4	20 16.3	21 10.5	22 26.3	23 25.9	24 26.8	25 14.8		

Use Bootstrap (M=50 replications) to construct sampling distribution of the coefficient of variation i.e. $T = \frac{s}{\bar{y}} \times 100$. (In financial terms this is risk per unit of average return. The reciprocal of this i.e. reward-to-risk ratio is called "Sharpe" ratio which is extensively used to measure and compare attractiveness of investment). Obtain Bias and Standard Error of this statistic if used as an estimator of the true coefficient of variation of the population data. Comment on your results. To draw sample with replacement you can code the observations as 01-25. Use a random number table or a calculator to draw a uniform random number with replacement i.e. accept a random number even if number is previously drawn.

Q-2: Using the Jackknife to construct sampling distribution: Consider the same sample of return on equity as in Q-1.
Use Jackknife method to construct sampling distribution of the coefficient of variation i.e. $T = \frac{s}{\bar{y}} \times 100$. Obtain Bias and Standard Error of this statistic if used as an estimator of the true coefficient of variation of the population data. Comment on your results.
Note: total number of Jackknife samples are M = n = 25 and each sample is of size n-1 = 24