ASSIGNMENT : SIMPLEX METHOD

Name: Shariq Faraz
Roll No: 17BME068

(12) let models : x_A, x_B, x_C
work done/process : P_1, P_2, P_3
Given no. of workers/process : 15, 30, 15.

Model	P_1	P_2	P_3	Profit
x_A	60	100	80	7500
x_B	100	240	100	15000
x_C	200	360	160	30000
n(worker)	15	30	15	

; Average worker on job = 200 days/year.

$$\text{Max. } Z = 7500x_A + 1500x_B + 30000x_C + 0s_1 + 0s_2 + 0s_3$$

Subject to :

$$60x_A + 100x_B + 200x_C \le 15 \times 200$$
$$100x_A + 240x_B + 360x_C \le 30 \times 200$$
$$80x_A + 100x_B + 160x_C \le 15 \times 200$$

$$\Rightarrow \quad 60x_A + 100x_B + 200x_C + s_1 + 0s_2 + 0s_3 = 3000$$
$$100x_A + 240x_B + 360x_C + 0s_1 + s_2 + 0s_3 = 6000$$
$$80x_A + 100x_B + 160x_C + 0s_1 + 0s_2 + s_3 = 3000$$

C : 7500 15000 30000 0 0 0

Basis	C_b	x_A	x_B	x_C	s_1	s_2	s_3	b	r
s_1	0	60	100	200	1	0	0	3000	15
s_2	0	100	240	360	0	1	0	6000	16.67
s_3	0	80	100	160	0	0	1	3000	18.75
z_j		0	0	0	0	0	0	0	
C_j		7500	15000	30000	0	0	0		

where, $C_j = c - z_j$, and $z_j = \sum C_B a_{ij}$
incoming var. = x_C ; outgoing var. = s_1

	C_j:	7500	15000	30000	0	0	0		
Basis	C_b	X_A	X_B	X_C	S_1	S_2	S_3	b	r
X_C	30000	0.3	0.5	1	0.005	0	0	15	30
S_2	0	-8.0	60	0	-1.8	1	0	600	10
S_3	0	32	20	0	-0.8	0	1	600	30
Z_j		9000	15000	30000	150	0	0	450000	
C_j		-1500	0	0	-150	0	0		

Since all $C_j < 0$, ∴ optimal sol^n is reached;

$Z = 30000 X_C$; $X_C = 15$

Max. $Z = ₹ 450000$.

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(14) let X_A, X_B, X_C be the no. of vehicles of A, B, C resp.

$$X_A + X_B + X_C \leq 30$$
$$X_A + 2X_B + 2X_C \leq 150/3$$
$$80{,}000 X_A + 130{,}000 X_B + 150{,}000 X_C \leq 400{,}00{,}00{,}00$$

$\Rightarrow$
$$X_A + X_B + X_C + S_1 + 0S_2 + 0S_3 = 30$$
$$X_A + 2X_B + 2X_C + 0S_1 + S_2 + 0S_3 = 50$$
$$8X_A + 13X_B + 15X_C + 0S_1 + 0S_2 + S_3 = 400$$

Set $X_A = X_B = X_C = 0$

Max. $Z = 6300X_A + 10800X_B + 11340X_C + 0S_1 + 0S_2 + 0S_3$

C_i

Basis	C_b	X_A	X_B	X_C	S_1	S_2	S_3	b	r
S_1	0	1	1	1	1	0	0	30	30
S_2	0	1	2	2	0	1	0	50	25
S_3	0	8	13	15	0	0	1	400	26.7
Z_j		0	0	0	0	0	0	0	
C_j		6300	10800	11340	0	0	0		

Incoming var. = X_C ; outgoing var. = S_2

Basis	C_b	X_A	X_B	X_C	S_1	S_2	S_3	b	r
S_1	0	0.5	0	0	1	-0.5	0	5	10
X_C	11340	0.5	1	1	0	0.5	0	25	50
S_3	0	0.5	-2	0	0	-7.5	1	25	50
Z_j		5670	11340	11340	0	5670	0	283500	
C_j		630	-540	0	0	-5670	0		

Incoming var. = X_A; outgoing var. = S_1

2. contd.

Basis	C_b	X_A	X_B	X_C	S_1	S_2	S_3	b	γ
X_A	6300	1	0	0	2	-1	0	10	10
X_C	11340	0	1	1	-1	1	0	20	∞
S_3	0	0	-2	0	-1	-7	1	20	∞
Z_j		6300	11340	11340	1260	5040	0	289800	
C_j		0	-540	0	-1260	-5040	0		

Optimal solⁿ is : $X_A = 10$; $X_C = 20$

∴ No. of drivers of A & C will b :

A : $3 \times 10 = 30$

C : $6 \times 20 = 120$

Max. $Z = 6300(10) + 0 + 11340(20)$

$= 289800$ Rupees.

(16) let $x_A, x_B, \& x_C$ be the no. of candy bars resp.

$\therefore$

$x_A \leq 900$

$12x_A + 6x_B + 10x_C \leq 25{,}000$

$4x_A + 10x_B + 2x_C \leq 15{,}000$

$15x_A + 8x_B + 15x_C \leq 30{,}000$

$x_A + 0x_B + 0x_C + S_1 + 0S_2 + 0S_3 + 0S_4 \leq 900$

$12x_A + 6x_B + 10x_C + 0S_1 + S_2 + 0S_3 + 0S_4 \leq 25{,}000$

$4x_A + 10x_B + 2x_C + 0S_1 + 0S_2 + S_3 + 0S_4 \leq 15{,}000$

$15x_A + 8x_B + 15x_C + 0S_1 + 0S_2 + 0S_3 + S_4 \leq 30{,}000$

$\text{Max. } Z = 2x_A + 2.5x_B + 1.5x_C + 0S_1 + 0S_2 + 0S_3 + 0S_4$

c :

Basis	C_b	x_A	x_B	x_C	S_1	S_2	S_3	S_4	b	γ
S_1	0	1	0	0	1	0	0	0	900	∞
S_2	0	12	6	10	0	1	0	0	25×10^3	4167
S_3	0	4	10	2	0	0	1	0	15×10^3	1500
S_4	0	15	8	15	0	0	0	1	30×10^3	3750
z_j		0	0	0	0	0	0	0	0	
c_j		2	2.5	1.5	0	0	0	0		

Incoming var. = $\max(C_j = c - z_j)$ = x_B

Outgoing var. = $\min(\gamma)$ = S_3

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Basis	C_b	x_A	x_B	x_C	S_1	S_2	S_3	S_4	b	r
S_1	0	1	0	0	1	0	0	0	900	900
S_2	0	9.6	0	8.8	0	1	-0.6	0	16×10^3	1666.7
x_B	2.5	0.4	1	0.2	0	0	0.1	0	1500	3750
S_4	0	11.8	0	13.4	0	0	-0.8	1	18000	1525.4
Z_j		1	2.5	0.5	0	0	0.25	0	3750	
C_j		1	0	1.0	0	0	-0.25	0		

Incoming var. = x_A
Outgoing var. = S_1

Basis	C_b	x_A	x_B	x_C	S_1	S_2	S_3	S_4	b	r
x_A	2	1	0	0	1	0	0	0	900	∞
S_2	0	0	0	8.8	-9.6	1	-0.6	0	7360	836
x_B	2.5	0	1	0.2	-0.4	0	0.1	0	1140	5700
S_4	0	0	0	13.4	-11.8	0	-0.8	1	7380	550.7
Z_j		2	2.5	0.5	1	0	0.25	0	4650	
C_j		0	0	1	-1	0	-0.25	0		

incoming var. = x_C ; outgoing var. = S_4

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Basis	C_b	X_A	X_B	X_C	S_1	S_2	S_3	S_4	b	r
X_A	2	1	0	0	1	0	0	0	900	900
S_2	0	0	0	0	-1.85	1	-0.075	-0.66	2513	∞
X_B	2.5	0	1	0	-0.224	0	0.112	-0.015	1029	∞
X_C	1.5	0	0	1	-0.881	0	-0.06	0.075	550	∞
Z_j		2	2.5	1.5	0.119	0	0.19	0.075	5200	
C_j		0	0	0	-0.119	0	-0.19	-0.075		

Optimal solution is: $X_A = 900$; $X_B = 1029$;
$X_C = 550$; $S_2 = 2513$

Max. $Z = 2(900) + 2.5(1029) + 1.5(550)$
$= 5197.5$ Rupees.

(12) let the markets be X_1, X_2 & X_3.

$X_1 \leq 5$
$X_1 + X_2 + X_3 \leq 12$
$800X_1 + 700X_2 + 500X_3 \leq 7000$

$\Rightarrow$ $X_1 + 0X_2 + 0X_3 + S_1 + 0S_2 + 0S_3 = 5$
$X_1 + X_2 + X_3 + S_1 + 0S_2 + 0S_3 = 12$
$800X_1 + 700X_2 + 500X_3 + 0S_1 + 0S_2 + S_3 = 7000$

Max. $Z = 150(40X_1 + 36X_2 + 25X_3) + 0S_1 + 0S_2 + 0S_3$

c : 6000 5400 3750 0 0 0

Basis	C_b	x_1	x_2	x_3	S_1	S_2	S_3	b	r
S_1	0	1	0	0	1	0	0	5	5
S_2	0	1	1	1	0	1	0	12	12
S_3	0	800	700	500	0	0	1	7000	8.75
Z_j		0	0	0	0	0	0	0	
C_j		6000	5400	3750	0	0	0		

incoming var. = x_1 ; outgoing var. = S_1

Basis	C_b	x_1	x_2	x_3	S_1	S_2	S_3	b	r
x_1	6000	1	0	0	1	0	0	5	∞
S_2	0	0	1	1	-1	1	0	7	7
S_3	0	0	700	500	-800	0	1	3000	4.286
Z_j		6000	0	0	6000	0	0	3×10^4	
C_j		0	5400	3750	-6000	0	0		

Incoming var. = x_2 ; outgoing var. = S_3

Basis	C_b	x_1	x_2	x_3	S_1	S_2	S_3	b	r
x_1	6000	1	0	0	1	0	0	5	5
S_2	0	0	0	0.286	0.143	1	-10^{-3}	2.7	19
x_2	5400	0	1	0.714	-1.143	0	10^{-3}	4.3	-3.75
Z_j		6000	5400	3857.1	-171.4	0	7.71	53142.86	
C_j		0	0	-107.1	171.4	0	-7.71		

incoming var. = S_1 ; outgoing var. = x_1

Basis	Cb	x_1	x_2	x_3	S_1	S_2	S_3	b	γ
S_1	0	1	0	0	1	0	0	5	∞
S_2	0	-0.143	0	0.286	0	1	-0.001	2	∞
x_2	5400	1.143	1	0.714	0	0	0.001	10	10
z_j		6171.4	5400	3857.1	0	0	7.714	54000	
c_j		-171.4	0	-167.1	0	0	-7.714		

Optimal solution is: $x_1 = 0$; $x_2 = 10$
$x_3 = 0$; $S_1 = 5$

$$\text{Max. } z = 6000(0) + 5400(10) + 3750(0)$$
$$= 54000 \text{ Rupees.}$$

(20) let x_A, x_B, x_C be the piano types.

$$2x_A + 4x_B + 2x_C \le 600$$
$$5x_A + 2x_B + 3x_C \le 400$$
$$5x_A + 2x_B + 10x_C \le 900$$

$$\text{Max. } z = 100(30x_A + 40x_B + 20x_C)$$

$$2x_A + 4x_B + 2x_C + S_1 + 0S_2 + 0S_3 = 600$$
$$5x_A + 2x_B + 3x_C + 0S_1 + S_2 + 0S_3 = 400$$
$$5x_A + 2x_B + 10x_C + 0S_1 + 0S_2 + S_3 = 900$$

	Cj:	3000	4000	2000	0	0	0		
Basis	Cb	x_A	x_B	x_C	S_1	S_2	S_3	b	γ
S_1	0	2	4	2	1	0	0	600	150
S_2	0	5	2	3	0	1	0	400	200
S_3	0	5	2	10	0	0	1	900	450
z_j		0	0	0	0	0	0	0	
c_j		3000	4000	2000	0	0	0		

incoming var. = x_B ; outgoing var. = s_1

Basis	C_b	x_A	x_B	x_C	s_1	s_2	s_3	b	r
x_B	4000	0.5	1	0.5	0.25	0	0	150	300
s_2	0	4	0	2	-0.5	1	0	100	25
s_3	0	4	0	9	-0.5	0	1	600	150
z_j		2000	4000	2000	1000	0	0	6×10^5	
c_j		1000	0	0	-1000	0	0		

incoming var. = x_A ; outgoing var. = s_2

Basis	C_b	x_A	x_B	x_C	s_1	s_2	s_3	b	r
x_B	4000	0	1	0.25	0.312	-0.125	0	137.5	∞
x_A	3000	1	0	0.5	-0.125	0.250	0	25	25
s_3	0	0	0	7	0	-1	1	500	∞
z_j		3000	4000	2500	875	250	0	625000	
c_j		0	0	-500	-875	-250	0		

Optimal soln. is : $x_A = 25$; $x_B = 137$; $x_C = 0$
$s_3 = 500$

Max. $Z = 3000(25) + 4000(137) + 2000(0)$
$= 623000$ Rupees.

Dual: of Que 20.

$$\text{Min. } W = 600y_1 + 400y_2 + 900y_3$$

$$2y_1 + 5y_2 + 5y_3 \geq 3000$$
$$4y_1 + 2y_2 + 2y_3 \geq 4000$$
$$2y_1 + 3y_2 + 10y_3 \geq 2000$$

$\Rightarrow$

$$2y_1 + 5y_2 + 5y_3 - S_1 + 0S_2 + 0S_3 + A_1 + 0A_2 + 0A_3 = 3000$$
$$4y_1 + 2y_2 + 5y_3 + 0S_1 - S_2 + 0S_3 + 0A_1 + A_2 + 0A_3 = 4000$$
$$2y_1 + 3y_2 + 10y_3 + 0S_1 + 0S_2 - S_3 + 0A_1 + 0A_2 + A_3 = 2000$$

$$\text{Max. } W' = -600y_1 - 400y_2 - 900y_3 + 0S_1 + 0S_2 + 0S_3 - MA_1 - MA_2 - MA_3$$

Set $y_1 = y_2 = y_3 = S_1 = S_2 = S_3 = 0$.

	C_j:	-600	-400	-900	0	0	0	-M	-M	-M		
Basis	C_b	y_1	y_2	y_3	S_1	S_2	S_3	A_1	A_2	A_3	b	r
A_1	-M	2	5	5	-1	0	0	1	0	0	3000	600
A_2	-M	4	2	2	0	-1	0	0	1	0	4000	2000
A_3	-M	2	3	10	0	0	-1	0	0	1	2000	200
Z_j		-8M	-10M	-17M	M	M	M	-M	-M	-M	-9000M	
C_j		-600 +8M	-400 +10M	-900 +17M	-M	-M	-M	0	0	0		

incoming var. = y_3 ; outgoing var. = A_3

Basis	C_b	y_1	y_2	y_3	S_1	S_2	S_3	A_1	A_2	A_3	b	r
A_1	-M	1	3.5	0	-1	0	0.5	1	0	-0.5	2000	571.43
A_2	-M	3.6	1.4	0	0	-1	0.2	0	1	-0.2	3600	2571.43
y_3	-900	0.2	0.3	1	0	0	-0.1	0	0	0.1	200	666.67
Z_j		-180 -4.6M	-270 -4.9M	-900	M	M	90 -0.7M	-M	-M	-90 +0.7M	-5600M -180,000	
C_j		-420 +4.6M	-130 +4.9M	0	-M	-M	-90 +0.7M	0	0	90 -1.7M		

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~~Dual~~: incoming var. = y_2; outgoing = A_1

Basis	C_b	y_1	y_2	y_3	s_1	s_2	s_3	A_1	A_2	A_3	b	r
y_2	-400	0.29	1	0	-0.29	0	0.143	0.29	0	-0.143	571.4	2000
A_2	-M	3.2	0	0	0.4	-1	0	-0.4	1	0	2800	875
y_3	-900	0.114	0	1	0.085	0	-0.143	-0.085	0	0.143	28.57	250
Z_j		-218.6 -3.2M	-400	-900	39.5 -0.4M	M	71.43	-39.5 +0.4M	-M	-71.4	-254,273 -2800M	
C_j		-381.4 +3.2M	0	0	-39.5 +0.4M	-M	-71.43	39.5 -1.4M	0	-M		

incoming var. = y_1; outgoing var. = y_3

Basis	C_b	y_1	y_2	y_3	s_1	s_2	s_3	A_1	A_2	A_3	b	r
y_2	-400	0	1	-2.5	-0.5	0	0.5	0.5	0	-0.5	500	1000
A_2	-M	0	0	-28	-2	-1	4	2	1	-4	2000	500
y_1	-600	1	0	8.75	0.75	0	-1.25	-0.75	0	1.25	250	-200
Z_j		-600	-400	-4250 +28M	-250 +2M	M	550 -4M	250 -2M	-M	-550 +4M	-350,000 -2000M	
C_j		0	0	3350 -28M	250 -2M	-M	-550 +4M	-250 +M	0	550 -5M		

incoming var. = s_3; outgoing var. A_2

Basis	C_b	y_1	y_2	y_3	s_1	s_2	s_3	A_1	A_2	A_3	b	r
y_2	-400	0	1	1	-0.25	0.125	0	0.25	-0.125	0	250	∞
s_3	0	0	0	-7	-0.5	-0.25	1	0.5	0.25	-1	500	∞
y_1	-600	1	0	0	0.125	-0.312	0	-0.125	0.3125	0	875	875
Z_j		-600	-400	-400	25	137.5	0	-25	-137.5	0	-625000	
C_j		0	0	-500	-25	-137.5	0	-M	-M	-M		

Optimal Sol^n is : Max. $W' = -625000$

$\therefore$ Min $W = -$ Max. $W' = 625000$

$\because$ Min $W =$ Max $Z = 625000$

Primal is verified by dual.

(22) Let x_A, x_B, x_C be the no. of products formed by lines A, B, C respectively.

$\therefore$

$$150 x_A + 200 x_B + 160 x_C \geq 2000$$
$$100 x_A + 160 x_B + 80 x_C \geq 3000$$
$$500 x_A + 760 x_B + 890 x_C \geq 3000$$
$$400 x_A + 400 x_B + 600 x_C \geq 6000$$

$$\text{Min.}(Z) = 20 \times 3000 x_A + 20 \times 5000 x_B + 18 \times 4000 x_C$$

Forming its Dual,
Here x_1, x_2, x_3, x_4 will be no. products formed of product 1, 2, 3, 4 respectively.

Dual:

$$150 x_1 + 100 x_2 + 500 x_3 + 400 x_4 \leq 20 \times 3000$$
$$200 x_1 + 100 x_2 + 760 x_3 + 400 x_4 \leq 20 \times 5000$$
$$160 x_1 + 80 x_2 + 890 x_3 + 600 x_4 \leq 18 \times 4000$$

$$\text{Max.}(W) = 2000 x_1 + 3000 x_2 + 3000 x_3 + 6000 x_4$$

$$\Rightarrow 150 x_1 + 100 x_2 + 500 x_3 + 400 x_4 + s_1 + 0s_2 + 0s_3 \leq 60{,}000$$
$$200 x_1 + 100 x_2 + 760 x_3 + 400 x_4 + 0s_1 + s_2 + 0s_3 \leq 10{,}0000$$
$$160 x_1 + 80 x_2 + 890 x_3 + 600 x_4 + 0s_1 + 0s_2 + s_3 \leq 72{,}000$$

$$\text{Max.}(W) = 2000 x_1 + 3000 x_2 + 3000 x_3 + 6000 x_4 + 0s_1 + 0s_2 + 0s_3$$

Set $x_1 = x_2 = x_3 = 0$

C : 2000 3000 3000 6000 0 0 0

Basis	C_b	X_1	X_2	X_3	X_4	S_1	S_2	S_3	b	r
S_1	0	150	100	500	400	1	0	0	60000	150
S_2	0	200	100	760	400	0	1	0	100000	250
S_3	0	160	80	890	600	0	0	1	72000	120
Z_j		0	0	0	0	0	0	0	0	
C_j		2000	3000	3000	6000	0	0	0		

incoming var. = X_4 ; outgoing var. = S_3

Basis	C_b	X_1	X_2	X_3	X_4	S_1	S_2	S_3	b	r
S_1	0	43.3	46.67	-93.3	0	1	0	-0.67	12000	257.1
S_2	0	93.3	46.67	166.67	0	0	1	-0.67	52000	1114.3
X_4	6000	0.267	0.133	1.483	1	0	0	0.002	120	900
Z_j		1600	800	8900	6000	0	0	10.00	720000	
C_j		400	2200	-5900	0	0	0	-10.0		

incoming var. = X_2 ; outgoing var. = S_1

Basis	C_b	X_1	X_2	X_3	X_4	S_1	S_2	S_3	b	r
X_2	3000	0.93	1	-2	0	0.021	0	-0.014	257.1	-18000
S_2	0	50	0	260	0	-1	1	0	40000	∞
X_4	6000	0.143	0	1.75	1	-0.003	0	0	85.7	24000
Z_j		3642.9	3000	4500	6000	47.143	0	-21.43	1286000	
C_j		-1642.9	0	-1500	0	-47.143	0	21.43		

incoming var. = S_3 ; outgoing var. = X_4

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Basis	C_b	X_1	X_2	X_3	X_4	S_1	S_2	S_3	b	γ
X_2	3000	1.5	1	5	4	0.01	0	0	600	600
S_2	0	50	0	260	0	-1	1	0	4000	∞
S_3	0	40	0	490	280	-0.8	0	1	24000	∞
Z_j		4500	3000	15000	12000	30	0	0	1800000	
C_j		-2500	0	-12000	-6000	-30	0	0		

Optimal Solⁿ is : $X_2 = 600$

From Rule I of dual concept, $X_A = 30$

$\text{Max. } W = 2000(0) + 3000(600) + 0 + 0$
$= 1800000$ Rupees.

$\text{Min. } Z = \text{Max. } W = 1800000$ Rupees.

(24) let x_1 be the deluxe m/c and x_2 be the standard m/c.

$x_2 \leq 250$
$18x_1 + 3x_2 \leq 800$
$9x_1 + 4x_2 \leq 600$

$0x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 250$
$18x_1 + 3x_2 + 0s_1 + s_2 + 0s_3 = 800$
$9x_1 + 4x_2 + 0s_1 + 0s_2 + s_3 = 600$

$Max. Z = 400x_1 + 200x_2 + 0s_1 + 0s_2 + 0s_3$

	C_j	400	200	0	0	0		
Basis	C_b	x_1	x_2	s_1	s_2	s_3	b	γ
s_1	0	0	1	1	0	0	250	∞
s_2	0	18	3	0	1	0	800	44.4
s_3	0	9	4	0	0	1	600	66.67
		0	0	0	0	0	0	
		400	200	0	0	0		

incoming var. = x_1, outgoing var. = s_2

Basis	C_b	x_1	x_2	S_1	S_2	S_3	b	r
S_1	0	0	1	1	0	0	250	250
x_1	400	1	0.167	0	0.056	0	44.4	266.7
S_3	0	0	2.5	0	-0.5	1	200	80
z_j	~~400~~	400	66.67	0	22.2	0	17777.78	
c_j	~~0~~	0	133.33	0	-22.2	6		

incoming var. = x2
outgoing var. = S3

Basis	C_b	x_1	x_2	S_1	S_2	S_3	b	r
S_1	0	0	0	1	0.2	-0.4	170	850
x_1	400	1	0	0	0.089	-0.067	31.1	350
x_2	200	0	1	0	-0.2	0.4	80	-400
z_j		400	200	0	-4.44	53.33	28444.4	
c_j		0	0	0	4.44	-53.33		

incoming var. = S2; outgoing var. = S_1

Basis	C_b	x_1	x_2	S_1	S_2	S_3	b	r
S_1	0	-2.25	0	1	0	-0.25	100	∞
S_2	0	11.25	0	0	1	-0.75	350	∞
x_2	200	2.25	1	0	0	0.25	150	150
z_j		450	200	0	0	50	30000	
c_j		-50	0	0	0	-50		

Optimal solⁿ is : $x_1 = 0$; $x_2 = 150$; $x_3 = 0$

Max. $z = 400(0) + 200(150)$
$= 30000$ Rupees.

(26) let X_1 be no. of trousers and X_2 be no. of shirts.

$X_1 \geq 400$

$\frac{X_1}{1000} + \frac{X_2}{3000} \leq 1 \Rightarrow 3X_1 + X_2 \leq 3000$

$\frac{X_1}{1500} + \frac{X_2}{2000} \leq 1 \Rightarrow 2X_1 + 1.5X_2 \leq 3000$

$\Rightarrow X_1 + 0X_2 - S_1 + A_1 + 0S_2 + 0S_3 = 400$

$3X_1 + X_2 + 0S_1 + S_2 + 0S_3 + 0A_1 = 3000$

$2X_1 + 1.5X_2 + 0S_1 + 0S_2 + S_3 + 0A_1 = 3000$

$\text{Max. } Z = 40X_1 + 15X_2 + 0S_1 + 0S_2 + 0S_3 - MA_1$

$X_1 = X_2 = S_1 = 0$

	C :	40	15	0	0	0	-M		
Basis	C_b	X_1	X_2	S_1	S_2	S_3	A_1	b	r
A_1	-M	1	0	-1	0	0	1	400	400
S_2	0	3	1	0	1	0	0	3000	1000
S_3	0	2	1.5	0	0	1	0	3000	1500
Z_j		-M	0	M	0	0	-M	-400M	
C_j		40+M	15	-M	0	0	0		

incoming var. = X_1

outgoing var. = A_1

Basis	C_b	X_1	X_2	S_1	S_2	S_3	A_1	b	r
X_1	40	1	0	-1	0	0	1	400	-400
S_2	0	0	1	3	1	0	-3	1800	600
S_3	0	0	1.5	2	0	1	-2	2200	1100
Z_j		40	0	-40	0	0	40	16000	
C_j		0	15	40	0	0	-M		

incoming var. = S_1 ; outgoing var. = S_2

Basis	C_b	X_1	X_2	S_1	S_2	S_3	A_1	b	r
X_1	40	1	0.333	0	0.333	0	0	1000	3000
S_1	0	0	0.333	1	0.333	0	-1	600	1800
S_3	0	0	0.833	0	-0.667	1	0	1000	1200
Z_j		40	13.33	0	13.33	0	0	40,000	
C_j		0	1.667	0	-13.33	0	-M		

incoming var. = X_2 ; outgoing var. = S_3

Basis	C_b	X_1	X_2	S_1	S_2	S_3	A_1	b	r
X_1	40	1	0	0	0.6	-0.4	0	600	600
S_1	0	0	0	1	0.6	-0.4	-1	200	∞
X_2	15	0	1	0	-0.8	1.2	0	1200	∞
Z_j		40	15	0	12.0	2	0	42000	
C_j		0	0	0	-12.0	-2.0	-M		

Optimal Solⁿ is : $X_1 = 600$; $X_2 = 1200$

Max. $Z = 40(600) + 15(1200)$
$= 42,000$ Rupees.

(28) Let x_A, x_B, x_C be the no. of products of A, B, C resp.

For 1 week:

$$2.4\,x_A + 3x_B + 2x_C \le 20 \times 60$$
$$5\,x_A + 0x_B + 2.5x_C \le 25 \times 60$$
$$0\,x_A + 2.5x_B + 1.5x_C \le 10 \times 60$$

$\Rightarrow$ $2.4x_A + 3x_B + 2x_C + s_1 + 0s_2 + 0s_3 = 1200$

$5x_A + 0x_B + 2.5x_C + 0s_1 + s_2 + 0s_3 = 1500$

$0x_A + 2.5x_B + 1.5x_C + 0s_1 + 0s_2 + s_3 = 600$

set $x_A = x_B = x_C = 0$.

Max. $Z = 0.6x_A + 0.7x_B + 0.5x_C + 0s_1 + 0s_2 + 0s_3$

	C :	0.6	0.7	0.5	0	0	0		
Basis	C_b	x_A	x_B	x_C	s_1	s_2	s_3	b	γ
s_1	0	2.4	3	2	1	0	0	1200	400
s_2	0	5.0	0	2.5	0	1	0	1500	∞
s_3	0	0	2.5	1.5	0	0	1	600	240
z_j		0	0	0	0	0	0	0	
c_j		0.6	0.7	0.5	0	0	0		

incoming var. = x_B ; outgoing var. = s_3.

Basis	C_b	x_A	x_B	x_C	S_1	S_2	S_3	b	γ
S_1	0	2.4	0	0.2	1	0	-1.2	480	200
S_2	0	5	0	2.5	0	1	0	1500	300
x_B	0.7	0	1	0.6	0	0	0.4	240	∞
Z_j		0	0.7	0.42	0	0	0.28	168	
C_j		0.6	0	0.08	0	0	-0.28		

incoming var. = x_A; outgoing var. = S_1

Basis	C_b	x_A	x_B	x_C	S_1	S_2	S_3	b	γ
x_A	0.6	1	0	0.083	0.417	0	-0.5	200	2400
S_2	0	0	0	2.083	-2.083	1	2.5	500	240
x_B	0.7	0	1	0.6	0	0	0.4	240	400
Z_j		0.6	0.7	0.47	0.25	0	-0.02	288	
C_j		0	0	0.03	-0.25	0	0.02		

incoming var. = x_C; outgoing var. = S_2

Basis	C_b	x_A	x_B	x_C	S_1	S_2	S_3	b	γ
x_A	0.6	1	0	0	0.5	-0.04	-0.6	180	180
x_C	0.5	0	0	1	-1	0.48	1.2	240	∞
x_B	0.7	0	1	0	0.6	-2.89	-0.32	96	∞
Z_j		0.6	0.7	0.5	0.22	0.014	0.016	295	
C_j		0	0	0	-0.22	-0.014	-0.016		

Optimal Sol^n is : $x_A = 180$; $x_B = 96$
$x_C = 240$.

Max. $Z = 0.6(180) + 0.7(96) + 0.5(240)$
$= 295$ Rupees / week.

For shadow prices of resources:

Looking at C_j column, we have.

$$Z = 295 - 0.22 S_1 - 0.014 S_2 - 0.016 S_3$$

Thus, shadow prices of resources 1, 2, and 3, are -0.22 Rupees/min, -0.014 ₹/min, and -0.016 ₹/min respectively.

(30) Let x_A, x_B, x_C be the no. of Grade A, Grade B, Grade C plywoods resp.

$$\Rightarrow \quad 2x_A + 5x_B + 10x_C \leq 900$$
$$2x_A + 5x_B + 3x_C \leq 400$$
$$4x_A + 2x_B + 2x_C \leq 600$$

$\Rightarrow$ $2X_A + 5X_B + 10X_C + S_1 + 0S_2 + 0S_3 = 900$
$2X_A + 5X_B + 3X_C + 0S_1 + S_2 + 0S_3 = 400$
$4X_A + 2X_B + 2X_C + 0S_1 + 0S_2 + S_3 = 600$

$\text{Max. } Z = 40X_A + 30X_B + 20X_C + 0S_1 + 0S_2 + 0S_3$

Set $X_A = X_B = X_C = 0$

	C :	40	30	20	0	0	0		
Basis	C_b	X_A	X_B	X_C	S_1	S_2	S_3	b	r
S_1	0	2	5	10	1	0	0	900	450
S_2	0	2	5	3	0	1	0	400	200
S_3	0	4	2	2	0	0	1	600	150
Z_j		0	0	0	0	0	0	0	
C_j		40	30	20	0	0	0		

Incoming var. = X_A ; outgoing var. = S_3

Basis	C_b	X_A	X_B	X_C	S_1	S_2	S_3	b	r
S_1	0	0	4	9	1	0	-0.5	600	150
S_2	0	0	4	2	0	1	-0.5	100	25
X_A	40	1	0.5	0.5	0	0	0.25	150	300
Z_j		40	20	20	0	0	10	6000	
C_j		0	10	0	0	0	-10		

incoming var. = X_B
outgoing var. = S_2

Basis	C_b	X_A	X_B	X_C	S_1	S_2	S_3	b	r
S_1	0	0	0	7	1	-1	0	500	∞
X_B	30	0	1	0.5	0	0.25	-0.125	25	∞
X_A	40	1	0	0.25	0	-0.125	0.312	137.5	137.5
Z_j		40	30	25	0	2.5	8.75	6250	
C_j		0	0	-5	0	-2.5	-8.75		

Optimal Solⁿ is : $X_A = 137.5$; $X_B = 25$

$X_C = 0$

Max. $Z = 40(137.5) + 30(25) + 0$

$= 6230$ Rupees.