IEOR E4630

Asset Allocation

Final Project

Submitted by:

Advait Varma *av2959*
Khalifa Altamimi *kna2120*
Sharique Merchant *sm5151*
Shashank Nathani *sn2954*

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1. Introduction

In this project we will try to convince the investor that the risk parity method performs better than the other methods in term of its sharp ratios. We compared the risk parity method with several other method for different time periods and we also perform a stress test on it to see how the strategy performs in crisis.

Firstly, we created portfolio of stocks depending on its correlation, then we implemented the different asset allocation method and reported the results.

2. Risk Parity

Risk parity is an asset allocation method which uses the risk to determine the allocation rather than the fully depending on the returns. So, the method is based on choosing the allocation of the assets in a way that each asset contributes to the total risk equally.

So, for a long-term investor, who is not planning to touch his portfolio for a while after investing, his/her portfolio should be more immune to the economic surprises and that what the risk parity tries to achieve.

Risk parity portfolio tries to be resilient to any major economic environment and shocks and tries to generate a steady premium over time.

3. Picking the Stocks

As this class is not about picking the stocks, we tried a simple way to pick a portfolio of about 200 stocks from the S&P 500:

1. Picked a time which had a trough and peak so we can capture the correlation between the stocks (from 2006 to 2011)
2. Got the Sharpe ratio of the 500 stocks through the time-period chosen
3. Performed k clustering so we can group the highly correlated stocks together
4. From the highly correlated stocks, we picked the highest sharp ratio

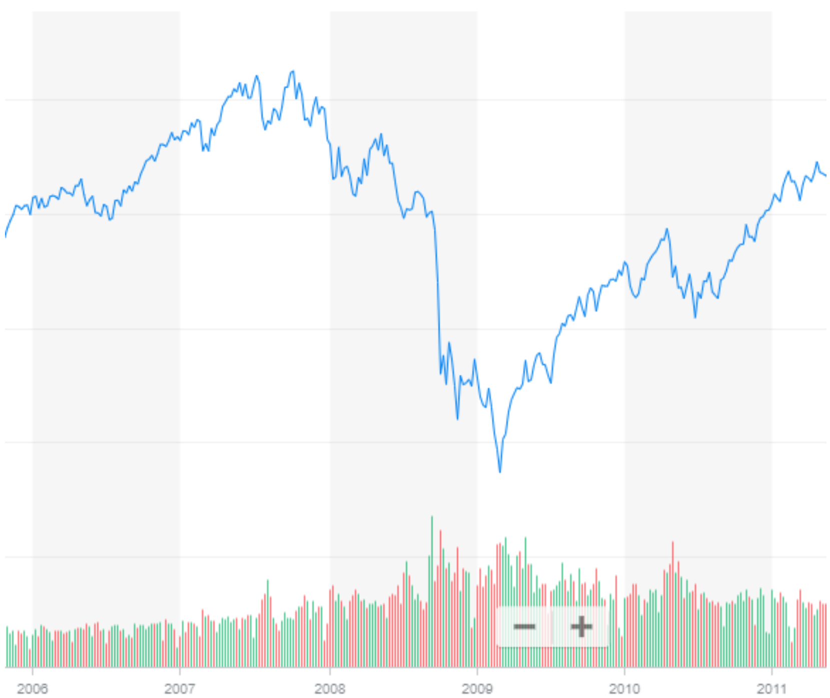

Fig 1. Historical Performance of S&P 500

4. Methods Implemented

4.1. 1/N with Rebalancing (ew)

This method is the simplest allocation method in which each asset is equally weighted.

$$\boldsymbol{a = 1/N}$$

4.2. Market Portfolio (mkt)

In this method, we set the asset allocation weight to be 100% for S&P 500 i.e., the Market.

$$\boldsymbol{a_f = 1}$$

4.3. Sample based Mean-Variance (mv)

We used the following equation to obtain the weights of the efficient portfolio with given required return:

$$\boldsymbol{a = \frac{k - r}{(u - re)^{-1}\omega^{-1}(u - re)}\,\omega^{-1}(u - re)}$$

4.4. Bayes-Stein

Bayesian method adds more degree of freedom and specify a prior, so it decreases it dependence to the in-sample returns using the following equations:

$$u^{js} = \alpha u_0 + (1 - \alpha) u$$

$$u_0 = \frac{u^T \omega^{-1} e}{e^T \omega^{-1} e} e$$

$$\alpha = \frac{d + 2}{d + 2 + (N - d - 2)(u - u_0)^T \omega^{-1} (u - u_0)}$$

4.5. Uncertainty in Mean with Box Uncertainty set (r-m-1)

In this method I used Scipy.minimize to minimize the objective:

$$\sqrt{a^T \omega^{-1} a}$$

Constraint:

$$u^T * a - \delta^T |a| \geq k$$

4.6. Risk Parity

Used Scipy to optimize the weights so that each asset contributes equally to the total risk:

$$\frac{1}{N} = a \frac{\omega a}{a \omega a}$$

5. Performance Comparison of Strategies

We have tested the risk parity strategy through several time periods and duration, and have also stressed test it through the 2008 Financial Crisis and the Covid-19 periods:

5.1. <u>3 Years (From October 2010 – October 2013)</u>

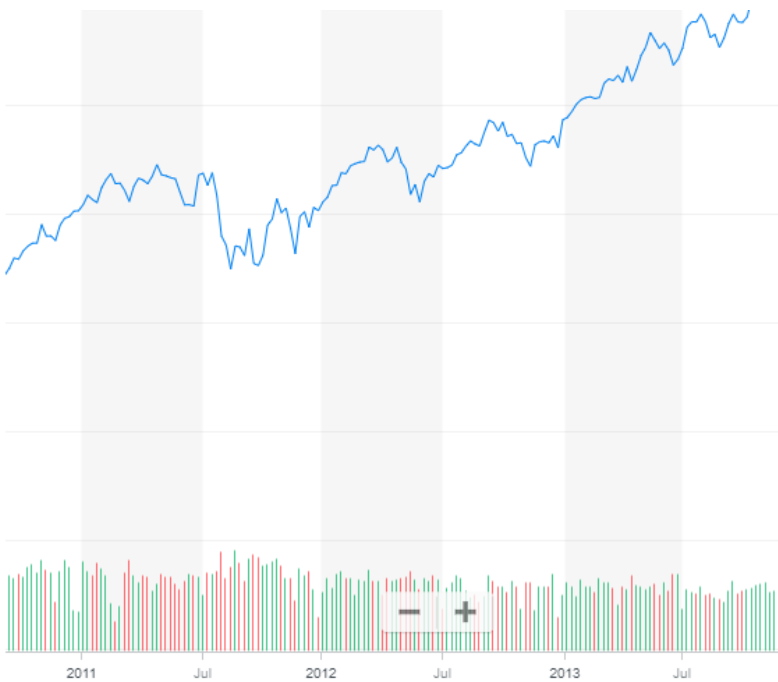

Fig 2. 3 Years Performance of S&P 500

Results:

	ISR	OSR	Change
ew	0.051	0.053	3%
mkt	0.040	0.041	3%
mv	0.548	- 0.224	-141%
bs	0.230	- 0.210	-191%
r-m-1	0.051	0.050	-2%
Risk Parity	0.052	0.053	3%

	OSR
Risk Parity	0.053
ew	0.053
r-m-1	0.050
mkt	0.041
bs	- 0.210
mv	- 0.224

Table 1. 3 Years Performance of S&P 500

5.2. 5 Years (From October 2010 – September 2015)

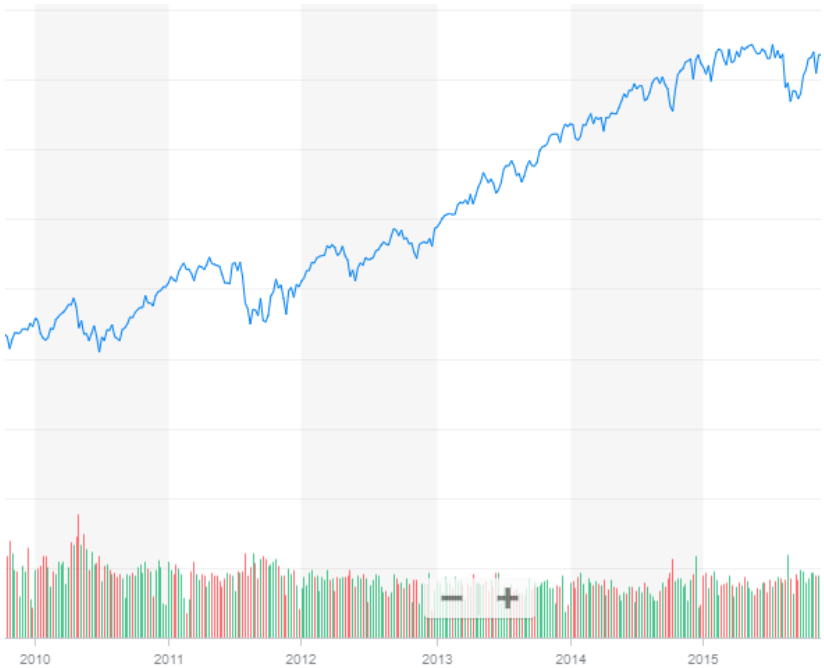

Fig 3. 5 Years Performance of S&P 500

Results:

	ISR	OSR	Change
ew	0.068	0.068	0%
mkt	0.056	0.057	2%
mv	0.677	- 0.227	-134%
bs	0.300	- 0.236	-179%
r-m-1	0.069	0.066	-5%
Risk Parity	0.057	0.073	27%

	OSR
Risk Parity	0.073
ew	0.068
r-m-1	0.066
mkt	0.057
mv	- 0.227
bs	- 0.236

Table 2. 5 Years Performance of S&P 500

5.3. 8 Years (From October 2010 – October 2018)

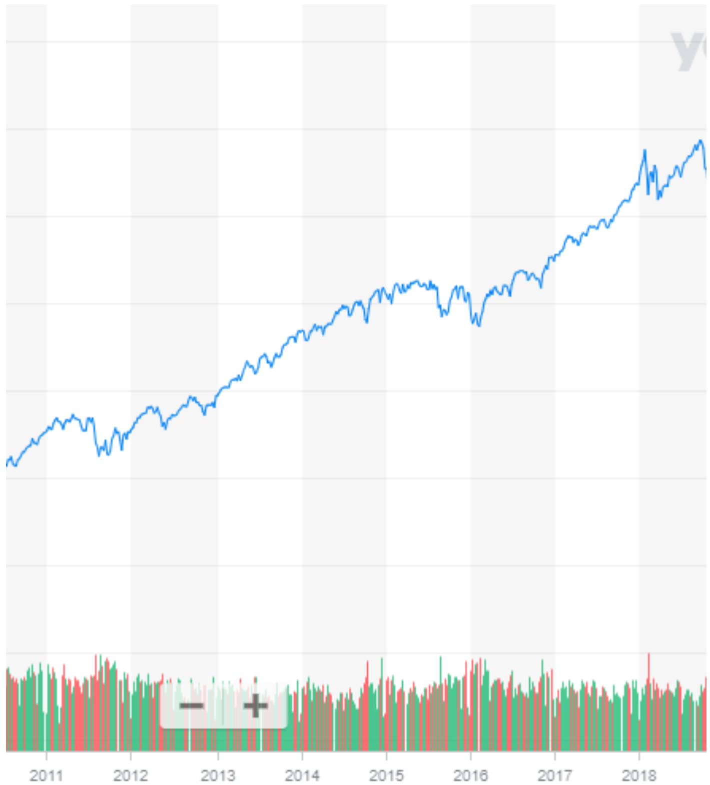

Fig 4. 8 Years Performance of S&P 500

Results:

	ISR	OSR	Change
ew	0.065	0.065	0%
mkt	0.055	0.055	0%
mv	0.879	- 0.167	-119%
bs	0.445	- 0.217	-149%
r-m-1	0.067	0.065	-2%
Risk Parity	0.069	0.067	-2%

	OSR
Risk Parity	0.067
r-m-1	0.065
ew	0.065
mkt	0.055
mv	- 0.167
bs	- 0.217

Table 3. 8 Years Performance of S&P 500

5.4. 10 Years (From October 2010 – October 2020)

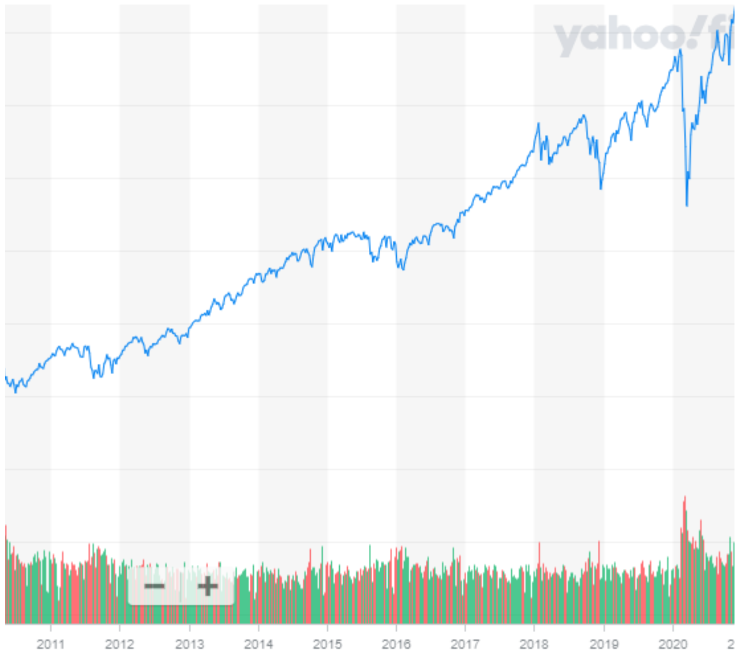

Fig 5. 10 Years Performance of S&P 500

Results:

	ISR	OSR	Change
ew	0.051	0.050	-2%
mkt	0.046	0.045	-2%
mv	0.936	- 0.138	-115%
bs	0.474	- 0.159	-134%
r-m-1	0.050	0.050	0%
Risk Parity	0.055	0.053	-4%

	OSR
Risk Parity	0.053
ew	0.050
r-m-1	0.050
mkt	0.045
mv	- 0.138
bs	- 0.159

Table 4. 10 Years Performance of S&P 500

5.5. Financial Crisis (From October 2007 – March 2009)

Fig 6. Financial Crisis Performance of S&P 500

Results:

	ISR	OSR	Change
ew	0.007	-	-100%
mkt	- 0.003	- 0.009	200%
mv	0.263	- 0.041	-116%
bs	0.156	- 0.007	-104%
r-m-1	0.005	0.001	-80%
Risk Parity	0.007	0.001	-86%

	OSR
r-m-1	0.001
Risk Parity	0.001
ew	-
bs	- 0.007
mkt	- 0.009
mv	- 0.041

Table 5. Financial Crisis Performance of S&P 500

5.6. Covid-19 (From October 2019 – October 2020)

Fig 7. Covid-19 Performance of S&P 500

Results:

	ISR	OSR	Change
ew	- 0.005	- 0.005	2%
mkt	- 0.007	- 0.007	-1%
mv	0.116	- 0.046	-140%
bs	0.015	- 0.008	-152%
r-m-1	- 0.005	- 0.005	-6%
Risk Parity	- 0.005	- 0.002	-60%

	OSR
Risk Parity	- 0.002
r-m-1	- 0.005
ew	- 0.005
mkt	- 0.007
bs	- 0.008
mv	- 0.046

Table 6. Covid-19 Performance of S&P 500

6. Final Comments

As it can be seen form the above results, on all the given period the risk parity beat the two benchmarks (S&P 500 and 1/N). Adding to that it also beat the other method! When stress testing it through the two crises (Covid-19 and Financial Crisis) even though the sharp ratios were negative, the risk parity decreased the least relative to the benchmark and the other methods.

Adding to that, we didn't execute the risk parity method properly as it should be more diversified than only stock where it should include commodities, gold, bonds to be more diversified and resilient to the any major economic environment.

So, if there are bonds, during the recession when the volatility of the stocks increases the shift will be higher to the bond and thus will result in a better performance than our results.

Finally, we highly recommend using the risk parity method as it shows a preferable performance.

The next step would be to introduce the Hierarchical Risk Parity (HRP) approach. HRP portfolios address three major concerns of quadratic optimizers in general and Markowitz's CLA in particular: Instability, concentration and underperformance. HRP applies modern mathematics (graph theory and machine learning techniques) to build a diversified portfolio based on the information contained in the covariance matrix. However, unlike quadratic optimizers, HRP does not require the invertibility of the covariance matrix. In fact, HRP can compute a portfolio on an ill-degenerated or even a singular covariance matrix, an impossible feat for quadratic optimizers. Monte Carlo experiments show that HRP delivers lower out-of-sample variance than CLA, even though minimum-variance is CLA's optimization objective. HRP also produces less risky portfolios out-of-sample compared to traditional risk parity methods.

References

- S&P 500 Graphs - https://finance.yahoo.com
- Risk parity - https://www.investopedia.com/terms/r/risk-parity