Valuing Levered Projects

Interactions between financing and investing

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Finance - investment interactions

Concern very common business decisions:

- Telenor wants to build new mobile network
  - but finance the investment with more debt
- Transport company considers fleet expansion
  - but wants to lease, not buy the trucks
- Oil company wants to diversify into green energy
  - project has very different risk characteristics
  - different debt ratio also

Structure of decision problem:

1. Accept project if its NPV>0
2. to calculate NPV we need:
    1. cash flows (are given here)
    2. discount rate (chapter's topic)
3. Discount rate depends on:
    1. business risk or, equivalently, the OCC
        1. calculated from existing operations if business risk is same
        2. otherwise, has to estimated from other companies
    2. financial risk, division over debt and equity
        1. depends on debt ratio
        2. and financing rule (predetermined of rebalanced)

Basic elements introduced in derivation MM proposition 2 with taxes:

- Costs of debt, equity increase with leverage
- taxes influence cost of capital

Elaborate in more detail here:

- Good exercise in systematic evaluation of financing decisions
- Usually avoided by making arbitrary assumptions, you'll get the full Monty (almost)
- Limited practical use: only 1 imperfection, taxes, can be incorporated in discount rate
- assume optimal capital structure 'externally' determined (not modelled)

Some important concepts

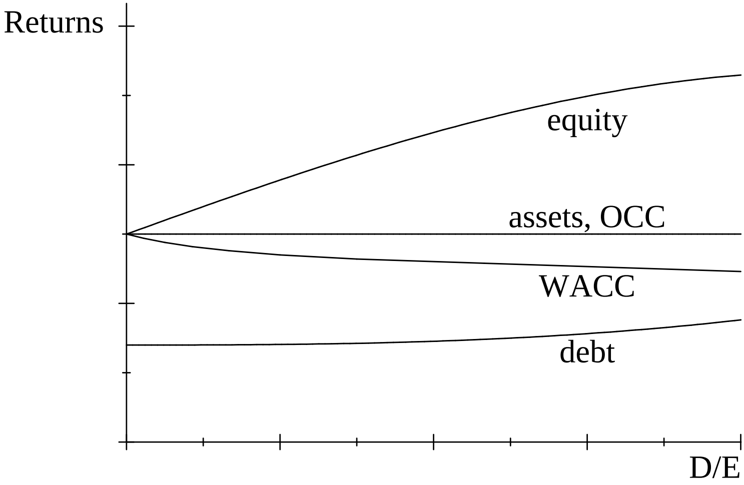
- Business risk
  uncertainty of cash flows generated by firm's assets
  risk of oil company, software house, construction company
- Opportunity costs of capital
  reward for bearing business risk
  is what shareholder expect if all equity financed
  set prices such that expected return equals OCC
- Financial risk
  if cash flow split in low risk-return and high risk-return part
  debtholders have priority over shareholders
  shareholders bear extra financial risk

Discount rates

- $r = r_a =$ *opportunity cost of capital* expected rate of return for equivalent risk all equity financed assets
- $r' = WACC$ *after tax weighted average cost of capital*
  - WACC calls for unlevered after tax cash flows
  - WACC is valid for assets with same risk and debt ratio

$$WACC = r_e \frac{E}{V} + r_d(1-\tau)\frac{D}{V} \tag{1}$$

- $r_d =$ cost of debt
- $r_e =$ cost of equity, subscript $._{u,l}$ for (un)levered
- $\tau =$ corporate tax rate (constant, no personal taxes)



Returns and leverage

Basic approaches

- When business risk changes:
  - calculate new opportunity cost of capital / new asset β
  - taking leverage into account (unlever βs, example later)
  - remember: proper β is project β (not necessarily company β)

- When debt ratio increases *given business risk*, two ways:
  1. Adjust the discount rate
     downwards, to include value of interest tax shields
  2. Adjust the present value with side effects
     called *Adjusted Present Value*, after Myers

To calculate Adjusted Present Value:

1. First calculate *base case* value of project as if all equity financed and without side effects
2. Then separetely calculate value of side effects and sum results.

Side effects can be anything: tax shields, issue costs, effects on other projects, agency costs, fees to stock exchange, etc.

In case of taxes:

1. first calculate value as if all equity financed
2. then calculate value of tax shields

Concentrate on tax shields here, but include some examples of other side effects

Value of tax shields depends on the financing rule followed:

1. Money amounts of debt *predetermined*, following a schedule
   1. repayments and interest follow schedule
   2. tax shields tied to interest payments, cost of debt appropriate discount rate
2. Debt *rebalanced* to a constant fraction of future project values
   1. money amount of debt goes up and down with project value
   2. tax shields also tied to fortunes of the project
      $\Rightarrow$ incorporate business risk
      $\Rightarrow$ discount at the opportunity cost of capital

Working with APV: some examples

Base case

- Project gives perpetual risky cash flow (EBIT) of 1562.5 per year
- Requires an investment of 8000
- Tax rate is 20%, risk of assets requires a return of 15%
- $r = r_a = r_{e,u} = .15$

Value of the 'unlevered' cash flows:

$$\frac{(1-.2)\times 1562.5 = 1250}{.15} = 8333$$

Base case NPV = 8333 - 8000 = 333

Issue costs

- Firm issues equity to finance the project
- Issue costs are 7.5%

Has to issue

$$\frac{100}{92.5} \times 8000 = 8649$$

to collect 8000

- Issue costs: 8649 - 8000 = 649
- APV = 333 - 649 = -316

Tax shields

- Project has debt capacity of 50%
- Take a perpetual loan of 4000, predetermined money amount
- Interest rate 10%, yearly interest charge 400
- Tax advantage interest: $.2 \times 400 = 80$

Debt fixed: discount at $r_d \Rightarrow$ value tax shields: $\frac{80}{.1} = 800$
Issue

$$4000 \times \frac{100}{92.5} = 4324$$

in equity, issue costs 324

- APV = 333 - 324 + 800 = 809

Rebalanced debt

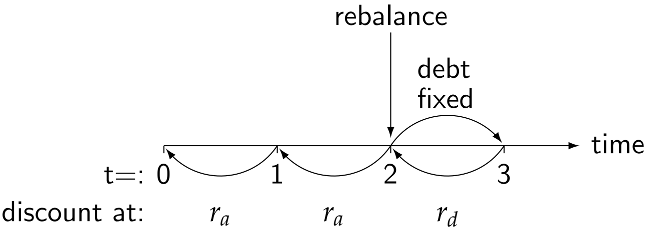
What if debt is rebalanced every year?

- We know the first year's tax shield: 80
  - Have to discount to present (t_0) at $r_d = .1$
- At the end of the first year, debt is rebalanced to 50% of project value (unknown now)
  - then second year's tax shield is know:
  - discount year 2 to year 1 (t_1) with r_d
  - but it is uncertain how project value develops in year 1:
  - discount year 1 to present (t_0) with r_a :

$$\frac{80}{\underbrace{(1+.15)}_{yr.1-rebal.} \times \underbrace{(1+.1)}_{yr.2-fixed}}$$

At the end of the second year, debt is rebalanced to 50% of project value (unknown now)

- then third year's tax shield is know:
- discount year 3 to year 2 (t_2) with r_d
- but project value development in years 1 and 2 uncertain:
- discount year 1 and 2 to present (t_0) with r_a



In formula: $\frac{80}{(1+.15)^2 \times (1+.1)}$ or more generally:

$$\frac{80}{1+r_d} + \frac{80}{(1+r_a)(1+r_d)} + \frac{80}{(1+r_a)^2(1+r_d)} + ...$$

Sum of this series calculated in 2 steps:

- discount at r_a, the opportunity cost of capital
- multiply result by: $\frac{1+r_a}{1+r_d}$

For our project:

$$\frac{80}{.15} = 533 \times \frac{1.15}{1.1} = 557$$

- APV = 333 + (- 324) + 557 = 566

Adjusting the discount rate

Structure of the problem is simple:

- we know the elements on the balance sheet
- need to express unknown returns as functions of known returns
  - by rewriting the balance sheet equality
- But: tax shields can have two returns/discount rates
- gives to sets of functions

Starting point is the balance sheet:

r_a	Value assets	$= V_a$	debt	$= D$	r_d
r_d or r_a	Value tax shields	PV(TS)	equity	$= E$	r_e
	total value	$= V$	total value	$= V$	

If debt is predetermined:

- risk of tax shields = risk of debt
- discount tax advantages at r_d
- use Modigliani-Miller formula, MM tax case

If debt is rebalanced:

- risk tax of shields = risk of assets
- discount tax advantages at r_a
- use Miles-Ezzell formula, MM no-tax case

(1) Predetermined debt

Gives the following balance sheet:

r_a	Value assets	$= \mathrm{V}_a$	debt	$= \mathrm{D}$	r_d
r_d	Value tax shields	PV(TS)	equity	$= \mathrm{E}$	r_e
	total value	$= \mathrm{V}$	total value	$= \mathrm{V}$	

Predetermined debt amounts mean:

- tax shields just as risky as debt itself
- $\Rightarrow$ discount at r_d

We can write the balance sheet in terms of weighted average costs of capital:

$$r_a V_a + r_d PV(TS) = r_e E + r_d D \tag{2}$$

Rearranging terms gives expressions for r_a and r_e :

$$\begin{aligned} r_a &= r_e \frac{E}{V_a} + r_d \frac{D - PV(TS)}{V_a} \\ r_a \frac{V_a}{V} &= r_e \frac{E}{V} + r_d \frac{D - PV(TS)}{V} \end{aligned} \tag{3}$$

and for r_e :

$$r_e = r_a + (r_a - r_d)\frac{D - PV(TS)}{E} \tag{4}$$

These are general expressions that can also be used for projects of limited life.

But they are not very practical:

- call for the value of tax shields
- usually not known before project value is calculated
- except under MM assumption that debt is also permanent

If debt is also permanent (as well as predetermined), the present value of the tax shields is

$$PV(TS) = \frac{\tau(r_d D)}{r_d} = \tau D \tag{5}$$

Substituting (5) in (3) and (4) gives the Modigliani-Miller expressions for r_e and r_a:

$$r_e = r_a + (r_a - r_d)\frac{D - PV(TS)}{E} = r_a + (r_a - r_d)\frac{D - \tau D}{E}$$

$$r_e = r_a + (r_a - r_d)(1 - \tau)\frac{D}{E} \tag{6}$$

i.e. MM proposition 2 with taxes

and for r_a:

$$
\begin{aligned}
r_a \frac{V_a}{V} &= r_e \frac{E}{V} + r_d \frac{D - PV(TS)}{V} \\
&= r_e \frac{E}{V} + r_d \frac{D - \tau D}{V}
\end{aligned}
$$

$$r_a \frac{V_a}{V} = r_e \frac{E}{V} + r_d (1 - \tau) \frac{D}{V} = WACC = r' \tag{7}$$

WACC formula (7) can be re-written in 2 ways:

1. Gives an explicit relation between r_a and r' (exact for fixed and permanent values)

$$
\begin{aligned}
r_a \frac{V_a}{V} &= WACC = r' \\
r_a \frac{V - \tau D}{V} &= r' \\
r_a \left(1 - \tau \frac{D}{V}\right) &= r'
\end{aligned}
$$

Defining $L = D/V$, i.e. the debt-value ratio, we get the Modigliani-Miller formula:

$$WACC = r' = r_a(1 - \tau L)$$

MM formula can be used to 'unlever' and 'relever':

- given the WACC, formula can be used to calculate r_a the opportunity cost of capital
  - in most given situations, r_e, r_d and τ are (in principle) observable
  - r_a is not
- r_a can then be used to calculate WACC for a different debt ratio

Modigliani-Miller formula can also be derived by substituting MM prop. 2 (6) into WACC (7)

2 Second way to rewrite WACC formula gives alternative expression for r_a :

$$r_a = r_d(1-\tau)\frac{D}{V-\tau D} + r_e\frac{E}{V-\tau D}$$

We can do the same analysis in terms of β:

$$\beta_e = \beta_a + (1-\tau)(\beta_a - \beta_d)\frac{D}{E}$$

$$\beta_a = \beta_d(1-\tau)\frac{D}{V-\tau D} + \beta_e\frac{E}{V-\tau D}$$

Adjusting the discount rate

(2) rebalanced debt

Continuous rebalancing

Gives the same balance sheet as before

r_a	Value assets	$= \mathrm{V}_a$	debt	$= \mathrm{D}$	r_d
r_a	Value tax shields	PV(TS)	equity	$= \mathrm{E}$	r_e
	total value	$= \mathrm{V}$	total value	$= \mathrm{V}$	

Continuous rebalancing means:

- tax shields just as risky as the assets
- proportion of total value in assets vrs. tax shields irrelevant
- taxes drop out of the equation
- opportunity cost of capital is simply the weighted average of the costs of debt and equity:

$$r_a \frac{V_a}{V} + r_a \frac{PV(TS)}{V} = r_a = r = r_d \frac{D}{V} + r_e \frac{E}{V}$$

Can also be rewritten in terms of r_e or β, gives MM prop.2 without taxes:

$$r_e = r + (r - r_d)\frac{D}{E}$$

$$\beta_e = \beta_a + (\beta_a - \beta_d)\frac{D}{E}$$

Periodical rebalancing

If debt is rebalanced once per period

- Tax shield over the next period is known: $\tau r_d D$
  - should be discounted at r_d : $\tau r_d D/(1 + r_d)$
- Tax shields further in future are uncertain
  - should be discounted with r_a

Gives following balance sheet identity in return terms:

$$V_a r_a + \underbrace{\frac{\tau r_d D}{1 + r_d} r_d}_{\textit{next period}} + \underbrace{(PV(TS) - \frac{\tau r_d D}{1 + r_d}) r_a}_{\textit{further periods}} = r_e E + r_d D$$

Rewriting (using $V_a = E + D - PV(TS)$) gives expression for r_e:

$$r_e = r_a + (r_a - r_d)\frac{D}{E}(1 - \frac{\tau r_d}{1 + r_d}) \tag{8}$$

equivalent to MM2 with discrete rebalancing

Substituting (8) into the formula for the WACC (1) gives (after extensive rewriting):

$$r' = WACC = r_a - \frac{D}{V} r_d \tau \left(\frac{1 + r_a}{1 + r_d}\right) \tag{9}$$

This formula is known as the Miles-Ezzell formula

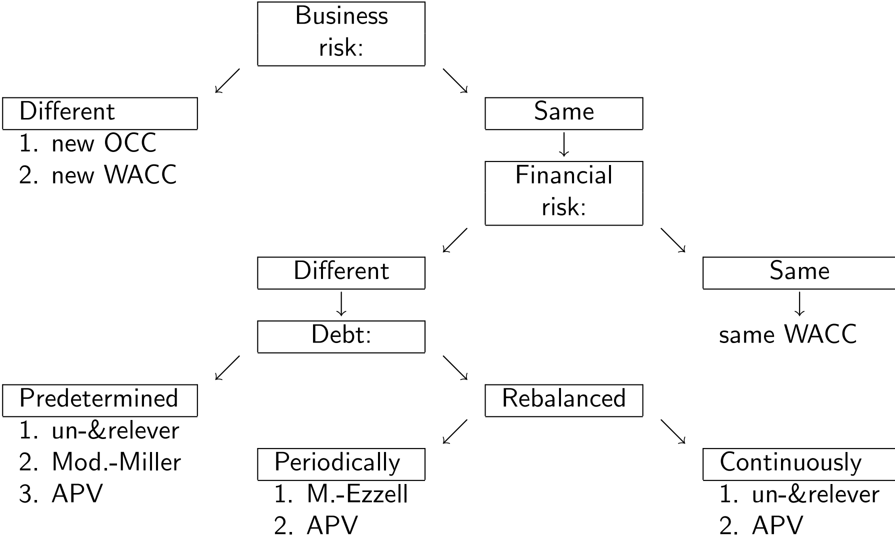
Miles-Ezzell formula equivalent to Modigliani-Miller if debt is rebalanced discretely.
Used in the same way for unlevering and relevering:

- for a given WACC, formula gives r_a the opportunity cost of capital
- given r_a formula can be used to calculate WACC for a different debt ratio (and different cost of debt)

Formula can also be derived using a backward iteration procedure, start with last period, work way to beginning (as originally done by Miles-Ezzell)

Project values with different debt ratios

- Recall that this requires business risk to remain the same
- Method depends on the characteristics of the project
- Main distinction is whether
  - debt amounts are predetermined
  - or vary with project value
- Formulas for perpetuities often used for short lived projects
- Distinction continuous - discrete rebalancing seldom used


Business risk:
Different
1. new OCC
2. new WACC
Same
Financial risk:
Different
Debt:
Predetermined
1. un-&relever
2. Mod.-Miller
3. APV
Rebalanced
Periodically
1. M.-Ezzell
2. APV
Continuously
1. un-&relever
2. APV
Same
same WACC

Procedure:

- we know returns: $\mathbf{r_e}$ and $\mathbf{r_d}$
- and relative sizes: $\mathbf{V_e/V}$ and $\mathbf{V_d/V}$
- of debt and equity in existing operations: data in **bold**

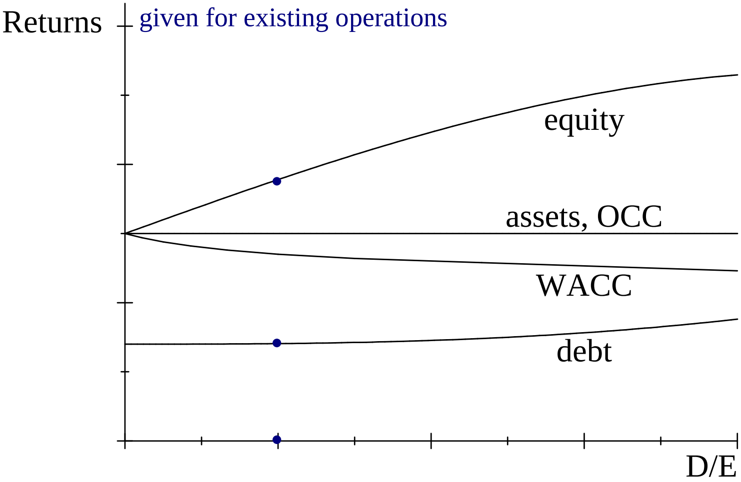
We also known

- the firm's financial policy (rebalanced or predetermined debt)
- the interest rate and D/E ratio for the new project

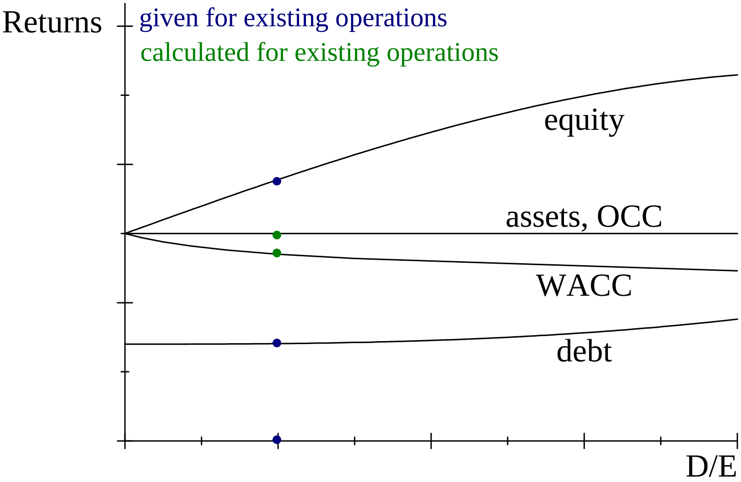
We can then calculate the project value

- by adjusting the WACC (stepwise or with a formula)
- or by using APV

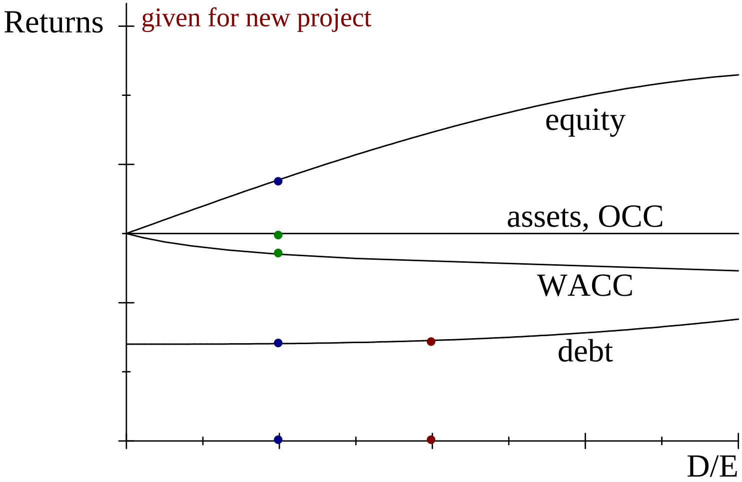
Graphical representation of procedure:



Returns and leverage



Returns and leverage



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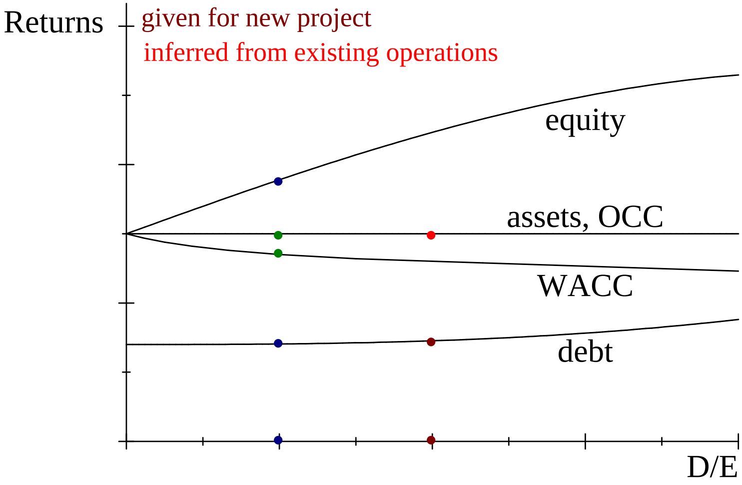
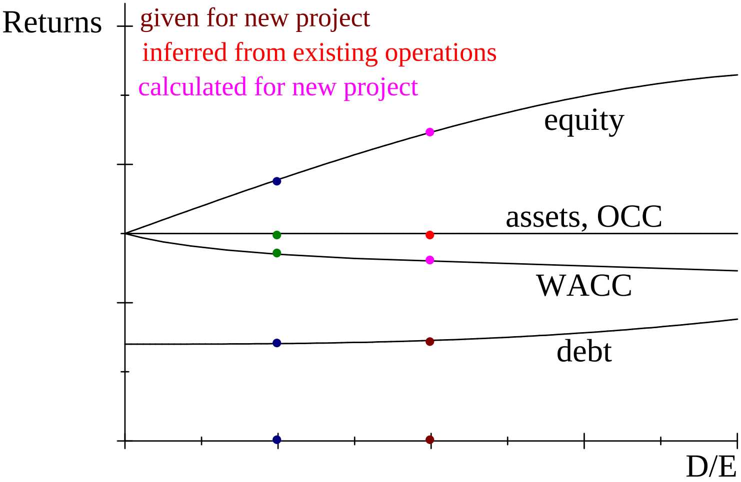


Returns and leverage



Returns and leverage

Procedure looks complicated, rationale is simple:

1. To calculate project value, we need the WACC
2. to calculate WACC, we need cost of equity, r_e
3. to calculate cost of equity, we need OCC $r = r_a$
4. OCC can be calculated from existing operations, since business risk is the same

So we start at the bottom, with the OCC

We also need project details:

- project's debt ratio (decided by management)
- project's financing rule (decided by management)
- project's cost of debt (bank will give an offer)

Debt rebalanced, 3 ways:

First way: stepwise adjust WACC
(this requires continuous rebalancing):

(a) Unlever: calculate opportunity cost of capital from the existing operations

$$r = \mathbf{r_d}\frac{\mathbf{D}}{\mathbf{V}}+\mathbf{r_e}\frac{\mathbf{E}}{\mathbf{V}}$$

(b) Use this OCC plus project's cost of debt and debt ratio to calculate project's cost of equity using:

$$r_e = r + (r - r_d)\frac{D}{E}$$

step (a) and (b) can be also be done in terms of β's, then use CAPM to calculate returns

(c) Relever: calculate after tax WACC using project's costs and weights:

$$WACC = r_e\frac{E}{V} + r_d(1-\tau)\frac{D}{V}$$

Second way: adjust WACC using Miles-Ezzell formula
(this requires discrete rebalancing):

(a) Unlever: use data from existing operations to calculate OCC

$$\mathbf{r}' = r - \tau \mathbf{r_d} \frac{\mathbf{D}}{\mathbf{V}} \left(\frac{(1+r)}{(1+\mathbf{r_d})} \right)$$

by solving Miles-Ezzell for r (use Miles-Ezzell 'in reverse')

(b) Relever: use Miles-Ezzell and OCC plus project's cost of debt and debt ratio to calculate project's WACC:

$$r' = r - \tau r_d \frac{D}{V} \left(\frac{(1+r)}{(1+r_d)} \right)$$

Third way: use Adjusted Present Value (APV):

1. Calculate OCC using 1 of methods above
2. discount project's cash flow to find base case NPV
3. Discount tax shields at opportunity cost of capital
4. Multiply PV with $(1+r)/(1+r_d)$ if debt is rebalanced periodically

Debt amounts predetermined, same 3 ways:

First way: stepwise adjust WACC:

(a) Unlever: calculate opportunity cost of capital $r = r_a$:

$$r_a = \mathbf{r_e}\frac{\mathbf{E}}{\mathbf{V_a}} + \mathbf{r_d}\frac{\mathbf{D} - \mathbf{PV}(\mathbf{TS})}{\mathbf{V_a}} \text{ or } r_a\frac{\mathbf{V_a}}{\mathbf{V}} = \mathbf{r_e}\frac{\mathbf{E}}{\mathbf{V}} + \mathbf{r_d}\frac{\mathbf{D} - \mathbf{PV}(\mathbf{TS})}{\mathbf{V}}$$

Not very practical, only used under the Modigliani-Miller assumption that cash flows are perpetuities:

$$r = r_a = \mathbf{r_d}(1 - \tau)\frac{\mathbf{D}}{\mathbf{V} - \tau\mathbf{D}} + \mathbf{r_e}\frac{\mathbf{E}}{\mathbf{V} - \tau\mathbf{D}}$$

(b) Use OCC and project's cost of debt and debt ratio to calculate project's cost of equity using:

$$r_e = r_a + (r_a - r_d)\frac{D - PV(TS)}{E}$$

or under the Modigliani-Miller assumptions:

$$r_e = r + (1-\tau)(r - r_d)\frac{D}{E}$$

step (a) and (b) can be also be done in terms of β's, use CAPM to calculate returns

(c) Relever: calculate after tax WACC using project's costs and weights

$$WACC = r_e\frac{E}{V} + r_d(1-\tau)\frac{D}{V}$$

Second way: adjust WACC using Modigliani-Miller formula (requires MM assumptions)

(a) Unlever: use data from existing operations to calculate OCC

$$\mathbf{r}' = r_a(1 - \tau\mathbf{L})$$

by solving MM for r_a (use MM 'in reverse')

(b) Relever: use MM again, with OCC (r_a) and project's debt-to-value ratio to calculate project's WACC:

$$r' = r_a(1 - \tau L)$$

- MM assumes debt is predetermined and permanent
- good approximation for projects with limited lives if debt is predetermined

Third way: adjusted present value (APV)

- calculate OCC
- calculate base case NPV
- use predetermined schedule for interest payments
- discount tax shield at the cost of debt

Example of unlevering β :

If project is in different line of business:

- different business risk
- different asset β
- different opportunity cost of capital

Find asset β from firms in same line of business:

- take average of a no. of firms after unlevering $\beta's$
- assumes disturbances cancel out.

Following 3 firms are considered representative of business risk (asset beta) in an industry. Calculate the asset beta.

Firm	Stock β	debt/total value
1	1.35	0.40
2	1.25	0.50
3	1.30	0.55

All debt is rebalanced and can be considered risk free.
The relation between asset β and equity beta is:

$$\beta_a = \beta_d \frac{D}{V} + \beta_e \frac{E}{V}$$

if debt is risk free, this is:

$$\beta_a = \beta_e \frac{E}{V}$$

The calculation becomes:

Firm	Stock β	equity/total value	Asset β
1	1.35	0.60	0.810
2	1.25	0.50	0.625
3	1.30	0.45	0.585
sum			2.020

Gives an average β of 2.02/3=0.67

A worked out example

company data

- Transport company, book value €90 million
- debt frequently adjusted and renegotiated
- capital structure constant (rebalanced)
- market value short term debt €20 mill., interest rate = 9%
- market value long term debt €20 mill., interest rate = 11%
- 10 million shares outstanding, priced at €6 to give 20% return
- tax rate 35%

Balance sheet ZXco

Property, plant & eq.	40		Equity		40
other fixed assets	20		Long term debt		20
total fixed assets		60	Accounts payable	10	
Cash	10		Short term debt	20	
Account receivable	10		*current liabilities*		30
Inventories	10				
current assets		30			
total assets		90	total liabilities & equity		90

Project data

- expansion to new geographical area, same business
- expansion has optimal capital structure at 60% debt
- apart from that financial policy unchanged
- debt available at 12%
- investment €50 million
- gives perpetual after tax cash flow of €7 million per year

Question:
should company accept project or not?

Analysis

- When should project be accepted?
  - Accept project if NPV > 0
- What do we need to calculate NPV?
  - Proper discount rate or APV
- Does project have different business risk?
  - No, expansion in same business, same risk
- Can we use company cost of capital (WACC)?
  - No, business risk is the same, financial risk is different, different debt ratio
- What procedure do we use?
  - Debt rebalanced $\Rightarrow$ stepwise adjust WACC, Miles-Ezzell formula or APV

Calculations

First, make some adjustments to balance sheet:

1. Calculate company capital structure at market prices:
    - Market value debt €40 million (20+20, freq. renegotiated)
    - Market value equity 6x10 million = €60 mill.
2. 'net out' accounts payable and current assets
    - accounts payable bear no interest
    - net working capital on left hand side
    - keep interest bearing short term debt on right hand side

Balance sheet's right hand side becomes:

Equity	60
Long term debt	20
Short term debt	20
total	100

Gives a company WACC of:

$$WACC = (1-.35) \times .11 \times .2 + (1-.35) \times .09 \times .2 + .2 \times .6 = .146$$

Miles-Ezzell calls for 1 cost of debt, use weighted average:

$$.5 \times .11 + .5 \times .09 = .1$$

We can use APV or find discount rate for project with 1 of the 2 methods

First method: stepwise adjust WACC
(this assumes continuous rebalancing):

1. Unlever: use data ZXco's existing operations to find OCC:

$$r = \mathbf{r_d}\frac{\mathbf{D}}{\mathbf{V}} + \mathbf{r_e}\frac{\mathbf{E}}{\mathbf{V}} = .1\frac{40}{100} + .2\frac{60}{100} = .16$$

2. use OCC and project's cost of debt (.12) to calculate project's cost of equity:

$$r_e = r + (r - r_d)\frac{D}{E} = .16 + (.16 - .12)\frac{30}{20} = .22$$

3. relever: calculate project's WACC:

$$WACC = (1 - .35) \times .12 \times .6 + .22 \times .4 = .1348 \text{ or } 13.5\%$$

Second method: use Miles-Ezzell
(this assumes discrete rebalancing):

1. Unlever: use ZXco's data to find OCC r:

$$\mathbf{r}' = r - \tau \mathbf{r_d} \frac{\mathbf{D}}{\mathbf{V}} \left(\frac{(1+r)}{(1+\mathbf{r_d})} \right)$$

$$.146 = r - .35 \times .1 \times .4 \times \left(\frac{(1+r)}{(1+.1)} \right) \Rightarrow r = .161$$

2. Relever: find project's WACC using OCC and project's r_d and L :

$$r' = .161 - .35 \times .12 \times .6 \times \left(\frac{1.161}{1.12}\right) = .1349 \text{ or } 13.5\%$$

Value of perpetual cash flow of €7

- with discount rate of $.135$
- is $7/.135 = 51.85$
- investment is €50,
- NVP $= 1.85 > 0$

Project should be accepted

Third method: use APV:

1. First calculate base case as if all equity financed

   using opportunity CoC: $7/.16 = 43.75$

2. Then calculate tax shield:

   $\tau r_d D = .35 \times .12 \times 30 = 1.26 \ (D = .6 \times 50)$

3. Discount at opportunity CoC:

   $1.26/.16 = 7.875$

4. Multiply PV with $(1+r)/(1+r_d)$:

   $((1+.16)/(1+.12)) \times 7.875 = 8.16$

5. Total APV is $43.75 + 8.16 = 51.91$, NPV is 1.91, same conclusion: accept project

If the project would be financed with a perpetual loan with

- *the same interest rate but*
- *predetermined amounts instead of the flexible loan used now*

would the value of the project go up or down? (no calculations necessary)

same interest $\longrightarrow$ same tax advantage
but fixed $\longrightarrow$ safer $\longrightarrow$ lower discount rate (r_d instead of r)
so higher project value

What methods are use in practice?

We have already seen 1 answer:

- Capital structure shows (slow) mean reversion
- means debt ratios are rebalanced

Found on a much wider scale:

- most firms have target debt ratios
- or a target range for their debt ratio

Rebalancing is dominant financial policy
WACC is extensively used in practice, together with CAPM