Real Options Analysis

Valuing the flexibility of investments in real assets

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The essential economic characteristic of options is:

the flexibility to exercise or not

- possibility to choose best alternative
- walk away from bad outcomes

Stocks and bonds are passively held, no flexibility

Investments in real assets also have flexibility, projects can be:

- delayed or speeded up
- made bigger or smaller
- abandoned early or extended beyond original life-time, etc.

Real Options Analysis

- Studies and values this flexibility
- Real options are options where underlying value is a real asset
- not a financial asset as stock, bond, currency

Flexibility in real investments means:

- changing cash flows along the way:
- profiting from opportunities, cutting off losses

Discounted cash flow (DCF) calculation cannot handle flexibility:

- assumes passive, not flexible, position
- accepts cash flows as they come

Example option analogy of investment decision:

- Company has exclusive 5 year license to develop a project say a natural resource (oil well)
- Investment can be estimated accurately
- Revenue is subject to price uncertainty

The license gives company flexibility to wait and see:

- defer development until some price uncertainty is resolved
- then make a better decision

Situation analogous to holding an American call option:

- company has right, not obligation, to 'buy' project's revenue
- by paying required investment within 5 years

Option characteristics can be represented as follows:

Determinant	Stock option	Real option
underlying	stock	project revenue
strike	exercise price	investment
time to maturity	maturity	license validity
volatility	stock σ	price volatility
interest rate	r_f	r_f

Option analogy not restricted to calls, some puts:

- abandon project and sell assets in second hand market
- down-size a project, etc.

Some more examples:

Call options	Put options
delay	default
expand	contract
extend	abandon
re-open	shut down

Notice: not every investment opportunity is a real option:

- there has to be a source of option value
    - either a lower exercise price than competitors
    - or a higher underlying value than competitors
- The opportunity to buy or sell at market prices on some future date is not a valuable option!

Value of financial options follows from contract:

- difference between
  - fixed exercise price
  - uncertain market value

Also the case for some real options:

- option to extend a rent or service contract
- at predetermined price

However, for most real options, value springs from:

- exclusiveness of investment opportunity
- which varies from
  - completely exclusive
  - completely shared (first come, first served)

Major sources of real option value are:

- Patents and copyrights
- Mineral (extraction) rights
- Surface (development) rights
- Other property rights
- The firm's know-how:
  - technical
  - commercial
  - managerial
- The firm's market position, reputation or size
- Market opportunities

Exclusive options are called *proprietary options*
the opposite is *shared options*

- Patents and copyrights are proprietary
- marked opportunities are shared

Distinction not as sharp as it seems:

- Patent value eroded by close substitutes
- Market opportunities not always easily exploitable:
  - who competes with MS-Windows, Intel, Oil companies?
  - or with SIT - Tapir (bok, mat, kantine)?

Option character of real options can be limited:

- Real options can be less clearly defined:
  - underlying value may be project in planning
  - no clear time to maturity
  - no clear exercise price
- Input data may be difficult to obtain
  - underlying not traded - value?
  - volatility of underlying even more difficult
  - exercise possibilities may be unclear
- Underlying project may be difficult to replicate

We look away from practical obstacles

- assume real options can be valued as financial ones
  - assume market is, at least locally, complete
  - and prices are arbitrage free
  - means projects and their options can be replicated

Not a wild assumption:

- very few, if any, projects expand the investment universe
- most projects are 13 in a dozen

Look at some common real options

The option to defer

- A very common real option
- usually discussed as possibility to postpone a project
- but notice proper option formulation:
  - is NOT decision to postpone accepted project
  - but postponement of decision to accept a project until more information is available

So valuing the option to defer means

- *valuing the opportunity to do a project!*

Re-use our old example and re-define it as real option

- Project is an oil well that can be taken into production
  - size of reserve is accurately measured
  - value depends on oil price
  - develops over time as binomial lattice
- Value is properly discounted present value of future production
  - not cash flows
  - after two periods price uncertainty is resolved
    $\Rightarrow$ stable values

$$
\begin{tikzcd}
 & & 625 \\
 & 500 \arrow[ur] \arrow[dr] & \\
400 \arrow[ur] \arrow[dr] & & 400 \\
 & 320 \arrow[ur] \arrow[dr] & \\
 & & 256 \\
\textit{t=0} & \textit{t=1} & \textit{t=2}
\end{tikzcd}
$$

Binomial tree for the value of an oil well

Further details:

- Market data:
  - risk free interest rate is 7%
  - real probability of upward movement is 80%
  - risk adjusted discount rate for oil production from wells like this is 16%.
- Investment
  - 375 to bring well into production
  - amount increases with the risk free rate over time
  - investment is irreversible
- Project is profitable from the start:
  - 400 - 375 = 25, a positive net present value

Firm has exclusive one-period license to develop the well
$\Rightarrow$ can defer decision to develop with one period

License gives firm a real option:

- has the right, but no obligation, to develop the well
- firm has flexibility, or future decision making opportunity, to profit from real option

Project has the ingredients that make options valuable:

- **Time**
  without time, becomes now-or-never decision as in DCF
- **Uncertainty**
  without uncertainty, situation in 1 year exactly same as today

Reformulate project as option:

- License gives the right to 'buy' the oil in the well
  - $\Rightarrow$ option is a call
- by paying development costs
  - $\Rightarrow$ exercise price $= 375$.
- License expires in 1 year
  - $\Rightarrow$ option has a time to maturity of 1 year
- Volatility follows from binomial process
- interest rate $= 7\%$

All ingredients necessary to value the option

The parameters of binomial process are:

$$
\begin{aligned}
u &= 1.25, \quad d = .8, \quad r = 1.07 \\
p &= \frac{r-d}{u-d} = \frac{1.07-.8}{1.25-.8} = .6 \quad (1-p) = .4
\end{aligned}
$$

Begin with values at maturity, i.e. at $t = 1$

- option value is $max[0, S - X]$
- exercise price is $375 \times 1.07 = 401.25$
  - $O_u = \max[0, 500 - 401.25] = 98.75$
  - $O_d = \max[0, 320 - 401.25] = 0.0$

$$
\begin{array}{ccc}
 & & \max[0, 500 - 401.25] = 98.75 \\
 & \nearrow & \\
55.374 & & \\
 & \searrow & \\
 & & \max[0, 320 - 401.25] = 0.0 \\
\text{t=0} & & \text{t=1}
\end{array}
$$

Value of the option is:

$$O = \frac{.6 \times 98.75 + .4 \times 0}{1.07} = 55.374$$

Opportunity to do project is more valuable than project itself!

How can option to defer be more valuable than project itself?

- Project very profitable in up node
- loss making in down node
- both included in t_0 value of 25.

Real option analysis values the flexibility to avoid losses in down node:

- wait a period, see how value develops:
  - If oil price goes up, develop well and profit from opportunity
  - If oil price goes down, do not develop and avoid loss
- Don't lose much by waiting one period
  - oil is still in well
  - license still valid

That is essence of real option valuation!

After one period, the license expires

- time runs out (option expires)
- source of option value disappears
- decision gets now-or-never character inherent in discounted cash flow approach

Similarly, if oil price would become fixed

- volatility would disappear
- source of option value would disappear
- option to defer would have no value

What is wrong with DCF valuation of flexibility?

It can be argued:

- DCF can capture value of flexibility
- by calculating NPV as if project started one period later
- decide to abandon project if NPV is negative
- gives same 98.75 in upper node and 0 in lower node

Result is a **decision tree**
Decision trees are analysed by:

- weighting branches with real probabilities of 0.8 and 0.2
- discounting expected value at risk adjusted discount rate of 16%

$$NPV = 500 - 401.25 = 98.75 \Rightarrow \text{accept project}$$

$$12.07 \quad \nearrow \quad \searrow$$

$$NPV = 320 - 401.25 = -81.25 \Rightarrow \text{refuse project}$$

t=0 t=1

Value of the project opportunity is:

$$\frac{.8 \times 98.75 + .2 \times 0}{1.16} = 68.103$$

Different from 55.374 we found with real options analysis!

How can we determine which value is correct?
Answer in modern finance:
by making a replicating portfolio

Assume payoff structure in tree can be constructed in market

- here we need assumption of locally complete market

Option's delta and D are:

$$\Delta = \frac{O_u - O_d}{(u-d)S} = \frac{98.75 - 0}{500 - 320} = 0.5486 \text{ and}$$

$$D = \frac{uO_d - dO_u}{(u-d)r} = \frac{1.25 \times 0 - .8 \times 98.75}{1.25 \times 1.07 - .8 \times 1.07} = -164.07$$

At time t_1 portfolio pays off either:

$$
\begin{aligned}
(0.5486 \times 500) - 164.07 \times 1.07 &= 98.745 \quad \text{or} \\
(0.5486 \times 320) - 164.07 \times 1.07 &= 0
\end{aligned}
$$

So payoff pattern is replicated.
Value of portfolio now is

$$(0.5486 \times 400) - 164.07 = 55.37$$

Real option value correct:

- no rational investor pays 68.10
- for payoff pattern that can be replicated for 55.37

Where does discounted cash flow approach go wrong?

- The error we made was:
  - applying risk adjusted discount rate for oil production from the well
  - to the opportunity to develop the well.
- The opportunity to do a project
  seldom has same risk as project itself,
- precisely because flexibility embedded in opportunity is used to change the risk:
  - upward potential is enhanced
  - downside risk is reduced

Once well is in production, then:

- fortunes tied to oil price
- proper discount rate for cfl's and values is 16%
- Values move through time with uncertainty of
  - up factor of 1.25
  - down factor of 0.8

Opportunity to do project has much larger uncertainty

- moves through time from 25 at t_0
  - to either 98.75 $\Rightarrow$ upfactor 3.95
  - or 0 $\Rightarrow$ down factor 0.

In principle, correct option value can be calculated using:

- real probabilities of 0.8 and 0.2
- risk adjusted discount rate

But that rate must be calculated from replicating portfolio:

- $\Delta S = 0.5486 \times 400 = 219.44$ in twin security S
- risk free loan of -164.07.

Gives weighted average portfolio return of:

$$\frac{219.44}{219.44 - 164.07} \times 0.16 + \frac{-164.07}{219.44 - 164.07} \times 0.07 = 0.427$$

i.e.$> 2.5\times$ discount rate for the project

Discounting exp. payoff with this rate gives correct value:

$$\frac{0.8 \times 98.75 + 0.2 \times 0}{1.427} = 55.36$$

However, with ΔS and D, we already know the option value:

$$O = \Delta S + D = 219.44 - 164.07 = 55.37$$

Boils down to calculating rate *given* option value:

$$55.37 = (0.8 \times 98.75)/r_{adj} \Rightarrow r_{adj} = 1.427$$

Moreover:

- Δ and D likely to differ between nodes
- must be calculated for each node in tree
- makes correct use of decision trees highly impractical

General conclusions:

- Discounted cash flow approach cannot properly capture dynamic aspects of risk
- Discounted cash flow represents passive attitude:
  - accept the cash flows as they come
  - without exploring the possibilities to change them
- Adapting approach to include flexibility (decision trees):
  - makes original discount rate useless
  - very cumbersome to calculate a new one
- Verdict on decision trees has to be:
  - they are outdated
  - should not be used for investment problems

Real option pricing is the proper approach to valuing flexibility.

Follow-up investments are very common

- Successfully completed projects give advantage over competition
- opportunity to exploit same technology, distribution channels, market base, know-how & experience
- clear real option value:
  - re-use gives lower exercise price
  - market base gives higher payoff
  - than competitors starting from scratch
- examples are abundant:
  - MicroSoft DOS, Windows -95. -NT, -XP, -7, ..
  - Intel 8086, 80286, 80386, 80486, Pentium, ..
  - Telenor's mobile networks in Russia, Pakistan, Bangla Desh, ..

To illustrate, adapt our old example in three ways:

1. Technology project instead of oil well
    - empty oil well has little follow-up
2. Investment required is 450
    - gives project negative NPV: $400 - 450 = -50$
3. Project can be repeated on double scale after 2 periods
    - first project is prerequisite for follow-up
      e.g. extension of technology
    - invest $2\times$ exercise price, get $2\times$ market value

In DCF terms, this simply doubles expected loss!

In real options terms, after 2 years:

- firm has opportunity to double value of underlying project
- by investing $2\times$ exercise price
- risk free rate is 7%
- as before exercise price increases with risk free rate:
  - investment is $2 \times 450 \times 1.07^2 = 1030.4$

Further details as before:

- real probabilities are .8 and .2
- risk adjusted discount rate 16%

Gives following value tree:

$$
\begin{array}{ccccccc}
 & & & & 625 & & \times 2 = 1250 \\
 & & & \nearrow & & & \\
 & & 500 & & & & \\
 & \nearrow & & \searrow & & & \\
400 & & & & 400 & & \times 2 = 800 \\
 & \searrow & & \nearrow & & & \\
 & & 320 & & & & \\
 & & & \searrow & & & \\
 & & & & 256 & & \times 2 = 512 \\
t{=}0 & & t{=}1 & & t{=}2 & &
\end{array}
$$

Follow-up project is loss making in DCF terms
expected payoff with real probabilities is:

$$(.8^2) \times 1250 + (2 \times .8 \times .2) \times 800 + (.2^2) \times 512 = 1076.5$$

$$\text{PV} = \frac{1076.5}{1.16^2} = 800 \text{ so the NPV} = 800 - (2 \times 450) = -100$$

Real options analysis gives different picture:

- models flexibility to profit from favourable market developments
- follow-up investment is out-of-the-money call option
- only exercised if profitable
- but out-of-the-money options are valuable

The parameters of binomial process are as before:

$$
\begin{aligned}
u &= 1.25, \quad d = .8, \quad r = 1.07 \\
p &= \frac{r-d}{u-d} = \frac{1.07-.8}{1.25-.8} = .6 \quad (1-p) = .4
\end{aligned}
$$

Follow-up option matures after 2 periods, at $t = 2$

- option value is $max[0, S - X]$
- exercise price is $2 \times 450 \times 1.07^2 = 1030.4$
- payoff at maturity are:
  - $\max[0, 1250 - 1030.4] = 219.6$
  - $\max[0, 800 - 1030.4] = 0.0$
  - $\max[0, 512 - 1030.4] = 0.0$

$$
\begin{array}{ccccc}
 & & & & \max[0, 1250 - 1030.4] = 219.6 \\
 & & & \nearrow & \\
 & & 123.14 & & \\
 & \nearrow & & \searrow & \\
69.05 & & & & \max[0, 800 - 1030.4] = 0.0 \\
 & \searrow & & \nearrow & \\
 & & 0 & & \\
 & & & \searrow & \\
 & & & & \max[0, 512 - 1030.4] = 0.0 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}
$$

Option value of follow-up project

Value of the option is

- at t=1: $(.6 \times 219.6)/1.07 = 123.14$
- at t=0: $(.6 \times 123.14)/1.07 = 69.05$

The follow-up opportunity is so valuable

- that it gives whole project NPV>0
- $-50 + 69.05 = 19.05$

Makes it a classic among real options

- particularly valuable in volatile (fast growing) markets
- used to be called 'strategic value'
- can now be priced properly!

Abandonment is another common option

- No need to continue loss making projects
- assets can be sold, used alternatively
- (cf. general purpose assets in bankruptcy)
- gives higher 'bottom' in project value

Abandonment option can be modelled in various ways

- separate tree for second hand value
  - lower starting point, less volatile
  - primary / secondary values cross in down nodes
  - more profitable to abandon
- simpler: fixed second hand value

Extend our example with abandonment option

- possibility to sell project's assets second hand
- at any time for a fixed price of 325

Thus formulated, option is American put

Valued by including exercise condition:

$$\max[\mathit{continue},\ \mathit{abandon}]$$

in all nodes of the value tree
Recall:

- tree contains project values, not cash flows
- $\Rightarrow$ exercise condition also in last node

Starting in the end nodes:

$$
\begin{array}{ccccc}
 & & & & \max[625, 325] = 625 \\
 & & & \nearrow & \\
 & & \max[\mathit{alive}, \mathit{dead}] & & \\
 & \nearrow & & \searrow & \\
\max[\mathit{alive}, \mathit{dead}] & & & & \max[400, 325] = 400 \\
 & \searrow & & \nearrow & \\
 & & \max[\mathit{alive}, \mathit{dead}] & & \\
 & & & \searrow & \\
 & & & & \max[256, 325] = 325 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}
$$

Option exercised lower node t=2 (alive < dead)

Then we calculate the values alive at t=1:
upper node is, of course:

$$\frac{.6 \times 625 + .4 \times 400}{1.07} = 500$$

lower node:

$$\frac{.6 \times 400 + .4 \times 325}{1.07} = 345.79$$

and compare them with the values dead:

$$625$$

$$\max[\underbrace{500}_{alive}, \underbrace{325}_{dead}] = 500$$

$$\max[409.64, 325]$$

$$400$$

$$\max[\underbrace{345.79}_{alive}, \underbrace{325}_{dead}] = 345.79$$

$$325$$

t=0 t=1 t=2

t=0 value found by repeating procedure:

$$\frac{.6 \times 500 + .4 \times 345.79}{1.07} = 409.64$$

and checking the t=0 values dead and alive: $\max[409.64, 325] = 409.64$

- Value without abandonment option is 400
- flexibility to abandon has value of 9.64

Option value can also be calculated separately:

- American put with an exercise price of 325
- exercise condition is $\max[0, 325 - \mathit{project\ value}]$

Values at t=2 are:

- $\max[0, 325 - 625] = 0$
- $\max[0, 325 - 400] = 0$
- $\max[0, 325 - 256] = 69$

The t=1 value alive is

$$\frac{.6 \times 0 + .4 \times 69}{1.07} = 25.794$$

and the t=0 value alive is

$$\frac{.6 \times 0 + .4 \times 25.794}{1.07} = 9.64$$

$$
\begin{array}{ccccc}
 & & & & 0 \\
 & & & \nearrow & \\
 & & \underbrace{0.0}_{alive}, \underbrace{\max[0, 325-500]}_{dead} & & \\
 & \nearrow & & \searrow & \\
\underbrace{9.64}_{alive}, \underbrace{\max[0, 325-400]}_{dead} & & & & 0 \\
 & \searrow & & \nearrow & \\
 & & \underbrace{25.794}_{alive}, \underbrace{\max[0, 325-320]}_{dead} & & \\
 & & & \searrow & \\
 & & & & 69 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}
$$

Phasing investments also gives flexibility

- Firms may not commit themselves to entire project at once
- but to successive stages, one at a time

Common practice for certain types of projects

- Construction industry:
  - preparation phase: licenses, groundwork
  - construction phase: building
  - finishing stage: fixtures, plumbing, etc.
- Very pronounced in pharmaceutical research:
  - basic research: search for potential drugs
  - preclinical tests (on rats)
  - clinical tests (on humans)
  - approval and production

Notice: option refers to

- *decision* to accept project's next phase
- *not* implementation of already accepted next phase
- means next phase can be rejected, project abandoned

With a project structured in phases:

- each phase is call option on the next
- accepting first phase buys option on second
- second phase buys option on third, etc.

Means they are compound options:

- compound options are options on options
- notice: value of option on 3^{rd} phase included in value of 2^{nd} phase

To illustrate, we adapt, again, our binomial example

- project's investment of 375
- can be made in two stages:
  - 50 now (preparation phase)
  - rest (325) next period
- as before, investment grows with risk free rate over time
  - investment t = 1 is $1.07 \times 325 = 347.75$

Option modelled by including in t=1 nodes:

$$\text{project value} = \max[\,0, \text{project value - investment}]$$

Looks obvious in option context, not in practice

Value tree for flexible (phased) project:

$$\begin{tikzcd}
 & & 625 \\
 & \max[0, 500 - 347.75] = 152.25 \arrow[ur] \arrow[dr] & \\
\max[0, 85.37 - 50] = 35.37 \arrow[ur] \arrow[dr] & & 400 \\
 & \max[0, 320 - 347.75] = 0 & \\
I = -50 & I = -347.75 & \\
\text{t=0} & \text{t=1} & \text{t=2}
\end{tikzcd}$$

t=0 value found with familiar procedure:

$$\frac{.6 \times 152.25 + .4 \times 0}{1.07} = 85.37 - 50 = 35.37$$

Value of flexibility is 10.37, project's value increase from 25

Option can also be modelled separately

- Is the option a call or put? How do we formulate the exercise condition?
- The option is a put
  - by not investing we 'keep' the investment amount
  - and give up the project value
- exercise condition:

$$\max[0, \text{ investment - project value}]$$

Option's value tree becomes

$$
\begin{array}{lll}
 & & \max[0, 347.75 - 500] = 0 \\
 & \nearrow & \\
10.37 & & \\
 & \searrow & \\
 & & \max[0, 347.75 - 320] = 27.75 \\
 & & I = 347.75 \\
\text{t=0} & & \text{t=1}
\end{array}
$$

Option value is:

$$\frac{.6 \times 0 + .4 \times 27.75}{1.07} = 10.37$$

Option has counter-intuitive elements:

- Why only consider investment for next stage?
    - we know later investments are required
    - why not include them in exercise decision?

That is what DCF does!
Real option analysis does not ignore later investments

- included as exercise prices of later options
- determine value of later options
- but exercise decision made later
- at expiration, when more information is available

Why not include previous investments in decision?

- If the project is abandoned
- would not they be wasted?

Is 'sunk cost fallacy'

- if previous investments are irreversible (they usually are)
- they are wasted already

If much is already invested, will not a small extra investment produce large project?

- that is precisely what Real Options Analysis models
- but project should be large in future cash flows, not past investments

Defaulting a loan is financial option

- underlying is financial asset
- often discussed with real options, part. phased investments
- not always correctly treated

Can easily be included in our binomial example
Assume initial investment of 375 financed with:

- a zero coupon loan of 300
- an equity investment of 75

Further details of the loan:

- nominal interest rate 9.5%
- matures after 2 periods
- lenders provide 300 today
- against promise of 359.71 (300×1.095^2) after 2 periods
- no interest payments in between

Assume owners have limited liability:

- means they have option to default the loan
- will do so when

$$\text{project value} < \text{debt obligations}$$

When owners default a loan:

- firm will be declared bankrupt
- lenders become the new owners
- receive the remaining project value

We model perfect markets:

- means transfer of ownership is costless
  - no bankruptcy costs
- no taxes or information asymmetry either

Means that capital structure must be irrelevant

Set up separate value trees for equity and debt:

$$\begin{array}{ccccc}
 & & & & \max[0, 625 - 359.71] = 265.29 \\
 & & & \nearrow & \\
 & & 163.82 & & \\
 & \nearrow & & \searrow & \\
100.31 & & & & \max[0, 400 - 359.71] = 40.29 \\
 & \searrow & & \nearrow & \\
 & & 22.59 & & \\
 & & & \searrow & \\
 & & & & \max[0, 256 - 359.71] = 0 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}$$

The value of levered equity

Loan is defaulted lower node at t = 2

t = 1 values calculated as usual:

$$\frac{.6 \times 265.29 + .4 \times 40.29}{1.07} = 163.82 \quad \frac{.6 \times 40.29 + .4 \times 0}{1.07} = 22.59$$

so that t = 0 value is

$$\frac{.6 \times 163.82 + .4 \times 22.59}{1.07} = 100.31$$

Project's NPV is $100.31 - 75 = 25.31$ as before (allowing 0.31 rounding)

Value of debt calculated in same way:

$$
\begin{tikzcd}[column sep=small, row sep=small]
 & & 359.71 \\
 & 336.18 \arrow[ur] \arrow[dr] & \\
299.69 \arrow[ur] \arrow[dr] & & 359.71 \\
 & 297.41 \arrow[ur] \arrow[dr] & \\
 & & 256 \\
t=0 & t=1 & t=2
\end{tikzcd}
$$

The value of debt

- Payment in lower node $t = 2$ is remaining project value
- lower than promised payment of 359.71

Total project value remains $100.31 + 299.69 = 400$

- changing capital structure does not add value
- divides it differently

We can calculate effective market interest rate for risky loan:

- calculate expected payoff with real probabilities:

$$.8^2 \times 359.71 + 2 \times .8 \times .2 \times 359.71 + .2^2 \times 256 = 355.56$$

- then solve $355.56/r^2 = 300$ for r, gives r$=1.089$

Does this mean that option to default has no value?
NO! option redistributes value

To illustrate, first calculate value of option to default separately

- reformulate option as a put
- owners can 'sell' project to lenders
- by keeping amount due to the lenders
- exercise condition is

$$\max[0, \textit{amount due} - \textit{project value}]$$

- option only exercised in lower node
- value at exercise is $\max[0, 359.71 - 256] = 103.71$

$$
\begin{array}{ccccc}
 & & & & \max[0, 359.71 - 625] = 0.0 \\
 & & & \nearrow & \\
 & & 0 & & \\
 & \nearrow & & \searrow & \\
14.49 & & & & \max[0, 359.71 - 400] = 0.0 \\
 & \searrow & & \nearrow & \\
 & & 38.77 & & \\
 & & & \searrow & \\
 & & & & \max[0, 359.71 - 256] = 103.71 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}
$$

The value of the option to default

The t=0 value is $(.4^2 \times 103.71)/1.07^2 = 14.49$

Economic interpretation:

- by giving the loan
- lenders have written put as part of the deal

But put is included with proper price:

- without default option, loan would be risk free
- value would be $359.71/1.07^2 = 314.18$
- lenders supplied only 300

Without the put, equity would have unlimited liability

- means it can have negative value
- can calculate value of unlimited liability equity:

$$
\begin{array}{ccccc}
 & & & & 625 - 359.71 = 265.29 \\
 & & & \nearrow & \\
 & & 163.82 & & \\
 & \nearrow & & \searrow & \\
85.81 & & & & 400 - 359.71 = 40.29 \\
 & \searrow & & \nearrow & \\
 & & -16.18 & & \\
 & & & \searrow & \\
 & & & & 256 - 359.71 = -103.71 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}
$$

The value of unlimited liability equity

We see that:

- lim. liab. equity = unlim. liab. equity + value default option

$$100.31 = 85.81 + 14.49$$

- risky debt = safe debt - value of default option

$$299.69 = 314.18 - 14.49$$

Default option transfers value from debtholders to equityholders

- but transfer is anticipated (perfect market!)
- and properly included in prices

Modelling corporate securities (debt and equity) as options has other, far reaching consequences

Can you guess which?

- option prices increase with volatility
- means equity holders prefer risky projects

Interacting real options

Background and problem

ZX Co is international dredging contractor
Core activities are:

- construction and maintenance of ports and waterways
- land reclamation, coastal defences and riverbank protection

ZX Co operates fleet of dredging & support vessels

Company negotiates harbour renovation project with port authority in Middle East

involves largest bucket dredger in fleet:

Project details:

- takes dredger $1\frac{1}{2}$ year to complete the project
- 3 periods of 6 months, standard time unit in dredging

The state of negotiations:

- port authority willing to pay current world market rate
  - €30 million per half year for the dredger
  - for the entire $1\frac{1}{2}$ year period
- ZX Co operates dredger for much less
  - is inclined to accept price
- Negotiations are about additional clauses

Port authority wants to include one or both of the following two clauses in the contract:

1. the clause that it can extend the contract with one half year period at the same rate; extra half year used to construct small marina adjacent to harbour;
2. the clause that it can terminate the contract at beginning of 2^{nd} and 3^{rd} half year period by paying penalty of €2.5 million per remaining period; reduction may be necessary because of budget cuts.

Required:

Help negotiating team of engineers by calculating how clauses affect project value, both separately and in combination.

Background information:

- world market rate for dredging very volatile, corresponding to yearly standard deviation of 25%
- rates only adjusted at beginning of each half year period
- constant until beginning of next period
- risk free interest rate is 6% per year
- market for dredging projects such that
  - vessels can be redeployed immediately at market prices
  - cost of idle capacity can be ignored

Analysis

The clause to extend:

- gives port authority the right, but not the obligation, to buy dredging services $1\frac{1}{2}$ years from now at price fixed today
- What kind of real option, if any, is this?
  - is a real option, long European call

The clause to terminate contract after six and twelve months:

- gives the right, but not the obligation:
  - to 'sell back' the obligation to pay €30 m. per $\frac{1}{2}$ year
  - in return for paying €2.5 million
  - plus market price for dredging services (have to be bought elsewhere if contract is terminated)
- Is also real option, a?long American put

Port authority asks ZX Co to write the two options

- project value changes with option values
- should be included at proper prices
  (is not always obvious to sales people)

We have the information necessary to value the options:

- underlying value is the contract
- maturities and exercise prices specified in clauses
- volatility is that of world market rate for dredging projects
- risk free rate is given

Rates only adjusted at beginning each $\frac{1}{2}$ year:

- binomial method proper way to calculate option values
- parameters of binomial process:

$$u = e^{.25\sqrt{.5}} = 1.193 \qquad d = 1/u = 0.838$$

$$r = \sqrt{1.06} = 1.03$$

$$p = \frac{1.03 - .838}{1.193 - .838} = 0.541 \qquad 1 - p = 0.459$$

Notice: volatility & interest rate re-scaled to half year values

Parameters give following value tree for half year dredging rates:

			50.94
		42.70	
	35.79		35.79
30		30	
	25.14		25.14
		21.07	
			17.65
t=0	*t=1*	*t=2*	*t=3*

Value tree for half year dredging rates

The option to extend

European call without complications

Option value found with familiar binomial procedure

- calculate pay-offs at maturity
- option only exercised if: marked rate $>$ contract rate
    - option ends in-the-money in 2 upper end nodes
- then working back through lattice

Gives option value now of 5.17

t=0	*t=1*	*t=2*	*t=3*
			max[50.94-30, 0]=20.94
		13.58	
	8.49		max[35.79-30, 0]=5.79
5.17		3.04	
	1.60		max[25.14-30, 0]=0
		0	
			max[17.65-30, 0]=0

Value tree for the option to extend

The option to terminate

American put, bit more complicated

- can be exercised on two moments
- penalty if exercised

Valuation procedure as American call with dividends:

- compare the values 'dead' and 'alive'

Start at end of tree, is at beginning 3^{rd} period
(exercise only possible after six and twelve months)

Inputs require extra attention

Consider lower node *t=2*, dredging rate is 21.07

- If option exercised, port authority
    - reverses obligation to pay €30m (saves €30m.)
    - instead has to pay €2.5m. penalty + market rate of 21.07
- payoff is then $max[(30 - (21.07 + 2.5)), 0] = 6.43$
- option is exercised

Same calculation 2 upper nodes gives zero values

- option is not exercised

Discounting values back gives t_1 values 'alive':
$(.459 \times 6.43)/1.03 = 2.87$ and 0

$$max[(30-(42.70+2.5)),0]$$
$$=0$$

$$\nearrow$$

$$\underset{\textit{dead or alive}}{(0)}$$

$$\nearrow \qquad \searrow$$

$$1.71 \qquad\qquad max[(30-(30+2.5)),0]=0$$

$$\searrow \qquad \nearrow$$

$$\max[\underset{\textit{dead}}{(3.84)},\underset{\textit{alive}}{(2.87)}]$$

$$\searrow$$

$$max[(30-(21.07+2.5)),0]$$
$$=6.43$$

t=0 *t=1* *t=2*

Value tree for the option to terminate

If the option is exercised in lower node *t=1*
there are 3 pay-offs:

1. price advantage over second period:
$30 - 25.14 = 4.86$.
2. PV(expected price advantage 3^{rd} period):
$(.459 \times (30 - 21.07))/1.03 = 3.98$.
3. penalty of $2 \times -2.5 = -5$ million

Total value 'dead' is $4.86 + 3.98 - 5 = 3.84$

Value dead $>$ value alive (2.87), option should be exercised

Gives a value now of $(.459 \times 3.84)/1.03 = 1.71$.

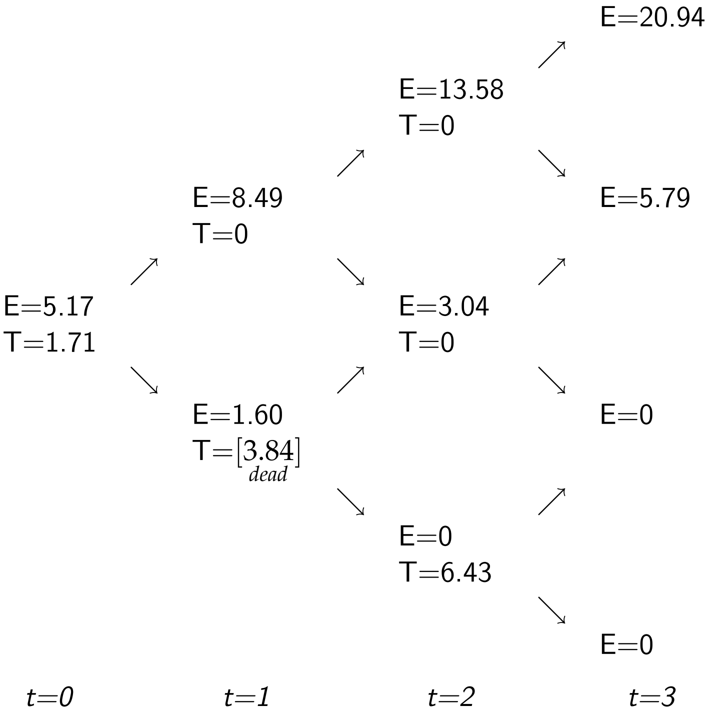
Combined option value

Value now ($t{=}0$) of both options combined
is NOT sum of the two $(1.71 + 5.17 = 6.88)$

- The options interact
- makes combined value *less* than sum of separate values

Easy to see why:

- optimal decision was: exercise option to terminate in lower node $t{=}1$
- comparing value trees for both options we see that option to extend ALSO has value in this node
- exercising option to terminate eliminates this value: cannot extend contract that was cancelled two periods ago.


E=20.94
E=13.58
T=0
E=8.49
T=0
E=5.79
E=5.17
T=1.71
E=3.04
T=0
E=1.60
T=[3.84] dead
E=0
E=0
T=6.43
E=0
t=0
t=1
t=2
t=3

Calculate combined option value:

- adjust 'dead' value of option to terminate at *t=1* with lost value of option to extend: $3.84 - 1.60 = 2.24$
- makes value 'dead' $<$ value 'alive'(of 2.87)
- optimal decision becomes *not* to exercise
- *t=0* value becomes $(.459 \times 2.87)/1.03 = 1.28$
- total value options combined is 1.28+5.17=6.45, 0.43 less than sum of separate values
- option to terminate at *t=2* only exercised in lower node *t=2* where option to extend has no value. So no correction needed.

$$
\begin{tikzcd}
 & & & \text{E}=20.94 \\
 & & \begin{array}{l}\text{E}=13.58\\ \text{T}=0\end{array} \arrow[ur] \arrow[dr] & \\
 & \begin{array}{l}\text{E}=8.49\\ \text{T}=0\end{array} \arrow[ur] \arrow[dr] & & \text{E}=5.79 \\
\begin{array}{l}\text{E}=5.17\\ \text{T}=1.28\end{array} \arrow[ur] \arrow[dr] & & \begin{array}{l}\text{E}=3.04\\ \text{T}=0\end{array} \arrow[ur] \arrow[dr] & \\
 & \begin{array}{l}\text{E}=1.60\\ \text{T}=max[\underset{dead}{2.24},\underset{alive}{2.87}]\end{array} \arrow[ur] \arrow[dr] & & \text{E}=0 \\
 & & \begin{array}{l}\text{E}=0\\ \text{T}=6.43\end{array} \arrow[ur] \arrow[dr] & \\
 & & & \text{E}=0
\end{tikzcd}
$$

Result highlights some general aspects of real options in contracting:

- real options in contract can constitute considerable value, should be treated with caution
- in this case, combined option value of €6.45 million likely to be lion's share of project's profits
- options increase in value with time and volatility:
  - makes them very valuable for unlikely events far in future
  - in contracting, these events tend to be neglected unless explicitly valued

Financial options seldom interact

- traded independently from underlying
- exercising doesn't affect
  - value underlying
  - value other options

Most real options interact, in several ways:

- option value > 0, increases value underlying
  - different effects on puts (-) and calls (+)
- exercising affects value underlying
  - calls increase value underlying
  - puts decrease value underlying
- this affects value later options
  - different effects on puts and calls:

2^{nd} option $\geqq 0$

$\downarrow$

underlying: $\Uparrow$

$\downarrow$

1^{st} option is

$\swarrow$	$\searrow$
call: $\Uparrow$	put: $\Downarrow$
if exercised	if exercised
$\downarrow$	$\downarrow$
underlying: $\Uparrow$	underlying: $\Downarrow$
$\downarrow$	$\downarrow$
2^{nd} option is	2^{nd} option is
$\downarrow$	$\downarrow$
put: $\Downarrow$	call: $\Downarrow$
call: $\Uparrow$	put: $\Uparrow$

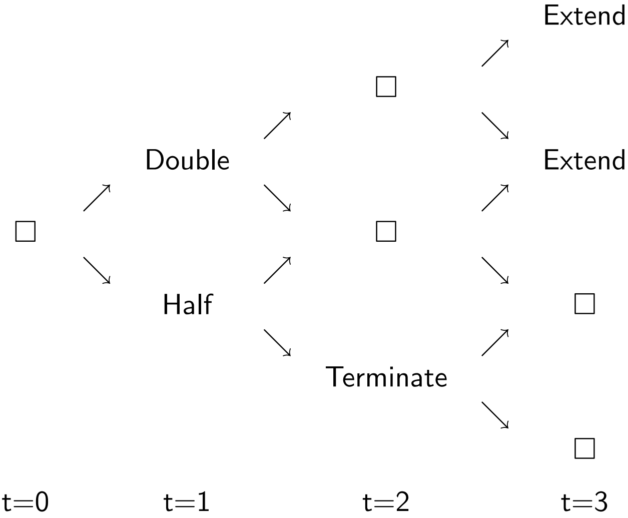
Theoretical (ceteris paribus) effects also apply to real options but without ceteris paribus condition

- real options not only change value underlying
- also specifications later options

To illustrate, look at dredging project again, suppose:

- can be doubled or halved after 1 period
- terminated after 2 periods
- extended after 3 periods

Gives following tree:



Real options attached to dredging project

If project is doubled at t=1

- extension option at t=3 refers to double project
- port authority can hire 2 dredgers at fixed rate
- not the same as 1 option on underlying with double value
- difference is double exercise price

Similarly, if project is halved at t=1

- only half project can be terminated at t=2
- $\pm$ halves option value

Endogenous competition: game theory

So far, we modelled price (market) uncertainty

- can partly spring from competitors' actions
- but competition not modelled explicitly

If real options are shared (and most are)

- exercise decision depends on competitors' decisions
- have to be included, option pricing no longer sufficient
- we need elements from *game theory*

Game theory founded in 1940s
John von Neumann and Oskar Morgenstern:
Theory of Games and Economic Behaviour

- game theory studies decision making behaviour
- when participants' choices depend on choices of other participants

Basic theory developed in 1950s and 1960s
Now a mature scientific discipline

- Nobel prizes in 1994 and 2005
- including John Nash (1994)

Link with real options elaborated by
Smit, H.T.J. and L. Trigeorgis:
Strategic Investment: Real Options and Games

Look at 1 example, *the prisoners dilemma:*

- 2 suspects, A and B, interrogated separately by police
- Each is offered following deal:
    - confess: sentence reduced to 10 years
    - confess while other denies:
        - confessor further reduction to 2.5 years
        - denier full sentence of 20 years
- If both deny, sentence will be 5 years (the evidence is weak)

Gives following choices and consequences

		B	
		confess	**deny**
A	confess	10, **10**	2.5, **20**
	deny	20, **2.5**	5, **5**

Data for B are **in bold**

Both suspects have *dominant strategy:*

- gives better result than other strategies
- no matter what other suspect chooses

In this example:

- If B confesses
  - A is better off confessing than denying (10 vs. 20 years)
- but if B denies
  - A is also better off confessing (2.5 vs. 5 years)

Same is true for B

Result: both confess (would be better off denying)

Resulting equilibrium known as *Nash equilibrium:*

- no participant wants to change strategy
- if strategy of other participant(s) become known

Choices with similar payoff structures

- found in many situations in economics and finance
- shared real options are good example

To illustrate, adapt our binomial example again

- look at option to defer once more
- but now as a shared option by two market participants, A & B

Assume value tree stays as before, but:

- investment increases over time with risk adjusted rate of 16%
- exercise now gives NPV 400-375=25, as before
- exercise price at t=1 is $1.16 \times 375 = 435$
- Value of option to defer becomes:

$$\begin{array}{ccc} & & \max[0, 500 - 435] = 65 \\ & \nearrow & \\ 36.45 & & \\ & \searrow & \\ & & \max[0, 320 - 435] = 0 \\ \text{t=0} & & \text{t=1} \end{array}$$

Assume market has *first mover advantage:*

- if a firm enters market alone
  it gets whole market
  - e.g. because technological leadership
  - or monopoly over distribution channels
  - or loyal customer base (high switching costs)
- if both firms enter simultaneously
  they equally share the market
  - if both invest now, each gets $25/2{=}12.5$
  - if both defer, each gets $36.45/2 = 18.22$

Gives following payoff structure

		B	
		invest	**defer**
A	invest	12.5, **12.5**	25, **0**
	defer	0, **25**	18.22, **18.22**

Data for B are **in bold**

Both firms have dominant strategy, leads to Nash equilibrium:

- If firm B invests
    - A is better off investing than deferring (12.5 vs. 0)
- But if firm B defers
    - A is also better off investing than deferring (25 vs. 18.22)

Same is true for firm B

- B is better off investing than deferring
- no matter whether A invests or defers

Both firms will invest now, although deferring would be better
Is more general result, using game theory leads to early exercise

We have seen 3 influences on investment decision:

1. Option nature (financial option analogy)
    - tends to favour late investment
    - keep flexibility, uncertainty resolves over time
2. Real options nature
    - tends to favour early exercise
    - sources of option value erode over time
3. Game theoretic extensions
    - tend to lead to early exercise
    - anticipation of action competitors leads to pre-emptive action