Chapter 4: Marked efficiency

Exercises

1. In September 1998 the investment banking and securities firm Goldman, Sachs & Company cancelled its plan to go public, i.e. to offer shares to the public. The decision was made after a sharp drop of 25% in the results over the third quarter. Goldman's co-chairman and chief executive Henry M. Paulson Jr. said to the New York Times: "With the volatility we have, the falling valuations (of other investment banks) and uncertainty of earnings going forward, I can't imagine that we would advise a client that this is a good time to go public for a financial service company". What does the EMH say about Goldman's decision to cancel the stock issue because of falling valuations?

2. What does the EMH imply about the Net Present Value (NPV) of the purchase or sale of a security on an efficient market?

3. Suppose that the stock prices of a fertilizer producer move in the same cycles as the fertilizing seasons, high in spring and summer, low in fall and winter. Explain how trading will eliminate the cyclical pattern.

4. It is sometimes argued that markets cannot be efficient because only a small proportion of investors follow the information on a stock and an even smaller proportion actively trade in a stock on a day or in a week. Is this argument correct?

5. Investors can disagree strongly about the implications of information for the price of a particular stock. Therefore, the information cannot be fully reflected in prices and markets are not efficient. Is this argument correct?

6. You are planning to visit Amsterdam and in preparation for the trip you collect (among other things) some data about the local stock market, viz. the returns of five major stocks and the index. You analyse the data in different ways. First, you plot the percentage daily returns of the stocks and the index against their return on the next day. Figure 1 shows two such plots, one for Air France-KLM (ticker: AF) and one for the AEX index (ticker: AEX). The AEX index consists of the 25 most actively traded securities on the Amsterdam Stock Exchange.

   (a) Do these plots reveal market inefficiency? Explain why.

   (b) As an aside, what else do these plots show?

   Next, you calculate autocorrelation coefficients for the stocks and the index, i.e. correlation coefficients between the returns today and tomorrow. The results are presented in Table 1.

   (c) Does Table 1 reveal market inefficiency? Explain why.

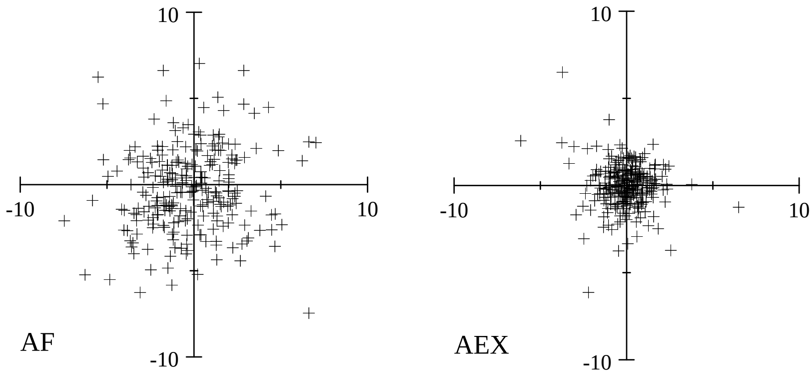

Figure 1: Return day t (x-axis) vs. day t+1 (y-axis)

Table 1: Autocorrelation coefficients

Stock	ticker	$\rho_{r_t,r_{t-1}}$
Air France-KLM	AF	.070
BosKalis-Westminster	BOKA	-.070
Philips	PHIA	-.043
Royal Dutch Shell	RDSA	-.011
Unilever	UNA	-.138*
AEX index (25 stocks)	AEX	-.107

*significantly ≠0 (5% level, 2-tailed test)

Finally, you run a regression in which, for each stock, the return today is explained by the returns from one to five days back:

$$r_t = \gamma_0 + \gamma_1 r_{t-1} + \gamma_2 r_{t-2} + \gamma_3 r_{t-3} + \gamma_4 r_{t-4} + \gamma_5 r_{t-5} + u_t$$

Table 2 shows the estimated coefficients, γ, of these regressions.

Table 2: Coefficients of time series regression, 5 lags

Ticker	γ_0 constant	γ_1 r_{t-1}	γ_2 r_{t-2}	γ_3 r_{t-3}	γ_4 r_{t-4}	γ_5 r_{t-5}	R^2
AF	-0.254	0.073	-0.121	-0.007	-0.083	-0.062	0.029
BOKA	0.291	-0.071	0.037	0.025	-0.126	-0.059	0.025
PHIA	-0.017	-0.032	-0.018	0.079	-0.058	-0.053	0.014
RDSA	-0.008	0.008	0.15*	-0.14*	0.054	0.079	0.046
UNA	0.036	-0.111	0.065	-0.090	0.042	-0.045	0.040
AEX	-0.029	-0.096	0.085	-0.027	0.039	-0.011	0.024

*significantly ≠0 (5% level, 2-tailed test)

(d) Does Table 2 reveal market inefficiency? Explain why.

7. Stocks are expected to earn (much) more than the risk free interest rate. This means that stock prices are expected to increase over time which, in turn, means that stock prices will be positively autocorrelated and that they are not a fair game or a martingale as the EMH claims. Is this reasoning correct?

8. In almost all countries there are a few people who became very rich by speculating on the stock market. This proves that excess returns can be earned and that the stock market is not efficient. Is this reasoning correct?

9. You are a student with good data skills and you decide to apply your talents to the stock market. After running a large number of regressions you find that the sign (+ or -) of the change in a company's stock price in one quarter is an accurate predictor of wether the company's earnings in the next quarter will increase or decrease.

   (a) Does this finding contradict the EMH?

   Next, you take daily return data of 100 stocks and test 10 different filter rules on each of them. You find that 27 stock-rule combinations earn significantly higher returns than a buy-and-hold strategy.

   (b) Does this finding contradict the EMH?

   You take a closer look at the stocks for which you found profitable filter rules and you see that they mainly belong to smaller, infrequently traded companies.

   (c) Is this finding relevant for the application of filter rules?

   Finally, you decide to apply an automatic function generator. You let your computer search through a very large number of functions that relate stock prices to variables in your dataset. You find that next month's stock prices are accurately predicted by the streeet number in the company's address plus the square root of the number of visitors to the company's website.

   (d) Do these results contradict the efficient market hypothesis?

10. A local mutual fund says it has expertise in identifying stocks that are undervalued because they are underresearched or unpopular. To prove its point, the fund produces evidence that its return over the past four years was 3% above the return on the market index.

    (a) What does the EMH say about undervalued stocks?
    (b) Does the fund's evidence contradict the EMH?

11. Many (financial) newspapers around the world regularly publish a ranking of mutual funds in their countries, based on the funds' performance, together with a relevant index as a benchmark. What would be the place of the benchmark index in the ranking in an efficient marked? Distinguish between performance over short and long periods.

12. It is sometimes said that market efficiency protects unknowledgeable investors, so that it does not matter what and how you buy and sell, you always pay and get a fair price. Comment on this statement.

13. It is often reported that the price of a stock has increased over some period *before* the announcement of good news such as higher earnings, dividend increases, etc. Does this contradict the EMH?

14. There are cases in which the price of a stock *dropped* after the firm announced some good news, e.g. an increase in quarterly earnings. Does this contradict the EMH?

15. The following message is taken from the Newsweb on Oslo Stock Exchange.
On 2009-06-18, Det norske oljeselskap ASA announced that it had discovered between 40 million and 130 million barrels of oil in the Grevling prospect. The appraisal well shows that the discovery is larger than first anticipated. Prior to drilling operations, Det norske estimated that Grevling could hold between 10 million and 80 million barrels of oil. The appraisal well also shows that the discovery is larger than indicated by the first discovery well. Det norske increased its ownership stake in Grevling from five percent to 30 percent prior to drilling operations. The company's net share of the discovery is thus between 12 million and 40 million barrels of oil, which means that Det norske's share of the discovery could match the volumes it sold in Goliat for MNOK 1,100 last autumn.

    (a) Calculate the abnormal return of Det norske oljeselskap on the announcement day. Use the market model and an estimation window of April-May 2009. The datafile (DetnorData.xls) is on the website.

16. Szewczyk et al.[1] analyzed a sample of companies announcing dividend omissions (announcements that no dividends will be paid). The CAAR (in %) on days relative to the announcement day (zero) are in the table below.

Day:	-6	-4	-2	0	2	4	6
CAAR %	.108	.032	-.483	-5.012	-5.183	-4.563	-4.685

    (a) Do the results of Szewczyk et al. contradict the Efficient Market Hypothesis (EMH)? If so, explain which form of the EMH it contradicts. Make additional assumptions if necessary.

    (b) It is sometimes argued that management of firms announcing dividend omissions know beforehand what they are going to announce, so that they could have shorted (sold short) the stock a week before. This would give them, on average, 5% return in a week $\rightarrow >200\%$ a year. This would be insider trading, but that happens, it is not illegal in some case and seldom discovered anyway. The conclusion is that the market is not strong form efficient. Is this argument correct?

[1] Samuel H. Szewczyk and George P. Tsetsekos and Zaher Z. Zantout, 'Do Dividend Omissions Signal Future Earnings or Past Earnings?', The Journal of Investing, Vol.6, n.1., pp-40-53, 1997.

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Exercises - solutions

1. Goldman's decision is an effort to 'time' the market, which assumes the ability to predict when valuations will go up again. In efficient markets prices cannot be predicted, the possibility that valuations will go up again is already included in today's price.

2. That the NPV is zero. On efficient markets securities are priced such that they earn their expected return, which makes their NPV zero.

3. Investors will sell and short sell the stock when prices are predictably high and buy when they are low. This will drive the prices down at the top of the cycle and up at the bottom of the cycle, and they will keep on buying and selling until the cycle has disappeared.

4. No, even if many investors do not follow the information on a stock and even less trade, the price can still reflect available information. Market efficiency requires that trades take place and at prices that reflect all available information, the number of investors involved is irrelevant.

5. No, investors can disagree and prices will reflect some average of opinions. However, market efficiency requires that no (groups of) investors are consistently better at evaluating the impact on prices than others. Investors can earn excess returns, but not consistently.

6. (a) The plots can visualize autocorrelation in the daily returns. Positive autocorrelation would give a clustering of the observations in the upper right and lower left quadrants of the graphs, showing that positive returns tend to be followed by positive ones and negative returns by negative ones. Negative autocorrelation would give a clustering of the observations in the lower right and upper left quadrants of the graphs, showing that positive returns tend to be followed by negative ones and negative returns by positive ones. The figure in the question shows neither pattern, the observations are scattered over all quadrants. Hence, the graphs show no autocorrelation in daily returns and do not reveal market inefficiency.

   (b) The plots show that the single stock AF has a much larger volatility than the index: the index returns are much more clustered around the graph's origin (coordinates 0,0).

   (c) The autocorrelation coefficients are all low, but the coefficient of Unilever is statistically significant. Significant autocorrelation rejects the hypothesis that the market is efficient. However, it is doubtful whether we can use such a low coefficient for a successful trading strategy. The square of the correlation coefficient measures the proportion of the variance of today's return that is explained by yesterday's return. For Unilever's coefficient this is less than 2%: $-.138^2 = .019$. Also, it is highly unlikely that Unilever will have the same autocorrelation in the next period.

   (d) As under (c), the coefficients are low, but two of them are significantly different from zero (those for r_{t-2} and r_{t-3} of Royal Dutch Shell (RDSA)). These two contradict the efficient market hypothesis but it is, again, doubtful whether this result persists in other periods and whether it is large enough to be exploitable. The proportion of variance in today's return that is explained by the regression (measured by the R^2 statistic) is lower than 5%. Note that the significant autocorrelation in UNA does not appear in the regression.

7. No, over short time intervals (e.g. days) the expected return is so small that it can be ignored in autocorrelation calculations. 20% return per year over 250 trading days means less than 0.1% per day, very small compared to daily price changes. The fair game model does not require returns (price changes) to have zero expectation, but the *excess returns*, or deviations from the expected returns. Similarly, the EMH does not require stock prices to be martingales but the *properly discounted* stock prices. The stock prices themselves are expected to increase with required rate of return on the stock.

8. No, the EMH does not say that excess returns are impossible, it says that excess returns cannot be systematically earned by using the available information set. In view of the very large number of investors, it is to be expected that a few will be lucky a number of times in a row. Similarly, a few people will be very unlucky and lose large amounts on the stock market.

9. (a) No, it is to be expected that news reaches the stock market before it materializes in earnings (see question 13). It would contradict the EMH if it were the other way around (that earnings predict nest quarter's stock returns).
   (b) No, you have tried 1000 different filter rules. Using a 5% confidence interval with a two-sided test you would expect to find $\pm$ 25 rules with a significantly positive return and the same number with a significantly negative return, based on pure chance. If you re-run the analysis on a different period you will probably find about the same number of significant results but in different stock-rule combinations.
   (c) Yes, the stocks of smaller, infrequently traded companies usually have higher transaction costs, e.g. a larger bid-ask spread.
   (d) Predictability contradicts the EMH but in this case it is obvious that the relation you found is spurious. Re-testing on a different dataset and/or period will almost certainly not reproduce similar results. However, the same conclusion would apply if your computer happened to find two variables that look plausible on first sight.

10. (a) That they do not exist.
    (b) No. There is no mention of risk, so the extra return relative to the index could be a premium for extra risk, relative to the index. Even if the 3% were truly excess returns, they could have been due to chance.

11. If the ranking is over a short period (0.5-1 year is typical), the benchmark index would be somewhere in the middle. Since excess returns are random, about equal proportions of the funds will over- and under-perform the index. As the period becomes longer, the index would move higher up on the ranking. Excess performance is not persistent so that good and bad years alternate. In the (very) long run few, if any, funds will outperform the index, so that the index will be (almost) on top.

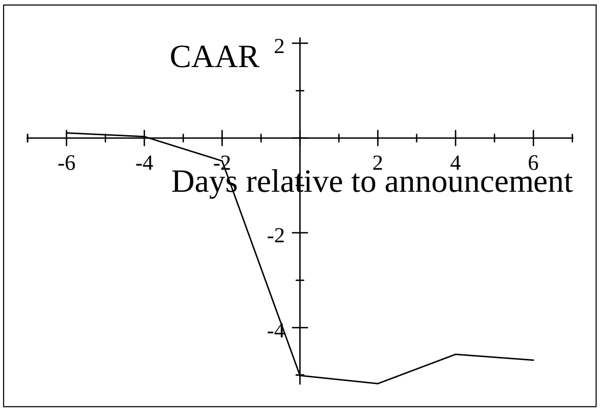
12. Unknowledgeable investors are indeed protected in an efficient market, but only to a certain degree. Buying and selling at market prices will be fair, i.e. there is no mispricing that other, knowledgeable investors can profit from. But if you offer to sell below market prices (e.g. by making a typing error) other investors will quickly profit from your mistake. Efficient markets offer no protection against excessive trading and no reward for the unsystematic risk of poorly diversified portfolios.

13. No, the good news can have reached the market gradually in other forms, e.g. news that large contracts were concluded, personal was recruited, new plants were being build, etc. The reflection of this news in the stock price does not contradict the EMH.

14. No, the market can have expected a larger increase in earnings based on the news that became available earlier.

15. To calculate the abnormal return we first have to calculate the normal return with the market model. Estimation of the market model for DetNor over April-May gives: $r_{DetNor} = 0.339 + 0.659 \times r_{OBX}$. The return of the OBXindex on the announcement day is 1.46% so DetNor's normal return is $0.339 + 0.659 \times 1.46 = 1.30$. DetNor's observed return on the announcements day is 8.65%, so the abnormal return $8.65 - 1.3 = 7.35\%$.

16. (a) Dividend (omission) announcements are public information other than prices and volumes: Szewczyk et al. test semi-strong market efficiency. Their results do not contradict the Efficient Market Hypothesis (EMH): CAAR are stable before the event, there is a sharp decline on the event date and again hardly any change in CAR after event date. This is shown in the data but it becomes more clear if we plot CAAR over time. The figure shows an efficient marked response to new information. The usual statistical assumptions apply.



CAAR for firms announcing dividend omissions

    (b) No! In order to say something about strong form market efficiency it has to be tested. For example by taking a sample of insiders, analyse their investments, look at the excess returns, etc. It is not possible to draw conclusions about strong form efficiency from a test of semi-strong efficiency.

Chapter 5: Capital structure and dividends

Exercises

1. Flotsam and Jetsam are two marine salvage companies. Both companies have the same assets that produce the same perpetual cash flow of €10 million. Both companies have 10 million shares outstanding. Flotsam has outstanding debt with a value of €20 million. Its shares are currently priced at €8 to give an annual return of 11.25%. Jetsam has outstanding debt with a value of €80 million. All debt is risk free and the risk free interest rate is 5%. Assume a Modigliani-Miller world without taxes.

   (a) Calculate the value of Flotsam's assets. Use an alternative calculation to check your results.

   (b) Calculate the price and return of Jetsam's shares. Check your results.

   (c) Mrs Grange owns 1 million shares of Flotsam. Mr Skelton wants to replicate Mrs Grange's investment, but he only wants to invest in Jetsam's shares and risk free borrowing or lending. What portfolio of Jetsam's shares and risk free borrowing or lending should Mr Skelton choose? Show that the portfolio requires the same investment amount and generates the same cash flow as Mrs Grange's 1 million shares of Flotsam.

2. Below is a list of characteristics that a stock can have or not have. State for each characteristic whether this will increase or decrease the value of the stock, other things equal.

   (a) voting rights

   (b) priority claim on profits

   (c) priority voting rights in merger and take-over decisions

   (d) a clause that gives the issuer of the stock the right to repay the amount received for the stock and redeem the share

   (e) a clause that gives the investor the right to convert a priority share to a common share

   (f) a clause that gives the issuer the right to convert a priority share to a common share

3. Below is a list of characteristics that a bond can have or not have. State for each characteristic whether this will increase or decrease the value of the bond, other things equal.

   (a) a clause that makes the interest payments dependent on the income earned by the issuer

   (b) a *call provision*, i.e. a clause that gives the issuer the right to buy back the bond before maturity at a pre-specified price (callable bond)

(c) a clause that gives the investor the right to sell the bond back to the issuer before maturity at a pre-specified price (putable bond)

(d) a clause that gives the investor the right to convert the bond into a common share

(e) a seniority ('me first') clause

4. A tricky question: suppose you have lent money to a company. You are the only party from which the company has borrowed money, i.e. there are no other creditors. The company wants to negotiate a lower interest rate and offers to secure the loan with its buildings. Is that a good deal?

5. Explain briefly what the advantages and dangers of securitization are (securitization is transforming internally held capital categories into securities that can be traded).

6. ZXco is financed with 80% equity and the β of its equity is 1.2. The risk free rate is 4% and the market risk premium, i.e. $r_m - r_f$, is 6%. All debt can be considered risk free and there are no taxes.

   (a) Calculate ZXco's WACC.

   ZXco decides to change its capital structure and increase debt to 60%. Assuming that debt remains risk free:

   (b) calculate ZXco's equity β, the required return on its equity and its WACC after the refinancing.

7. AG Goldmünzen & Verschuldung has plans to open a new mine. The plan is to finance the mine with 80% debt. Other mining companies are more conservative and finance their operations with, on average, 40% debt. The shares of these companies are priced such that they generate an expected return of, on average, 11.4%. Since the mining industry is very safe (at least, for investors) all debt can be considered risk free. The risk free interest rate is 7%. All mining firms use the same technology and generate the same return on assets. The tax rate is 30% and all the assumptions of the Modigliani-Miller tax case apply.

   (a) What is the required return on AG Goldmünzen & Verschuldung's equity investment in the planned mine?

8. TechCon is a technical construction company run by engineers. It is financed entirely with equity because, as the leading engineers put it, construction is a risky business. A newly employed engineer, who followed a course in finance for science and technology students, suggests to finance the company partly with debt because that would increase its value. TechCon has earnings before interest and taxes (EBIT) of 100 per year. Its equity is priced to give an expected return of 10% and the corporate tax rate is 20%. The company can borrow at the risk free rate of 4%. Its earnings can be treated as a perpetuity and all the assumptions of the Modigliani-Miller tax case apply.

   (a) With how much will the value of TechCon change if it refinances of 50% of its current value with a perpetual loan?

9. A consequence of the so-called clientele effect is that dividends are 'sticky' i.e. that dividend payments tend to be stable over time.

   (a) Explain what the clientele effect is and why it leads to sticky dividends.

(b) What is the effect of sticky dividends on the payout ratio (fraction of earnings paid out as dividends) of firms with very volatile earnings?

(c) To what erroneous conclusion can that lead?

(d) What is the effect of sticky dividends on the payout ratio of firms that were hit by a strike and had an occasional year of low earnings?

(e) To what erroneous conclusion can that lead?

10. MacroHard is a successful software company that has been generating cash at a phenomenal rate over the past few years. Now it has decided to pay out $250 million of it as cash dividend to its shareholders. The total market value of MacroHard is $1250 million, including $550 million of debt. The company has 2.5 million shares outstanding.

    (a) Calculate the change in MacroHard's stock price due to the dividend payment. Assume perfect markets.

    (b) State the changes in investment policy and financial policy due to the dividend payment.

    (c) How can MacroHard pay out the same amount in dividends without changing its investment and financial policy? Be precise in number of shares and price per share.

    (d) Now suppose that MacroHard uses the money to buy back shares instead of paying dividends. Calculate the effects of the buyback on the firms total market value, the share price and the number of shares oustanding.

11. Exodus Biofuel N.V. converts agricultural waste into diesel fuel. The market value of its equity is €200 million and since investors regard the biofuel business (or its managers) as very risky, it is priced to give a return of 16.5%. To strengthen investor confidence, Biofuel pays out all its earnings as dividends to its shareholders, no earnings are retained for new investments. For next year, Biofuel announced a dividend of €2.50 per share and the expectation is that earnings and dividends will grow at the inflation rate of 4%.

    (a) Explain why paying out all earnings as dividends may strengthen confidence in a firm (or a firm's management).

    (b) Calculate the price per share based on the expected return and growth rate.

Chapter 5: Capital structure and dividends

Exercises - solutions

1. (a) The value of Flotsam's debt is €20 million. The value of its equity is 10 million shares $\times$ €8 $=$ €80 million, so its assets have a value of €100 million. Alternatively, the cash flow to Flotsam's shareholders is €10 million - (20 $\times$ 0.05) $=$ €9 million. With a required return of 11.25%, the value of this cash flow is $9/0.1125 = 80$. Still another way is to calculate the opportunity cost of capital (or r_a): $r_a = 0.05 \times \frac{20}{100} + 0.1125 \times \frac{80}{100} = 0.1$ and use this to calculate the value of the total cash flow: $10/0.1 = 100$.

   (b) In a Modigliani-Miller world without taxes Flotsam and Jetsam have to have the same value. So Jetsam's total value is also €100 million, its debt has a value €80 million, which leaves €20 million for its equity, i.e. €2 per share. The cash flow to Jetsam's shareholders is €10 million - (80 $\times$ 0.05) $=$ €6 million or 0.6 per share. Solving $(0.6/r) = 2$ for r gives $r = 0.3$. We can check this by calculating the required return on equity with MM proposition 2:

   $$r_e = r_a + (r_a - r_d)\frac{D}{E} = 0.1 + (0.1 - 0.05)\frac{80}{20} = 0.3$$

   (c) Mrs Grange owns 10% of Flotsam's shares. The value of that investment is $0.1 \times 80 =$ €8 million and it produces an annual cash flow of $0.1 \times (10 - (20 \times 0.05)) =$ €0.9 million. Mr Skelton can replicate this investment by buying 10% of Jetsam's shares and 'undo' the difference in borrowing by the two companies. The difference in borrowing is $80 - 20 =$ €60 and 10% of this is €6 million. So Mr Skelton should lend €6 million risk free and buy 10% of Jetsam's shares, which costs €2 million, a total investment of €8 million. This investment produces a cash flow of $0.1 \times (10 - (80 \times 0.05)) =$ €0.6 million from the shares and $0.05 \times 6 =$ €0.3 million from the risk free lending, in total $0.6 + 0.3 =$ €0.9 million, identical to Mrs Grange's investment.

2. (a) increase

   (b) increase

   (c) increase

   (d) decrease

   (e) no effect if priority share has same rights as common shares plus priority

   (f) decrease

3. (a) decrease, income bonds only receive coupon (i.e. interest) payments if the issuing company has enough earnings to make the payment.

   (b) decrease

   (c) increase

   (d) increase

(e) increase

4. No, if you are the only lender, the loan is already secured with all of the company's assets. If the interest is not paid, you can ask for the company to be declared bankrupt which may lead to the sale of the company's assets to repay the loan. Securing a loan is only meaningful if there are also unsecured loans.

5. The advantages are that the companies whose capital was tied up in internal capital categories get access to capital that can be invested in other projects and that investors get access to new securities. The pooling of internally held capital of many companies also gives an insurance and diversification effect. The dangers are that the internal control and risk management procedures, that applied to the internal capital categories, no longer apply to external capital and have to be replaced by new, probably costly, procedures.

6. (a) The CAPM gives the return on ZXco's equity before refinancing: $r_e = r_f + \beta_e(r_m - r_f) = .04 + 1.2(.06) = 0.112$. The WACC is then: $.2 \times .04 + .8 \times .112 = 0.0976$

   (b) First, we calculate β_a before refinancing: $\beta_a = D/V \times \beta_d + E/V \times \beta_e = .2 \times 0 + .8 \times 1.2 = 0.96$. With this asset β we can calculate the equity β after refinancing: $\beta_a = D/V \times \beta_d + E/V \times \beta_e \Rightarrow 0.96 = .6 \times 0 + .4 \times \beta_e \Rightarrow \beta_e = 2.4$. The CAPM gives $r_e = .04 + 2.4 \times .06 = 0.184$. The WACC remains: $.6 \times .04 + .4 \times .184 = 0.0976$.
   Alternatively, we could have used MM proposition 2 to calculate $r_e : r_e = r_a + D/E(r_a - r_f) = .0976 + .6/.4(.0976 - .04) = 0.184$. In the absence of taxes, the WACC equals r_a. We can check this with CAPM and $\beta_a : r_a = .04 + .96 \times .06 = 0.0976$

7. (a) First, we use MM proposition 2 and the data of the mining industry on average to calculate r_a, the return on assets:

$$\begin{aligned} r_e &= r_a + (1-\tau)(r_a - r_d)\frac{D}{E} \\ .114 &= r_a + (1-.3)(r_a - .07)\frac{.4}{.6} \qquad \Rightarrow r_a = .1 \end{aligned}$$

   Alternatively, we can use the formula for r_a under the MM assumptions:

$$r_a = r_d(1-\tau)\frac{D}{V - \tau D} + r_e\frac{E}{V - \tau D}$$

   The amounts D, E and V are not known, but we only need the proportions, which are known:

$$r_a = 0.07(1-0.3)\frac{0.4}{1 - 0.3 \times 0.4} + 0.114\frac{0.6}{1 - 0.3 \times 0.4} = 0.1$$

   With this r_a we can calculate the required return on AG Goldmünzen & Verschuldung's equity investment in the planned mine:

$$\begin{aligned} r_e &= r_a + (1-\tau)(r_a - r_d)\frac{D}{E} \\ r_e &= .1 + ((1-.3)(.1 - .07)\frac{.8}{.2} = .184 \text{ or } 18.4\% \end{aligned}$$

8. (a) TechCon's value before refinancing is calculated as follows: over its EBIT of 100 it pays 20 in taxes. The net earnings have a value of 80/.1=800. Refinancing with 50% debt means issuing 400 debt, with a yearly interest bill of $.04 \times 400 = 16$.

The division of the earnings (cash flow) then becomes:

EBIT	100.0
interest	-16.0
EBT	84.0
taxes	-16.8
net earnings	67.2

To calculate the value of these earnings we need the cost of equity after refinancing, which can be calculated with MM proposition 2

$$r_e = r_a + (1-\tau)(r_a - r_d)\frac{D}{E}$$

This calls for E, but since the cash flows are perpetuities we can use the balance sheet identity $V_a + tD = E + D$

$$\begin{aligned} r_e &= r_a + (1-\tau)(r_a - r_d)\frac{D}{V_a + \tau D - D} \\ r_e &= .1 + (1-.2)(.1-.04)\frac{400}{800 + .2 \times 400 - 400} = 0.14 \end{aligned}$$

this gives an equity value of $67.2/.14 = 480$. The total company value is $480+400 = 880$. The change in company value is thus $880 - 800 = 80$

A shorter calculation uses MM proposition 1: $V_l = V_u + \tau D$. The change in value is τD or $.2 \times 400 = 80$

9. (a) The clientele effect says that investors select their portfolios in such a way that the dividend payments they receive are enough to cover their needs for cash. This does not create value for (non-)dividend paying stocks if there are enough dividend paying and non dividend paying stocks to choose from. A change in dividend policy forces investors to make costly adjustments to their portfolios so that they prefer companies with stable, i.e. sticky, dividend policies.

   (b) Firms with very high earnings in some periods and very low earnings in other periods can only maintain a stable (sticky) payout ratio if it is low.

   (c) To the extent that the volatility is market synchronous (i.e. that it represents systematic risk) it reduces the value of the earnings. This may lead to the erroneous conclusion that a low payout ratio gives a low value.

   (d) If earnings fall sharply because of a strike but dividends remain stable, the payout ratio will rise.

   (e) A fall in earnings will be reflected in the stock price but the percentage decrease will be less than the percentage decrease in earnings, since an occasional strike will not affect the long term earnings potential. This means that the price-earnings ratio will go up. An erroneous conclusion would be that an increase in the payout ratio leads an increase in the price-earnings ratio, i.e. that investors would be willing to pay more for earnings that are paid in dividend than for earnings that are retained in the company. As a numerical example, consider a firm with perpetual earnings of 20; the firm pays 10 in dividends, so the payout ratio is 0.5. Its return on equity is 10% so the value of the earnings is $20/0.1 = 200$, which gives a price/earnings ratio of $200/20 = 10$. This year the firm is hit by a strike, and its earnings are halved. Dividends are sticky and remain 10, so that the payout ratio becomes 1. The strike is one-time event, so the firm's value becomes:

$$\frac{10+200}{1.1} = 190.9$$

   The price/earnings ratio becomes: 190.9/10=19. The erroneous conclusion would be: if the payout ratio goes up, the P/E goes up.

10. (a) Before the dividend payment, MacroHard holds assets worth \$1250 million, including the \$250 to be paid as dividends. Its debt is worth \$550 million, so its equity has a value of $1250 - 550 = 700$ million or $700/2.5 = 280$ per share. After the dividends are paid the firm holds assets worth \$1000 million, debt is still worth \$550 million, so equity is $1000 - 550 = 450$ million or $450/2.5 = 180$ per share. The change in stock price is thus \$100, exactly equal to the amount of dividend per share: 250/2.5.

    (b) The dividend payment reduces the value of the assets that MacroHard holds from \$1250 to \$1000 million. Equity is reduced from \$700 to \$450 million, this changes the debt-equity ratio from 550/700 to 550/450.

    (c) If MacroHard wants to pay dividend without changing investment and financial policy it has to issue shares to pay for the dividends. After the dividends are paid the price per share is \$180. So the firm has to issue $250/180 = 1.3889$ million shares. This leaves the values on the balance sheet unaltered but the firm now has $2.5 + 1.3889 = 3.8889$ million shares @ 180 instead of 2.5 million @ 280.

    (d) If MacroHard uses the \$250 million to buy back shares it has to buy $250/280 = 0.89286$ million shares. The total value of the firm will become \$1000 of which \$550 is debt and \$450 equity. The number of outstanding shares becomes $2.5 - 0.89286 = 1.6071$ million, the share price remains $450/1.6071 = 280$.

11. (a) Internally generated funds as retained earnings slip the control of the market because management does not have to ask the market to provide them, they are already under management's control. This increases the risk that these funds will be used on projects that increase management's wealth rather than shareholders'. Paying out all earnings eliminates this risk.

    (b) Since there are no capital gains we can use Gordon's growth model to calculate the present value of the growing dividend stream:

$$P_0 = \frac{A}{r - g} = \frac{2.5}{.165 - .04} = 20$$

Chapter 6: Valuing levered projects

Exercises

1. ZXco is a mature, established company. Over the years, it acquired capital from different sources and the right hand side of its balance sheet currently has the following items:

   - Common shares: 10 million outstanding shares, currently priced at €6 to give 12% return.
   - Preferred shares: 1 million outstanding shares, currently priced at €5 to give 15% return.
   - Bonds: 4 million outstanding zero coupon bonds; the bonds have a face value of €10, and mature 5 years from now; they currently trade at €6.806.
   - Long term bank loan: €20 million, secured by ZXco's buildings; interest rate is 7% but is adjusted every year.
   - Short term bank loan: €20 million in a roling line of credit (i.e. automatically renewed), 9% interest.
   - Accounts payable: €10 million outstanding.

   (a) ZXco pays 25% taxes. Calculate its WACC.

2. TeleSør considers investing in a new mobile broadband network in Trøndermark. The company wants to finance the project with 25% debt, which it can borrow at 5% interest. The rest would be financed with equity, on which TeleSør expects a 15% return. Together with the corporate tax rate of 28% this gives the project a WACC of 12.15% ($WACC = (1-\tau)r_d\frac{D}{V}+r_e\frac{E}{V} = (1-.28)\times.05\times.25+.15\times.75 = .1215$). However, calculations show that the project is unprofitable if the discount rate is higher than 10%. The project's chief engineer suggests to finance the project with 50% debt because that would bring the WACC below 10%: $(1-.28)\times.05\times.5+.15\times.5 = 0.093$.

   (a) Is the chief engineer's argument correct? Explain.

3. A firm is considering investing in a new line of business. As preparation for the decision, a junior financial manager collected the following data on the 4 main competitors in the business:

Firm	r_e	r_d	D/V
1	16.0%	5.2%	0.6
2	14.5%	4.9%	0.5
3	13.6%	4.6%	0.5
4	12.4%	4.3%	0.4

   All debt is continuously rebalanced.

   (a) Use the data of the 4 firms to calculate the Opportunity Cost of Capital (r_a) in the industry.

4. A company obtained a €400 000 loan from its bank. The loan has to be paid back in amounts of €100 000 after each of the following four years. The interest on the loan is 10% and the company has a tax rate of 30%

   (a) Calculate the value of the tax savings on the loan, assuming that the interest and taxes are paid at the end of each year.

   Suppose the company knows it will have to settle an old conflict with the tax authorities. It therefore agreed with the bank to double the loan in year 2 on the same conditions. The extra money will be paid back in year 3. This will make the tax advantage go up and down strongly.

   (b) Is this volatility in the tax savings a reason to adjust the discount rate? Explain.

5. E-razor Corp. is considering the introduction of a new product. The introduction requires an immediate investment of €1000 and the product is expected to generate an after tax cash flow of €333 per year for 4 years starting 1 year from now. E-razor estimates the risk of the project to be such that the company would require a return of 12.5% if it financed the projects exclusively with equity. For this project, E-razor plans to raise 50% of the investment by issuing new shares. The issue costs are 5%. The remaining 50% of the investment is financed with a loan. E-razor agreed with its bank to pay 8% interest at the end of each of the following 4 years. The loan will be redeemed in 1 payment together with the last interest payment. E-razor has a tax rate of 30%.

   (a) Should E-razor go ahead with the project or not? Use APV calculations to support your answer.
   (b) Explain why APV is the preferred method in this case.

6. Korkla AS is a large conglomerate active in, among other things, soft drinks, heavy metals and financial services. As a well diversified firm that has excellent relations with its banks, it has a low company average costs of capital. Although 60% of its total value is financed with debt, its cost of debt is only 7%, or 1% over the risk free rate of 6%. It is now considering a project to enter the NO-WITS (Norwegian wireless internet telephone service) business, the latest development in telecommunication. This will require an immediate investment of 750 million and is expected to produce a perpetual after tax cash flow of 100 million per year starting 1 year from now. Korkla plans to finance the investment with 25% debt, for which the bank has made an offer at 8% interest. At present there is only one firm active in the NO-WITS industry, the Checkers company. Checkers is financed with equal parts of debt and equity, its debt has a 9% interest rate and its equity has a β of 1.4. The market risk premium is 8% and the corporate tax rate for all firms is 40%. All debt is rebalanced.

   (a) Should Korkla accept the NO-WITS project or not? Use calculations to support your answer and make additional assumptions if necessary.

Chapter 6: Valuing levered projects

Exercises - solutions

1. (a) To calculate the WACC we need the value of each capital category and its required return. The value of the common shares is $10 \times 6 = 60$ million @ 12%; the value of the preferred shares is $1 \times 5 = 5$ million @ 15%. The value of the zero coupon bonds is $4 \times 6.806 = 27.224$. Their required return can be calculated from the face value, to be paid when the bond matures: $6.806 = 10/(1+r)^5$ solving for r gives 1.08 or 8%. The values and interest rates of the bank loans are given. Accounts payable do not bear interest, they can be netted out against current assets to give net working capital. Total equity and liabilities is thus $60 + 5 + 27.224 + 20 + 20 = 132.22$. The WACC then is:

$$\frac{60}{132.22} \times 0.12 + \frac{5}{132.22} \times 0.15 + (1-0.25) \times \frac{27.224}{132.22} \times 0.08 + ..$$
$$.. + (1-0.25) \times \frac{20}{132.22} \times 0.07 + (1-0.25) \times \frac{20}{132.22} \times 0.09 = 0.091$$

2. (a) No! If the proportion of debt doubles the interest rate will not remain the same, nor will the required return on equity.

3. (a) With continuous rebalancing taxes drop from equation and $r_a = r_e \frac{E}{V} + r_d \frac{D}{V}$ so for the 4 firms:

$$\begin{aligned}
.16 \times .4 + .052 \times .6 &= 0.0952 \\
.145 \times .5 + .049 \times .5 &= 0.097 \\
.136 \times .5 + .046 \times .5 &= 0.091 \\
.124 \times .6 + .043 \times .4 &= 0.0916
\end{aligned}$$

The average of the 4 rates is $(.0952 + .097 + .091 + .0916)/4 = 0.0937$ or 9.4%.

4. (a) Calculations are shown in the table below:

year	Loan at year end	Interest $r_d D$	tax saving $\tau r_d D$	PV(tax saving)
1	400 000	40 000	12 000	12 000/1.1=10 909
2	300 000	30 000	9 000	9 000/1.1^2= 7 438
3	200 000	20 000	6 000	6 000/1.1^3= 4 508
4	100 000	10 000	3 000	3 000/1.1^4= 2 049
sum				24 904

(b) No, the point is not that the tax savings are stable, but that they are predetermined, i.e. their size does not depend on uncertain future developments. The tax settlement is already known, not uncertain.

5. (a) To calculate APV we start with the base case, the value of the project as if it were all equity financed. The OCC is given as 12.5% so the NPV of the cash flows is:

$-1000 + \frac{333}{1.125} + \frac{333}{1.125^2} + \frac{333}{1.125^3} + \frac{333}{1.125^4} = 0.88$. The project has 2 side effects, issue costs and tax shields.

To raise 500, the company will have to issue $\frac{100}{95} \times 500 = 526.32$ so the issue costs are $526.32 - 500 = 26.32$

The amount of debt will be available throughout the project's life so the yearly interest payments are $.08 \times 500 = 40$. This gives a tax advantage of $.3 \times 40 = 12$. Since debt is predetermined, the discount rate for the tax advantage is the cost of debt. The PV is $\frac{12}{1.08} + \frac{12}{1.08^2} + \frac{12}{1.08^3} + \frac{12}{1.08^4} = 39.75$.

The project's APV is $0.88 - 26.32 + 39.75 = 14.31 > 0$, E-razor should go ahead with the project.

(b) APV is the preferred method in this case because it involves issue costs that cannot be included in the discount rate. Moreover, because debt is predetermined and fixed, the D/E ratio will be different in each of the project's 4 years and, hence, the cost of equity, r_e, and the WACC will be different too.

6. Korkla should accept the project if it has a positive NPV; to calculate NPV we need the cash flow and investment (both given) and the proper discount rate. The discount rate is determined by the characteristics of the project; the background data on Korkla are irrelevant. The business risk and the corresponding opportunity cost of capital can be calculated from Checkers' data, since this is the only company active in the NO-WITS industry. We proceed as follows:

(I) First we calculate the return on assets, or the opportunity cost of capital (OCC), r_a, for Checkers. For this we need the return on equity, r_e, which we can find with the CAPM: $r_e = r_f + \beta_e(r_m - r_f) = .06 + 1.4 \times .08 = .172$

Return on assets can be found in three ways:

i. By using the formula for r_e in reverse:
$r_e = r_a + (r_a - r_d)D/E$
$.172 = r_a + (r_a - .09).5/.5 \Rightarrow r_a = .131$

ii. Alternatively, using unlevering:
$r_a = .5 \times .09 + .5 \times .172 = .131$.

iii. Finally, it is also possible to calculate β_d from the CAPM: $.09 = .06 + \beta_d \times .08 \Rightarrow \beta_d = .375$ and then use the β version of the formula to find β_a :
$\beta_e = \beta_a + (\beta_a - \beta_d)D/E \Rightarrow 1.4 = \beta_a + (\beta_a - .375).5/.5 \Rightarrow \beta_a = .8875$.
The CAPM then gives: $r_a = .06 + .8875 \times .08 = .131$

(II) Given the opportunity costs of capital in the NO-WITS industry and the financing characteristics for Korkla's project ($D/E = .25/.75$, $r_d = 8\%$) we can calculate the project's WACC in two ways:

i. Calculate r_e for the project: $r_e = r_a + (r_a - r_d)D/E = .131 + (.131 - .08).25/75 = .148$, which gives a project WACC of: $(.75 \times .148) + (.25 \times (1 - .4) \times .08) = .123$

ii. Alternatively, we could have used Miles-Ezzell's formula: $r' = r_a - \tau r_d L \left(\frac{1+r_a}{1+r_d}\right) = .131 - .4 \times .08 \times \frac{.25}{1} \times \frac{1.131}{1.08} = .123$

(III) With this WACC the value of the cash flow becomes: 100/.123=813 so that NPV= $813 - 750 = 63 > 0$, $\Rightarrow$ Korkla should accept the project.

(IV) APV can also be used. We proceed as follows:

i. Discount the cash flow at the OCC: $100/.131 = 763.359$

ii. The tax advantage is $\tau r_d D = .4 \times .08 \times (.25 \times 750) = 6$

iii. Discounting this at the OCC gives $6/.131 = 45.80$

iv. Multiply by $(1 + r_a)/(1 + r_d) = 1.131/1.08 = 1.0472.. \times 45.80 = 47.96$.

v. APV is then: $763.359 + 47.96 - 750 = 61.32 > 0$, so Korkla should accept the project.

Chapter 7: Option pricing foundations

Exercises

1. On June 23, shares of ZX Co. were traded in Frankfurt for 97.70. Call options on this share with a time to maturity of one year and a strike price of 80 were traded for 23.20. Puts on the same stock with the same time to maturity and the same strike price traded for 4.16. The relevant interest rate for the options' lifetime was 3.15%.

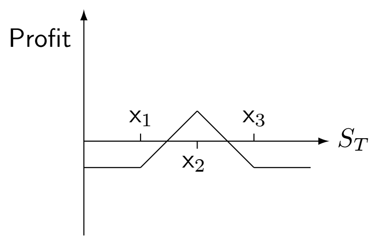
    (a) Discuss the arbitrage opportunities on this market from a pricing point of view and from a practical point of view. Explicitly mention the assumptions on which arbitrage opportunities, or the lack thereof, rest.

2. You have done some option trading and you now hold the following option contracts: you have written (sold) a put with an exercise price of 75, you have bought a put with an exercise price of 100, you have bought a call with an exercise price of 75 and you have written a call with an exercise price of 100. All options are on the same underlying share and have the same maturity.

    (a) Construct the payoff diagram (not the profit diagram) for your total position over the share price interval from 0 to 150.

3. It is sometimes said that the prices of at-the-money puts and calls on the same underlying and with the same maturity have to be the same, since simultaneously buying and selling cancels out. Is this reasoning correct? Assume European options on non dividend paying stocks.

4. The option position depicted in the figure below is known as a butterfly spread.



    (a) Work out a combination of call options that produces the butterfly spread in the figure above.

    (b) Work out a combination of put options that produces the butterfly spread in the figure above.

5. On November 1^{th}, 3 months European call options on a share ZXco with an exercise price of 460 cost 20.75. The same options with an exercise price of 480 resp. 500 cost 11.75 resp. 6. The share price of ZXco is 462.5 and the 3 months risk free interest rate is 1.5%.

(a) Calculate the initial investment that is required to set up a butterfly spread.

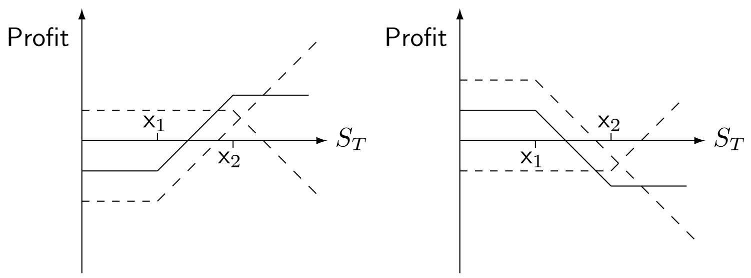
(b) Show that the same butterfly spread set up with put options requires the same initial investment.

(c) According to a business newspaper, the prices of 3 months put options ZXco are as follows:

strike	bid	ask
460	15.25	17.50
480	26.00	29.00
500	40.00	43.25

Explain why these prices are the same as, or different from, the prices you used in question b.

6. The figure below repeats the profit diagrams for option positions known as spreads that was used in the main text.



(a) Bull spread (b) Bear spread

(a) Construct the spreads in this figure using put options.

(b) What happens to the initial investment (initial balance of the option premiums) required to set up the position, compared to the same position in calls? Hint: look at the prices of options in relation to the exercise prices.

7. In Bound 6 the following maximum value for European put options is formulated: a European put option cannot be worth more than the present value of the exercise price. Explain why this bound is or is not valid for American options.

8. A financial market can have 2 possible future states. Both states are equally likely. In the market 2 securities, A and B, are traded. The securities' pay-offs in the future states are as follows:

	A	B
$State\ 1$	6	4
$State\ 2$	12	14

The required rates of return on the securities are 8% for A and 12% for B.

(a) Calculate the value of the securities A and B using the risk adjusted discount rates.

(b) Calculate the state prices and the risk free interest rate on the market. Check your results.

(c) Re-calculate the values of the securities A and B:

i. using the state prices

ii. using the risk neutral valuation formula

(d) Calculate the values of call options on the two securities A and B with an exercise price of 10 and that mature in the future state (of course).

9. In a financial market a stock is traded at a price of 100. At the end of the period, the stock price can either increase with 20 to 120 or decrease with 20 to 80. Risk free debt is available at 5% per period. Using state prices, calculate the value of an at the money call option on the stock which matures at the end of the period.

10. In a financial market a stock is traded with a current price of 100. The price of the stock can either go up with 50% next period or go down with 50% next period. Risk free debt is available with an interest rate of 25%. Also traded is a European call option on the stock with an exercise price of 110 and a time to maturity of 2 (i.e. the option matures at end of the second period on the third moment).

    (a) Discuss the completeness and the arbitrage properties of this market
    (b) Calculate the value of the option assuming that:
        i. The stock pays no dividend
        ii. The stock pays out 25% of its value in dividends on the second moment (end of the first period)
        iii. Repeat step b(i) for an American call option
        iv. Repeat step b(ii) for an American call option
        v. Repeat step b(i) for a European put option
        vi. Repeat step b(ii) for a European put option

11. In a financial market a stock is traded with a current price of 50. Next period the price of the stock can either go up with 30% or go down with 25%. Risk free debt is available with an interest rate of 8%. Also traded are European options on the stock with an exercise price of 45 and a time to maturity of 1, i.e. they mature next period.

    (a) Calculate the price of a call option by constructing and pricing a replicating portfolio.
    (b) Calculate the price of a put option by constructing and pricing a replicating portfolio.

12. On page 202 of the book, a self-financing strategy is defined as a strategy that requires no extra cash along the way, i.e. all additional outlays must be part of the strategy. On page 212-213, the dynamic hedging portfolio of the two-period example is shown to give a perfect hedge with a self-financing strategy. Demonstrate that the dynamic hedge in this example is indeed self-financing. Hint: think of what self-financing means for rebalancing the portfolio.

Chapter 7: Option pricing foundations

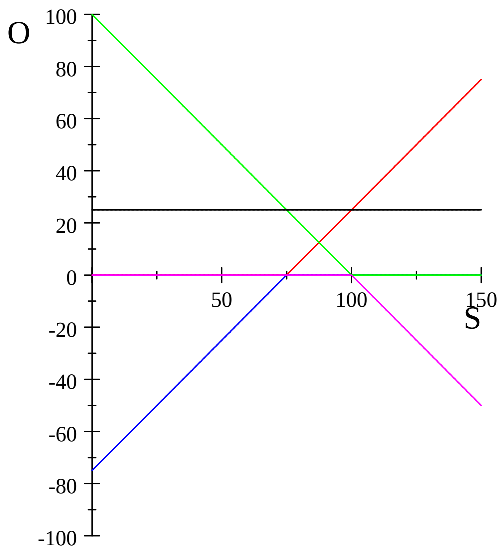
Exercises - solutions

1. (a) We use the put-call parity: Share + Put = Call + PV(X) or
Share + Put - Call $= 97.70 + 4.16 - 23.20 = 78.66$
and $PV(X) = 80 \times e^{-0.0315} = 77.519$.
We see that the put-call parity did not precisely hold. This probably does not mean that are any arbitrage opportunities since:
   - PCP is only valid for European options, and most traded options are American options.
   - ZX Co. may pay a dividend before expiry.
   - We ignored bid – ask spreads in trading and any other transaction costs.
   - There may have been non synchronous trading, i.e. put price may refer to a different time than the call and/or the share.

2. (a) It may be helpful to calculate the options' pay-offs on different points in the interval:

Share price (S)	0	50	75	100	125	150
short put (X=75)	- 75	- 25	0	0	0	0
long put (X=100)	100	50	25	0	0	0
long call (X=75)	0	0	0	25	50	75
short call (X=100)	0	0	0	0	- 25	- 50
Total position (O)	+ 25	+ 25	+ 25	+ 25	+ 25	+ 25

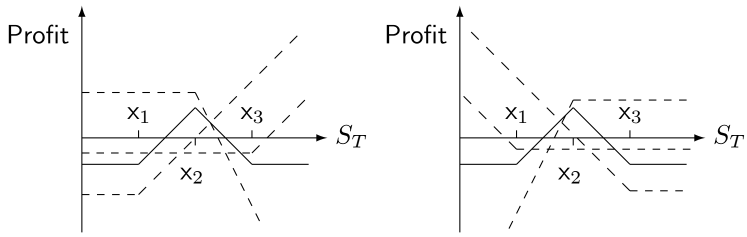
The positions are plotted in the figure below; the plotted functions are $\min[0, x - 75]$, $\max[0, 100 - x]$, $\max[0, x - 75]$, $\min[0, 100 - x]$ and their sum.



3. No, puts and calls do not cancel out, but buying a put is cancelled out by selling a put. Similarly, a short call is cancelled out by a long call. Also, the prices of otherwise identical at-the-money puts and calls are not the same, as is easily verified with the put-call parity: share + long put = long call + PV(X) or long put - long call = PV(X) - share. If the put and call have the same value, the share price has to be equal to the present value of the exercise price (so not the exercise price).

4. (a) A butterfly with calls consists of 1 long call with exercise price x1, 2 short calls with exercise price x2 and 1 long call with exercise price x3.

   (b) The same position with puts consists of 1 long put with exercise price x3, 2 short puts with exercise price x2 and 1 long put with exercise price x1. The positions are depicted in Figure 1 below.



(a) Using calls (b) Using puts

Figure 1: Profit diagrams for a butterfly spread

5. A butterfly spread involves buying 1 call with a low exercise price, selling two calls with a higher exercise price and buying 1 call with an even higher exercise price.

   (a) The initial investment required is:

	strike	price	amount
buy 1	460	20.75	-20.75
sell 2	480	11.75	23.50
buy 1	500	6.00	-6
total			-3.25

   (b) We first have to calculate the put prices using the put-call parity:

call	+PV(X)	-share	=price put
20.75	460/1.015=453.2	-462.50	=11.45
11.75	480/1.015=472.9	-462.50	=22.15
6	500/1.015=492.6	-462.50	=36.10

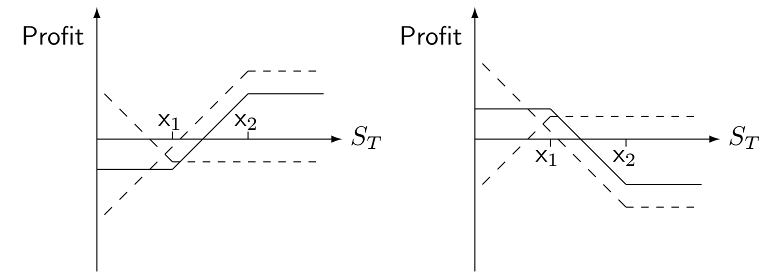
Then we can set up the butterfly with puts:

	strike	price	amount
buy 1	500	36.10	-36.10
sell 2	480	22.15	44.30
buy 1	460	11.45	-11.45
total			-3.25

which gives the same initial investment.

(c) These prices are different because traded options are (almost) always American options and by using the put-call parity we calculated prices of European options. European puts trade at a lower price range than American puts because the right of early exercise is valuable (as well as frequently used).

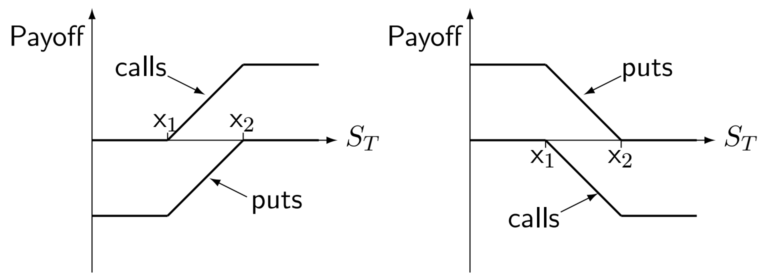
6. (a) The bull spread is constructed by selling 1 put with exercise price x2 and buying 1 put with exercise price x1. In the bear spread the positions are reversed, i.e. a put with exercise prise x1 is sold and a put with exercise price x2 is bought. The positions are depicted in Figure 2 below



(a) Bull spread (b) Bear spread

Figure 2: Profit diagrams for spreads

(b) The initial balances of option premiums (initial investments) are reversed compared with the same positions constructed with calls. The price of a put increases with the exercise price (call prices decrease). In a bull spread constructed with puts, the more expensive put with a high exercise price is sold and the cheaper one is bought so that the initial balance is positive (negative investment). On the other hand, the payoff at maturity is either zero or negative. In a bear spread constructed with puts the more expensive option is bought so that the initial balance is negative (initial investment required). However, the payoff at maturity is either zero or positive. The payoffs at maturity at depicted in the payoff diagram below. The differences



(a) Bull spread (b) Bear spread

Figure 3: Payoff diagrams of spreads

are summarized in the table below.

		Bull	Bear
Calls decrease with X		buy low X (costs more) sell high X (costs less)	sell low X (costs more) buy high X (costs less)
		price <0 payoff at maturity >0	price >0 payoff at maturity <0
Puts increase with X		buy low X (costs less) sell high X (costs more)	sell low X (costs less) buy high X (costs more)
		price >0 payoff at maturity <0	price <0 payoff at maturity >0

7. The bound states that a European put option cannot be worth more than the present value of the exercise price and this bound is *not* valid for American puts. The arbitrage strategy that enforces the bound for European puts does not always hold for American puts because of early exercise. If the put is exercised early the interest earned on PV(X) is not enough to cover the exercise price so that additional cash is required. The easiest way to illustrate this is by assuming that the stock price drops to zero right after the put was sold:

	Now	At exercise $S_T = 0 < X$
Sell put	$+O_p^A$	$-X$
Lend $PV(X)$	$-PV(X)$	$PV(X)$
Total position	>0	$PV(X) - X < 0$

8. (a) We calculate the values as the expected pay-offs discounted at the risk adjusted rate of return; since both states are equally likely:

$$\begin{aligned} A &= \frac{0.5 \times 6 + 0.5 \times 12}{1.08} = 8.33 \\ B &= \frac{0.5 \times 4 + 0.5 \times 14}{1.12} = 8.04 \end{aligned}$$

(b) We can calculate state price by making portfolios of A and B that pay off 1 in only one state and 0 in the other. For the first state this means solving the equations:

$$\begin{aligned} 6 \times a + 4 \times b &= 1 \\ 12 \times a + 14 \times b &= 0 \end{aligned}$$

for the weights a and b. The solution is: $a = \frac{7}{18},\ b = -\frac{1}{3}$. This gives a state price of $\frac{7}{18} \times 8.33 - \frac{1}{3} \times 8.04 = 0.559$. For the second state price we solve the equations (re-using the symbols a and b):

$$\begin{aligned} 6 \times a + 4 \times b &= 0 \\ 12 \times a + 14 \times b &= 1 \end{aligned}$$

The solution is: $a = -\frac{1}{9},\ b = \frac{1}{6}$, so the second state price is $-\frac{1}{9} \times 8.33 + \frac{1}{6} \times 8.04 = 0.414$. We can also use matrix algebra:

$$\left[\begin{array}{cc} 8.33 & 8.04 \end{array}\right] \left[\begin{array}{cc} 6 & 4 \\ 12 & 14 \end{array}\right]^{-1} = \left[\begin{array}{cc} 0.559 & 0.414 \end{array}\right]$$

To calculate the risk free interest rate we have to make a portfolio of A and B that pays off the same in both states. This means solving the equations:

$$\begin{aligned} 6 \times a + 4 \times b &= 1 \\ 12 \times a + 14 \times b &= 1 \end{aligned}$$

which gives $a = \frac{5}{18}$, $b = -\frac{1}{6}$. The present value of this portfolio is $\frac{5}{18} \times 8.33 - \frac{1}{6} \times 8.04 = 0.9739$. We can check this with the sum of the state prices, which has to be the same: $0.559 + 0.414 = 0.973$. The risk free interest rate is $1/0.9739 = 1.027$.

(c) The values of the securities A and B are re-calculated as follows:

i. using state prices:
$A = 6 \times 0.559 + 12 \times 0.414 = 8.32$
$B = 4 \times 0.559 + 14 \times 0.414 = 8.03$

ii. to use the risk neutral valuation formula we first have to calculate the risk neutral probabilities. The are defined as the state prices $\times(1 + r_f)$ so $p_1 = 0.559 \times 1.027 = 0.574$ and $p_2 = 0.414 \times 1.027 = 0.426$. The risk neutral valuation formula then gives:

$$\begin{aligned} A &= \frac{0.574 \times 6 + 0.426 \times 12}{1.027} = 8.33 \\ B &= \frac{0.574 \times 4 + 0.426 \times 14}{1.027} = 8.04 \end{aligned}$$

(d) At maturity the options pay off the maximum of zero and the difference between the security price and the exercise price: $\max[0, S - X]$, so:

	A	B
$State1$	$\max[0, 6 - 10] = 0$	$\max[0, 4 - 10] = 0$
$State2$	$\max[0, 12 - 10] = 2$	$\max[0, 14 - 10] = 4$

The state price for the second state is 0.414, so the options prices are $2 \times 0.414 = 0.828$ and $4 \times 0.414 = 1.656$.
We can also use the risk neutral valuation formula and the risk neutral probabilities we calculated above (which is, of course, basically the same thing):

$$\begin{aligned} Option\ on\ A &= \frac{0.574 \times 0 + 0.426 \times 2}{1.027} = 0.830 \\ Option\ on\ B &= \frac{0.574 \times 0 + 0.426 \times 4}{1.027} = 1.659 \end{aligned}$$

9. State prices can be calculated in 3 ways (which all boil down to the same thing, of course):

(I) We can construct the payoff matrix for the securities in this market, take its inverse to get the weights of the portfolios that produce the state securities and then multiply by the values of the securities now to get the state prices.

(II) We can write out the system of 2 equations with 2 unknowns and solve it.

(III) We can calculate the risk neutral probabilities and then discount them to get the state prices.
We present all 3 methods.

(I) The payoff matrix is:

$$\Psi = \begin{bmatrix} 1.05 & 80 \\ 1.05 & 120 \end{bmatrix}$$

and its inverse

$$\Psi^{-1} = \begin{bmatrix} 2.8571 & -1.9048 \\ -0.025 & 0.025 \end{bmatrix}$$

The prices of the securities now are:

$$v = \begin{bmatrix} 1 & 100 \end{bmatrix}$$

so that the state prices are:

$$v\Psi^{-1} = \left[\begin{array}{cc} 0.357\,14 & 0.595\,24 \end{array}\right]$$

The option's pay-offs are

$$O = \left[\begin{array}{c} 0 \\ 20 \end{array}\right]$$

multiplying these by the state prices gives the option price:

$$v\Psi^{-1}O = 11.905$$

(II) The system of 2 equations with 2 unknowns is:

$$\begin{array}{rcl} 1 & = & \psi_1 \times 1.05 + \psi_2 \times 1.05 \\ 100 & = & \psi_1 \times 80 + \psi_2 \times 120 \end{array}$$

and its solution is:

$$[\psi_1 = 0.357\,14, \psi_2 = 0.595\,24]$$

so that the option price becomes

$$0.357\,14 \times 0 + 0.595\,24 \times 20 = 11.905$$

(III) State prices are discounted risk neutral probabilities:

$$p = \frac{r-d}{u-d} = \frac{1.05 - .8}{1.2 - .8} = 0.625\,/1.05 = 0.595\,24$$
$$1 - p = 1 - 0.625 = 0.375/1.05 = 0.35714$$

and the calculation proceeds as under (b).

10. (a) Since $u \neq r \neq d$ the stock and risk free debt are linearly independent, so there are as many independent securities as there are states of the world, which means that the market is complete. Since $d < r < u$ there are no arbitrage opportunities on this market.

(b) We first calculate the value tree for the stock without and with dividends:

$$\begin{array}{ccccc} & & & & 225 \\ & & & \nearrow & \\ & & 150 & & \\ & \nearrow & & \searrow & \\ 100 & & & & 75 \\ & \searrow & & \nearrow & \\ & & 50 & & \\ & & & \searrow & \\ & & & & 25 \end{array} \qquad \begin{array}{ccccc} & & & & 168.75 \\ & & 150 & \nearrow & \\ & \nearrow & 112.5 & & \\ 100 & & & \searrow & \\ & \searrow & & & 56.25 \\ & & 50 & \nearrow & \\ & & 37.5 & & \\ & & & \searrow & \\ & & & & 18.75 \end{array}$$

The parameters of the binomial process are: $u = 1.5$, $d = 0.5$, $r = 1.25$, $p = (r-d)/(u-d) = 0.75$

i. European call, no dividends:
$O_{uu} = \max[0, 225 - 110] = 115$
$O_{ud} = \max[0, 75 - 110] = 0$
$O_{dd} = \max[0, 25 - 110] = 0$
$O_u = (.75 \times 115) + (.25 \times 0)/1.25 = 69$
$O_d = (.75 \times 0) + (.25 \times 0)/1.25 = 0$
$O = (.75 \times 69) + (.25 \times 0)/1.25 = 41.4$

ii. European call, 25% dividends:
$O_{uu} = \max[0, 168.75 - 110] = 58.75$
$O_{ud} = \max[0, 56.25 - 110] = 0$
$O_{dd} = \max[0, 18.75 - 110] = 0$
$O_u = (.75 \times 58.75) + (.25 \times 0)/1.25 = 35.25$
$O_d = (.75 \times 0) + (.25 \times 0)/1.25 = 0$
$O = (.75 \times 35.25) + (.25 \times 0)/1.25 = 21.15$

iii. American call on a non dividend paying stock is the same as a European call

iv. American call, 25% dividend:
$O_{uu} = \max[0, 168.75 - 110] = 58.75$
$O_{ud} = \max[0, 56.25 - 110] = 0$
$O_{dd} = \max[0, 18.75 - 110] = 0$
$O_u - alive = (.75 \times 58.75) + (.25 \times 0)/1.25 = 35.25$
$O_u - dead = \max[0, 150 - 110] = 40$
$O_u = \max[alive, dead] = 40$
$O_d = 0$ both dead and alive
$O - alive = (.75 \times 40) + (.25 \times 0)/1.25 = 24$
$O - dead = \max[0, 100 - 110] = 0$
$O = \max[alive, dead] = 24$

v. European put, no dividends:
$O_{uu} = \max[0, 110 - 225] = 0$
$O_{ud} = \max[0, 110 - 75] = 35$
$O_{dd} = \max[0, 110 - 25] = 85$
$O_u = (.75 \times 0) + (.25 \times 35)/1.25 = 7$
$O_d = (.75 \times 35) + (.25 \times 85)/1.25 = 38$
$O = (.75 \times 7) + (.25 \times 38)/1.25 = 11.80$

vi. European put, 25% dividends:
$O_{uu} = \max[0, 110 - 168.75] = 0$
$O_{ud} = \max[0, 110 - 56.25] = 53.75$
$O_{dd} = \max[0, 110 - 18.75] = 91.25$
$O_u = (.75 \times 0) + (.25 \times 53.75)/1.25 = 10.75$
$O_d = (.75 \times 53.75) + (.25 \times 91.25)/1.25 = 50.50$
$O = (.75 \times 10.75) + (.25 \times 50.50)/1.25 = 16.55$

11. Next period the stock price is either $50 \times 1.3 = 65$ or $50 \times .75 = 37.5$. The value trees for the options are:

$$\begin{array}{lll} & & \max[0, 65 - 45] = 20 \\ & \nearrow & \\ O_c & & \\ & \searrow & \\ & & \max[0, 37.5 - 45] = 0 \end{array} \qquad \begin{array}{lll} & & \max[0, 45 - 65] = 0 \\ & \nearrow & \\ O_p & & \\ & \searrow & \\ & & \max[0, 45 - 37.5] = 7.5 \end{array}$$

(a) For a call option, the option Δ and D are:

$$\begin{aligned} \Delta &= \frac{O_u - O_d}{(u-d) \times S} = \frac{20 - 0}{(1.3 - 0.75) \times 50} = 0.72727 \\ D &= \frac{u \times O_d - d \times O_u}{(u-d) \times r} = \frac{1.3 \times 0 - 0.75 \times 20}{(1.3 - 0.75) \times 1.08} = -25.253 \end{aligned}$$

The price of the call is $O_c = S \times \Delta + D = 50 \times 0.72727 - 25.253 = 11.111$

(b) For a put option, the option Δ and D are:

$$\begin{aligned}
\Delta &= \frac{O_u - O_d}{(u-d)\times S} = \frac{0-7.5}{(1.3-0.75)\times 50} = -0.272\,73 \\
D &= \frac{u\times O_d - d\times O_u}{(u-d)\times r} = \frac{1.3\times 7.5 - 0.75\times 0}{(1.3-0.75)\times 1.08} = 16.414
\end{aligned}$$

The price of the put $O_P = S\times\Delta + D = 50\times -0.27273 + 16.414 = 2.7775$

12. The self-financing property means that, when the portfolio is rebalanced, the new portfolio has exactly the same value as the old one. So we could sell the old portfolio and use the money to buy the new one without needing any additional cash. In the two-period example on page 212-213 we start with a hedging portfolio of 0.753 shares and 212.11 in borrowing. If the stock price rises to 500 at t_1 the value of this old portfolio is:

$$0.753\times 500 + (-212.11\times 1.07) = 149.54$$

The amount of borrowing has increased with the risk free interest rate over the period. With this stock price the new hedge ratio becomes 1, so we need to buy $1-0.753 = 0.247$ of the stock at a price of 500, which costs $0.247\times 500 = 123.50$. We borrow this amount, so the value of the new portfolio is:

$$1\times 500 - (123.50 + 212.11\times 1.07) = 149.54$$

exactly the same as the old portfolio.
If the stock price falls to 320 at t_1 the value of the old portfolio is:

$$0.753\times 320 + (-212.11\times 1.07) = 14$$

With this stock price the new hedge ratio becomes 0.174, so we have to sell $0.753 - 0.174 = 0.579$ of the stock at a price of 320. This brings in $0.579\times 320 = 185.28$ and we use this amount to reduce the borrowing. The value of the new porfolio thus becomes:

$$0.174\times 320 - ((212.11\times 1.07) - 185.28) = 14$$

again exactly the same as the old porfolio.

Chapter 8: Black and Scholes option pricing

Exercises

1. It is sometimes said that if the current share price equals the exercise price, then call options on the share have a 50% chance of ending up worthless (if the share price falls) and a 50% chance of ending up "in the money" (if the share price rises). Explain in simple terms or with an example why this statement is or is not correct.

2. A stock has an annual volatility (standard deviation) of 34%. Calculate the standard deviation of the daily return. Assume that a year has 252 trading days and that the returns are independently and identically distributed (iid).

3. In the discussion of the properties of log returns we made the assumption that these returns are independently and identically distributed, which means that they follow a random walk. Does this mean that stock prices also follow a random walk?

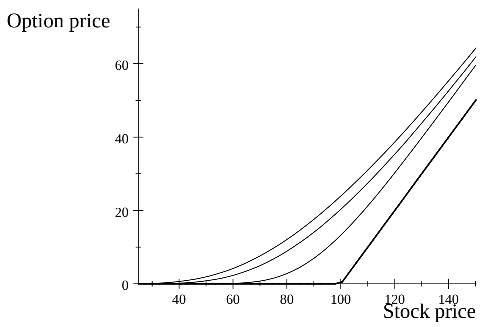
4. The iid assumption means that the distribution of stock returns is stable over time. Does this stability mean that past stock returns can be used to predict future ones?

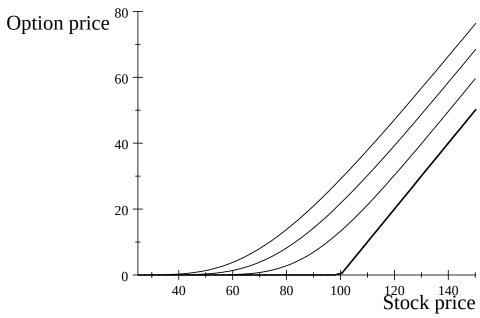
5. Explain why the value of options increases with the volatility of the underlying.

6. Explain why the value of a call increases with the risk free interest rate while the value of a put decreases.

7. Use the put-call parity to work out a relation between the Greek 'delta' of puts and calls.

8. The graphs below (copies from the main text) plot call option prices for different values of their determinants. All options have an exercise price of 100, a risk free interest rate of 10% and are written on a stock that pays no dividends. The graph on the left plots options with a time to maturity of 1 year and three different volatilities: $\sigma = .5$ (top), $.4$ and $.2$ (bottom). The graph on the right plots options with a volatility of $.2$ and three different maturities: $T = 3$ (top), 2 and 1 (bottom). Explain briefly why the options with different volatilities converge to a common value as the stock price increases and why the options with different maturities do not converge to a common value as the stock price increases.





9. On a financial market a stock is traded at a price of €240. The stock has an annual volatility of 25%. Call options on the stock with an exercise price of €250 and a time

to maturity of 1 year are also traded. The risk free interest rate is 6%. Calculate the price of the option.

10. Suppose the price of a share Norske Skog at some point in the future is NOK 100. Over each of the next two periods of half a year, the price can either increase with 7.5% or decrease with 7%, corresponding to a yearly standard deviation of 10.228%. After the first half year, Norske Skog pays out a dividend of NOK 10. The 6 months risk free rate is 2.5%, so slightly over 5% per year.

    (a) Calculate the value of an American call option on the stock, that matures in 1 year and has an exercise price of 102.5

    (b) Calculate the hedge portfolio of the option for the first half year period and show that it gives a perfect hedge.

    (c) Explain in general terms how the call option delta changes as the stock prices changes.

11. The following information on option prices, stock prices and interest rates was published in Finansavisen (a Norwegian financial newspaper) of 5 Sept. 2005.

Option price quotes

			call option		put option	
Ticker	T	X	bid	ask	bid	ask
NHY	nov.5	620	68.00	70.00	9.25	10.00
"	"	680	28.75	30.25	29.00	31.25
"	feb.6	620	82.00	83.75	19.00	20.75
"	"	680	44.00	47.00	41.50	44.25
ORK	jan.6	240	25.50	26.75	5.00	5.50
NSG	dec.5	100	10.00	11.00	2.85	3.35

Stock price quotes

	stock price	
Ticker	bid	ask
NHY	677.00	678.50
ORK	259.00	259.50
NSG	108.75	109.50

NIBOR rates	
1 month	2.180
2 month	2.235
3 month	2.290
4 month	2.313
5 month	2.337

NIBOR is the Norwegian InterBank Offer Rate that can be used as the yearly risk free interest rate for the different maturities. Ask prices are prices at which a dealer is willing sell, bid prices at prices at which a dealer is willing to buy. The ticker marks are for Norsk Hydro (NHY), Orkla (ORK) and Norske Skog (NSG).

    (a) Is there any sign of mispricing on the market? If so, how would you profit from it?

    (b) Is there an alternative explanation for price differences, if you find any?

Chapter 8: Black and Scholes option pricing

Exercises - solutions

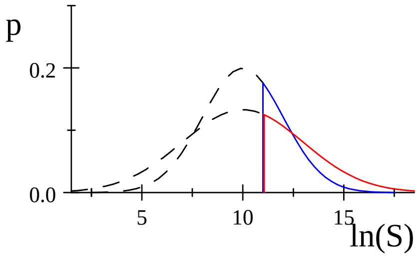
1. The statement is not correct. The share price is expected to increase during the option's life time, whereas the exercise price remains constant. So if the current share price equals the exercise price, the expected share price at maturity will be larger than the exercise price. The growth rate of the stock depends on the probability measure that is used to model stock prices. Under the equivalent martingale measure, returns are equalized into the risk free interest rate, so that the share grows with r_f. The example in section 8.3.3 of the book illustrates this case. It values an at the money call option on a non-dividend paying share with a current price of €100 and a volatility of 20%. The option's time to maturity is 1 and the risk free interest rate is 10%. The calculations in the book (easily verified with the option price calculator accompanying the book) show that the equivalent martingale probability that the option will be exercised, or $N(d_2)$, is 0.655. Under the real probability measure the share will grow with μ, the expected, continuously compounded return of the share.

2. Under the iid assumption, variance increases with time, so that standard deviation increases with the square root of time. An annual standard deviation of 34% thus gives a daily standard deviation of $\sigma\sqrt{T} = 0.34\sqrt{1/252} = 0.02142$ or 2.14%. In section 3.2.2 (Home-made portfolio optimization) we used the same procedure 'in reverse' to calculate the annual volatility of the stocks in uncle Bob's portfolio from their daily returns.

3. As was pointed out by Fama, if *stock returns* are iid, then *stock prices* will not follow a random walk since price changes will depend on the price level.

4. Stability over time means that past returns are the best information to assess the distributional properties of the returns. However, they *cannot* be used the predict future returns because they contain no information on the *sequence* of future returns.

5. Volatility increases the probability of large price changes, both price increases and price decreases. For stockholders these price movements will tend to cancel out. But option holders have an exposure to only one of these price movements: call holders to price increases and put holders to price decreases. For option holders it doesn't matter how far out of the money the option ends, the option is worthless if it ends out of the money. So options profit from the upward potential of price movements but have a limited exposure to the downside risk, hence their values increase with volatility. This is illustrated in the graph below with the truncated normal distribution we used in the derivation of the Black and Scholes formula. The truncated distribution with the higher standard deviation has a higher expectation than the one with a lower standard deviation (14.5 vs. 11.8).



Lognormally distributed stock prices ($ln(S) \backsim N(10, 2 \text{ and } 3)$, dashed), and their left truncations at $ln(S) = 11$ (solid)

6. Holders of a long call have a claim on the upward potential of stock, but they don't pay for it until maturity, if at all (the option may also expire out of the money). The possibility to delay payment is more valuable the higher the interest rate is (the holder can earn interest over the exercise price). Holders of a long put can sell the stock at the fixed exercise price in the future. The value of a future payment decreases with the interest rate.

7. The put-call parity states:
$$put = call + PV(X) - S$$
If the stock price increases with 1, the call increases (by definition) with $\Delta_c \times 1 = \Delta_c$. The $PV(X)$ is unaffected by changes in the stock price. So the right hand side of the equation changes with
$$\Delta_c \times 1 + 0 - 1 = \Delta_c - 1$$
This must be the delta of the put: $\Delta_p = \Delta_c - 1$.

8. Economically, holding an extremely far in the money call option is equivalent to holding a share that is not yet paid for, i.e. $S - PV(X)$. Differences in volatility (practically) do not matter any more as the options are (almost) certain to be exercised anyway. The options in left hand figure are paid for on the same date, so their PV(X) are the same and, consequently, they have a common value as a function of the stock price. The options in right hand figure are not paid for on the same date, so their PV(X) are different and they do not have a common value as a function of the stock price.
Technically, as the stock price S get larger and larger, both the option delta N(d_1) and the probability of exercise N(d_2) approach 1. The Black and Scholes price then approaches $O_{c,0} = S_0 - Xe^{-rT}$. This price is independent of σ, but not of T, hence options with different volatilities converge to a common value, but options with different maturities do not.

9. We use the Black and Scholes option pricing formula:
$$O_{c,0} = S_0 \times N(d_1) - X \times e^{-rT} \times N(d_2)$$
with
$$d_1 = \frac{\ln(S_0/X) + (r + \frac{1}{2}\sigma^2) \times T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$
The input data are: $S = 240$, $X = 250$, $\sigma = 25\%$, $r = 6\%$, $T = 1$
$$d_1 = \frac{\ln(240/250) + (0.06 + 0.5 \times 0.25^2) \times 1}{0.25 \times \sqrt{1}} = 0.20171$$
$N(d_1) \rightarrow \text{NormalDist}(0.20171) = 0.579\,93$
$$d_2 = 0.20171 - 0.25 \times \sqrt{1} = -0.04829$$

$N(d_2) \rightarrow \text{NormalDist}(-0.04829) = 0.480\,74$

$$O_{c,0} = 240 \times 0.57993 - 250 \times e^{-0.06\times 1} \times 0.48074 = 25.997$$

10. (a) The parameters of the binomial process are:

$$u = 1.075 \qquad d = .93 \qquad r = 1.025 \qquad p = \frac{1.025 - .93}{1.075 - .93} = .655$$

With these parameter the binomial tree can be constructed as in Lattice 1. Note that the tree no longer recombines after a fixed amount of dividend payments. The option values are obtained by calculating their values at maturity, taking their risk neutral expectation and discounting this back in time with the risk neutral rate. This gives the values of the option alive, and since this is an American option the values alive have to be compared with the values dead, i.e. if exercised. Lattice 2 gives the results, the calculations are below.

$$\begin{array}{ccccc}
 & & 107.5 & & 104.8\\
 & \nearrow & \downarrow & \nearrow & \\
 & & 97.5 & & \\
100 & & & \searrow & \\
 & \searrow & & & 90.7\\
 & & 93 & & 89.2\\
 & & \downarrow & \nearrow & \\
 & & 83 & & \\
 & & & \searrow & \\
 & & & & 77.2\\
\textit{t=0} & & \textit{t=1} & & \textit{t=2}
\end{array}$$

Lattice 1 Value NSG stock

The options only ends in the money in the upper node at *t=2*. Its value at expiration in that node is

$$104.8 - 102.5 = 2.3$$

This gives a *t=1* value alive of:

$$(.655 \times 2.3)/1.025 = 1.47$$

The value dead is $107.5 - 102.5 = 5$ so the option should be exercised which gives a *t=0* value of

$$(.655 \times 5)/1.025 = 3.2$$

$$\begin{array}{ccccc}
 & & & & O_{uu} = 2.3\\
 & & & \nearrow & \\
 & & O_u = \max[\underbrace{(1.47)}_{alive}, \underbrace{\max[0, (107.5 - 102.5)]}_{dead}] = 5 & & \\
 & \nearrow & & \searrow & \\
3.2 & & & & O_{ud} = 0\\
 & & & & O_{du} = 0\\
 & \searrow & & \nearrow & \\
 & & O_d = \max[\underbrace{(0)}_{alive}, \underbrace{\max[0, (93 - 102.5)]}_{dead}] = 0 & & \\
 & & & \searrow & \\
 & & & & O_{dd} = 0\\
\textit{t=0} & & \textit{t=1} & & \textit{t=2}
\end{array}$$

Lattice 2 Value call option on NSG stock

(b) To construct the hedge portfolio we first calculate Δ and D:

$$\Delta = \frac{O_u - O_d}{S_u - S_d} = \frac{5 - 0}{107.5 - 93} = .345$$

$$D = \frac{uO_d - dO_u}{(u - d)r} = \frac{1.075 \times 0 - .93 \times 5}{(1.075 - .93)1.025} = -31.287$$

As always, the hedging portfolio for a call is a levered long position in the stock. We sell the option at 3.2 and we buy the hedge portfolio to cover the obligations from the call. The fraction of the stock costs: $.345 \times 100 = 34.5$. We borrow 31.287 and the difference of $34.5 - 31.287 = 3.2$ is covered by the received premium from selling the call. If at *t=1*:

i. The stock price is 107.5: the option will be exercised and we have to sell the stock at the exercise price of 102.5. We have .345 stock in the hedge portfolio, so we have to buy $1 - .345 = .665$ stock, which costs $.665 \times 107.5 = 70.41$. Hence, we receive $102.5 - 70.41 = 32.09$. This is exactly enough to pay off the debt, which now amounts to $1.025 \times 31.287 = 32.07$. We have a perfect hedge.

ii. The stock price is 93: the option is worthless. The debt amounts to the same 32.07, which is exactly covered by the fraction of the share: $.345 \times 93 = 32.09$. Again, we have a perfect hedge.

(c) The call option delta increases with the stock price, all other things equal. As the stock price increases, the option becomes more likely to be exercised and becomes more like a stock that has not yet been paid for. Ultimately, when the option is so far in the money that it is certain to be exercised the call option delta becomes 1. Conversely, when the stock price falls and the call is farther and farther out of the money, it becomes less and less likely to be exercised. Ultimately, when the option is so far out of the money that it is certain not to be exercised the call option delta becomes 0. The call has lost its value, no matter what happens to the stock price. This is the case in the lower at *t=1* in Lattice 2: the option cannot get in the money from this node and is worthless and no longer sensitive to stock price changes.

11. (a) We use the put-call parity to construct a synthetic put and check for any mispricing:

$$long\ put = long\ call + PV(X) - share\ price$$

First we calculate the PV(X) using the appropriate NIBOR rate and period:
NHY nov.5: $620e^{-.02235 \times 2/12} = 617.69$
NHY nov.5: $680e^{-.02235 \times 2/12} = 677.47$
NHY feb.6: $620e^{-.02337 \times 5/12} = 613.99$
NHY feb.6: $680e^{-.02337 \times 5/12} = 673.41$
ORK jan.6: $240e^{-.02313 \times 4/12} = 238.16$
NSG dec.5: $100e^{-.0229 \times 3/12} = 99.429$
We construct a synthetic put by buying a call, putting PV(X) in the bank and selling the share. So we use the ask price for the call and the bid price for the share.

Synthetic and market option prices

Ticker	T	X	call ask	PV(X)	share bid	syn. put	put option bid	put option ask
NHY	nov.5	620	70.00	617.69	677.00	10.69	9.25	10.00
„	„	680	30.25	677.47	677.00	30.72	29.00	31.25
„	feb.6	620	83.75	613.99	677.00	20.74	19.00	20.75
„	„	680	47.00	673.41	677.00	43.41	41.50	44.25
ORK	jan.6	240	26.75	238.16	259.00	5.91	5.00	5.50
NSG	dec.5	100	11.00	99.429	108.75	1.68	2.85	3.35

The rows for NHY nov.5-620, ORK jan.6 and NSG dec.5 in Table 4 show that the prices of the synthetic puts for these options lie outside the bid-ask spread on the market. But the first two are not arbitrage opportunities: you can buy a synthetic put NHY nov.5-620 at 10.69 or an ordinary put at 10, but you cannot sell at 10.69, only at 9.25. The same applies to ORK jan.6. They are outside the bid-ask spread, but on wrong side from an arbitrage point of view. The NSG option is outside the bid-ask spread on the other side and that offers an arbitrage opportunity: we can buy the synthetic put at 1.68 and sell the ordinary put at 2.85. This gives an arbitrage profit of 1.17. If we manage to close a million of these contracts we have become millionaires overnight.

Such arbitrage opportunities are in practice not open to investors who get their information from a newspaper and on closer examination it appears that we made a typing error in Table 2: the bid-ask prices for NSG are 106.75 and 107.5. With the proper stock price, the price of the synthetic put becomes:

call + PV(X) - share = 11+99.429-106.75=3.679

The price of the synthetic put now lies outside the bid-ask spread on the other side and the arbitrage opportunity has disappeared. We can make sure by checking the relation the other way around by constructing a synthetic short put. Then we write a call, borrow pv(X) and buy the share. That costs: -10 + (-99.429) + 107.5 = -1.929 i.e. brings in 1.929. Note that we use the bid price for the call and the ask price for the share. If we sell the put on the market we get 2.85, so the synthetic price is on the wrong side of the bid-ask spread from a an arbitrage point of view.

(b) Differences between implied and observed prices can occur because we apply the put-call parity, which is only valid for European options on non-dividend paying stocks, to traded American options on stocks that may pay dividends. Further, price differences can occur because of nonsynchronous trading. The prices in newspapers are generally closing prices, but we do not know when the last trade of the day took place. If the last option trade was at 13.00 hours and the last stock trade at 15.00 hours, the option trade could be based on a different stock price than the one we read in the newspaper.

Chapter 9: Real options analysis

Exercises

1. As an avid beer brewer, you invented a brewing process that allows you switch from malted barley to malted wheat and back again without significant cost. You think this could be an important commercial advantage and you ask your assistant to collect and analyse data on the prices of barley and wheat. Which price behaviour of barley and wheat would make the possibility to switch valuable? Be specific in with regard to price level, volatility and correlation.

2. Discuss the following cases from a (real) options point of view:

   (a) In an article on the financial crisis published in The Guardian of October 16 2008, the well known economist Joseph Stiglitz compared the terms of the rescue packages for banks provided by investor Warren Buffet and the US government: "Buffett got a warrant (=option) - the right to buy in the future at a price that was even below the depressed price at the time. Paulson got for the US a warrant to buy in the future - at whatever the prevailing price at the time."

   (b) Norsk Hydro considers to expand its aluminium production by installing an aluminium smelter. Smelter installations use standard technology that is available on the market in any quantity. The produced aluminium is a bulk product that is sold in commodity markets by large numbers of producers.

   (c) The presence of vacant plots of land in city centres.

   (d) Hewlett-Packard decided to adapt its inkjet printers to local languages in local depositories. Although adaptation in central production facilities is much cheaper, HP's decision proved to be very profitable.

3. In the section about bounds on option prices we saw that American call options on non-dividend paying stocks are never exercised before maturity (bound 9). Explain why real options with a call nature are often exercised early.

4. State Drilling AG has a concession that gives it the right to develop a small natural gas field. The concession expires in one year. The field will produce 100 million m^3 gas per year for 4 years. The investment required to develop the field is \$30 million, to be paid immediately. If development is postponed the investment amount will increase over time with the risk free interest rate. Production and cash flows from selling the gas will start 1 year after the investment. The current gas price is \$0.08 per m^3 but that price develops over time according to a binomial process: after each period the price can go up with 25% or down with 20%. The probability of an upward movement in price is 80%. The risk adjusted discount rate for cash flows from gas production from the field is 16% and the risk free interest rate is 7%.

   (a) What is the value of the gas reserve in the field?

   (b) What is the value of the opportunity to develop the field and when should it be developed?

5. The well known investor Peter Smalldale owns a chain of hotels. He plans to expand his activities into a city that has announced its candidacy to host the European football championship 2 years from now. If the city gets the championship, the new hotel's (and the city's) reputation will be made and it will generate a perpetual cash flow of €20 million per year. However, if the city does not get the championship, the hotel will fall into oblivion and the perpetual cash flow will only be €10 million per year. The football association will decide 1 year from now where the championship is to be held. There is general agreement among insiders that the probability that the city in question will get the championship is 37.5%. Peter Smalldale has an offer from CCI, Construction Consortium Inc. to build the hotel for a price of €120 million, to be paid when the offer is accepted. Construction will start immediately and the hotel will be ready 2 years from now, in time for the championship. However, the offer is only valid if accepted immediately.

   (a) If the proper discount rate for cash flows and values from the hotel is 10%, what is the NPV of the project if it is accepted immediately?

   Smalldale wants to postpone his investment decision until it becomes known where the championship will be held. CCI can build the hotel in 1 year for the same price of €120 million by allocating a double work force to the project. However, CCI's director followed a course in finance for science and technology students and anticipates that Smalldale will cancel the project if the championship goes to another city. To compensate for the profit that CCI misses if the project is cancelled, it asks an immediate payment of €12 million. When that payment is made, CCI is willing to build the centre in 1 year, starting 1 year from now, for €120 million (to be paid at the start).

   (b) If the risk free rate is 5%, is postponement profitable for Smalldale with this offer from CCI? Calculate the increase or decrease in project value compared with the now-or-never project under (a).

   (c) What is the risk adjusted discount rate for the project opportunity under (b)?

6. In June 2007 the Norwegian company Aker ASA signed a cooperation agreement with the Norwegian Ministry of Trade and Industry and the Swedish Wallenberg group. The cooperation was organized in a new company, Aker Holding, in which Aker ASA held 60% of the shares, the Norwegian state 30% and the Wallenberg group 10%. When the agreement was signed, the total value of the shares was 16 billion (10^9) Norwegian kroner. The agreement stipulated that Aker ASA and the Norwegian state would hold their position in Aker Holding for at least ten years, but the Wallenberg group had the right to sell its shares to Aker ASA after four years at their original price plus a return of 10%. In addition, Aker ASA had the right to buy the Wallenberg group's shares after four years at their original price plus a return of 40%. (Data are based on a Press Release from Aker ASA dated 22 June 2007.)

   (a) Describe the position of the Wallenberg group when the deal was signed in terms of option positions. Be precise in style, moneyness and maturity.

   (b) Assuming no dividends, a risk free interest rate of 5% per year, and an annual volatility of Aker Holding returns of 20%, what was the value of the Wallenberg group's right to sell its shares in Aker Holding when the deal was signed?

   (c) Using the same assumptions, what was the value of Aker ASA's right to buy the Wallenberg group's shares when the deal signed?

(d) On January 1 2011 the value of Aker Holding had dropped to 12.5 billion. What was the value of the Wallenberg group's re-sell right then? Assume 6 months to maturity and that the interest rate and volatility remain the same.

7. ShortSight Ltd. is the investment vehicle of three optometrists. It specializes in short term optometry projects of the now or never type. It came across the following project: by investing 10 now (t_0) in the design of spectacles, it can let subcontractors produce a party of spectacles which will be ready 1 period later at t_1. The spectacles will be sold immediately after delivery. The production costs are 67, to be paid upon delivery at t_1. The price of the spectacles now (at t_0) is 75, but that price will increase over the next period with $\frac{2}{3}$ if the design is well received in the market. On the other hand, if the design is not well received, the price will decrease with $\frac{1}{3}$ over the next period. Both possibilities have equal probabilities. The risk free interest rate is 10%.

   (a) What is the value of the project if accepted completely on a now or never basis?

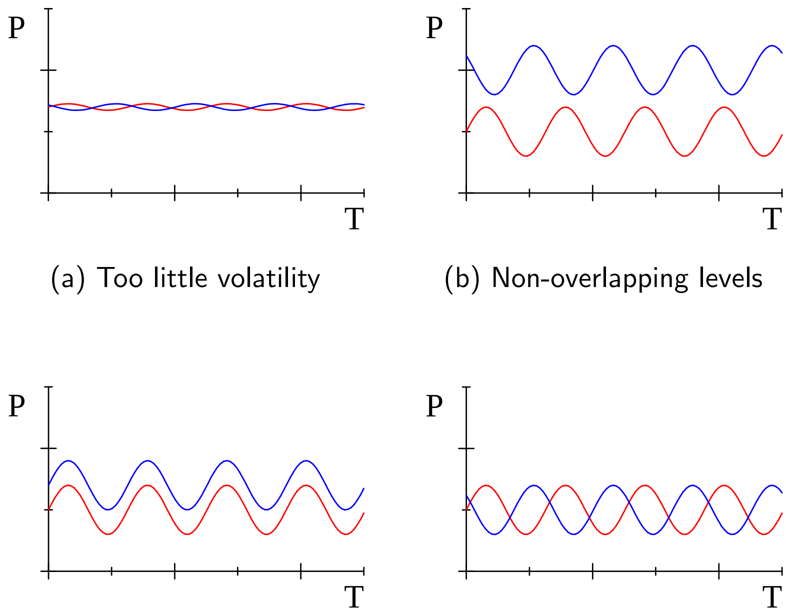
   A newcomer at ShortSight wants to introduce long term thinking by postponing the production decision to next period.

   (b) What is the value of the project if the production decision is postponed with 1 period? Assume that the t_1 prices remain the same in t_2 and that the production costs remain constant at 67.

Chapter 9: Real options analysis

Exercises- solutions

1. The prices have to be volatile, not strongly positively correlated and not on very different levels, so that intermittently one is significantly cheaper than the other. The various possibilities are schematically depicted in Figure 1. The option to switch is only valuable if the prices of barley and wheat behave as in panel (d); the option has little or no value if the price patterns are as in panel (a), (b) and (c).



(a) Too little volatility (b) Non-overlapping levels

(c) Too highly correlated (d) High switch value

Figure 1: Prices of barley (red) and wheat (blue) over time (T)

2. (a) The option to buy or sell at market prices has no value, everybody can do that. Buffett got a valuable option, the US did not. In an op-ed article in the New York Times of Oct. 17, 2008 Warren Buffett encouraged investors to but American stocks ("Buy American. I am"). Increased demand would make his options, that were already in-the-money, even more valuable.

   (b) If both the input market (smelters) and the output market (bulk aluminium) are characterized by perfect competition (large numbers of buyers and sellers, none of which can individually influence prices), Norsk Hydro has no lower exercise price nor a higher underlying value than its competition so that there is no source of real option value.

(c) City centre building plots are very valuable and by developing the plot, *the option to develop* is given up. If one development is chosen (say, offices), alternative developments (say, a hotel) are given up and the latter may prove to be very valuable in the near future. The option to develop may be more valuable than any particular development today.

(d) By assembling locally, HP introduced the flexibility to adapt to variations in local demand, e.g. to increase the number of French printers and decrease the German number. If demand is volatile, flexible production (although by itself more expensive) can be more profitable than other methods to match supply and demand (e.g. maintaining large buffer stocks).

3. Real call options are often exercised before maturity because the sources of option value tend to be eroded over time. This is plain to see in a patent that has a limited life time and has to be used (= exercised) before it expires if its value is to be realized. The value of patents and other real options is further eroded over time because competitors will develop close substitutes. Also, if the option is shared in some degree, the game-theoretic anticipation of competitors' actions generally leads to an earlier exercise.

4. (a) The value of the gas reserve is $4 \times 100 \times 0.08 = \$32$ million. The cash flows from gas production occur on 4 future points in time, but both the expected return on gas and the time value of money are included in the binomial process. Feel free to check this using either the risk neutral probabilities and risk free rate or the real probabilities and the risk adjusted rate.

   (b) If the field is developed immediately, its NPV is 32-30=\$2 million. The concession gives State Drilling the flexibility to wait and see for 1 year. After 1 year, the investment amount is $30 \times 1.07 = 32.1$. The gas price is either $0.08 \times 1.25 = 0.1$ or $0.08 \times 0.8 = 0.064$, so the value of the gas reserve is either $4 \times 100 \times 0.1 = 40$ or $4 \times 100 \times 0.064 = 25.6$. Of course, the field will not be developed if the gas price becomes \$0.064, so the opportunity to develop the field in one year is $\max[0, 40 - 32.1] = 7.9$ in the up state and $\max[0, 25.6 - 32.1] = 0$ in the down state. The risk neutral probability that the up state occurs is $(r-d)/(u-d) = (1.07-0.8)/(1.25-0.8). = 0.6$. So the value of the opportunity to develop the field is $(0.6\times 7.9)/1.07 = 4.4299$ This is higher than the value if developed immediately, so development should be postponed and the value of the opportunity to develop the field is the option value, \$4.43 million.

5. (a) We can calculate the value of the inflexible project with the real probabilities and the risk adjusted discount rate. The project will generate either €20 or €10 million per year, starting 2 years from now at t=2. The t=1 value of this perpetual cash flow is either 20/0.1=200 or 10/0.1=100

$$\begin{array}{lll} & & Value = 20/0.1 = 200 \quad cfl = 20 \\ & \nearrow & \\ 125 & & \\ & \searrow & \\ & & Value = 10/0.1 = 100 \quad cfl = 10 \\ I = -120 & & \\ t_0 & t_1 & t = 2, \infty \end{array}$$

   the real probabilities are 0.375 and 0.625 so the t=0 value is

$$\frac{0.375 \times 200 + 0.625 \times 100}{1.1} = 125$$

   and the NPV is 125-120=€5 million.

(b) With the flexible project, Smalldale can decide 1 year from now whether to build the centre or not, when the football association's decision is known. Of course, he will only invest if the project is profitable and abandon the project if it is not profitable.

$$\begin{array}{lll}
 & Value = \max[(200-120),0] = 80 & cfl = 20 \\
 & \nearrow & \\
23.81 & & \\
 & \searrow & \\
 & Value = \max[(100-120),0] = 0 & cfl = 10 \\
I = -12 & I = -120 & \\
t_0 & t_1 & t = 2, \infty
\end{array}$$

To value the flexible project, we use the risk neutral probabilities and the risk free interest rate. Given the value of the inflexible project of 125, the up-factor is $200/125 = 1.6$ and the down-factor is $125/100 = 0.8$, so that $p = (1.05 - 0.8)/(1.6 - 0.8) = 0.3125$ and $1 - p = 0.6875$. The t=0 value is

$$\frac{0.3125 \times 80 + 0.6875 \times 0}{1.05} = 23.81$$

The NPV is thus $23.81 - 12 = 11.81$. So postponement is profitable, the project value increases with $11.81 - 5 = 6.81$.

(c) The risk adjusted discount rate can be calculated from the replicating portfolio. The ingredients of this portfolio are calculated with Δ and D:

$$\Delta = \frac{O_u - O_d}{S_u - S_d} = \frac{80 - 0}{200 - 100} = 0.8$$

and

$$D = \frac{uO_d - dO_u}{(u - d)r} = \frac{1.6 \times 0 - 0.8 \times 80}{(1.6 - 0.8) \times 1.05} = -76.19$$

So the replicating portfolio consists of $0.8 \times 125 = 100$ in the inflexible project and a loan of 76.19. The weighted average return of this portfolio is

$$\frac{100}{100 - 76.19} \times 0.10 + \frac{-76.19}{100 - 76.19} \times 0.05 = 0.26$$

A shorter calculation is solving:

$$\frac{0.375 \times 80 + 0.625 \times 0}{r} = 23.81$$

for r, which gives the same risk adjusted rate of 26%.

6. (a) The Wallenberg group held a long position in the shares of Aker Holding plus a long position in in-the-money European put options and a short position in out-of-the money European call options on these shares, both with a time to maturity of four years. It is a 'share plus protective put' position, plus a short call position. The combined result is that the Wallenberg group is guaranteed to earn at least 10% over four years, but cannot earn more than 40%.

(b) The Wallenberg group held 10% of 16 billion, or 1.6 billion and had the right to sell it for $1.6 \times 1.1 = 1.76$ billion. The inputs for the calculation of the option's value are: $S_0 = 1.6$, $X = 1.76$, $\sigma = 0.2$, $T = 4$ and $r = 0.05$. The Black and Scholes formula then gives:

$$O_{p,0} = Xe^{-rT}N(-d_2) - S_0 N(-d_1)$$

with

$$d_1 = \frac{\ln(S_0/X) + (r + \frac{1}{2}\sigma^2) \times T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln(1.6/1.76) + (0.05 + 0.5 \times 0.2^2) \times 4}{0.2 \times \sqrt{4}} = 0.46172$$

$N(-d_1) \to \mathrm{NormalDist}(-0.46172) = 0.32214$

$$d_2 = 0.46172 - 0.2 \times \sqrt{4} = 0.06172$$

$N(-d_2) \to \mathrm{NormalDist}(-0.06172) = 0.47539$

$$O_{p,0} = 1.76 \times e^{-0.05\times 4} \times 0.47539 - 1.6 \times 0.32214 = 0.1696$$

or 169.6 million Norwegian kroner.

(c) The call option position is valued along similar lines. The exercise price is $1.6\times 1.4 = 2.24$ so that the inputs for the calculation of the option's value are: $S_0 = 1.6$, $X = 2.24$, $\sigma = 0.2$, $T = 4$ and $r = 0.05$. The Black and Scholes formula then gives:

$$O_{c,0} = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

with

$$d_1 = \frac{\ln(S_0/X) + (r + \frac{1}{2}\sigma^2) \times T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln(1.6/2.24) + (0.05 + 0.5 \times 0.2^2) \times 4}{0.2 \times \sqrt{4}} = -0.14118$$

$N(d_1) \to \mathrm{NormalDist}(-0.141\,18) = 0.44386$

$$d_2 = -0.14118 - 0.2 \times \sqrt{4} = -0.54118$$

$N(d_2) \to \mathrm{NormalDist}(-0.54118) = 0.29419$

$$O_{c,0} = 1.6 \times 0.44386 - 2.24 \times e^{-0.05\times 4} \times 0.29419 = 0.17064$$

or 170.64 million Norwegian kroner.

(d) The inputs for the calculation of the option's value on January 1 2011 are: $S_0 = 1.25$, $X = 1.76$, $\sigma = 0.2$, $T = 0.5$ and $r = 0.05$. The Black and Scholes formula then gives:

$$O_{p,0} = Xe^{-rT} N(-d_2) - S_0 N(-d_1)$$

with

$$d_1 = \frac{\ln(S_0/X) + (r + \frac{1}{2}\sigma^2) \times T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln(1.25/1.76) + (0.05 + 0.5 \times 0.2^2) \times 0.5}{0.2 \times \sqrt{0.5}} = -2.172$$

$N(-d_1) \rightarrow \text{NormalDist}(2.172) = 0.98507$

$$d_2 = -2.172 - 0.2 \times \sqrt{0.5} = -2.3134$$

$N(-d_2) \rightarrow \text{NormalDist}(2.3134) = 0.98965$

$$O_{p,0} = 1.76 \times e^{-0.05 \times 0.5} \times 0.98965 - 1.25 \times 0.98507 = 0.46744$$

or nearly half a billion Norwegian kroner.

7. (a) The value of the project without flexibility can be calculated with the risk neutral probabilities. Since $u = 1.667,\ d = 0.667$ and $r = 1.1$ so

$$p = \frac{1.1 - 0.667}{1.667 - 0.667}. = 0.433 \text{ and } 1 - p = 0.567$$

The pay-offs without flexibility are $(1\frac{2}{3} \times 75) - 67 = 58$ and $(\frac{2}{3} \times 75) - 67 = -17$, so the present value is:

$$\frac{0.433 \times 58 + 0.567 \times -17}{1.1} = 14.07$$

and the NPV is $14.07 - 10 = 4.07$

(b) If the production decision is made at t_1, the pay-offs are $\max[0, 125 - 67] = 58$ and $\max[0, 50 - 67] = 0$, but they will be available 1 period later so their present value is:

$$\frac{0.433 \times 58 + 0.567 \times 0}{1.1^2} = 20.76$$

and the NPV is $20.76 - 10 = 10.76$. The price dynamics and investments are shown in the lattice below.

$$\begin{array}{lcccl}
 & & 125 & \rightarrow & 125 - 67 = 58 \\
 & \nearrow & & & \\
75 & & & & \\
 & \searrow & & & \\
 & & 50 & \rightarrow & 50 - 67 = -17 \\
I = -10 & & & & I = -67 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}$$