

Chapter 8: Option pricing in continuous time

Answers to self test questions

1. Correct answers are:

- (a) That they are independently distributed True
- (b) That they are identically distributed True
- (c) That they follow a random walk True
- (d) That they are lognormally distributed False

If returns are iid, they follow a random walk. Normality follows from the iid assumption and the Central Limit Theorem. If (log-)returns are normally distributed, stock prices are lognormally distributed. Black and Scholes themselves made the equivalent assumption that: *"The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is lognormal."*

- 2. (a) Increase proportionally with time
- 3. (a) Increases proportionally with time
- 4. (b) Increases proportionally with the square root of time
- 5. (c) Is independent of time

6. Correct answers are:

- (a) Very well, real life stock returns are also normally distributed False
- (b) Real life stock returns have a higher expectation False
- (c) Real life stock returns have fatter tails True
- (d) Real life stock returns are skewed True

7. Correct answers are:

- (a) They are independent True
- (b) They are lognormally distributed False
- (c) They are stationary True

The increments of Brownian motion are *normally* distributed.

8. Correct answers are:

- (a) The instantaneous return of the stock False
- (b) The drift in stock prices over time False
- (c) The random element in stock prices over time True
- (d) The arrival of new information on stock prices True

9. (b) A deterministic drift and a stochastic diffusion term

10. Correct answers are:

- (a) Sudden jumps False
- (b) Mean reversion False
- (c) Rapid growth to very high prices True
- (d) Negative prices False
- (e) Momentum in prices False

11. (c) Minus the market price of risk: $-(\mu - r)/\sigma$
12. (c) The option delta
The option delta reflects both the probability of exercise and the moneyness, but is neither
13. (a) The risk neutral probability that the option will be exercised
14. Correct answers are:

| | |
|---|------|
| (a) The hedge ratio | True |
| (b) The option price's sensitivity for changes in the stock price | True |
| (c) The partial derivative $\partial O_{c,0}^E / \partial S_0$ | True |
| (d) $N(d_1)$ | True |
15. (c) Decreases with the exercise price
16. (a) Increases with the exercise price
17. (a) Increases with the stock price
18. (c) Decreases with the stock price
19. (a) Increases with the stock price volatility
20. (a) Increases with the stock price volatility
21. (a) Increases with the time to maturity
B&S assume European options and non-dividend paying stocks; if dividends are paid in the 'extra' time, the price of a European call may decrease with maturity
22. (d) Cannot say without more information
Generally, puts increase in price with time to maturity, but deep in the money European puts, that are almost certain to be exercised, may decrease with time to maturity
23. (a) Increases with the risk free interest rate
24. (c) Decreases with the risk free interest rate
25. (b) Is independent of in- and out of-the-moneyness
26. (a) Increases with in- and out of-the-moneyness
This is called volatility smile