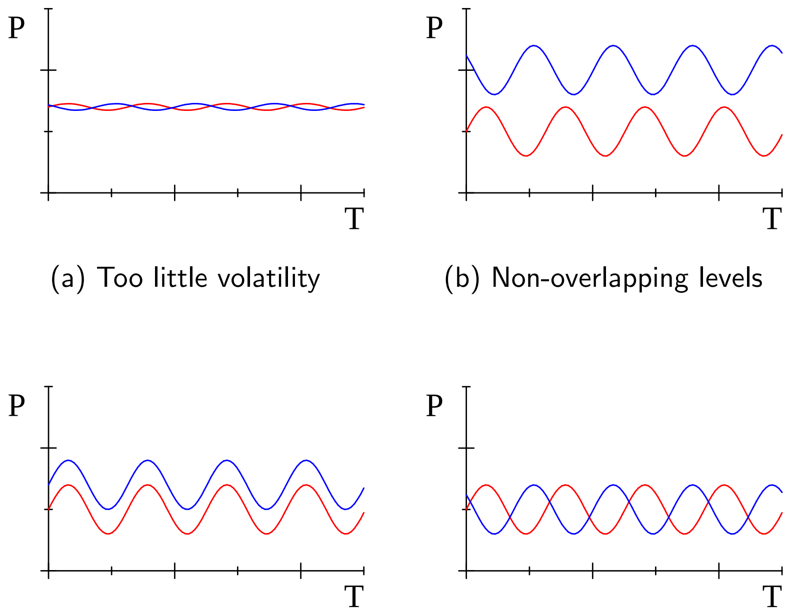
Chapter 9: Real options analysis

Exercises- solutions

1. The prices have to be volatile, not strongly positively correlated and not on very different levels, so that intermittently one is significantly cheaper than the other. The various possibilities are schematically depicted in Figure 1. The option to switch is only valuable if the prices of barley and wheat behave as in panel (d); the option has little or no value if the price patterns are as in panel (a), (b) and (c).



(a) Too little volatility (b) Non-overlapping levels

(c) Too highly correlated (d) High switch value

Figure 1: Prices of barley (red) and wheat (blue) over time (T)

2. (a) The option to buy or sell at market prices has no value, everybody can do that. Buffett got a valuable option, the US did not. In an op-ed article in the New York Times of Oct. 17, 2008 Warren Buffett encouraged investors to but American stocks ("Buy American. I am"). Increased demand would make his options, that were already in-the-money, even more valuable.

   (b) If both the input market (smelters) and the output market (bulk aluminium) are characterized by perfect competition (large numbers of buyers and sellers, none of which can individually influence prices), Norsk Hydro has no lower exercise price nor a higher underlying value than its competition so that there is no source of real option value.

(c) City centre building plots are very valuable and by developing the plot, *the option to develop* is given up. If one development is chosen (say, offices), alternative developments (say, a hotel) are given up and the latter may prove to be very valuable in the near future. The option to develop may be more valuable than any particular development today.

(d) By assembling locally, HP introduced the flexibility to adapt to variations in local demand, e.g. to increase the number of French printers and decrease the German number. If demand is volatile, flexible production (although by itself more expensive) can be more profitable than other methods to match supply and demand (e.g. maintaining large buffer stocks).

3. Real call options are often exercised before maturity because the sources of option value tend to be eroded over time. This is plain to see in a patent that has a limited life time and has to be used (= exercised) before it expires if its value is to be realized. The value of patents and other real options is further eroded over time because competitors will develop close substitutes. Also, if the option is shared in some degree, the game-theoretic anticipation of competitors' actions generally leads to an earlier exercise.

4. (a) The value of the gas reserve is $4 \times 100 \times 0.08 = \$32$ million. The cash flows from gas production occur on 4 future points in time, but both the expected return on gas and the time value of money are included in the binomial process. Feel free to check this using either the risk neutral probabilities and risk free rate or the real probabilities and the risk adjusted rate.

   (b) If the field is developed immediately, its NPV is 32-30=$2 million. The concession gives State Drilling the flexibility to wait and see for 1 year. After 1 year, the investment amount is $30 \times 1.07 = 32.1$. The gas price is either $0.08 \times 1.25 = 0.1$ or $0.08 \times 0.8 = 0.064$, so the value of the gas reserve is either $4 \times 100 \times 0.1 = 40$ or $4 \times 100 \times 0.064 = 25.6$. Of course, the field will not be developed if the gas price becomes $0.064, so the opportunity to develop the field in one year is $\max[0, 40 - 32.1] = 7.9$ in the up state and $\max[0, 25.6 - 32.1] = 0$ in the down state. The risk neutral probability that the up state occurs is $(r - d)/(u - d) = (1.07 - 0.8)/(1.25 - 0.8). = 0.6$. So the value of the opportunity to develop the field is $(0.6 \times 7.9)/1.07 = 4.429\,9$ This is higher than the value if developed immediately, so development should be postponed and the value of the opportunity to develop the field is the option value, $4.43 million.

5. (a) We can calculate the value of the inflexible project with the real probabilities and the risk adjusted discount rate. The project will generate either €20 or €10 million per year, starting 2 years from now at t=2. The t=1 value of this perpetual cash flow is either 20/0.1=200 or 10/0.1=100

$$
\begin{array}{lll}
 & & Value = 20/0.1 = 200 \quad cfl = 20 \\
 & \nearrow & \\
125 & & \\
 & \searrow & \\
 & & Value = 10/0.1 = 100 \quad cfl = 10 \\
I = -120 & & \\
t_0 & t_1 & t = 2, \infty
\end{array}
$$

the real probabilities are 0.375 and 0.625 so the t=0 value is

$$\frac{0.375 \times 200 + 0.625 \times 100}{1.1} = 125$$

and the NPV is 125-120=€5 million.

(b) With the flexible project, Smalldale can decide 1 year from now whether to build the centre or not, when the football association's decision is known. Of course, he will only invest if the project is profitable and abandon the project if it is not profitable.

$$\begin{array}{lll} & Value = \max[(200-120),0] = 80 & cfl = 20 \\ & \nearrow & \\ 23.81 & & \\ & \searrow & \\ & Value = \max[(100-120),0] = 0 & cfl = 10 \\ I = -12 & I = -120 & \\ t_0 & t_1 & t = 2, \infty \end{array}$$

To value the flexible project, we use the risk neutral probabilities and the risk free interest rate. Given the value of the inflexible project of 125, the up-factor is $200/125 = 1.6$ and the down-factor is $125/100 = 0.8$, so that $p = (1.05 - 0.8)/(1.6 - 0.8) = 0.3125$ and $1 - p = 0.6875$. The t=0 value is

$$\frac{0.3125 \times 80 + 0.6875 \times 0}{1.05} = 23.81$$

The NPV is thus $23.81 - 12 = 11.81$. So postponement is profitable, the project value increases with $11.81 - 5 = 6.81$.

(c) The risk adjusted discount rate can be calculated from the replicating portfolio. The ingredients of this portfolio are calculated with Δ and D:

$$\Delta = \frac{O_u - O_d}{S_u - S_d} = \frac{80 - 0}{200 - 100} = 0.8$$

and

$$D = \frac{uO_d - dO_u}{(u - d)r} = \frac{1.6 \times 0 - 0.8 \times 80}{(1.6 - 0.8) \times 1.05} = -76.19$$

So the replicating portfolio consists of $0.8 \times 125 = 100$ in the inflexible project and a loan of 76.19. The weighted average return of this portfolio is

$$\frac{100}{100 - 76.19} \times 0.10 + \frac{-76.19}{100 - 76.19} \times 0.05 = 0.26$$

A shorter calculation is solving:

$$\frac{0.375 \times 80 + 0.625 \times 0}{r} = 23.81$$

for r, which gives the same risk adjusted rate of 26%.

6. (a) The Wallenberg group held a long position in the shares of Aker Holding plus a long position in in-the-money European put options and a short position in out-of-the money European call options on these shares, both with a time to maturity of four years. It is a 'share plus protective put' position, plus a short call position. The combined result is that the Wallenberg group is guaranteed to earn at least 10% over four years, but cannot earn more than 40%.

(b) The Wallenberg group held 10% of 16 billion, or 1.6 billion and had the right to sell it for $1.6 \times 1.1 = 1.76$ billion. The inputs for the calculation of the option's value are: $S_0 = 1.6$, $X = 1.76$, $\sigma = 0.2$, $T = 4$ and $r = 0.05$. The Black and Scholes formula then gives:

$$O_{p,0} = Xe^{-rT}N(-d_2) - S_0N(-d_1)$$

with

$$d_1 = \frac{\ln(S_0/X) + (r + \frac{1}{2}\sigma^2) \times T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln(1.6/1.76) + (0.05 + 0.5 \times 0.2^2) \times 4}{0.2 \times \sqrt{4}} = 0.46172$$

$N(-d_1) \rightarrow \text{NormalDist}(-0.46172) = 0.32214$

$$d_2 = 0.46172 - 0.2 \times \sqrt{4} = 0.06172$$

$N(-d_2) \rightarrow \text{NormalDist}(-0.06172) = 0.47539$

$$O_{p,0} = 1.76 \times e^{-0.05 \times 4} \times 0.47539 - 1.6 \times 0.32214 = 0.1696$$

or 169.6 million Norwegian kroner.

(c) The call option position is valued along similar lines. The exercise price is $1.6 \times 1.4 = 2.24$ so that the inputs for the calculation of the option's value are: $S_0 = 1.6$, $X = 2.24$, $\sigma = 0.2$, $T = 4$ and $r = 0.05$. The Black and Scholes formula then gives:

$$O_{c,0} = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

with

$$d_1 = \frac{\ln(S_0/X) + (r + \frac{1}{2}\sigma^2) \times T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln(1.6/2.24) + (0.05 + 0.5 \times 0.2^2) \times 4}{0.2 \times \sqrt{4}} = -0.14118$$

$N(d_1) \rightarrow \text{NormalDist}(-0.141\,18) = 0.44386$

$$d_2 = -0.14118 - 0.2 \times \sqrt{4} = -0.54118$$

$N(d_2) \rightarrow \text{NormalDist}(-0.54118) = 0.29419$

$$O_{c,0} = 1.6 \times 0.44386 - 2.24 \times e^{-0.05 \times 4} \times 0.29419 = 0.17064$$

or 170.64 million Norwegian kroner.

(d) The inputs for the calculation of the option's value on January 1 2011 are: $S_0 = 1.25$, $X = 1.76$, $\sigma = 0.2$, $T = 0.5$ and $r = 0.05$. The Black and Scholes formula then gives:

$$O_{p,0} = Xe^{-rT} N(-d_2) - S_0 N(-d_1)$$

with

$$d_1 = \frac{\ln(S_0/X) + (r + \frac{1}{2}\sigma^2) \times T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\ln(1.25/1.76) + (0.05 + 0.5 \times 0.2^2) \times 0.5}{0.2 \times \sqrt{0.5}} = -2.172$$

$N(-d_1) \rightarrow \text{NormalDist}(2.172) = 0.98507$

$$d_2 = -2.172 - 0.2 \times \sqrt{0.5} = -2.3134$$

$N(-d_2) \rightarrow \text{NormalDist}(2.3134) = 0.98965$

$$O_{p,0} = 1.76 \times e^{-0.05\times 0.5} \times 0.98965 - 1.25 \times 0.98507 = 0.46744$$

or nearly half a billion Norwegian kroner.

7. (a) The value of the project without flexibility can be calculated with the risk neutral probabilities. Since $u = 1.667,\ d = 0.667$ and $r = 1.1$ so

$$p = \frac{1.1 - 0.667}{1.667 - 0.667}. = 0.433 \text{ and } 1 - p = 0.567$$

The pay-offs without flexibility are $(1\frac{2}{3} \times 75) - 67 = 58$ and $(\frac{2}{3} \times 75) - 67 = -17$, so the present value is:

$$\frac{0.433 \times 58 + 0.567 \times -17}{1.1} = 14.07$$

and the NPV is $14.07 - 10 = 4.07$

(b) If the production decision is made at t_1, the pay-offs are $\max[0, 125 - 67] = 58$ and $\max[0, 50 - 67] = 0$, but they will be available 1 period later so their present value is:

$$\frac{0.433 \times 58 + 0.567 \times 0}{1.1^2} = 20.76$$

and the NPV is $20.76 - 10 = 10.76$. The price dynamics and investments are shown in the lattice below.

$$\begin{array}{lllll}
 & & 125 & \rightarrow & 125 - 67 = 58 \\
 & \nearrow & & & \\
75 & & & & \\
 & \searrow & & & \\
 & & 50 & \rightarrow & 50 - 67 = -17 \\
I = -10 & & & & I = -67 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}$$