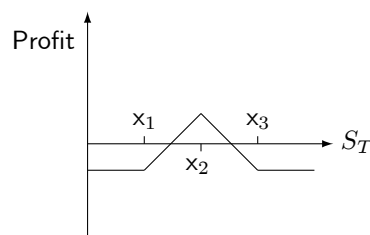


## Chapter 7: Option pricing foundations

### Exercises

- On June 23, shares of ZX Co. were traded in Frankfurt for 97.70. Call options on this share with a time to maturity of one year and a strike price of 80 were traded for 23.20. Puts on the same stock with the same time to maturity and the same strike price traded for 4.16. The relevant interest rate for the options' lifetime was 3.15%.
  - Discuss the arbitrage opportunities on this market from a pricing point of view and from a practical point of view. Explicitly mention the assumptions on which arbitrage opportunities, or the lack thereof, rest.
- You have done some option trading and you now hold the following option contracts: you have written (sold) a put with an exercise price of 75, you have bought a put with an exercise price of 100, you have bought a call with an exercise price of 75 and you have written a call with an exercise price of 100. All options are on the same underlying share and have the same maturity.
  - Construct the payoff diagram (not the profit diagram) for your total position over the share price interval from 0 to 150.
- It is sometimes said that the prices of at-the-money puts and calls on the same underlying and with the same maturity have to be the same, since simultaneously buying and selling cancels out. Is this reasoning correct? Assume European options on non dividend paying stocks.
- The option position depicted in the figure below is known as a butterfly spread.



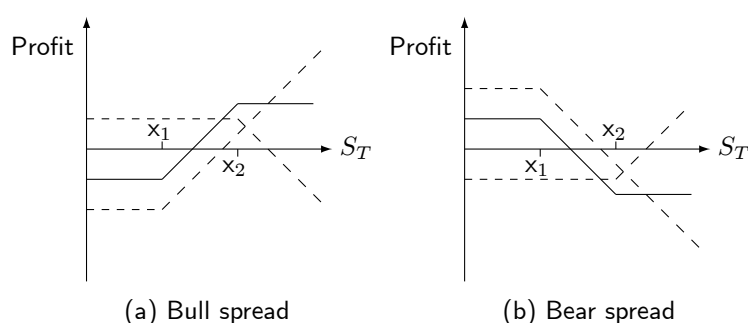
- Work out a combination of call options that produces the butterfly spread in the figure above.
  - Work out a combination of put options that produces the butterfly spread in the figure above.
- On November 1<sup>th</sup>, 3 months European call options on a share ZXco with an exercise price of 460 cost 20.75. The same options with an exercise price of 480 resp. 500 cost 11.75 resp. 6. The share price of ZXco is 462.5 and the 3 months risk free interest rate is 1.5%.

- (a) Calculate the initial investment that is required to set up a butterfly spread.
- (b) Show that the same butterfly spread set up with put options requires the same initial investment.
- (c) According to a business newspaper, the prices of 3 months put options ZXco are as follows:

strike	bid	ask
460	15.25	17.50
480	26.00	29.00
500	40.00	43.25

Explain why these prices are the same as, or different from, the prices you used in question b.

6. The figure below repeats the profit diagrams for option positions known as spreads that was used in the main text.



- (a) Construct the spreads in this figure using put options.
  - (b) What happens to the initial investment (initial balance of the option premiums) required to set up the position, compared to the same position in calls? Hint: look at the prices of options in relation to the exercise prices.
7. In Bound 6 the following maximum value for European put options is formulated: a European put option cannot be worth more than the present value of the exercise price. Explain why this bound is or is not valid for American options.
  8. A financial market can have 2 possible future states. Both states are equally likely. In the market 2 securities, A and B, are traded. The securities' pay-offs in the future states are as follows:

	A	B
State 1	6	4
State 2	12	14

The required rates of return on the securities are 8% for A and 12% for B.

- (a) Calculate the value of the securities A and B using the risk adjusted discount rates.
- (b) Calculate the state prices and the risk free interest rate on the market. Check your results.
- (c) Re-calculate the values of the securities A and B:
  - i. using the state prices
  - ii. using the risk neutral valuation formula
- (d) Calculate the values of call options on the two securities A and B with an exercise price of 10 and that mature in the future state (of course).

9. In a financial market a stock is traded at a price of 100. At the end of the period, the stock price can either increase with 20 to 120 or decrease with 20 to 80. Risk free debt is available at 5% per period. Using state prices, calculate the value of an at the money call option on the stock which matures at the end of the period.
10. In a financial market a stock is traded with a current price of 100. The price of the stock can either go up with 50% next period or go down with 50% next period. Risk free debt is available with an interest rate of 25%. Also traded is a European call option on the stock with an exercise price of 110 and a time to maturity of 2 (i.e. the option matures at end of the second period on the third moment).
  - (a) Discuss the completeness and the arbitrage properties of this market
  - (b) Calculate the value of the option assuming that:
    - i. The stock pays no dividend
    - ii. The stock pays out 25% of its value in dividends on the second moment (end of the first period)
    - iii. Repeat step b(i) for an American call option
    - iv. Repeat step b(ii) for an American call option
    - v. Repeat step b(i) for a European put option
    - vi. Repeat step b(ii) for a European put option
11. In a financial market a stock is traded with a current price of 50. Next period the price of the stock can either go up with 30% or go down with 25%. Risk free debt is available with an interest rate of 8%. Also traded are European options on the stock with an exercise price of 45 and a time to maturity of 1, i.e. they mature next period.
  - (a) Calculate the price of a call option by constructing and pricing a replicating portfolio.
  - (b) Calculate the price of a put option by constructing and pricing a replicating portfolio.
12. On page 202 of the book, a self-financing strategy is defined as a strategy that requires no extra cash along the way, i.e. all additional outlays must be part of the strategy. On page 212-213, the dynamic hedging portfolio of the two-period example is shown to give a perfect hedge with a self-financing strategy. Demonstrate that the dynamic hedge in this example is indeed self-financing. Hint: think of what self-financing means for rebalancing the portfolio.