Chapter 2: Fundamental concepts and techniques

Exercises - solutions

1. The time value of money and the risk premium, that together constitute discount rates, strongly reduce values over time. With a discount rate of 10%, €1000 to be received 10 years from now has a present value of $1000/1.1^{10} = 385.54$, if the amount is to be received after 20 years the present value is $1000/1.1^{20} = 148.64$ and after 30 years $1000/1.1^{30} = 57.31$. The benefits have to be very large indeed to justify projects on such a time scale from an economic point of view. This effect also works for negative benefits. The demolition costs of a nuclear power plant can be enormous, but since they are incurred after the plants life time of 30 years or more, their present value is small. The effect is illustrated in Figure 1. The economic viability of visionary projects can be enhanced by lowering interest rates and lowering risk premiums, for example by guaranteeing a high, fixed price for green electricity for 20 or 30 years. With a 5% discount rate, the present values are $1000/1.05^{10} = 613.91$, $1000/1.05^{20} = 376.89$ and $1000/1.05^{30} = 231.38$.

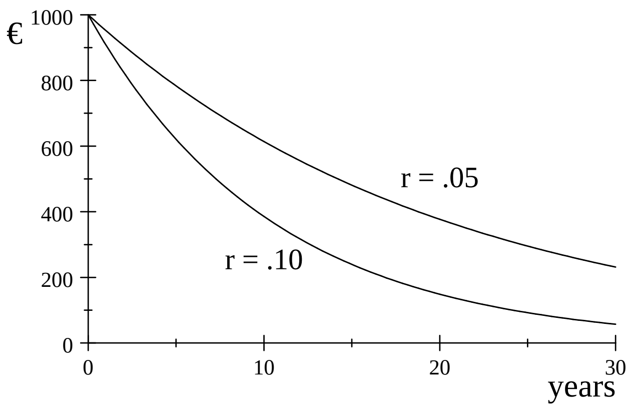

Figure 1: Value as function of time

2. The formula of an annuity given its future value is (see p. 16 of the book):

$$A = FV\frac{r}{(1+r)^n - 1}$$

Inserting the numbers gives

$$A = 35000\frac{0.07}{(1+0.07)^7 - 1} = 4044.4$$

3. To make amounts in different periods comparable, we have to discount or compound them to the same point in time. Here, discounting to the present is the easier way. Note that the annuity and perpetuity begin now, not at the end of the period.

(a) 300 000 AED

(b) $425000/1.1^4 = 290\,281$

(c) using formula (2.2) we get:

$$PV = A\frac{1-\left(\frac{1}{1+r}\right)^n}{1-\frac{1}{1+r}} \Rightarrow 65000\frac{1-\left(\frac{1}{1+.1}\right)^6}{1-\frac{1}{1+.1}} = 311\,400$$

(d) $30000 + 30000/.1 = 330000$

So the perpetuity in (d) is the most valuable alternative. If the interest rate is 12.5%, the amount now has the highest value:

(a) 300 000 AED

(b) $425000/1.125^4 = 265\,325$

(c)

$$65000\frac{1-\left(\frac{1}{1+.125}\right)^6}{1-\frac{1}{1+.125}} = 296\,440$$

(d) $30000 + 30000/.125 = 270000$

4. (a) The project should be accepted if it has a positive net present value. To calculate NPV we first have to calculate the proper cash flows and then discount them to the present. The calculations are shown in Table 1.

Table 1: NPV calculations

Year	0	1	2	3	4
Income		300	400	450	350
Production cost		-150	-175	-200	-150
operating costs		-50	-75	-65	-60
operating income		100	150	185	140
Depreciation		-87.5	-87.5	-87.5	-87.5
Income before tax		12.5	62.5	97.5	52.5
tax @ 28%		-3.5	-17.5	-27.3	-14.7
Income after tax		9	45	70.2	37.8
Income after tax		9	45	70.2	37.8
Depreciation		87.5	87.5	87.5	87.5
Change in working capital	-20	-15	-15	-10	60
Investment	-350				
Cash flow	-370.0	81.5	117.5	147.7	185.3
PV of cash flows	408.7				
NPV	38.7				
IRR	14 %				

The present value of the cash flows is calculated as

$$\frac{81.5}{1.1} + \frac{117.5}{1.1^2} + \frac{147.7}{1.1^3} + \frac{185.3}{1.1^4} = 408.73$$

(b) The internal rate of return is calculated by solving

$$-370 + \frac{81.5}{r} + \frac{117.5}{r^2} + \frac{147.7}{r^3} + \frac{185.3}{r^4} = 0$$

for r, which gives $r = 1.1415$ and $r = -0.73482$, corresponding to discount rates of 14% and -173%

5. To answer the question we calculate the Arrow-Pratt coefficients of absolute risk aversion: $U'_A = 3 - .04W$ and $U''_A = -.04$; similarly $U'_B = 2 - .02W$ and $U''_B = -.02$. The coefficients are: $ARA_A = \frac{.04}{3-.04W}$ and $ARA_B = \frac{.02}{2-.02W}$. Multiplying the latter by $\frac{2}{2}$ makes is easy to see that $\frac{.04}{3-.04W} > \frac{.04}{4-.04W}$. Multiplication by W to find the RRA leaves the inequality unaltered, so person A is more risk averse than B. The utility curves are plotted in Figure 2; we see that the line representing the first function is more curved, while the second is flatter.

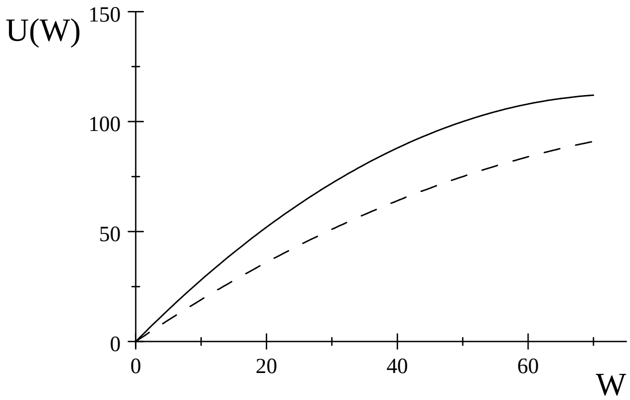

Figure 2: The utility functions $3W - .02W^2$ (solid) and $2W - .01W^2$ (dashed)

6. The budget line becomes more steeply downward sloping. If the opportunity cost of capital is higher, less projects are taken into production.

7. The optimum is reached with the following steps:

   - At t_0, borrow the maximum against the t_1 budget; this gives a total t_0 budget of 19.
   - Of this 19, invest 19-B01 in productive assets, this leaves B01-0 in t_0.
   - Of this available budget in t_0, lend B01-B02, this leaves B02 for consumption, the optimal amount in t_0.
   - The productive investment pay off B11-0 in t_1; the B01-B02 lent in t_0 has a t_1 value of B12-B11. Total budget in t_1 is thus (B11-0)+(B12-B11)=B12, the optimal amount.

8. (a) $0.04^{100} = 1.6 \times 10^{-140} = 0$

   (b) The number is found by solving $x \times .10 + (1-x) \times -1 = 0$ for x, which gives $x \approx 0.91$ so 9 defaulted loans and 91 loans fully paid.

   (c) For a 100 loan portfolio the expected number of defaults is

$$\mu = n \times p = 100 \times 0.04 = 4$$

The standard deviation is

$$\sigma = \sqrt{n \times p \times (1-p)} = \sqrt{100 \times 0.04 \times 0.96} = 1.9596$$

To calculate the probability, we have have to transform to a standard normal distribution:

$$z = \frac{w - \mu}{\sigma}, \quad z \backsim N(0,1)$$

where w is the number we want to test. For 9 defaults the z-value is $z = (9 - 4)/1.9596 = 2.55$. The table in the appendices of Chapter 8 gives a probability of 0.995 for this z-value (the table uses the symbol d instead of z). So there is a 99.5% probability that the bank will at least break even, and a 0.5% probability that the bank will make a loss (compared with 4% for a single loan).