Chapter 8: Black and Scholes option pricing

Exercises

1. It is sometimes said that if the current share price equals the exercise price, then call options on the share have a 50% chance of ending up worthless (if the share price falls) and a 50% chance of ending up "in the money" (if the share price rises). Explain in simple terms or with an example why this statement is or is not correct.

2. A stock has an annual volatility (standard deviation) of 34%. Calculate the standard deviation of the daily return. Assume that a year has 252 trading days and that the returns are independently and identically distributed (iid).

3. In the discussion of the properties of log returns we made the assumption that these returns are independently and identically distributed, which means that they follow a random walk. Does this mean that stock prices also follow a random walk?

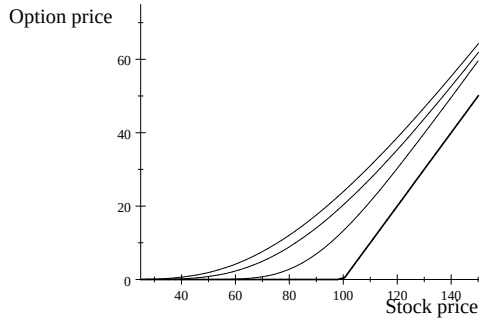
4. The iid assumption means that the distribution of stock returns is stable over time. Does this stability mean that past stock returns can be used to predict future ones?

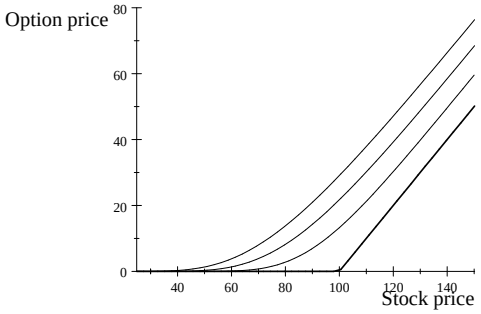
5. Explain why the value of options increases with the volatility of the underlying.

6. Explain why the value of a call increases with the risk free interest rate while the value of a put decreases.

7. Use the put-call parity to work out a relation between the Greek 'delta' of puts and calls.

8. The graphs below (copies from the main text) plot call option prices for different values of their determinants. All options have an exercise price of 100, a risk free interest rate of 10% and are written on a stock that pays no dividends. The graph on the left plots options with a time to maturity of 1 year and three different volatilities: $\sigma = .5$ (top), $.4$ and $.2$ (bottom). The graph on the right plots options with a volatility of $.2$ and three different maturities: $T = 3$ (top), 2 and 1 (bottom). Explain briefly why the options with different volatilities converge to a common value as the stock price increases and why the options with different maturities do not converge to a common value as the stock price increases.





9. On a financial market a stock is traded at a price of €240. The stock has an annual volatility of 25%. Call options on the stock with an exercise price of €250 and a time

to maturity of 1 year are also traded. The risk free interest rate is 6%. Calculate the price of the option.

10. Suppose the price of a share Norske Skog at some point in the future is NOK 100. Over each of the next two periods of half a year, the price can either increase with 7.5% or decrease with 7%, corresponding to a yearly standard deviation of 10.228%. After the first half year, Norske Skog pays out a dividend of NOK 10. The 6 months risk free rate is 2.5%, so slightly over 5% per year.

    (a) Calculate the value of an American call option on the stock, that matures in 1 year and has an exercise price of 102.5

    (b) Calculate the hedge portfolio of the option for the first half year period and show that it gives a perfect hedge.

    (c) Explain in general terms how the call option delta changes as the stock prices changes.

11. The following information on option prices, stock prices and interest rates was published in Finansavisen (a Norwegian financial newspaper) of 5 Sept. 2005.

Option price quotes

			call option		put option	
Ticker	T	X	bid	ask	bid	ask
NHY	nov.5	620	68.00	70.00	9.25	10.00
„	„	680	28.75	30.25	29.00	31.25
„	feb.6	620	82.00	83.75	19.00	20.75
„	„	680	44.00	47.00	41.50	44.25
ORK	jan.6	240	25.50	26.75	5.00	5.50
NSG	dec.5	100	10.00	11.00	2.85	3.35

Stock price quotes

	stock price	
Ticker	bid	ask
NHY	677.00	678.50
ORK	259.00	259.50
NSG	108.75	109.50

NIBOR rates

1 month	2.180
2 month	2.235
3 month	2.290
4 month	2.313
5 month	2.337

NIBOR is the Norwegian InterBank Offer Rate that can be used as the yearly risk free interest rate for the different maturities. Ask prices are prices at which a dealer is willing sell, bid prices at prices at which a dealer is willing to buy. The ticker marks are for Norsk Hydro (NHY), Orkla (ORK) and Norske Skog (NSG).

    (a) Is there any sign of mispricing on the market? If so, how would you profit from it?

    (b) Is there an alternative explanation for price differences, if you find any?