Options as securities

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> *In my view, derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal.*

Warren Buffett,
Berkshire Hathaway Annual report for 2002

Options

are financial contracts that give their holders:

- the right, but not the obligation, to buy or sell something
- on a future date at a price determined today

Distinction between right and obligation, which gives the holder a choice, is the essential characteristic

Practical use of options is very old:

- oldest sources go back to Thales of Miletus (62?-546 BC), as related in Aristotle's (384-322 BC) work *Politics*
- in the 1600s options on rice were traded in Japan and options on tulips in The Netherlands

Use is very old, organised trade is not:

- First option exchange opened in 1973: Chicago Board of Options Exchange (CBOE)
- 1978: Standardized options trade in Europe (European Option Exchange (EOE), Amsterdam)

Enormous growth since, exchanges make trading easy by:

- operating clearinghouses:
  - transactions are properly effectuated
  - payment guaranteed (no counterparty risk)
- standardization of contracts w.r.t.
  - quantity: options on 100 shares, bonds, ounces gold
  - expiration dates
  - exercise prices

Example of standardization

- All Apple call and put options on Nasdaq maturing in:
  - December 2014 expire on the 20^{th}
  - January 2015 expire on the 17^{th}
- Traded Apple calls and puts on Nasdaq have wide range of exercise prices:
  - ranging from \$27.86 to \$150 in steps of $\pm$\$0.70
  - but no values in between
- Most of them are actively traded
(situation early October 2014, price AAPL is $\pm$\$99.62
Apple split its stock 7 to 1 in June 2014)

- Standard options available on stocks, bonds, gold, etc.
- Special, large deals can be negotiated with banks

Some option terminology:

- Call option
    - right to *buy* 'something' (the underlying)
    - at specified price (=*exercise* or *strike* price)
        - *on* a specified date (exercise date or maturity): *European* call option
        - *before* a specified date (exercise date or maturity): *American* call option

- Put option
    - right to *sell* 'something' (the underlying)
    - at specified price (=*exercise* or *strike* price)
        - *on* a specified date (exercise date or maturity): *European* put option
        - *before* a specified date (exercise date or maturity): *American* put option

Rights and obligations attached to options:

	Buyer (long position)	Seller (short position)
call options	right to buy	obligation to sell
put options	right to sell	obligation to buy

Exercise possibilities also called *style:*

- *American-style* and *European-style* options
- not geographical
- most traded options are American, also in Europe

There are many different kinds of options:

- ordinary 'plain vanilla' European and American options

Plus wide range of *exotic* options, e.g.:

- Bermudan options:
  American options with limited number of exercise dates
- Asian options
  payoff depends on average price underlying
- Barrier options
  payoff depends on whether underlying reaches threshold level
  - Knock-out options cease to exist if threshold is reached
  - Knock-in options come into existence
- Binary options
  cash-or-nothing call pays fixed amount if underlying ends in the money

Some more terminology:

- Price of an option is also called the *option premium*.
- To sell an option is also called to *write* an option
- *Moneyness* describes the value of an option if it would be exercised immediately:

moneyness	Call	Put
in the money	$underlying > strike$	$underlying < strike$
at the money	$underlying = strike$	$underlying = strike$
out of the money	$underlying < strike$	$underlying > strike$

Example of an option

- You have bought a European call on a share of Apple, strike price \$100, maturity 17 January 2015
- Gives you the right, not the obligation, to buy that share on that date at that price
  - If share price of Apple on 17 January $>$ \$100
    - you *exercise* the option (you have to *do* something)
    - and earn difference between share price and strike
  - If share price of Apple on 17 January $<$ \$100
    - you will *not* exercise the option (do nothing): let it expire worthlessly.
- European put (long) gives you comparable right to sell
- With American options (long) you can do same things, but on any date before maturity

Main economic characteristics

- A long option is a *limited liability* investment:
  - gives the *right*, not the *obligation* to buy/sell
- Economically options represent *flexibility:*
  - possibility to choose best alternative
  - walk away from bad outcomes
  - also found in real investments (real options)
- Options are almost always riskier than underlying values
- *Redistribute risk* at market prices
- Are *zero sum game:* one's losses are someone else's profits

Reasons to use options:

- to insure against
- to profit from
- to speculate

Insure against e.g. a fall in stock price: buy a put:

- gives 'bottom' in price
- disadvantages: temporary and expensive

Reverse position: selling a put

- is like collecting insurance premiums without proper insurance
- very risky: only use when really want to buy

Speculation possibilities enhanced by leverage effect (need less money to control large positions)

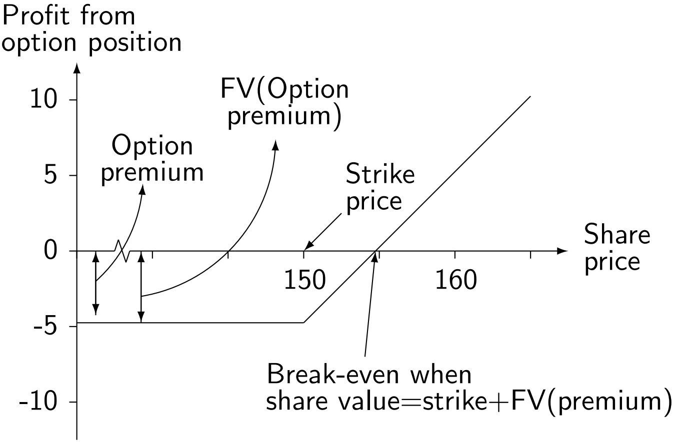
Example: speculating with options

- Early October call options on Apple shares, strike price \$100, maturity 17 January, cost $\pm$\$4.72
- Share price Apple same date is $\pm$\$99.62
- $\pm$\$100 buys you 1 share or 21 options
  - If, on 17 January, the price of Apple share = \$109.50
    - share pays off $(109.50 - 100)\,/100 = 0.095$ or 9.5%
    - investment in options pays off $((21 \times (109.50 - 100)) - (21 \times 4.72))\,/100 \approx 1$ or 100%
      you have doubled your money
  - If, on 17 January, the price of Apple share = \$97.50
    - share investment pays off $(97.50 - 100)\,/100 = -0.025$ or -2.5%, you have 97.5% left
    - investment in options pays off $((21 \times 0) - (21 \times 4.72))\,/100 \approx -0.99$ or -100%,
      you have lost your whole investment

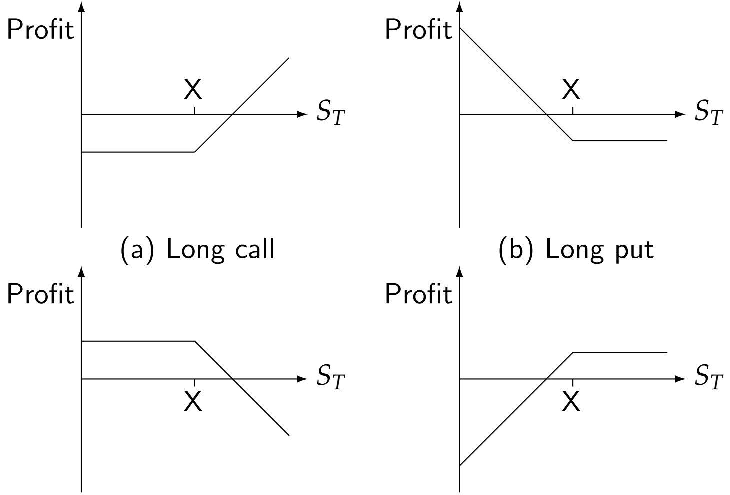
Option positions

Advantages and disadvantages of holding positions in options given in diagrams

- Usually depicted at maturity in:
    - *payoff diagrams* (or position diagrams) ignoring premium
    - *profit diagrams* including premium
- Option positions can be:
    - simple (or naked) option positions (1 option)
    - combined with other options and securities in:
        - strips, straps, straddles, spreads, butterflies, etc.



Detailed profit diagram for a call, strike=150, O_c= 4.5



(a) Long call

(b) Long put

(c) Short call

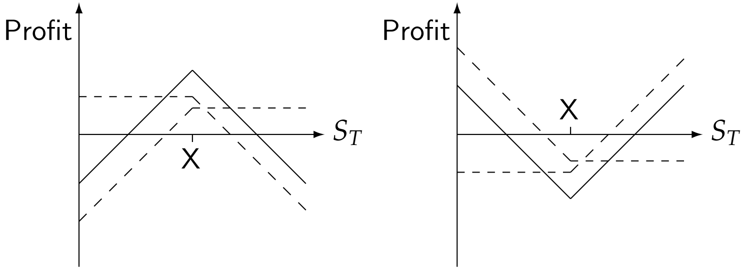
(d) Short put

Profit from simple option positions as a function of the share price at maturity, S_T, and the strike price, X

Combined option positions

Straddles are combinations of options that are constructed as bets on volatility

- *Long straddle* is long put + long call with same strike
  - profits from large price changes
  - e.g. important news expected, but nature of the news (good or bad) unknown
- *Short straddle* is short put + short call with same strike
  - profits from small price changes
  - no news expected, collect double premium, but possibly large loss if expectation is wrong
  - (Nick Leeson sold those before Kobe earthquake)

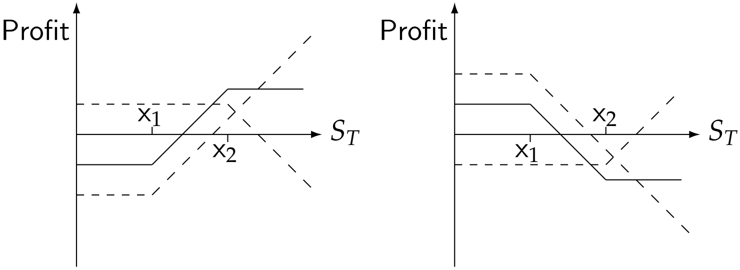


(a) Short straddle (b) Long straddle

Profit diagrams for straddles

Spreads are limited bets on stock price movements

- A *bull spread* bets on increasing stock price
  - long call and short call with higher strike on same stock
  - short call cheaper $\Rightarrow$ initial balance is negative
- A *bear spread* bets on decrease in stock price
  - long and short positions reversed, lower strike sold, higher strike bought
  - initial balance of option premiums positive
- Payoffs limited on up- and downside
- Price and riskiness vary with moneyness:
  - out of the money calls $\Rightarrow$ cheap, low prob. of payoff
  - in the money calls $\Rightarrow$ less risky, more expensive



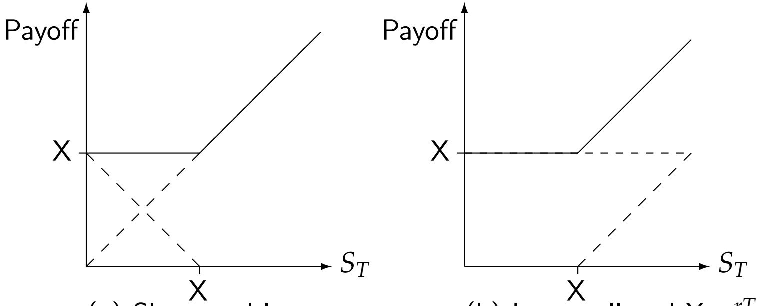
(a) Bull spread (b) Bear spread

Profit diagrams for spreads

Next picture shows 2 *payoff* diagrams

$\Rightarrow$ option premia not depicted, only values at maturity

1. Payoff of a share and long put
    - the protective put we saw earlier
    - gives a floor in combined position
2. Payoff of a call and the PV(strike) invested risk free
    - risk free investment gives floor in position
    - upward potential from long call



(a) Share and long put (b) Long call and Xe^{-rT}

Payoff diagrams for the put-call parity

Payoffs at maturity are the same:

- $\Rightarrow$prices of combinations have to be the same

$$\begin{aligned} share + long\ \ put &= long\ \ call + pv(x) \\ long\ \ put &= long\ \ call + pv(x) - share \end{aligned}$$

Expression for put, called *put-call parity*

- only valid for European options on stocks that don't pay dividends
- parity relation invalidated by:
  - early exercise
  - dividend payments

Bounds on option prices

- Simple arbitrage arguments limit the range of option prices
- Only assumption made is greedy investors $\Rightarrow$ bounds are independent of pricing model
- But all pricing models must stay within these bounds to be acceptable
- Bounds formulated for stock options:
  - S is stock price, X is strike price, T maturity
  - have wider validity
- Some bounds formally proven with arbitrage portfolio
- Most should be intuitively clear, and should give good intuition of option prices

Bound 1 A call option cannot be worth more than the stock

Intuition: Call gives right to buy stock, cannot be worth more than stock itself, obviously!

Proof.

if $O_c > S$, writing covered call is arbitrage opportunity:

- selling the call and buying the stock
  - gives positive cash flow now: $O_c - S > 0$
  - and at maturity or exercise:
    - either strike price (if exercised)
    - or stock value (if not exercised)
    - both >0
- positive cash flow now + later is money machine

$\square$

Bound 2 A put option cannot be worth more than the strike price

Proof/intuition

- Put gives the right to sell stock for the strike price
- cannot be worth more than strike

Bound 3 A European put cannot be worth more than the present value of the strike price

Proof/intuition

- Put at maturity not worth more than strike
- European put cannot be exercised early
- hence value now cannot be higher than pv(strike)

Bound 4 The minimum value of a European call option, O_c, on a stock that pays no dividends is $\max[0, S - PV(X)]$

If $0 < O_c < S - PV(X)$ this arbitrage strategy exists:

- buy the call, short sell the stock and lend PV(X)
- costs $-O_c + S - PV(X) > 0$ if bound is violated
- payoff of the option at maturity:
  - $(S_T - X)$ if exercised (i.e. if $S_T > X$)
  - 0 if not exercised (i.e. if $S_T < X$)
- payoff of the short stock at maturity: $-S_T$
- payoff of lending at maturity: X
- Total payoff:
  - $(S_T - X) - S_T + X = 0$ if exercised
  - $0 - S_T + X > 0$ if not exercised

Arbitrage strategies usually summarized in tables:

Proof.

If $0 < O_c < S - PV(X)$
then following arbitrage possibility exists:
buy call, shortsell stock and lend PV(strike)

	Now	At expiration	
		$S_T > X$	$S_T < X$
Buy call	$-O_c$	$+(S_T - X)$	0
Sell stock	$+S$	$-S_T$	$-S_T$
Lend $PV(X)$	$-PV(X)$	X	X
Total position	> 0	0	$-S_T + X > 0$

$\square$

Positive cash flow now and either no or positive cash flow later is money machine

Bound 5 The minimum value of an American call option on a stock is $\max[0, S - X]$

Proof/intuition

- American call can be exercised immediately, which gives $S - X \Rightarrow$ option value cannot be less

Bound 6 An American call option is worth at least as much as a comparable European call option

Proof/intuition

- With an American call you do everything that you can do with a European call, plus exercise early
- the right to exercise early cannot have negative value

Together, these imply an exercise bound on American call options:

Bound 7 An American call option on a stock that pays no dividends will not be exercised before maturity

Proof.

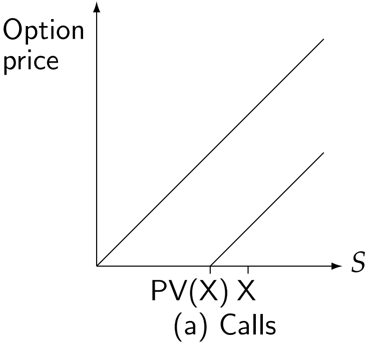
By the previous bounds:

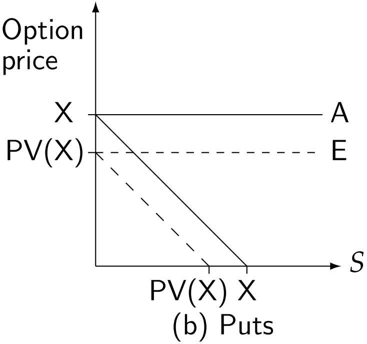
- American call $\geq$ European call $\geq \max[0, S - PV(X)]$
- If exercised now, American call is $\max[0, S - X]$
- since $X > PV(X) \Rightarrow [S - X] < [S - PV(X)]$

$\square$

Intuition:

- Exercise now $\Rightarrow$ paying now $\Rightarrow$ give up interest on X, end up with same share, $+$ accept risk that price falls below X
- Option worth more alive than dead, so sell, don't exercise





Arbitrage bounds on option prices before maturity, non-dividend paying stocks; A=American, E=European