Homework #1

Due on Monday, October 7, at 6:00pm.

The Harvard Management Company and Inflation-indexed Bonds [HBS 9-201-053].

1 HMC's Approach

This section is not graded, and you do not need to submit your answers. But you are expected to consider these issues and be ready to discuss them.

1. The HMC framing of the portfolio allocation problem.

   (a) Why does HMC focus on real returns when analyzing its portfolio allocation? Is this just a matter of scaling, or does using real returns versus nominal returns potentially change the MV solution?

   (b) There are thousands of individual risky assets in which HMC can invest. Explain why MV optimization across 1,000 securities is infeasible.

   (c) Rather than optimize across all securities directly, HMC runs a two-stage optimization. First, they build asset class portfolios with each one optimized over the securities of the specific asset class. Second, HMC combines the asset-class portfolios into one total optimized portfolio.

   In order for the two-stage optimization to be a good approximation of the full MV-optimization on all assets, what must be true of the partition of securities into asset classes?

   (d) Should TIPS form a new asset class or be grouped into one of the other 11 classes?

2. Portfolio constraints.

   The case discusses the fact that Harvard places bounds on the portfolio allocation rather than implementing whatever numbers come out of the MV optimization problem.

   (a) Similar to what we did in the lecture, write down the mathematical optimization problem which corresponds to the bounded solutions given in Exhibits 5 and 6. Do not try to solve it.

   In the lecture, we solved these optimization problems to get explicit formulas. Explain why we cannot similarly get closed-form solutions for these bounded optimizations.

   (b) Exhibits 5 shows zero allocation to domestic equities and domestic bonds across the entire computed range of targeted returns, (5.75% to 7.25%). Conceptually, why is the constraint binding in all these cases? What would the unconstrained portfolio want to do with those allocations and why?

   (c) Exhibit 6 changes the constraints, (tightening them in most cases.) How much deterioration do we see in the mean-variance tradeoff? Do you think this deterioration is worse at a targeted return of 10% or at 5% . Why?

2 Mean-Variance Optimization

- The exhibit data is in a spreadsheet posted on Canvas,but you do not need to use it; I provide it only in case you wish to do extra comparisons to the case data.
- For our analysis, we use more current data found in `assetclass_data_monthly.xlsx`.[1]
- The time-series data gives monthly returns for the 11 asset classes and "Cash" from Jan 2000 to Sep 2019.
- There are missing values, as some of the assets do not have return data until 2003, 2007, or even 2009. You can still work with this; just be sure that your computations ignore missing values when calculating the mean returns and that they calculate pairwise covariances[2] using only the rows for which both assets have a listed return.[3]
- Assume that the risk-free rate is .01/12 in months without data on Cash' returns, and that otherwise it equals the Cash return.[4]
- We will be working with the risky MV frontier for 11 risky asset classes, and we will use the excess-return formulation and frontier. To do the analysis below, you will want to subtract the risk-free rate from each of the other 11 security returns to get 11 time-series of excess returns.
- These are nominal returns—they are not adjusted for inflation, and we only worry about optimizing nominal returns.

In the questions below, annualize the statistics you report.

1. Summary Statistics
   (a) Calculate and display the mean and volatility of each asset's excess return. (Recall we use volatility to refer to standard deviation.)
   (b) Which assets have the best and worst Sharpe ratios?
2. The MV frontier.
   (a) Compute and display the weights of the tangency portfolios: $\boldsymbol{w}^{\texttt{tan}}$.
   (b) Compute the mean, volatility, and Sharpe ratio for the tangency portfolio corresponding to $\boldsymbol{w}^{\texttt{tan}}$.[5]
3. The allocation.

[1] The case does not give time-series data, so this data has been compiled outside of the case, and it intends to represent the main asset classes under consideration via liquid investment vehicles. For details on the specific securities/indexes, check the "Info" tab of the data.

[2] Instead of calculating the covariance matrix only using dates for which all asset returns are defined, calculate $\sigma_{i,j}$ using all dates for which i and j have defined returns, even if other asset returns are not defined on that day. This will not guarantee a positive definite covariance matrix, but in our case it works out without further adjustment.

[3] Python, R, and Matlab can easily handle missing data in computing means and covariances. Use the Canvas discussion board if you need advice on how to do it.

[4] In the lecture-note we considered a constant risk-free rate. It is okay that our risk-free rate changes over time, but the assumption is that investors know it's value one-period ahead of time—making it risk-free.

[5] Given this is monthly data, you may wish to display annualized results after running your calculations. Simply scale the mean by 12 and the standard deviation by $\sqrt{12}$.

(a) Compute and display the weights of MV portfolios with target returns of $\mu^p = .005$.[6]

(b) What is the mean, volatility, and Sharpe ratio for $\boldsymbol{w}^p$?

(c) Discuss the allocation. In which assets is the portfolio most long? And short?

(d) Does this line up with which assets have the strongest Sharpe ratios?

4. The allocation without inflation-indexed bonds.

   (a) Drop the inflation-indexed bonds from your return array, and recompute $\boldsymbol{w}^p$ as an 10×1 vector allocating to the remaining assets.

   (b) How does the portfolio compare to the allocation above where inflation-indexed bonds were available?

   (c) Calculate the Sharpe ratio. How much did it change?

   (d) Do you think inflation-indexed bonds are a significant expansion of the investment opportunity set for an allocator that is optimizing nominal returns, as we are here? Or should it only be relevant to HMC given their focus on inflation-adjusted returns?

5. Long-short positions.

   (a) Consider an allocation between only domestic and foreign equities. (Drop all other return columns and recompute $\boldsymbol{w}^p$ for $\mu^p = .005$.)

   (b) What is causing the extreme long-short position?

   (c) Make an adjustment to $\mu^{\text{foreign equities}}$ of $+0.001$, ($+0.012$ annualized.) Recompute $\boldsymbol{w}^p$ for $\mu^p = .005$ for these two assets.

   How does the allocation among the two assets change?

   (d) What does this say about the statistical precision of the MV solutions?

6. Robustness

   (a) Recalculate the two-asset allocation, again with the unadjusted $\mu^{\text{foreign equities}}$ and again for $\mu^p = 0.005$. This time, make one change: in building $\boldsymbol{w}^{\tan}$ and $\boldsymbol{w}^{\vee}$, do not use $\boldsymbol{\Sigma}$ as given in the formulas in the lecture. Rather, use a diaganolized $\boldsymbol{\Sigma}^D$, which zeroes out all non-diagonal elements of the full covariance matrix, $\boldsymbol{\Sigma}$.

   How does the allocation look now?

   (b) What does this suggest about the sensitivity of the solution to estimated means and estimated covariances?

   (c) HMC deals with this sensitivity by using explicit constraints on the allocation vector. Conceptually, what are the pros/cons of doing that versus modifying the formula with $\boldsymbol{\Sigma}^D$?

7. Let's divide the sample to both compute a portfolio and then check its performance out of sample.

   (a) Using only data through the end of 2016, compute $\boldsymbol{w}^p$ for $\mu^p = .005$, allocating to all 11 assets.

   (b) Calculate the portfolio's Sharpe ratio within that sample, through the end of 2016.

[6] This is monthly data, so the annualized target return is, (using simple compounding,) 8%.

(c) Calculate the portfolio's Sharpe ratio based on performance in 2017-2019.

(d) How does this out-of-sample Sharpe compare to the 2000-2016 performance of a portfolio optimized to μ^p using 2000-2016 data?

(e) How does this out-of-sample Sharpe compare to the 2017-2019 performance of a portfolio optimized to μ^p using 2017-2019 data?

(f) Recalculate $\boldsymbol{w}^p$ on 2000-2016 data using the diaganolized covariance matrix, $\boldsymbol{\Sigma}^D$. What is the performance of this portfolio in 2017-2019? Does it do better out of of sample than the portfolio constructed on 2000-2016 data using the full covariance matrix?

3 Theory Preview

These problems will not be graded, so it is optional to hand in a solution.

1. Use your base solution for the full-sample data calculated in the previous section: $\boldsymbol{w}^{\text{tan}}$. Denote the return series of this tangency portfolio as $\tilde{r}_t^{\text{tan}}$.

   (a) Calculate the correlation between $\tilde{r}_t^{\text{tan}}$.and each of the 11 underlying assets: $\rho_{i,\texttt{tan}}$ for $i = 1 \ldots 11$.

   (b) Calculate the Sharpe ratio of the tangency portfolio returns $\tilde{r}_t^{\text{tan}}$.

   (c) For each asset, i, calculate the following comparison for each asset and determine whether it is $>, =, <$:

$$\frac{\tilde{\mu}_i}{\sigma_i} \quad \substack{\geq \\ = \\ <} \quad \rho_{i,\texttt{tan}} \frac{\tilde{\mu}_{\texttt{tan}}}{\sigma_{\texttt{tan}}}$$

   (d) What do you see? Do you think this is a coincidence?