Random Variables:

A random variable is a *numerical value* that is determined by the outcome of a random experiment. It assigns a number to each possible outcome.

Example:

Toss a coin : If Heads = 1, Tails = 0
Here, the outcome of the coin toss is random, so the number we assign (0 or 1) is a random variable.

Example of rolling dice:

The experiment have random possibilities: Y=[1,2,3,4,5,6] = **Random Variables.**

Types Of Random Variables:

1. **Discrete Random Variables:**

   The Random variables are based on discrete data.

   Example: Head or Tail, Rolling Dice.

2. **Continuous Random Variables:**

   The Random variables are based on continuous data.

   Example: Height, Distance, Temperature.

Probability Distributions:

A probability distribution is a list of all of the possible outcomes of a random variable

along with their corresponding probability values.

Example: Toss of a Coin with probability

Coin Toss	**Head**	**Tail**
Probability	**1/2**	**1/2**

Problem with Distribution?

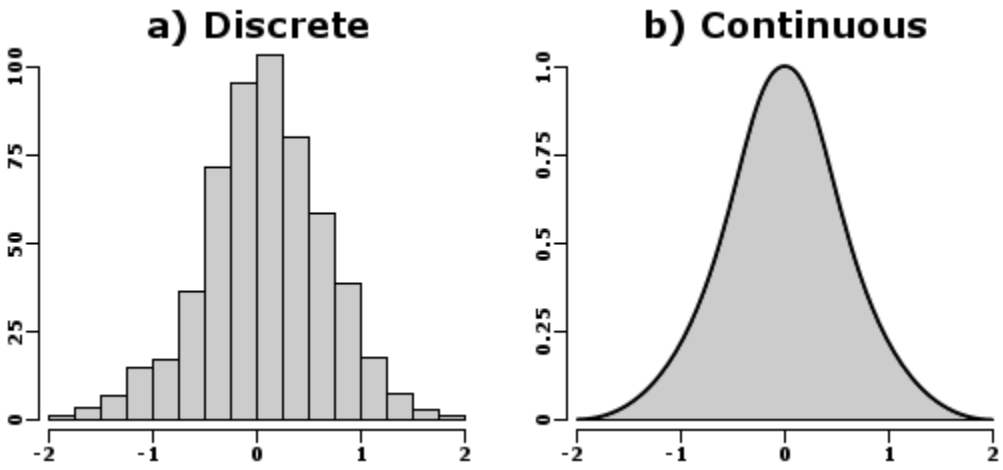
In many scenarios, the number of outcomes can be much larger and hence a table would

be tedious to write down. Worse still, the number of possible outcomes could be infinite,

Example : Height of people, Rolling 10 dice together.

So, we create a mathematical function that establishes the relationship between outcomes and probabilities which is know as probability distribution function.

Types of Probability Distributions:



Why Probability Distribution Is Important:

- Gives an idea about the shape/distribution of the data.

Probability Distribution Function:

A probability distribution function is a mathematical function that describes the probability of obtaining different values of a random variable in a particular probability distribution.

Types Of Probability Distribution Function:

1. Probability Mass Function (PMF)
2. Probability Density Function (PDF)
3. Commulitative Distribution Function of PMF
4. Commulitative Distribution Function of PDF

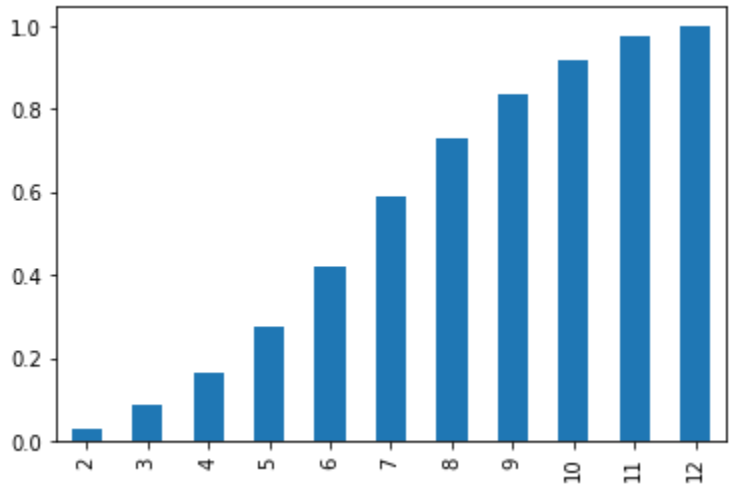
Probability Mass Function (PMF)

PMF stands for Probability Mass Function. It is a mathematical function that describes the probability distribution of a discrete random variable. The PMF of a discrete random variable assigns a probability to each possible value of the random variable. The probabilities assigned by the PMF must satisfy two conditions:

1. The probability assigned to each value must be non-negative (i.e., greater than or equal to zero).
2. The sum of the probabilities assigned to all possible values must equal 1.

Commulitative Distribution Function(CDF) of PMF:

A CDF is a function that shows the probability that a random variable can take a value up to x. We keep adding the PMF probabilities step by step, and that gives us the CDF.



Probability Density Function (PDF)

PDF stands for Probability Density Function. It is a mathematical function that describes the probability distribution of a continuous random variable.

Why Probability density instead of Probability mass function:

For continuous random variables, the probability of any exact value is 0 or closer to 0. To overcome this, we use the probability density function (PDF), which allows us to calculate the probability over an interval.

Density In PDF:

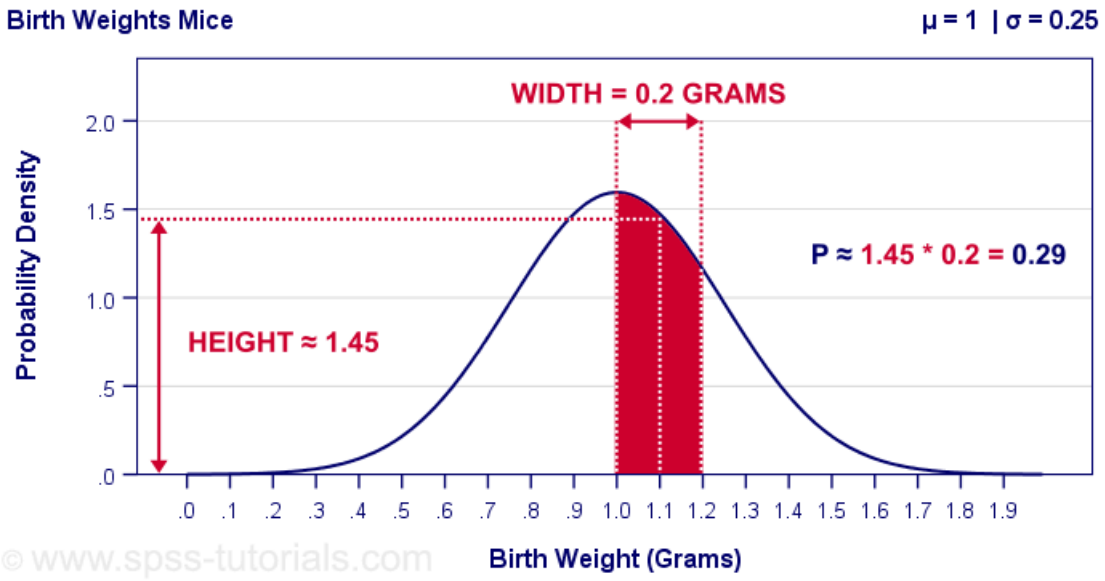
Density is a function that shows how probability is spread out over the values of a continuous random variable.

Example:

The probability that a student's exact height is 165.234 cm is 0. But the chance of having a height around 165 cm can be higher or lower. This intensity of chance is what we call density.

Formula : $\text{Density} = \frac{Count}{N \times Bin\ width}$ N = Total no of values

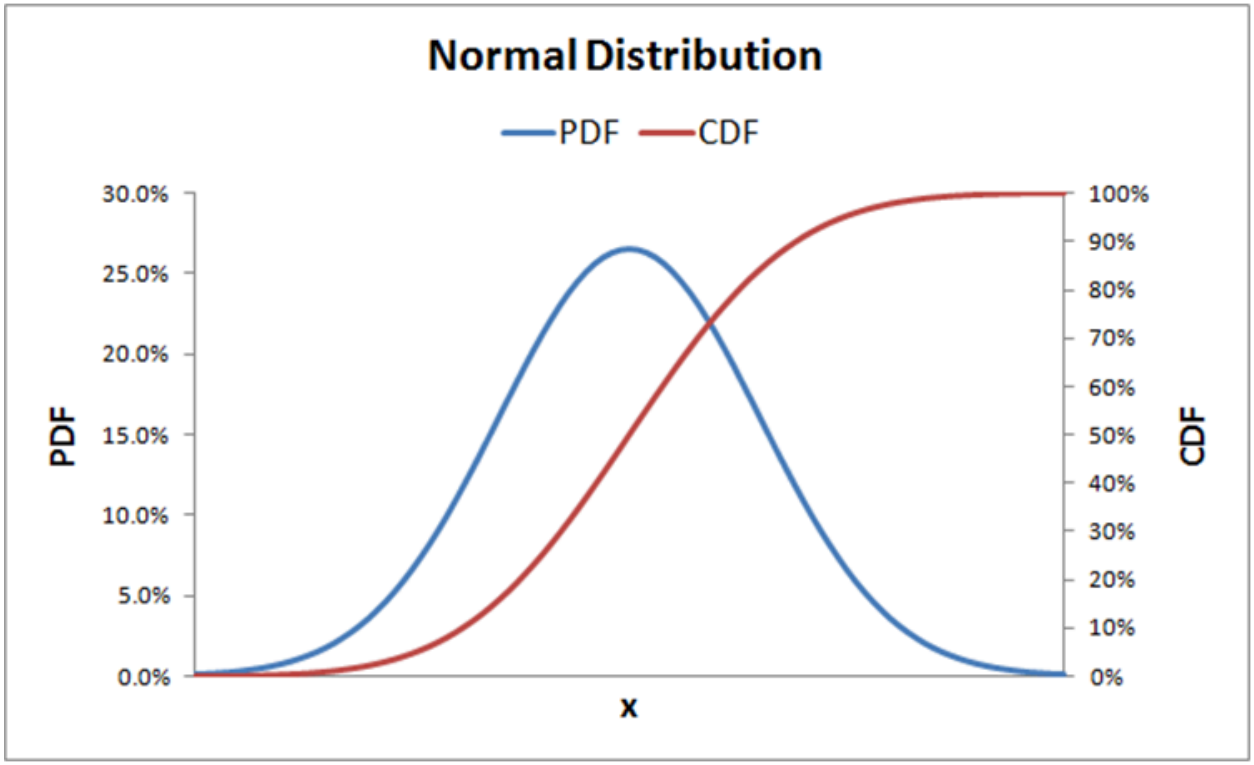
Present The Actual Probability Finding:



Probability Formula : **Prob=Density×bin width**

Commulitative Distribution Function of PDF:

A CDF is a function that shows the probability that a continuous random variable can take a value up to x. We keep adding (integrating) the PDF values step by step over the interval, and that gives us the CDF.

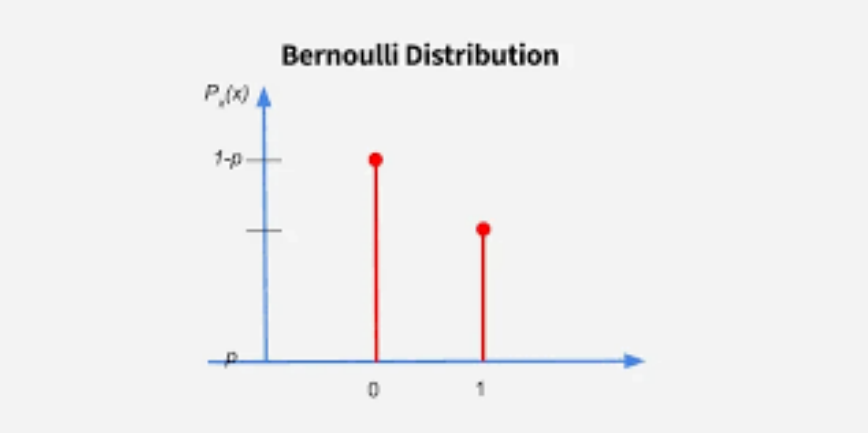


PMF Common Distributions:

1. Bernoulli Distribution:

   A Bernoulli distribution models a single trial with only two outcomes — success (1) or failure (0).

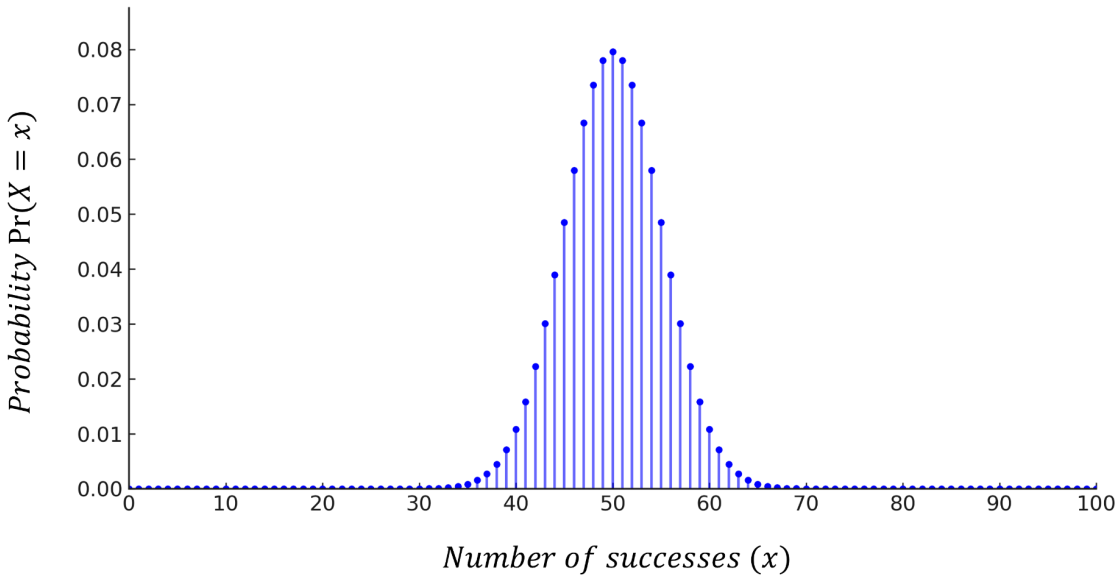
   Example: `Clickadd one time` : 1 = clicked , 0 = not clicked



2. Binomial Distribution:

Binomial distribution models the **number of successes in n independent Bernoulli trials**.

Example : 50 students Exam: (Pass , Fail)



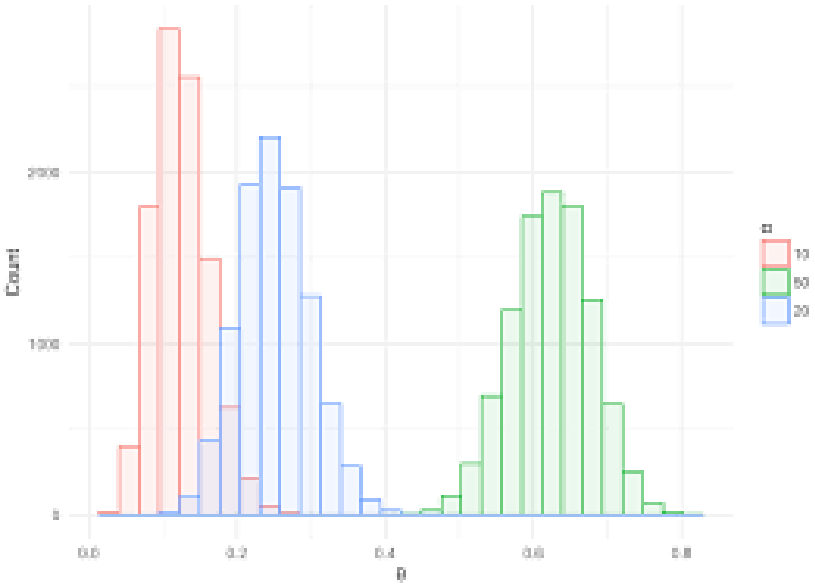
3. Multinomial Distribution:

A multinomial distribution models multiple independent trials, where each trial can have more than two possible outcomes, and it describes the counts of each outcome over all the trials.

Example :

`DiceRollCounts` : counts of 1, 2, 3, 4, 5, 6 when dice is rolled n times

`SurveyResponses` : counts of responses: Yes, No, Maybe



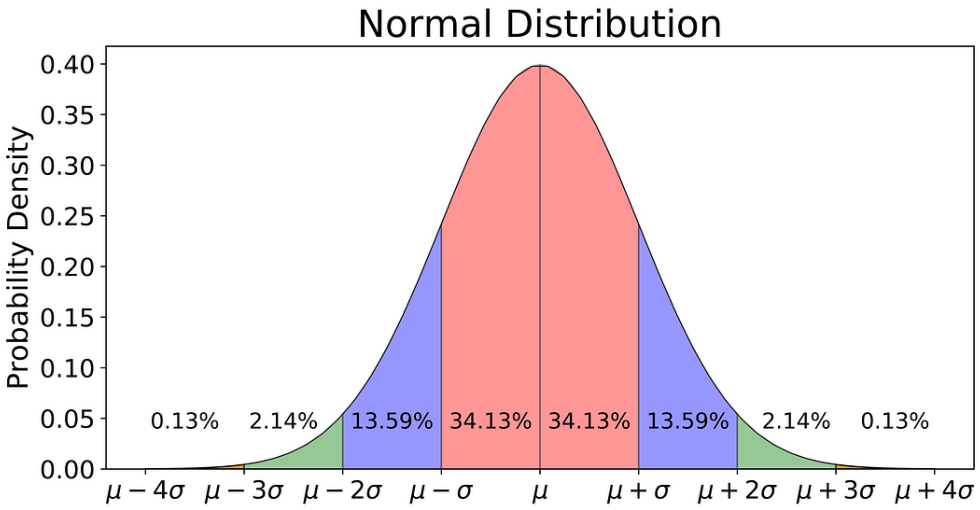
PDF Common Distributions:

1. Normal Distribution:

A normal distribution is a continuous probability distribution that is symmetric around its mean, forming a bell-shaped curve, where most of the data is concentrated around the mean, and probabilities for values further away from the mean decrease smoothly.

Characteristics:

- Symmetric around mean (μ)
- Mean = Median = Mode



2. Exponential Distribution:

Exponential distribution is a continuous probability distribution that models the time between events, where events occur continuously and independently at a constant average rate.

It is right-skewed, with most values near zero and a long tail extending to the right.

Example:

- Time between arrivals of customers at a bank.
- Waiting time in a queue.
- Mean > Median > Mode.