2023 Linear Algebra Team4 Final Project

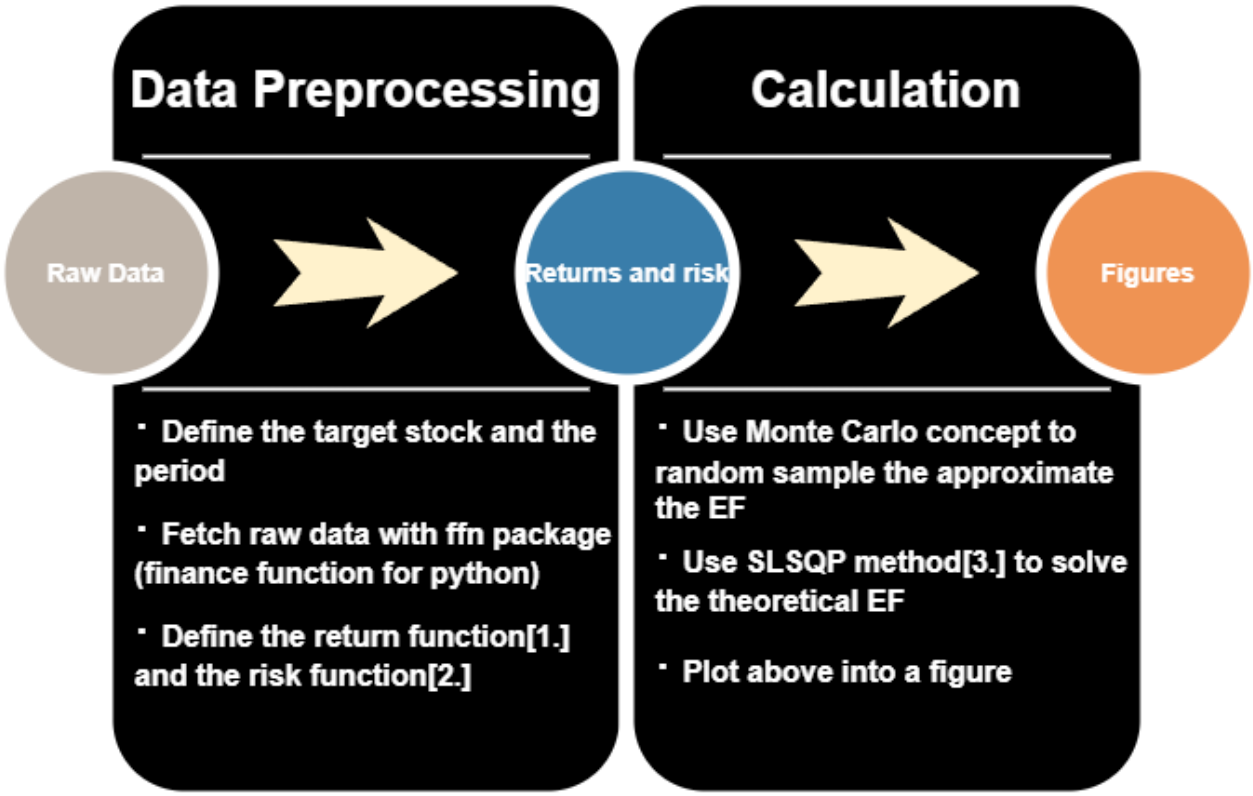
Abstract

Given an investing stock list. Find a set of portfolios that can either get best return with the same risk or get the same return with the lowest risk.

Introduction

The concept of the efficiency frontier (EF) was first introduced in the Modern Portfolio Theory (MPT) proposed by Markowitz in 1952. It states that every weight combination under the EF curve can be optimized to either maximize the return or lower the risk.
In this project we implement the EF in the MPT with python code to for investors to optimize their portfolio. We also managed to find the lowest risk weight combination on the EF and plot the EF as a graph.

Material and methods



[1.] The return of the portfolio equals to the weighted average of the individuals' $\boldsymbol{R_p} = \sum_{i=1}^{n} \boldsymbol{W_i R_i}$

[2.] The standard deviation of the portfolio can be represented as the square root of the variance shown below.

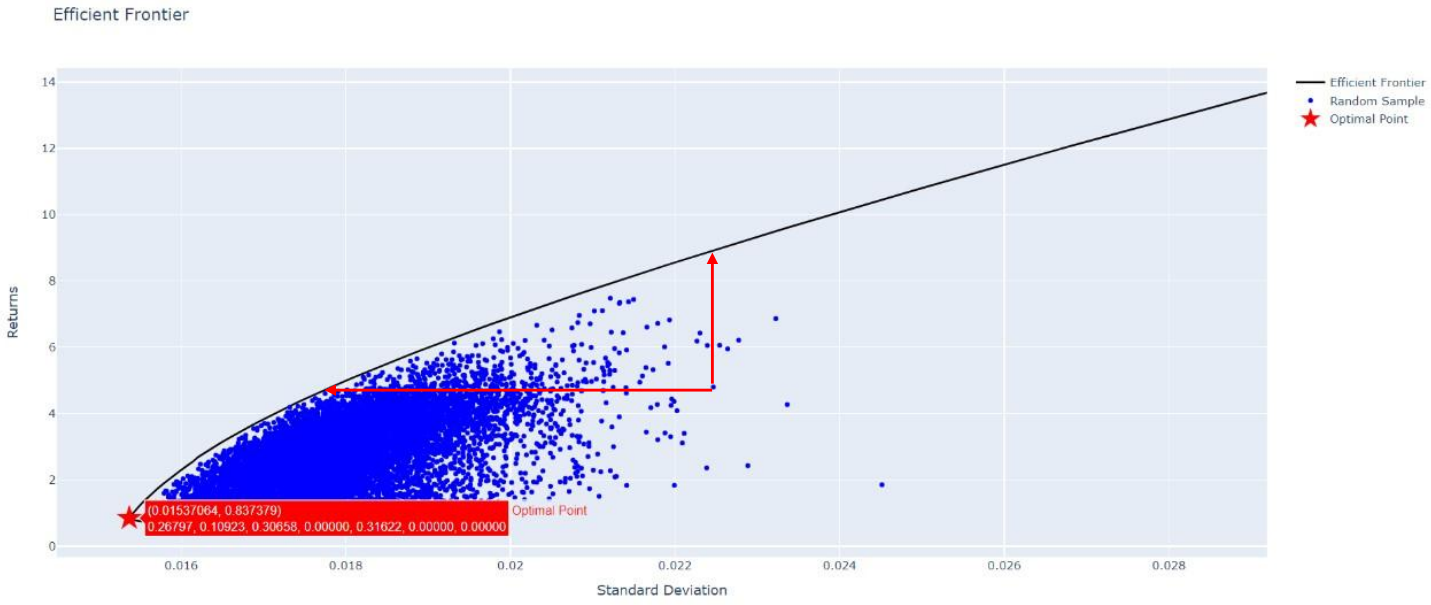
$$\boldsymbol{Var(P) = W^T * \sigma * W} \quad \textbf{where} \quad \boldsymbol{W = [W_a\, W_b \ldots W_i]}$$

$$\boldsymbol{\sigma = \begin{bmatrix} \sigma_{aa} & \cdots & \sigma_{ai} \\ \vdots & \ddots & \vdots \\ \sigma_{ia} & \cdots & \sigma_{ii} \end{bmatrix}}$$

[3.] The SLSQP method leverages the idea of Lagrange multiplier to solve the equation and Gradient Descent to speed up the process.

Results and Discussion

Example: AAPL, ASML, GOOG, META, MSFT, NVDA, TSLA in past decade
The weight combination of lowest risk/return
[0.334, 0.104, 0.267, 0, 0.294, 0, 0]
Demo link: https://youtu.be/IRMFLBSN2zQ



1. **The higher return comes with higher risk. When it comes to the top right corner of the EF, the NVDA dominants the whole portfolio since it has the largest risk and return in the past decade.**
2. **Every blue point (random sample) under the EF curve can be moved leftward to lower the risk or moved upward to higher the return until it touched the EF curve.**

Conclusion

We can now input the stocks we want to investigate and select the weight combinations on the EF according to our risk tolerance!

Appendix

Code is submitted on the E3 system
Work Distribution Chart

Name	Work distribution	% account for
張仲瑜	Topic concept Code implementation Report completion	28%
何昕叡	Introduction presentation Report completion	18%
徐睿廷	Methods presentation Report completion	18%
楊竣傑	Methods presentation Report completion	18%
陳炤宇	discussion presentation Report completion	18%
黃亦勤	None	0%

References

Formula derivation
https://vocus.cc/article/60f2edc4fd89780001629c67