Josephson Junctions and the Superconducting Quantum Interference Device

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Abstract

This experiment focuses on studying the behavior of Josephson junctions in superconducting quantum interference devices (SQUIDs). Using the Mr. SQUID educational kit, we explore three main objectives: measuring the current-voltage characteristics (IVC) of the SQUID, studying the voltage vs. flux (V-Φ) behavior, and observing Shapiro steps under microwave radiation. These objectives provide insights into the quantum properties of the Josephson junction and its practical applications in highly sensitive magnetic field measurements. The experimental data were collected through sweeping currents and magnetic flux, with key features like flux quantization and phase dynamics analyzed through non-linear curve fitting.

1 Introduction

Josephson junctions plays a crucial role in modern quantum technology, particularly in applications like quantum computing and highly sensitive magnetometers. A Josephson junction is a quantum mechanical device consisting of two superconductors separated by a thin insulating barrier. The junction allows a current to flow without resistance under specific conditions, governed by the Josephson effect [1].

Superconducting Quantum Interference Devices (SQUIDs) are circuits that utilize Josephson junctions to measure extremely small changes in magnetic fields. The SQUID operates based on the principle of flux quantization, where the magnetic flux through a superconducting loop is quantized in units of the flux quantum ($\Phi_0 = \frac{h}{2e}$).

The behavior of Josephson junctions, particularly in SQUIDs, is fundamental to understanding quantum mechanics in superconducting circuits. This experiment aims to investigate the key characteristics of Josephson junctions, such as their current-voltage relationship, sensitivity to magnetic flux, and response to external microwave radiation. These measurements are crucial for applications in precision metrology and quantum computation, where understanding the quantum dynamics of such devices is essential.

We first aim to measure the Current-Voltage Characteristic (IVC) of the SQUID to determine the critical current (I_c) and SQUID resistance in non-superconducting state. We analyse the transition from the superconducting state to the resistive state.

Next, we measure the Voltage vs. Flux (V-Φ) relation at fixed bias current. This objective involves observing how the voltage across the SQUID varies periodically with applied magnetic flux, demonstrating flux quantization and phase coherence in the superconducting loop.

Under external signal of high-frequency, the Josephson junction exhibits quantized voltage steps, known as Shapiro steps, due to the AC Josephson effect. We will measure and analyze these steps to verify the synchronization between Josephson oscillations and external signal.

2 Measurements

2.1 Current-Voltage Characteristic (IVC) Measurement

In this measurement, we apply a triangular waveform to sweep the current through the SQUID while recording the voltage across the junction. Reason to use the triangular waveform is to observe the behaviour change in continuous and controlled way. The main objective is to observe the critical current, I_c, at which the SQUID transitions from zero voltage (superconducting state) to a finite voltage (resistive state). For the schematic diagram of the Mr. SQUID electronics box see the fig. 1. This '4-point' measurement circuit consists of a source (bias) to supply the current and amplifier to measure the voltage. Here, V, the voltage drop across the SQUID has a gain of 10,000. V_I, the voltage drop across the bias resistor R_b has a gain of 1000[3].

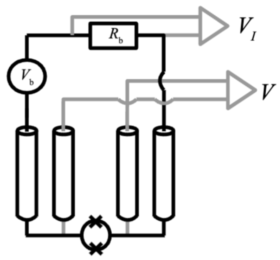

Figure 1: Schematic of the Mr. SQUID electronic toolbox[2].

The current is swept using a function generator, and the voltage is measured across the SQUID with an amplifier. We observe the voltage signal across the SQUID remains zero until the bias current reaches a certain value and then it becomes non-zero (shown in fig. 2). This demonstrate the transition of SQUID from superconducting state to resistive state.

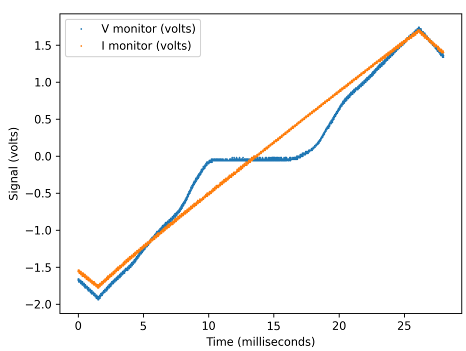

Figure 2: **SQUID current-voltage characteristic.** Signal data from monitor plotted against time.

2.2 Voltage vs. Flux (V-Φ) Measurement

Here, we apply a constant current bias and vary the magnetic flux through the SQUID loop using a flux bias coil. As the magnetic flux is swept, we measure the voltage response of the SQUID. This measurement is aimed at demonstrating the periodic voltage oscillations due to flux quantization.

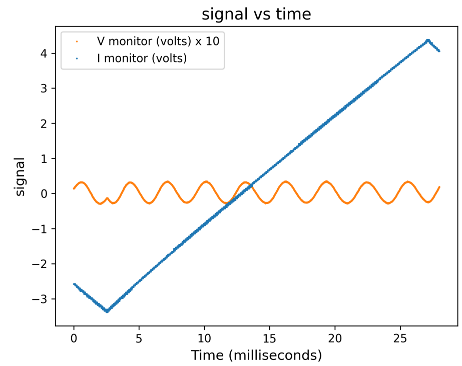

Figure 3: **SQUID voltage vs flux curve.** Signal data from monitor plotted against time. Voltage signal has been multiplied by 10 to improve visibility.

A constant current is applied to the SQUID, and the external magnetic flux is varied through the coil. The resulting voltage vs. flux curve is recorded and plotted, showing periodic behavior (shown in fig. 3).

2.3 Shapiro Steps Observation

In this step, the SQUID is irradiated with microwave radiation at a frequency of 44 GHz. The objective is to observe Shapiro steps in the IVC, which result from the synchronization of the Josephson oscillations with the external microwave signal.

We apply microwave radiation via an antenna positioned near the SQUID and sweep the current through the device. Voltage steps are observed at quantized values given by $\bar{V_n} = n\Phi_0 f_{\text{ext}}$, where f_{ext} is the microwave frequency and Φ_0 is flux quantum. Signal data from monitor can be seen in fig. 4

3 Analysis

3.1 Current-Voltage Characteristic (IVC) Measurement

The IVC data was analyzed by fitting the experimental results to the theoretical model for Joseph-

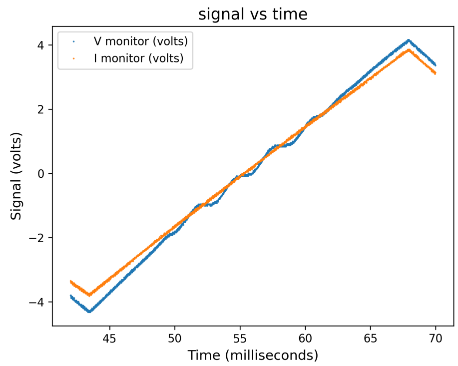

Figure 4: **IVC curve for Shapiro steps.** Signal data current and voltage from monitor plotted against time.

son junctions in the over-damped regime. Theoretically, voltage across the junction is related to the current by the expression for $i > 1$:

$$V(I) = I_c R\sqrt{i^2 - 1} \tag{1}$$

where, $V(I)$ is the voltage, I_c is the critical current, R is the resistance in the normal state, $i = I/I_c$ is the normalized current. Non-linear curve fitting was applied to the experimental data to determine the values of I_c & R.

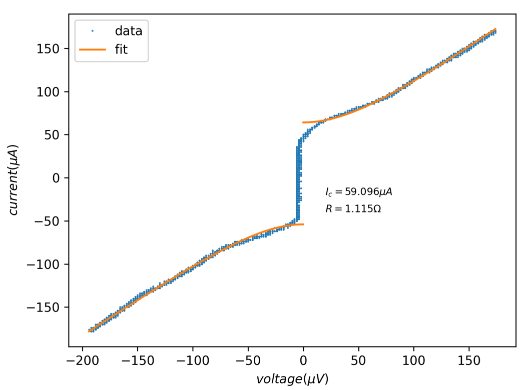

Figure 5: **SQUID current-voltage characteristic.** Approximated values of I_c & R were obtained from the curve fitting.

We fitted the curve for $V > 0$ & $V < 0$ separately and took the average of the two values of I_c & R we obtained from the two curves.

3.2 Voltage vs. Flux (V-Φ) Measurement

The V-I_{coil} data was fitted to a sine wave function, as the voltage across the SQUID varies periodically with applied flux coil current:

$$V(I_{\text{coil}}) = Asin(BI_{\text{coil}} + C) + D \tag{2}$$

We then use the period of I_{coil} to determine the mutual inductance. The mutual inductance can be calculated from the period of coil current $(I_{\text{coil}}) = 2\pi/\text{B} = 2\pi/0.06613 = 95\ \mu\text{A}$.

$$M = \frac{\Phi_0}{\Delta I} = \frac{2.07 \times 10^{-15}}{95} = 0.14nH \tag{3}$$

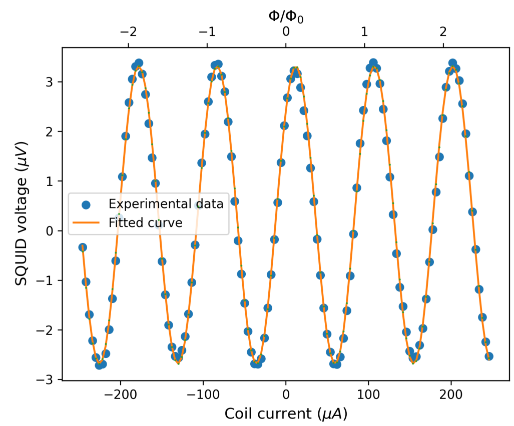

Figure 6: **The SQUID voltage vs Flux characteristic.**

In the fig. 6, we see the sinusoidal change in the the voltage across SQUID as we increase the current in the flux coil.

3.3 Shapiro Steps Observation

Shapiro steps were observed at quantized voltage intervals. The voltage steps were compared to the theoretical values:

$$\bar{V_n} = n\Phi_0 f_{\text{ext}} \tag{4}$$

where, $f_{\text{ext}} = 44\text{GHz}$ and Φ_0 can be calculated as below:

$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15} V \cdot s \tag{5}$$

Using equation (4) & (5), we can get the theoretical value of one Shapiro step $(\bar{V_1})$ as follows,

$$\bar{V_1} = 2.07 \times 10^{-15} \cdot 44 \times 10^9 = 91.08\mu V \quad (6)$$

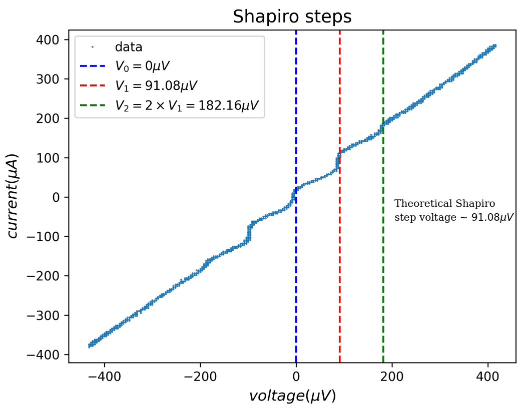

Figure 7: **Shapiro steps.** The current-voltage characteristic of an under-damped Josephson junction drive with microwave radiation. Annotations on the plot showing theoretical values of step positions.

In the fig. 7, Shapiro steps are separated by approximately $91\mu V$ which is the theoretical value ($91.08\mu V$) calculated above. The step heights were found to match the theoretical values, confirming the presence of the AC Josephson effect.

4 Conclusion

In this experiment, we successfully explored the fundamental properties of Josephson junctions and SQUIDs by examining their current-voltage characteristics (IVC), voltage response to varying magnetic flux, and the emergence of Shapiro steps under microwave radiation.

Key findings include the clear observation of periodic oscillations in the voltage as a function of magnetic flux, validating the quantized behavior of flux through a superconducting loop. Additionally, the Shapiro steps, appearing as quantized voltage plateaus under microwave irradiation, confirmed the transformer like behavior of the SQUIDs.

Looking ahead, improvements could be made by using a more sensitive measurement system to reduce noise. In particular, we had difficulty obtaining the Shapiro steps due to external electromagnetic radiations interfering with our setup. This could be improved by adding more insulation to the circuit.

5 Methods

The python code used for data analysis can be found in the github link. We used the Python package scipy.optimize.curve_fit[4] for fitting the experimental data collected during the experiments.

References

[1] B.D. Josephson. Possible new effects in superconductive tunnelling. Physics Letters, 1(7):251 – 253, 1962.

[2] Josephson Junctions and the Superconducting Quantum Interference Device. Haviland, D. B. (2024)

[3] Mr. SQUID User's Guide. Version 6.7, Star Cryoelectronics, LLC. Santa Fe, NM, USA. https://starcryo.com/mr-squid/

[4] https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html