

# Technical Talk - PhD Project

Kamal Sharma

Department of Mathematics, University of Hamburg

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Universität Hamburg

DER FORSCHUNG | DER LEHRE | DER BILDUNG

# Introduction

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# What is it about?

Development of a structure-preserving idealized stochastic climate model

- ▶ **Climate model:** set of equations modelling coupled ocean-atmosphere dynamics
- ▶ **Idealized:** 2D model  $\rightarrow$  solves for velocity, temperature, and pressure
- ▶ **Stochastic:** has stochastic/random terms (think of Brownian motion)
- ▶ **Structure-preserving:** preserves underlying geometric and physical structure

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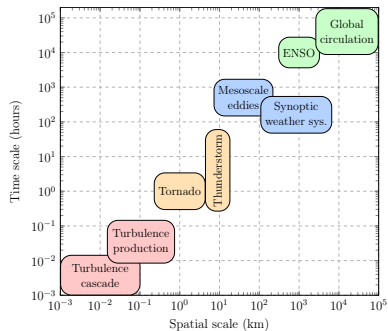
**We are solving stochastic PDEs which model ocean-atmosphere interactions!**

# Introduction

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Why add stochastic terms?

# Multiscale phenomena

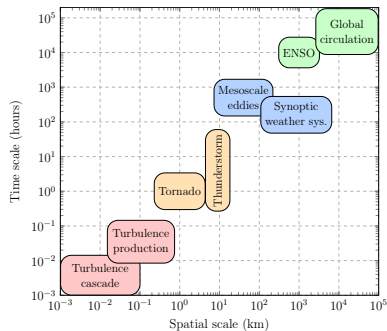


Spatial and temporal scales of various meteorological phenomena<sup>1</sup>

- ▶ Numerical models have fixed resolution
- ▶ Processes occurring below the model resolution are not captured → leads to prediction error!

<sup>1</sup>Roland B. Stull. *Practical Meteorology: An Algebra-Based Survey of Atmospheric Science*. 2017.

# Multiscale phenomena



Spatial and temporal scales of various meteorological phenomena<sup>1</sup>

- ▶ Numerical models have fixed resolution
- ▶ Processes occurring below the model resolution are not captured → leads to prediction error!
- ▶ **Solution:** Parameterization or subgrid-scale modeling

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# Parameterization

- ▶ Additional terms are added to account for the missing effect of unresolved/small scales
- ▶ Parameterization techniques can be divided into two categories:
  1. Deterministic
  2. Stochastic



# Parameterization

- ▶ Additional terms are added to account for the missing effect of unresolved/small scales
- ▶ Parameterization techniques can be divided into two categories:
  1. Deterministic
  2. Stochastic
- ▶ We explored **stochastic parameterization** : stochastic advection by Lie transport (SALT)
- ▶ Why SALT?
  - ▶ preserves physical structure → reliable long simulations
  - ▶ better forecast skills → uncertainty due to unresolved transport processes can be quantified
  - ▶ conducive to data-driven approaches → can be estimated from observations/satellite data

## Research objectives

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# Main objectives

1. Explore the efficacy of stochastic parameterization (in terms of UQ skills) for a climate model
  - ▶ Previous studies focused on Euler, quasi-geostrophic, and shallow water equations
2. Numerically solve the idealized stochastic climate model

# The climate model

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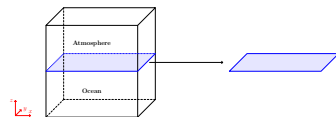
# Model equations

2D climate model equations:

$$\begin{aligned}
 \text{Atmosphere : } d\mathbf{u}^a + ((\mathbf{u}^a dt + \sum_i \xi_i \circ dW^i) \cdot \nabla) \mathbf{u}^a + \frac{1}{Ro^a} \hat{\mathbf{z}} \times (\mathbf{u}^a dt + \sum_i \xi_i \circ dW^i) \\
 + \sum_i (u_1^a \nabla \xi_{i,1} + u_2^a \nabla \xi_{i,2}) \circ dW^i = (-\frac{1}{C^a} \nabla \theta + \frac{1}{Re^a} \Delta \mathbf{u}^a) dt, \\
 d\theta^a + \nabla \cdot (\theta^a (\mathbf{u}^a dt + \sum_i \xi_i \circ dW^i)) = (\gamma(\theta^a - \theta^o) + \frac{1}{Pe^a} \Delta \theta^a) dt,
 \end{aligned}$$

$$\begin{aligned}
 \text{Ocean : } \frac{\partial \mathbf{u}^o}{\partial t} + (\mathbf{u}^o \cdot \nabla) \mathbf{u}^o + \frac{1}{Ro^o} \hat{\mathbf{z}} \times \mathbf{u}^o + \frac{1}{Ro^o} \nabla p^a &= \sigma(\mathbf{u}^o - \mathbb{E} \mathbf{u}_{sol}^a) + \frac{1}{Re^o} \Delta \mathbf{u}^o, \\
 \nabla \cdot \mathbf{u}^o &= 0, \\
 \frac{\partial \theta^o}{\partial t} + \mathbf{u}^o \cdot \nabla \theta^o &= \frac{1}{Pe^o} \Delta \theta^o.
 \end{aligned}$$

Incompressible/compressible Navier-Stokes + Advection-diffusion equations



Model domain

$\mathbf{u}$  : velocity,  
 $\theta$  : temperature,  
 $p$  : pressure

# How to get the stochastic terms?

- ▶ Difference between Lagrangian trajectories at different resolutions:

$$\sum_i \xi_i dW^i \approx \mathbf{u}_{true} dt - \mathbf{u} dt$$

$\mathbf{u}_{true}$  : true velocity

$\mathbf{u}$  : mean flow velocity i.e. the large-scale component of  $\mathbf{u}_{true}$

- ▶ Ways to estimate the true velocity  $\mathbf{u}_{true}$ :
  1. Observation data (for example, from satellites)
  2. Synthetic data (from high-resolution numerical simulations)

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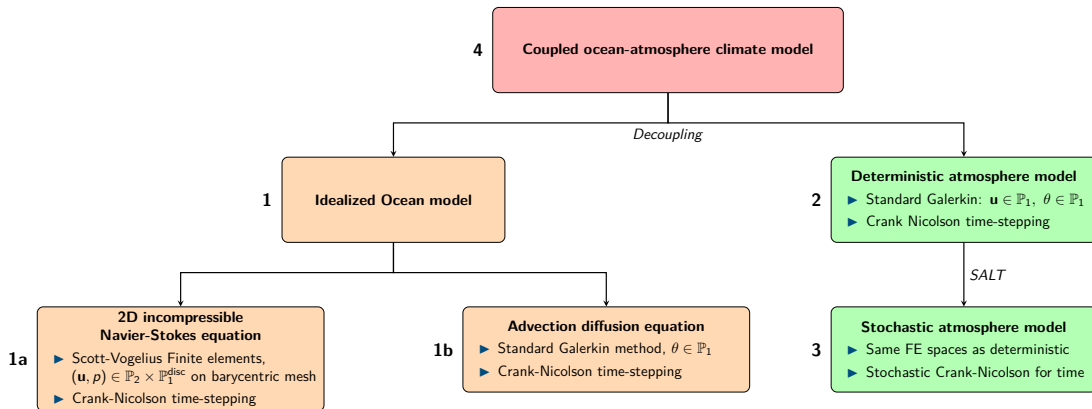
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  2. Synthetic data (from high-resolution numerical simulations)
- ▶ **We use synthetic data (proof of concept)!**

# Our methodology



Incremental validation approach - solve simpler components first, verify at each step!



# Numerical simulation of the climate model

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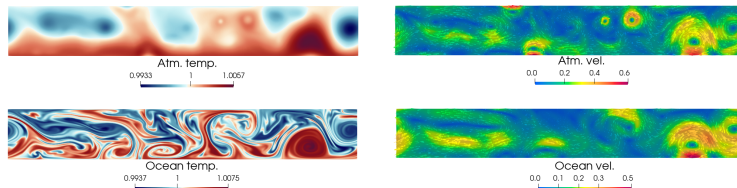
# Data collection and model calibration

- We run the deterministic climate model in high-res. ( $\Delta x = 1/128 \sim 30$  km)

Initial condition



↓  
At t=45



- Analyze data → extract small-scale features (using statistical algorithms) → obtain  $\sum_i \xi_i dW^i$

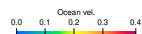
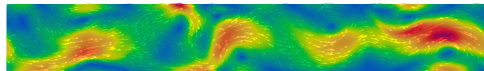
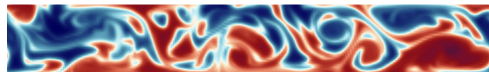
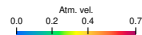
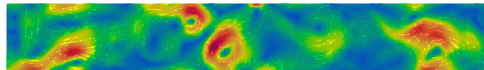
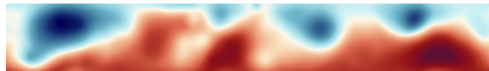
# Numerical simulation of the climate model

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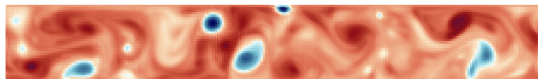
Stochastic model simulation

# Stochastic model simulation

- ▶ We run the stochastic model on a coarse grid ( $\Delta x = 1/32 \sim 120$  km)
- ▶ Initial conditions correspond to coarse grained velocity and temperature fields at  $t = 25$



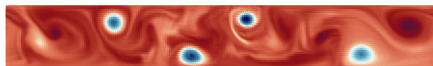
Atmospheric vorticity at  $t = 25$



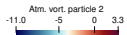
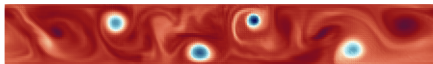
# Simulation results

- ▶ Stochastic model is simulated for 20 time units ( $t = 25$  to  $t = 45$ )
- ▶ Stochastic vs. deterministic (without parameterization) model results at  $t = 35$

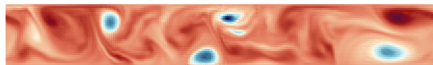
Particle 1



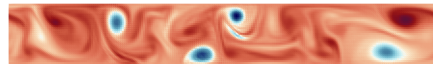
Particle 2



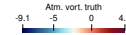
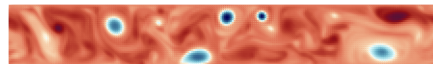
Particle 3



Deterministic solution



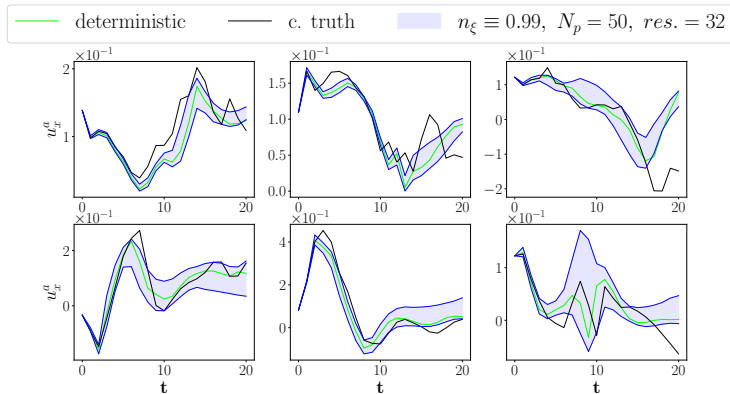
True solution



# Simulation results

- Comparison between the stochastic ( $\mu \pm 1\sigma$ ) and deterministic sol. at 6 grid points:

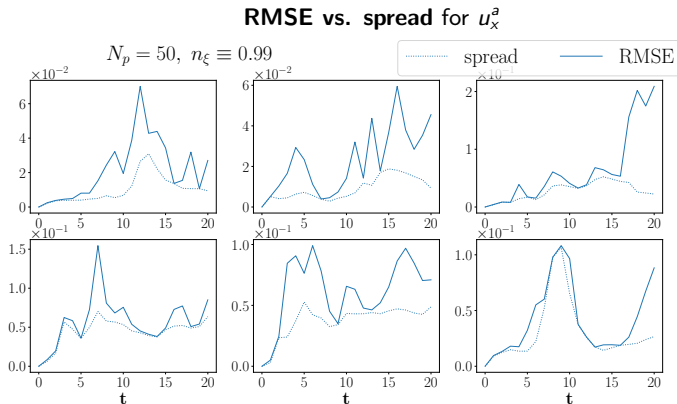
## Atmospheric velocity (x component)



1. Ensemble spread increases over time
2. Stochastic model captures the truth for 5 to 10 time units

# Uncertainty quantification

- Plots for the evolution of ensemble spread and RMSE at six different locations on the grid



- Ensemble spread  $\propto$  RMSE (for at least 15 time units)
- Error in the stochastic solution can be estimated by its own spread

# Summary

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# Summary & outcomes

- ▶ **Aim:** solve a stochastic coupled ocean-atmosphere climate model
- ▶ **Novelty:** data-driven model
  - ▶ Stochastic terms (modelling unresolved processes) are obtained from observation data
  - ▶ Exhibit desired conservation properties
  - ▶ High-res. model sim. data is used to estimate  $\xi_i \rightarrow$  data analysis & statistical modelling
- ▶ **Methodology** for solving the model equations:
  - ▶ Solve simpler models first
  - ▶ build the climate model step-by-step
  - ▶ verify numerical schemes at each step
- ▶ **Results:**
  - ▶ Stochastic model solution captures the truth for some time units
  - ▶ Shows promising UQ test results: ensemble spread size  $\propto$  RMSE
- ▶ **Outcome:** first step towards using SALT in a highly complex coupled system!

Thank you for your attention!