

Technical Talk - PhD Project

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Introduction

What is it about?

Development of a **structure-preserving idealized stochastic climate model**

- ▶ **Climate model:** set of equations modelling coupled ocean-atmosphere dynamics
- ▶ **Idealized:** 2D model → solves for velocity, temperature, and pressure
- ▶ **Stochastic:** has stochastic/random terms (think of Brownian motion)
- ▶ **Structure-preserving:** preserves underlying geometric and physical structure

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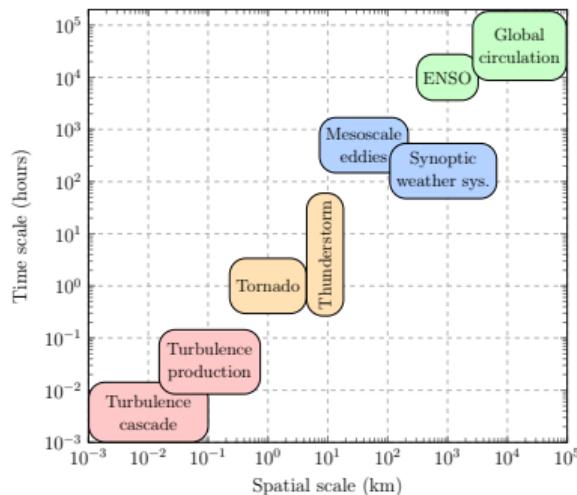
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We are solving stochastic PDEs which model ocean-atmosphere interactions!

Introduction

Why add stochastic terms?

Multiscale phenomena

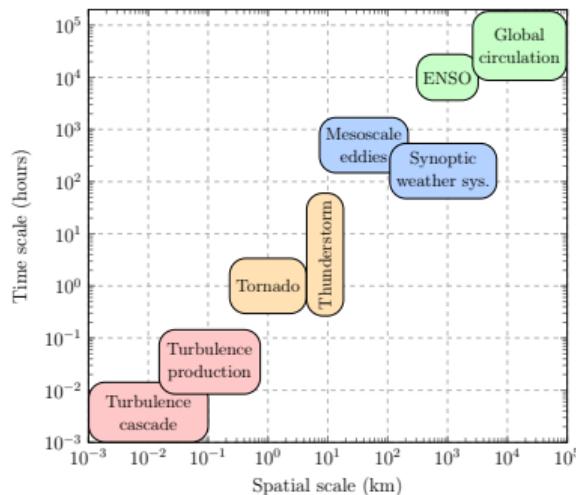


Spatial and temporal scales of various meteorological phenomena¹

- ▶ Numerical models have fixed resolution
- ▶ Processes occurring below the model resolution are not captured → leads to prediction error!

¹Roland B. Stull. *Practical Meteorology: An Algebra-Based Survey of Atmospheric Science*. 2017.

Multiscale phenomena



Spatial and temporal scales of various meteorological phenomena¹

- ▶ Numerical models have fixed resolution
- ▶ Processes occurring below the model resolution are not captured → leads to prediction error!
- ▶ **Solution:** Parameterization or subgrid-scale modeling

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Parameterization

- ▶ Additional terms are added to account for the missing effect of unresolved/small scales
- ▶ Parameterization techniques can be divided into two categories:
 1. Deterministic
 2. Stochastic

Parameterization

- ▶ Additional terms are added to account for the missing effect of unresolved/small scales
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 1. Deterministic
 2. Stochastic
- ▶ We explored **stochastic parameterization** : stochastic advection by Lie transport (SALT)
- ▶ Why SALT?
 - ▶ preserves physical structure → reliable long simulations
 - ▶ better forecast skills → uncertainty due to unresolved transport processes can be quantified
 - ▶ conducive to data-driven approaches → can be estimated from observations/satellite data

Research objectives

Main objectives

1. Explore the efficacy of stochastic parameterization (in terms of UQ skills) for a climate model
 - ▶ Previous studies focused on Euler, quasi-geostrophic, and shallow water equations
2. Numerically solve the idealized stochastic climate model

The climate model

Model equations

2D climate model equations:

$$\text{Atmosphere : } d\mathbf{u}^a + ((\mathbf{u}^a dt + \sum_i \boldsymbol{\xi}_i \circ dW^i) \cdot \nabla) \mathbf{u}^a + \frac{1}{Ro^a} \hat{\mathbf{z}} \times (\mathbf{u}^a dt + \sum_i \boldsymbol{\xi}_i \circ dW^i)$$

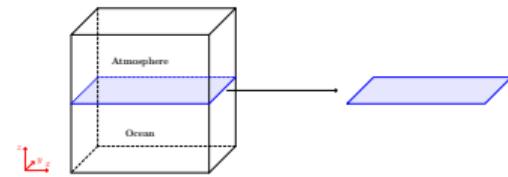
$$+ \sum_i (u_1^a \nabla \xi_{i,1} + u_2^a \nabla \xi_{i,2}) \circ dW^i = (-\frac{1}{C^a} \nabla \theta + \frac{1}{Re^a} \Delta \mathbf{u}^a) dt,$$

$$d\theta^a + \nabla \cdot (\theta^a (\mathbf{u}^a dt + \sum_i \boldsymbol{\xi}_i \circ dW^i)) = (\gamma(\theta^a - \theta^o) + \frac{1}{Pe^a} \Delta \theta^a) dt,$$

$$\text{Ocean : } \frac{\partial \mathbf{u}^o}{\partial t} + (\mathbf{u}^o \cdot \nabla) \mathbf{u}^o + \frac{1}{Ro^o} \hat{\mathbf{z}} \times \mathbf{u}^o + \frac{1}{Ro^o} \nabla p^a = \sigma(\mathbf{u}^o - \mathbb{E}\mathbf{u}_{sol}^a) + \frac{1}{Re^o} \Delta \mathbf{u}^o,$$

$$\nabla \cdot \mathbf{u}^o = 0,$$

$$\frac{\partial \theta^o}{\partial t} + \mathbf{u}^o \cdot \nabla \theta^o = \frac{1}{Pe^o} \Delta \theta^o.$$



Model domain

- \mathbf{u} : velocity,
- θ : temperature,
- p : pressure

Incompressible/compressible Navier-Stokes + Advection-diffusion equations

How to get the stochastic terms?

- ▶ Difference between Lagrangian trajectories at different resolutions:

$$\sum_i \xi_i dW^i \approx \mathbf{u}_{true} dt - \mathbf{u} dt$$

\mathbf{u}_{true} : true velocity

\mathbf{u} : mean flow velocity i.e. the large-scale component of \mathbf{u}_{true}

- ▶ Ways to estimate the true velocity \mathbf{u}_{true} :

1. Observation data (for example, from satellites)
2. Synthetic data (from high-resolution numerical simulations)

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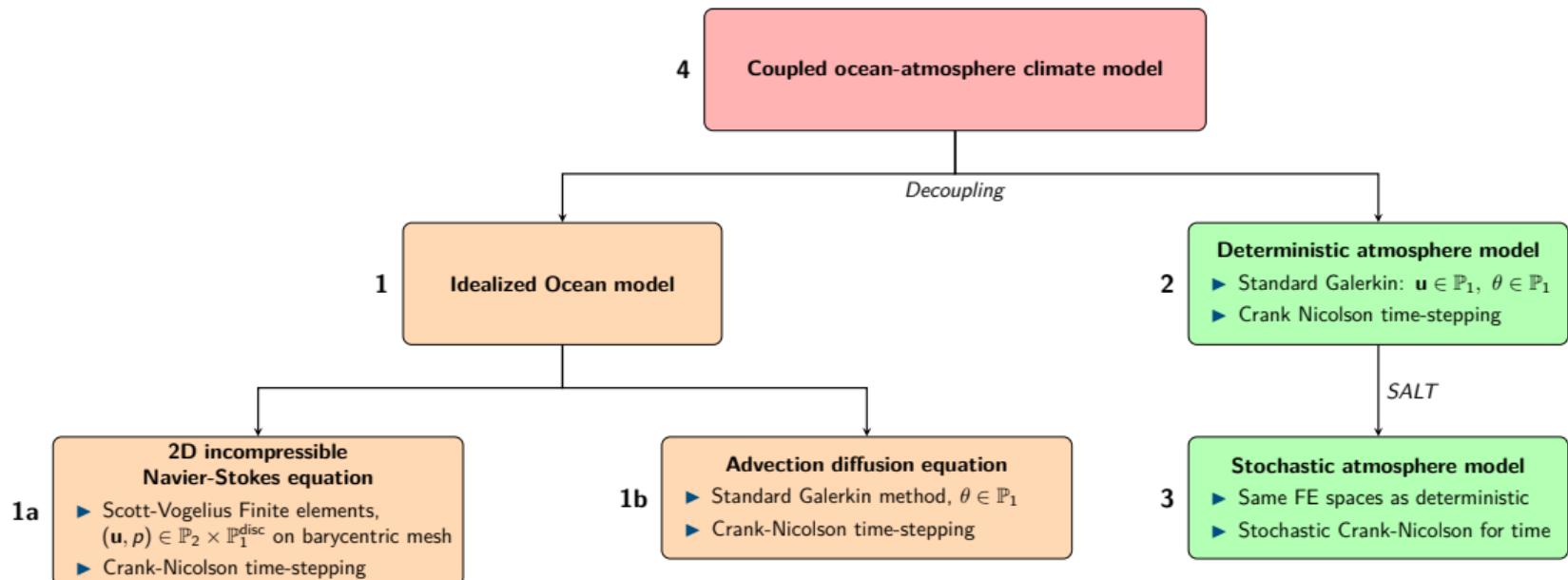
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- ▶ Ways to estimate the true velocity \mathbf{u}_{true} :
 1. Observation data (for example, from satellites)
 2. Synthetic data (from high-resolution numerical simulations)
- ▶ **We use synthetic data (proof of concept)!**

Our methodology



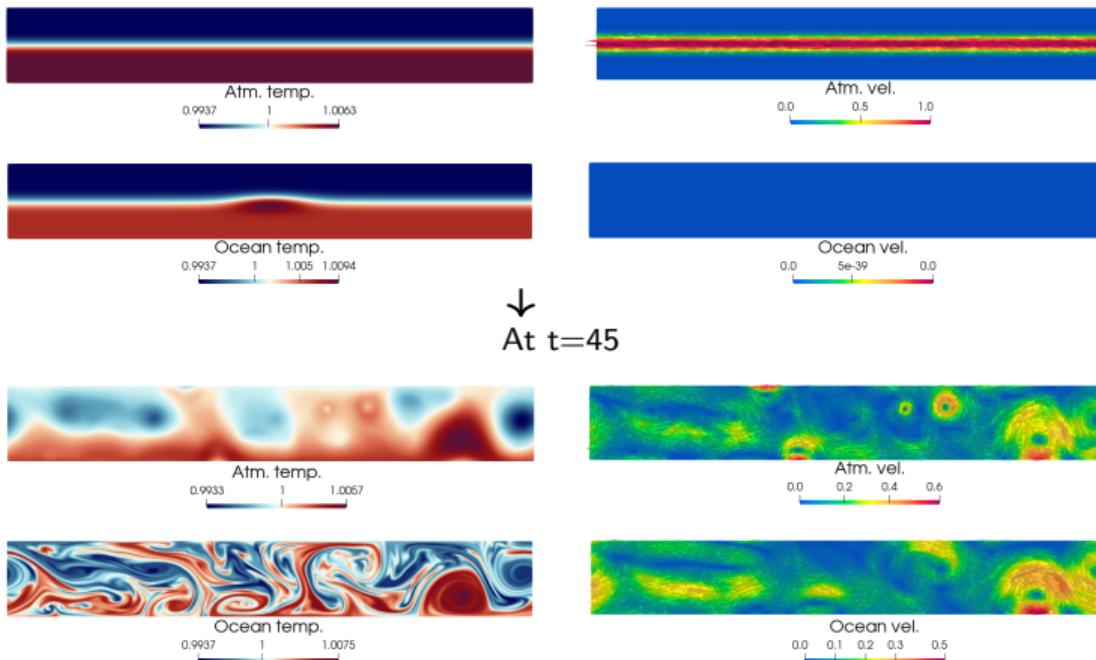
Incremental validation approach - solve simpler components first, verify at each step!

Numerical simulation of the climate model

Data collection and model calibration

- We run the deterministic climate model in high-res. ($\Delta x = 1/128 \sim 30 \text{ km}$)

Initial condition



- Analyze data → extract small-scale features (using statistical algorithms) → obtain $\sum_i \xi_i \, dW^i$

Numerical simulation of the climate model

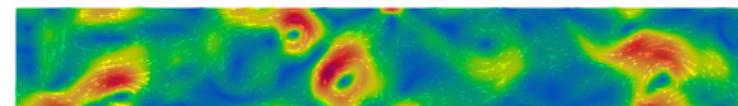
Stochastic model simulation

Stochastic model simulation

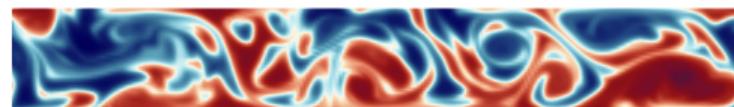
- We run the stochastic model on a coarse grid ($\Delta x = 1/32 \sim 120$ km)
- Initial conditions correspond to coarse grained velocity and temperature fields at $t = 25$



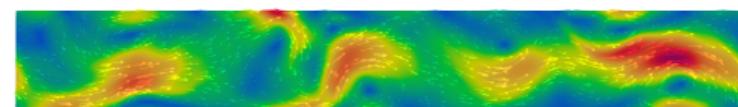
Atm. temp.
0.9934 1 1.0054



Atm. vel.
0.0 0.2 0.4 0.7

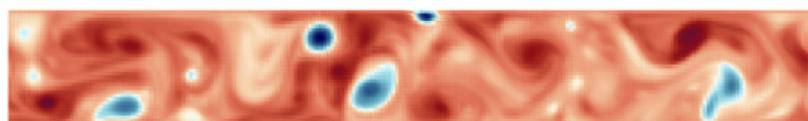


Ocean temp.
0.9938 1 1.0072



Ocean vel.
0.0 0.1 0.2 0.3 0.4

Atmospheric vorticity at $t = 25$

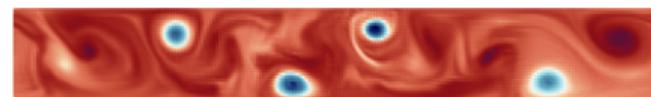


Atm. vort.
-12.1 -5 0 6.4

Simulation results

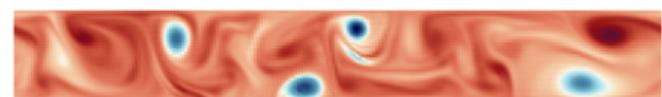
- ▶ Stochastic model is simulated for 20 time units ($t = 25$ to $t = 45$)
- ▶ Stochastic vs. deterministic (without parameterization) model results at $t = 35$

Particle 1



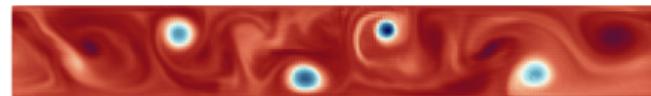
Atm. vort. particle 1
-9.5 -5 0 3.4

Deterministic solution



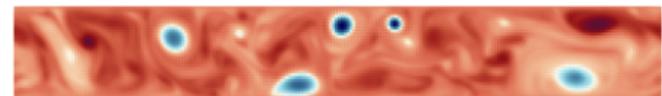
Atm. vort. deterministic
-7.6 -5 0 4.0

Particle 2



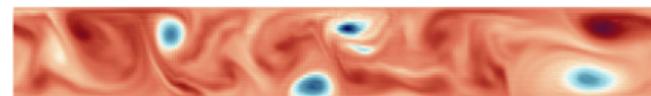
Atm. vort. particle 2
-11.0 -5 0 3.3

True solution



Atm. vort. truth
-9.1 -5 0 4.4

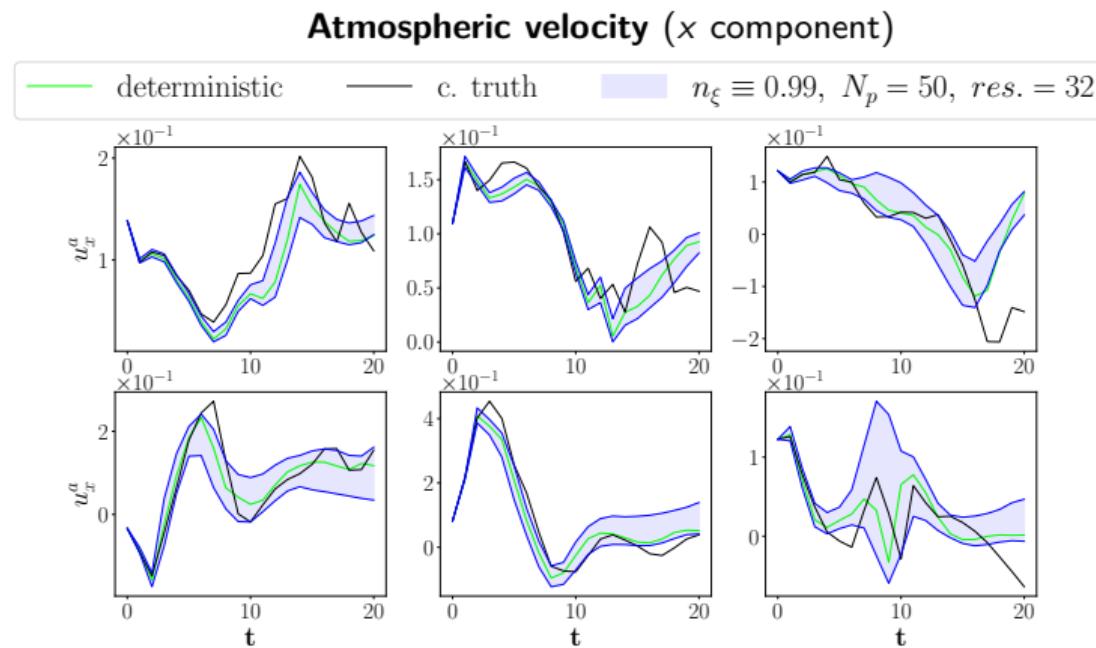
Particle 3



Atm. vort. particle 3
-7.5 -5 0 3.9

Simulation results

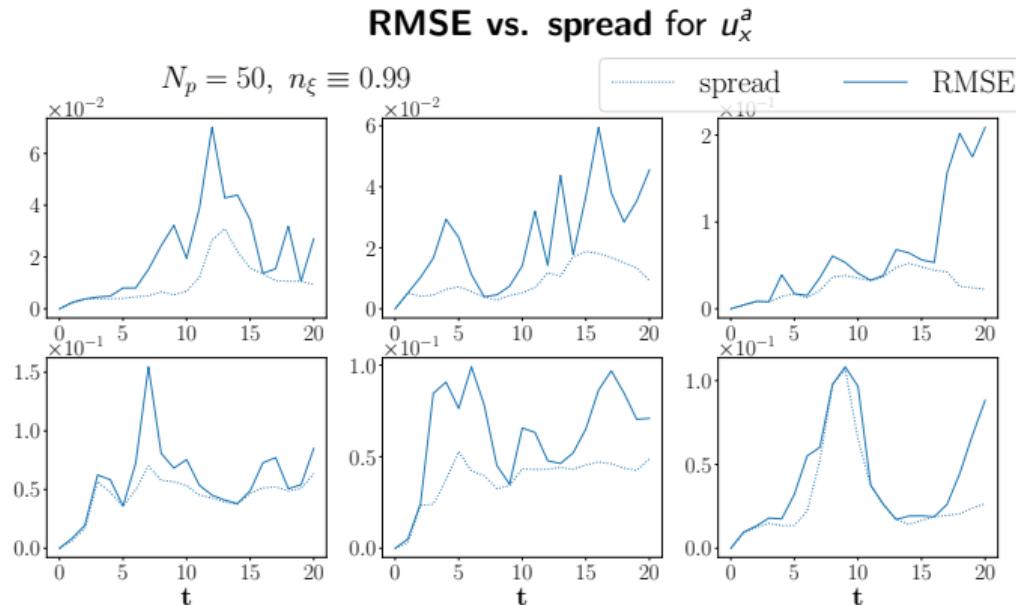
- ▶ Comparison between the stochastic ($\mu \pm 1\sigma$) and deterministic sol. at 6 grid points:



1. Ensemble spread increases over time
2. Stochastic model captures the truth for 5 to 10 time units

Uncertainty quantification

- ▶ Plots for the evolution of ensemble spread and RMSE at six different locations on the grid



- ▶ Ensemble spread \propto RMSE (for at least 15 time units)
- ▶ Error in the stochastic solution can be estimated by its own spread

Summary

Summary & outcomes

- ▶ **Aim:** solve a stochastic coupled ocean-atmosphere climate model
- ▶ **Novelty:** data-driven model
 - ▶ Stochastic terms (modelling unresolved processes) are obtained from observation data
 - ▶ Exhibit desired conservation properties
 - ▶ High-res. model sim. data is used to estimate $\xi_i \rightarrow$ data analysis & statistical modelling
- ▶ **Methodology** for solving the model equations:
 - ▶ Solve simpler models first
 - ▶ build the climate model step-by-step
 - ▶ verify numerical schemes at each step
- ▶ **Results:**
 - ▶ Stochastic model solution captures the truth for some time units
 - ▶ Shows promising UQ test results: ensemble spread size \propto RMSE
- ▶ **Outcome:** first step towards using SALT in a highly complex coupled system!

Thank you for your attention!