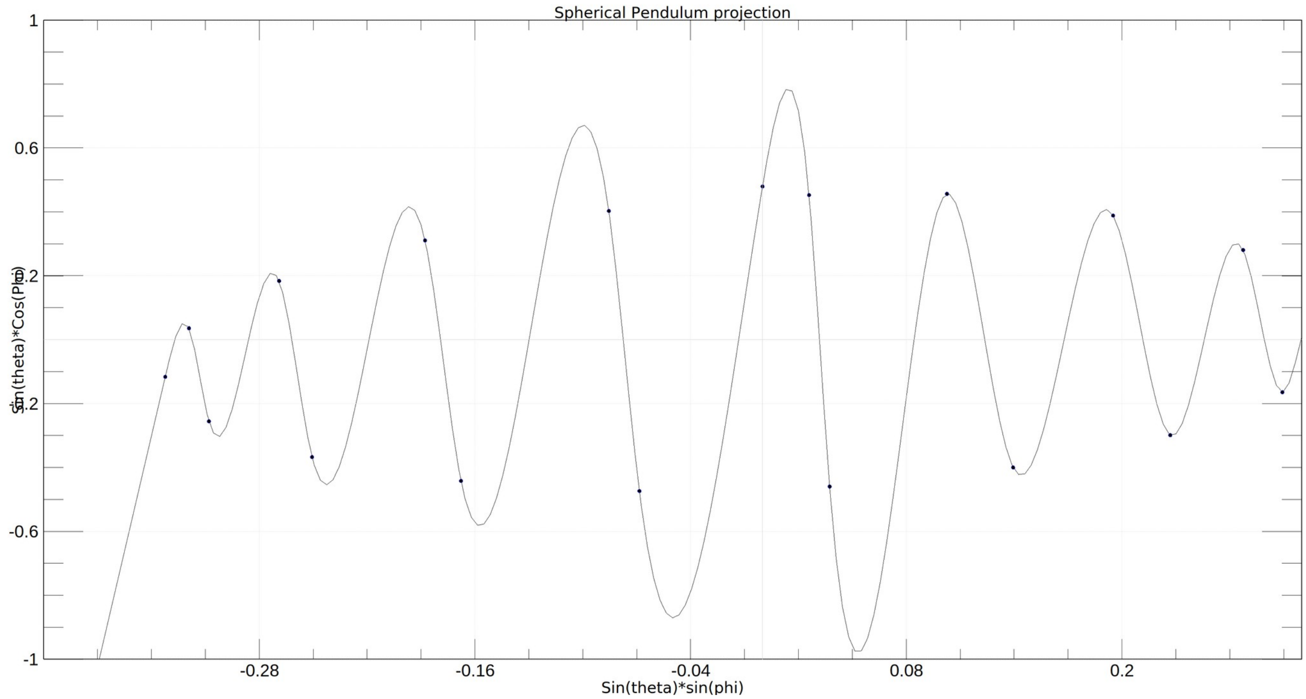
<u>Assignment-11</u>

4. The problem was solved till t= 2 seconds. The plot observed is as follows:

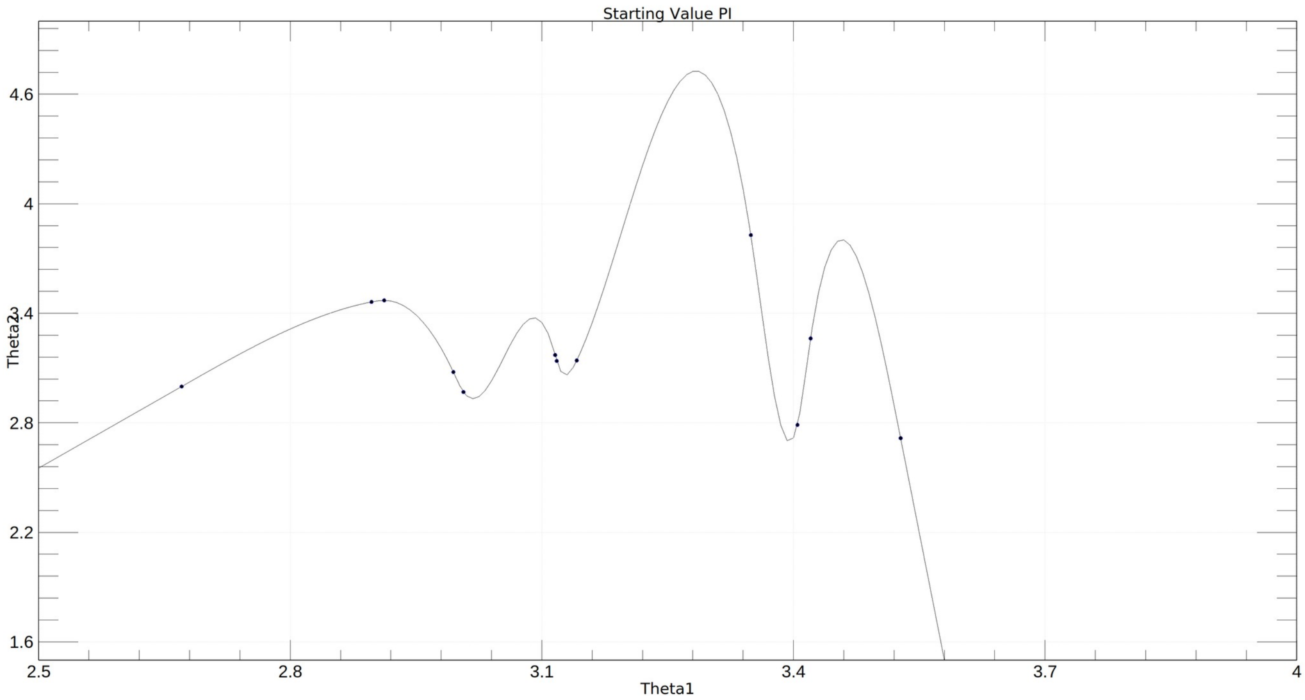


From observation, the procession rate is 0.09 units.

10. The program is successfully working with all the needed integration rules as well as fixed and adaptive setting. I was not able to find the analytical solution to compare to.

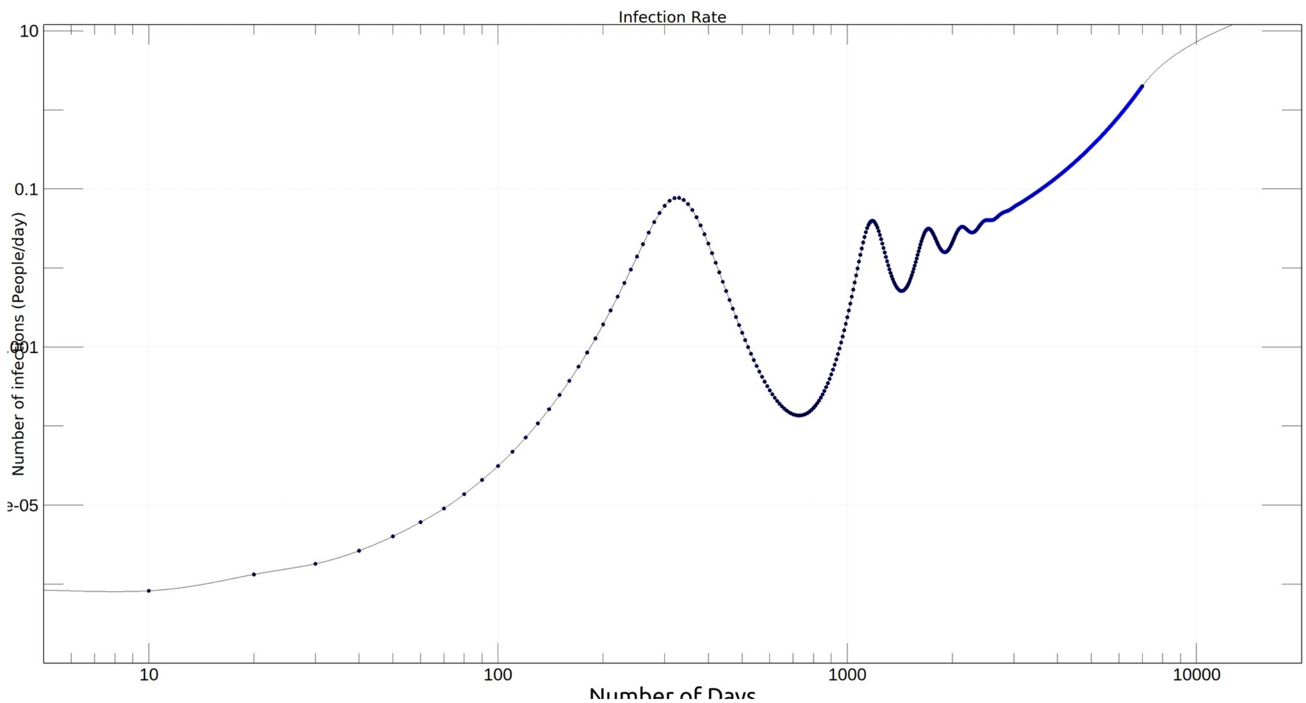
14.
(a) , (b), (c), Kindly find the solution in hand-written form at the end.
(d) For the case of initial conditions of PI/12, the program was taking too long to solve for various combinations of parameters

For the initial conditions for PI, the program was run for 12 seconds of time. Beyond this time, the program took too long to solve.



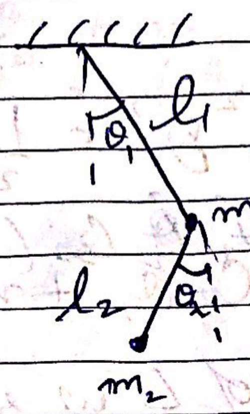
15. (a) , (b), (c), Kindly find the solution in hand-written form at the end.

(d) The Infection rate to number of days log-log plot is obtained as follows:



Infection Rate
10
0.1
0.001
1e-05
Number of infections (People/day)
10
100
1000
10000
Number of Days

14.



The general lagrangian expression is given by

$L = T - V$, where T is the kinetic energy of the system and V is potential energy of the system.

~~Here~~ ~~V=~~ The position of mass m_1 are given as

$x_1 = l_1 \sin\theta_1$, $y_1 = -l_1 \cos\theta_1$

$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$, $y_2 = -l_1\cos\theta_1 - l_2 \cos\theta_2$

The potential energy $(V) = m_1 g y_1 + m_2 g y_2$

$= -(m_1 + m_2) g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2$

$T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$= \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \left[l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \right.$

$\therefore L = T - V$

$$L = \frac{1}{2}(m_1+m_2) l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + (m_1+m_2) g l_1 \cos\theta_1 + m_2 g l_2 \cos\theta_2$$

(ii) Taking derivative of L w.r.t $\dot{\theta}_1$ we get,

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_1 l_1^2 \dot{\theta}_1 + m_2 l_1^2 \dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = (m_1+m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2 l_1 l_2 \dot{\theta}_2 \sin(\theta_1 - \theta_2)(\dot{\theta}_1 - \dot{\theta}_2)$$

$$\frac{\partial L}{\partial \theta_1} = -l_1 g (m_1+m_2) \sin\theta_1 - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2)$$

So the Euler Lagrangian equation for θ_1 becomes,

$$(m_1+m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + l_1 g (m_1+m_2) \sin\theta_1 = 0$$

Divide both sides by l_1,

$$(m_1+m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1-\theta_2) + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1-\theta_2) + g(m_1+m_2)\sin\theta_1 = 0$$

Similarly for θ_2,

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1-\theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 l_2 \dot{\theta}_1 \sin(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2)$$

$$\frac{\partial L}{\partial \theta_2} = m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1-\theta_2) - l_2 m_2 g \sin\theta_2$$

$\therefore$ The Euler-Lagrange equation becomes

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_1-\theta_2) - m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_1-\theta_2) + l_2 m_2 g \sin\theta_2 = 0$$

Divide both sides by l_2,

$$m_2 l_2 \ddot{\theta}_2 + m_2 l_1 \ddot{\theta}_1 \cos(\theta_1-\theta_2) + m_2 l_1 \dot{\theta}_1^2 \sin(\theta_2-\theta_1) + m_2 g \sin\theta_2 = 0$$

Now writing these equations in Hamiltonian formulation,

$$p_{\theta_1} = \frac{\partial L}{\partial \dot{\theta}_1} = (m_1+m_2) l_1^2 \dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1-\theta_2)$$

$$p_{\theta_2} = \frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1-\theta_2)$$

The Hamiltonian is given as,

$$H = \dot\theta_i p_i - L$$

$$= \frac{1}{2}(m_1+m_2) l_1^2 \dot\theta_1^2 + \frac{1}{2} m_2 l_2^2 \dot\theta_2^2$$
$$+ m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \cos(\theta_1 - \theta_2)$$
$$\cancel{-(m_1+m_2) g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2}$$

Putting values of $p\theta_1$, $p\theta_2$ & L we get

$$H = \frac{l_2^2 m_2 p\theta_1^2 + l_1^2 (m_1+m_2) p\theta_2^2 - 2 m_2 l_1 l_2 p\theta_1 p\theta_2 \cos(\theta_1 - \theta_2)}{2 l_1^2 l_2^2 m_2 (m_1 + \sin^2(\theta_1-\theta_2) m_2)}$$
$$-(m_1+m_2) g l_1 \cos\theta_1 - m_2 g l_2 \cos\theta_2$$

The Hamiltonian differential equations are obtained as

$$\dot\theta_1(t) = \frac{\partial H}{\partial p\theta_1} = \frac{l_2 p\theta_1 - l_1 p\theta_2 \cos(\theta_1-\theta_2)}{l_1^2 l_2 [m_1 + m_2 \sin^2(\theta_1-\theta_2)]}$$

$$\dot\theta_2(t) = \frac{\partial H}{\partial p\theta_2} = \frac{l_1(m_1+m_2) p\theta_2 - l_2 m_2 p\theta_1 \cos(\theta_1-\theta_2)}{l_1 l_2^2 m_2 [m_1 + m_2 \sin^2(\theta_1-\theta_2)]}$$

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$$\dot{p}_{\theta_1} = -\frac{\partial H}{\partial \theta_1} = -(m_1+m_2) g l_1 \sin\theta_1 - C_1 + C_2$$

$$\dot{p}_{\theta_2} = -\frac{\partial H}{\partial \theta_2} = -m_2 g l_2 \sin\theta_2 + C_1 - C_2$$

where $C_1 = \dfrac{p_{\theta_1} p_{\theta_2} \sin(\theta_1-\theta_2)}{l_1 l_2 [m_1 + m_2 \sin^2(\theta_1-\theta_2)]}$

& $C_2 = \dfrac{l_2^2 m_2 p_1^2 + l_1^2 (m_1+m_2) p_2^2 - l_1 l_2 m_2 p_1 p_2 \cos(\theta_1-\theta_2)}{2 l_1^2 l_2^2 [m_1 + m_2 \sin^2(\theta_1-\theta_2)]^2} \sin(2(\theta_1-\theta_2))$

From observation it can be seen that all the four equations are linear differential equations

(d) (i) $\theta_1 = \frac{\pi}{12}$, $\theta_2 = \frac{\pi}{12}$

(ii) $\theta_1 = \pi$, $\theta_2 = \pi$

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(a) $$\frac{dS}{dt} = b - \lambda S - dS + \partial R$$

$$\frac{dE}{dt} = \lambda S - aE$$

$$\frac{dI}{dt} = aE - \gamma I - dI$$

$$\frac{dR}{dt} = \gamma I - dR$$

(b) If $b = d$,

$$\frac{dS}{dt} = b - \lambda S - dS + \partial R \quad [d = b]$$

$$\frac{dS}{dt} = b - \beta \frac{IS}{N} - bS + \partial R$$

$$\frac{dE}{dt} = \beta \frac{IS}{N} - aE$$

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The rate equation do not become zero, if birth rates are equal to death rate, hence, the population does not become static.

(c) The model assumes that disease mortality ~~is equal~~ or the death rate because of the disease is equal in all cases of compartments (S, I, E, R)

(d) Solved above

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