**Game:** If you have to use coins of 1 and 2 only.There are two player each can play one after another. The game is to make a stack of coins those who play for sum of coin 10 is the winner. E.G:

Player 1 Player 2 Sum

1 1

2 3

1 4

1 5

2 7

1 8

2 10 (Now sum is 10 and this is made by the player 1 hence player 1 is winner).

NOTE: Strategy to win: Observe the player who make 7 is winner. Now to make (definitely) 7 the player must have to make 4. similarly by recurrsive the player who make 1 is winner. Hence winning startegy is to play first and make 1 hence after that just play opposite of the other player.

(Link: <http://www.math.ucla.edu/~tom/Game_Theory/comb.pdf>)

**Game2:**

Here are the rules of a very simple impartial combinatorial game of removing chips from a pile of chips.

(1) There are two players. We label them I and II.

(2) There is a pile of 21 chips in the center of a table.

(3) A move consists of removing one, two, or three chips from the pile. At least one chip must be removed, but no more than three may be removed.

(4) Players alternate moves with Player I starting.

(5) The player that removes the last chip wins. (The last player to move wins. If you can’t move, you lose.)

We analyze this game from the end back to the beginning. This method is sometimes called **backward induction**.

If there are just one, two, or three chips left, the player who moves next wins simply by taking all the chips. Suppose there are four chips left. Then the player who moves next must leave either one, two or three chips in the pile and his opponent will be able to win. So four chips left is a loss for the next player to move and a win for the previous player, i.e. the one who just moved. With 5, 6, or 7 chips left, the player who moves next can win by moving to the position with four chips left. With 8 chips left, the next player to move must leave 5, 6, or 7 chips, and so the previous player can win. We see that positions with 0,4,8,12,16,...chips are target positions; we would like to move into them. We may now analyze the game with 21 chips.Since 21 is not divisible by 4, the first player to move can win. The unique optimal move is to take one chip and leave 20 chips which is a target position

**Subtraction Games**

Let S be a set of positive integers. The subtraction game with subtraction set S is played as follows. From a pile with a large number, say n , of chips, two players alternate moves. A move consists of removing s chips from the pile where s ∈ S . Last player to move wins. The above game 2 is subtraction game with S = {1,3,4}.

**Analysis of game2:**

Def: P position: The position which are winning for the previous player.

N position: position which are winning for the next position.

Here is exactly one terminal position, namely 0. Then 1, 3, and 4 are N-positions, since they can be moved to 0. But 2 then must be a P-position since the only legal move from 2 is to 1, which is an N-position. Then 5 and 6 must be N-positions since they can be moved to 2. Now we see that 7 must be a P-position since the only moves from 7 are to 6, 4, or 3, all of which are N-positions.

Now we continue similarly: we see that 8, 10 and 11 are N-positions, 9 is a P-position, 12 and 13 are N-positions and 14 is a P-position. This extends by induction. We find that the set of P-positions is P = { 0,2,7,9,14,16,...}, the set of nonnegative integers leaving remainder 0 or 2 when divided by 7. The set of N-positions is the complement, N={1,3,4,5,6,8,10,11,12,13,15,...}.

x 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14...

position P N P N N N N P N P N N N N P..

The pattern PNPNNNN of length 7 repeats forever. Who wins the game with 100 chips, the first player or the second? The P-positions are the numbers equal to 0 or 2 modulus 7. Since 100 has remainder 2 when divided by 7,100 is a P-position; the second player to move can win with optimal play.

**Game of Nim:**

There are three piles of chips containing x1,x2 and x3 chips respectively. Two players take turns moving. Each move consists of selecting one of the piles and removing chips from it. You may not remove chips from more than one pile in one turn, but from the pile you selected you may remove as many chips as desired, from one chip to the whole pile. The winner is the player who removes the last chip.

**Analysis:**

Terminal Position: (0,0,0) P position

Now for any position with exactly one non empty pile is N position (0,0,x) where x > 0