HW 09: PRATA SPOJ

Problem Statement:

IEEE is having its AGM next week and the president wants to serve cheese prata after the meeting.

The subcommittee members are asked to go to food connection and get **P (P<=1000) pratas** packed for the function.

The stall has **L cooks (L<=50)** and each cook has **a rank R (1<=R<=8)**.

A cook with a rank R can cook 1 prata in the first R minutes 1 more prata in the next 2R minutes, 1 more prata in 3R minutes and so on (he can only cook a complete prata)

For example: if a cook is **ranked 2**, he will cook
1st prata in 2 mins
2nd prata in the next 4 mins
3rd prata in the next 6 mins
hence he cooks 3 pratas in total 12 minutes

🖊**MUJHE FIND KYA KARNA HAI:** The webmaster wants to know the **minimum time** to get the order done.

📌**Observation:**

Number of pratas = P <=1000
Number of cooks = L <=50
Each cook has a rank = R ▢ [1,8]

Input:
3 --> Total Cases
10 --> nP
4 1 2 3 4 --> nC and remaining 4 are R [1,2,3,4]
8 --> nP
1 1 --> nC and remaining 1 are R [1]
8 -- nP
8 1 1 1 1 1 1 1 1 --> nC and remaining 8 are R [1,1,1,1,1,1,1,1]

Output:
12
36
1

Number of pratas = nP *= 10*
Number of cooks = nC *= 4*
Each cook has a rank = R *= [1,2,3,4]*

Example 01:
input: nP = 10, nC = 4, R = [1,2,3,4]
Output: 12

Example 02:
input: nP = 8, nC = 1, R = [1]
Output: 36

Example 03:
input: nP = 8, nC = 8, R = [1,1,1,1,1,1,1,1]
Output: 1

OPTIMAL APPROACH: *Define search space and predicate function*
Step 01: Find highest ranked cooked time to create search space's end point (end = highest maximum time to complete the order)
Step 02: Now, Applying Binary Search on search space BinarySearch()
Step 03: create predicate function isPossibbleSol()

Time Complexity: *O(N*log(end)), Here N is size of array R and end is the highest maximum time to complete the order*
Space Complexity: *O(1), no extra space used*

Resource: *https://www.spoj.com/problems/PRATA/*

OBSERBATION

Number of pratas = nP = 10
Number of cooks = nC = 4
Each cook has a rank = R = [1,2,3,4]

$$R = \begin{array}{|c|c|c|c|} \hline C_1 & C_2 & C_3 & C_4 \\ 1 & 2 & 3 & 4 \\ \hline \end{array} \longrightarrow \text{COOK'S NAME}$$

1st Cook
Rank = 1

2nd Cook
Rank = 2

3rd Cook
Rank = 3

4th Cook
Rank = 4

DRY RUN

STEP 01 Find highest ranked cooked time

$C_4 \Rightarrow$ nPrata $*$ Rtime

$\Rightarrow$ $1*4 + 2*4 + 3*4 + 4*4 + 5*4 + 6*4 + 7*4$

$8*4 + 9*4 + 10*4$

$\Rightarrow 4[1+2+3+\cdots+9+10] \Rightarrow 4[10(\frac{10+1}{2})] = 4(55)$

$\Rightarrow 220$ minutes

This is a highest maximum time to complete the order but we have to find minimum time to get order done.

Min-Time ●————————————● max-Time

0   22

Time taken by cook 4rt highest ranked who all the pratas given to him

STEP 02

$\boxed{\text{Iteration 1}}$

$hp = 10$

$start = 0$

$End = \cancel{220}$ $\;\;109$

$mid = \dfrac{0 + 220}{2}$

$= 110$

$Ans = \cancel{-1}\;\; 110$
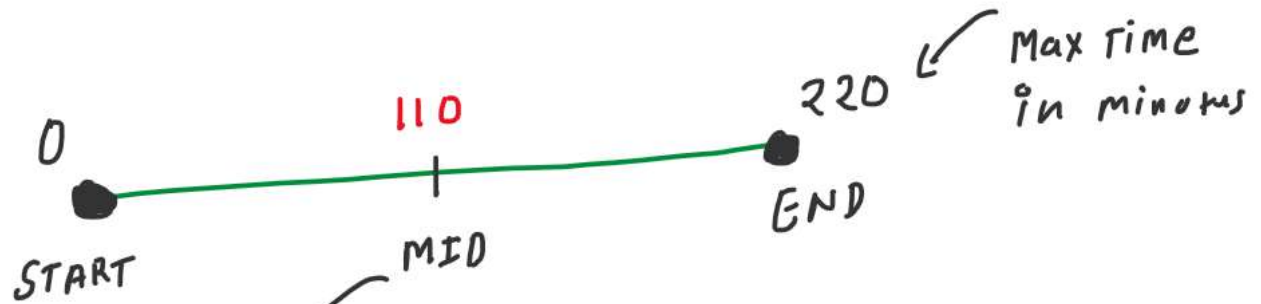
0   110   220 ← Max time in minutes

START   MID   END

Kya all cooks mujhe **mid time** me ya **less then mid time** me mera 10 prata ka order complete kar payenge?

STEP:03

$C_1 \Rightarrow 1 * 1 + 2 * 1 + 3 * 1 + 4 * 1 + 5 * 1 + 6 * 1 + 7 * 1 +$

$8 * 1 + 9 * 1 + 10 * 1$

$\Rightarrow 1 \left[10 \left(\dfrac{10 + 1}{2} \right) \right] = 55 \text{ mins}$

possible sol.^{n}

$(Time <= mid)$ TRUE

$65 <= 110$

→ ANS = mid

END = mid-1

STEP 02

Itumation 2

$h^p = 10$

$start = 0$

$End = \cancel{109} \quad 53$

$mid = \dfrac{0 + 109}{2}$

$\quad = 54$

$Ans = \cancel{110} \quad 47$
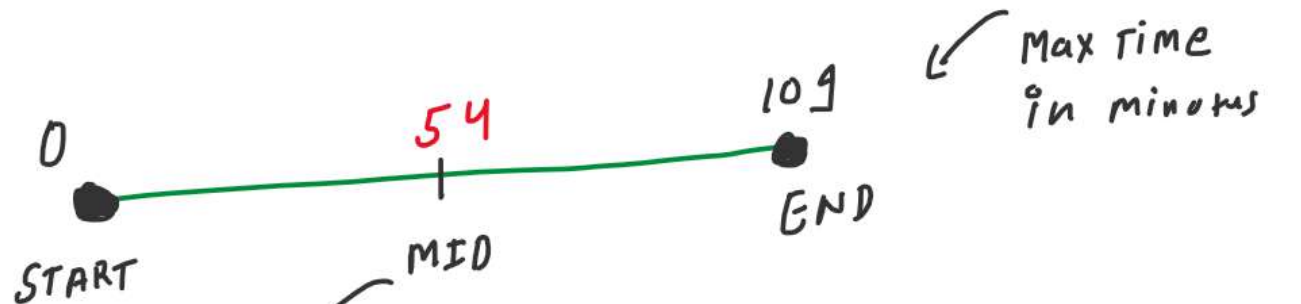
0 —————— 54 ————— 109

START   MID   END

Max Time in minutes

Kya all cooks mujhe **mid time** me ya **less then mid time** me mera 10 prata ka order complete kar payenge?

possible sol.ⁿ

$(Time <= mid)$ TRUE

$45 <= 54$

$\hookrightarrow ANS = mid$

$END = mid - 1$

$C_1 \Rightarrow 1*1 + 2*1 + 3*1 + 4*1 + 5*1 + 6*1 + 7*1 +$

$8*1 + 9*1 + \boxed{10*1} \times$

$\Rightarrow 1\left[9\left(\dfrac{9+1}{2}\right)\right] = 45$ mins

$C_2 \Rightarrow 1*2 \Rightarrow 2$ mins

$C_1 \Rightarrow$ 9 Pratas are computed in 45 mins by C_1

$C_2 \Rightarrow$ 1 Prata is computed in 2 mins by C_2
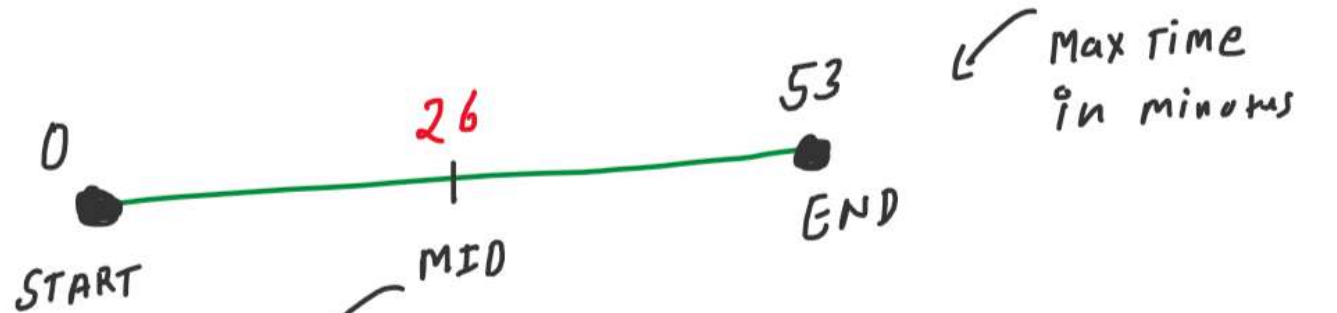
STEP 02

Iteration 3

$hp = 10$

$start = 0$

$End = \cancel{53}\ 25$

$mid = \dfrac{0+53}{2}$

$= 26$

$Ans = \cancel{47}\ 26$

```
0                26                53
●————————————————|————————————————●
START            MID               END
```

53
26

END

Max Time
in minutes

> Kya all cooks mujhe **mid time** me ya **less then mid time** me mera 10 prata ka order complete kar payenge?

possible sol.ⁿ

(Time $\leq$ mid) TRUE

$21 \leq 26$

ANS = mid

END = mid-1

$C_1 \Rightarrow 1 \times 1 + 2 * 1 + 3 * 1 + 4 * 1 + 5 * 1 + 6 * 1 + \boxed{7 * 1} \; X$

$\Rightarrow 6 \left[\dfrac{6+1}{2} \right] = 21 \text{ mins}$

$C_2 \Rightarrow 1 * 2 + 2 * 2 + 3 * 2 + 4 * 2$

$\Rightarrow 2 + 4 + 6 + 8$

$\Rightarrow 20 \text{ mins}$

PARALLEL
KAM HO
RHA Hai
$C_1 C_2 C_3 C_4$

$C_1 \Rightarrow 6$ Pratas are computed in 21 mins by C_1

$C_2 \Rightarrow 4$ Prata are computed in 20 mins by C_2
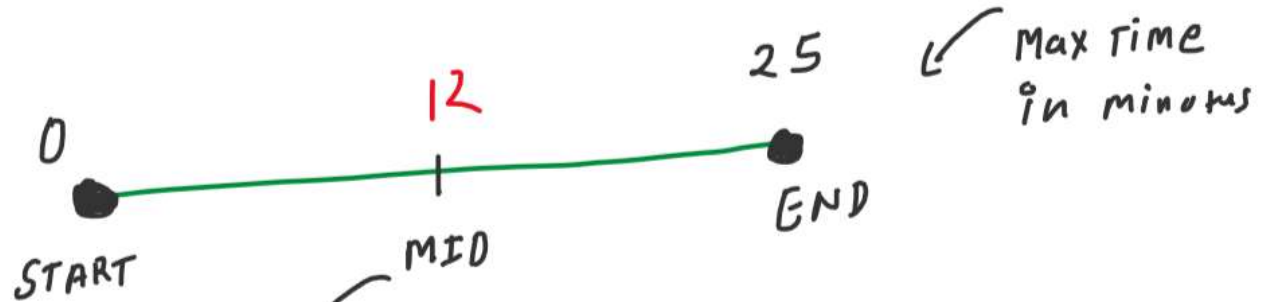
STEP 02

Iteration 4

$hp = 10$

start $= 0$

End $= \cancel{25}$ 11

mid $= \dfrac{0+25}{2}$

$= 12$

Ans $= \cancel{25}$ 12

---

0 START

12 MID

25

END

Max time in minutes

Kya all cooks mujhe **mid time** me ya **less then mid time** me mera 10 prata ka order complete kar payenge?

possible sol.ⁿ

$(Time <= mid)$ TRUE

$12 <= 12$

ANS $=$ mid

END $=$ mid-1

PARALLEL
KAM HO
RHA Hai
C_1 C_2 C_3 C_4

$C_1 \Rightarrow 1 * 1 + 2 * 1 + 3 * 1 + 4 * 1 + \boxed{5 * 1}$ ✗

$\Rightarrow 4\left[\dfrac{4+1}{2}\right] = 10$ mins

$C_2 \Rightarrow 1 * 2 + 2 * 2 + 3 * 2 + \boxed{4 * 2}$ ✗

$\Rightarrow 2 + 4 + 6 \Rightarrow 12$ mins

$C_3 \Rightarrow 1 * 3 + 2 * 3 + \boxed{3 * 3}$ ✗

$\Rightarrow 3 + 6 \Rightarrow 9$ mins

$C_4 \Rightarrow 1 * 4 \Rightarrow 4$ mins

$C_1 \Rightarrow$ 4 Pinatas are
completed
in 10 mins by C_1

$C_2 \Rightarrow$ 3 Pinata are
Completed in 12 mins by C_2

$C_3 \Rightarrow$ 2 Pinata are completed in
9 mins by C_3

$C_4 \Rightarrow$ 1 pinata is completed in
4 mints by C_4

STEP 02

Itemation 5

$hp = 10$

$start = \cancel{0} \quad 6$

$End = 11$

$mid = \dfrac{11 + 0}{2}$

$\quad = 5$

$Ans = 12$
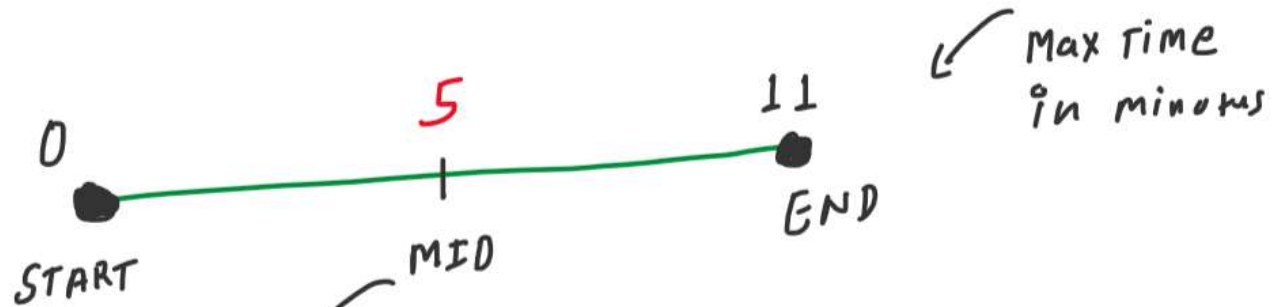

```
        5
0                     11
●━━━━━━━|━━━━━━━●
START    MID        END
```

Max time
in minutes

Kya all cooks mujhe **mid time** me ya **less then mid time** me mera 10 prata ka order complete kar payenge?

No Possible solution

Total Paratas = 5 only so

$hp : = 10$

$\longrightarrow Start = mid + 1$

PARALLEL
KAM HO
RHA Hai
C_1, C_2, C_3, C_4

$C_1 \Rightarrow 1 \times 1 + 2*1 + \boxed{3*1}\times$

$\Rightarrow$ 3 mins

$C_2 \Rightarrow 1*2 + \boxed{2*2}\times$

$\Rightarrow$ 2 mins

$C_3 \Rightarrow 1*3 + \boxed{2*3}\times$

$\Rightarrow$ 3 mins

$C_4 \Rightarrow 1*4 \Rightarrow$ 4 mins

Total
panata = 5

$C_1 \Rightarrow$ 2 Panatas are completed in 3 mins by C_1

$C_2 \Rightarrow$ 1 Panata is completed in 2 mins by C_2

$C_3 \Rightarrow$ 1 panata is completed in 3 mins by C_3

$C_4 \Rightarrow$ 1 panata is completed in 4 mints by C_4
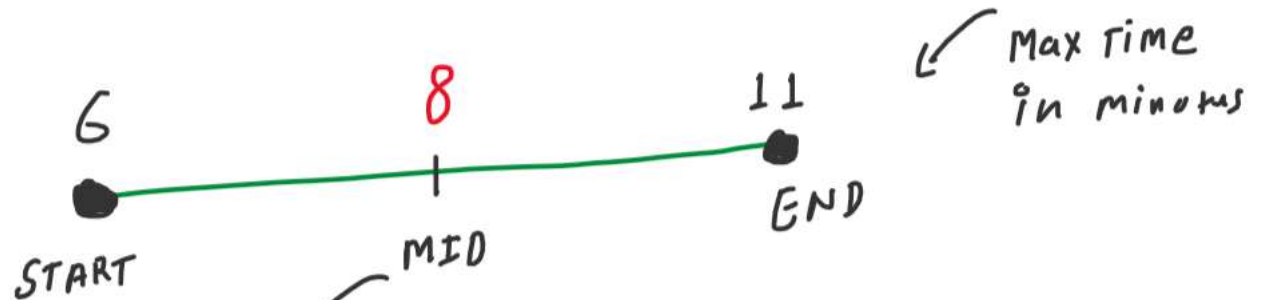
STEP 02 | Iteration 6 |

$hp = 10$

$start = \cancel{6} \; 9$

$End = 11$

$mid = \dfrac{6+11}{2}$

$\qquad = 8$

$Ans = 12$

6   8   11

START   MID   END

Max time in minutes

Kya all cooks mujhe **mid time** me ya **less then mid time** me mera 10 prata ka order complete kar payenge?

No possible solution

Total paratas = 7 only so

$hp_o = 10$

$\longrightarrow Start = mid + 1$

$C_1 \Rightarrow 1 \times 1 + 2 * 1 + 3 * 1 + \boxed{4 * 1}\ X$

$\Rightarrow 6$ mins

$C_2 \Rightarrow 1 * 2 + 2 * 2 + \boxed{3 * 2}\ X$

$\Rightarrow 6$ mins

$C_3 \Rightarrow 1 * 3 + \boxed{2 * 3}\ X$

$\Rightarrow 3$ mins

$C_4 \Rightarrow 1 * 4 \Rightarrow 4$ mins

Total
panata = 7

PARALLEL
KAM HO
RHA HAI
C_1, C_2, C_3, C_4

$C_1 \Rightarrow 3$ pnatas aru
completed
in 6 mins by C1

$C_2 \Rightarrow 2$ pnata aru
completed in 6 mins by C2

$C_3 \Rightarrow 1$ pnata is computed in
3 mins by C3

$C_4 \Rightarrow 1$ panata is completed in
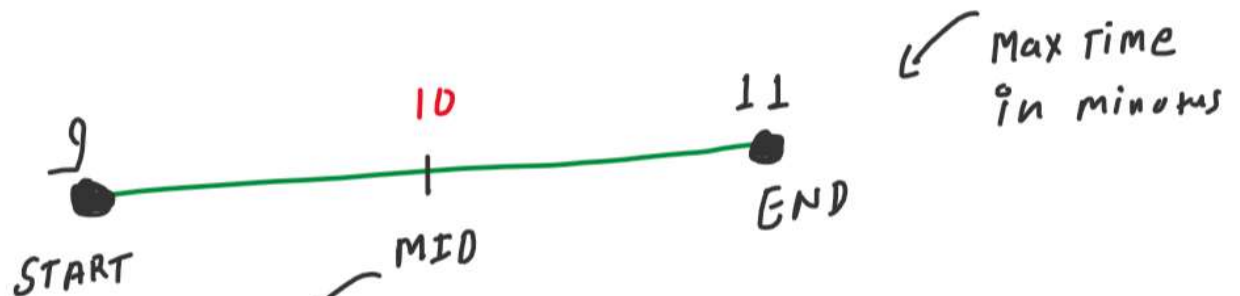4 mints by C4

STEP 02

Iteration 7

$hp = 10$

$start = \cancel{9} \quad 11$

$End = 11$

$mid = \dfrac{9+11}{2}$

$\quad = 10$

$Ans = 12$

10

9 ————————|——————— 11

START   MID   END

Max time in minutes

Kya all cooks mujhe **mid time** me ya **less then mid time** me mera 10 prata ka order complete kar payenge?

NO possible solution

Total paratas = 9 only so

$hp \neq 10$

$\longrightarrow start = mid+1$

$C_1 \Rightarrow 1*1 + 2*1 + 3*1 + 4*1 + \boxed{5*1}$ ✗

$\Rightarrow 10$ mins

$C_2 \Rightarrow 1*2 + 2*2 + \boxed{3*2}$ ✗

$\Rightarrow 6$ mins

$C_3 \Rightarrow 1*3 + 2*3 + \boxed{3*3}$ ✗

$\Rightarrow 9$ mins

$C_4 \Rightarrow 1*4 \Rightarrow 4$ mins

Total
panata = 9

PARALLEL
KAM Ho
RHA Hai
C_1, C_2, C_3, C_4

$C_1 \Rightarrow$ 4 Pnatas are
computed
in 10 mins by C_1

$C_2 \Rightarrow$ 2 Pnata are
computed in 6 mins by C_2

$C_3 \Rightarrow$ 2 Pnata are computed in
9 mins by C_3

$C_4 \Rightarrow$ 1 panata is completed in
4 mints by C_4

STEP 02   Iteration 8

$hp = 10$

start = ~~11~~ 12

End = 11

mid = $\frac{11+11}{2}$

= 11

Ans = 12

Max time
in minutes

START   MID   END

Kya all cooks mujhe **mid time** me ya **less then mid time** me mera 10 prata ka order complete kar payenge?

NO possible solution

Total Paratas = 9 only so

$hp_0! = 10$

Start = mid+1

$C_1 \Rightarrow 1 * 1 + 2 * 1 + 3 * 1 + u * 1 + \boxed{5 * 1}$ ✗

$\Rightarrow$ 10 mins

$C2 \Rightarrow 1 * 2 + 2 * 2 + \boxed{3 * 2}$ ✗

$\Rightarrow$ 6 mins

$C3 \Rightarrow 1 * 3 + 2 * 3 + \boxed{3 * 3}$ ✗

$\Rightarrow$ 9 mins

$C_4 \Rightarrow 1 * u \Rightarrow u$ mins

Total
panata = 9

$C_1 \Rightarrow$ u Pnatas are
computed
in 10 mins by C1

$C2 \Rightarrow$ 2 Pnata are
computed in 6 mins by C2

$C3 \Rightarrow$ 2 Pnata are computed in
9 mins by C3

$C_4 \Rightarrow$ 1 panata is compled in
u mints by C4

PARALLEL
KAM Ito
RHAHai
C1 C21 C31 C4

STEP 02   Iteration 9

$hp = 10$

$start = 12$

$End = 11$ } STOP $\longrightarrow$ Start > END

$ANS = 12$

{ Minimum
Time TO compute
The ORder is **12 mins**
By all cooks
$c_1, c_2, c_3,$ and c_4 }

```cpp
// HW 09: PRATA SPOJ
#include<iostream>
#include<vector>
#include<algorithm>
using namespace std;

// predicate function
bool isPossibleSol(vector<int> cooksRanks,int nP,int mid){
    int currP = 0; // initial cooked prata count

    for(int i=0;i<cooksRanks.size();i++){
        int R = cooksRanks[i], cP=1; // cP = completed prata
        int timeTaken = 0; // initial time taken by each cooks

        while(true){
            if(timeTaken + cP*R <= mid){
                currP++;
                timeTaken += cP*R;
                cP++;
            }
            else{
                // while loop se bahar nikal jaunga or dusre cook ko pakdunga
                // order complete karne ke liye
                break;
            }
        }
        if(currP>=nP){
            // agar 10 ya 10 jyada prate given time (mid) se kam
            // time me honge to return true kardo(Possible case)
            return true;
        }
    }
    // agar kisi bhi cases me diye huee prate given time(mid) ya usse kam
    // time me nhi ho paate hai to return false kardo(Not Possible case)
    return false;
}

// Binary Search
int BinarySearch(vector<int> cooksRanks,int nP,int end){
    int start = 0, mid = start + (end - start)/2;
    int ans = -1; // minimum time stored in ans

    while(start<=end){
        // Step 03: create predicate function isPossibbleSol()
        if(isPossibleSol(cooksRanks,nP,mid)){
            ans = mid;
            end = mid - 1;
        }
        else{
            start = mid + 1;
        }
        mid = start + (end - start)/2;
    }
    return ans;
}

int minTimeToCompleteOrder(vector<int> cooksRanks,int nP){
    // Step 01: Find highest ranked cooked time
    int maxCooksRank = *max_element(cooksRanks.begin(), cooksRanks.end());
    int end = maxCooksRank * (nP * (nP + 1) / 2);

    // Step 02: Now, Applying Binary Search on search space
    int finalAns = BinarySearch(cooksRanks,nP,end);
    return finalAns;
}

int main(){
    int T;
    cin>>T;
    while(T--){
        int nP, nC;
        cin >> nP >> nC;
        vector<int> cooksRanks;
        while(nC--){
            int R;
            cin>>R;
            cooksRanks.push_back(R);
        }
        int ans = minTimeToCompleteOrder(cooksRanks,nP);
        cout<<ans<<endl;
    }
    return 0;
}
```