

HW 08: EKO SPOJ

Problem Statement:

LAKSHAY BHAIYA needs to chop down M metres of wood.
It is an easy job for him since he has a nifty new wood-cutting machine that can take down forests like wildfire.
However, LAKSHAY BHAIYA is only allowed to cut a single row of trees.

LAKSHAY BHAIYA's machine works as follows:

He sets a height parameter H (in meters), and the machine raises a giant sawblade to that height and cuts off all tree parts higher than H (of course, trees not higher than H meters remain intact).
He then takes the parts that were cut off.

He is ecologically minded, so he doesn't want to cut off more wood than necessary.
That's why he wants to set his sawblade as high as possible.
Help LAKSHAYA find the 'maximum integer height of the sawblade' that still allows him to cut off 'at least M metres of wood'.

Example 01:

Input:
4 7
20 15 10 17

Output:
15

📌 Observation:

Number of trees/Array's size $\Rightarrow N = 4$
At least M metres of wood $\Rightarrow M = 7$
Trees Array \Rightarrow trees [20, 15, 10, 17]

1st tree ki height = trees[0] = 20 meters
2nd tree ki height = trees[1] = 15 meters
3rd tree ki height = trees[2] = 10 meters
4th tree ki height = trees[3] = 17 meters

 **PROBLEM KYA HAI:** We have to find maximum integer height of sawblade that still allows to cut off at least M meters of wood.

hame sawblade ki height ko aisa rakhna hai ki trees ko iss tarike se kate jo required height of wood se kam to bilkul na ho or usse bahut jyada bhi na ho.

To meaning yeh niklta hai ki hame sawblade ki highest maximum height me se (Possible) least maximum height btani hai.

✦ agar saw blade ki height = 0 meter hai to => ek bhi tree jungle nhi rahega to hum kah skte hai ki yeh least minimum height hai
(wood height = 20+15+10+17 = 62 meters)

✦ agar saw blade ki height = 20 meter hai to => all trees will save tu hum kah skte hai ki yeh highest maximum height hai
(wood height = 0 meters)

Required (saw blade height) answer always less than or equal to 20 meters in this case Janha par M meters of wood ya usse thodi jyada M meters of wood milegi. DRY RUN KARNE SE OR BHI SAMJH AYEGA ABHI...

OPTIMAL APPROACH: Define search space and predicate function

Step 01: Find maximum height tree to create search space's end point (highest maximum height of sawblade)

Step 02: Now, Applying Binary Search on search space BinarySearch()

Step 03: create predicate function isPossibleSol()

Time Complexity: $O(N \cdot \log(\text{end}))$, Here N is size of array trees and end is the maximum heighted tree in array

Space Complexity: $O(1)$, no extra space used

Resource: <https://www.spoj.com/problems/EKO/>

Example: 01

Example 01:

Input:

4 7

20 15 10 17

Output:

15

sawblade
ki
possible
height



OBSERVATION

Number of trees = $N = 4$

At Least Required wood = $M = 7$ meters

Trees = [20, 15, 10, 17]

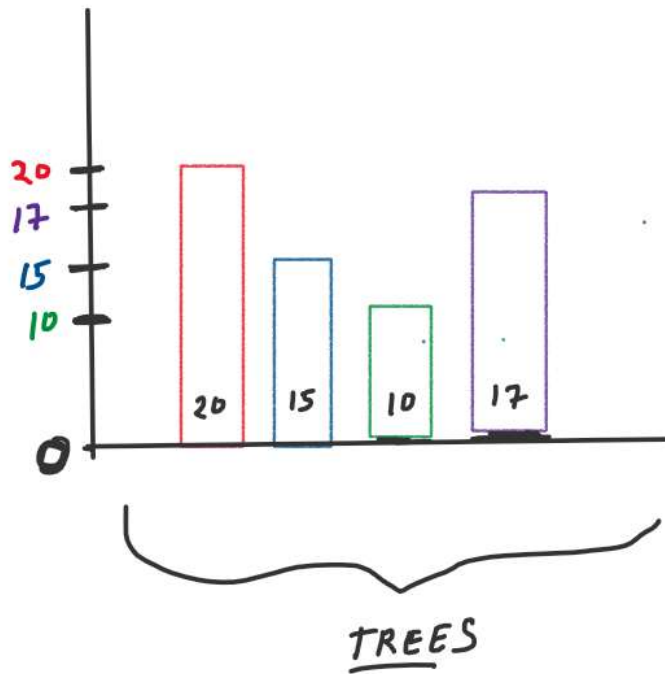
↑
1st Tree
Height 20

↑
2nd Tree
Height 15

↓
3rd Tree
Height 10

→
4th Tree
Height 17

METERS {

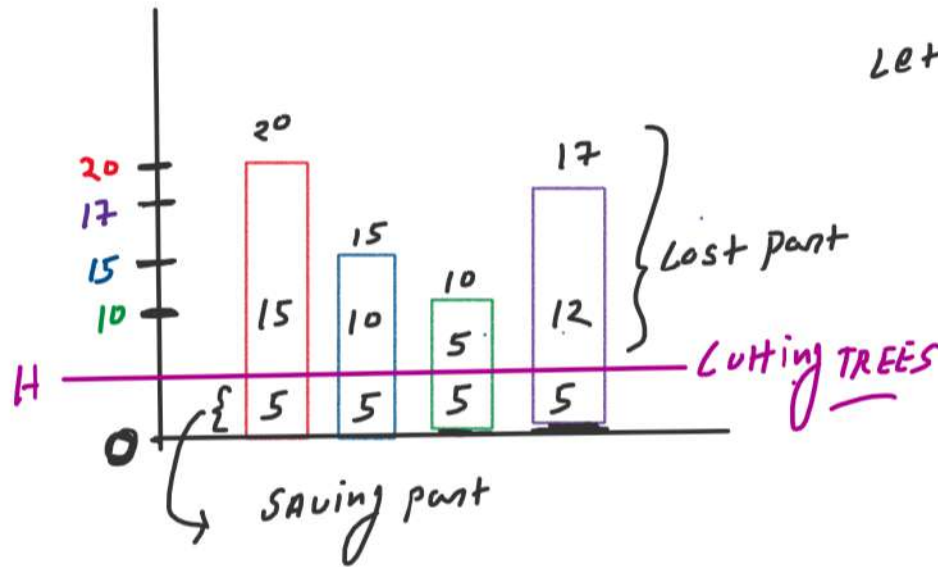


{ Total Wood = Note W when sawblade height H }

EXPLANATION

CASE 01

METERS



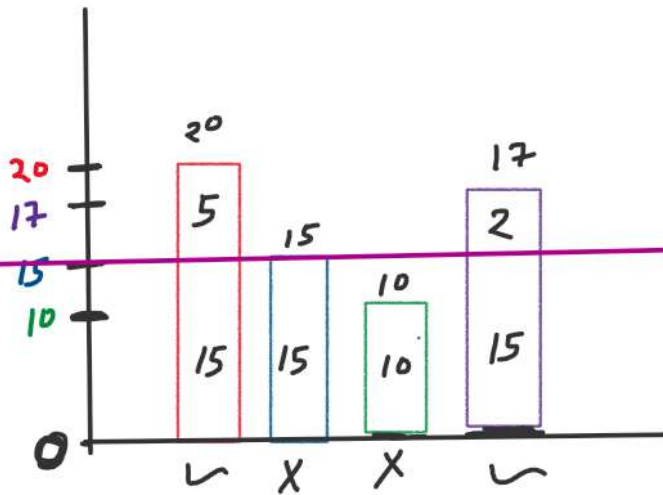
Let $\Rightarrow H = 5$ meters

$$\begin{aligned}
 \text{Total wood} &= (20 - 5) + (15 - 5) + (10 - 5) \\
 &\quad + (17 - 5) \\
 &= 15 + 10 + 5 + 12 \\
 &= 42 \text{ meters wood}
 \end{aligned}$$

CASE 02

METERS

H



CUTTING TREES

Let $\Rightarrow H = 15$ meters

$$\text{Total wood} = (20 - 15) + (15 - 15) + (10 - 15) + (17 - 15)$$

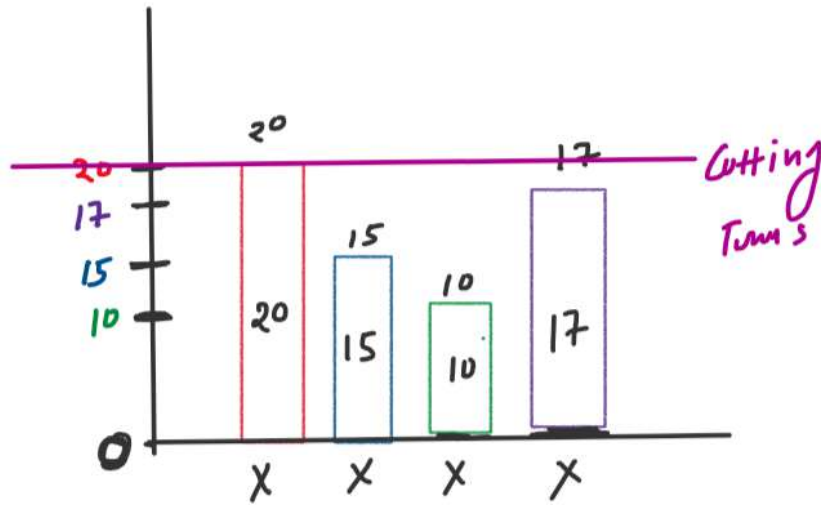
$$= 5 + 0 + 0 + 2$$

$$= 7 \text{ meters wood}$$

CASE 03

METERS

14



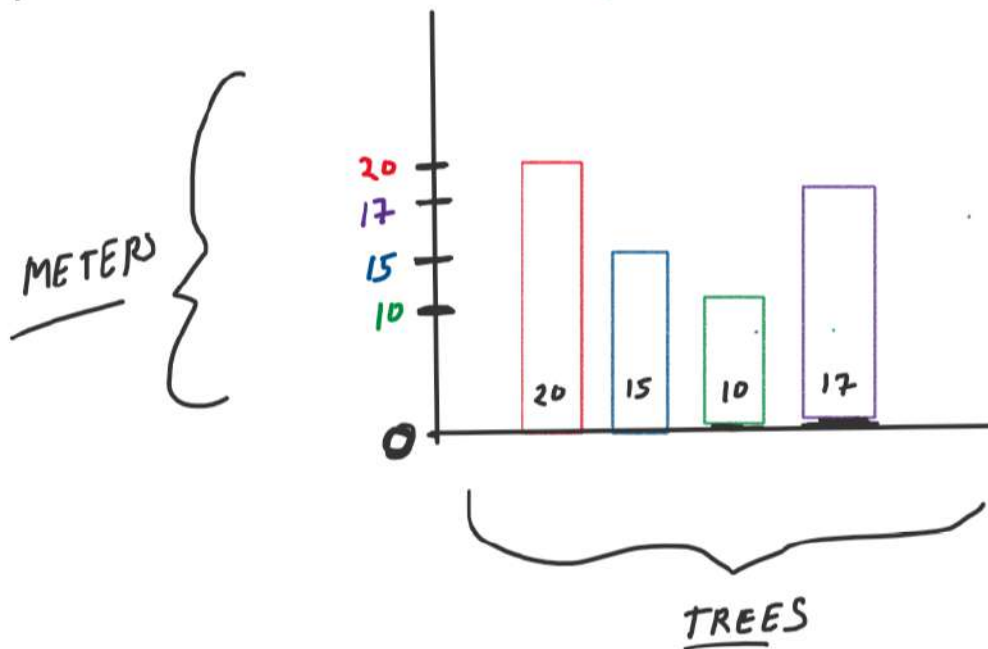
All trees will saw
in jungle

Let $\Rightarrow H = 20$

Not possible saw blade height

$$\begin{aligned}
 \text{Total wood} &= (T_1 - 20) + (T_2 - 20) + (T_3 - 20) + (T_4 - 20) \\
 &= 0 + 0 + 0 + 0 \\
 &= 0 \text{ meter wood}
 \end{aligned}$$

DRY RUN



$M = 7$ and $N = 4$ $H = ?$

arr

20	15	10	17
0	1	2	3

STEP 01

Find maximum height of tree

$END = arr[0]$
 $= 20 \text{ M}$

SEARCH
SPACE

0
START

Maximum No. of
WOOD

20
END

Least No. of
WOOD

max
Height
tree

STEP 02

Now, Applying Binary Search on search space

Iter: 1

$M = 7$

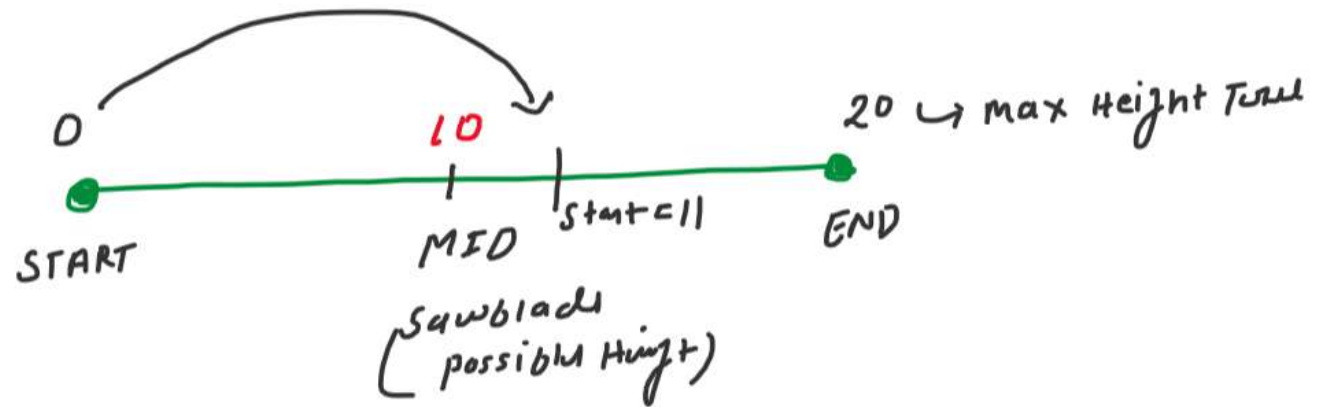
$ANS(H) = \cancel{11} 10$

$START = \cancel{0} 11$

$END = 20$

$$MID = \frac{0+20}{2}$$

$$= 10$$



$$\text{Total wood} = (20-10) + (15-10) + (10-10) + (17-10)$$
$$= 10 + 5 + 0 + 7 = 22 \text{ Meter}$$

possible sol.ⁿ when

$(\text{Total wood} \geq M)$ TRUE

$START = mid + 1$ and $ANS = mid$

STEP 02

Now, Applying Binary Search on search space

Iter: 2

$M = 7$

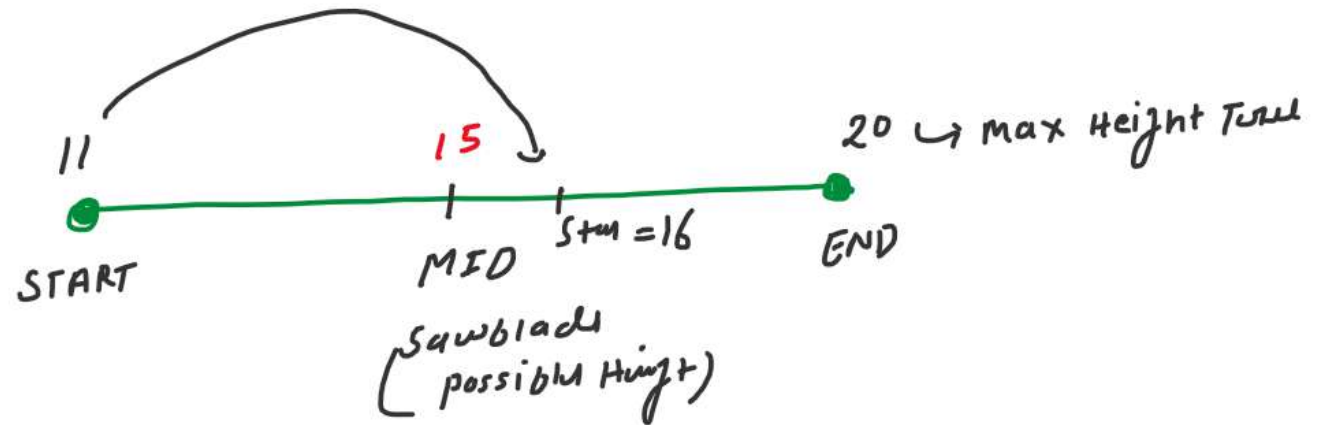
$ANS(H) = 15$

$START = 16$

$END = 20$

$$MID = \frac{11 + 20}{2}$$

$$= 15$$



$$\begin{aligned} \text{Total wood} &= (20 - 15) + (15 - 15) + (10 - 15) + (17 - 15) \\ &= 5 + 0 + 0 + 2 = 7 \text{ Meter} \end{aligned}$$

possible sol.ⁿ when

$(\text{Total wood} \geq M)$ TRUE

$START = mid + 1$ and $ANS = mid$

STEP 02

Now, Applying Binary Search on search space

Iter: 3

$M = 7$

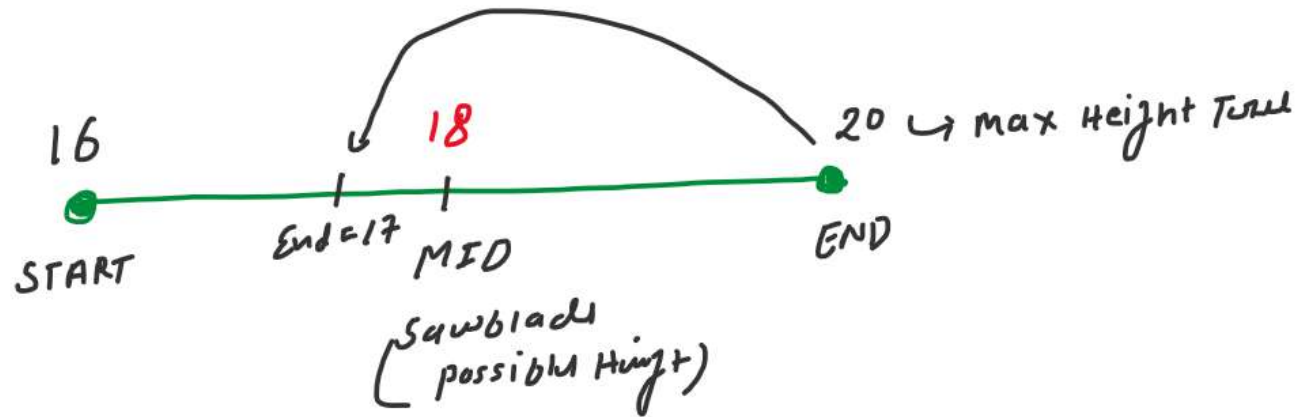
$ANS(H) = 15$

$START = 16$

$END = \cancel{20} \ 17$

$$MID = \frac{16 + 20}{2}$$

$$= 18$$



$$\text{Total wood} = (20 - 18) + (15 - 18) + (10 - 18) + (17 - 18)$$

$$= 2 + 0 + 0 + 0 = 2 \text{ Meter}$$

possible sol.ⁿ when

$(\text{Total wood} > M)$ False

$$END = mid - 1$$

STEP 02

Now, Applying Binary Search on search space

Item: 4

$$M = 7$$

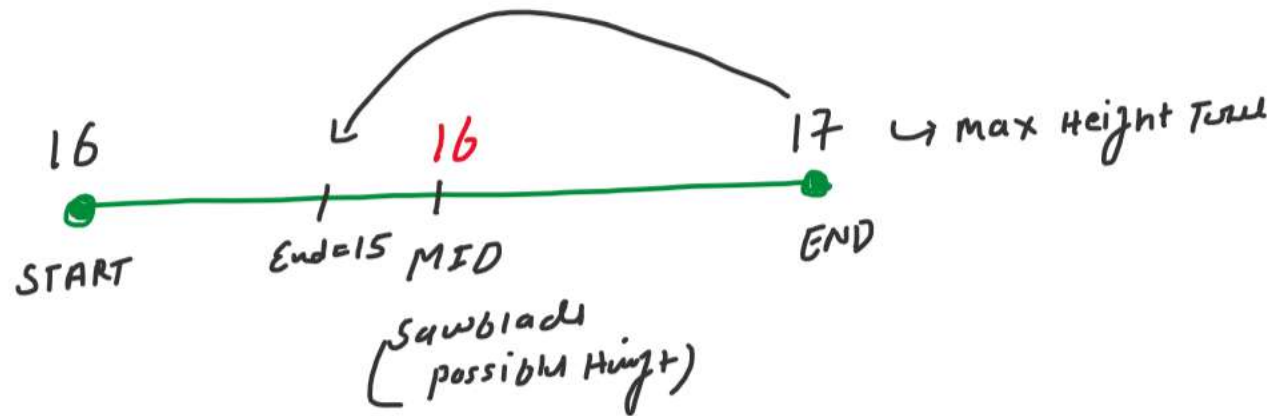
$$\text{ANS}(H) = 15$$

$$\text{START} = 16$$

$$\text{END} = 17$$

$$\text{MID} = \frac{16+17}{2}$$

$$= 16$$



$$\text{Total wood} = (20 - 16) + (15 - 16) + (10 - 16) + (17 - 16)$$

$$= 4 + 0 + 0 + 0 = 4 \text{ meter}$$

possible sol.ⁿ when

(Total wood \geq M) False

$$\text{END} = \text{mid} - 1$$

STEP 02

Now, Applying Binary Search on search space

$I_{\text{tree}} = 5$

$M = 7$

$\text{ANS}(H) = 15$

$\text{START} = 16$

$\text{END} = 15$ } $\text{STOP} \rightarrow \text{start} > \text{end}$



Output {

possible (max) maximum height of
saw blade is $= H \Rightarrow 15$

```

#include<iostream>
#include<vector>
#include<algorithm>
using namespace std;

// Step 03: create predicate function isPossibleSol()
bool isPossibleSol(vector<long long int> trees, long long int m, long long int mid){
    long long int totalWood = 0;
    for(int i=0; i<trees.size(); i++){
        if(trees[i] > mid){
            totalWood = totalWood + (trees[i]-mid);
            if(totalWood >= m){
                return true;
            }
        }
    }
    return false;
}

// Step 02: Now, Applying Binary Search on search space BinarySearch()
int BinarySearch(vector<long long int> trees, long long int m, long long int end){
    long long int start = 0, ansH = -1, mid = start + (end - start)/2;

    while(start<=end){
        // Step 03: create predicate function isPossibleSol()
        if(isPossibleSol(trees, m, mid)){
            ansH = mid;
            start = mid + 1;
        }
        else{
            end = mid - 1;
        }
        mid = start + (end - start)/2;
    }
    return ansH;
}

int maxSawBladeHeight(vector<long long int> trees, long long int m){
    // Step 01: Find maximum height of tree
    long long int end = *max_element(trees.begin(), trees.end());

    // Step 02: Now, Applying Binary Search
    long long int sawbladeMaxHeight = BinarySearch(trees, m, end);

    return sawbladeMaxHeight;
}

int main(){
    long long int n, m;
    cin>>n>>m;
    vector<long long int> trees;
    while(n--){
        long long int height;
        cin>> height;
        trees.push_back(height);
    }

    cout <<maxSawBladeHeight(trees, m);
    return 0;
}

```