Theorem Proving in First Order logic

- Resolution is a theorem proving technique that proceeds by building Refutation proofs, i.e., proofs by contradictions.
- It was invented by a Mathematician John Alan Robinson in the year 1965.
- Resolution is used, if there are various statements are given, and we need to prove a conclusion of those statements.
- Unification is a key concept in proofs by Resolution
- Resolution is a single inference Rule which can efficiently operate on the conjuctive Normal form or clausal form

* clause: Disjunction of literals (an atomic sentence) is called a clause. It is also known as Unit clause.

* conjuctive Normal Form: A sentence Represented as a conjuction of clauses is said to be conjuctive normal form or CNF.

Steps of Resolution

1) Conversion of facts into First Order logic

2) Convert First Order logic statements in Conjuctive Normal Form (CNF)

3) Negate the statement which needs to prove (proof by contradiction)

4) Draw Resolution graph (Unification).

Example :

Consider the following sentences :

1. Raju likes all kinds of food.
2. Apple and Vegetables are food.
3. Anything anyone eats and not killed is food.
4. Rani eats Peanuts and still Alive.
5. Amit eats everything that Rani eats.

Then Proof by Resolution that

Raju likes peanuts

Solution:

Step 1 Conversion of facts into FOL.

1. $\forall x : Food(x) \rightarrow Likes(Raju, x)$

2. $Food(Apple) \wedge Food(Vegetables)$

3. $\forall x \; \forall y : Eats(x, y) \wedge \neg Killed(x) \rightarrow Food(y)$

4. $Eats(Rani, Peanuts) \wedge Alive(Rani)$

5. $\forall x : Eats(Rani, x) \rightarrow Eats(Amit, x)$

Due to Granularity of Representation, we have to add some Additional facts.

6. $\forall x : \neg Killed(x) \rightarrow Alive(x)$

7. $\forall x : Alive(x) \rightarrow \neg Killed(x)$.

8. $Likes(Raju, Peanuts)$

Step 2 : Conversion of FOL into CNF

1) Eliminate all Implication (→) and rewrite.
2) Move Negation (¬) inwards and rewrite
3) Rename Variables or Standardize Variables
4) Eliminate Existential Instantiation Quantifier by Elimination.
5) Drop Universal Quantifiers.
6) Distribute Conjunction ∧ over disjunction ∨.

→ In first Order Logic Resolution, it is required to convert the FOL into CNF as CNF form makes easier for Resolution proofs.

→ Here we eliminate → using the fact that $a \rightarrow b$ is equivalent to $\neg a \vee b$ and move Negation inwards if required

(1) Raju likes all kind of food.

in FOL : $\forall x : Food(x) \rightarrow Likes(Raju, x)$

in CNF : $\forall x : \neg Food(x) \vee Likes(Raju, x)$

(2) Apples and Vegetables are food

in FOL : $Food(Apple) \wedge Food(Vegetable)$

In CNF : $Food(Apple) \wedge Food(Vegetables)$

3) Anything anyone eats and not killed is food

in FOL : $\forall x \forall y : eats(x,y) \wedge \neg killed(x) \rightarrow Food(y)$

in CNF : ① $\forall x \forall y : \neg(eats(x,y) \wedge \neg killed(x)) \vee Food(y)$.

② $\forall x \forall y : \neg eats(x,y) \vee killed(x) \vee Food(y)$

4) Rani eats peanuts and still alive

in FOL : $eats(Rani, Peanuts) \wedge Alive(Rani)$

in CNF : " " " (same as above)

5) Amit eats everything that Rani eats

in FOL : $\forall x : eats(Rani, x) \rightarrow eats(Amit, x)$

in CNF : $\forall x : \neg eats(Rani, x) \vee eats(Amit, x)$

6) Newly Added Fact

In FOL : $\forall x : \neg killed(x) \rightarrow Alive(x)$

in CNF : $\forall x : (\neg killed(x)) \vee Alive(x)$

$\forall x : killed(x) \vee Alive(x)$

7) $\forall x : Alive(x) \rightarrow \neg Killed(x)$

in CNF : $\forall x : \neg Alive(x) \vee \neg Killed(x)$

8) $likes(Raju, Peanuts)$ - same in CNL

Step 2 : Conversion of FOL into CNF.

⇒ Rename Variables or Standardize Variables.

1) $\forall x : \neg Food(x) \vee likes(Raju, x)$

2) $Food(Apple) \wedge Food(Vegetables)$

3) $\forall x \forall y : \neg eats(x, y) \vee Killed(x) \vee Food(y)$

(here we have already used x

After Renaming

$x = y$, $y = z$

$\forall y \forall z : \neg eats(y, z) \vee Killed(y) \vee Food(z)$

4) $eats(Rami, Peanuts) \wedge Alive(Rami)$

5) $\forall x : \neg eats(Rami, x) \vee eats(Anil, x)$

x is already used

⟶ After Standardizing

$x = w$

$\forall w : \neg eats(Rami, w) \vee eats(Anil, w)$

6) $\forall x : Killed(x) \vee Alive(x)$

here $x = g$.

$\forall g : Killed(g) \vee Alive(g)$

7) $\forall x : \neg Alive(x) \vee \neg Killed(x)$

here $x = k$

$\forall k : \neg Alive(k) \vee \neg Killed(k)$

8) likes (Raju , Peanuts.

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Step 2: Conversion of FOL into CNF

- Drop Universal Quantifier
- In this step we will drop all Universal Quantifier since all the statements are not implicitly Quantified so we don't need it.

1) $\forall x : \neg Food(x) \vee likes(Raju, x)$

$\rightarrow \neg Food(x) \vee likes(Raj, x)$

2) $Food(Apple) \wedge Food(Vegetable)$

If $\wedge$ symbol comes then we seperate each and every statement

Food (Apple)

Food (Vegetable)

3) $\forall y \forall z : \neg \text{Eats}(y, z) \vee \text{Killed}(y) \vee \text{Food}$

$\Rightarrow \neg \text{Eats}(y, z) \vee \text{Killed}(y) \vee \text{Food}(z)$

4) $\text{Eats}(\text{Rani}, \text{Peanuts}) \wedge \text{Alive}(\text{Rani})$

$\Rightarrow \text{Eats}(\text{Rani}, \text{Peanuts})$

$\Rightarrow \text{Alive}(\text{Rani})$

5) $\forall w : \neg \text{Eats}(\text{Rani}, w) \vee \text{Eats}(\text{Amit}, w)$

$\Rightarrow \neg \text{Eats}(\text{Rani}, w) \vee \text{Eats}(\text{Amit}, w)$

6) $\forall g : \text{Killed}(g) \vee \text{Alive}(g)$

$\Rightarrow \text{Killed}(g) \vee \text{Alive}(g)$

7) $\forall k : \neg \text{Alive}(k) \vee \neg \text{Killed}(k)$

$\Rightarrow \neg \text{Alive}(k) \vee \neg \text{Killed}(k)$

8) $\text{likes}(\text{Raju}, \text{Peanuts})$

Step 2 : Conversion of FOL into CNF

Distribute conjuction $\wedge$ over disjunction $\neg$.

→ This step will not make any changes in the problem

Step 3 : Negate the statement to be proved

- In this statement, we will apply Negation to the conclusion statements.

Therefore it will be written as ¬ likes (Raju, Peanuts)

~~Thus we have~~.

Step 4 : Draw Resolution graph

- Now In this step, we will solve the problem by Resolution tree using Substitution

For the Above problem, it will be given as follows

¬ likes (Raju, Peanuts)

- ① Fact we will se ¬ Food (x) V likes (Raju, x)

¬ likes (Raju, Peanuts) ¬ Food (x) V likes (Raju, x)

{Peanuts, x}

① ¬ Food (Peanuts)

② ¬ eats (y, z) V Killed (y) V Food (z)

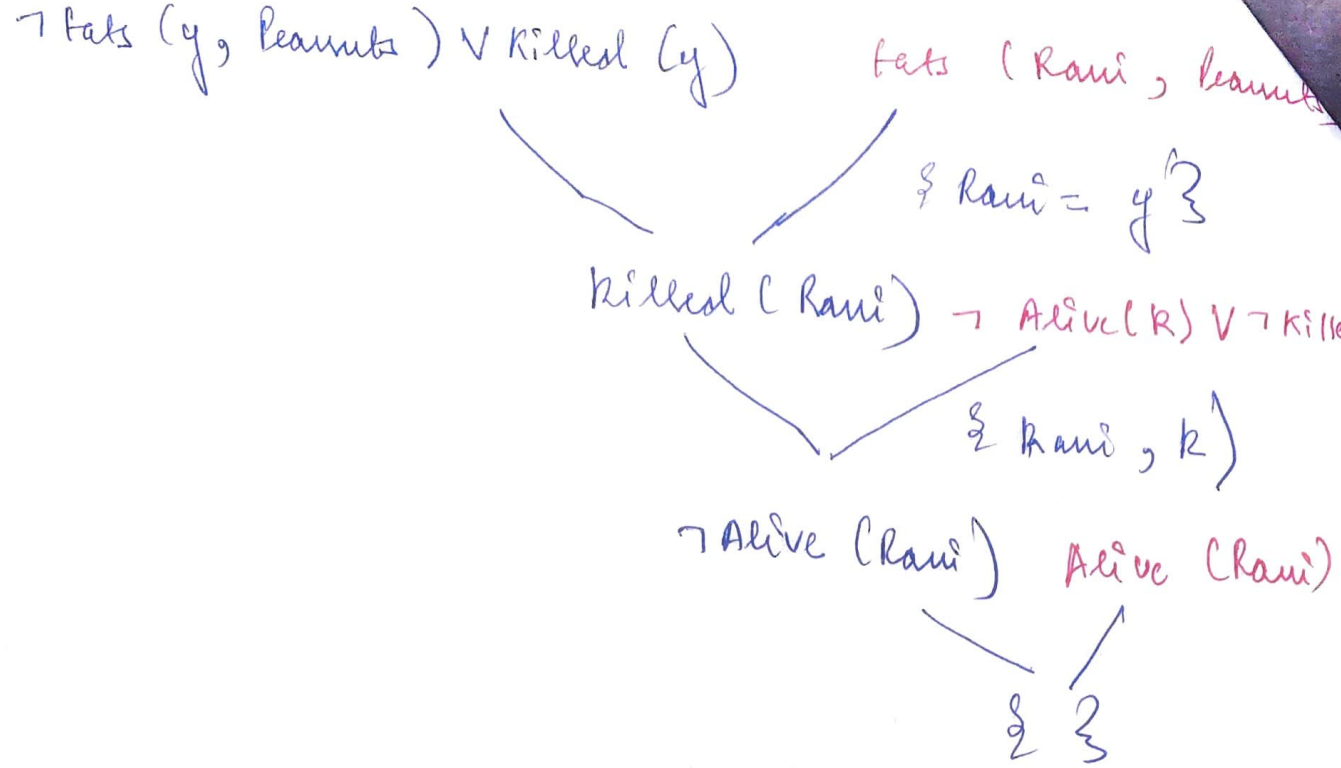
{Peanuts, z}

¬ eats (y, Peanuts) V Killed (y)

Stopping condn

① Till Resolution can be performed

② If you don't get the NIL set output.



As, output of Resolution is Null set

Hence, ¬ likes (Raju, Peanuts) = FALSE

Therefore Like (Raju, Peanuts) = True

↓

Raju likes Peanuts.

Example :

1) Amit only likes easy courses
2) Science courses are Hard
3) All the courses in computer department are easy.
4) AI is computer department course

Then use the Resolution to ensure the Quest^n.

What courses would Amit like?

→ Step1 : conversion of facts into FOL

1) $\forall x$: Easy Course $(x) \rightarrow$ likes (Amit, x)
2) $\forall x$: Science Course $(x) \rightarrow$ Hard Course (x)
3) $\forall x$: Computer dep artment course $(x) \rightarrow$ Easy course (x)
4) Computer Dept. Course (AI)

We have to proof

→ we need to add an Additional fact

likes (Amit, S)

Step 2 : conversion of FOL into CNF

1) Eliminate the $\rightarrow$ and rewrite
2) Move Negation ($\neg$) inwards and rewrite.
3) Rename Variables or Standardize Variables.
4) Eliminate Existential Quantifier

5) Drop Universal Quantifier.

6) Distribute Conjunction $\wedge$ over disjunction $\vee$.

1) Eliminate all $\rightarrow$ and rewrite

$a \rightarrow b \quad = \neg a \vee b.$

1) $\forall x : \text{easyCourse}(x) \rightarrow \text{likes}(\text{Amit}, x)$

$\Rightarrow \forall x : \neg \text{easyCourse}$

Solution

Step - 2 : Conversion of FOL into CNF

- Eliminate all implication (→) and rewrite.
- Here we eliminate →, using the fact that a → b is equivalent to ¬ a V b.

1. ∀x : EasyCourse(x) → Likes(Amit, x)
2. ∀x : ScienceCourse(x) → HardCourse(x)
3. ∀x : ComputerDepartmentCourse(x) → EasyCourse(x)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. Likes(Amit, s)

1. ∀x : ¬ EasyCourse(x) V Likes(Amit, x)
2. ∀x : ¬ ScienceCourse(x) V HardCourse(x)
3. ∀x : ¬ ComputerDepatmentCourse(x) V EasyCourse(x)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. Likes(Amit, s)

Solution

Step - 2 : Conversion of FOL into CNF

- Move negation (¬) inwards and rewrite
- In this problem there is no need to apply this step.
- Therefore the statements will remain same in this step.

1. ∀x : ¬ EasyCourse(x) V Likes(Amit, x)
2. ∀x : ¬ ScienceCourse(x) V HardCourse(x)
3. ∀x : ¬ ComputerDepatmentCourse(x) V EasyCourse(x)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. Likes(Amit, s)

Solution

Step - 2 : Conversion of FOL into CNF

- Rename variables or standardize variables

1. ∀x : ¬ EasyCourse(x) V Likes(Amit, x)
2. ∀x : ¬ ScienceCourse(x) V HardCourse(x)
3. ∀x : ¬ ComputerDepatmentCourse(x) V EasyCourse(x)
4. ComputerDepartmentCourse(Artificial Intelligence)

1. $\forall x_1$: ¬ EasyCourse(x_1) V Likes(Amit, x_1)
2. $\forall x_2$: ¬ ScienceCourse(x_2) V HardCourse(x_2)
3. $\forall x_3$: ¬ ComputerDepatmentCourse(x_3) V EasyCourse(x_3)
4. ComputerDepartmentCourse(Artificial Intelligence)

Solution

Step - 2 : Conversion of FOL into CNF

- Rename variables or standardize variables

1. $\forall x : \neg$ EasyCourse(x) V Likes(Amit, x)
2. $\forall x : \neg$ ScienceCourse(x) V HardCourse(x)
3. $\forall x : \neg$ ComputerDepatmentCourse(x) V EasyCourse(x)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. Likes(Amit, s)

1. $\forall x_1 . \neg$ EasyCourse(x_1) V Likes(Amit, x_1)
2. $\forall x_2 : \neg$ ScienceCourse(x_2) V HardCourse(x_2)
3. $\forall x_3 : \neg$ ComputerDepatmentCourse(x_3) V EasyCourse(x_3)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. Likes(Amit, s)

Solution

Step - 2 : Conversion of FOL into CNF

- Eliminate existential quantifier.
- In this step, we will eliminate existential quantifier ∃, and this process is known as Skolemization.
- But in this problem since there is no existential quantifier so all the statements will remain same in this step.

1. $\forall x_1 : \neg$ EasyCourse(x_1) V Likes(Amit, x_1)
2. $\forall x_2 : \neg$ ScienceCourse(x_2) V HardCourse(x_2)
3. $\forall x_3 : \neg$ ComputerDepatmentCourse(x_3) V EasyCourse(x_3)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. Likes(Amit, s)

Solution

Step - 2 : Conversion of FOL into CNF

- Drop Universal quantifiers.
- In this step we will drop all universal quantifier since all the statements are not implicitly quantified so we don't need it.

1. $\forall x_1 : \neg$ EasyCourse(x_1) V Likes(Amit, x_1)
2. $\forall x_2 : \neg$ ScienceCourse(x_2) V HardCourse(x_2)
3. $\forall x_3 : \neg$ ComputerDepatmentCourse(x_3) V EasyCourse(x_3)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. Likes(Amit, s)

1. $\neg$ EasyCourse(x_1) V Likes(Amit, x_1)
2. $\neg$ ScienceCourse(x_2) V HardCourse(x_2)
3. $\neg$ ComputerDepatmentCourse(x_3) V EasyCourse(x_3)
4. ComputerDepartmentCourse(Artificial Intelligence)

Solution

Step - 2 : Conversion of FOL into CNF

- Distribute conjunction ∧ over disjunction ¬.
- This step will not make any change in this problem.

1. ¬ EasyCourse(x_1) V Likes(Amit, x_1)
2. ¬ ScienceCourse(x_2) V HardCourse(x_2)
3. ¬ ComputerDepatmentCourse(x_3) V EasyCourse(x_3)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. Likes(Amit, s)

Solution

Step - 3 : Negate the statement to be proved

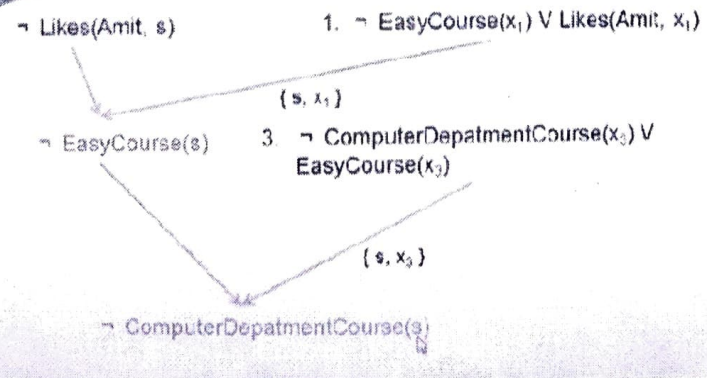
- In this step, we will apply negation to the conclusion statements.
- Therefore it will be written as ¬Likes(Amit ,s)
- Thus , we have

1. ¬ EasyCourse(x_1) V Likes(Amit, x_1)
2. ¬ ScienceCourse(x_2) V HardCourse(x_2)
3. ¬ ComputerDepatmentCourse(x_3) V EasyCourse(x_3)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. ¬ Likes(Amit, s)

Solution

Step - 4 : Draw Resolution graph:

- Now in this step, we will solve the problem by resolution tree using substitution. For the above problem, it will be given as follows:



1. ¬ EasyCourse(x_1) V Likes(Amit x_1)
2. ¬ ScienceCourse(x_2) V HardCourse(x_2)
3. ¬ ComputerDepatmentCourse(x_3) V EasyCourse(x_3)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. ¬ Likes(Amit, s)

Solution

Step - 4 : Draw Resolution Graph:

¬ ComputerDepatmentCourse(s)

4. ComputerDepartmentCourse(Artificial Intelligence)

{ s , Artificial Intelligence }

{ }

1. ¬ EasyCourse(x_1) V Likes(Amit, x_1)
2. ¬ ScienceCourse(x_2) V HardCourse(x
3. ¬ ComputerDepatmentCourse(x_3) V EasyCourse(x_3)
4. ComputerDepartmentCourse(Artificial Intelligence)
5. ¬ Likes(Amit, s)

- As ,output of Resolution is NULL Set.
- Hence , ¬ Likes(Amit, s) = FALSE
- Therefore Likes(Amit, s) = TRUE ....
- Amit likes Artificial Intelligence...

Move ¬ inward

- $\neg(\forall x\, P) = \exists x\, \neg P$
- $\neg(\exists x P) = \forall x\, \neg P$
- $\neg(a \vee b) = \neg a \wedge \neg b$
- $\neg(a \wedge b) = \neg a \vee \neg b$
- $\neg\neg a = a$