Probabilities / Probabilistic Reasoning in AI

Uncertainity :

- In FOL, we were sure about the predicates.
With the Knowledge Repⁿ, we might write A → B, which means if A is true then B is true, but consider a situation where we are not sure about whether A is true or not then we cannot express this statement, this situation is called Uncertainity.

So to represent Uncertain Knowledge, where we are not sure about the predicates, we need uncertain Reasoning or Probabilistic Reasoning

Causes of uncertainity:

1) Information occured from Unreliable Sources
2) Experimental Errors.
3) Equipment fault.
4) Temperature Variation
5) Climate Change.

Probabilistic Reasoning

It is a way of Knowledge Repn where we apply the concept of probability to indicate the Uncertainity in Knowledge

→ In Probabilistic Reasoning, we combine probability theory with logic to handle the Uncertainity

Q (It will Rain today?

Q Behaviour of someone at some situation

Need of Probabilistic Reasoning in AI

- When there are unpredictable Outcomes
- When Specifications or possibilities of predicates becomes too large to handle.
- When an Unknown error occurs during an Experiment.

⇒ In PR, there are 2 ways to solve problems with Uncertain Knowledge

- Bayes Rule
- Bayesian Statistics

Probability : It can be defined as a chance that an uncertain event will occur. The value of Probability always remain b/w 0 and 1 that represent ideal uncertainities.

$0 \leq P(A) \leq 1$, where $P(A)$ is the probability of an event A.

$P(A) = 0$, indicates total uncertainity in an event A.

$P(A) = 1$, indicates total certainity in an event A.

$$\text{Probability of occurance} = \frac{\text{No. of desired outcomes}}{\text{Total No. of outcomes}}$$

- $P(\neg A)$ = Probability of not happening event.

$P(\neg A) + P(A) = 1$

Conditional Probability : It is a probability of occ[urrence of]
an event when another event has already happen[ed]

lets suppose, we want to calculate the event A when event B has already occurred, " the probability of A under the condition of B ", it can be written as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Joint probability of A and B

Marginal probability of B

Q In a class, there are 70% of students who like English and 40% of students who like English & Maths. then what is % of students those who like English also like Mathematics.

Ans let A is an event that student like maths
B is an event that a student likes English.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.7} = 57\%$$

Hence 57% are the students who like English also like Maths.

Bayesian Networks in AI

It defines Probabilistic independencies and dependencies among the Variables in the Network.

↳ It is a Probabilistic graphical model which represents a set of Variables and their conditional dependencies using a directed acyclic graph (DAG)

→ Built from Probability distribution

→ Consists of
- DAG
- Table of conditional Probabilities.

→ Arc / Directed Arrows
↓
Represents Causal Relationship or conditional Probabilities among Random variables

→ Node
↓
corresponds to a random variable
- continuous
- discrete

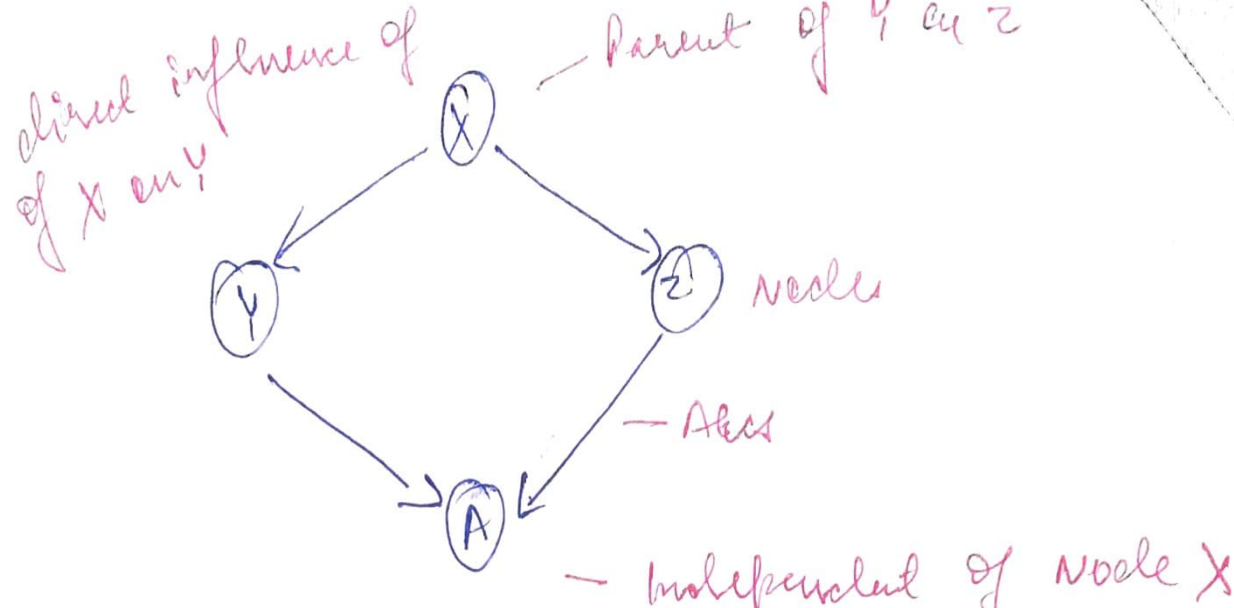
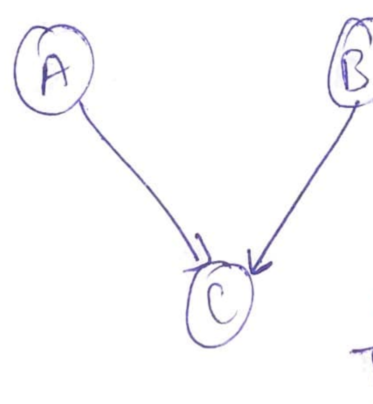


Table : Conditional Probability of all nodes with effect of Parent Nodes.

To Propagate belief in Bayesian N/w, initial "Directed acyclic Graph" is converted into an undirected graph in which the arcs can be used to transmit probabilities in direct of evidence

T	.002
F	.998



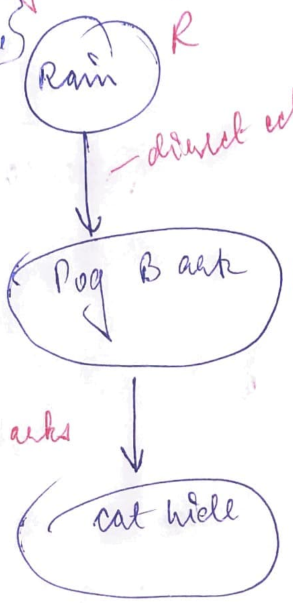
T	0.001
F	0.999

A	B	P(C=T)	P(C=F)
T	T	–	–
T	F	–	–
F	T	–	–
F	F	–	–

I Directed Acyclic graph (DAG)

Parent node

R



– directed Arrow represents conditional probability

P (Dog Bark | Rains)

P (X / P)

Child node

B

- You need to take out the probability with respect to its parent node

Cat hide | Dog Barks

- convenient for Representing probabilistic relation between multiple events

II Conditional Probability Table

	R	~R
B	9/48	18/48
~B	3/48	18/48

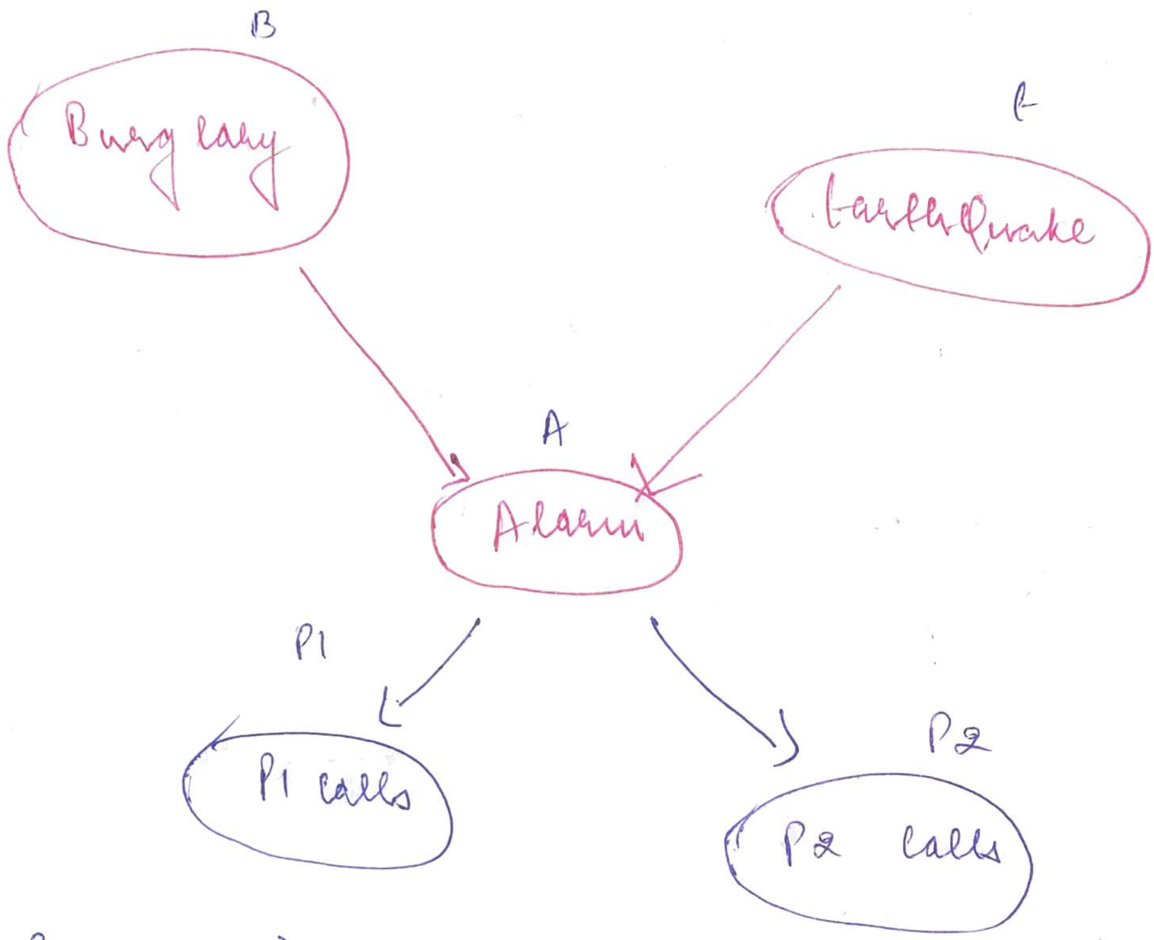
$(B = T \;\&\; R = T) = 0.19$

$(B = T \;\&\; R = F) = 0.375$

$(B = F \;\&\; R = T) = 0.06$

$(B = F \;\&\; R = F) = 0.375$

DAG :-



$P = (B = T) = 0.001$

$P = (B = F) = 0.999$

$P(E = T) = 0.002$

$P(E = F) = 0.998$

B	E	P(A=T)	P(A=F)
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
T	F	0.001	0.999

→ Observed Values

Problem : Calculate the Probability that Alarm has sounded, but there is neither a burglary nor an earthquake occurred, and P1 and P2 both called the Owner.

Example : Owner installed a burglar Alarm at his home to detect burglary. The Alarm Reliably responds at detecting a burglary but also responds for minor earthquakes. ~~Harry~~ Owner has 2 neighbors P1 & P2, who have taken a responsibility to inform owner at work when they hear the Alarm. P1 always calls Harry if he hear the Alarm, but sometimes he got confused with the phone Ringing & calls at that time too. On the other hand, P2 likes to listen to high music, so sometimes she misses to hear the Alarm. Here we would like to compute the Probability of Burglary Alarm.

A	P(P1 = T)	P(P1 = F)
T	0.90	0.10
F	0.05	0.95

A	P(P2 = T)	P(P2 = F)
T	0.70	0.30
F	0.01	0.99

$P = P(P1, P2, A, \sim B, \sim E)$

$P(P1|A) \cdot P(P2|A) \cdot P(A| \sim B, \sim E) \cdot P(\sim B)\, P(\sim E)$

$0.90 \times 0.70 \times 0.001 \times 0.999 \times 0.998$

$= 0.00062$

Hence a Bayesian Network can answer any query about the domain by using Joint Distribution

The Semantic of Bayesian Network

Ways :-

1) To understand the Network as the Representation of Joint Probability distribution

→ Helpful to understand How to construct a N/w

2) To Understand the Network as an encoding of a collection of conditional independence statements

→ Helpful in designing inference Procedure