



# Poornima

COLLEGE OF ENGINEERING

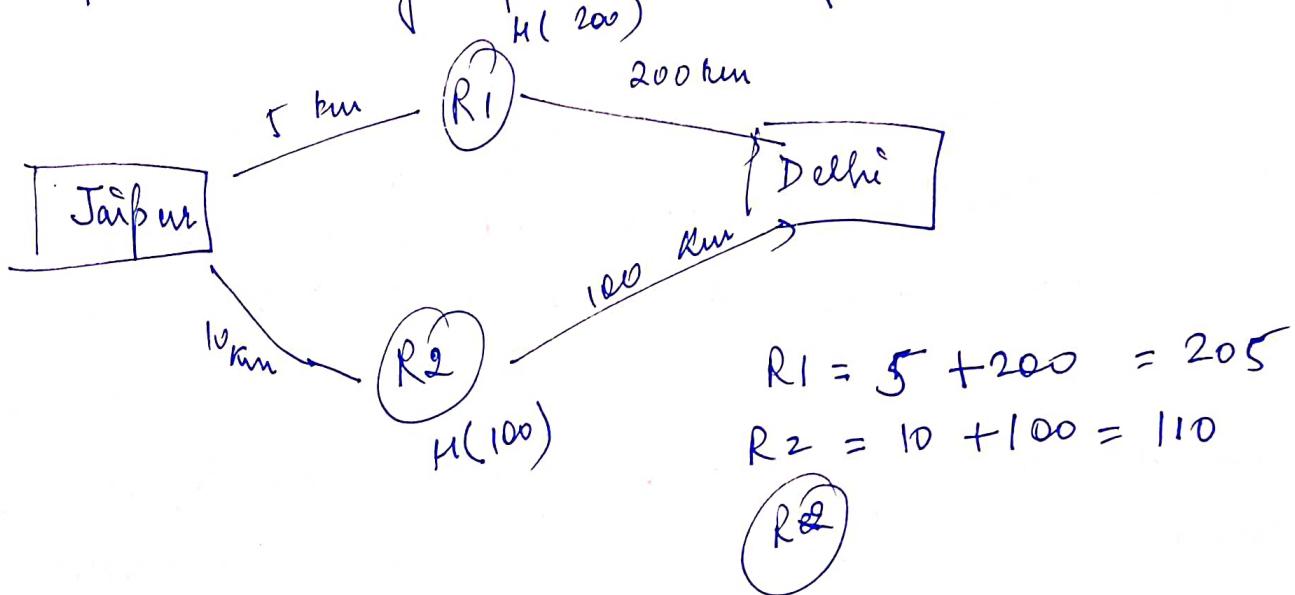
## DETAILED LECTURE NOTES

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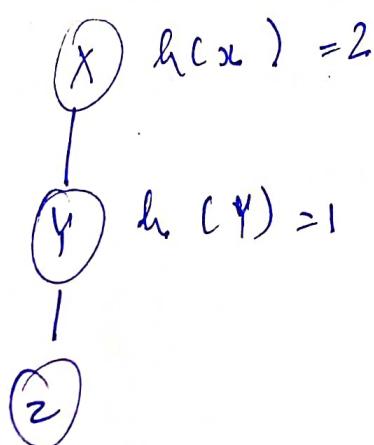
### Heuristic Search Techniques

1. It is a type of a search performed by AI that looks to find a good solution, not necessarily a perfect one, out of the available options.
2. This technique makes decisions by ranking every available choices at each branch of a search and then chooses the Best Option of those presented.
3. Rather than focusing on finding an Optimal Solution like other search methods, heuristic searching is designed to be quick, and therefore finds the most acceptable option within a reasonable time limit. There are several types of heuristic search Algorithms that can be used depending on the task.

- It is a technique designed to solve a problem quickly.
  - Tries to optimize a problem (this is to some problem) using heuristic function in minimum steps/cost.
- Heuristic function**
- It is a function  $h(n)$  that gives an estimation on the cost of getting from node  $n$  to the GOAL STATE.
  - Helps in selecting optimal node for expansion.



→ It informs that how far would be your goal state.





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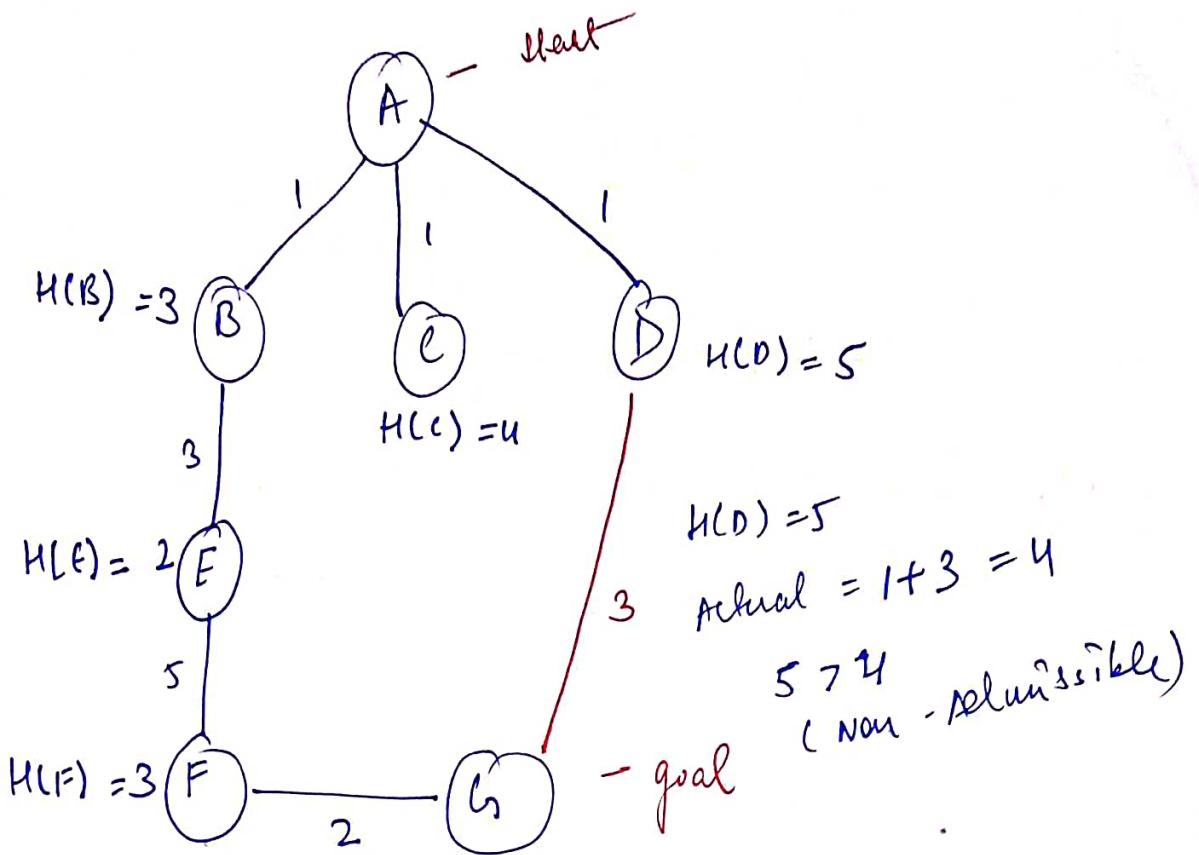
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### Types of Heuristic Search :

- 1) Admissible : In this heuristic function , never overestimates the cost of reaching the goal.  
If underestimates the cost of reaching the goal.  
 $\rightarrow h(n)$  is always less than or equal to actual cost of lowest cost path from ' $n$ ' to goal.  
$$h(n) \leq h'(n) \& \text{goal}$$
- 2) Non Admissible : It overestimates the cost of reaching the goal.  
 $\rightarrow h(n)$  is always greater than actual cost of lowest cost path from ' $n$ ' to goal.  
$$h(n) > h'(n)$$



$$f(n) = h(n) + g(n)$$

$$B = 3 + 1 = 4 \text{ (select)}$$

$$C = 4 + 1 = 5$$

$$D = 5 + 1 = 6$$

$$\text{Actual} = 1 + 3 + 5 + 2 = 11 -$$

$$3 < 11 \text{ (Admissible)}$$

- Particularly useful in solving tough and complex problems, solutions of which require infinite time for far longer.



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### Generate and Test

- 1) It is an informed search technique or Heuristic search technique.
- 2) Uses DFS with backtracking
- Steps Involved / Algorithm : Generator  
Test
- 1) Generate a possible solutions
  - 2) Test to see if this is a Actual solution.
  - 3) If a solution is found , quit . Otherwise go + step 1

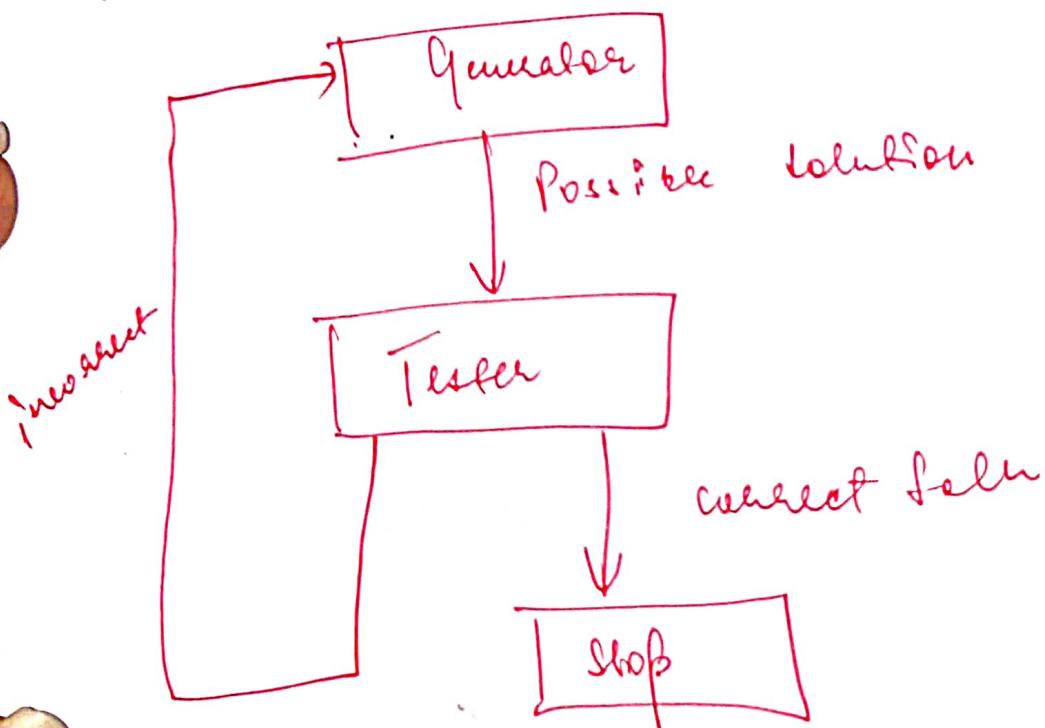
Also known as British Museum Search Algorithm finding an object in the British museum by wandering properties of good generator: randomly

- 1) Complete
- 2) non- Redundant
- 3) Informed

If problem space is large, It may take very long time.

Systematic generate and test

- While generating complete random solutions are 2 extremes there will be another approach that lies in b/w.
- The Approach is that the search process proceeds systematically but some paths that unlikely to lead the solution are not considered.
- This valuation is performed by heuristic fn.
- DFS with Backtracking can be used to implement Systematic generate and test.





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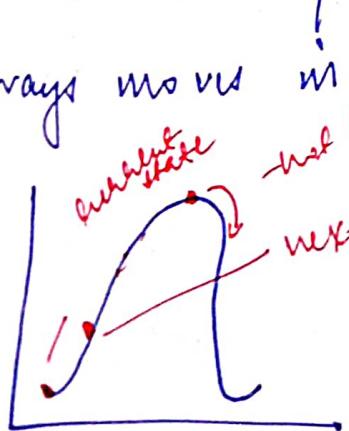
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### Hill Climbing

- Local search Algorithm which continuously moves in the direction of increasing elevation / value to find the peak of mountain or best solution to the problem. It terminates when it reaches a peak value where no neighbor has a higher value.
- It is a technique used for the mathematical problems.
- It is also known as greedy local search as it only looks to its good immediate neighbor state and not beyond that.
- Variant of generate and test in which feedback from test procedure is used to help generator decide which direction to move in search space.
- NO Backtracking
- Always moves in single direction.
  - current state
  - next / new state
  - if new state is better than current state, new state = current state
  - not better - stop



Type of hill climbing →

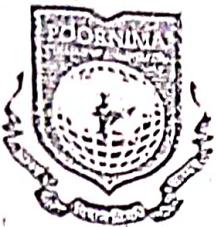
- 1) Simple hill climbing : It only checks its one neighbor or successor state, and if it finds better than the current state, then move else be in the same state.

#### Features

- less time consuming
- less optimal sol<sup>n</sup> and sol<sup>n</sup> is not guaranteed

Algorithm for simple hill climbing.

1. Evaluate the initial state, if it is a goal state then return success and stop.
2. loop until a sol<sup>n</sup> is found or there is no new operator left to apply.
3. Select and apply one operator to the current state.
4. Check new state
  - a. If it is goal state , then return success and quit
  - b. Else if it is better than current state then assign new state as a current state



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- Else if not better than the current state  
then return to step 2.

Step 5 Exit

**Steepest - Ascent Hill Climbing**

- Variation of Simple Hill Climbing.
- It examines all neighboring node of current state and selects one Neigbor node which is closest to the goal state.
- It consumes more time as it searches for multiple Neighbors.

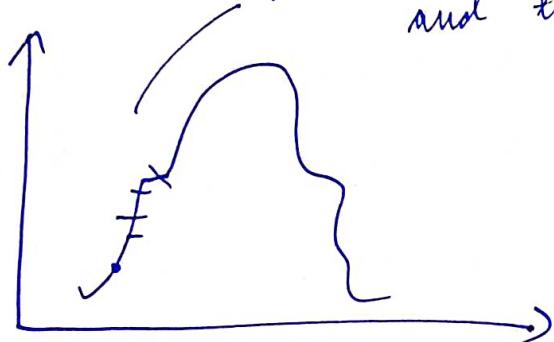
**Algorithm for Steepest Ascent Hill Climbing**

Step 1: Evaluate the initial state , if it is a goal state then return success and Stop , else make current state as initial state.

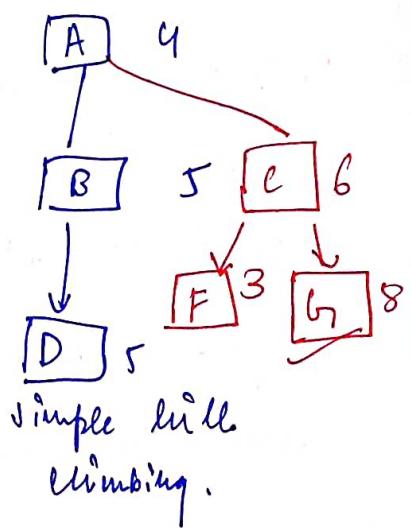
Step 2: Loop until a soln is found or the current state does not change.

- a. let succ be a state such that any successor of the current state does not change. will be better than it.
- b. For each operator that applies to current state.
- apply a new operator and generate a new state
  - Evaluate the New State.
  - If it is a goal state, then return it and quit, else compare it to succ.
  - If it is better than succ, then set new state as succ.
  - If the succ is better than the current state then set current state to succ.

Step 5 : exit.



multiple point are checked  
and then jump to best state



-Deep mind / Alpha go

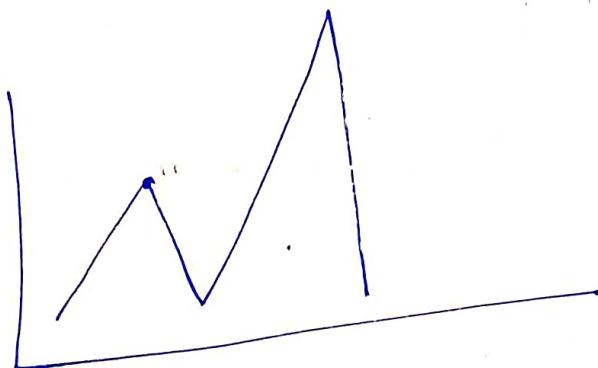


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Problems in Hill climbing:

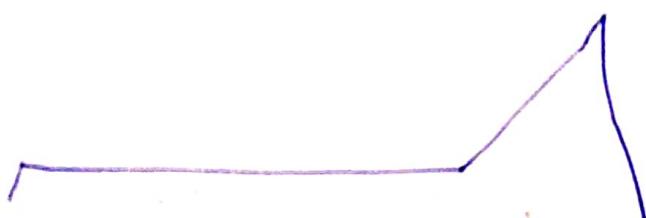
- 1) local Maximum: peak state in the landscape which is better than each of its neighbouring states , but there is another state also present which is higher than the local maximum



Solution: Backtracking technique  $\rightarrow$  Maintain a list of visited states . If the search state reaches the Unclimbable state , it can backtrack to the previous configuration and explore a new path.

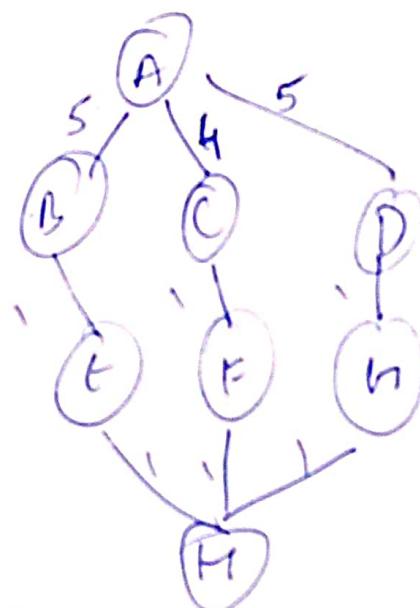
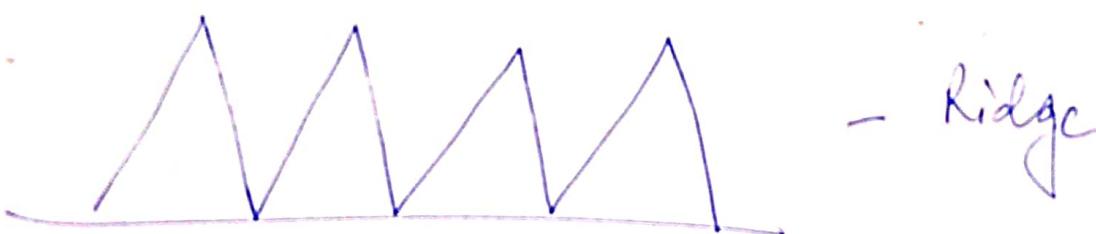
2) Plateau : Flat area of search space in which all Neighbour states of the current state contains the same value, because of this algorithm does not find any best direction to move.

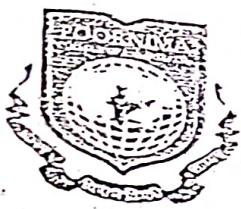
Solution: Make a Big jump.



3) Ridges: Special form of local maximum. It has an area which is higher than its surrounding areas, but itself has a slope and cannot be reached in single move

Solution: Move in different direction.





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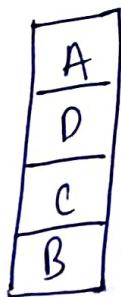
Block World problem using = Kill climbing =

=> What is Block World?

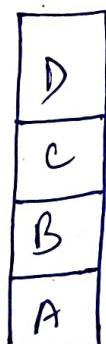
The world comprises of

- A Flat surface such as Tabletop.
- An Adequate set of Identical blocks which are identified by letters
- The Blocks can be stacked one on one to form towers of apparently unlimited height.
- The stacking is achieved using a robot arm which has fundamental operations and states.
- The Robot can hold one block at a time and only 1 block can be moved at a time.

Initial State



Goal State



## Local Heuristic function

- +1 for each block that is resting on the thing it is supposed to be resting on.
- 1 for each block i.e. resting on a wrong thing.

### Initial State

-1	A
+1	D
+1	C
-1	B



After Applying  
Simple Hill  
Climbing

### Initial State

+1	P
+1	C
+1	B
+1	A

### Goal State

$$h = +4.$$

$$h = 0$$

the initial state  
will become  
current state.

### New State

+1	D
+1	C
-1	B



$$h = +2$$

Better

Then we  
will make  
it as current  
state

New state is better state on ↑  
that event Net?

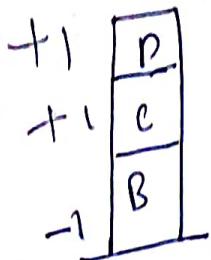


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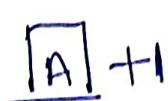
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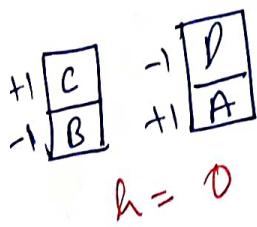
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Current State



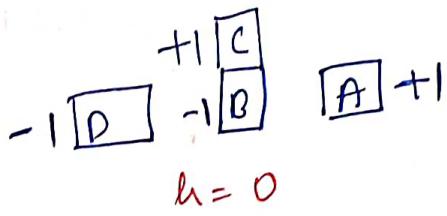
$$h = +2$$



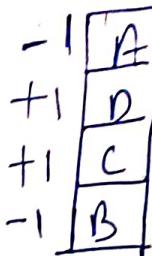
$$h = 0$$



$$h = 0$$



$$h = 0$$



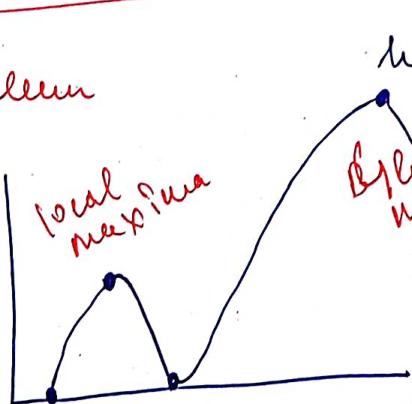
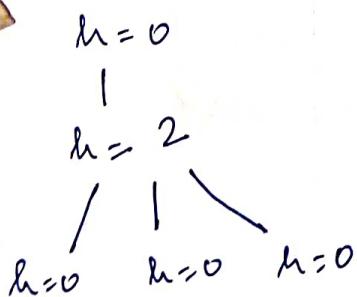
$$h = 0$$

(Backtrack)

Which one is Better choice?

→ Trapped

Overall Problem



Goal State =  $h = 4$

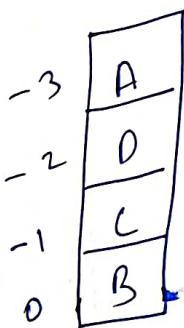
$h = 4$   
Global maxima  
→ 10 local  
Maxima

So now we need to see a New path / Backtrack.

Note: intended while designing heuristic for it should not consider local parameter but it should consider global parameter.

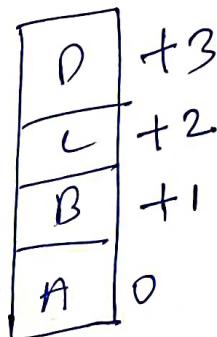
### Global Heuristic function?

- For each block that has the correct support structure: +1 so every block in the support structure.
- For each block that has a wrong support structure: -1



$$h = -6$$

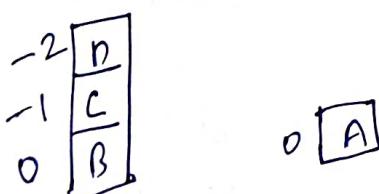
Initial State



$$h = +6$$

Goal State

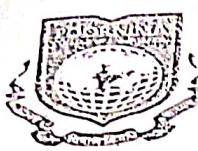
This will become  
as current state



$$h = -3$$



New State is Better?  
if then make this as  
current state.



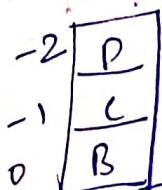
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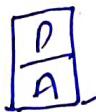
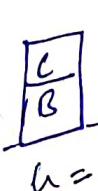
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Current State:



$$h = -3$$



$$h = -2$$



$$h = -1$$

(Better state)



$$h = -6$$

(Backtracking)

Which one is better choice?

better choice?

Keep A and D

$$h = -1$$

Keep P and A

$$h = -1$$



$$h = -1$$

Keep C and A

$$h = -1$$

Keep D and C

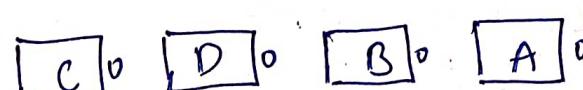
$$h = -2$$

Keep A and C

$$h = -2$$

Keep P and C

$$h = -1$$



$$h = 0$$

C better state, so make  
this as current state

### Current State

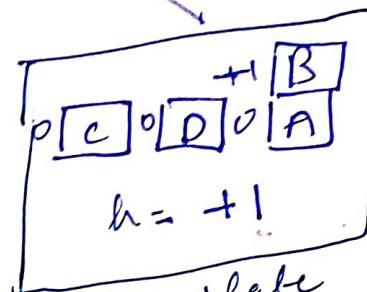
o [c] o [D] o [B] o [A]

$h = 0$

Keep C on D or  
B on A  
( $h = -1$ )

Keep D  
on C or  
B on A  
( $h = -1$ )

Keep A  
on B or  
D or C  
 $h = -1$



Better State

Keep B  
on C or  
D  
 $h = -1$



New State

Keep D  
on C  
 $h = 0$   
Keep D on  
B  
 $h = -1$

Keep C  
on D  
 $h = 0$

+1 [B]  
0 [A]  $h = +1$

Keep B on  
Table  
 $h = 0$

Keep B on  
D  
 $h = -1$   
Keep B  
on C  
 $h = -1$

+2 [C]  
+1 [B]  
0 [A]

$h = +3$

Better State

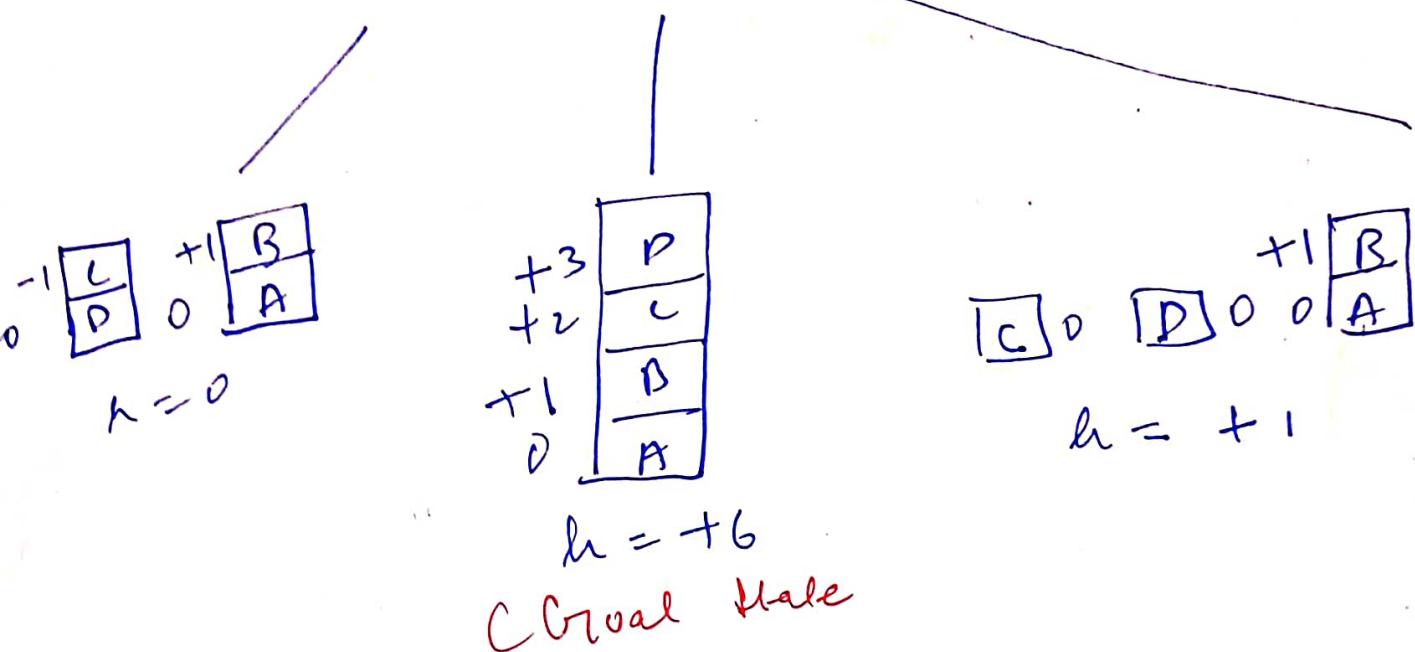
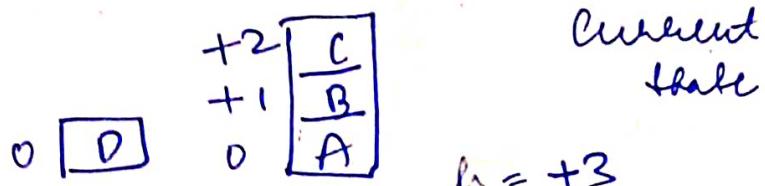


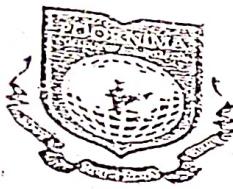
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i) Best - First search Algorithm (Greedy search)

- Greedy Best first search Algorithm selects the path which appears best at moment.
- It is the combination of DFS and BFS.
- It uses heuristic  $f^n$  and search.
- With the help of Best first search, at each step, we can choose the most promising node.
- In the Best First search Algorithm, we expand the node which is closest to goal node and the closest cost is estimated by heuristic  $f^n$  i.e.,  
$$f^n(n) = g(n).$$
- Or we can also say that it uses Evaluation function to decide which adjacent node is most promising and then explore.
- Lies in the category of heuristic or informed search.
- Priority queue is used to store list of nodes.

Algorithm

Priority Queue PQ containing initial states

loop  
if  $PQ = \text{Empty}$  Return FAIL

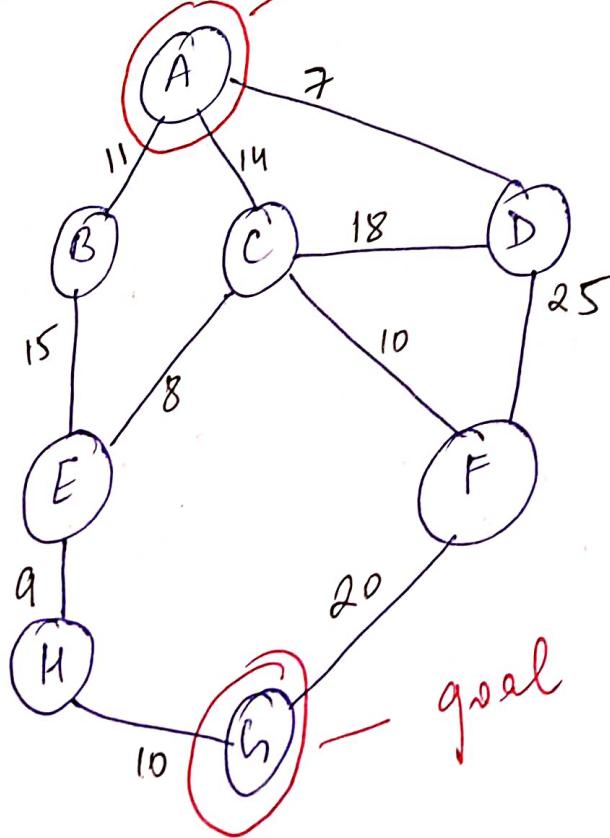
else  
NODE  $\leftarrow$  Remove - First ( $PQ$ )

If  $NODE = GOAL$   
Return Path from Initial to NODE

else  
Generate all successors of NODE and insert  
newly generated NODE into PQ according  
to cost Value

End loop

shortest line  
distance



$A \rightarrow G$	$= 40$
$B \rightarrow G$	$= 32$
$C \rightarrow G$	$= 25$
$D \rightarrow G$	$= 35$
$E \rightarrow G$	$= 19$
$F \rightarrow G$	$= 17$
$H \rightarrow G$	$= 0$
$H \rightarrow G$	$= 10$



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open

[A]

[C, B, D] // here we  
will see straight line  
distance and put it  
in sorted order.

closed

[ ]

[A]

[B, D]

[A, C]

[F, E, B, D]

[A, C]

[G, E, B, D]

[A, C, F]

[E, B, D]

[A, C, F, G]

Path = A → C → F → G = (44)

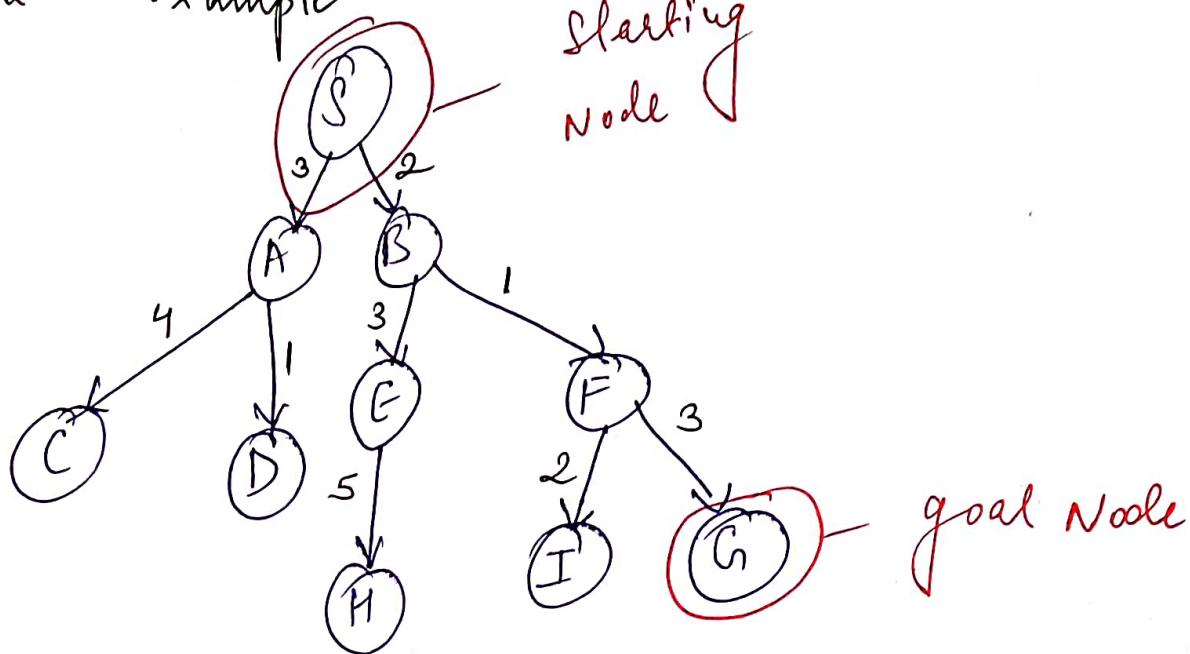
Space complexity =  $O(b^d)$

Time complexity =  $O(b^d)$

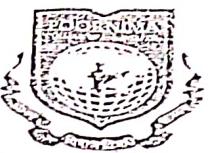
b: branching factor

d: depth

2<sup>nd</sup> example



Node	$H(n)$	Open	Closed
A	12	[S]	[ ]
B	4	[B, A]	[S]
C	7		[S, B]
D	3	[F, E, A]	
E	8	[G, I, E, A]	[S, B, F]
F	2	[I, E, A]	
G	0		[S, B, F, G]
H	4		
I	9		
S	13		
		Path: $\rightarrow S \rightarrow B \rightarrow F \rightarrow G.$	



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### Best First Search Algorithm

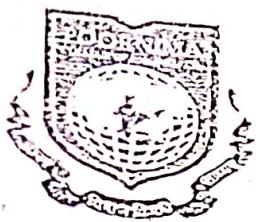
- 1) Place the starting Node into the OPEN list.
- 2) If the Open list is empty , stop and return Failure.
- 3) Remove the Node  $n$  , from the open list which has the lowest Value of  $f(n)$  and places it in the CLOSED list.
- 4) Expand the Node  $n$  , and generate the successors of Node  $n$  .
- 5) Check each successor of Node  $n$  , and find whether any Node is a Goal Node or not .  
If Any successor Node is a Goal Node , then Return Success and terminate the search ,  
else proceed to step 6.
- 6) For each successor Node , Algorithm checks for Evaluation  $f(n)$  and then check if a Node was been in either OPEN or CLOSED list . If the Node has Not been in both list , then add it to Open list .
- 7) return to step 2.

### Advantages:

- Best First Search can switch b/w BFS and DFS by giving the advantages of both the Algorithms.
- More Efficient than BFS and DFS.

### Disadvantages

- Stuck in loop as DFS.
- Not Optimal.



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Unit - 1 : Topic : A\* search Algorithm

- uses heuristic function  $h(n)$  and cost to reach the node ' $n$ ' from start state  $g(n)$ )
- Finds shortest path through search space
- Fast and Optimal Result.

$$f(n) = g(n) + h(n)$$

- It is a form of Best First Search.

$$f(n) = g(n) + h(n) \rightarrow \text{cost to reach from node } n \text{ to goal}$$

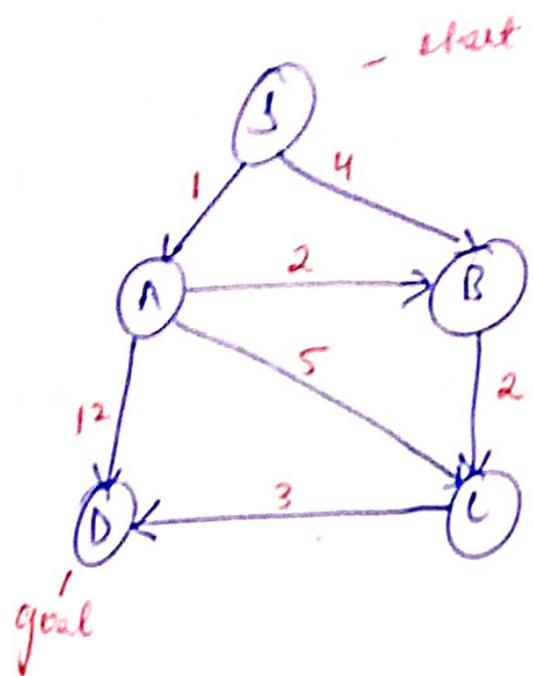
$\downarrow$

Fitness No:

Estimated cost  
of the cheapest  
soln.

cost to reach node  
 $n$  from start  
state

Note: At each point in the search space,  
only those Node is Expended which have the  
lowest value of  $f(n)$ , and the Algorithm



Start	$\rightarrow$	h(S)
S	$\rightarrow$	7
B	$\rightarrow$	2
C	$\rightarrow$	1
A	$\rightarrow$	6
D	$\rightarrow$	0

$$S \rightarrow A = 1 + 6 = 7 \quad \text{or}$$

$$S \rightarrow B = 4 + 2 = 6$$

$$S \rightarrow B \rightarrow C = (4 + 2) + 1 = 7. \quad \text{closed}$$

$$S \rightarrow B \rightarrow C \rightarrow D = (4 + 2 + 3) + 0 = 9 \quad \text{closed}$$

We will keep true to compare it with other paths.

$$S \rightarrow A \rightarrow B = (1 + 2) + 2 = 5$$

$$S \rightarrow A \rightarrow C = (1 + 5) + 1 = 7$$

$$S \rightarrow A \rightarrow D = (1 + 12) + 0 = 13 \quad \text{closed} \rightarrow \text{not 2}$$

$$S \rightarrow A \rightarrow B \rightarrow C = (1 + 2 + 2) + 1 = 6$$

$$S \rightarrow A \rightarrow B \rightarrow C \rightarrow D = (1 + 2 + 2 + 3) + 0 = 8 \quad \text{closed} \rightarrow \text{not 2}$$

$$S \rightarrow A \rightarrow C \rightarrow D = (1 + 5 + 3) + 0 = 9 \quad \text{closed} \rightarrow \text{not 2}$$

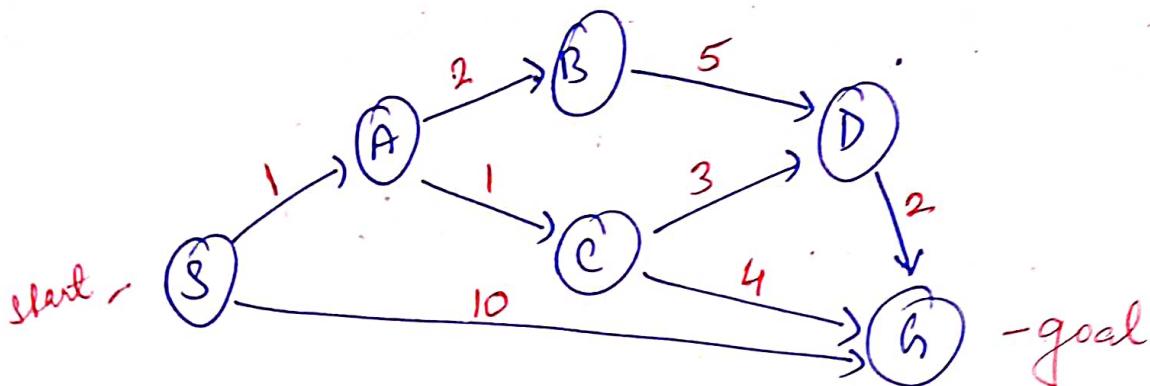


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State                           $H(n)$

$$S \longrightarrow 5$$

$$A \longrightarrow 3$$

$$B \longrightarrow 4$$

$$C \longrightarrow 2$$

$$D \longrightarrow 6$$

$$G \longrightarrow 0$$

$$S \rightarrow A = 1 + 3 = 4$$

$$\boxed{S \rightarrow G = 10 + 0 = 10} - \text{eq1}$$

$$S \rightarrow A \rightarrow B = (1+2) + 4 = 3 + 4 = 7$$

$$\boxed{S \rightarrow A \rightarrow C = (1+1) + 2 = 2 + 2 = 4}$$

$$\hookrightarrow S \rightarrow A \rightarrow C \rightarrow D = (1+1+3) + 6 = 5 + 6 = 11$$

$$\boxed{22 \quad S \rightarrow A \rightarrow C \rightarrow G = (1+1+4) + 0 = 6}$$

$$S \rightarrow A \rightarrow B \rightarrow D = (1+2+5)+6 = 14$$

Now here we exploring this path as:

$$S \rightarrow A \rightarrow C \rightarrow D = (1+1+3)+6 = 11$$

$$S \rightarrow A \rightarrow C \rightarrow D \xrightarrow{h} (1+1+3+2)+0 = 7.$$

R.B

$$S \rightarrow A \rightarrow B \rightarrow D \xrightarrow{h} (1+2+5+2)+0 = 10$$

We will get as

$$S \rightarrow A \rightarrow C \rightarrow n \xrightarrow{f} (1+3+4)+0 = 6.$$

Algorithm of A\* search

Step 1: Place the starting node in OPEN LIST.

Step 2: Check if the OPEN list is empty or not, if the list is empty then return failure and stop.

Step 3: Select the node from the OPEN list which has the smallest value of  $f(g+h)$ . If the node  $n$  is the goal node then return success and stop, otherwise.

Step 4: Expand Node  $n$  and generate all of its successors, and put  $n$  into the closed list. For each successor  $n'$ , check whether  $n'$  is already



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in the OPEN or CLOSED list. If not then compute valuation  $f_n$  for  $n$  and then place into Open list.

Step 5: Here if node  $n$  is already in OPEN and CLOSED, then it should be attached to the back pointer which reflects the lowest  $g(n')$  value.

Step 6: Return to Step 2.

**Advantages :**

- Best searching Algorithm
- Optimal and complete
- Solving Complex problems.

**Disadvantages :**

- Does not always produce the shortest path as it mostly based on Heuristics and Approximation.
- Some complexity issues.
- Requires memory



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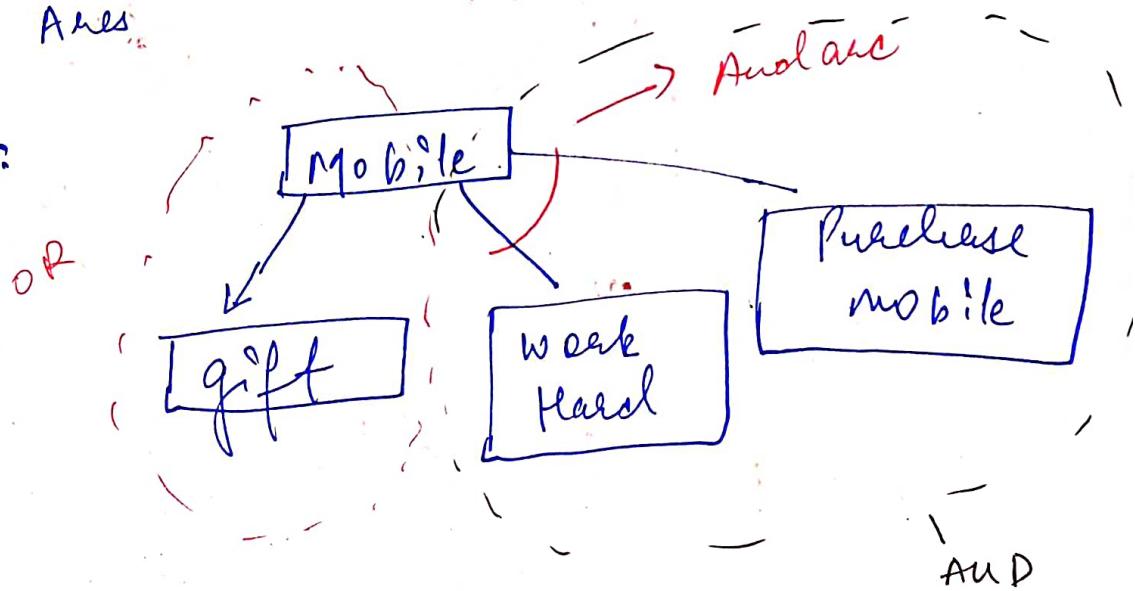
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#### AO \* Algorithm for AND-OR Graphs

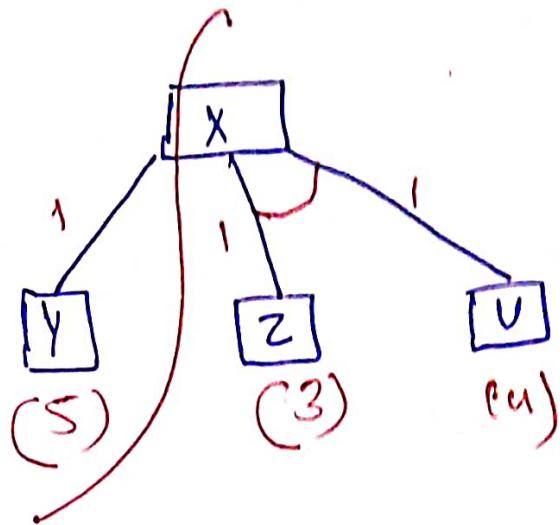
- And-Or graphs is useful for representing the problem solution that can be solved by decomposing them into smaller set of subproblems, all of which must be solved.
- AND ARCS : May point to 'n' no. of Successor Nodes.
- It is an algo like Best-fit search can be used that has the ability to handle AND Arcs.

Example :



\* FUTILITY : Should be chosen to correspond to a threshold value.

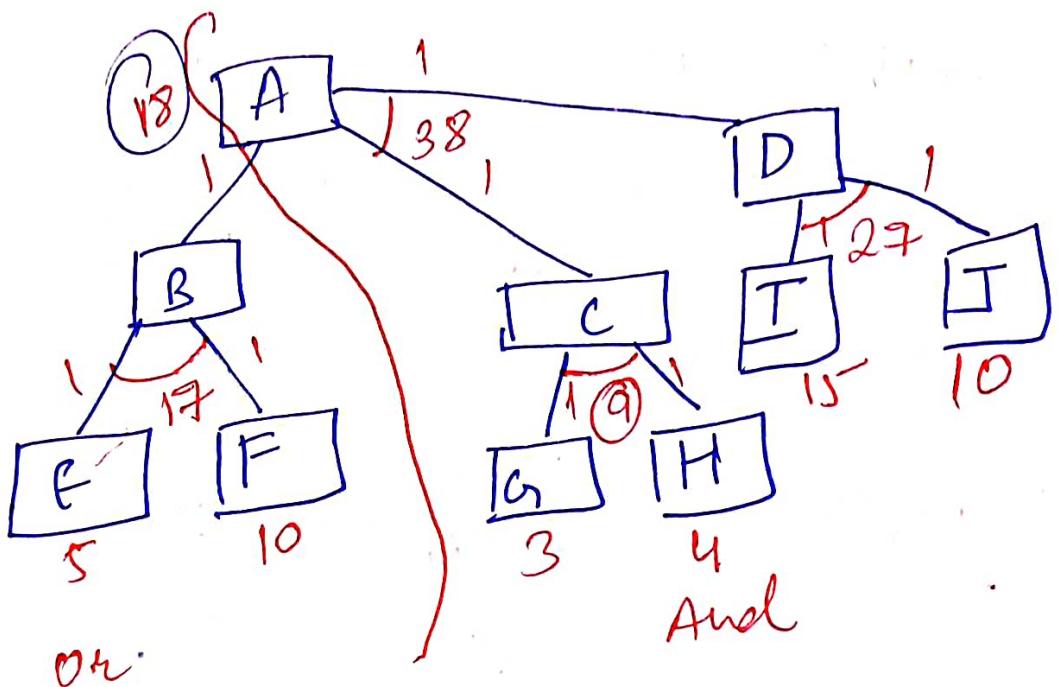
→ If  $est \cdot test > Futility \Rightarrow$  Stop Search



$$(\text{OR}) \quad x \rightarrow y + 5 = 6$$

$$(\text{AND}) \quad x \rightarrow z \rightarrow x \rightarrow u = 1 + 3 + 1 + 4 = 9$$

Example 2



$$A \rightarrow B \rightarrow E \text{ and } F. \quad 217 + 1^{18}$$



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- It is useful for representing the solution of problems that can be solved by decomposing them into a set of smaller problems, all of which must be solved.
- The decomposition, or reduction generates arcs that we call AND arcs. One arc may point to any no. of successor nodes all of which must be solved in order for the arc to point to a sol<sup>n</sup>.
- Just as in an OR graph, several arcs may emerge from a single node, indicating a variety of ways in which the original problem might be solved. That is why the structure is called Not simply an AND Graph but rather an AND - OR Graph.



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### Constraint Satisfaction Problem C (CSP)

- It is a search procedure that operates in a space of constraint sets.
- Constraint Satisfaction Problems in AI have goal of discovering some problem state that satisfies a given set of constraints

### Process

- i) Constraints are discovered and propagated throughout the system.  
is No feasible , search begins
- ii) If still , true  
↳ A guess is made about something and modeled as a new constraint.

$$\text{CSP} \rightarrow V [ \text{Variable Set } \subseteq V_1, V_2, V_3 \dots V_n ]$$

$$\rightarrow D [ \text{Domain } D_1, D_2 \dots D_n ]$$

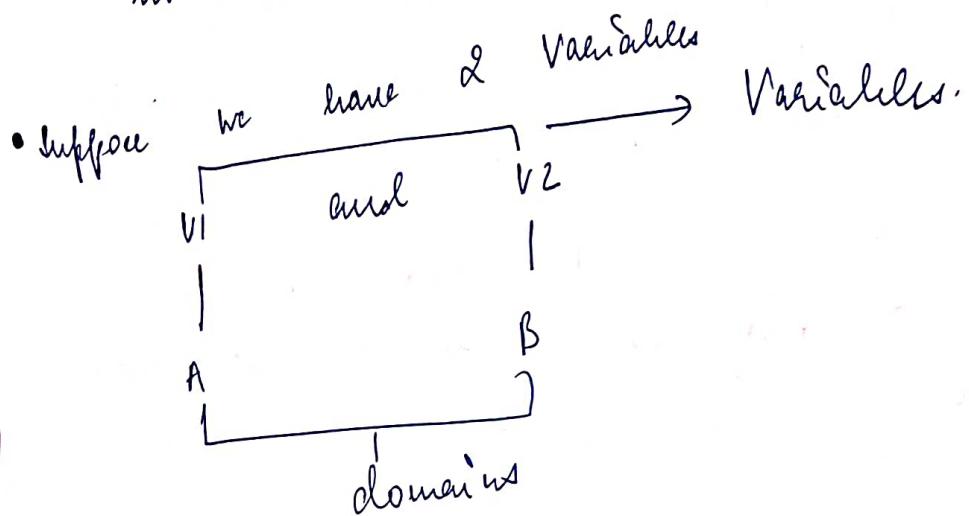
↳ one domain for each Variable.

$$C ( \text{constraints} )$$

→ Specify allowable combination of values

$C_i = (\text{Scope}, \text{Relation})$   
 ↓  
 list of variables  
 that part of pair  
 in constraint

Refines values that  
 a variable can  
 take.



constraint

Values of  $v_1$  and  $v_2$  can be same.

$C_i = \{ (v_1, v_2) \mid v_1 \neq v_2 \}$  (Relation)

To solve constraint satisfaction problem,  
we use intelligent backtracking method.

Only Backtrack when  
Conflict occurs



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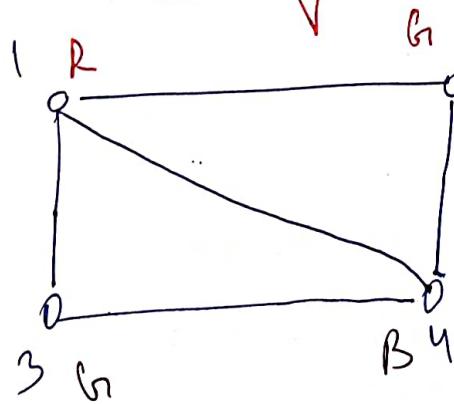
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Example of Constraint Satisfaction Problem

Node      Colouring



$G = \{1, 2, 3, 4\}$   
 $D = \{\text{Red, Green, Blue}\}$

$c = \{\text{adjacent Nodes}\}$   
 $\text{several not have same color}\}$

	1	2	3	4
Initial	R, G, B	R G, B	R G, B	R G, B
$1 = R$	R	G, B	G, B	B, B
$2 = G$	R	G	G, B	B

$3 = B$

- this particular state is giving me an error  
For this we need to Backtrack till where  
conflict has occurred

$3 = B$	R	G	G	B
---------	---	---	---	---

Example 2 of constraint satisfaction problem

### Crypt Arithmetic Problem

- It is an example or type of constraint satisfaction problem. (CSP)
- Constraints
  - Digits that can be assigned to a word / Alphabet  $\{0-9\}$   
Range
  - No 2 letters have same value
  - Sum of digits must be as shown in the problem.
  - There should be only 1 carry forward

$$\begin{array}{r} \text{TO} \\ + \text{GO} \\ \hline \text{OUT} \end{array}$$

start from  
leftmost digit =

(1)

(0-9)

$$9+8=17$$

(Here you will get maximum carry as 1) Also leftmost digit cannot be 0.

Letter	Digit
T	2
O	1
G	8
U	0



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$$\begin{array}{r} 2 \quad 1 \\ 8 \quad 1 \\ \hline 1 \quad 0 \quad 2 \end{array}$$

$$2 + 6 = 8 + 10$$

$$\boxed{2 + 9 = 11}$$
$$\boxed{2 + 8 = 10}$$

select.

this can't be taken  
as V value will  
become 1 and we  
have already assigned  
1 to 0.

Second example of constraint satisfaction  
problem.

$$\begin{array}{c} \text{S} \text{ } \text{F} \text{ } \text{N} \text{D} \\ \text{M} \text{ } \text{O} \text{ } \text{R} \text{E} \\ \hline \text{M} \text{ } \text{O} \text{ } \text{N} \text{ } \text{T} \text{Y} \end{array} + \begin{array}{c} \text{(C3(i))} \quad \text{(C2(i))} \quad \text{(C1(i))} \\ \boxed{S} \boxed{9} \quad \boxed{6} \boxed{5} \quad \boxed{N} \boxed{6} \quad \boxed{D} \boxed{7} \\ \hline \boxed{M} \boxed{1} \quad \boxed{O} \boxed{0} \quad \boxed{R} \boxed{8} \quad \boxed{E} \boxed{5} \\ \hline \boxed{M} \boxed{1} \quad \boxed{O} \boxed{0} \quad \boxed{N} \boxed{6} \quad \boxed{E} \boxed{5} \quad \boxed{Y} \boxed{2} \end{array}$$

Now we will check  $S + M$ , if  $m = 1$

$$\boxed{S + M \geq 10} \quad S = 9$$

Now  $E + O = N$  ] if  $c_2 = 0 \times \rightarrow c_2 = 1$   
 $E + O = N$

• Let  $E = 5$

$$E + 0 + C_2 = N$$

$$5 + 0 + 1 = 6$$

• Let  $N + R = E$

$$6 + R = 5$$

$$6 + 9 = \underline{15} \quad \text{carry.}$$

Let  $R = 9$

not possible as  
we already given  
the value of 9 to S.

• Now to do this we can do as

$$6 + 8 + 1 =$$

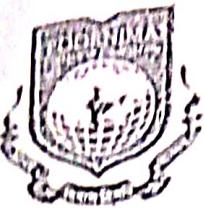
(C1)

• Now  $D + E = 4$

$$7 + 5 = 12$$

1 goes to carry  
 $y = 2$ .

$$\begin{array}{r} p \\ \diagdown \quad \diagup \\ 1 \quad 1 \\ 9 \times 8 \times 7 \\ \diagup \quad \diagdown \\ g \times x \times R \end{array}$$



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3) Example of Augt. Arithmetic Problem.

$$\begin{array}{r}
 & B & A & L & E \\
 + & B & A & L & L \\
 \hline
 & 6 & A & M & E S
 \end{array}$$

in (1)

$$\begin{array}{r}
 \boxed{B} \quad \boxed{A} \quad \boxed{L} \quad \boxed{E} \\
 \boxed{7} \quad \boxed{4} \quad \boxed{8} \quad \boxed{3} \\
 + \quad \boxed{B} \quad \boxed{A} \quad \boxed{L} \quad \boxed{5} \\
 \hline
 \boxed{1} \quad \boxed{4} \quad \boxed{9} \quad \boxed{3} \quad \boxed{8}
 \end{array}$$

$$\begin{aligned}
 & E + L = S \quad - (1) \quad \text{No carry} \\
 & E + L = S + 10 \quad \text{carry or } S - E = L \\
 & \qquad \qquad \qquad S - E = 5
 \end{aligned}$$

$$E = S - L + 10$$

$$(5, 0) \times L$$

$$(6, 1) \times$$

$$(7, 2) \times$$

$$(8, 3) \times$$

$$(9, 4) \times$$

$$\begin{aligned}
 & S + L = E \\
 & S + L = S - L + 10 \quad \text{or} \\
 & 2L = 10 \\
 & L = 5
 \end{aligned}$$

$$B + B = A + \text{array}$$

$B \leftarrow 5 \times L$

The diagram shows a stack frame with local variables. At the top,  $B \leftarrow 5 \times L$  is written. Below it, there is a stack frame structure with three slots labeled  $S$ ,  $E$ , and  $B$ . The slot  $S$  contains the value  $7$ , and the slot  $E$  contains the value  $2$ . The slot  $B$  contains the value  $6$ . Red annotations show the stack growing downwards: a red bracket under the stack frame is labeled  $9$ ,  $8$ , and  $7$  from bottom to top. A red arrow points from the  $B$  slot to the  $6$  value. To the right of the stack frame, the equation  $6 + 6 = 12$  is written.

let  $S, E = 7, 2$  (discards)  
     $\uparrow$   
 $B = 6$   
 $6 + 6 = 12$

• let  $S, E = 8, 3$   
 $B = 7$

Q. Try Cross + Locals = DAWDR DAWBR