



POORNIMA

COLLEGE OF ENGINEERING

DETAILED LECTURE NOTES

PAGE 17

Basic Elements of First-Order Logic

- 1) Constants : 1, 2, A, Amër, Nagpur, Cat
- 2) Variables : x, y, z, a, b, \dots
- 3) Predicates : Brother, Father, etc.
- 4) Functions : Sqrt, leftof, Right of \dots
- 5) Connectives : $\wedge, \vee, \neg, \Rightarrow, \Leftarrow$
- 6) Equality : $=$
- 7) Quantifier : \forall, \exists

Free and Bound Variables

Types of Variables are as follows:

- 1) Free Variable : A Variable is said to be free Variable in a formula if it occurs outside the scope of a Quantifier.

Example

$\forall x \exists (y) [P(x, y, z)]$, where z is a free Variable.
(\forall quantifier ke range ke Bahar hai)

- 2) Bound Variable : A Variable is said to be Bound Variable if it occurs within the scope of the Quantifier.

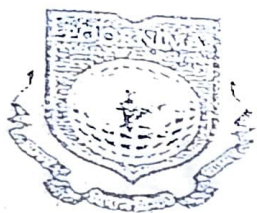
- here x , and y will be Bound Variable.
- $(x: \forall x \forall y [A(x) B(y)])$, here x and y are the bound variables

Points to Remember.

- The main connective for universal quantifier \forall is implication \rightarrow .
- The main connective for existential quantifier \exists is and (\wedge).
- In universal quantifier, $\forall x \forall y$ is similar to $\forall y \forall x$.
- In existential quantifier, $\exists x \exists y$ is similar to $\exists y \exists x$.
- $\exists x \forall y$ is not similar to $\forall y \exists x$.

Predicate logic examples

- 1) Marcus was a man
Man (Marcus)
- 2) Marcus was a Pompeian
Pompeian (Marcus)
- 3) All Pompeians were Romans.
 $\forall x: \text{Pompeian}(x) \rightarrow \text{Roman}(x)$
- 4) Caesar was a ruler
Ruler (Caesar)



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5) All Romans were either loyal to Caesar or hated him. PAGE NO

$\forall x : \text{Roman}(x) \rightarrow \text{loyalto}(x, \text{Caesar}) \vee \text{Hate}(x, \text{Caesar})$

6) Everyone is loyal to someone.

$\forall x : \exists y : \text{loyalto}(x, y)$

In this fact the predicate is $\text{loyalto}(x, y)$ where $x = \text{Everyone}$ and $y = \text{someone}$. Since there is someone so we will use \exists , and it will be represented above.

7) People only try to assassinate rulers they are not loyal to.

In this fact the predicates are "Person", "Try-assassinate (x, y) ", "ruler" and " $\text{loyalto}(x, y)$ ".

And since people only try to assassinate rulers they are not loyal to so it will be represented as follows:

$\forall x : \forall y : \text{Person}(x) \wedge \text{Ruler}(y) \wedge \text{Tryassassinate}(x, y) \rightarrow \neg \text{loyalto}(x, y)$

8) Marcus tried to assassinate Caesar.

Try assassinate (Marcus, Caesar)

9) All men are people

$\forall x : \text{Man}(x) \rightarrow \text{Person}(x)$

Questions to Practice :

1) Marcus was born in 40 AD.

2) All men are Mortal

3) All Pompeians died when the Volcano ~~er~~ erupted in 79 AD.

4) No mortal lives longer than 150 years.

5) It is now 2022.

6) Alive means not dead.

7) If someone dies, then he is dead at all later times