

# Linear Regression →

1

Linear Regression is a Method to Predict dependent variable ( $y$ ) based on values of independent variable ( $x$ ). It can be used for the cases where we want to predict some continuous quantity.  
for Example

Suppose you have to find out annual Sale of your Company.

So factors that affect the Annual Sale known as dependent variables.

like \*

- \* Employee
- \* Product Sale
- \* Loss
- \* Profit

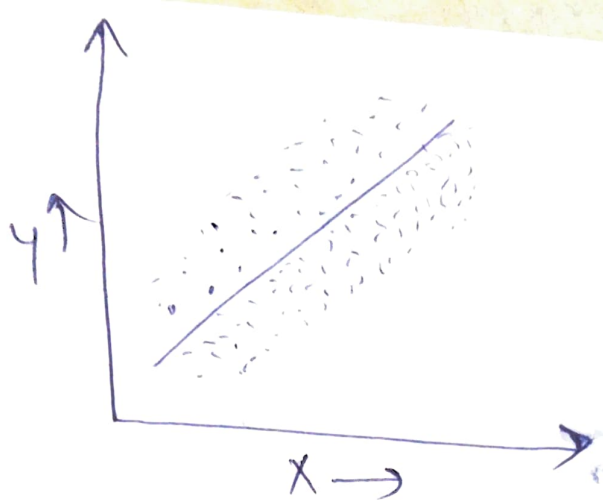
Dependent Variables

\* Annual Sale — Independent variable

Regression analysis is used for establishing the relationship between the dependent & independent variables.

OR

When dependent variable or output is in numeric or Number then we will use regression.



linear Means — When  $x$  increase  $y$  is also increase this is linear nature.

\* Simple linear Regression →

if we have only one dependent Variable and one independent variable then we can say that it is Simple linear regression.

$$y = \alpha_0 + \alpha_1 x_1$$

↓  
dependend  
variable
↓  
independent  
variable

\* Multiple linear Regression →

if we have one dependent variable and More than one independent variable then we can say that it is Multiple linear Regression.

$$y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n$$

$\alpha_1$  = Regression Coefficient

$x_1$  = independent variable

$y$  = dependent variable

$$y = 0.9 + 1.2x_1 + 2x_2 + 4x_3 + 1x_4$$

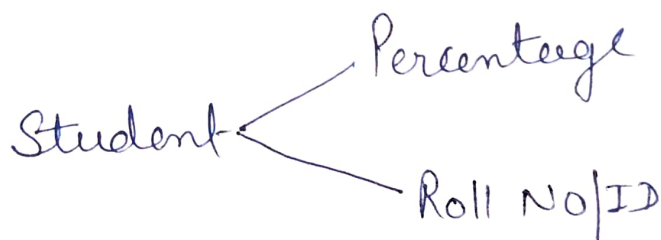
$\alpha_1$                    $\alpha_2$                    $\alpha_3$                    $\alpha_4$                    $\alpha_5$

The value of coefficient shows that the features related to the coefficient. How much important. it is important for output.

It shows the importance of the features.

In the above equation the most important feature is  $\alpha_4$  because its value is equal to 4 that is maximum.

for example



if we have Percentage then we can decide that Student is fail or Pass. So Percentage feature is more important to determine the result. So it is more effective feature.

from where  $\alpha$  value comes  $\rightarrow$

4

Whole data set to training phase then learning process starts then Model automatically decide which feature is putting more impact on the output. and How much it is important for the output... according to this it decides  $\alpha$  values.

Ex if we have only student's ID and roll no then we can not predict the result of the student. So these features are not important for output.



# Given Dataset.

X	Y	X <sup>2</sup>	XY
1	3	1	3
2	4	4	8
3	5	9	15
4	7	16	28
10	19	30	54

features  
attributes  
parameter  
Variable

5

$$a = \frac{(\sum Y)(\sum X^2) - (\sum X)(\sum XY)}{n(\sum X^2) - (\sum X)^2}$$

→ No of observation

→ No of Cases

→ No of values

→ No of Points

\*You can say anything

$$b = \frac{n(\sum XY) - (\sum X)(\sum Y)}{n(\sum X^2) - (\sum X)^2}$$

$$Y = bX + a$$

$$a = \frac{(19)(30) - (10)(54)}{(4)(30) - (100)}$$

$$= \frac{570 - 540}{120 - 100}$$

$$= 30/20$$

$$= 3/2 \Rightarrow 1.5$$

$$b = \frac{(4)(54) - (10)(19)}{(4)(30) - (100)}$$

6

$$\Rightarrow \frac{216 - 190}{120 - 100}$$

$$= \frac{26}{20}$$

$$= \frac{13}{10} \Rightarrow 1.3$$

$$Y = 13X + 15$$

Alternative formula of (a)

$$a \Rightarrow \frac{\sum Y - b(\sum x)}{n}$$

$$\frac{19 - (1.3)(10)}{4} \Rightarrow \frac{6}{4} = \frac{3}{2} = \underline{1.5}$$

Multiple Linear Regression →

Q Predict the value of  $Y$  given  $x_1$  and  $x_2$

Subject       $Y$        $x_1$        $x_2$

1      -3.7      3      8

2      3.5      4      5

3      2.5      5      7

4      11.5      6      3

5      5.7      2      1

6      ?      3      2

there are five instances

Testing Example →

here five instances given So using these 7 fine instances we need to Model and then use that Particular Model ~~to need to~~ find the ~~the~~ value of  $y$ .

$$a = \bar{y} - b_1 \bar{x}_1 - b_2 \bar{x}_2$$

$$b_1 = \frac{(\sum x_2^2)(\sum x_1 y) - (\sum x_1 x_2)(\sum x_2 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 x_2)^2}$$

$$b_2 = \frac{(\sum x_1^2)(\sum x_2 y) - (\sum x_1 x_2)(\sum x_1 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 x_2)^2}$$

Subject	$y$	$x_1$	$x_2$	$x_1 x_1$	$x_2 x_2$	$x_1 x_2$	$x_1 y$	$x_2 y$
1	-3.7	3	8	9	64	24	-11.1	-29.6
2	3.5	4	5	16	25	20	14	17.5
3	2.5	5	7	25	49	35	12.5	17.5
4	11.5	6	3	36	9	18	69	34.5
5	5.7	2	1	4	1	2	11.4	5.7
$\Sigma$	19.5	20	24	90	148	99	95.8	45.6

$x_2$  is Not a simple  $x_2^2$  it is the variance <sup>8</sup>  
with respect to  $x_2$ .  $\sum x_i^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{N}$

$$i=1 \quad \sum x_1^2 = \sum x_1 x_1 - \frac{(\sum x_1)(\sum x_1)}{N} \Rightarrow 10$$

$$i=2 \quad \sum x_2^2 = \sum x_2 x_2 - \frac{(\sum x_2)(\sum x_2)}{N} = 32.8$$

$$\sum x_1 y = \sum x_1 y - \frac{(\sum x_1)(\sum y)}{N} = 17.8$$

$$\sum x_2 y = \sum x_2 y - \frac{(\sum x_2)(\sum y)}{N} = -48$$

$$\sum x_1 x_2 = \sum x_1 x_2 - \frac{(\sum x_1)(\sum x_2)}{N} = 3$$

$$b_1 = \frac{32.8 \times 17.8 - 3 \times (-48)}{10 \times 32.8 - 3 \times 3} = 2.28$$

$$b_2 = \frac{10 \times (-48) - 3 \times 17.8}{10 \times 32.8 - 3 \times 3} = -1.67$$

$$\hat{y} = \bar{y} - b_1 \bar{x}_1 - b_2 \bar{x}_2$$

$$= \frac{19.5}{5} - \frac{2.28 \times 20}{5} - \frac{-1.67 \times 24}{5}$$

$$\Rightarrow 2.796$$