



# POORNIMA FOUNDATION

## LECTURE NOTES

Campus: PCE Course: BTECH in CSE Class/Section: III Yr. Section- A Date: 9/3/21  
Name of Faculty: Praveen Kumar Yadav Name of Subject: Machine Learning Code: 6CS4-02  
Date (Prep.): 9/3/21 Date (Del.): 3-4-21 Unit No.: II Lect. No.: 12

**OBJECTIVE:** To be written before taking the lecture (Pl. write in bullet points the main topics/concepts etc., which will be taught in this lecture)

Probabilistic clustering.

**IMPORTANT & RELEVANT QUESTIONS:**

1. what is gaussian distribution?

**FEED BACK QUESTIONS (AFTER 20 MINUTES):**

How a Gaussian distribution help in Probabilistic clustering.

**OUTCOME OF THE DELIVERED LECTURE:** To be written after taking the lecture (Pl. write in bullet points about students' feedback on this lecture, level of understanding of this lecture by students etc.)

good

**REFERENCES:** Text/Ref. Book with Page No. and relevant Internet Websites:

scikit with ML, youtube video



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## COLLEGE OF ENGINEERING

### DETAILED LECTURE NOTES

PAGE NO.

Probabilistic clustering:- In Bayesian Hierarchical clustering instead of physical distance metrics such as Euclidean and Manhattan distance, we use probabilistic distance between the data points are used.

→ The probabilistic distance is given by Bayesian posterior probability and hence the name is, BHC.

BHC Algorithm:-

Let  $D = \{x^{(1)}, x^{(2)}, \dots, x^{(n)}\}$  represents the

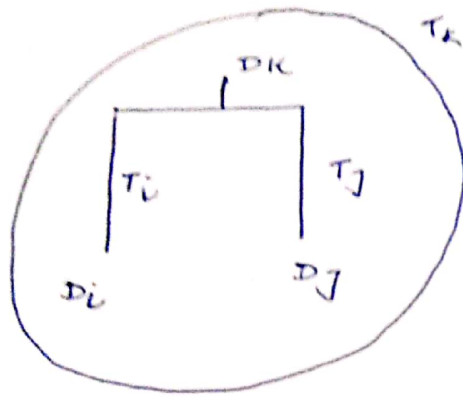
dataset of  $n$  elements.

Initially, all each data elements  $x^{(i)}$ , is a single cluster, yielding a set of cluster at leaf-level, i.e.  $D_i \subset D$

These leaf level clusters can also be denoted as subsequence  $\{T_i : 1 \dots n\}$



(we have 'n' subtraces initially denoting 'n' clusters and 'n' data elements.)



$$D_k = D_i \cup D_j$$

Here subtraces are merged until a single trace is formed uniting all subtraces.

The Merging Criterion :- Here we have two Hypothesis

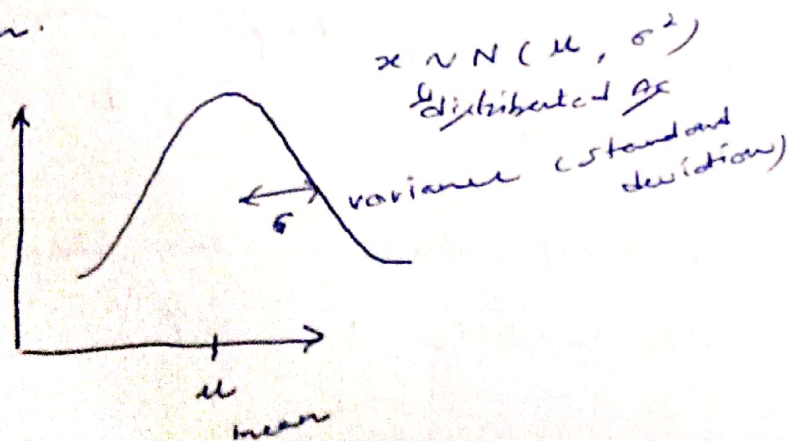
$H_1 + H_2$  to merge clusters (subtraces)

Hypothesis  $H_1$  :- All data elements belonging to the merged clusters ( $D_k$ ) were independently and identically from the same probabilistic model (Gaussian Model) which has mean and variance.

that is  $P(x/\theta) = \theta$  are unknown parameter

where  $\theta = (\mu, \sigma^2)$   
 $\downarrow$  mean  $\downarrow$  variance

Gaussian distribution - (Normal Dist.) Say  $x$  is random variable. If ' $x$ ' is a distributed gaussian.

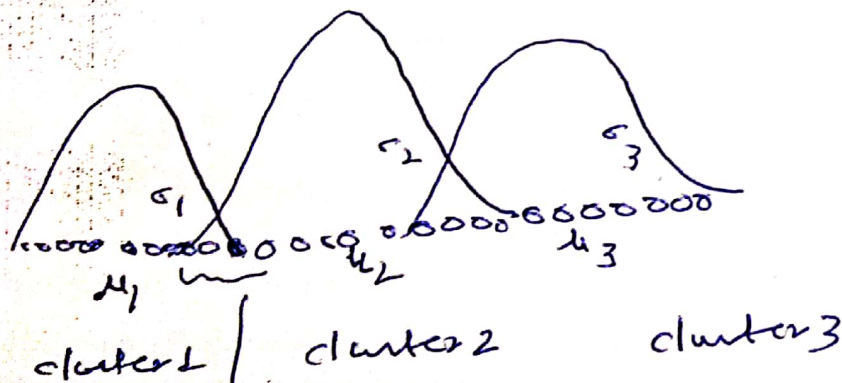


$$P(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{(1-\mu)^2}{2\sigma^2}\right)$$

for a given dataset:  $\{x^1, x^2, \dots, x^n\}$

$$\mu = \frac{1}{n} \sum_{i=1}^n x^i \quad \sigma^2 = \frac{1}{n} \sum_{i=1}^n (x^i - \mu)^2 \quad (2)$$

"The intuition is that all the "x" value in single Gaussian distribution is supposed to forming be in a single cluster.  
 ↳ Like we have single Gaussian distribution we have also Gaussian Mixture models.



soft clustering  
of some points

$$P(D_K | H_1^K) = \int P(D_K | \theta) P(\theta | \beta)$$

$$\theta \leftarrow \mu, \sigma^2$$

$$\beta \leftarrow \text{Hyper parameters (features)}$$

Hypothesis  $H_2$  - The data subset in  $D_K$  has two or more clusters in  $H$ .

$$P(D_K | H_2^K) = P(D_i | T_i) P(D_J | T_J)$$

product of two  
subtrees (Joint prob.  
of the  
subtrees)

so we put all the two prob- in Bayes theorem.

$$P(D_K | T_K) = \underbrace{\pi_K}_{H_1} P(D_K | H_1^K) + (1 - \pi_K) \underbrace{P(D_i | T_i)}_{H_2} P(D_J | T_J)$$

where  $\pi_K = P(H_1^K)$  prior probability that a  
points in  $D_K$  belongs  
to single cluster

$$\pi_2 = P(H_2^K)$$