Decision Tree using CART algorithm →

$$\text{Gini Index (Attribute = value)} = 1 - \sum_{i=1}^{N} (P_i)^2$$

$$\text{Gini (Attribute)} = \sum_{V=\text{value}} P_V * GI(V)$$

Given data set there 14 instances of golf playing decision based on outlook.

Outlook	Temprature	Humidity	wind	Decision
SUNNY	HOT	HIGH	WEAK	NO
SUNNY	HOT	HIGH	STRONG	NO
OVERCAST	HOT	HIGH	WEAK	YES
RAIN	MILD	HIGH	WEAK	YES
RAIN	COLD	NORMAL	STRONG	YES
RAIN	COLD	NORMAL	STRONG	NO
OVERCAST	COLD	NORMAL	STRONG	YES
SUNNY	MILD	HIGH	WEAK	NO
SUNNY	COLD	NORMAL	WEAK	YES
RAIN	MILD	NORMAL	WEAK	YES
SUNNY	MILD	NORMAL	STRONG	YES
OVERCAST	MILD	HIGH	STRONG	YES
OVERCAST	HOT	NORMAL	WEAK	YES
RAIN	MILD	HIGH	STRONG	NO

Now Create a table for outlook

Outlook	Yes	NO	Number of instant
Sunny	2	3	5
Overcast	4	0	4
Rain	3	2	5

$$Gini(outlook=sunny) = 1 - \sum_{i=1}^{N} (P_i)^2$$

$$\Rightarrow 1 - (2/5)^2 - (3/5)^2$$

$$\Rightarrow 0 - 0.16 - 0.36$$

$$\Rightarrow 0.48$$

$$Gini(outlook=overcast) = 1 - (4/4)^2 - (0/4)^2$$

$$\Rightarrow 1 - 1$$

$$\Rightarrow 0$$

$$Gini(outlook=Rain) = 1 - (3/5)^2 - (2/5)^2$$

$$\Rightarrow 1 - 0.36 - 0.16$$

$$\Rightarrow 0.48$$

Now Calculate weighted sum of Gini Index -

$$Gini(outlook) = (5/14) \times 0.48 + (4/14) * 0 + (5/14 * 0.48)$$

$$\Rightarrow 0.171 + 0 + 0.171$$

$$\Rightarrow 0.342$$

* Now for Temprature →

Temprature	Yes	NO	No of Instance
HOT	2	2	4
COLD	3	1	4
MILD	4	2	6

$$\text{Gini}(\text{Temprature}-\text{HOT}) = 1-\left(\frac{2}{4}\right)^2-\left(\frac{2}{4}\right)^2$$
$$\Rightarrow 0.5$$

$$\text{Gini}(\text{Temprature}-\text{COLD}) = 1-\left(\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2$$
$$\Rightarrow 0.375$$

$$\text{Gini}(\text{Temprature}-\text{MILD}) = 1-\left(\frac{4}{6}\right)^2-\left(\frac{2}{6}\right)^2$$
$$= 0.445$$

$$\text{Gini}(\text{Temprature}) = \left(\frac{4}{14}\right)*0.5+\left(\frac{4}{14}\right)*0.375+\left(\frac{6}{14}\right)*0.445$$

$$\Rightarrow 0.439$$

Now for WIND

WIND	YES	NO	NO of Instances
Weak	6	2	8
Strong	3	3	6

Gini (WIND-Weak) ⇒ 0.375

Gini (WIND-STrong) ⇒

Gini (WIND) ⇒ 0.428

Now for Humidity →

Humidity	Yes	NO	NO of Instances
High	3	4	7
Normal	6	1	7

GINI (Humidity-High) ⇒ $1-\left(\frac{3}{7}\right)^2-\left(\frac{4}{7}\right)^2 \Rightarrow$

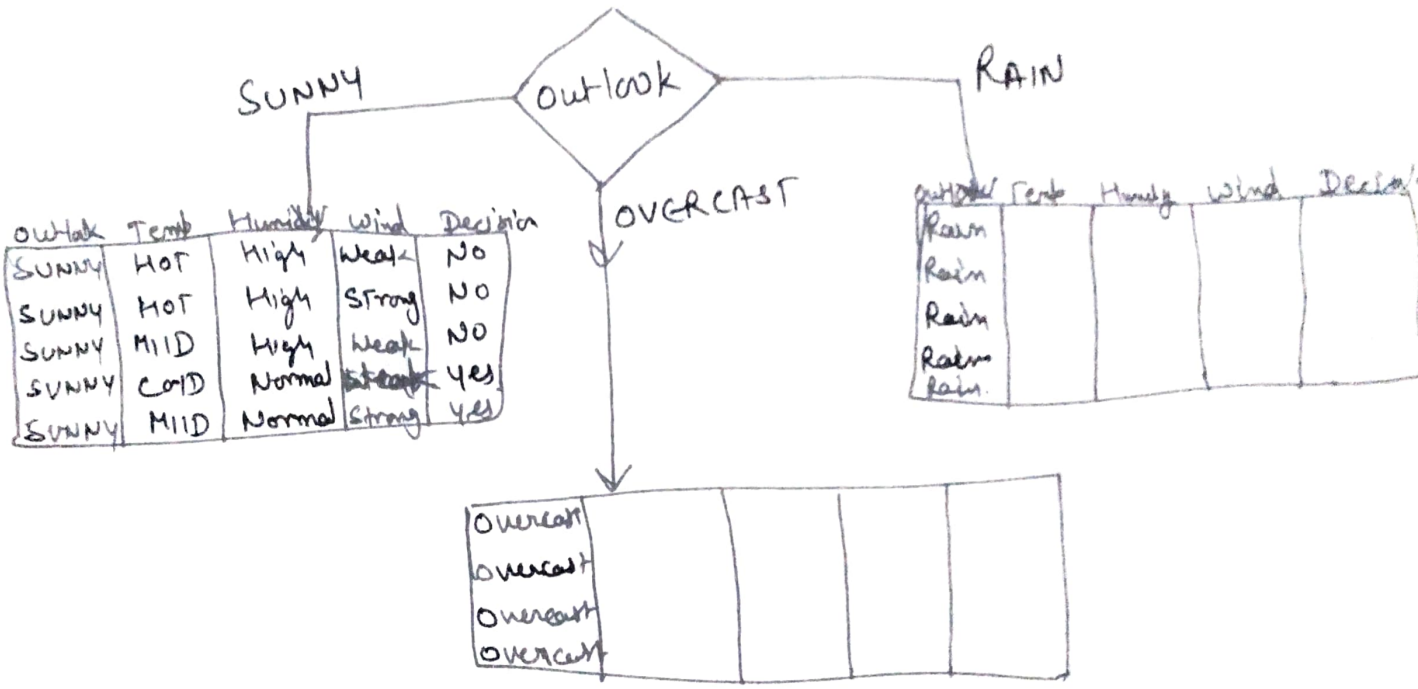
GINI (Humidity-Normal) $= 1-\left(\frac{6}{7}\right)^2-\left(\frac{1}{7}\right)^2 \Rightarrow$

GINI (Humidity) ⇒ 0.367

Time to decide Root Node

feature	GINI Index
Outlook	0.342
Temperature	0.439
Humidity	0.367
WIND	0.428

Minimum



Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	HOT	High	Weak	NO
2	Sunny	HOT	High	Strong	NO
8	Sunny	MILD	High	Weak	NO
9	Sunny	COLD	Normal	Weak	YES
11	Sunny	MILD	Normal	Strong	YES

Gini of Temprature for Sunny outlook (5)

Temprature	Yes	No	No of Instances
Hot	0	2	2
COOL	1	0	1
MILD	1	1	2

Gini (Outlook = Sunny and Temp. = HOT)

$$\Rightarrow 1-\left(\frac{0}{2}\right)^2-\left(\frac{2}{2}\right)^2 = 0$$

Gini (Outlook = Sunny and Temp = COOL)

$$\Rightarrow 1-\left(\frac{1}{1}\right)^2-\left(\frac{0}{1}\right)^2 = 0$$

Gini (Outlook = Sunny and Temp. = MILD)

$$\Rightarrow 1-\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2$$

$$= 1-0.25-0.25$$

$$= 0.5$$

Gini (Outlook = Sunny and Temp) $\Rightarrow$

$$\left(\frac{2}{5}\right)*0+\left(\frac{1}{5}\right)*0+\left(\frac{2}{5}\right)*0.5$$

$$\Rightarrow 0.2$$

Gini of Humidity for Sunny outlook

Humidity	Yes	No	No of Instances
High	0	3	3
Normal	2	0	2

Gini (outlook = Sunny and Humidity = High)

$$\Rightarrow 1-\left(\frac{0}{3}\right)^2-\left(\frac{3}{3}\right)^2=0$$

Gini (outlook = Sunny and Humidity = Normal)

$$\Rightarrow 1-\left(\frac{2}{2}\right)^2-\left(\frac{0}{2}\right)^2=0$$

Gini (outlook = Sunny and Humidity)

$$\Rightarrow \left(\frac{3}{5}\right)*0+\left(\frac{2}{5}\right)*0$$

$$=0$$

Gini of wind for Sunny outlook

wind	yes	NO	No of Instances
weak	1	2	3
strong	1	1	2

Gini (outlook = Sunny & wind = weak)

$$\Rightarrow 1-\left(\frac{1}{3}\right)^2-\left(\frac{2}{3}\right)^2=0.266$$

Gini (outlook = Sunny & wind = strong)

$$\Rightarrow 1-\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2=0.2$$

Gini (outlook = Sunny & wind)

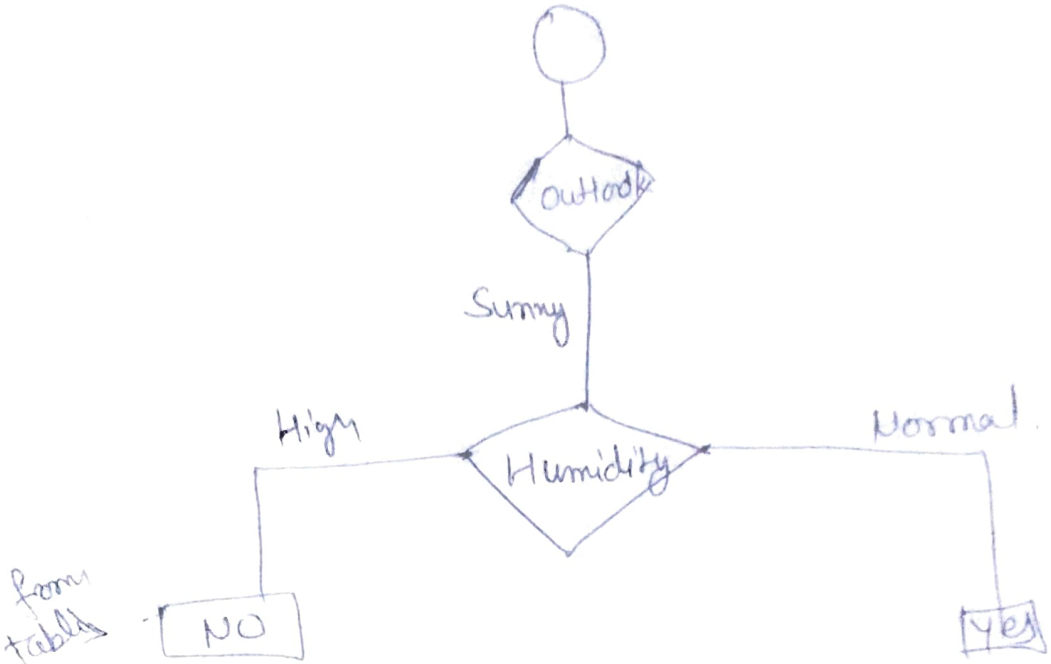
$$\Rightarrow \left(\frac{3}{5}\right)*0.266+\left(\frac{2}{5}\right)*0.2$$

$$\Rightarrow 0.466$$

Decision for Sunny outlook

feature	Gini Index
Temprature	0.2
Humidity	0
Wind	0.466

Humidity — Minimum.



Rain

Day	Outlook	Temp	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cold	Normal	Weak	Yes
6	Rain	COLD	Normal	Strong	No
10	Rain	MILD	Normal	weak	Yes
14	Rain	MILD	High	Strong	Yes