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Energy-Efficiency Study for Two-tier Heterogeneous Networks (HetNet) Under Coverage Performance Constraints

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Abstract Recently, heterogeneous networks (HetNet) have been widely studied as an effective approach to provide high network capacity and coverage, which jointly utilizes the technologies of cognitive radio and cooperative communications. However, the dense and random deployment of small-cells (e.g., micro, pico and femtocells) raises important questions about the energy consumption for HetNets. In this paper, we study the optimal energy efficiency of a two-tier heterogeneous network consists of a macro-cell and many small-cells under coverage performance constraints for different spectrum deployments (including orthogonal and co-channel spectrum deployments). Firstly, we derive the closed-form expressions of coverage performance for each tier based on stochastic geometry. Then the relationship between energy efficiency and the density of small-cells for the two-tier network is evaluated, the optimal density of small-cells that maximize energy efficiency under coverage performance constraints for the two-tier network is obtained. The theoretical analysis is validated by simulations. The results show that the energy efficiency of the two-tier networks with orthogonal spectrum deployment is better than that with co-channel spectrum deployment. The results also show that the optimal density of small-cells for maximal energy efficiency is only dependent on the coverage performance of small-cells in orthogonal spectrum deployment scenario. However, in co-channel spectrum deployment scenario, the optimal density of small-cells for maximal energy efficiency is jointly decided by the coverage performance of both macro-cell and small-cell. This work provides an essential understanding for successful deployment of green heterogeneous networks.

Keywords Energy-efficiency · Two-tier networks · Heterogeneous networks (HetNet) · Coverage performance

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1 Introduction

During recent years with the development of technologies in 3G and LTE/4G telecommunication systems and smart phones such as iPhone/Android, wireless data and multimedia traffic has been increasing exponentially, particularly at hotspot/indoor areas. Also, as services migrate from voice centric to data centric, more users will operate in indoor environment, which requires increased link budget and coverage extension to provide uniform user experience. Traditional networks optimized for homogeneous traffic face unprecedented challenges to meet the demand cost effectively. Most recently, Third Generation Partnership Project (3GPP) Long Term Evolution (LTE)-Advanced has started a new study item to investigate heterogeneous network (HetNet) [1, 5] deployments as an efficient way to improve system capacity as well as effectively enhance network coverage. In [1], the authors presents a comprehensive overview for the hierarchical cooperative relay based heterogeneous networks (HetNet). Unlike traditional heterogeneous networks that deal with the interworking of wireless local area networks and cellular networks, which the research community has already studied for more than a decade, in this new paradigm regarding the cellular network domain, HetNet is a network containing nodes with different characteristics such as transmission power and radio frequency (RF) coverage area. The objective of HetNet targets the improvement of overall capacity as well as a cost-effective coverage extension and green radio solution by deploying additional network nodes within the local area range, such as low-power micro/pico network nodes, home-evolved Node-Bs (HeNBs)/closed subscriber group (CSG) cells, and femto and relay nodes.

In the architecture of HetNet, two types of nodes co-exist in the new topology, one is the Macro-cell Base Station (MBS), and the other is customer-deployed Small-cell Base Station (SBS) distributed among the coverage of the Macro-cell. SBS has the characteristics of flexible, inexpensive and low-power and usually serves the indoor users. The introduction of SBS facilitates the indoor penetration and spatial frequency reuse yielding the enhanced wireless capacity. It is expected that an extensive deployment of Small-cells will take place currently and in the near future [2, 5], for example, nanocell as an extended version of Small Cell have received considerable attention recently. The future cellular networks will be divided into two clearly separated layers or denoted two-tier, the large-coverage: Macro-cells layer and small-coverage: Small-cell layer.

However, the topology of two-tier networks also come up with new problems and challenges. For example, the interference coordination between Macro-cells and Small-cells which is known as cross-tier interference will seriously decrease the capacity of Macro-cell Mobile Stations (Macro-MS). The problem emerges when MBS and SBS use the same radio resource simultaneously. Cognitive radio and cooperative communications have been widely studied to overcome the problems in the utilization of HetNets. Extensive studies have focused on this problem [6, 7]. In a general way, there are two kinds of spectrum deployments for heterogeneous networks, orthogonal channel deployment and co-channel deployment [6]. Co-channel deployment reuses the spectrum resources to improve the system capacity while causing cross-tier interferences. On the other hand, orthogonal channel deployment can be utilized to avoid the cross-tier interferences.

The other more important challenge is the greater energy consumption in HetNet because of the dense and randomly deployment of small base stations. Traditional deployment of cellular system does not consider the power consumed by wireless devices and networks which creates a gap between the energy a wireless network needs to operate and the battery capacity of the wireless devices. Hence, the requirement of Energy-Efficiency [8–10] appears as an extremely important property of new protocols for designing of wireless networks with battery-powered mobile nodes. Moreover, Energy-Efficiency is the tool to realize the vision of green wireless networks, which are deemed important in recent years due to the increasing share of wireless systems of the total energy expended in communications and networking systems. It is reported that wireless and mobile communications account for more than 9 % of global carbon dioxide emissions originated from *information and communication technologies* (ICT) [11]. For wireless cellular communications system, according some statistics, more than 70 % energy of the total wireless networks (including terminal, radio access network and core network, etc.) are consumed in the base station subsystem (BSS). Thus in order to design an energy-efficient mobile networks, one important effort should be focused on BSS. A concept of Green Radio (GR) [12] is proposed to study the energy efficiency of wireless communication instead of traditional pursuit of higher capacity and coverage.

Nowadays researchers from various fields have realized the importance of energy-efficient design for HetNet, for example, [13, 14] studied the energy efficient high capacity HetNet through utilization of femto cells and energy saving strategy for HetNet, respectively. In [15, 16] the authors studied the energy efficiency of an open-access two-tier networks comprised of macro-micro and macro-pico and the performance is evaluated by simulations. Meanwhile energy efficiency on macro-femto two tier networks is studied in [17, 18]. In summary, most existing studies on the energy efficiency of two-tier HetNet networks only consider the orthogonal channel deployment [19]. In this paper, we consider both co-channel and orthogonal resource allocation scenarios, analyze the relationship between energy efficiency and the density of small-cells for the two-tier networks. As a result, the major contributions of this paper can be summarized as follows,

1. A method using Stochastic Geometry is proposed to analyze the energy-efficiency of HetNet and the optimal density of small-cells for the optimal network energy efficiency is obtained, which satisfy the coverage performance constraints for both MBS and SBS.
2. Both theoretical and simulation results show that the energy efficiency of the two-tier networks with orthogonal spectrum deployment is better than that with co-channel spectrum deployment.
3. The optimal density of small-cells for maximal (optimal) energy efficiency is only dependent on the coverage performance of small-cells in orthogonal spectrum deployment scenario.

The remaining of this paper is outlined as follows. Section 2 presents the system model and proposes the energy efficiency function. And then in Section 3 we analyze the the coverage and capacity of the networks under orthogonal channel and co-channel scenarios. Meanwhile, the exact cumulative distributions function (CDF) expression of the downlink SINR (i.e. signal-to-interference and noise ratio) distribution for both orthogonal channel and co-channel scenarios. Section 4 evaluates the relationship between energy efficiency and the density of small-cells under different spectrum deployment. In Section 5, numerical simulation and analysis is given, followed by the conclusions in Section 6.

2 System model

First the framework of the two-tier HetNets and spectrum deployments discussed in this paper are given in this section. Then the energy efficiency expression is derived.

2.1 Architecture of two-tier heterogeneous networks (HetNet)

As shown in Fig. 1, a typical two-tier heterogeneous networks consist of one macro-basestation (MBS) and several small-basestations (SBS). Both macro- and small-cells adopt Orthogonal Frequency Division Multiple Access (OFDMA). The *closed-access* policy is assumed, which means that any unregistered mobile stations cannot access to a small-cell base-station even if they are more closed to the small-cells. Perfect synchronization is assumed in both time and frequency domain.

The coverage of macro-cell is assumed as a circle coverage with radius R_m, denoted as H, which consists of multiple macro mobile-stations (denoted as macro-MS) and one MBS located in the center of H. The transmitting power of MBS is denoted as P_{mbt}. The macro-cell is overlaid with SBSs with radius R_s spreading randomly in H according to a homogeneous spatial Poisson Point Process (SPPP) with intensity λ_s. The location set of SBSs is denoted of

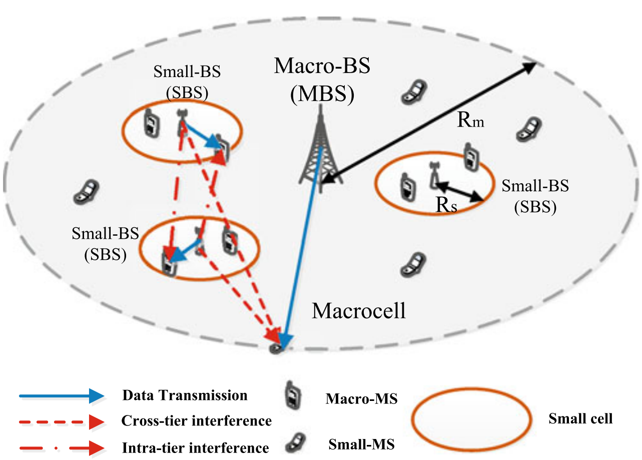

Fig. 1 System model for two-tier heterogeneous networks (HetNet)

$\Phi_s = \{Y_i\}$, where Y_i corresponds to the location of the i-th SBS. The transmitting power of SBS is P_{sbt}. Assume that the radius of macro-cell is much bigger than that of small-cell, thus the distance of two small-cell mobile stations (denoted as small-MS) from different small-cells can be approximated as the distance of corresponding SBSs.

2.2 Channel model

For the channel model, in this paper path loss, penetration loss, fastfading and noise N_0 are considered. The path loss exponent of transmission is denoted as α. The fastfading model is assumed to be Rayleigh fading with unit average power G.

We use triplets $(R_m, \Gamma_m, \epsilon_m)$[1] to describe the coverage performance of macro-cell system. In the triplet $(R_m, \Gamma_m, \epsilon_m)$, R_m is the radius of macro-cell, ϵ_m is the outage probability of the macro-MS located at the cell boundary when its corresponding received Signal-to-Interference and Noise Ratio (SINR) is Γ_m. Similarly, the triplet $(R_s, \Gamma_s, \epsilon_s)$ denotes the coverage performance of small-cell system. In general (hereinafter, the script m and s are neglected for simplicity), the relationship between SINR and outage probability can be denoted as

$$\Pr\{SINR \geq \Gamma\} = 1 - \epsilon \tag{1}$$

2.3 Spectrum resource deployment

To improve the spectrum efficiency, spectrum reuse is adopted for the spectrum allocation between various SBSs, that is to say, different SBSs can use the same spectrum simultaneously, which will introduce intra-frequency interference.

Between MBS and SBS system, there exists two different spectrum deployment schemes, co-channel and orthogonal channel deployment. In co-channel deployment, MBS and SBSs are permitted to use the same spectrum for data transmission at any time, which will cause severe cross-tier interference accounting for the severely signal quality of macro-MSs. In comparison for orthogonal spectrum allocation, spectrum resource is divided into two orthogonal parts, one part for MBS and the other for SBSs. Although for orthogonal spectrum allocation cross-tier interference can be avoided, its spectrum efficiency is greatly reduced. To bring a balance between performance and spectrum efficiency, by introducing the avoidance region, only the users located outside this region can reuse the spectrum, in this way, orthogonal channel is utilized and cross-tier interference is effectively avoided.

[1] in the following of this paper, the scripts m and s denote the macro-cell and small-cell, respectively.

2.4 Power consumption model of base station

The power consumption of base station can be denoted as $P = P_c + \xi P_t$ according to [20], where P_c denotes the static (fixed) part of power consumed by BS due to signal processing, analog-to-digital and digital-to-analog converting, cooling, etc. P_t is the data transmitting power, ξ represents the power consumption factor which scales with the transmitting power. In this way the total power consumption of MBS and SBS are:

$$\begin{aligned} P_{mb} &= P_{mc} + \xi_m P_{mbt} \\ P_{sb} &= P_{sc} + \xi_s P_{sbt} \end{aligned} \tag{2}$$

2.5 Definition of energy efficiency

Network energy efficiency can be defined as the ratio of the total amount of throughput and total power consumption in the network. The energy efficiency function can be written as

$$\eta_{EE} = \frac{C_m + \lambda_s \pi R_m^2 C_s}{P_{mb} + \lambda_s \pi R_m^2 P_{sb}} \tag{3}$$

where $\lambda_s \pi R_m^2$ is the number of small-cells in the coverage of macro-cell; C_m, C_s represent the throughput of MBS and SBS, respectively. Based on the system model, we could describe the throughput of macro- and small-cell as follows,

$$\begin{aligned} C_m &= (1 - \epsilon_m) \log_2 (1 + \Gamma_m) \\ C_s &= (1 - \epsilon_s) \log_2 (1 + \Gamma_s) \end{aligned} \tag{4}$$

where$(1 - \epsilon_m)$, $(1 - \epsilon_s)$ are the successful probability of transmission of the macro-cell and small-cell regarding the prescribed SINR threshold Γ_m, Γ_s.

3 Coverage performance analysis under different spectrum deployment

In this section, we use Stochastic Geometry theory to analyze the coverage performance of MBS and SBS system under different spectrum allocation strategies. For convenience, some notations used in this paper are listed in Table 1.

Table 1 Parameter notations used in this paper

P_{mbt},P_{sbt}	Transmitting power of MBS and SBS
P_{mc},P_{sc}	Static or fixed power consumption of MBS and SBS
ξ_m,ξ_s	Constant coefficient of power consumption of MBS and SBS
R_m,R_s	Radius of macro- and small-cell
Γ_m,Γ_s	SINR threshold of macro- and small-MS
γ_m,γ_s	The received SINR of macro- and small-MS
ϵ_m,ϵ_s	Outage probability of macro- and small-MS
N_0	Power of background noise
α	Path loss exponent
d	Radius of avoidance region

3.1 Orthogonal spectrum deployment

Under orthogonal spectrum deployment, the spectrum allocated for MBS and SBS is orthogonal, which avoids the cross-tier interference. The received SINR of macro-MS located at the cell boundary is $\gamma_m = \frac{P_{mbt} G_m R_m^{-\alpha}}{N_0}$, where G_m is the channel gain between macro-MS and MBS following exponential distribution with unit mean. To guarantee the coverage performance of macro-cell (Eq. 1), the received SINR of the MS at the macro-cell edge should satisfy the following equation,

$$\Pr \left\{ \frac{P_{mbt} G R_m^{-\alpha}}{N_0} \geq \Gamma_m \right\} = 1 - \epsilon_m \tag{5}$$

While for small-cells there's no interference coordination, inter-tier interference (denoted as I_{ss}) will occur among those small-cells using the same spectrum simultaneously. The received SINR of MS at small-cell edge is written as $\gamma_s = \frac{P_{sbt} G_s R_s^{-\alpha}}{N_0 + I_{ss}}$, where G_s accounts for the channel gain between other SBSs and the reference small-MS following exponential distribution with unit mean. Similarly to the situation in macro-cell scenario, to avoid the interference from SBSs which will violate the outage constraints for small-cell MS (SMS), the γ_s shall guarantee

$$\Pr \left\{ \frac{P_{sbt} G_s R_s^{-\alpha}}{N_0 + I_{ss}} \geq \Gamma_s \right\} = 1 - \epsilon_s \tag{6}$$

The inter-tier interference from surrounding SBSs is $I_{ss} = \sum_{Y_i \in \Phi_s \backslash \{Y_0\}} G_{Y_i} \beta_{ss} P_{sbt} \|Y_i\|^{-\alpha}$, β_{ss} is the penetration loss due to inner walls, G_{Y_i} and $\|Y_i\|$ are the channel gain and distance between BS located at Y_i and reference small-MS, respectively. Y_0 is the serving BS corresponding to reference small-MS. From [21], the moment generating function (MGF) of I_{ss} is

$$\begin{aligned} \mathrm{E}_{I_{ss}} \left[\exp(-s I_{ss}) \right] &= \exp \left(-2\pi \breve{\lambda}_s \int_0^\infty \frac{x}{1 + \frac{x^\alpha}{s P_{sbt} \beta_{ss}}} \mathrm{d}x \right) \\ &= \exp \left[-\breve{\lambda}_s (P_{sbt} \beta_{ss} s)^{\frac{2}{\alpha}} K_\alpha \right] \end{aligned} \tag{7}$$

where $K_\alpha = \frac{2\pi^2}{\alpha \sin(2\pi/\alpha)}$. To derive $\check{\lambda}_s$, Eq. 6 can be re-written as

$$\Pr\left\{G_s \geq \frac{\Gamma_s R_s^\alpha}{P_{sbt}}(N_0 + I_{ss})\right\} = \exp\left(-\frac{\Gamma_s N_0 R_s^\alpha}{P_{sbt}}\right)\exp\left[-\check{\lambda}_s R_s^2 (\Gamma_s \beta_{ss})^{\frac{2}{\alpha}} K_\alpha\right] \quad (8)$$

We thus have

$$1-\epsilon_s = \exp\left(-\frac{\Gamma_s N_0 R_s^\alpha}{P_{sbt}}\right)\exp\left[-\check{\lambda}_s R_s^2 (\Gamma_s \beta_{ss})^{\frac{2}{\alpha}} K_\alpha\right] \quad (9)$$

$$\check{\lambda}_s = \frac{-\ln(1-\epsilon_s) - \frac{\Gamma_s R_s^\alpha N_0}{P_{sbt}}}{(\Gamma_s \beta_{ss})^{\frac{2}{\alpha}} R_s^2 K_\alpha} \quad (10)$$

Under this strategy, the density of small-cells must be less then $\check{\lambda}_s$ to guarantee the coverage performance. Meanwhile, the density is only determined by the coverage performance of small-cell $(R_s, \Gamma_s, \epsilon_s)$, instead of that of macro-cell.

3.2 Co-channel spectrum deployment

Since co-channel deployment does not have interference coordination, MBS and SBSs use the same spectrum simultaneously; though it can achieve high spectrum utilization co-channel spectrum deployment will introduce severe cross-tier interference, denoted as I_{sm}. Considering the macro-MS coverage performance requirement, the MS at macro-cell edge must satisfy the outage constraints as follows,

$$\Pr\left\{\frac{G_m P_{mbt} R_m^{-\alpha}}{N_0 + I_{sm}} \geq \Gamma_m\right\} = 1-\epsilon_m \quad (11)$$

The cross-tier interference caused by surrounding SBSs is $I_{sm} = \sum_{Y_i \in \Phi_s} G_{Y_i} \beta_{s,m} P_{sbt} \|Y_i\|^{-\alpha}$, where β_{sm} is the penetration loss due to the outer walls. The left of Eq. 11 can be written as

$$\Pr\left\{G_m \geq \frac{\Gamma_m R_m^\alpha}{P_{mbt}}(N_0 + I_{sm})\right\} = \exp\left(-\frac{\Gamma_m R_m^\alpha N_0}{P_{mbt}}\right)\exp\left[-\tilde{\lambda}_s K_\alpha R_m^2 \left(\frac{\beta_{sm}\Gamma_m P_{sbt}}{P_{mbt}}\right)^{\frac{2}{\alpha}}\right] \quad (12)$$

We thus have

$$\tilde{\lambda}_s = \frac{\left[-\ln(1-\epsilon_m) - \frac{\Gamma_m R_m^\alpha N_0}{P_{mbt}}\right](P_{mbt})^{\frac{2}{\alpha}}}{(\Gamma_m P_{sbt}\beta_{sm})^{\frac{2}{\alpha}} R_m^2 K_\alpha} \quad (13)$$

If we want to guarantee the coverage performance of MBS, the density of small-cell deployed in the macro-cell must be less than $\tilde{\lambda}_s$.

Then we consider the coverage performance constraints of small-MS, the cell-edge small-MS must guarantee

$$\Pr\left\{\frac{G_s P_{sbt} R_s^{-\alpha}}{N_0 + I_{ss} + I_{ms}} \geq \Gamma_s\right\} = 1-\epsilon_s \quad (14)$$

Small-MSs are not only interfered by surrounding SBSs, but also the MBS. So the interference includes inter- and cross-tier interference. We can rewrite the left of Eq. 14.

$$\Pr\left\{G_s \geq \frac{\Gamma_s R_s^\alpha}{P_{sbt}}(N_0 + I_{ss} + I_{ms})\right\} = \exp\left(-\frac{\Gamma_s R_s^\alpha N_0}{P_{sbt}}\right)\cdot \mathrm{E}_{I_{ss}}\left[\exp\left(-\frac{\Gamma_s R_s^\alpha I_{ss}}{P_{sbt}}\right)\right] \times \mathrm{E}_{I_{ms}}\left[\exp\left(-\frac{\Gamma_s R_s^\alpha I_{ms}}{P_{sbt}}\right)\right] \quad (15)$$

The part $\mathrm{E}_{I_{ss}}\left[\exp\left(-\frac{\Gamma_s R_s^\alpha I_{ss}}{P_{sbt}}\right)\right]$ can be calculated according to Eq. 7

$$\mathrm{E}_{I_{ss}}\left[\exp\left(-\frac{\Gamma_s R_s^\alpha I_{ss}}{P_{sbt}}\right)\right] = \exp\left[-\bar{\lambda}_s R_s^2 K_\alpha (\Gamma_s \beta_{ss})^{\frac{2}{\alpha}}\right] \quad (16)$$

Since SBSs are randomly distributed in the area according to a homogeneous SPPP, the probability density function (PDF) of the distance between MBS and the reference small-MS is

$$f(r) = \frac{2r}{R_m^2} \quad (17)$$

Thus we obtain the moment generating function (MGF) as

$$\mathrm{E}_{I_{ms}}\left[\exp\left(-\frac{\Gamma_s R_s^\alpha I_{ms}}{P_{sbt}}\right)\right] = \int_0^{R_m} \frac{x^\alpha}{1 + s P_{mbt}\beta_{sm}} \frac{2x}{R_m^2} \mathrm{d}x = \frac{2R_m^\alpha P_{sbt} F\left(1,1;2+\frac{2}{\alpha};\frac{R_m^\alpha P_{sbt}}{R_m^\alpha P_{sbt} + R_s^\alpha \beta_{sm}\Gamma_s P_{mbt}}\right)}{(2+\alpha)(R_m^\alpha P_{sbt} + R_s^\alpha \beta_{sm} P_{mbt}\Gamma_s)} \quad (18)$$

where the function $F(,;;)$ is the Gaussian hyper-geometric function. We thus have

$$\bar{\lambda}_s = \frac{-\ln\left[\frac{(1-\epsilon_s)(2+\alpha)(R_m^\alpha P_{sbt} + R_s^\alpha \beta_{sm}\Gamma_s P_{mbt})}{2R_m^\alpha P_{sbt} F\left(1,1;2+\frac{2}{\alpha};\frac{R_m^\alpha P_{sbt}}{R_m^\alpha P_{sbt}+R_s^\alpha\beta_{sm}\Gamma_s P_{mbt}}\right)}\right]}{K_\alpha R_s^2 (\beta_{ss}\Gamma_s)^{\frac{2}{\alpha}}} - \frac{R_s^\alpha \Gamma_s N_0}{P_{sbt} K_\alpha R_s^2 (\beta_{ss}\Gamma_s)^{\frac{2}{\alpha}}} \quad (19)$$

To guarantee the coverage performance of SBS, the density of SBSs deployed in the macro-cell must be less than $\bar{\lambda}_s$.

Taking both MBS and SBS's coverage performance into consideration, the upper bound of density of small-cells is $\check{\lambda}_s = \min(\tilde{\lambda}_s, \bar{\lambda}_s)$.

Without loss of generality, we assume that $\tilde{\lambda}_s < \bar{\lambda}_s$. And any λ_s larger than $\tilde{\lambda}_s$ will make the outage probability of users at macro-cell edge larger than ϵ_m, which violates the requirement of the outage performance constraints.

Considering the severe cross-interference for co-channel deployment especially when macro-MS is closed to the SBS, because of the closed access strategy, the interference is very huge. One of the solutions is introducing the *avoidance region* around the macro-MS, which means that within this region the SBSs cannot re-use the spectrum originally allocated to that macro-MS.

3.3 Co-channel spectrum deployment with avoidance region

Next the analysis of the coverage performance of macro-MS under co-channel deployment with avoidance region is presented as follows. According to the outage constraints, we have

$$\Pr\left\{\frac{G_m P_{mbt} R_m^{-\alpha}}{N_0 + I'_{sm}} \geq \Gamma_m\right\} = 1 - \epsilon_m \tag{20}$$

The avoidance region is assumed to be a circle with radius d which is denoted as $B(0, d)$, and in this region there will be no users allocated the co-channel spectrum thus the interference will be avoided in a certain distance. In this way the distance between macro-MS and SBSs interfering it is in $[d, \infty]$, in Eq. 20 the cross-tier interference is $I'_{sm} = \sum_{Y_i \in \Phi_s \backslash B(0,d)} G_{Y_i} \beta_{s,m} P_{sbt} \|Y_i\|^{-\alpha}$. Rewrite the left of Eq. 20 as

$$\begin{aligned}
\Pr\left\{G_m \geq \frac{\Gamma_m R_m^{\alpha}}{P_{mbt}}(N_0 + I'_{sm})\right\} &= \exp\left(-\frac{\Gamma_m R_m^{\alpha} N_0}{P_{mbt}}\right) \\
&\times \exp\left(-2\pi \hat{\lambda}_s \int_d^{\infty} \frac{x}{1 + \frac{P_{mbt} x^{\alpha}}{\Gamma_m R_m^{\alpha} P_{sbt} \beta_{sm}}} \mathrm{d}x\right) \\
&= \exp\left[\frac{-2\pi \hat{\lambda}_s d^2 F\left(1, 1; 2 - \frac{2}{\alpha}; \frac{R_m^{\alpha} \Gamma_m \beta_{sm} P_{sbt}}{R_m^{\alpha} \Gamma_m \beta_{sm} P_{sbt} + d^{\alpha} P_{mbt}}\right)}{\alpha\left(1 - \frac{2}{\alpha}\right)\left(1 + \frac{d^{\alpha} P_{mbt}}{R_m^{\alpha} \Gamma_m \beta_{sm} P_{sbt}}\right)}\right] \\
&\times \exp\left(-\frac{\Gamma_m R_m^{\alpha} N_0}{P_{mbt}}\right)
\end{aligned} \tag{21}$$

We thus have

$$\begin{aligned}
\hat{\lambda}_s = &\frac{\alpha\left(1 - \frac{2}{\alpha}\right)\left(1 + \frac{d^{\alpha} P_{mbt}}{R_m^{\alpha} \Gamma_m \beta_{sm} P_{sbt}}\right)}{2\pi d^{\alpha} F\left(1, 1; 2 - \frac{2}{\alpha}; \frac{R_m^{\alpha} \Gamma_m \beta_{sm} P_{sbt}}{R_m^{\alpha} \Gamma_m \beta_{sm} P_{sbt} + d^{\alpha} P_{mbt}}\right)} \\
&\times \left[-\ln(1 - \epsilon_m) - \frac{R_m^{\alpha} \Gamma_m N_0}{P_{mbt}}\right]
\end{aligned} \tag{22}$$

Similarly, the density of small-cells deployed in the macro-cell should be less than $\hat{\lambda}_s$ to satisfy the constraints on the outage probability of macro-MS. Further, to satisfy both layers' outage constraints, the upper bound of the density of small-cells should be the smaller one between $\hat{\lambda}_s$ and $\bar{\lambda}_s$.

Under both strategies of co-channel deployment with and without avoidance region, the density of small-cells is restrained by the coverage performance of both macro-cell $(R_m, \Gamma_m, \epsilon_m)$ and small-cell $(R_s, \Gamma_s, \epsilon_s)$.

4 Analysis of optimal energy efficiency

Combining the function of energy efficiency in Eq. 3 with the coverage performance constraints analyzed in Section 3, we will analyze the optimal density of small-cells that maximize the energy efficiency under a desired coverage requirement for the two-tier network. Firstly, the optimization target is written as,

$$\begin{aligned}
&\max \left\{\eta(\lambda_s) = \frac{(1-\epsilon_m)\log_2 1+\Gamma_m+}{}\right\} \\
&\quad s.t.\ \ \epsilon_m \leq \epsilon_{mth}, \epsilon_s \leq \epsilon_{sth}
\end{aligned} \tag{23}$$

where ϵ_{mth} and ϵ_{sth} are respectively the outage probability threshold constraints, which reflect the requirement of quality of service (QoS). The expressions of ϵ_m, ϵ_s under different spectrum allocation strategies are derived in Section 3.

The following theorem demonstrates the quasi-concavity of the target function $\eta(\lambda_s)$. For any continuous and strictly quasi-concave function, there is always a unique global maximum over a finite domain [22]. Thus, according to Theorem 1, there always exists a optimal λ_s make $\eta(\lambda_s)$ maximum globally.

Theorem 1 *The energy efficiency function $\eta(\lambda_s)$ is a strictly quasi-concave function in $\mathbf{R}^+$. Moreover, in the λ_s region $(0, a]$, $\eta(\lambda_s)$*

(i) *strictly increases with λ_s and is maximized at $\lambda_s = a$ if*

$$\left.\frac{\mathrm{d}\eta(\lambda_s)}{\mathrm{d}\lambda_s}\right|_{\lambda_s=a} \geq 0$$

(ii) *first strictly increases and then strictly decreases with λ_s and is maximized at $\lambda_s = \lambda_s^{max}$ if*

$$\left.\frac{\mathrm{d}\eta(\lambda_s)}{\mathrm{d}\lambda_s}\right|_{\lambda_s=a} < 0$$

where λ_s^{max} is the density of small-cell maximize the $\eta\lambda_s$ in region $\mathbf{R}^+$.

$\widehat{\lambda}_s$, the upper bound of λ_s is calculated with equation in Section III according to different spectrum allocation strategies. $\lambda_s^{(1)} \leftarrow \widehat{\lambda}_s$, $d^* \leftarrow \frac{\mathrm{d}\eta(\lambda_s)}{\mathrm{d}\lambda_s}\Big|_{\lambda_s=\lambda_s^{(1)}}$

if $d^* \geq 0$ **then**
  return $\lambda_s^{opt} \leftarrow \lambda_s^{(1)}, \quad \eta^{opt} = \eta(\lambda_s = \lambda_s^{opt})$;
else
  $\lambda_s^{(2)} \leftarrow \lambda_s^{(1)}, \quad \lambda_s^{(1)} \leftarrow \kappa\lambda_s(1)$, where $0 < \kappa < 1$, $\quad d^* \leftarrow \frac{\mathrm{d}\eta(\lambda_s)}{\mathrm{d}\lambda_s}\Big|_{\lambda_s=\lambda_s^{(1)}}$;
  while $d^* < 0$ **do**
    $\lambda_s^{(1)} \leftarrow \kappa\lambda_s^{(1)}, \quad d^* \leftarrow \frac{\mathrm{d}(\lambda_s)}{\mathrm{d}\lambda_s}\Big|_{\lambda_s=\lambda_s^{(1)}}$;
  end while
end if
while no convergence **do**
  $\lambda_s^{opt} \leftarrow \frac{\lambda_s^{(1)}+\lambda_s^{(2)}}{2}, \quad d^* \leftarrow \frac{\mathrm{d}(\lambda_s)}{\mathrm{d}\lambda_s}\Big|_{\lambda_s=\lambda_s^{opt}}$;
  if $d^* \geq 0$ **then**
    $\lambda_s^{(1)} \leftarrow \lambda_s^{opt}$
  else
    $\lambda_s^{(2)} \leftarrow \lambda_s^{opt}$
  end if
end while
return $\lambda_s^{opt}, \quad \eta^{opt} = \eta(\lambda_s^{opt})$

Proof It will be further demonstrated in the simulation section. As the result of strict quasi-concavity of Eq. 23, it can be solved iteratively by the following two steps:

1. For a given outage probability threshold, find out the upper bound of the density of small-cells that satisfy the coverage performance of both tiers and its corresponding derivative of $\eta(\lambda_s)$;
2. Find out the optimal energy efficiency η^{opt} and its corresponding small-cells' density λ_s^{opt}, by bisection search.

Here is the algorithm to solve the optimization problem. □

5 Simulation analysis

In this section, simulations are given to validate the theoretical analysis. Meanwhile, we discuss the impact of coverage performance constraints on optimal energy efficiency. The simulation parameters are given in Table 2.

Figs. 2, 3 and 4 respectively shows the relationship among energy efficiency, density of small-cells and system outage performance for different spectrum deployment schemes. From the simulation results it is found that the the outage probability of macro-MS is much smaller than that of small-MS, which results from avoidable cross-tier interference. Also it can be learned that the optimal density of small-cells that maximize the overall energy efficiency under a desired coverage only depends on the coverage performance of small-cell. Similar with the analysis in the previous section, there exists a global maximum in the energy efficiency curve. The optimal energy efficiency can be discussed in two cases.

– In case 1, the requirement of coverage performance is strict, the upper-bound value of small-cells' density that satisfy the coverage performance of both two tiers reaches the optimal energy efficiency;
– In case 2, the constraints of coverage performance is looser, the density λ_s^{max} as shown in Fig. 2 can be obtained to achieve the maximum energy efficiency.

In Fig. 3, the energy-efficiency is very similar to that in Fig. 2. The differences between these two figures are that the coverage performance in Fig. 3 is very severe and the

Table 2 Parameters setup

	Macro-cell	Small-cell
Transmitting power	$P_{mbt} = 20W$	$P_{sbt} = 0.1W$
Static power consumption of base station	$P_{mc} = 354.44W$	$P_{sc} = 71.5W$
Constant coefficient of power consumption of base station	$\xi_m = 21.54$	$\xi_s = 7.84$
Radius of cell	$R_m = 300m$	$R_s = 15$
SINR threhold	$\Gamma_m = 1$	$\Gamma_s = 1$
Power of background noise	$N_0 = -110dBm$	
Path loss exponent	$\alpha = 4$	
Distance of avoidance region	$d = 50, 100, 150m$	

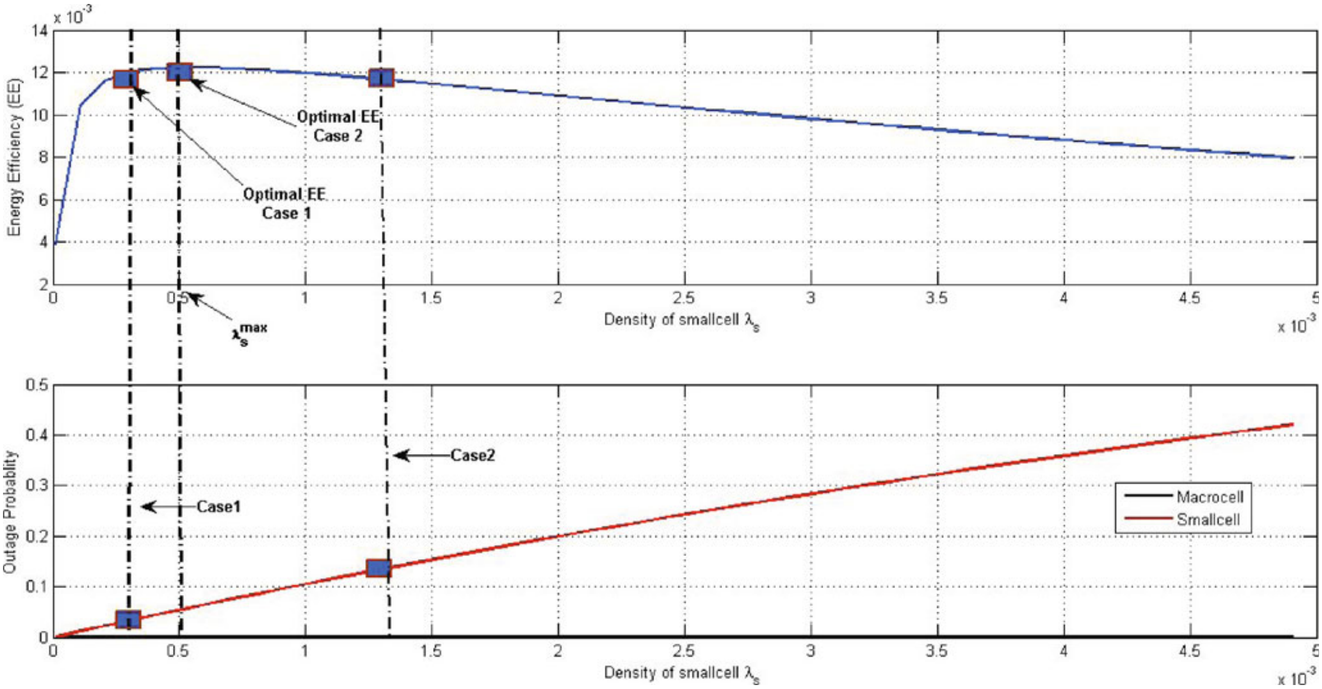

Fig. 2 Relationship between energy efficiency and outage performance under orthogonal channel deployment

outage probability is much larger than that of small-MS. The coverage requirement limits the energy-efficient small-cell deployment, which is due to the severe interference from SBSs around the macro-MS. Thus technique must be done to decrease the cross-tier interference and introduce the avoidance region.

Figure 4 shows the effects of radius of avoidance region d on the coverage performance. The outage probability of macro-MS decreases as d increases, especially it decreases to be smaller than that of small-MS when d increase to 100 m. Thus the optimal energy efficiency under a desired coverage is enlarged, since more

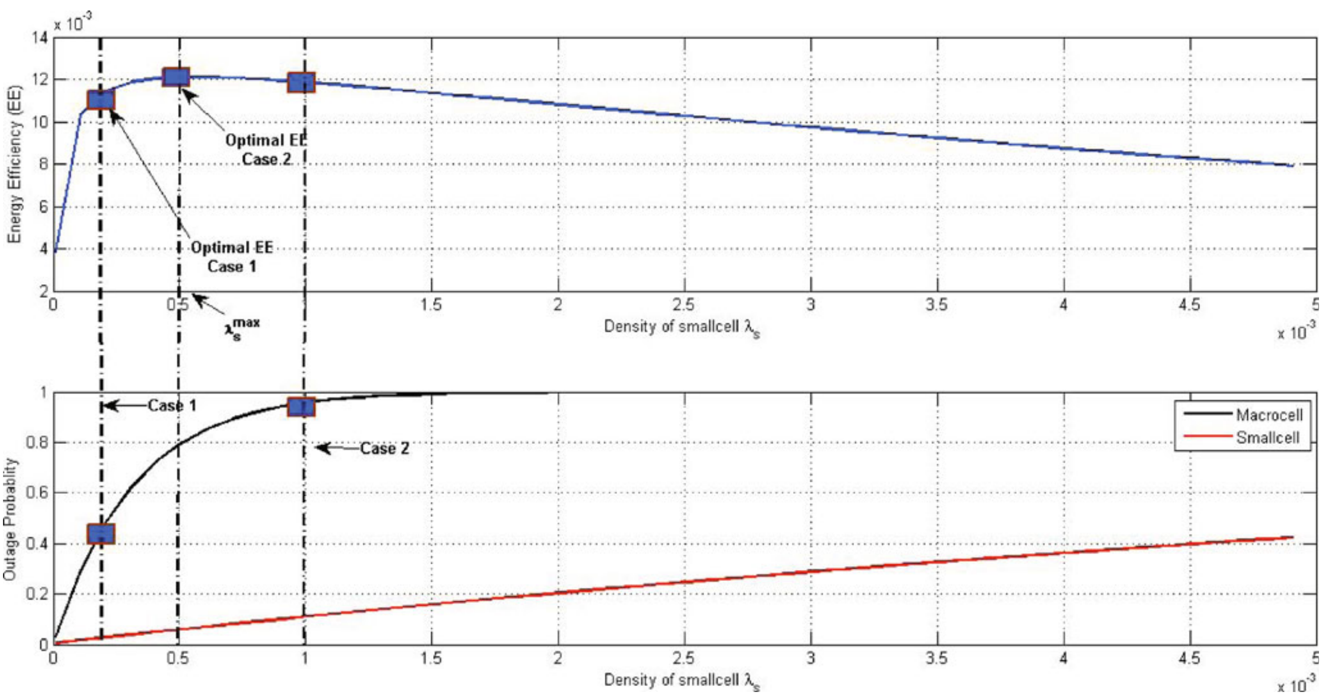

Fig. 3 Relationship between energy efficiency and outage performance under co-channel deployment without avoidance region

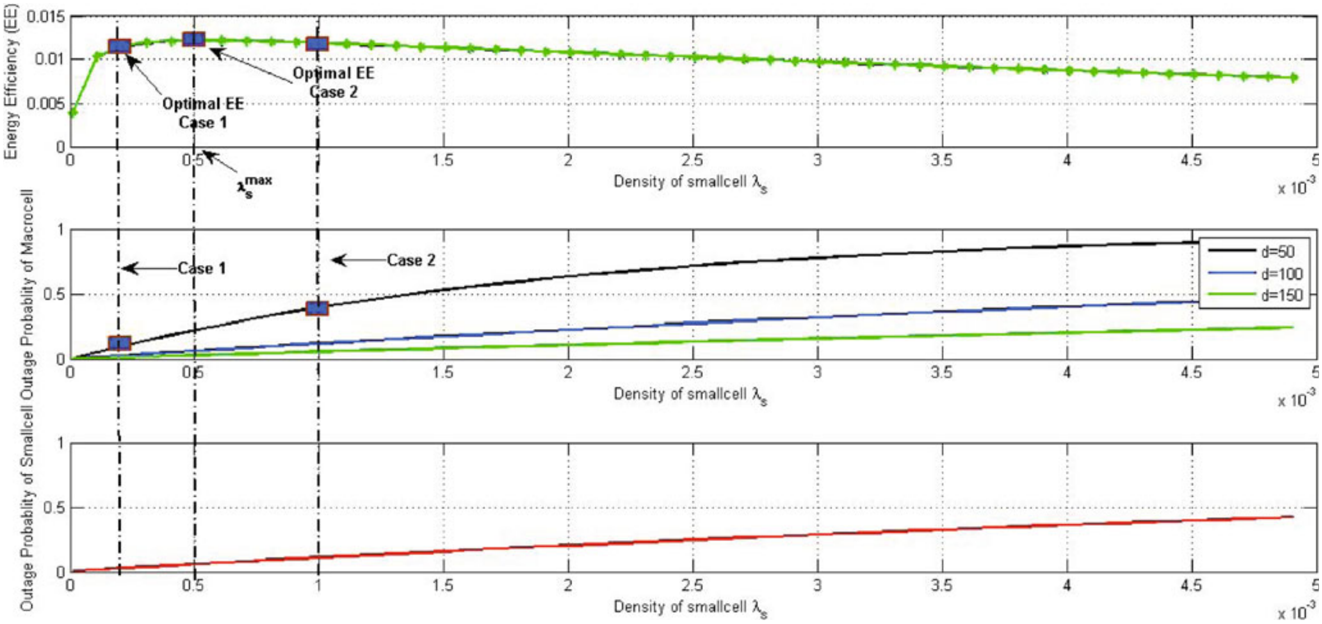

Fig. 4 Relationship between energy efficiency and outage performance under co-channel deployment without avoidance region

small-cells can be deployed under a fixed requirement of coverage performance. However the introduction of d is not able to improve the coverage performance of small-cells.

The comparison of the three spectrum deployment strategies is depicted in Fig. 5. For a certain density λ_s of small cells, the energy-efficiency for the three allocation schemes are almost the same, while the coverage performance (denoted by outage probability) is not the same, which can be seen from the lower two figures of Fig. 5. That is to say, for a certain coverage performance (this is a prerequisite for the network's performance), the density of small cells for the three spectrum allocation schemes will be different, thus the energy-efficiency is not the same as well. Meanwhile, under the same coverage performance, it can be seen that the energy efficiency of orthogonal spectrum allocation the the largest, and that of co-channel spectrum allocation is the lowest. When the avoidance distance is large enough, the energy-efficiency of co-channel spectrum allocation with avoidance region is very close to that of orthogonal spectrum allocation, thus co-channel

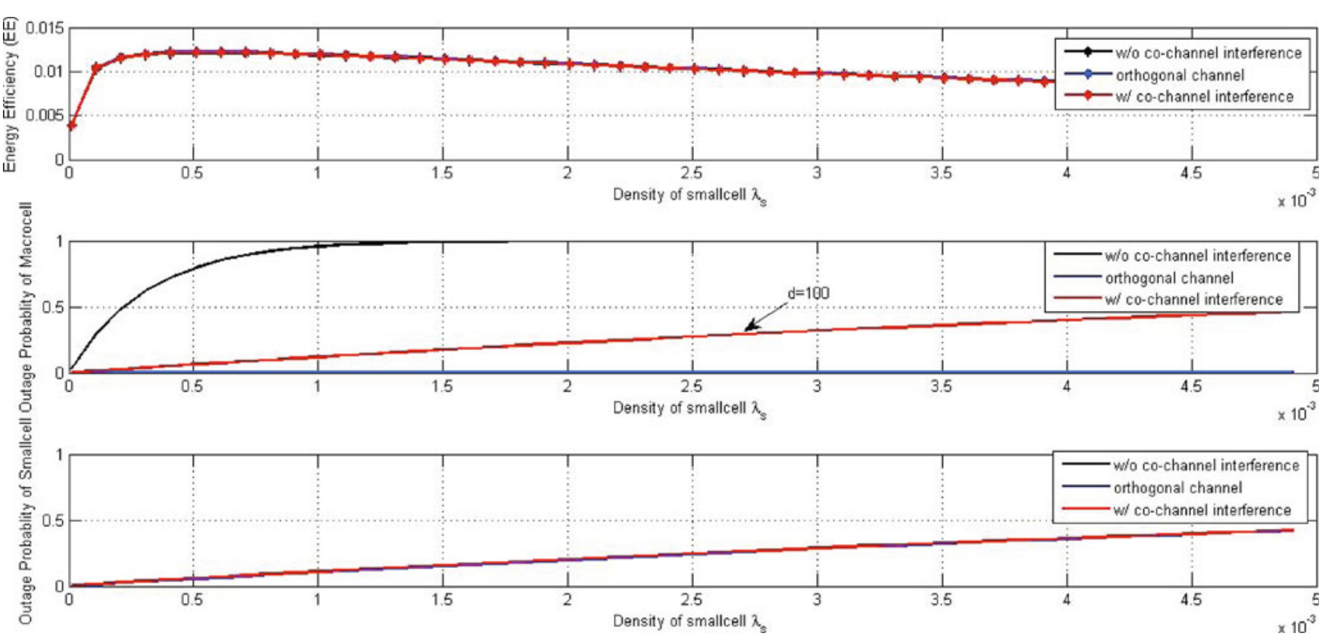

Fig. 5 Relationship between energy efficiency and outage performance under different spectrum allocation schemes

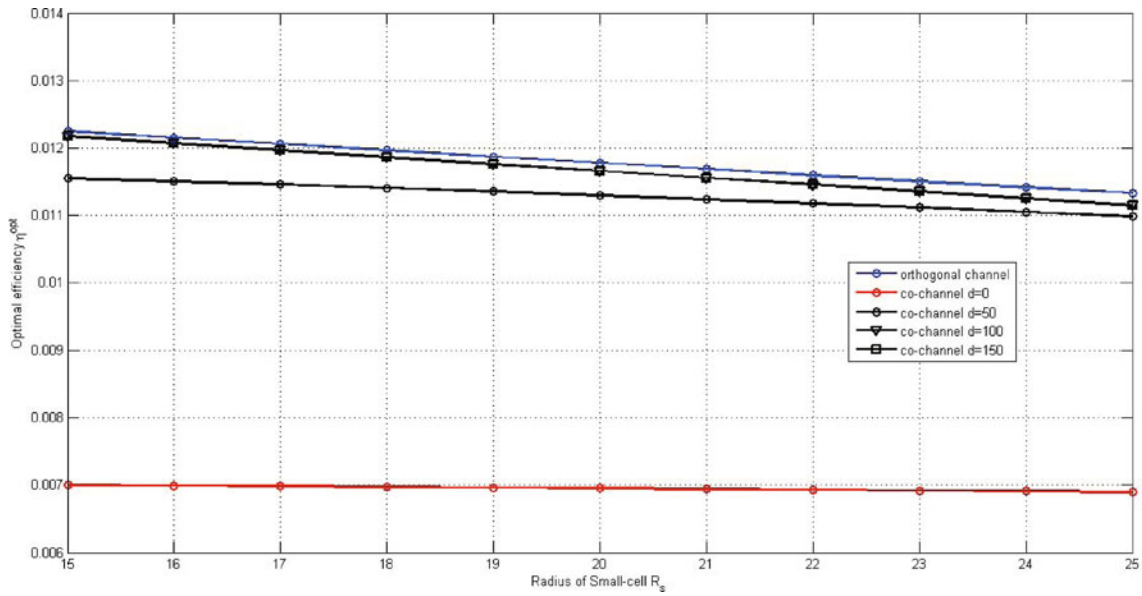

Fig. 6 The optimal energy efficiency versus R_f for different spectrum deployment under given coverage performance constraints. The probability of outage threshold is $\epsilon_{mth} = 0.1$,$\epsilon_{sth} = 0.1$

spectrum allocation with avoidance region is recommended to be used.

Figure 6 shows the effect of R_s on optimal energy efficiency. As R_s increases, the optimal energy efficiency of the three strategies decreases. As R_f increases, the small-MS located at the boundary of small-cell receives weaker signal from SBS and the density of small-cells should decrease to guarantee the outage constraints at the reference small-MS. In general we find that

1. The optimal energy efficiency with orthogonal channel deployment is better than that with co-channel.
2. As d increase, the optimal energy efficiency increase. However, after d increases to a certain value, the optimal energy efficiency no longer increases, this is because that at this time the outage probability of macro-MS is smaller than that of small-MS.
3. A discrepancy exists between orthogonal channel and co-channel with $d = 150\ m$ mainly due to the fact that cross-tier interference still exists in co-channel with avoidance region though it is very slight.

6 Conclusions

This paper analyzes and compares the optimal energy efficiency of a two-tier heterogeneous network (HetNet) which consists of macro-cell and small-cells under coverage performance constraints for different spectrum deployments (e.g., orthogonal and co-channel spectrum allocations). The relationship between energy efficiency and density of small-cells for the two-tier network is evaluated based on the analysis of coverage performance. And then an algorithm is proposed to obtain the optimal density of small-cells that maximize the energy efficiency under a desired coverage requirement. The theoretical analysis is validated by simulations. The results show that the energy efficiency of the two-tier networks with orthogonal spectrum deployment is better than that with co-channel spectrum deployment. Meanwhile it is also showed that the optimal density of small-cells for maximal energy efficiency is only dependent on the coverage performance of small-cells in orthogonal spectrum deployment scenario. However, in co-channel spectrum deployment scenario, the optimal density of small-cells for maximal energy efficiency is jointly decided by the coverage performance of both macro-cell and small-cell. This work provides essential understanding for successful deployment of green heterogeneous networks, and has the guidance to deploy a energy efficient heterogeneous networks.

Appendix: Proof of Theorem 1

Only the proof of quasi-concavity under co-channel deployment is derived, since the other two situations are the special cases of it. According to the property of quasi-concave function, $\eta(\lambda_s)$ is strictly quasi-concave in λ_s if the super-level sets of $\eta(\lambda_s)$ is always strictly convex. The super-level sets of $\eta(\lambda_s)$ is denoted as

$$P_a = \{\lambda_s | \eta\lambda_s \geq a,\ \lambda_s > 0\}$$

For any real number a, P_a is strictly convex, then $\eta(\lambda_s)$ is strictly quasi-concave function. If $a < 0$, since $\eta(\lambda_s) > 0$ for any $\lambda_s > 0$, P_a is apparently convex. If $a \geq 0$, we will prove the theorem in the following way. We put the outage constraints condition into $\eta(\lambda_s)$ and obtain

$$\eta(\lambda_s) = \frac{\widetilde{C}_m \mathrm{e}^{-A\lambda_s} + \lambda_s \widetilde{C}_s \mathrm{e}^{-B\lambda_s}}{P_{mb} + \lambda_s \widetilde{P}_{sb}}$$

where

$$\begin{aligned}
P_{mb} &= \xi_m P_{mbt} + P_{mc} \\
\widetilde{P}_{sb} &= \pi R_m^2 (\xi_s P_{sbt} + P_{sc}) \\
\widetilde{C}_m &= \exp\left(-\frac{\Gamma_m N_0 R_m^\alpha}{P_{mbt}}\right) \log_2(1+\Gamma_m) \\
\widetilde{C}_s &= \pi R_m^2 \exp\left(-\frac{\Gamma_s R_s^\alpha N_0}{P_{sbt}}\right) \log_2(1+\Gamma_s) \\
&\quad \times \frac{2R_m^\alpha P_{sbt} F\left(1, 1; 2+\frac{2}{\alpha}; \frac{R_m^\alpha P_{sbt}}{R_m^\alpha P_{sbt} + R_s^\alpha \beta_{sm} \Gamma_s P_{mbt}}\right)}{(2+\alpha)(R_m^\alpha P_{sbt} + R_s^\alpha \beta_{sm} P_{mbt} \Gamma_s)}
\end{aligned} \tag{24}$$

$$\begin{aligned}
A &= K_\alpha \left(\frac{\beta_{sm} \Gamma_m P_{sbt}}{P_{mbt}}\right)^{\frac{2}{\alpha}} R_m^2 \\
B &= (\beta_{ss} \Gamma_s)^{\frac{2}{\alpha}} R_s^2 K_\alpha
\end{aligned}$$

It is easy to know that P_{mb}, $\widetilde{P}_{sb}$, $\widetilde{C}_m$, $\widetilde{C}_s$, A and B are positive constant. Moreover, according to the parameter setup following typical two-tier networks, we have these relationships

$$(i)\ A^2 \widetilde{C}_m < B\widetilde{C}_s;\ (ii) A > B$$

The super-level set P_a is equivalent to $P_a = \{\lambda_s | \widetilde{C}_m \mathrm{e}^{-A\lambda_s} + \lambda_s \widetilde{C}_s \mathrm{e}^{-B\lambda_s} - a(P_{mb} + \lambda_s \widetilde{P}_{sb}) \geq 0,\ \lambda_s > 0\}$. Meanwhile, we denote $H(\lambda_s) = \widetilde{C}_m \mathrm{e}^{-A\lambda_s} + \lambda_s \widetilde{C}_s \mathrm{e}^{-B\lambda_s} - a(P_{mb} + \lambda_s \widetilde{P}_{sb})$. If $H(\lambda_s)$ is strictly monotonic or concave, P_a is strictly convex. $H(\lambda_s)$ is continuous in $\lambda_s > 0$ and its derivative is $H'(\lambda_s) = -A\widetilde{C}_m \mathrm{e}^{-A\lambda_s} + (1 - B\lambda_s)\widetilde{C}_s \mathrm{e}^{-B\lambda_s} - a\widetilde{P}_{sb}$, its second derivative is $H''(\lambda_s) = A^2 \widetilde{C}_m \mathrm{e}^{-A\lambda_s} + \widetilde{C}_s B(B\lambda_s - 2)\mathrm{e}^{-B\lambda_s}$. If $\lambda_s \geq \frac{1}{B}$, $H(\lambda_s)$ is strictly monotone decreasing for $H'(\lambda_s) < 0$; If $0 < \lambda_s < \frac{1}{B}$, according to the relationship (i),(ii), $H''(\lambda_s) < 0$ and $H(\lambda_s)$ is strictly concave. Thus continuous function $H(\lambda_s)$ is divided into two parts, a strictly concave function following a strictly monotone decreasing function. So P_a is strictly convex, and energy efficiency function $\eta(\lambda_s)$ is strictly quasi-concave.

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