

Modeling of Energy Efficiency in Heterogeneous Network

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Abstract: Cellular networks are undergoing transformation from conventional homogeneous macro Base Stations (BSs) to Heterogeneous Network (HetNet). This new paradigm not only offers a significant improvement in the overall network capacity or user data rate; it also promises an improvement in the overall network Energy Efficiency (EE). In this study, a theoretical model for evaluating the EE in HetNet is developed. A HetNet generally consists of different types of Base Stations (BSs) which operate in harmony towards a set of common goals defined by the network operator such as coverage and capacity improvement. Each BS may differ in terms of transmit power, achievable data rate, coverage, BS density and EE, under different network deployment scenarios. The results show that the picocell strongly impacts the Energy Efficiency (EE) of the HetNet as compared to microcell. More specifically it is observed that certain ratios of microcells and picocells per macro BS will result in sub-optimal of Area Energy Efficiency (AEE). It is shown that the AEE of HetNet also increases as the percentage area of macro BS overlaid by smaller cells and the density of micro/picocells increases.

Keywords: Energy efficiency, HetNet, LTE

INTRODUCTION

According to Cisco (2012) the number of mobile-connected devices will exceed global population by the end of 2012 and by 2016, there will be 1.4 mobile devices per person. At the same time, mobile data traffic will grow at a Compound Annual Growth Rate (CAGR) of 78 percent from 2011 to 2016, indicating the need for a new infrastructure to cope with the capacity requirement (Ling and Chizhik, 2012). It is certain that smaller base stations or Low Power Nodes (LPN) will play a major role in such infrastructure since they have higher capacity per area. On the other hand, the amount of energy consumed by this dramatic increase of traffics poses great challenges to network operators in preserving energy cost as well as environmental preservation. To address both capacity and energy challenges, future cellular networks will need to consider Network Energy Efficiency (EE) as a key performance indicator. EE not only considers energy consumed by the base station, but also features and properties related to capacity and coverage of the network (Tao *et al.*, 2010). Some research works on cellular network planning mainly focus on the practical deployment algorithm design (Panayiotis *et al.*, 2012; Amaldi *et al.*, 2008). A survey of green mobile networks and an overview of some promising approaches and methods for improving the EE in HetNet is provided in Ayad *et al.* (2012a) and Xiaofei *et al.* (2012). This study extends the work to cover multi-user and multicell cases as well as considering the practical issues. In this regard, Richter *et al.* (2009)

introduce a metric known as area power consumption *APC*. For a given average area spectral efficiency, the authors show that moderate power saving can be achieved through heterogeneous deployments using micro and macro base stations. In their study however, only two types of BSs were considered, a uniform user distribution was assumed under full traffic load scenarios and there was no optimization on the specific location of the micro-cells (Richter *et al.*, 2009). The study in Ayad *et al.* (2012b) looks into the relationship between percentage requirements of coverage area and optimal inter site distances (or macrocell radius) in homogeneous macrocell network. The authors in Miao *et al.* (2011) investigate multi-cell interference-limited uplink OFDMA cellular network scenarios and develop a non-cooperating game for energy-efficient power optimization. Wang and Shen (2010) analyzes the energy efficiency and area energy efficiency AEE of two-tier networks with macro and pico cells. In their study, the maximum achievable data transmission rate for each user is obtained with the knowledge of receiving SNR. In this study, a theoretical model of energy efficiency that reinforces the key deployment solutions in heterogeneous cellular networks considering three types of base stations has been derived and the network performance in terms of EE has been evaluated. The EE of HetNet is defined as the ratio of the total sum of achievable data rate at user's terminals within all BSs to the sum of power consumed by these BSs. The achievable data rate is calculated as a function of the received power at User Equipment (UE) for different locations.

SYSTEM MODELS

Propagation model: Commonly, deterioration of signal quality due to propagation is related to several factors such as Path-Loss (PL), penetration loss (β), antenna pattern (Ah) in the case of directional antennas and shadowing effect (Ψ). The received signal power, P_{rx} , of the n th user from a base station BS at a distance r and angle θ from the antenna's main lobe can be modeled in term of BS transmitted power as Tesfay *et al.* (2011):

$$Pr x_n = Ptx_n - ALF_n \quad (1)$$

where, Ptx is the transmitted power and ALF is the aggregate signal attenuation factor due to Path Loss (PL), outdoor-indoor penetration loss (β) and radiation pattern (Ah) as well as shadowing effect, ALF for the n th user can be written as:

$$ALF_n(r, \theta) = PL_{dB}(r) + \beta_{dB} + Ah_n(\theta) + \Psi_{dB} \quad (2)$$

These factors can be obtained by choosing the appropriate path loss model (e.g., (3GPP, 2009)). The propagation model (1) provides the basis for the more realistic models presented in 3GPP (2009), which incorporate path loss dependency on carrier frequency, Line of Sight (LOS) conditions as well as shadowing deviations. Furthermore, they also consider UE and BS height, where the latter differs significantly between macro, micro and pico cells (Richter *et al.*, 2009).

Coverage model: A cellular coverage is designed based on an average received power $P_{rx}(R)$ at the cell boundary with cell radius R . The cell coverage area can be defined as the fraction of cell area where received power is above a given level P_{min} , which is also referred as receiver sensitivity. The cell coverage area C can be written as (Goldsmith, 2005):

$$C = Q(a) + \exp\left(\frac{2-2ab}{b^2}\right) Q\left(\frac{2-ab}{b}\right) \quad (3)$$

where, the Q -function is defined as the probability that a Gaussian random variable X with mean 0 and variance 1 is greater than z (see Appendix A)

$$Q(z) = \text{prob}(X \geq z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} \exp\left(-\frac{x^2}{2}\right) dx$$

and

$$a = \frac{P_{min} - P_{rx}(R)}{\sigma_{\Psi dB}}, \quad b = \frac{10 \alpha \log_{10}(e)}{\sigma_{\Psi dB}}$$

P_{min} is the minimum power received at which a throughput requirement is fulfilled. The throughput is equal or higher than 95% of the maximum throughput for a specified reference measurement channel and can be expressed as Stefania *et al.* (2011):

$$P_{min} = kTB + NF + SINR_{req} + IM - G_d \quad (4)$$

where, kTB represents the thermal noise level in a specified noise bandwidth BW , where $BW = N_{RB} * 180$ (kHz) as defined in 3GPP (Stefania *et al.*, 2011). N_{RB} is the number of Resource Blocks (RB) and 180 kHz is the bandwidth of one RB. NF is the prescribed maximum noise figure for the receiver. $SINR_{req}$ is the signal to interference plus noise ratio requirement for the chosen Modulation and Coding Scheme (MCS). IM is the implementation margin and the G_d represents the diversity gain (Stefania *et al.*, 2011). Note that $a = 0$, when the minimum received power, P_{min} is equal to target average power, $P_{rx}(R)$ at the cell boundary with radius R . $\sigma_{\Psi dB}$ is the standard deviation of shadow fading. The outage probability of the cell is defined as the percentage of area within the cell that does not meet its minimum power requirement P_{min} ; namely $P_{out} = p(P_{rx}(r) < P_{min}) = 1 - C$ (Goldsmith, 2005).

Hence, the coverage area of a cell is a function of receiver sensitivity P_{min} , carrier frequency f , transmitted power P_{tx} , path losses exponent α and shadowing standard deviation $\sigma_{\Psi dB}$. By limiting the coverage area to a certain size, the network performance in terms of achievable data rates and efficiencies within each base station can be estimated. The lower and upper bounds for coverage area, C can be written as:

$$\frac{2 \exp\left(\frac{-a^2}{2}\right)}{(1+a^2)[b^2 + (2-ab)^2]\sqrt{2\pi}} < C < \frac{2 \exp\left(\frac{-a^2}{2}\right)}{a(2-ab)\sqrt{2\pi}}, \quad \frac{2}{b} > a > 0 \quad (5)$$

The detailed proof of this derivation is given in Appendix A. Also by optimizing the coverage area according to actual user distribution within the cellular network, cell congestion can be reduced.

Power model: Analyses of data traffic in wireless networks reveal that for current network design and operation, the power consumption is mostly insensitive and can be considered independent of the traffic load carried by the network (Auer *et al.*, 2011; Micallef *et al.*, 2012; Skillermark *et al.*, 2011). Assuming static power consumption across all traffic loads, the average power consumption of a base station P_{ci} can be defined as Richter *et al.* (2009):

$$P_{ci} = N_{sec} N_{ant} (A_i P_{tx} + B_i + P_{BH_i}) \quad (6)$$

where, N_{sec} and N_{ant} denote the BS's number of sectors and the number of antennas per sector, respectively. P_{ci}

is the average total power per base station and P_{tx} is the power fed to the antenna as defined in (1). The coefficient A_i accounts for the part of the power consumption that is proportional to the transmitted power (e.g., Radio Frequency (RF) amplifier power including feeder losses), while B_i denotes the power that is consumed independent of the average transmit power (e.g., signal processing, site cooling). These factors are the most important elements that provide energy efficiency within the stations (Bambos and Rulnick, 1997). For small cell site which has an optical Small-form Factor Pluggable (SFP) interface connected to an Ethernet switch port at the aggregation site, a coefficient P_{BHi} is added to represent the power consumption of the SFP used to transmit over the backhauling fiber if exist (Tombaz *et al.*, 2011). In order to evaluate the power consumption of the network corresponding to its size and for a given cell power consumption P_C , the APC is used and can be expressed as Richter *et al.* (2009):

$$APC = \frac{P_C}{AC} \quad (7)$$

where, AC is the cell area

Energy efficiency model : Energy Efficiency (EE) is defined as the ratio of the total amount data delivered and total power consumed PC_T (Chockalingam and Zorzi, 1998), which is expressed as:

$$EE = \frac{\text{Overall data rate}}{\text{Total power consumed}} = \frac{R_T}{PC_T} \quad (8)$$

The unit of EE is then bits per sec per Watt, namely bits per Joule, which has been frequently used in literature for energy efficient communications. Bit/Joule is expected as the main EE metric for next generation cellular systems and beyond (Tao *et al.*, 2010). R_T is the overall data rate which can be defined for one user as:

$$R_n = \sum_{k=1}^K r_n^k \quad (9)$$

where, K is the total number subchannels assigned for user n (in LTE, subchannels can be considered as resource blocks, RB), hence the total data rate for all users can be written as Miao *et al.* (2011):

$$R_T = \sum_{n=1}^N R_n \quad (10)$$

Since the maximum data for each user is a function of the received power, i.e.,

$$R_n = BW_n \log_2 \left(1 + \frac{Pr x_n}{IN_n} \right) \quad (11)$$

where, $Pr x_n$ is the received signal and IN_n represents interference plus noise and BW_n is the assigned bandwidth for the n th user. The Shannon capacity bound in Eq. (11) cannot be reached in practice due to several limitations in the implementation. To represent these loss mechanisms, the following modified Shannon capacity formula for LTE is used (Mogensen *et al.*, 2007):

$$R_n = \eta_{BW} \eta_{SNR} BW_n \log_2 \left(1 + \frac{SNR_n}{\eta_{SNR}} \right) \quad (12)$$

where, η_{BW} accounts for the system bandwidth efficiency of LTE and η_{SNR} accounts for the SNR implementation efficiency of LTE. LTE generally achieves 1.6~2 dB lower than the Shannon capacity bound because η_{SNR} is not constant and it changes with the geometry factor (G-factor). It was shown that this impact can be accounted for using the fudge factor, η , multiplied with the η_{BW} parameter. The factor η is a correction factor which nominally should be equal to one (Harri and Antti, 2009). Eq. (12) can also be rewritten as:

$$R_n = \eta_{BW} \eta_{RB,n} BW_{RB} \log_2 \left(1 + \frac{SNR_n}{\eta_{SNR}} \right) \quad (13)$$

where, $N_{RB,n}$ is the number of allocated RBs to the n th user and BW_{RB} is the RB bandwidth (Quintero, 2008). The resource block is the smallest entity that can be scheduled in the frequency domain (in downlink and uplink). This means in the frequency domain, one UE can receive or transmit in one resource block or integer multiples of one resource blocks only. In other words, it is not possible to assign less than 12 subcarriers to one terminal. The resource block size of 12 subcarriers (180 kHz) is the same for all bandwidths. A feasible starting point is to allocate equal RBs to each user, i.e. $RB_k = N_{RB}/N$ and the sum of RBs allocated to all users equals N_{RB} :

$$\sum_{k=1}^K RB_k = N_{RB} \quad (14)$$

For a 20 MHz channel, there are 100 Resource Blocks (RB) that can carry user traffic. This means the BS can theoretically serve a maximum number of 100 simultaneous users every 1 ms interval. In practice, different users will have different dynamic bandwidth requirements so that the number will be less 100. For simplicity, it is assumed that each RB is assigned for one user. Let $\eta_i = \eta_{BW} \eta_{RB,n}$ and substitute (1) in (11):

$$R_n = \eta_i N_{RB,n} \log_2 \left(1 + \frac{P_{tx} - ALF_n}{IN_n} \right) \quad (15)$$

Hence, the total data rate of all users within a specific BS, i can be written as:

$$R_{T,i} = \eta_i \sum_{n=1}^N N_{RB,n} \log_2 \left(1 + \frac{P_{Tx} - ALF_n}{IN_n} \right) \quad (16)$$

From here the EE of a specific base station with consumed power P_c can then be written as:

$$EE_i = \frac{R_{T,i}}{P_{c_i}} \quad (17)$$

Therefore, the EE of a HetNet with one macro BS, M micro BSs and P pico BSs can be written as:

$$EE_{Het} = \frac{R_{macro} + \sum_{m=1}^M R_{micro,m} + \sum_{p=1}^P R_{pico,p}}{P_{c_{macro}} + \sum_{m=1}^M P_{c_{micro,m}} + \sum_{p=1}^P P_{c_{pico,p}}} \quad (18)$$

In order to assess the EE the network relative to its size, the notion of Area Energy Efficiency (AEE) which is defined as the bit/Joule/unit area is introduced. The AEE for a certain base station can be expressed as:

$$AEE_i = \frac{EE_i}{A_{BS,i}} \quad (19)$$

where, EE_i and $A_{BS,i}$ denote the energy efficiency in bit/Joule and the area covered by a certain station with the unit of km^2 respectively (Wang and Shen, 2010). The AEE of HetNet can be defined as the number of bits delivered per Joule of energy per unit area supported by the whole network. The area energy efficiency of heterogeneous networks AEE_{Het} can be expressed as:

$$AEE_{Het} = \frac{EE_{Het}}{A_{Het}} \quad (20)$$

where, A_{Het} denotes the total area of the heterogeneous cellular network.

RESULTS AND ANALYSES

In the following sections, the simulation setup and the results are analyzed.

Simulation setup: A straightforward model for HetNet would consist of a mix of macro, micro and pico BSs as shown in Fig. 1. BSs are uniformly placed in the area according to the cell radius; where each BS may differ in terms of transmit power, achievable data rate, coverage and EE. The cell radius for each type of BS is calculated based on a cell coverage requirement of $C = 95\%$ by setting the transmit powers for macro, micro and pico to 46, 35 and 30dBm, respectively. UEs are assumed to be uniformly distributed within the cell and the bandwidth for the investigated LTE downlink scenario is set to 10 MHz at carrier frequency of 2.6 GHz. It is worth mentioning that the path loss model

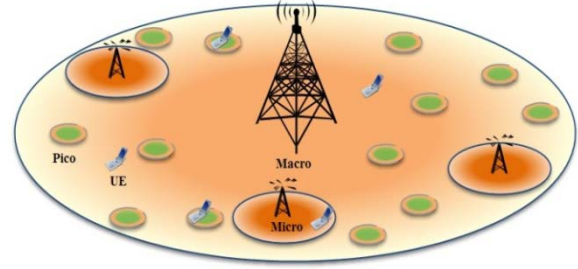


Fig. 1: Network Architecture

Table 1: Simulation Parameters

Parameter	Value
Carrier frequency	2.6 GHz
Bandwidth	10MHz
P_{Tx} for macro, micro and pico BS	46dBm, 35dBm, 30dBm
Channel model	3GPP Typical Urban
Penetration loss	20 dB
MCS	15 different sets defined in (Stefania <i>et al.</i> , 2011)
No. of sectors for macro, micro and pico	3,1,1
Thermal noise	-174dBm/Hz
Noise figure	9 dB
No. of resource blocks	50
Coverage degree, C	95%
Power consumption parameters	macro: $A_i = 21.45$; $B_i = 354.44$ micro: $A_i = 7.84$; $B_i = 71.5$ pico: $A_i = 5.5$; $B_i = 38$

follows that in 3GPP TR 25.814 (3GPP, 2009). The QPSK 1/3 is used in the simulation to represent the MCS at the cell edge. Other parameters used in the simulation can be found in Table 1. The micro and pico BSs are located in macro BS coverage area for capacity enhancement without overlapping between them (i.e., $M \cdot A_{micro} + P \cdot A_{pico} \leq A_{macro} = A_{Het}$ where, A_{macro} , A_{micro} and A_{pico} are the areas of macro, micro and pico BS respectively). The cell edge data rate as well as EE for each BS has been calculated for one UE assuming that all the resources are allocated to that UE.

Simulation results: In this section, some simulation results to verify the theoretical analysis and the effectiveness of the suggested approaches are presented. The performance of the HetNet from EE perspective is presented. The proposed model for energy efficiency of the heterogeneous network has been verified through extensive simulations under various practical configurations. The methodology for evaluating the energy efficiency, considering heterogeneous cellular network can be more understood through the flow chart in Fig. 2.

Effects on area power consumption: The area power consumption of a network with different types of BSs is shown in Fig. 3. In order to achieve $C=95\%$ in correspondence to their transmitted powers, the radius of macro, micro BS and pico BS are found to be

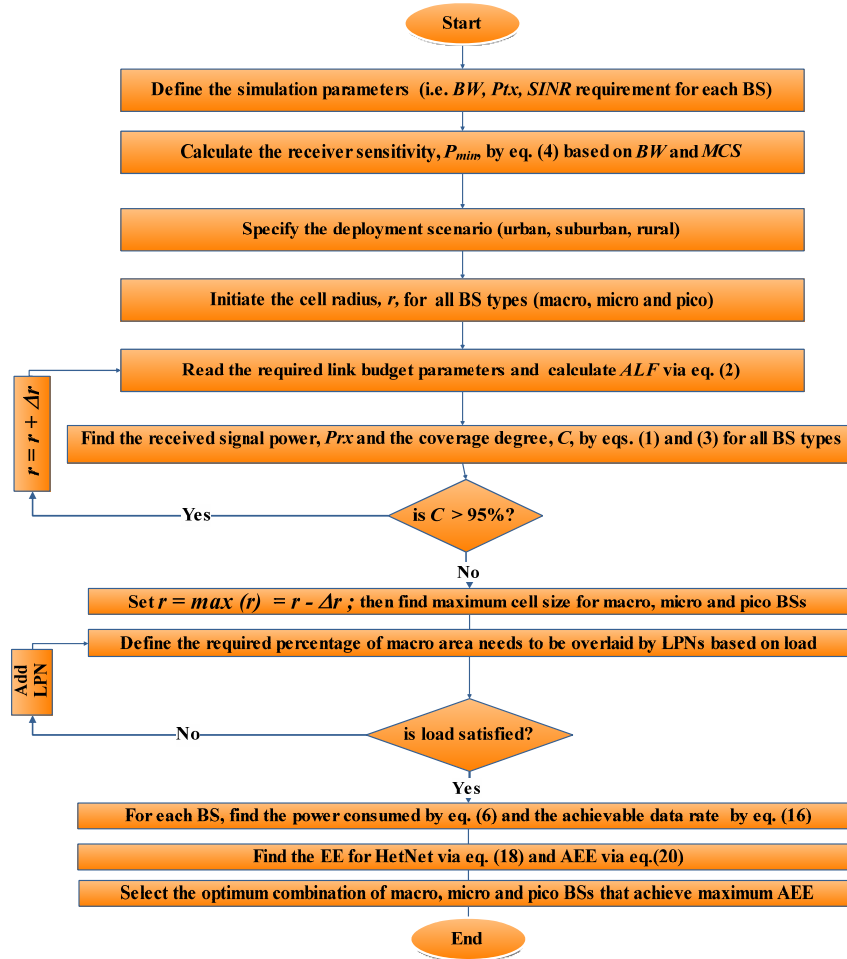


Fig. 2: Flow chart of EE Evaluation Methodology

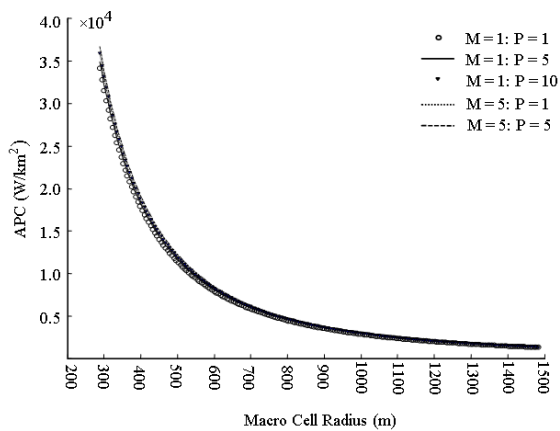


Fig. 3: Area power consumption as a function of macrocell radius for different deployments

1484m, 570m and 150m, respectively. It is clear that the APC decreases when the cell radius of macro Bs (or inter site distance) increases. The significant point in this plot is that there are some curves are much closer

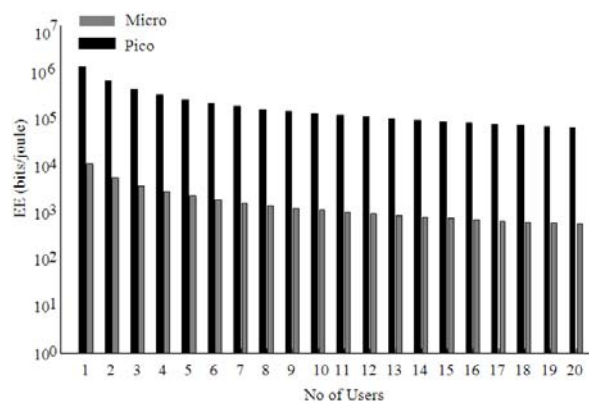


Fig. 4: Energy efficiency of microcell and picocell at cell edge

together. On other hand, the pico BS provides better bit per joules than micro BS within the coverage area. For instance, the energy efficiency of pico BS is greater than the micro BS at their cell edge as shown in Fig. 4. Hence, it can be inferred that the APC metric cannot

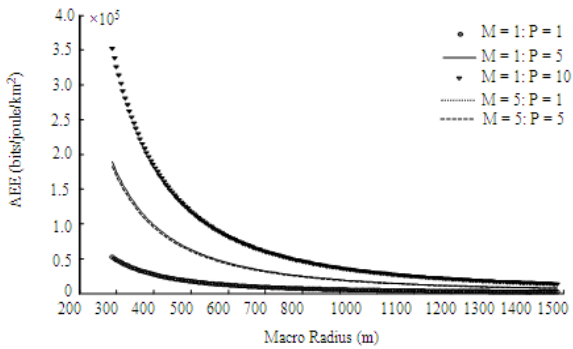


Fig. 5: Area Energy efficiency as a function of macrocell radius for different deployments

capture all the EE characteristics in heterogeneous cellular networks and cannot be the exclusive metric describing the EE since it does not take into account the provided additional network capacity and higher system spectral efficiency. Nevertheless, this metric makes it possible to evaluate different network topologies with similar performance figures with regard to EE.

Effects on area energy efficiency: While there are different cell sizes in a HetNet which comprises of macro, micro and picocells, there are different data rates for each base station according to its size and therefore EEs.

Moreover, the coverage area of each cell varies according to the implementation environment. Thus, the Area Energy Efficiency (*AEE*) is used to evaluate the EE of HetNet relative to its size. The EE of the network corresponding to its size and deployment can be assessed by comparing *AEE* performance under

different sector radius and scenarios. The *AEE* of HetNet as a function of macro BS' radius with different deployments is shown in Fig. 5. It is clearly shown that *AEE* decreases as the macrocell BS' radius increases. When considering the *APC* metric, the difference of *AEE* between a HetNet with 5 micros and one pico and HetNet with one micro and 10 picos becomes evident. For instance, at macro cell radius of 1000 m, the *AEE* of the network with 5 micros and one pico is about 4340 bites/joule/km² while for one micro and 10 picos the *AEE* is equal to 29347 bites/joule/km² which is greater than the previous scenario. The number of picocell in HetNet has a more dramatic impact on the *AEE* of the network as compared with microcell because picocells provide data rate within their coverage higher than the macro and micro BSs.

In addition, picocells provide better cell-edge performance than large cells because of their size are much more power efficient in providing broadband coverage. The *AEE* improves as the number of microcells and picocells increases this is because the capacity improves as the number of pico stations increases. The *AEE* as a function of the number of micro and picos is shown in Fig. 6. It is obvious that the *AEE* improves as the number of picos increases. Also, for each combination of micro and pico BSs, there is a certain *AEE*. Moreover, as the microcell is larger than a picocell, the number of picos is more influencing on *AEE* than the micros and maximum the *AEE* is achieved with minimum number of micros and maximum number of pico BS. However, the number of micros and picos overlay traditional macrocell networks should be chosen according to the required percentage of the macro BS area which needed to be covered and according to the user density.

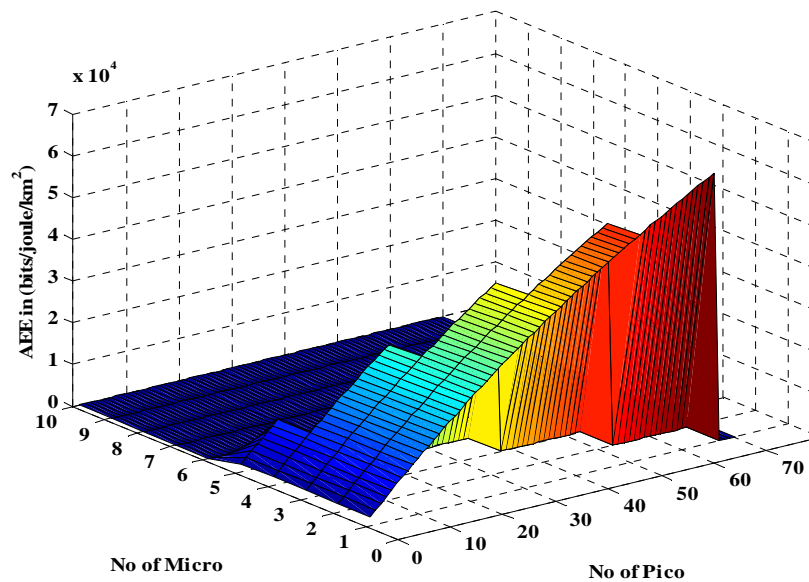


Fig. 6: Area Energy efficiency as a function of number of microcells and picocells

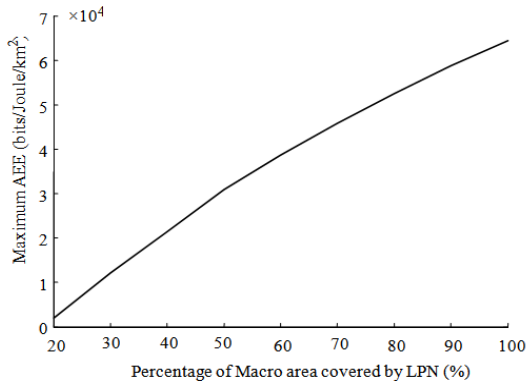


Fig. 7: AEE as a function of the macro area percentage covered by LPN

Table 2: Possible maximum number of micros and picos and the corresponding AEE and number of users when 100% of macro area is overlaid by LPN

No. of Micros	No. of Picos	No. of possible simultaneous users	AEE
1	66	3400	64439
2	52	2750	53574
3	37	2050	40611
4	23	1400	26986
5	8	700	10478

Table 3: AEE as a function of LPN number for different percentage of macro covered area by LPN

Parameter	Percentage of macro area covered by LPN			
	<= 25%	<= 50%	<= 75%	<= 100%
No. of Micros	1	1	1	1
No. of Picos	5	26	46	66
Maximum AEE	7165	30929	49244	64439
No. of Possible simultaneous	350	1400	2400	3400

The *AEE* of the heterogeneous networks is plotted in Fig. 7 in correspondence to different percentage of overlapping i.e. the needed percentage of the macro coverage area to be overlaid with multiple smaller cells or LPN.

It can be observed that *AEE* increases as the percentage area of macro BS overlaid by LPN increases. For example, the maximum *AEE* is less than 1973 (bits/joule/km²) when 20% of macro coverage area overlaid by LPN while the *AEE* becomes 58682 (bits/joule/km²) when the coverage area of the macro BS is around 90%. Table 2. shows all the possible maximum number of micros and picos that can be deployed to the macro area and the corresponding *AEE* of the whole network. From Table 2 it can be deduced that *AEE* decreases as the number of micro increases and the number of pico decreases. Basically the maximum *AEE* can be achieved with a minimum number of micros and maximum number of picos. In other hand, *AEE* decreases as the required percentage of macro area decrease, the maximum *AEE* as well as the maximum number of simultaneous users can be obtained when the original macro area is 100% filled by small cells or LPNs as it is demonstrated in Table 3.

CONCLUSION

In this study, the Energy Efficiency (EE) of a heterogeneous cellular network HetNet has been investigated, particularly for a LTE network of macrocell base stations overlaid by randomly distributed microcells and picocells. The main contribution is the simulation results which are reinforced by the theoretical expressions that reinforce the key deployment solutions. First, the analytical expressions of the EE have been derived. Then the network performance in terms of EE has been evaluated. From the numerical results which support the analytical framework, it can be concluded that there exists an optimal micro-pico per macro density that maximizes the overall EE of HetNet. It has been shown that the *APC* metric cannot be the exclusive metric describing the EE in HetNet since it does not take into account the provided higher network capacity. The picocells have a dramatic impact on the AEE of the network as compared with microcell because the picocells provide higher data rate within their coverage than the macro and micro BS. Also, there is an optimal number of microcells and picocells per macro BS that achieve the maximum AEE. Moreover, the AEE of the HetNet increases as the percentages area of macro BS overlaid by smaller cells is increases. Using this model, the EE of different deployment scenarios involving HetNet can be obtained and hence providing insights on how to deploy a greener HetNet. Interference issues are not taken into account in this study, which needs to be addressed in future works.

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Appendix A: In this appendix, the derivation of the analytical form of the expected percentage of locations within a cell where the received power at these locations is above a given minimum has been given. Precisely, formula (2) has been derived. The aim is to prove that:

$$C = \frac{2}{R^2} \int_0^R r Q(a + b \ln(\frac{r}{R})) dr = Q(a) + \exp\left(\frac{2-2ab}{b^2}\right) Q\left(\frac{2-ab}{b}\right) \quad (1)$$

where,

$$a = \frac{P_{\min} - P_{rx}(R)}{\sigma_{\Psi dB}}, \quad b = \frac{10 \alpha \log_{10}(e)}{\sigma_{\Psi dB}}$$

The received power at cell edge can be expressed as:

$$P_R = P_{tx} + 10 \log_{10} K - 10 \alpha \log_{10}\left(\frac{R}{r_o}\right) \quad (2)$$

$$Q(a) = \frac{1}{\sqrt{2\pi}} \int_a^\infty \exp\left(-\frac{x^2}{2}\right) dx = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{a}{\sqrt{2}}\right). \quad (3)$$

Now by using the following substitution: $y = a = b \ln(r/R)$
Therefore, at $r = 0 \rightarrow y = -\infty$, also at $r = R \rightarrow y = a$. Moreover we
have $r = R \exp(y - a/b)$ and $dr = R/b \exp(y - a/b) dy$. Substituting all
these elements in the first line of Eq. (1) the result will be arrived at:

$$Q(a) = \frac{2}{b} \int_{-\infty}^a \exp\left[\frac{2(y-a)}{b}\right] Q(y) dy \quad (4)$$

Next, using integration by parts, which can be explained as follows: For any two differentiable functions u, v :

$$\int_{x_0}^{x_1} u dv = u v \Big|_{x_0}^{x_1} - \int_{x_0}^{x_1} v du \quad (5)$$

In order to apply the rule (5) to (4), it can be assumed that $u = Q(y)$ and $v = (b/2) \exp(2y/b)$:

$$\begin{aligned} C &= \exp\left[\frac{-2a}{b}\right] \int_{-\infty}^a Q(y) d \exp\left(\frac{2y}{b}\right) = \exp\left[\frac{-2a}{b}\right] \left\{ Q(y) \exp\left(\frac{2y}{b}\right) \Big|_{-\infty}^a - \int_{-\infty}^a \exp\left(\frac{2y}{b}\right) dQ(y) \right\} \\ &= \exp\left[\frac{-2a}{b}\right] \left\{ Q(a) \exp\left(\frac{2a}{b}\right) - \int_{-\infty}^a \exp\left(\frac{2y}{b}\right) dQ(y) \right\} \quad (6) \end{aligned}$$

It is worth reminding that $\exp(-1) = 0$. The next task is to find the last integral in (6). For doing so, next step is to find the explicit form of $dQ(y)$ as:

$$\begin{aligned} dQ(y) &= \frac{d}{dy} Q(y) dy = \frac{d}{dy} \left[\frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{y}{\sqrt{2}}\right) \right] dy \\ &= -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy \quad (7) \end{aligned}$$

Where the following identity has been used:

$$\frac{d}{dy} \operatorname{erf}(y) = \frac{1}{\sqrt{\pi}} \exp(-y^2) \quad (8)$$

By substituting (7) into (6), the following can be obtained:

$$C = Q(a) + \frac{\exp\left(\frac{-2a}{b}\right)}{\sqrt{2\pi}} \int_{-\infty}^a \exp\left[-\frac{1}{2}\left(y^2 - \frac{4y}{b}\right)\right] dy \quad (9)$$

It is easy to prove:

$$\frac{1}{2}\left(y^2 - \frac{4y}{b}\right) = \frac{1}{2}\left(y - \frac{2}{b}\right)^2 - \frac{2}{b^2}$$

Therefore,

$$C = Q(a) + \frac{\exp\left(\frac{-2a}{b}\right)}{\sqrt{2\pi}} \exp\left(\frac{2}{b^2}\right) \int_{-\infty}^a \exp\left[-\frac{1}{2}\left(y - \frac{2}{b}\right)^2\right] dy \quad (10)$$

For the last integral we substitute $\beta = -(y - 2/b)$. This leads to that at $y = -\infty \rightarrow \beta = \infty$; at $y = a \rightarrow \beta = (2-ab)/b$. Also $dy = -d\beta$. Invoking all these elements into (10) we obtain:

$$C = Q(a) + \frac{\exp\left(\frac{2-2a}{b^2}\right)}{\sqrt{2\pi}} \int_{\infty}^{\frac{2-2a}{b}} \exp\left(-\frac{\beta^2}{2}\right) d\beta \quad (11)$$

For the last integral, by means of the definition of Q function (2) it can be simplified as:

$$\int_{\infty}^{\frac{2-2a}{b}} \exp\left(-\frac{\beta^2}{2}\right) d\beta = - \int_{\frac{2-2a}{b}}^{\infty} \exp\left(-\frac{\beta^2}{2}\right) d\beta = -\sqrt{2\pi} Q\left(-\frac{2-ab}{b}\right) \quad (12)$$

By substituting (12) into (11) relation (1) can be obtained. The Q -function is not an elementary function. However, the bounds:

$$\frac{x}{1+x^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-x^2/2} < Q(x) < \frac{1}{x} \cdot \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, x > 0 \quad (13)$$

Become increasingly tight for large x and are often useful. Applying these inequalities for (1) the following formula can be obtained:

$$\frac{2 \exp(-a^2/2)}{(1+a^2)[b^2 + (2-ab)^2] \sqrt{2\pi}} < C < \frac{2 \exp(-a^2/2)}{a(2-ab) \sqrt{2\pi}}, \frac{2}{b} > a > 0$$

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