CSE578: Computer Vision

3D Reconstruction:
Structure from Motion using Bundle Adjustment

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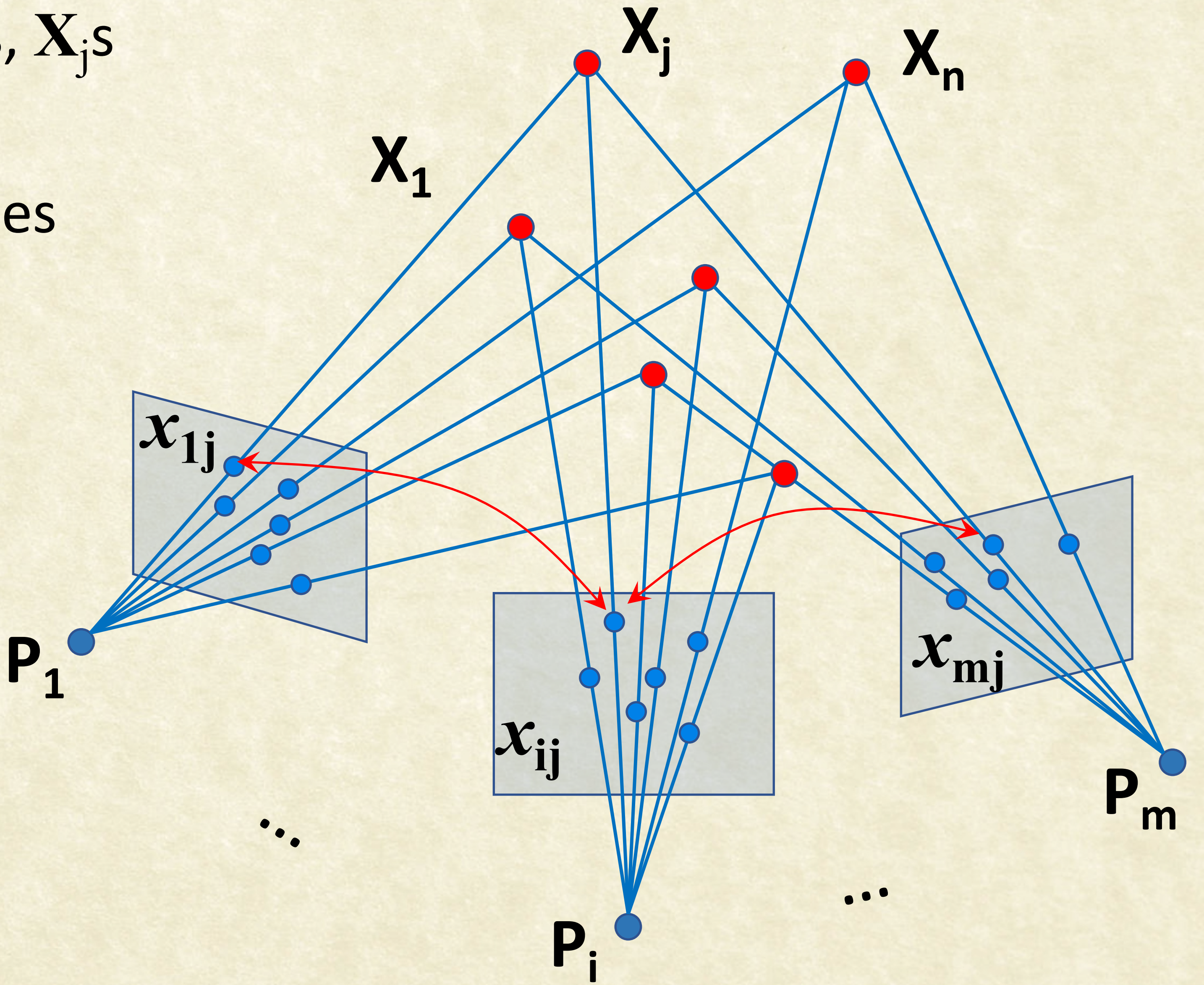
Points in Multiple Views

- Unknowns: $\mathbf{P}_i$s, $\mathbf{X}_j$s
- Knowns: x_{ij}s, Correspondences



Structure from Motion

- m cameras and n points, with correspondences
- Unknown: m matrices $\mathbf{P}_i$ and n coordinates, $\mathbf{X}_j$
- We have: $\boldsymbol{x}_{ij} = \mathbf{P}_i\mathbf{X}_j$, $1 \leq i \leq m$, $1 \leq j \leq n$
- $2mn$ equations in total (2 for each visible point)
- Can be solved if $2mn > 11m + 3n$
- However, under projective transformation, $\mathbf{PX} = (\mathbf{PQ})(\mathbf{Q}^{-1}\mathbf{X})$, a projective ambiguity will remain
- Projective structure if $2mn > 11m + 3n - 15$
- Affine structure if $2mn > 11m + 3n - 12$
- Metric structure if $2mn > 11m + 3n - 7$
- Affine/Metric structure only by enforcing affine/metric constraints

Bundle Adjustment

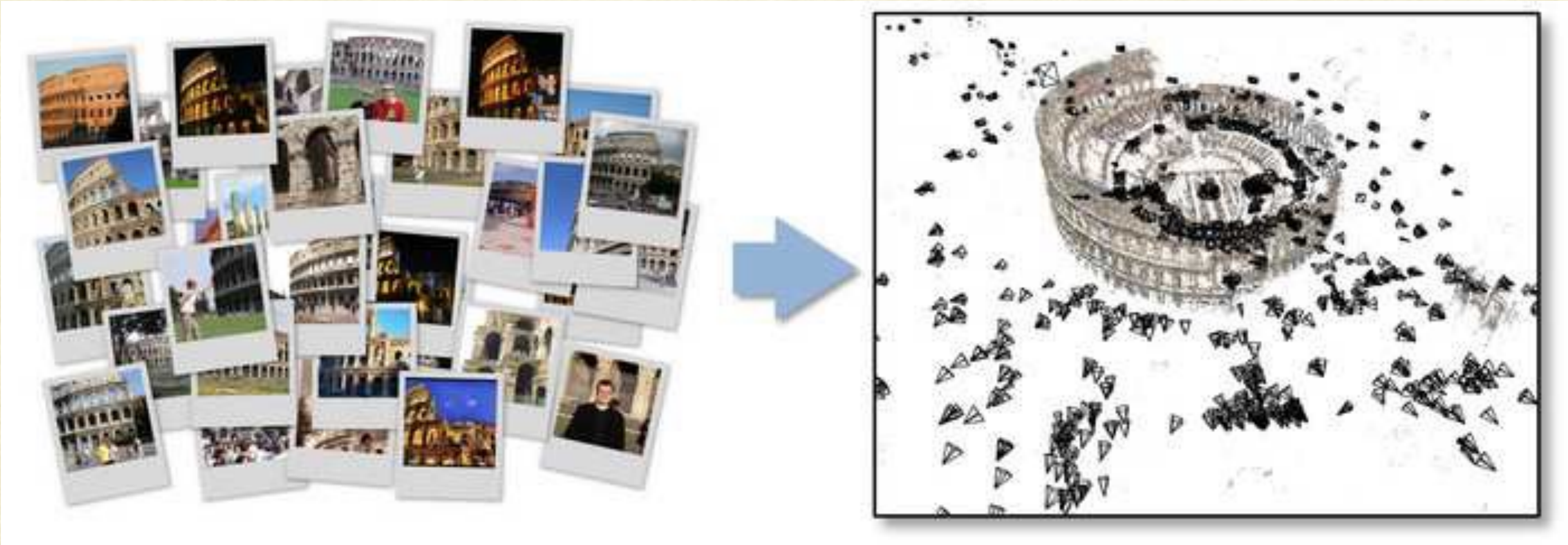
- Given m views of n 3D points, with unknown $\mathbf{P}_i$ and $\mathbf{X}_j$. Ideally, $\boldsymbol{x}_{ij} = \mathbf{P}_i\mathbf{X}_j$
- Minimize the re-projection error over all cameras/views:

$$\min_{P_i X_j} \sum_{i=1}^{m} \sum_{i=1}^{n} dist(\boldsymbol{x}_{\boldsymbol{ij}}, P_i \boldsymbol{X}_j)^2$$

- A non-linear optimization problem. Can be solved using the Levenberg-Marquardt procedure directly.
- Called bundle adjustment. Known to photogrammetry community for a long time
- Needs good initialization as the complex non-linear optimization problem can get stuck in local minima

Photo Tourism or Photo-Synth

- An automatic process, starting with independent images of a scene/monument/object. The images could be from a video sequence.
- Of particular interest has been SfM from Community Photo Collections (CPC), which are images that can be downloaded from flickr/shutterfly by giving a keyword like "Taj Mahal".



SfM Steps

1. Download images for the place of interest!!
2. Extract descriptors from interest points on all images
3. Match points in pairs of images using Approximate Nearest Neighbours
4. Refine matches using Geometric Verification: Epipolar constraint, etc.
5. Form tracks of points across images. Transitively connect matches to get long matching “tracks”
6. Build image connectivity graph based on common points
7. Perform incremental SfM using the image connectivity graph and bundle adjustment

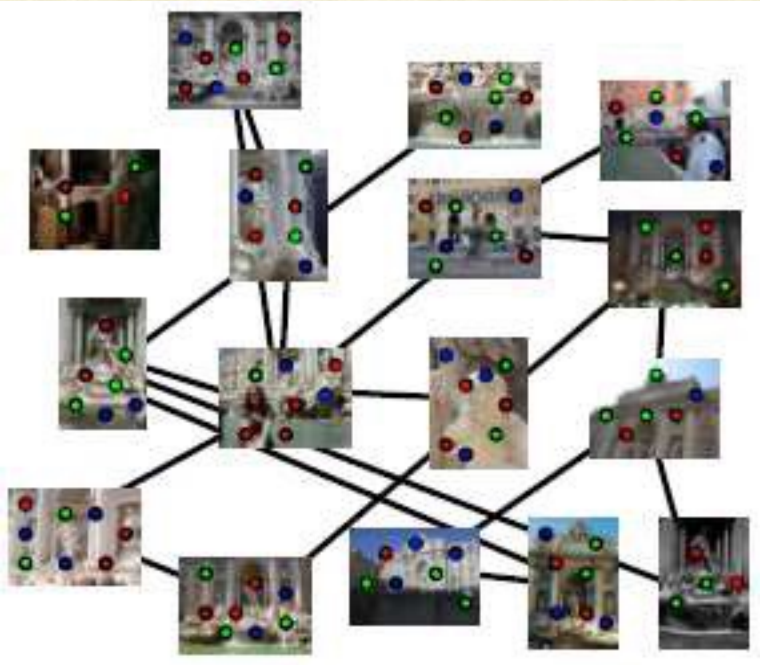
Matching Points across Images

- Extract interest points x_{ij} in each image I_i and descriptors s_{ij} for it. SIFT is popular. A few thousand in a typical image.
- Match interest points in image pairs. An approximate nearest neighbour approach is used, with a ratio test
- Point x_{ij} matches point x_{kl} iff:
  1. $\mathrm{dist}(s_{ij}, s_{kl})$ is minimum over all points in I_k and
  2. $\mathrm{dist}(s_{ij}, s_{kl}) < r \times \mathrm{dist}(s_{ij}, s_{km})$ where x_{km} is the second closest point in I_k. r is typically 0.6
- Discard all points involved in case of multiple matches

Geometric Verification and Tracks

- Find fundamental matrix between pairs of cameras using RANSAC
- Refine matches by eliminating those not satisfying epipolar relation
- Propagate matches using transitivity to generate tracks, which represent the same world point in multiple images
- Form image connectivity graph. Two images have an edge if they share a point
- Densely connected regions represent parts of the scene that are visible to a large number of views. Sparse, leaf regions denote low sampling of parts of the scene

Features and Graph Match



Form **tracks** of points by transitively connecting matches.
They represent the same 3D point in multiple images.

Track Statistics

- For a typical large data set with approximately 3000 images:
  - 1.5 million tracks
  - 75-80% with of length 2
  - 98% of length less than 10
  - A few tracks of length more than 100
- Remember: Only 2D feature matching and verification has been done so far, but we seem to have come far!!

Structure from Motion

- 11m + 3n parameters for m cameras and n points. 2mn equations mapping each point in each camera
- Recover camera and structure. Minimize reprojection error across all of them using a non-linear minimization step. This needs good initialization
- Not possible to do them all together. So, start with one pair of cameras and incrementally add more cameras
- Adjust points and cameras to reduce global reprojection error after new cameras are added

Modern digital cameras store a lot of metadata in the images as EXIF tags.

Assume: Only focal length is the unknown intrinsic parameter!

Incremental SfM

- Find a strong starting pair of cameras. These should have a large number of points in common and a large baseline
- Find a pair with a large number of matches. Compute a planar homography from the matches. The pair is good if the error from the homography is high!
- Select the pair with the lowest percentage of inliers to homography using RANSAC
- Estimate the essential matrix for the camera pair
- Reconstruct cameras and common points using the essential matrix
- Perform bundle adjustment to minimize reprojection error

Adding Views

- While there are more connected cameras
  - Pick an image that sees the highest number of 3D points so far
  - Estimate pose of the camera using DLT and known 3D points. Perform a local bundle adjustment to correct new camera pose
  - Triangulate new points (if any) and add to the collection
  - Perform a global bundle adjustment on all cameras and points, using a non-linear optimization step
- Can remove outlier tracks altogether
- Can add a small group of camera views together, instead of one at a time

Bundle Adjustment

- Find P, X that minimizes (with visibility indicator w_{ij})

$$g(P,X)=\sum_{i}^{m}\sum_{j}^{n}w_{ij}\left\|x_{ij}-P_iX_j\right\|^2$$

- Write it as $g(P,X)=\|A-\mathrm{C}(P,X)\|^2$, where C is the non-linear camera projection function
- Linearize
- Iterative solution using Levenberg-Marquardt method
- A sparse problem as the indicator w_{ij} of point j being visible in camera/image i is sparse.

Practical Aspects

- Heavy computations. Several days to reconstruct 500 images. About half of that time is for the bundle adjustment step
- Several optimizations have been worked on in the past.
- Interesting papers:
  - “Building Rome in a Day”, ICCV 2009 (U of W)
  - “Building Rome on a Cloudless Day”, ECCV 2010 (UNC)
  - “Reconstructing the World in Six Days (As Captured by the Yahoo 100 Million Image Dataset)”, CVPR 2015 (UNC)
  - “Pixel-Perfect SFM with Featuremetric Refinement”, ICCV 2021 (ETH-Z)
- Combinatorics of pairwise matching is also huge. Use image search approaches to reduce the potential numbers

Building Rome in a Day

Agarwal, Simon, Seitz, Szeliski. ICCV 2009

- Over a million images of the city of Rome
- Pair-wise matching can take 15 years at 2 pairs/sec
- Find 40 most similar words (fast matching)
- Query expansion to increase graph density
- Full bundle adjustment may run till end of time (nearly!)
- Use skeletal graphs to capture overall structure; perform bundle adjustment in local clusters
- Reconstructed Rome in 24 hours on a 1000-node cluster!!

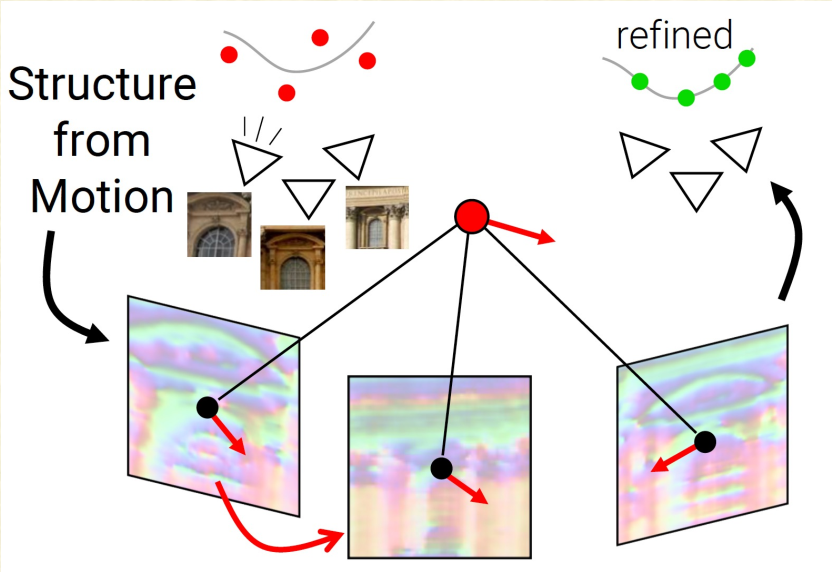
- A local experiment on a 400-image Hampi dataset:
  - Extracting SIFT: 54 minutes,
  - Image matching: 17.2 hrs
  - Bundle adjustment: 12.6 hrs!!

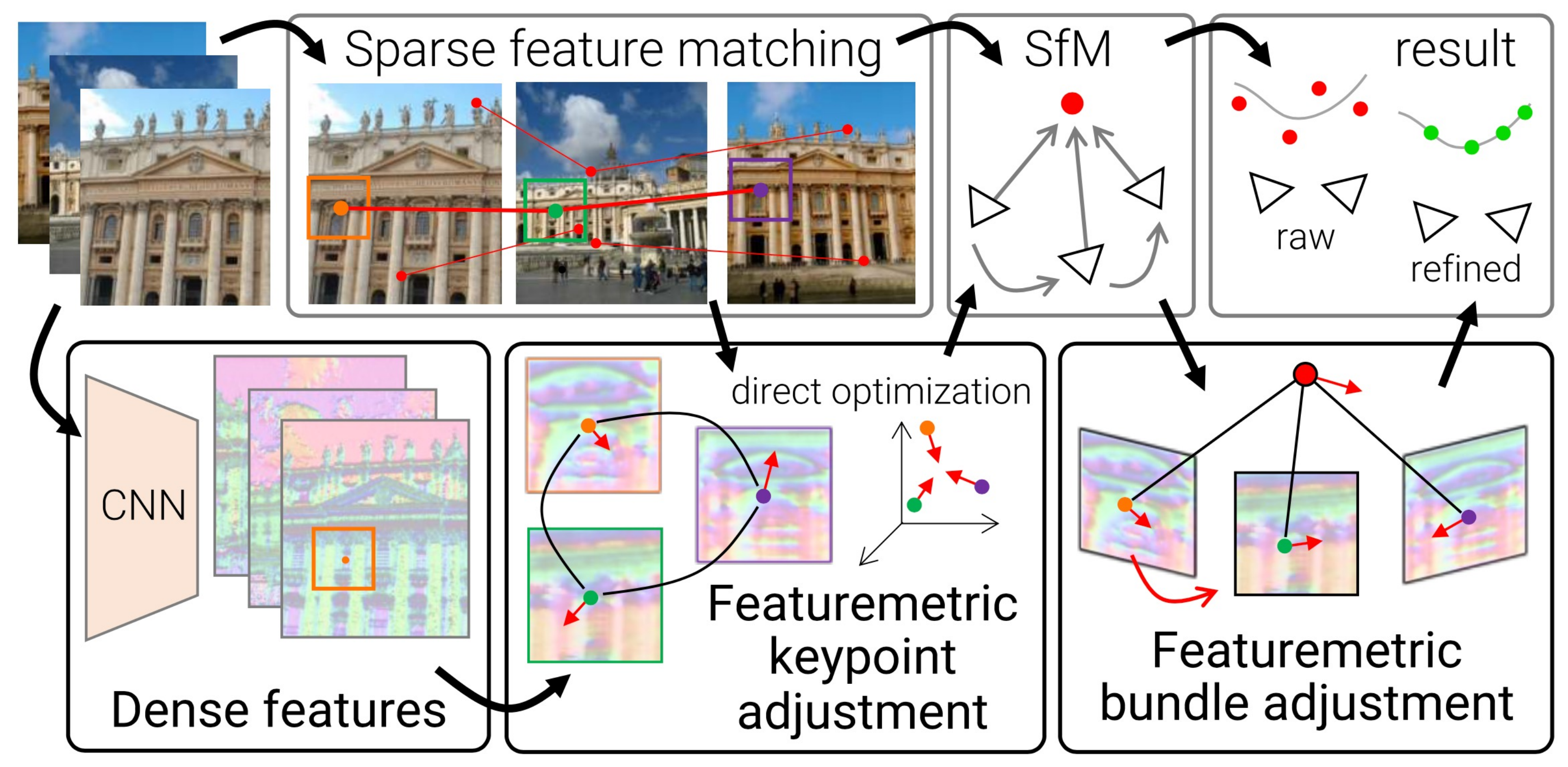
Rome Reconstructions

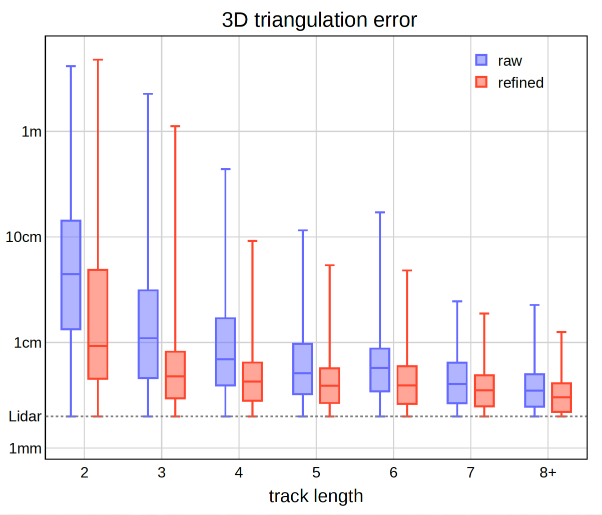
http://www.cs.cornell.edu/projects/bigsfm/

Pixel-Perfect SFM

- Use deep features for refining 2D & 3D points, and Camera locations
- The pipeline uses 2D key-point adjustment before SFM and 3D point and camera location refinement after SFM







Paper and Code:

https://github.com/cvg/pixel-perfect-sfm

3D Reconstruction in Practice

A few practical setups

Studios to Record Events

- Virtualized Reality (CMU, 1995)
  - A 51-camera recording setup
  - Off-line digitization
  - Multi-baseline stereo
  - Merge depth maps to get structure
- Free-Viewpoint Video (ETH, MPI)
  - Multicamera setup, all digital
  - Visual hull for quick structure
- 123D from Autodesk
  - Submit your photographs
  - Get a 3D model!!

Structure Recovery: Conclusions

- A problem that has been solved somewhat well
- Many challenges remain, but many have been tackled
- Next generation movies: Watch it from a viewpoint of your choice, decided at view time!!
- Integrating structure recovered by geometry-based techniques such as SFM and appearance-based techniques such as shape from shading and single-image reconstruction are promising.

Resources

- Lecture 10 from:
  - https://www.youtube.com/channel/UC8wqMjG6rQNN_1EGLmOfNnA
- Multiple-View Geometry: Lectures by Prof. Daniel Cremers
  - https://www.youtube.com/watch?v=RDkwklFGMfo&list=PLTBdjV_4f-EJn6udZ34tht9EVIW7lbeo4
- Structure from Motion: First Principles of CV course by Shree Nayar
  - https://fpcv.cs.columbia.edu
- Other Related Topics:
  - SLAM: Simultaneous Localization and Mapping (Robotics)
- Code:
  - VisualSFM: http://ccwu.me/vsfm/
  - COLMAP: https://colmap.github.io

Questions?