Introduction:

In mathematics, a system of linear equations is a collection of equations in which each equation is a linear combination of the same variables. These systems are fundamental in various areas of mathematics and science, providing a framework for solving problems involving multiple linear relationships.

A system of linear equations can be represented in matrix form as A = B, where:

- A is the coefficient matrix containing the coefficients of the variables,
- x is the column vector containing the variables,
- B is the column vector containing the constants.

Example:

Consider the following system of linear equations:

$$\begin{cases} x + 2y = 5 \\ 3x + 4y = 6 \end{cases}$$

In matrix form, this system can be represented as:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Solving Systems of Linear Equations without Numpy:

Method 1: Substitution: We can solve the first equation for x and then substitute this expression into the second equation to solve for y.

From the first equation: $x = 5 - 2y$

Substituting this into the second equation:

$3(5 - 2y) + 4y = 6$

$15 - 6y + 4y = 6$

$15 - 2y = 6$

$-2y = 6 - 15$

$-2y = -9$

$y = -9/-2 = 4.5$

Now that we have found y, we can substitute it back into the first equation to find x:

$x = 5 - 2(4.5) = -4$

So, the solution is $x = -4$ and $y = 4.5$.

Method 2: Elimination: We can eliminate one variable by adding or subtracting multiples of the equations to create a new equation with only one variable.

Multiplying the first equation by 3 and the second equation by 1, we get:

$3x + 6y = 15$

$3x + 4y = 6$

Subtracting the second equation from the first equation:

$(3x + 6y) - (3x + 4y) = 15 - 6$

$2y = 9$

$y = 9 / 2 = 4.5$

Now that we have found y, we can substitute it into any of the original equations to find x. Let's use the first equation:

$x + 2(4.5) = 5$

$x + 9 = 5$

$x = 5 - 9 = -4$

So, the solution is $x = -4$ and $y = 4.5$.

Solving Systems of Linear Equations with Numpy:

Numpy, a powerful library for numerical computing in Python, provides the **'linalg.solve'** function to solve systems of linear equations efficiently.

Example Implementation:

```
import numpy as np

A = np.array([[1,2],[3,4]])

B= np.array([[5],[6]])

solution = np.linalg.solve(A,B)

print(solution)
```

Output

```
[[-4. ]

 [ 4.5]]
```

Interpretation: The solution obtained indicates that x=−4 and y=4.5 satisfy both equations of the system.

Generalization: The **'linalg.solve'** function can handle systems of linear equations with any number of variables and equations. It utilizes numerical methods based on matrix operations to find solutions efficiently.

Conclusion:
Systems of linear equations play a vital role in various fields of science and engineering. With the help of Numpy's **'linalg.solve'** function, solving such systems becomes straightforward and efficient in Python, enabling practitioners to tackle complex problems with ease. Understanding and mastering this concept opens doors to a wide range of applications across different domains.