

Oppenheim CH20BTECH11032

YASH K SHARMA

October 2022

1 Problem: 2.44(a)

Let $X(e^{j\omega})$ denote the Fourier Transform of the signal $x[n]$ shown in the figure P2.44-1. Evaluate $X(e^{j\omega})$ at $\omega = 0$.

Solution:

At $\omega = 0$,

$$X(e^{j\omega}) = \sum_{n=-\infty}^{n=+\infty} x[n]e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{n=+\infty} x$$

$$X(e^{j\omega}) = 6$$

2 Problem: 3.44(d)

When the input to an LTI system is

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}2^n u[-n-1]$$

the output is:

$$y[n] = \frac{1+z^{-1}}{(1-z^{-1})(1+\frac{1}{2}z^{-1})(1-2z^{-1})}$$

Is the system stable? Is it casual?

Solution:

The ROC is $\frac{1}{2} < |z| < 2$. Since $x(z)$ has poles at 0.5 and 2, the poles at 1 and -0.5 are due to $H(z)$.

Since $H(z)$ is casual, its ROC is $|z| > 1$.

$$H(z) = \frac{X(z)}{Y(z)} = 1 + \frac{\frac{2}{3}}{1-z^{-1}} + \frac{-\frac{2}{3}}{1+\frac{1}{2}z^{-1}}$$

Since $H(z)$ has a pole on the unit circle, the system is not stable.