Trajectory Optimization of a Wheelie

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***Abstract*—A wheelie is a maneuver in which the front wheel of a bike comes off the ground due to sufficient torque being applied about the rear wheel. We seek to understand what kinds of parameters are important to a wheelie by modeling a bike and performing trajectory optimization. The person on the bike was modeled with a point mass on an inverted pendulum and contact dynamics were implemented to allow rolling contact between the bike wheels and the ground. Two different trajectory optimization formulations were tested: direct transcription and hybrid collocation, with varying objectives and varying pendulum lengths and masses. Results indicated that there were less oscillations when costs were added, suggesting that an optimal wheelie would start with high input wheel and hip torques and then settle at a lower value. Results also suggested that shorter riders would need less hip torque to perform a wheelie.**

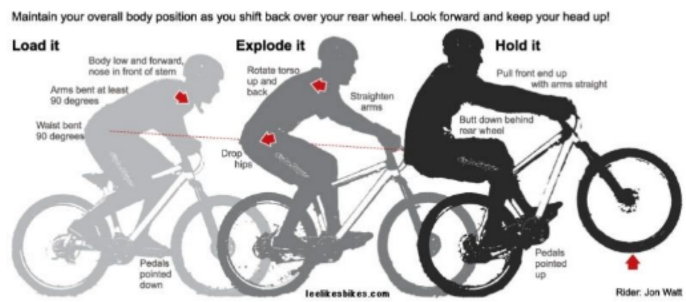

Fig. 1. Step by step process explaining how experts perform a wheelie [2]. Note that the hands remain outstretched during the wheelie and that the center of mass moves over the back wheel.

I. Introduction

Whether it be on television or right next to you on the road, we have all seen bikes performing a rather impressive bike trick known as a wheelie. A wheelie is a maneuver in which the front wheel of a bike comes off the ground due to sufficient torque being applied about the rear wheel. Although it may look simple, the rider is performing a precarious balancing act - balancing the forces and torques on the bike with their center of mass.

The premise of this project is to understand what kinds of parameters are important to a wheelie. There is little literature that methodically studies what contributes to an efficient wheelie. According to several sources [1] [2], when a human performs the trick, they load the bike by moving their center of mass forwards and then quickly backwards. They push their hips back while keeping their arms straight to keep their center of mass aligned with the back wheel (Fig. 1). This allows them to balance on the back wheel. We can say that to complete a wheelie, humans use two "actuators": their hips to control the popping of the front wheel and their legs to control the speed of the bike.

We wish to see how modeling a bike using Drake and optimizing the trajectory of a bike wheelie would compare to how humans typically perform a wheelie. How would varying the costs on the two "actuators" impact the trajectory of the wheelie. This comparison may offer insight into what a human is optimizing for when performing a wheelie. Is it energy, or some other quantity? Furthermore, how does the mass and length of the person on the bike impact how difficult it is to perform the wheelie? By varying the masses and lengths of the modeled bike, we aim to understand how the trajectory of a wheelie changes based on these parameters.

This study requires modeling contact between geometries which inherently leads to difficult and non-smooth dynamics. Two different formulations, direct transcription and hybrid collocation, were used to produce solutions via different contact modeling techniques. This allowed us to understand how different formulations affected the optimal trajectories found.

II. Related Work

To model a wheelie, we must capture the dynamics of a bike as well as the true dynamics of the rider to lift the front wheel off the ground while maintaining some linear and rotational velocity along the ground with the back wheel. Prior research has been done on how the wheelie is performed on different vehicles including motorcycles and wheelchairs. In a paper studying the power transmission of a motorcycle wheelie, the center of masses of the person and wheels are modeled to determine the torque and power required to perform the wheelie [5]. The locations of these masses are used in this work to model the bike.

In past work that focused on optimal control of a wheelchair wheelie, the dynamics of the wheelie were split up into two phases. Phase 1 was described as the stable phase where all four wheels of the wheelchair had contact with the ground. Phase 2 was the unstable phase where the front two wheels would break contact with the ground using some torque from the rider [3]. In this second phase, the mechanical model describes the dynamics of the rider to be analogous to that of an inverted pendulum attached to the wheelchair, moving through an angle range with some torque to lift the wheelchair. This model from the wheelchair work informed the decision to use an inverted pendulum as a ”rider” in our bike model. Additionally work done investigating optimal control of the wheelchair found that the optimal position of stabilization in phase 2 was to have the center of mass of the person directly over the rear wheel axle in the vertical direction. Though the goal of this work is to optimize the trajectory of achieving the wheelie, this optimal position finding was referred to when analyzing experimental results.

Though there is limited prior work in simulating wheelies using Drake, the crux of being able to model the wheelie dynamics rests on the ability to model rolling contact, for which there exists working examples. One key project referred to for modeling rolling contact on the back wheel is Russ Tedrake's Ballbot model done using a URDF (Unified Robotics Description Format) [4]. The Ballbot is a model used to mimic the dynamics of a Segway and consists of a ball link, bot link, and a dummy ballx link.

Rolling contact was created between the ball and the ground using the world, ballx, and ball links. A prismatic joint was created between the world and ballx link and the ballx acted as parent link to the ball link in a continuous joint. This way, coupled lateral and rotational motion was achieved by the ball. Additionally, to keep contact between the ball and the ground, collision dynamics were written into the URDF itself, under the ground link and the ball link, along with frictional properties which were necessary to create rolling. This same framework was carried over into the bike URDF used in this work, specifically for the back wheel which we modeled as maintaining contact with the ground at all times.

III. Methods

A. Bike Setup

A bike was modeled as two circular wheels connected by a rectangular frame. The person on the bike was modeled as a point mass at the end of an inverted pendulum. The person's pedaling effort was modeled as a torque on the center of the back wheel. The back wheel was considered the driving wheel whereas the front wheel would be passive and spin according the back wheel when both wheels of the bike were on the ground. The person's effort to change the location of their center of mass was modeled as a torque on the base of the inverted pendulum.

A bike URDF was created using this simplified model (Fig. 2). The default mass of the point-mass-person was 40kg. The wheels were initially modeled as cylinders but the final model had sphere wheels due to simpler contact dynamics. The frame of the bike was modeled as a box and the wheel to wheel frame length was 1m. The pendulum consisted of a cylinder of length 0.6m connected to a point mass sphere. Each wheel was 0.66m in diameter based on average bike wheel sizes [1]. It was important to define the correct joints within the URDF between each of the individual geometries in the bike. Similar to the Ballbot example, the back wheel was modeled with a prismatic dummy x-link for translation connected to a continuous joint to provide wheel rotation. The passive front wheel and pendulum were both simply modeled with a continuous joint. A wheelie is mostly a 2D phenomenon. Therefore, the bike frame was defined with a planar joint, allowing the model of the bike to be constrained to the x-z plane.

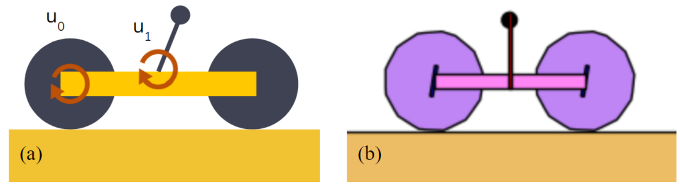

Fig. 2. Image a shows how the bike was modeled in this study and image b shows the URDF generated by implementing this model in code. There are two input torques: u_0 at the back wheel and u_1 at the pendulum base.

B. Contact

For modeling the bike dynamics, contact was necessary between the ground and both wheels. The back wheel contact was permanently constrained to rolling on the ground while the front wheel's contact was necessary only for the trajectory pre-wheelie. Initially, the wheel links were represented as thin cylinders in the URDF and collision dynamics between both wheels and the ground link were also written directly in the URDF. However, with this strategy there were issues running the bike simulation in Drake as collisions between cylinders and box links (the ground link) are not yet fully supported. From this finding, the cylindrical links were swapped for spherical links that were constrained to remain planar to each other to create the same rolling motion as the two cylinders. Since the visualizer we would use later on used a 2D side view of the bike, having spheres instead of cylinders would not change the view of the bike in the later simulations either.

With different optimization strategies (described in later sections) approaches to contact modeling also changed. For hybrid collocation, dual collision dynamics (collision dynamics on both wheels) worked well, however, with the initial experiments with direct collocation, it seemed that having dual collision dynamics on the wheels was leading to errors in the resulting trajectories. As a result, the collision block in the front wheel URDF was swapped for a force vector. This force vector was used as a decision variable in the trajectory optimization and encapsulated both tangential friction force and normal force on the front wheel.

This approach worked to lift the front wheel up from the ground however the force would not drop to zero once the front wheel was in the air. To resolve this, the force decision variables were removed and instead a constrained force was added in the manipulator equation that was governing the dynamics of the optimization. To do this, the point on the front wheel that would be making contact with the ground (Fig. 3) had to be expressed using the state parameters of the bike: $[f_x, w_{1\theta}, f_\theta]$, which are the back wheel theta and the frame theta respectively.

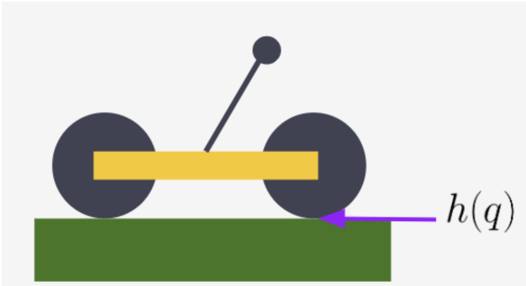

Fig. 3. Location of the h(q) ground point with respect to the bike.

Then using this expression for the contact point, an adjusted jacobian, H, with respect to $[\dot{q}]$ was calculated explicitly, along with the constraint force, λ [4]. Both λ and H were solved using (2, 3), knowing that $h(q)$ was constrained to be zero to keep the front wheel on the ground. This adjusted manipulator equation, was only used for the ground phase of the trajectory optimization when both wheels had to maintain contact with the ground. Below is the derivation of the adjusted manipulator equation Where M is the mass matrix, C is the Coriolis matrix, τ is the vector of generalized forces, and B is the actuation matrix from the URDF plant:

$$h(q) = 0 \tag{1}$$

$$H(q) = \frac{dh}{dq} I \tag{2}$$

$$\lambda = -(HM^{-1}H^T)^{+}(HM^{-1}\tau + \dot{H}v) \tag{3}$$

$$M(q)\dot{v} + C(q,v)v = \tau(q) + Bu + H^T(q)\lambda \tag{4}$$

For breaking contact in the trajectories the original manipulator equation was used.

$$M(q)\dot{v} + C(q,v)v = \tau(q) + Bu \tag{5}$$

C. Direct Transcription

The direct transcription formulation uses both the state and inputs as decision variables. The position state vector for the bike model is defined by the x position of the frame, the angle of the back wheel, the angle of the frame, the angle of the front wheel, and the angle of the pendulum: $q = [f_x, w_{1\theta}, f_\theta, w_{2\theta}, p_\theta]$. The input torque of the back wheel and pendulum were defined as u_0 and u_1 respectively.

Originally, the wheelie was defined as a two-stage process where the first stage was taken to be the ground phase and the second was the wheelie phase, similar to the wheelchair wheelie model of a two phase trajectory. However, the constraints required for each of these phases prevented the solver from finding a solution for the trajectory at the transition between these two phases. Instead of two stages, the trajectory was defined as a five-stage process. In the first stage, the bike was required to maintain rolling contact with the ground, next the bike was allowed to break contact; this transition was the second stage. The third stage was the full wheelie itself, and the fourth stage was another transition after which the bike was constrained to rolling contact once again in the fifth stage. Each phase of the trajectory required its own constraints.

First, constraints required for all the stages and therefore all the time steps were defined:

1) The back wheel must maintain rolling contact. Rolling contact occurs when $v = \omega r$ which can be written using the velocity state vector, $\dot{q}$.
2) The point-mass-person must not go below the frame. The position of the pendulum was constrained between $\frac{\pi}{2}$ and $-\frac{\pi}{2}$.
3) The speed of the frame was constrained to be between 0 and 6 $\frac{m}{s}$. This was to decrease the area of the solution space so the solver could more easily find an optimal solution.

Next, constraints for each of the 5 phases were defined. As described previously, two different manipulator equations were used based on whether the phase involved rolling contact on both wheels or just the back wheel. The manipulator equation for the first is given by $M(q)\dot{v} + C(q,v)v = \tau(q) + Bu + H^T(q)\lambda$ and the equation for the latter is $M(q)\dot{v} + C(q,v)v = \tau(q) + Bu$. When both wheels are on the ground, we expect the angular velocity of wheel 1 and wheel 2 to be equal. Furthermore, when contact is occurring the frame must be flat and we must specify $h(q) = 0$. Note that the frame angle f_θ is defined with respect to the back wheel. The max angle of the wheelie was defined as $(\frac{\pi}{2} - 0.1)$. The constraints for each phase are organized below:

1) Phase 1: Beginning Rolling Contact
   a) $\dot{w_{1\theta}} = \dot{w_{2\theta}}$
   b) $h(q) = 0 \rightarrow f_\theta + w_{1\theta} = 0$
   c) $M(q)\dot{v} + C(q,v)v = \tau(q) + Bu + H^T(q)\lambda$
2) Phase 2: Transition to Breaking Contact
   a) $0 \leq f_\theta + w_{1\theta} \leq \frac{\pi}{2} - 0.1$.
   b) $M(q)\dot{v} + C(q,v)v = \tau(q) + Bu$
3) Phase 3: Full Wheelie
   a) $\frac{\pi}{6} \leq f_\theta + w_{1\theta} \leq \frac{\pi}{2} - 0.1$.
   b) $M(q)\dot{v} + C(q,v)v = \tau(q) + Bu$
4) Phase 4: Transition to Remaking Contact
   a) $0 \leq f_\theta + w_{1\theta} \leq \frac{\pi}{2} - 0.1$.
   b) $M(q)\dot{v} + C(q,v)v = \tau(q) + Bu$
5) Phase 5: Ending with Rolling Contact
   a) $\dot{w_{1\theta}} = \dot{w_{2\theta}}$
   b) $h(q) = 0 \rightarrow f_\theta + w_{1\theta} = 0$
   c) $M(q)\dot{v} + C(q,v)v = \tau(q) + Bu + H^T(q)\lambda$

The time steps at which each of these transitions would occur were pre-determined, however the duration of the time step itself was a decision variable. Below is a list of the decision variables:

1) $q, \dot{q}, \ddot{q}$ at each time step
2) u_0, u_1 at each time step
3) h at each time step [0.005, 0.5]

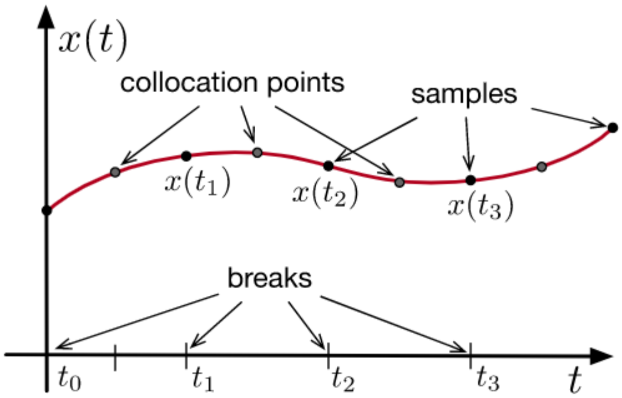

Fig. 4. Cubic spline parameters used in the direct collocation method [4]

The initial state of the system for the optimization consisted of all zeros other than an initial frame velocity of 5 $\frac{m}{s}$ to ensure the bike was moving at a decent speed to perform the wheelie.

D. Hybrid Collocation

The hybrid collocation approach is a modification on the original direct collocation algorithm for trajectory optimization. Direct collocation creates a piece-wise cubic polynomial based on sample points in the trajectory (Fig. 4). Using these sample points the optimizer attempts to pick points in between each sample (collocation points) in order to create a smooth trajectory. The decision variables are the state variables, inputs, and time-step [4]. In the case of our bike model the state vector x included: $[f_x, w_{1\theta}, f_\theta, w_{2\theta}, p_\theta, \dot{f_x}, \dot{w_{1\theta}}, \dot{f_\theta}, \dot{w_{2\theta}}, \dot{p_\theta}]$. The input vector was $u = [u_0, u_1]$ which are the back wheel torque and pendulum torque, and the time-step h was constrained to be [0.08,0.5].

Originally, direct collocation was used to find optimal wheelie trajectories, however the optimizer could not find any trajectories that showed a wheelie. This was a result of the direct collocation method's inability to model contact and non contact in the same trajectory, a necessary requirement for the wheelie. Thus, a new optimization strategy, called hybrid collocation was implemented. A Drake template, developed by Professor Russ Tedrake at MIT, was used to implement this strategy which separates the original collocation optimization problem into two parts: a ground collocation and an aerial collocation, emulating the wheelchair wheelie model of a two phase trajectory. This optimization solves both the ground collocation and aerial collocation (where the front wheel is off the ground) and ensures that the last state of the ground phase is the first state of the aerial phase. With this strategy, the states, inputs, and time step were used as decision variables (just as in the original direct collocation), with an added decision variable of contact force for inputted contact pairs which are specified in the URDF via collisions. In this setup, the contact pairs for the front and back wheel were inputted and for the aerial phase, just the back wheel contact was inputted, in order to ensure both wheels on the ground for the ground phase and only the rear wheel on the ground for the aerial phase.

Having two separate collocation problems in the hybrid trajectory allowed the use of two separate sets of constraints, similar to the multi-phase approach of the direct transcription. The constraints held for all time included the following:

1) The back wheel must maintain rolling contact. Rolling contact occurs when $v = \omega r$ which can be written using the velocity state vector, $\dot{q}$.
2) The point-mass-person must not go below the frame. The position of the pendulum was constrained between $\frac{\pi}{2}$ and $-\frac{\pi}{2}$.
3) The speed of the frame was constrained to be ≥ 0. $\frac{m}{s}$

Then constraints for solely the ground state were added. These were the following:

1) The front wheel must maintain rolling contact, occurring when $v = \omega r$, written using the velocity state vector, $\dot{q}$.
2) The bike must start at a specified initial condition, $q_0 = [0, 0, 0, 0, 0, 2, 2, 0, 0, 0]$, to give the bike some initial x and back wheel velocity.

No additional constraints were needed for the aerial phase. The contact force pair of the front wheel and ground was taken away from the collocation problem to make sure that there was no contact between the ground and the front wheel.

IV. Experiments

After setting up the structure for direct transcription and hybrid collocation, physical parameters were changed and objectives were added to the optimizations to see how they would affect the wheelie trajectories. These parameter modifications were done with both formulations.

In terms of objectives, costs were added for both the wheel torque and pendulum torque to see how input penalties would effect the efficiency of the wheelie/see how the bike trajectory would change with minimal input torques. Additionally the objective was changed to put penalties on ONLY the wheel torque and ONLY the pendulum torque to see how or if the trajectory would change to favor the non-penalized input. In all cases a squared cost was added, e.g. u_0^2 for the wheel torque and u_1^2 for the pendulum torque.

With respect to the physical parameters of a wheelie, the effects of different rider masses and torso heights were investigated via changes to the pendulum mass and pendulum lengths respectively.

V. Results

For all simulations, the x velocity, pendulum angle as well as input torques were recorded as a function of time. The results for each set of simulations for each optimization strategy are explained.

A. Dual-Input Cost

A squared cost was added for each of the inputs in both optimizations and the resulting optimized trajectories were plotted (Fig. 5). The URDFs for each optimizer were identical. For the direct transcription approach, the magnitudes of linear

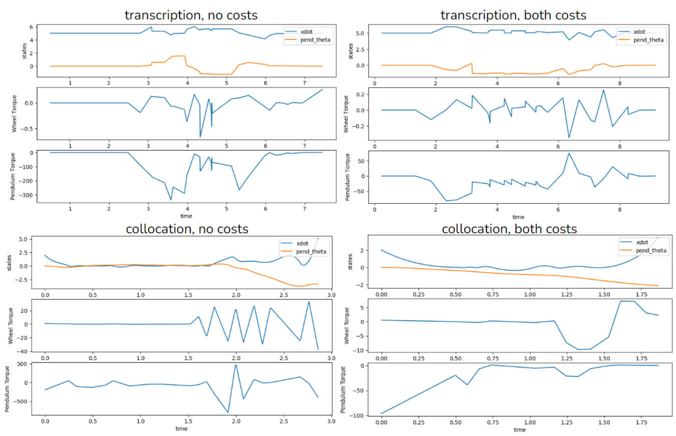

Fig. 5. The top-left and top-right plots show the states $\dot{f}_\theta, p_\theta$ as well as both input torques for the case when there were no costs in the optimization and when both input torques had associated costs. The top two plots show results from the direct transcription formulation. The bottom-left and bottom-right plots show the same variables for the hybrid collocation formulation.

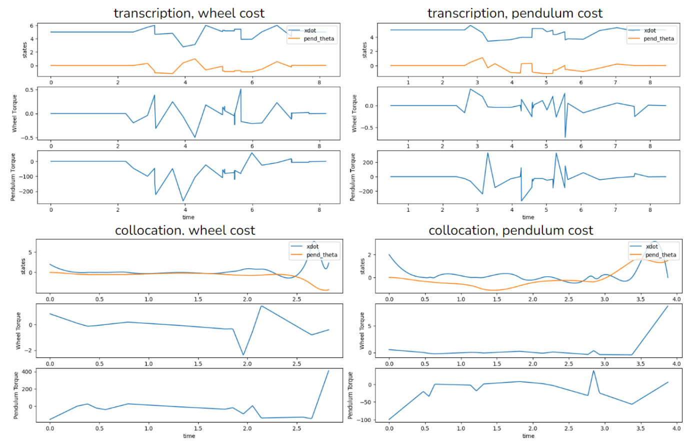

Fig. 6. The top two plots show the states $\dot{f}_\theta, p_\theta$ as well as both input torques for the case when there was only a cost on the back wheel input torque (top-left) and when there was only a cost on the pendulum input torque (top-right). The top two plots show results from the direct transcription formulation. The bottom-right and bottom-left plots show the same variables plotted but for the hybrid collocation formulation.

bike speed and pendulum angle showed little difference. In both cost and no cost trials the bike speed hovered around 5m/s while the pendulum angle stayed at about 0 degrees, meaning the pendulum was rotated toward the front wheel. The main difference between trials was that the magnitude of oscillations around the average values was much less for the cost trial. This can be seen visually in the two graphs while taking note that the scale is slightly smaller for the cost trial results. The maximum magnitude of pendulum torque for the both costs trial was 75% less than that of the no cost trial. Similarly, the maximum magnitude of the wheel torque for the both costs trial was 40% less than that of the no cost trial. For both input torques, less oscillations occurred with squared cost on each torque compared to no costs.

The same dual cost simulations were done with hybrid collocation. In these results, similar to direct transcription, the magnitudes of bike speed and pendulum angle stayed the same between the cost and no cost trials. The inputted wheel torque maximum magnitude decreased from 40Nm for no cost trials to 10 Nm for both cost trials, a 75% decrease, similar to direct transcription. The maximum pendulum torque decreased by 80% going from 500Nm in the no cost trial to 100 Nm in the both costs trial. Overall both inputs oscillate around zero, however, oscillation magnitudes and frequency decreased after adding cost.

B. Single-Input Cost

The next set of simulations involved adding only one squared cost at a time, first a wheel only cost and then pendulum only cost (Fig. 6) .

For wheel torque only costs, the direct transcription strategy showed little change in wheel torque compared to the no cost run in Fig. 5. Running the trial with costs only on the pendulum torque showed little change in input torques from the trial with only wheel torque cost, besides the fact that the pendulum torque curve was centered on 0Nm instead of -100Nm. Oscillations in the plots for the input torques did not show a significant change when adding cost one at a time compared to having both costs added simultaneously.

Looking at the hybrid collocation results, the magnitudes changed for each input when a cost was added. Wheel torque spiked from 2 to 5 Nm when costs were removed while pendulum torque spiked from 100 to 400 Nm when costs were removed. All values given here are max magnitudes reached in the trajectory.

C. Rider Mass

In simulations with changing rider mass, the mass value was changed in the URDF in both optimizations. For direct transcription a mass of 40kg with equal squared input costs on both inputs was simulated along with a trial where mass was changed to 60kg. Ideally the mass would have been doubled to 80kg although this number caused the optimizer to fail at solving the optimization.

In comparing the 40kg to 60kg simulations, it was shown that the wheel torques increased slightly, from 0.2-0.5 Nm in maximum magnitude whereas the pendulum torque maximum increased by 4 times (Fig. 7). In comparing a 40kg to 80kg mass with hybrid collocation, opposite changes were observed. Input wheel torque decreased on average although maximum magnitude increased slightly while input pendulum torque decreased from 500 to 200 Nm with increasing weight. With the 80kg trial, the hybrid collocation constraint of maintaining positive x velocity was broken. This may have been due to the added mass of the pendulum being more difficult to stabilize over the rear wheel.

D. Rider Length

The last set of simulations done involved replacing the 0.6m pendulum length for 0.3 meters (0.4m in direct transcription because 0.3m gave errors which are unresolved) to simulate

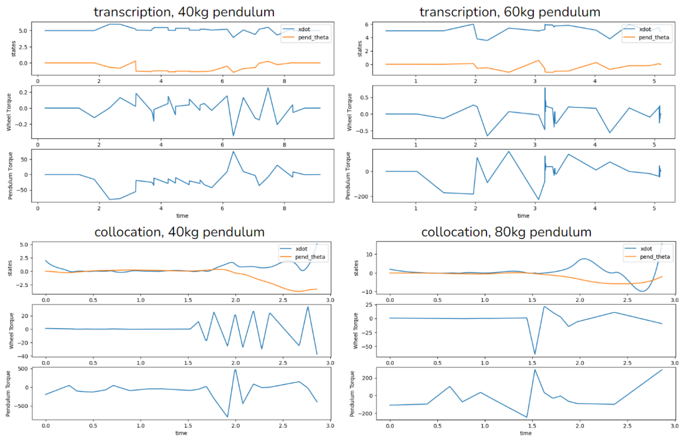

Fig. 7. The top two plots show the states and inputs while varying the mass from 40kg (top-left) to 60kg (top-right) for the direct transcription formulation. In both of the top plots, the costs in the optimization were on both input torques. The bottom two plots show the states and inputs for masses of 40kg (bottom-left) and 80kg (bottom-right) that were generated from the optimal trajectory solved using a hybrid collocation formulation. The optimization for the bottom plots was done with no costs.

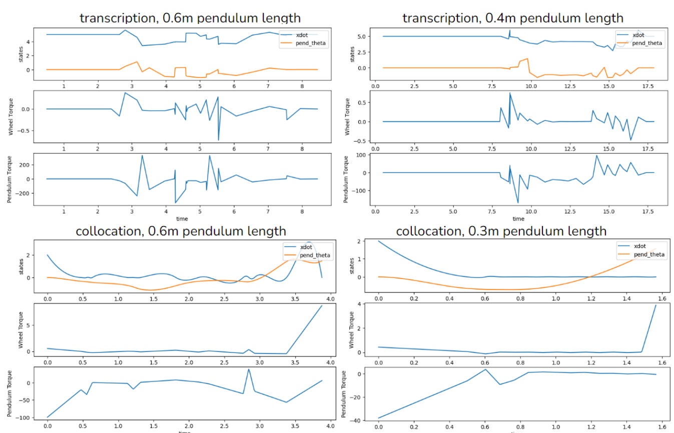

Fig. 8. The top two plots show the optimal trajectory generated via a direct transcription formulation for varying lengths of the pendulum. The top-left shows the results for a pendulum length of 0.6m whereas top-right shows the results for a pendulum length of 0.4m. The bottom two plots show the optimal trajectory generated via a hybrid collocation approach while varying pendulum lengths. Lengths of 0.6m and 0.3m are shown in the bottom-left and bottom-right plots respectively. In all of the optimizations, the cost was only placed on the pendulum input torque.

a shorter rider attempting a wheelie. The mass was reverted back to 40kg for all trials.

In both direct transcription and hybrid collocation, the shorter pendulum required less pendulum torque, both max magnitudes decreased by 50% when going from the longer to shorter pendulum (Fig. 8). This may be because, to move the shorter pendulum about the same range of angles, less torque would be needed as the moment arm is essentially halved. The wheel torque had little change for both strategies.

VI. Conclusion

Looking at the results from both the direct transcription and hybrid collocation methods, there are some common conclusions to draw from the two methods about how cost affects trajectories. Overall, cost on pendulum torque caused the optimized pendulum torque to drop by roughly 75% when looking at maximum magnitudes, however this was only when wheel torque was also added as a cost. Individual costs did not affect the outcomes as much for either optimization method, although generally, for optimizations where the input had costs, the optimal trajectory showed less oscillation and a smoother curve toward the settling value. This implies that for a wheelie to be achieved with least effort from the rider, the input torque on back wheel and torque applied by the hip should spike at first, then settle at some value (settling value is at zero for most of these simulations).

In analyzing results for the physical parameter experiments (mass and length) there were conflicting simulations. When mass was added, direct transcription showed a 400% increase in pendulum torque maximum magnitude while the hybrid collocation about a 40% decrease in torque. This difference in result could be explained by the difference in optimization methods. i.e. it is possible that the hybrid collocation found a better trajectory while the transcription fell into a local minimum based on the initial condition it was given. For the length changing simulations, both methods saw a decrease in pendulum torque by about 50% (in maximum magnitude) from the longer to the shorter length. This suggests that a shorter rider may require less hip torque to wheelie a bike than a taller rider.

VII. Next Steps

One aspect that can be added to this project is a Linear Quadratic Regulator (LQR) for the fixed point of the bike when performing a wheelie. Using LQR would allow control of the fixed point in a certain range to hopefully balance the bike on the back wheel for longer. It would be interesting to see how well LQR would do on this system and what its range of validity would be.

Furthermore, it would be interesting to study how modeling a person as a distributed mass instead of a point mass would affect the dynamics. Not only that, but allowing this mass to move within a certain bounding box instead of constraining it to only be able to move along the circular trajectory of the pendulum could result in a more accurate simulation.

Another future direction to study would be varying parameters of the bike model and seeing how they affect the optimal trajectory. Bikes vary in many ways, but particularly in their shape, mass, and height. It would be interesting to model several bikes that are available on the market based on these parameters and see which bike would be the easiest to do a wheelie with.

Contributions

Both Maheera and Sharmi worked on the URDF and the different approaches for contact modeling. Both worked on

the direct collocation as well as direct transcription and hybrid collocation approaches. We each had our own notebook to try different solutions for each step we took along the project and used a "leap-frog" strategy. By this we mean, if one person got stuck at a certain point of the implementation, they would learn what worked from the other's notebook and apply it so that in total we could try double the solutions (and witness double the errors). This collaboration allowed quicker progress and allowed more of the solution space to be explored.

We diverged at the end of the project when we started working on hybrid collocation. Sharmi worked on varying parameters and coming up with trajectories in the direct transcription formulation and Maheera worked on varying parameters and finding trajectories using the hybrid collocation formulation.

External Research Overlap

Neither author has done prior research in this area nor is it related to projects being done for other classes or UROPs.

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