PES UNIVERSITY

ELECTRONIC CITY CAMPUS

(Established under Karnataka Act no. 16 of 2013)

Hosur Road, Near Electronic City,Bangalore-100

SCI LAB

Subject: LINEAR ALGEBRA AND ITS APPLICATIONS

Subject Code: UE19MA251

Name: R SHARMILA
SRN: PES2UG19CS309
Section: E
Branch: CSE

Marks awarded: **Name of the faculty: Dr.Girish. V.R.**

Class Number	Topic
1	Gaussian Elimination
2	The LU Decomposition
3	Inverse of a Matrix by the Gauss- Jordan Method
4	The Span of Column Space of a Matrix
5	The Four Fundamental Subspaces
6	Projections by Least Squares
7	The Gram-Schmidt Orthogonalization
8	Eigen values and Eigen Vectors of a Matrix

Topic: Gaussian Elimination

PROBLEM 1: Solve the following system of equations by Gaussian Elimination. Identify the pivots.

2x – 3y = 3, 4x – 5y + z = 7, 2x – y – 3z = 5.

CODE:

P1-Guassian elemination_PES2UG19CS309.sce (C:\Users\sharm\P1-Guassian elemination_PE

File Edit Format Options Window Execute ?

P1-Guassian elemination_PES2UG19CS309.sce (C:\Users\sharm\P1-Guassian elemination_PES2UG19CS

P1-Guassian elemination_PES2UG19CS309.sce | P2-LU Decomposition_PES2UG19CS309.sce

```
clc;clear;
A = [2,-3,0;4,-5,1;2,-1,-3], b = [3;7;5];
disp("Matrix before Gaussian Elimination: ")
disp(A);
Ab = [A b];
a = Ab;
n = 3;
for i = 2:n
    for j=2:n+1
        a(i,j) = a(i,j) - a(1,j)*a(i,1)/a(1,1);
    end
    a(i,1) = 0;
end
for i=3:n
    for j=3:n+1
        a(i,j) = a(i,j)-a(2,j)*a(i,2)/a(2,2);
    end
    a(i,2) = 0;
end
x(n) = a(n,n+1)/a(n,n);
for i=n-1:-1:1
    sumk = 0;
    for k=i+1:n
        sumk = sumk+a(i,k)*x(k);
    end
    x(i) = (a(i,n+1) - sumk)/a(i,i);
end
disp("Values of x,y,z:")
disp(x);
disp("Matrix after Gaussian Elimination: ")
disp(a);
disp("The pivots are: ");
disp(a(3,3),a(2,2),a(1,1));
```

OUTPUT:

Scilab 6.1.0 Console

```
 "Matrix before Gaussian Elimination: "

  2.  -3.   0.
  4.  -5.   1.
  2.  -1.  -3.

 "Values of x,y,z:"

  3.
  1.
  0.

 "Matrix after Gaussian Elimination: "

  2.  -3.   0.   3.
  0.   1.   1.   1.
  0.   0.  -5.   0.

 "The pivots are: "

 -5.

  1.

  2.

-->
```

Topic: LU decomposition of a matrix

PROBLEM 2: Factorize the following matrices as A = LU
$2x + 3y + z = 8$, $4x + 7y + 5z = 20$, $-2y + 2z = 0$

CODE:

P2-LU Decomposition_PES2UG19CS309.sce

```
clear;clc;;
A = [2 3 1;4 7 5; 0 -2 2];
U = A;
disp(A, "The given matrix is:");
m = det(U(1,1));
n = det(U(2,1));
a = n/m;
U(2,:)= U(2,:) - U(1,:)/(m/n);
n = det(U(3,1));
b = n/m;
U(3,:) = U(3,:) - U(1,:)/(m/n);
m = det(U(2,2));
n = det(U(3,2));
c = n/m;
U(3,:) = U(3,:) - U(2,:)/(m/n);
disp(U,"The upper triangular matrix is:");
L = [1,0,0;a,1,0;b,c,1];
disp(L,"The lower triangular matrix is:");
```

OUTPUT:

Scilab 6.1.0 Console

```
   2.   3.   1.
   4.   7.   5.
   0.  -2.   2.

 "The given matrix is:"

   2.   3.   1.
   0.   1.   3.
   0.   0.   8.

 "The upper triangular matrix is:"

   1.   0.   0.
   2.   1.   0.
   0.  -2.   1.

 "The lower triangular matrix is:"
```

Topic: The Gauss - Jordan method of calculating A -1

PROBLEM 3: Find the inverse of the following matrix A={[2,-1,0],[-1,2,-1],[0,-2,2]}

CODE:

P3-Gauss-Jordan for inverse_PES2UG19CS309.sce

```
clc;clear;
A = [2 -1 0;-1 2 -1;0 -1 2];
n = length(A(1,:));
Aug = [A,eye(n,n)];
//Forward Elimination
for j=1:n-1
    for i=j+1:n
        Aug(i,j:2*n) = Aug(i,j:2*n)-Aug(i,j)/Aug(j,j)*Aug(j,j:2*n);
    end
end
//Backward Elimination
for j = n:-1:2
    Aug(1:j-1,:) = Aug(1:j-1,:)-Aug(1:j-1,j)/Aug(j,j)*Aug(j,:);
end
//Diagonal Normalization
for j=1:n
    Aug(j,:) = Aug(j,:)/Aug(j,j);
end
B = Aug(:,n+1:2*n);
disp("The inverse of A is:");
disp(B);
```

OUTPUT:

Scilab 6.1.0 Console

```
 "The inverse of A is:"

  0.75    0.5    0.25
  0.5     1.     0.5
  0.25    0.5    0.75

-->
```

Topic: Span of the Column Space of A

PROBLEM 4: Identify the columns that are in the column space of A where A={[2,4,6,4],[2,5,7,6],[2,3,5,2]}

CODE:

P4-Span of column space_PES2UG19CS309.sce (C:\Users\sharm\P4-Span c

File Edit Format Options Window Execute ?

P4-Span of column space_PES2UG19CS309.sce

```
clc;clear;
a = [2 4 6 4;2 5 7 6;2 3 5 2];
disp("The given matrix is:");
disp(a);
a(2,:) = a(2,:)-(a(2,1)/a(1,1))*a(1,:);
a(3,:) = a(3,:)-(a(3,1)/a(1,1))*a(1,:);
disp(a);
a(3,:) = a(3,:)-(a(3,2)/a(2,2))*a(2,:);
disp(a);
a(1,:) = a(1,:)/a(1,1);
a(2,:) = a(2,:)/a(2,2);
disp(a);
for i=1:3
    for j=i:4
        if(a(i,j)<>0)
            disp("is a pivot column",j,"column");
            break;
        end
    end
end
```

OUTPUT:

Scilab 6.1.0 Console

```
"The given matrix is:"

  2.   4.   6.   4.
  2.   5.   7.   6.
  2.   3.   5.   2.

  2.   4.   6.   4.
  0.   1.   1.   2.
  0.  -1.  -1.  -2.

  2.   4.   6.   4.
  0.   1.   1.   2.
  0.   0.   0.   0.

  1.   2.   3.   2.
  0.   1.   1.   2.
  0.   0.   0.   0.

"is a pivot column"

  1.

"column"

"is a pivot column"

  2.

"column"

-->
```

<u>Topic: The Four Fundamental Subspaces</u>

PROBLEM 5: Find the four fundamental subspaces of A={[1,2,0,1],[0,1,1,0],[1,2,0,1]}

<u>CODE:</u>

P5-Four fundamental subspaces_PES2UG19CS309.sce (C:\Users\sharm\P5-Four fundamental subs

File Edit Format Options Window Execute ?

P5-Four fundamental subspaces_PES2UG19CS309.sce (C:\Users\sharm\P5-Four fundamental subspaces_PES

P4-Span of column space_PES2UG19CS309.sce | P5-Four fundamental subspaces_PES2UG19CS309.sce

```
clc;clear;
A = [1 2 0 1;0 1 1 0;1 2 0 1];
disp("The given matrix is:");
disp(A);
[m,n] = size(A);
disp(m,"m = ");
disp(n,"n = ");
[v,pivot] = rref(A);
disp(rref(A),"Row Reduced Echelon Form: ");
r = length(pivot);
disp(r,"Rank: ");
colspace = A(:,pivot);
disp(colspace,"Column Space: ");
nullspace = kernel(A);
disp(nullspace,"Null Space: ");
rowspace = v(1:r,:)';
disp(rowspace,"Row Space: ");
leftnullspace = kernel(A');
disp(leftnullspace,"Left Null Space: ");
```

<u>OUTPUT:</u>

Scilab 6.1.0 Console

```
 "Row Reduced Echelon Form: "

  2.

 "Rank: "

  1.   2.
  0.   1.
  1.   2.

 "Column Space: "

  3.909D-17  -0.8660254
 -0.4082483   0.2886751
  0.4082483  -0.2886751
  0.8164966   0.2886751

 "Null Space: "

  1.   0.
  0.   1.
 -2.   1.
  1.   0.

 "Row Space: "

 -0.7071068
  1.106D-16
  0.7071068

 "Left Null Space: "
```

Topic: Projections by Least Squares

PROBLEM 6: Find the solution x = (C , D) of the system Ax = b and the line of best fit C + Dt = b given
A={[1,0],[0,1],[1,1]} and b={[1,1,0]}

CODE:

P6-Projection of least squares_PES2UG19CS309.sce

```
clc;clear;
A = [1 0;0 1;1 1];
b = [1;1;0];
disp("The given matrix A is:")
disp(A);
disp(b, "b: ");
x = (A'*A)\(A'*b)
C = x(1,1);
D = x(2,1);
disp(C,"C: ");
disp(D,"D: ");
disp("The best fit line is b = C+Dt")
```

OUTPUT:

Scilab 6.1.0 Console

```
 "The given matrix A is:"

  1.   0.
  0.   1.
  1.   1.

  1.
  1.
  0.

 "b: "

  0.3333333

 "C: "

  0.3333333

 "D: "

 "The best fit line is b = C+Dt"

-->
```

Topic: The Gram- Schmidt Orthogonalization

PROBLEM 7: Apply the Gram – Schmidt process to the following set of vectors and find the orthogonal matrix:
(1, 0, 1) , (1, 1, 0) , (2, 1, 1)

CODE:

P7-Gram-schmidt orthogonalisation_PES2UG19CS309.sc

File Edit Format Options Window Execute ?

P7-Gram-schmidt orthogonalisation_PES2UG19CS309.sce (C:\Us

P7-Gram-schmidt orthogonalisation_PES2UG19CS309.sce

```
clc;clear;
A= [1 0 1;1 1 0;2 1 1];
disp(A, "The given matrix A is:");
[m,n] = size(A);
for k=1:n
    V(:,k) = A(:,k);
    for j=1:k-1
        R(j,k) = V(:,j)'*A(:,k);
        V(:,k) = V(:,k)-R(j,k)*V(:,j);
    end
    R(k,k) = norm(V(:,k));
    V(:,k) = V(:,k)/R(k,k);
end
disp(V,"Q: ");
```

OUTPUT:

Scilab 6.1.0 Console

```
   1.   0.   1.
   1.   1.   0.
   2.   1.   1.

  "The given matrix A is:"

   0.4082483  -0.7071068  -0.842701
   0.4082483   0.7071068   0.2407717
   0.8164966  -3.140D-16  -0.4815434

  "Q: "

-->
```

Topic: Eigen values and Eigen vectors of a given square matrix

PROBLEM 8: Find the Eigen values and the corresponding Eigen vectors of A={[8,-6,2],[-6,7,-4],[2,-4,3]}

CODE:

P8-Eigen values and vectors_PES2UG19CS309.sce (C:\Users\sharm\P8-Eigen valu

```
clc;clear;
A = [8 -6 2;-6 7 -4;2 -4 3];
disp(A,"The given matrix A is: ")
lam = poly(0,"lam");
charMat = A-lam*eye(3,3);
disp(charMat,"The Characteristic Matrix is: ");
charPoly = poly(A,"lam");
disp(charPoly,"The Characteristic Polynomial is:");
lam = spec(A);
disp(lam,"Eigen  Values: ");
function[x,lam] = eigenvectors(A)
    [n,m] = size(A);
    lam = spec(A)';
    x = [];
    for k=1:3
        B = A-lam(k)*eye(3,3);
        C = B(1:n-1,1:n-1);
        b = -B(1:n-1,n);
        y = C\b;
        y = [y;1];
        y = y/norm(y);
        x = [x y];
    end
endfunction
[x,lam] = eigenvectors(A);
disp(x,"Eigen Vectors of A: ");
```

OUTPUT:

Scilab 6.1.0 Console

```
  8.  -6.   2.
 -6.   7.  -4.
  2.  -4.   3.

 "The given matrix A is: "

 8 -lam  -6      2
 -6      7 -lam  -4
 2       -4      3 -lam

 "The Characteristic Matrix is: "

 -7.128D-14 +45lam -18lam² +lam³

 "The Characteristic Polynomial is:"

  1.584D-15
  3.
  15.

 "Eigen  Values: "

  0.3333333  -0.6666667   0.6666667
  0.6666667  -0.3333333  -0.6666667
  0.6666667   0.6666667   0.3333333

 "Eigen Vectors of A: "

-->
```