



ELECTRONIC CITY CAMPUS

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Hosur Road, Near Electronic City, Bangalore-100

SCI LAB

Subject: LINEAR ALGEBRA AND ITS APPLICATIONS

Subject Code: UE19MA251

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SRN: PES2UG19CS309

Section: E

Branch: CSE

Marks awarded:

Name of the faculty: Dr.Girish. V.R.

Class Number	Topic
1	Gaussian Elimination
2	The LU Decomposition
3	Inverse of a Matrix by the Gauss- Jordan Method
4	The Span of Column Space of a Matrix
5	The Four Fundamental Subspaces
6	Projections by Least Squares
7	The Gram-Schmidt Orthogonalization
8	Eigen values and Eigen Vectors of a Matrix

Topic: Gaussian Elimination

PROBLEM 1: Solve the following system of equations by Gaussian Elimination. Identify the pivots.

$$2x - 3y = 3, 4x - 5y + z = 7, 2x - y - 3z = 5.$$

CODE:

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P1-Gaussian elimination_PES2UG19CS309.sce (C:\Users\sharm\P1-Gaussian elimination_PE
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P1-Gaussian elimination_PES2UG19CS309.sce (C:\Users\sharm\P1-Gaussian elimination_PES2UG19CS
P1-Gaussian elimination_PES2UG19CS309.sce P2-LU Decomposition_PES2UG19CS309.sce
1: clc;clear;
2: A = [2,-3,0;4,-5,1;2,-1,-3], b = [3;7;5];
3: disp("Matrix before Gaussian Elimination: ")
4: disp(A);
5: Ab = [A b];
6: a = Ab;
7: n = 3;
8: for i = 2:n
9:     for j=2:n+1
10:         a(i,j) = a(i,j) - a(i,1)*a(1,1)/a(1,1);
11:     end
12:     a(i,1) = 0;
13: end
14: for i=3:n
15:     for j=3:n+1
16:         a(i,j) = a(i,j) - a(2,2)*a(i,2)/a(2,2);
17:     end
18:     a(i,2) = 0;
19: end
20: x(n) = a(n,n+1)/a(n,n);
21: for i=n-1:-1:1
22:     sumk = 0;
23:     for k=i+1:n
24:         sumk = sumk+a(i,k)*x(k);
25:     end
26:     x(i) = (a(i,n+1) - sumk)/a(i,i);
27: end
28: disp("Values of x,y,z:")
29: disp(x);
30: disp("Matrix after Gaussian Elimination: ")
31: disp(a);
32: disp("The pivots are: ");
33: disp(a(3,3),a(2,2),a(1,1));
34:
```

OUTPUT:

```
Scilab 6.1.0 Console

"Matrix before Gaussian Elimination: "

2.  -3.   0.
4.  -5.   1.
2.  -1.  -3.

"Values of x,y,z:"

3.
1.
0.

"Matrix after Gaussian Elimination: "

2.  -3.   0.   3.
0.   1.   1.   1.
0.   0.  -5.   0.

"The pivots are: "

-5.

1.

2.

--> |
```

Topic: LU decomposition of a matrix

PROBLEM 2: Factorize the following matrices as $A = LU$

$$2x + 3y + z = 8, 4x + 7y + 5z = 20, -2y + 2z = 0$$

CODE:

```
P2-LU Decomposition_PES2UG19CS309.sce (C:\Users\sharm\P2-LU Decomposition_PES2UG19CS309.sce)
File Edit Format Options Window Execute ?
P2-LU Decomposition_PES2UG19CS309.sce (C:\Users\sharm\P2-LU Decomposition_PES2UG19CS309.sce)
P1-Gaussian elimination_PES2UG19CS309.sce P2-LU Decomposition_PES2UG19CS309.sce
1 clear;clc;
2 A = [2 3 1; 4 7 5; 0 -2 2];
3 U = A;
4 disp(A, 'The given matrix is:');
5 m = det(U(1,1));
6 n = det(U(2,1));
7 a = n/m;
8 U(2,:) = U(2,:) - U(1,:)/(m/n);
9 n = det(U(3,1));
10 b = n/m;
11 U(3,:) = U(3,:) - U(1,:)/(m/n);
12 m = det(U(2,2));
13 n = det(U(3,2));
14 c = n/m;
15 U(3,:) = U(3,:) - U(2,:)/(m/n);
16 disp(U, 'The upper-triangular matrix is:');
17 L = [1, 0, 0; a, 1, 0; b, c, 1];
18 disp(L, 'The lower-triangular matrix is:');
19
```

OUTPUT:

```
Scilab 6.1.0 Console

2.    3.    1.
4.    7.    5.
0.   -2.    2.

"The given matrix is:"

2.    3.    1.
0.    1.    3.
0.    0.    8.

"The upper triangular matrix is:"

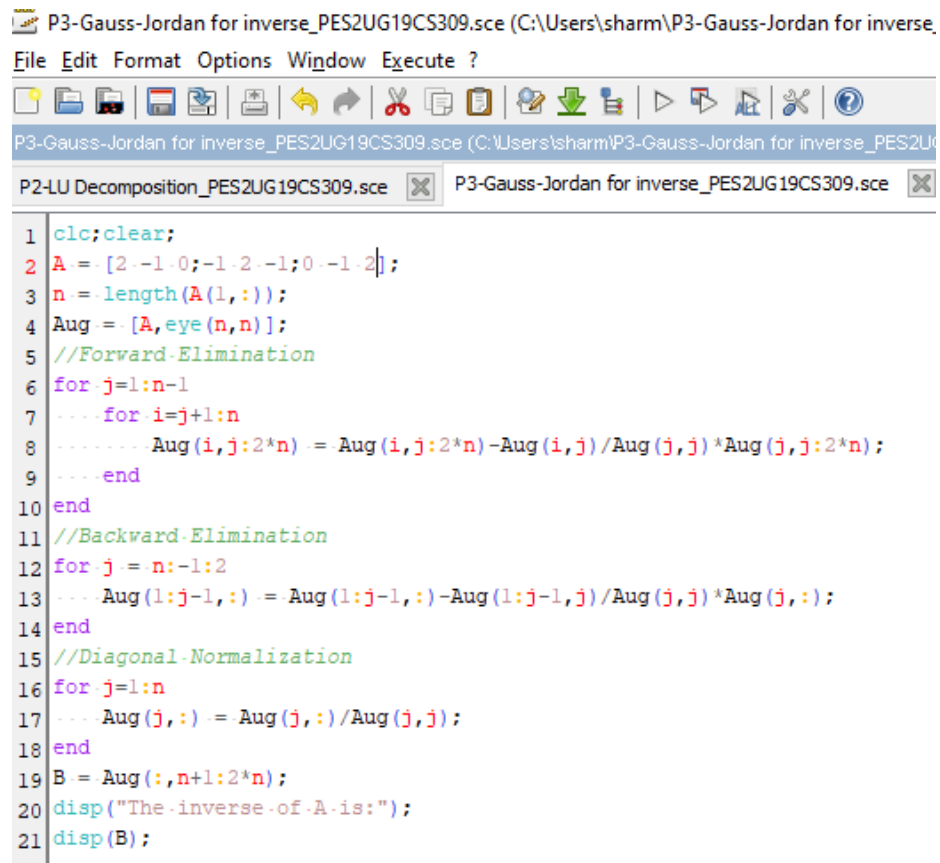
1.    0.    0.
2.    1.    0.
0.   -2.    1.

"The lower triangular matrix is:"
```

Topic: The Gauss - Jordan method of calculating A⁻¹

PROBLEM 3: Find the inverse of the following matrix
 $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -2 & 2 \end{bmatrix}$

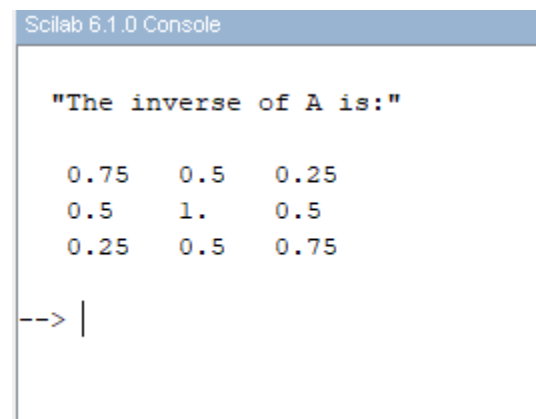
CODE:



```
P3-Gauss-Jordan for inverse_PES2UG19CS309.sce (C:\Users\sharm\P3-Gauss-Jordan for inverse_PES2UG19CS309.sce)
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P3-Gauss-Jordan for inverse_PES2UG19CS309.sce (C:\Users\sharm\P3-Gauss-Jordan for inverse_PES2UG19CS309.sce)
P2-LU Decomposition_PES2UG19CS309.sce P3-Gauss-Jordan for inverse_PES2UG19CS309.sce

1 clc;clear;
2 A = [2 -1 0; -1 2 -1; 0 -2 2];
3 n = length(A(1,:));
4 Aug = [A, eye(n,n)];
5 //Forward-Elimination
6 for j=1:n-1
7     for i=j+1:n
8         Aug(i,j:2*n) = Aug(i,j:2*n) - Aug(i,j)/Aug(j,j)*Aug(j,j:2*n);
9     end
10 end
11 //Backward-Elimination
12 for j=n:-1:2
13     Aug(1:j-1,:) = Aug(1:j-1,:) - Aug(1:j-1,j)/Aug(j,j)*Aug(j,:);
14 end
15 //Diagonal-Normalization
16 for j=1:n
17     Aug(j,:) = Aug(j, :)/Aug(j,j);
18 end
19 B = Aug(:,n+1:2*n);
20 disp("The inverse of A is:");
21 disp(B);
```

OUTPUT:



```
Scilab 6.1.0 Console

"The inverse of A is:"

0.75    0.5    0.25
0.5     1.     0.5
0.25    0.5    0.75

--> |
```

Topic: Span of the Column Space of A

PROBLEM 4: Identify the columns that are in the column space of A where $A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}$

CODE:

```
P4-Span of column space_PES2UG19CS309.sce (C:\Users\sharm\P4-Span of column space_PES2UG19CS309.sce)
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P4-Span of column space_PES2UG19CS309.sce (C:\Users\sharm\P4-Span of column space_PES2UG19CS309.sce)
P4-Span of column space_PES2UG19CS309.sce X

1 clc;clear;
2 a = [2 4 6 4;2 5 7 6;2 3 5 2];
3 disp("The-given-matrix-is:");
4 disp(a);
5 a(2,:) = a(2, :)-(a(2,1)/a(1,1))*a(1, :);
6 a(3,:) = a(3, :)-(a(3,1)/a(1,1))*a(1, :);
7 disp(a);
8 a(3,:) = a(3, :)-(a(3,2)/a(2,2))*a(2, :);
9 disp(a);
10 a(1,:) = a(1, :)/a(1,1);
11 a(2,:) = a(2, :)/a(2,2);
12 disp(a);
13 for i=1:3
14     for j=i:4
15         if(a(i,j)<>0)
16             disp("is-a-pivot-column",j,"column");
17             break;
18         end
19     end
20 end
21
```

OUTPUT:

```
Scilab 6.1.0 Console

"The given matrix is:"

2.  4.  6.  4.
2.  5.  7.  6.
2.  3.  5.  2.

2.  4.  6.  4.
0.  1.  1.  2.
0. -1. -1. -2.

2.  4.  6.  4.
0.  1.  1.  2.
0.  0.  0.  0.

1.  2.  3.  2.
0.  1.  1.  2.
0.  0.  0.  0.

"is a pivot column"

1.

"column"

"is a pivot column"

2.

"column"

-->
```

Topic: The Four Fundamental Subspaces

PROBLEM 5: Find the four fundamental subspaces of $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

CODE:

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P5-Four fundamental subspaces_PES2UG19CS309.sce (C:\Users\sharm\P5-Four fundamental subs
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P5-Four fundamental subspaces_PES2UG19CS309.sce (C:\Users\sharm\P5-Four fundamental subspaces_PES
P4-Span of column space_PES2UG19CS309.sce P5-Four fundamental subspaces_PES2UG19CS309.sce
1 clc;clear;
2 A = [1 2 0 1; 0 1 1 0; 1 2 0 1];
3 disp("The given matrix is:");
4 disp(A);
5 [m,n] = size(A);
6 disp(m,"m = ");
7 disp(n,"n = ");
8 [v,pivot] = rref(A);
9 disp(rref(A),"Row-Reduced-Echelon-Form: ");
10 r = length(pivot);
11 disp(r,"Rank: ");
12 colspace = A(:,pivot);
13 disp(colspace,"Column-Space: ");
14 nullspace = kernel(A);
15 disp(nullspace,"Null-Space: ");
16 rowpace = v(1:r,:);
17 disp(rowpace,"Row-Space: ");
18 leftnullspace = kernel(A');
19 disp(leftnullspace,"Left-Null-Space: ");
20
```

OUTPUT:

```
Scilab 6.1.0 Console

"Row Reduced Echelon Form: "

2.

"Rank: "

1. 2.
0. 1.
1. 2.

"Column Space: "

3.909D-17 -0.8660254
-0.4082483 0.2886751
0.4082483 -0.2886751
0.8164966 0.2886751

"Null Space: "

1. 0.
0. 1.
-2. 1.
1. 0.

"Row Space: "

-0.7071068
1.106D-16
0.7071068

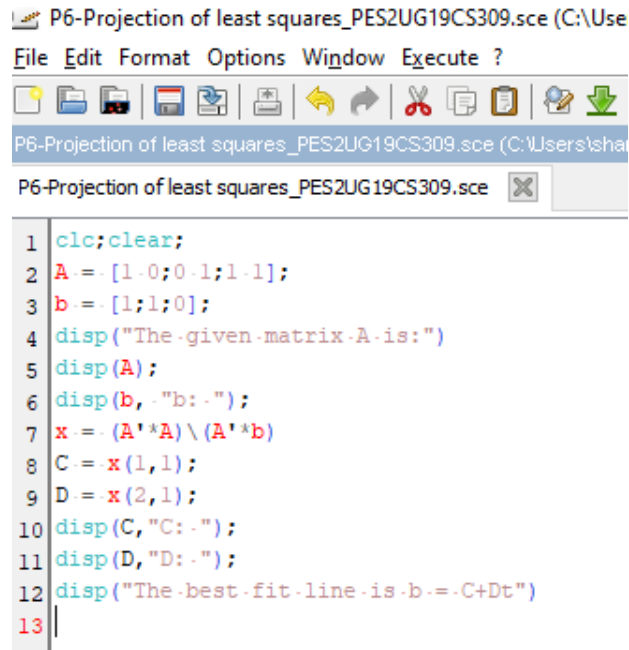
"Left Null Space: "
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Topic: Projections by Least Squares

PROBLEM 6: Find the solution $x = (C, D)$ of the system $Ax = b$ and the line of best fit $C + Dt = b$ given

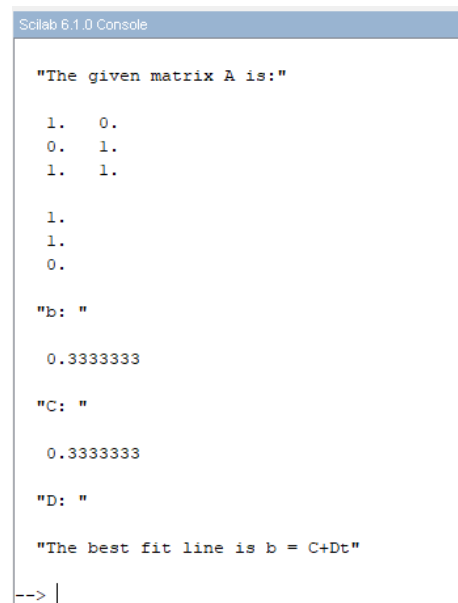
$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

CODE:



```
1 clc;clear;
2 A = [1 0; 0 1; 1 1];
3 b = [1; 1; 0];
4 disp('The given matrix A is:');
5 disp(A);
6 disp(b, 'b: ');
7 x = (A'*A) \ (A'*b);
8 C = x(1,1);
9 D = x(2,1);
10 disp(C, 'C: ');
11 disp(D, 'D: ');
12 disp('The best fit line is b = C+Dt');
13 |
```

OUTPUT:



```
Scilab 6.1.0 Console

"The given matrix A is:"

1.   0.
0.   1.
1.   1.

1.
1.
0.

"b: "

0.3333333

"C: "

0.3333333

"D: "

"The best fit line is b = C+Dt"

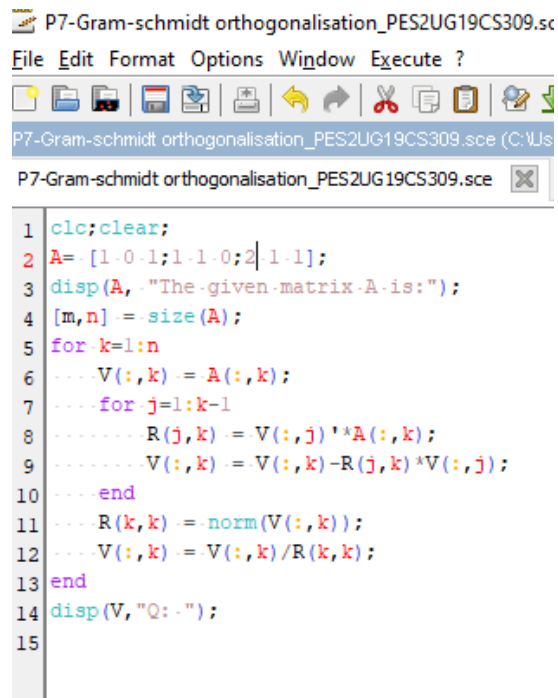
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```


Topic: The Gram- Schmidt Orthogonalization

PROBLEM 7: Apply the Gram – Schmidt process to the following set of vectors and find the orthogonal matrix:

$(1, 0, 1), (1, 1, 0), (2, 1, 1)$

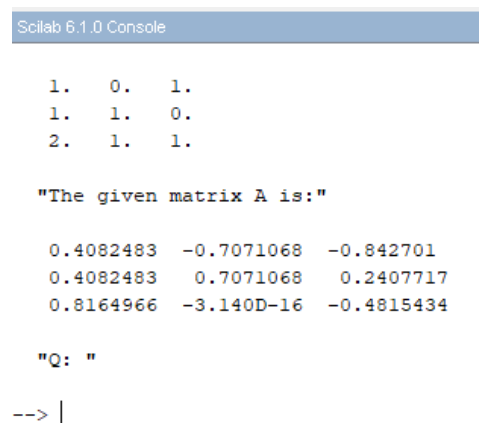
CODE:



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P7-Gram-schmidt orthogonalisation_PES2UG19CS309.sc
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P7-Gram-schmidt orthogonalisation_PES2UG19CS309.sce (C:\J...
P7-Gram-schmidt orthogonalisation_PES2UG19CS309.sce X

1 clc;clear;
2 A=[1 0 1;1 1 0;2 1 1];
3 disp(A, "The given matrix A is:");
4 [m,n] = size(A);
5 for k=1:n
6     V(:,k) = A(:,k);
7     for j=1:k-1
8         R(j,k) = V(:,j)'*A(:,k);
9         V(:,k) = V(:,k)-R(j,k)*V(:,j);
10    end
11    R(k,k) = norm(V(:,k));
12    V(:,k) = V(:,k)/R(k,k);
13 end
14 disp(V, "Q: ");
15
```

OUTPUT:



```
Scilab 6.1.0 Console

1.    0.    1.
1.    1.    0.
2.    1.    1.

"The given matrix A is:"

0.4082483  -0.7071068  -0.842701
0.4082483   0.7071068   0.2407717
0.8164966  -3.140D-16  -0.4815434

"Q: "

--> |
```

Topic: Eigen values and Eigen vectors of a given square matrix

PROBLEM 8: Find the Eigen values and the corresponding Eigen vectors of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

CODE:

```
P8-Eigen values and vectors_PES2UG19CS309.sce (C:\Users\sharm\P8-Eigen val
File Edit Format Options Window Execute ?
P8-Eigen values and vectors_PES2UG19CS309.sce (C:\Users\sharm\P8-Eigen values and v
P8-Eigen values and vectors_PES2UG19CS309.sce

1 clc;clear;
2 A = [8 -6 2; -6 7 -4; 2 -4 3];
3 disp(A, "The given matrix A is: ")
4 lam = poly(0, "lam");
5 charMat = A - lam * eye(3, 3);
6 disp(charMat, "The Characteristic Matrix is: ");
7 charPoly = poly(A, "lam");
8 disp(charPoly, "The Characteristic Polynomial is: ");
9 lam = spec(A);
10 disp(lam, "Eigen Values: ");
11 function [x, lam] = eigenvectors(A)
12     [n, m] = size(A);
13     lam = spec(A)';
14     x = [];
15     for k = 1:3
16         B = A - lam(k) * eye(3, 3);
17         C = B(1:n-1, 1:n-1);
18         b = -B(1:n-1, n);
19         y = C\b;
20         y = [y; 1];
21         y = y / norm(y);
22         x = [x y];
23     end
24 endfunction
25 [x, lam] = eigenvectors(A);
26 disp(x, "Eigen Vectors of A: ");
27
```

OUTPUT:

```
Scilab 6.1.0 Console

8.  -6.  2.
-6.  7.  -4.
2.  -4.  3.

"The given matrix A is: "

8 -lam  -6      2
-6      7 -lam  -4
2      -4      3 -lam

"The Characteristic Matrix is: "

-7.128D-14 +451am -181am^2 +lam^3

"The Characteristic Polynomial is:"

1.584D-15
3.
15.

"Eigen Values: "

0.3333333  -0.6666667  0.6666667
0.6666667  -0.3333333  -0.6666667
0.6666667  0.6666667  0.3333333

"Eigen Vectors of A: "

--> |
```

