

Phase Lead Compensation of an Inverted Pendulum

3.1 Position control

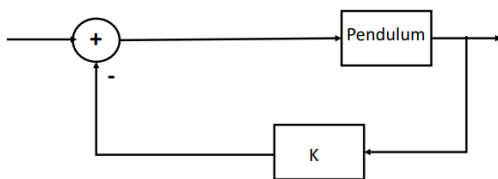
$$Y = X + L\sin(\theta) \approx Y = X + L\theta$$

X position represented by V_x

$$\therefore V_y = V_x + aV\theta$$

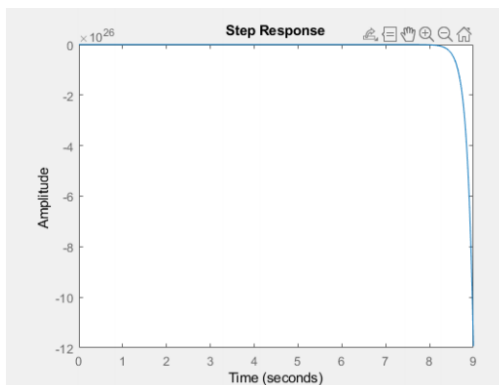
3.2 Proportional control

5. Negative feedback

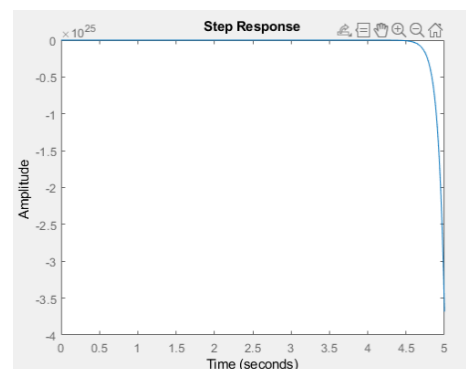


pendulum = tf(-49.05,[1,0,-49.05])

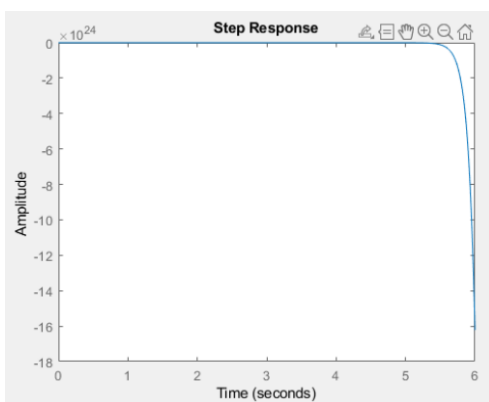
- K=0



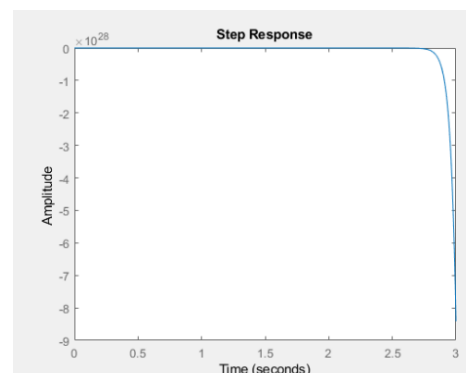
- K=2



- K=1

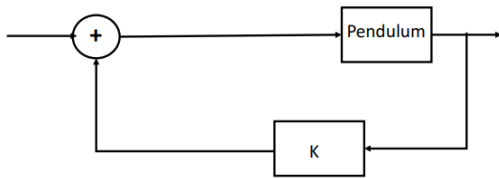


- K=10

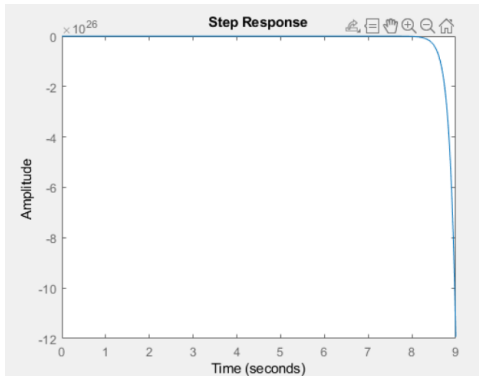


➔ System is unstable

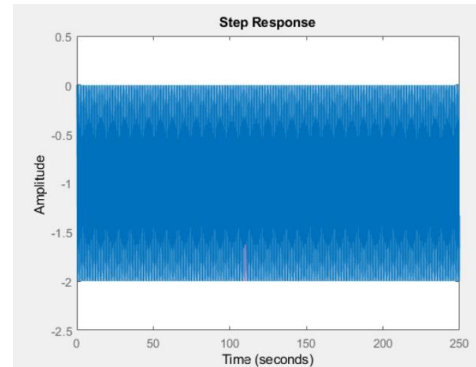
6. Positive feedback



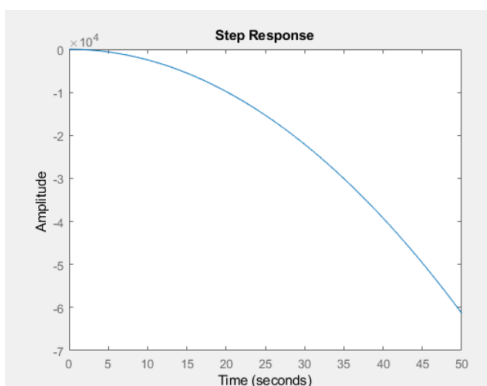
- K=0



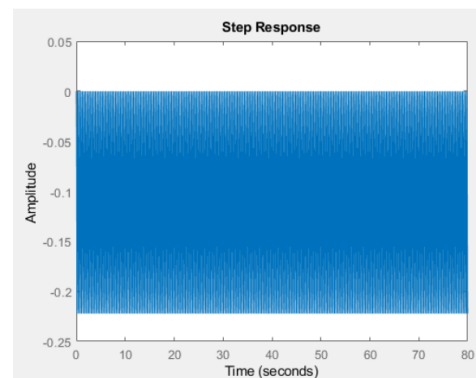
- K=2



- K=1



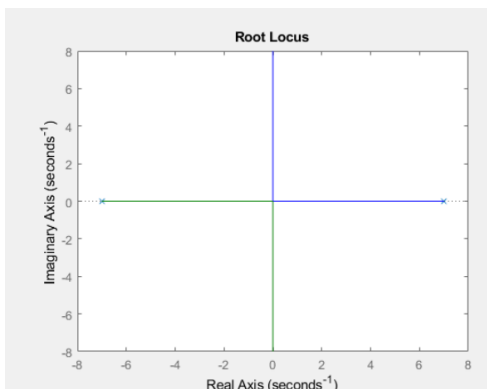
- K=10



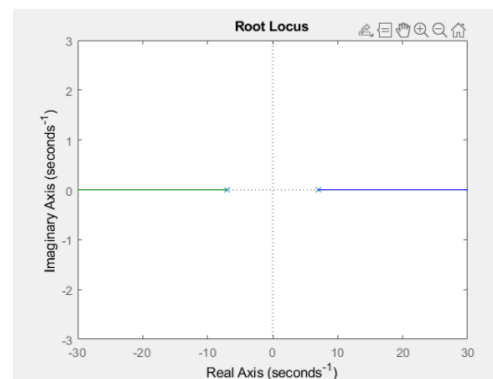
→ System becomes more stable, as k increases, the system no longer tends towards $\pm\infty$

7.

Positive feedback



Negative feedback



8.

- System is unstable
- Not all roots are stable
- Blue pole becomes more unstable
- Green pole becomes more stable

- System becomes (marginally) stable
- Both poles move towards stable/unstable: marginal stability to $\pm\infty$

9. Compare the behaviour qualitatively for different values of the gain. Is it consistent with the root-locus predictions?

As gain increases, pendulum accelerates increasingly to the side the pendulum leans.

Large gain: pendulum becomes erratic and unstable.

Corresponding to root loci, showing as K increases → system= unstable

3.3 Lead compensation

$$H(s) = \frac{1 + c\tau s}{1 + \tau s}$$

Place zero of compensator. Directly cancel stable pole of plant, choosing

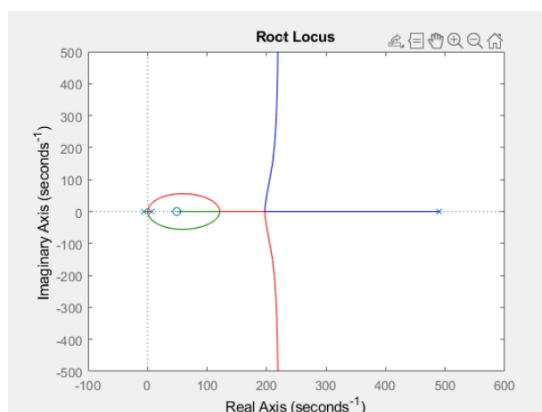
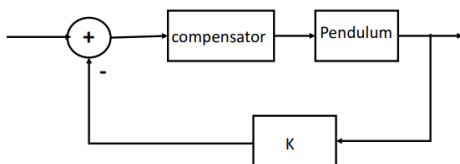
$$\omega = \frac{1}{c\tau}$$

Use 'high' value of c, e.g. c=10

Use compensator of the form

$$H_1(s) = \frac{1 + 0.1s}{1 + 0.01s}$$

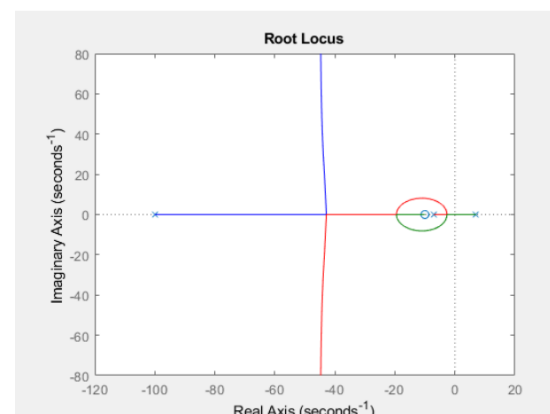
10.



- System is initially unstable
- System remains unstable
- 2 poles are marginally stable but become unstable

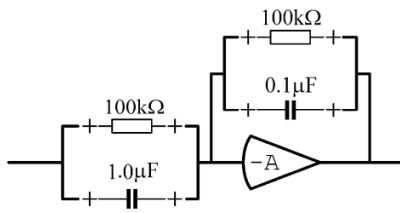
11.

compensator2 = tf([0,0.1,1],[0,0.01,1])



- 2 poles circle 1 root, ending at green point
- Another starts becoming stable, tends to ∞ (red)
- Blue pole starts becoming unstable, then tends to ∞

$H_1(s)$ = op-amp compensator circuit



V_0 = output voltage

V_1 = input voltage

Z_f = feedback impedance

Z_1 = impedance between V_1 and summing junction

$$V_0 / V_1 = Z_f / Z_1$$

For parallel circuit

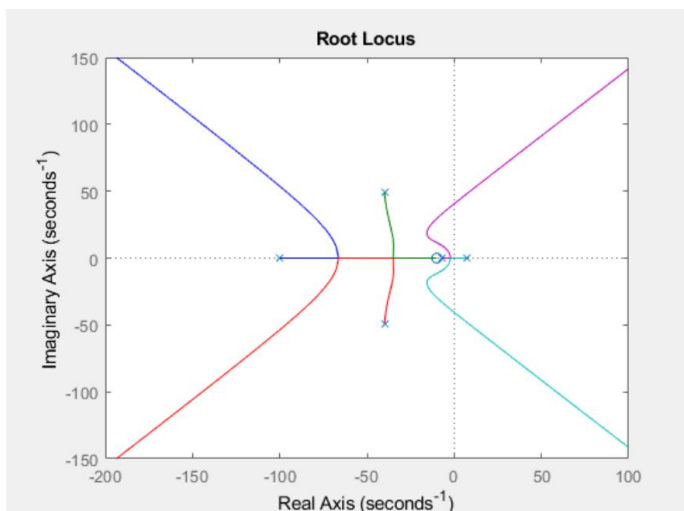
R = resistor

C_1 = capacitor

$$1/Z_1 = (1/R) + C_1 s \quad \text{and} \quad 1/Z_f = (1/R) + C_f s$$

$$\therefore V_0 / V_1 = - (1 + R C_1 s) / (1 + R C_f s)$$

12.Explain using the root-locus plot why small values of the gain P_2 do not lead to a stable closed loop.



- ➔ Small gain, green pole= unstable
- ➔ \therefore whole system = unstable
- ➔ System becomes stable after 1 pole crosses y axis
- ➔ All other poles are stable in the beginning

3.4 The neglected servo dynamics

$$G(s) = \frac{1}{0.00025s^2 + 0.02s + 1}$$

Natural frequency: $\omega = 10\text{Hz}$

Damping ratio $\zeta = 0.7$

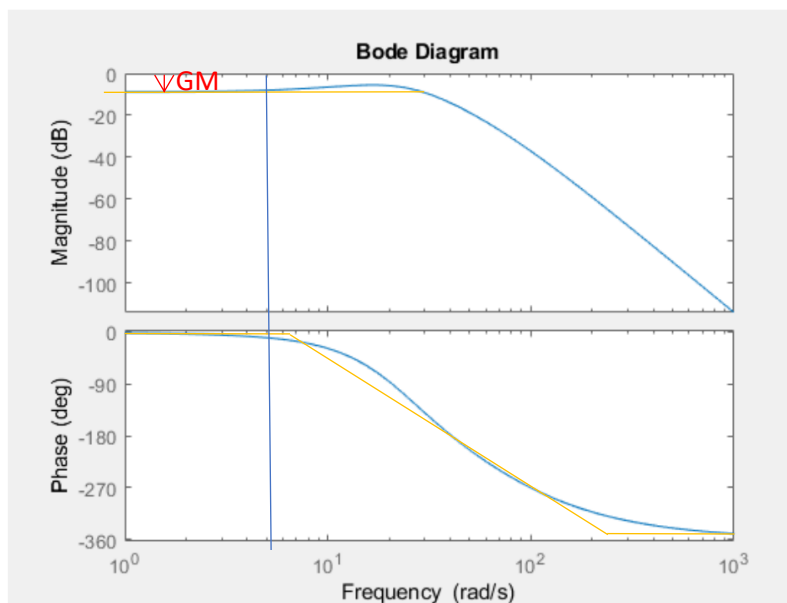
13.

Upper limit: 8.7965

Lower limit: 0.9915

When gain > upper limit, pendulum becomes unstable and fails to balance

14.



Gain k:

(max) Gain margin: 8.83db

Phase margin: 344 degrees (?)

15.

Tapping the weight: system responds more aggressively to tapping

Makes small sudden changes before adjusting

16.

Moving the weight to the bottom of the rod causes the system to vibrate a lot and moves very quickly to adjust to the tap.