WORKED EXAMPLE C5.2

a Calculate the change of frequency which will be detected by a police speed gun (see Figure C5.10) using a frequency of 20.6 GHz, when it is directed at a car moving directly away at a speed of $130\,\text{km h}^{-1}$ ($36.1\,\text{m s}^{-1}$).

■ **Figure C5.10** A police speed check

b State what change of frequency would be detected if the vehicle was moving directly *towards* the speed gun, at the same speed.

Answer

a $\frac{\Delta f}{f_0} = \frac{2v}{c}$

$\Delta f = 2 \times 36.1 \times \frac{20.6 \times 10^9}{3.00 \times 10^8} = 4.96 \times 10^3\,\text{Hz}$ (decrease)

b The same magnitude as in part **a**, but the frequency would increase, rather than decrease.

ATL C5A

Research and communication skills

Use a variety of internet websites to learn about *Doppler weather radar*.

Evaluate your sources for reliability.

Plan a short presentation of the key information for other IB physics students.

5 The speed limit in a town is $50\,\text{km h}^{-1}$ ($13.9\,\text{m s}^{-1}$). In a safety check by the police, using the Doppler effect, a frequency increase of 2.85 kHz was detected from a car moving along a straight road.

- **a** State whether the car was moving towards, or away from, the check point.
- **b** If the speed gun used a frequency of 32.8 GHz, show that the car was travelling slower than the legal limit.

6 An airport radar system using microwaves of frequency 2.72 GHz sends out a pulse of waves that is reflected off an aircraft that is within its control area.

A reflected signal is received back at the airport 1.374×10^{-4} s later, at a frequency of 1050 Hz more than the emitted wave. At that time the aerial was pointing exactly north.

- **a** Determine the distance between the aircraft and the radar aerial.
- **b** **i** Use the equation:

  $\frac{\Delta f}{f_0} = \frac{2v}{c}$

  to determine a value of v for the aircraft.

  ii Was the aircraft getting closer to, or further away from, the airport?

■ **Figure C5.11** Air traffic control uses the Doppler effect

c Your answer to part **b** should be less than the true value of the aircraft's speed ($120\,\mathrm{m\,s^{-1}}$) because it was not flying directly to the aerial at the airport at that moment. v was the component of its velocity towards the airport.

Make a sketch showing the positions of the airport and the aircraft, and the direction to north. Add vector arrows to represent the true speed of the aircraft and its component towards the airport.

7 Suggest how it might be possible for a military aircraft to avoid being detected by radar.

Tool 3: Mathematics

Propagate uncertainties in processed data

A motorist accused of driving over the speed limit might claim that their speed was within the 'margin of error' of the equipment used by the police. Consider again the data provided in Question 5.

If the value of the microwave frequency used was believed to be accurate to $\pm$ 0.2 GHz, and the change of frequency accurate to $\pm$ 0.05 kHz, what was the uncertainty in the determination of the car's speed (in $\mathrm{km\,h^{-1}}$)?

LINKING QUESTION

- What gives rise to emission spectra and how can they be used to determine astronomical distances?

This question links to understandings in Topics B.1 and E.5.

Doppler effect with light received from distant stars and galaxies

SYLLABUS CONTENT

- Shifts in spectral lines provide information about the motion of objects like stars and galaxies in space.

Top tip!

Everything emits thermal radiation, and we have seen in Topic B.1 that this emitted radiation is in the form of a *continuous spectrum,* with a wide range of frequencies. In Topic C.2 we saw that a continuous light spectrum can be displayed on a screen using a prism to disperse the radiation.

Individual atoms and simple molecules emit electromagnetic radiation of certain precise frequencies, rather than a continuous range of frequencies. Topic E.1 explains how this is connected to changing energy levels within the atoms. When elements (in the form of gases) are given enough energy, the spectra of the light that they emit are seen as a series of bright lines on black backgrounds – called line spectra. See Figure C5.12. Each line corresponds to a precise frequency.

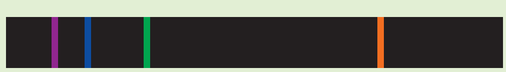


Figure C5.12 The principal lines of the line spectrum of hydrogen

When a continuous spectrum passes through a gas, the atoms in the gas will absorb the same frequencies as they would emit when given energy. This results in a spectrum with black absorption lines, as seen in Figure C5.13.

Atoms of the elements present in the outer layers of a star absorb light of certain frequencies from the continuous spectrum of radiation emitted from the star. Each different element produces a unique set of lines and frequencies, and this can be used to identify the element emitting the radiation.

◆ **Star** Massive sphere of plasma held together by the forces of gravity. Because of the high temperatures, thermonuclear fusion occurs and radiation is emitted.

◆ **Galaxy** A very large number of stars (and other matter) held together in a group by the forces of gravity.

◆ **Expansion of the universe** The redshift of light (similar to the Doppler effect) from distant galaxies provides evidence of an expanding universe.

◆ **Redshift, (Doppler effect)** Increase in wavelengths of electromagnetic radiation due to the fact that the distance between the observer and the source is increasing.

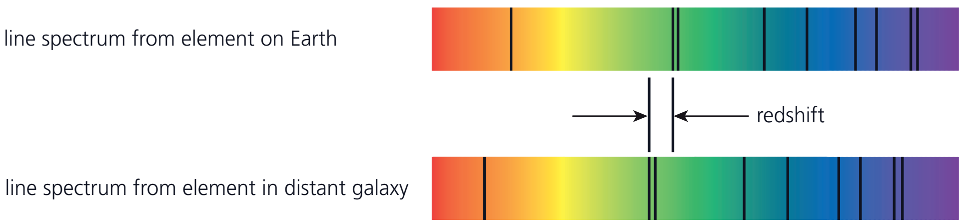
◆ **Blueshift** Decrease in wavelength of electromagnetic radiation (from a 'nearby' star) due to the fact that the observer and the source are moving closer together.

◆ **Recession speed** The speed with which a galaxy (or star) is moving away from Earth.

The line spectra received from **stars** within distant **galaxies** are received at lower frequencies than light from similar elements on Earth. This can be considered to be a Doppler effect and it suggests that the source and the observer are moving apart from each other.

Measurements of the Doppler effect in line spectra from stars in distant galaxies show that most galaxies are moving apart from each other – the Universe is **expanding**.

The lines on the upper spectrum seen in Figure C5.13 are produced from an element on Earth. The lower spectrum is received from the same element in a distant galaxy. Although the continuous spectra are identical, the black lines, which are characteristic of a certain element(s), have all been 'shifted' towards the red end of the spectrum, although their overall pattern is unchanged. This change to lower frequencies (larger wavelengths) is called a **redshift**. A change to higher frequency is called a **blueshift**. (Blueshifts only occur for a few stars relatively close to the Earth for reasons that need not be understood.)



■ **Figure C5.13** Redshift in line spectra

The equation:

$$\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$$

can be used to determine the speed with which a star or galaxy is moving away from the Earth.

WORKED EXAMPLE C5.3

A line in the hydrogen spectrum has a wavelength of 4.86×10^{-7} m. When detected on Earth from a distant galaxy, the same line has a wavelength of 5.21×10^{-7} m. Determine the speed with which the galaxy is moving away from Earth. This is commonly called its **recession speed**.

Answer

$\Delta\lambda = (5.21 \times 10^{-7}) - (4.86 \times 10^{-7}) = 3.5 \times 10^{-8}$ m

$$\frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$$

$$\frac{3.5 \times 10^{-8}}{4.86 \times 10^{-7}} = \frac{v}{3.00 \times 10^{8}}$$

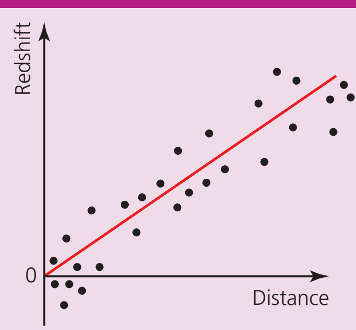
$v = 2.16 \times 10^{7}\,\text{m s}^{-1}$

Inquiry 3: Concluding and evaluating

Figure C5.14 shows a sketch graph which summarizes how the amount of redshift detected varies with the distance of the galaxies or stars from Earth. There are significant experimental uncertainties involved, which are not shown, so that many points are not close to the line of best fit.

1 What conclusion can be drawn from this graph (for positive values of redshift)?

2 There are a few points below the horizontal axis. They are not errors. Suggest a possible explanation.



■ **Figure C5.14** Variation of redshift

◆ **Big Bang model** Currently accepted model of the Universe, in which matter, space and time began at a point 13.7 billion years ago. The Universe has been expanding ever since.

Measurements on a large number of galaxies confirm that those with the greatest speeds are those which are furthest away. There is an obvious conclusion: they are further away *because* they are travelling faster. Moving back in time, they all started at the same place and time. This is the central concept of the **Big Bang model**. The Universe was created about fourteen billion years ago and has been expanding ever since.

Astronomers believe that all space itself is expanding, rather than stars and galaxies moving apart from each other into pre-existing space. This means that the full explanation of the Doppler effect with light from distant galaxies is not the same as the Doppler effect used to describe wave effects confined to Earth.

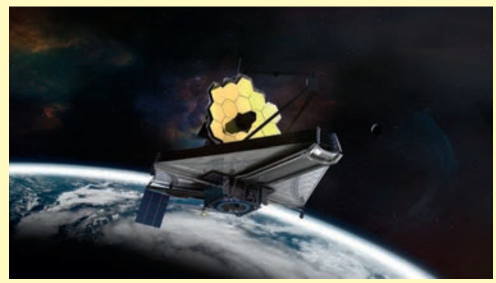
TOK

Knowledge and the knower

- How do the tools that we use shape the knowledge that we produce?

Nearly everything that we know about the Universe has been deduced only from electromagnetic radiation arriving at the Earth from space. This radiation has been detected by various types of telescopes and analysed using the techniques of **spectroscopy**. Astronomy is based solely on observation; we can choose what to observe, but we cannot design and carry out the type of laboratory experiments that characterize much of the rest of science.

Our knowledge of the Universe is limited by the instruments that we design to collect and analyse the radiation reaching the Earth.



■ **Figure C5.15** The James Webb Telescope was launched in December 2021

◆ **Spectroscopy** The analysis of spectra using instruments called spectroscopes or *spectrometers*.

8 A certain line on the helium spectrum has a well-known wavelength, but when observed from a distant galaxy it has a redshift of 1.85×10^{-8} m away from that value.
If the galaxy is receding from Earth at a speed of $7.84 \times 10^{6}\,\text{m s}^{-1}$, determine the original wavelength of the wave.

9 **a** Calculate the size of the redshift in frequency of a wave of frequency 6.17×10^{16} Hz (from the hydrogen spectrum) received from a galaxy which has a recession speed of:
  i $2.20 \times 10^{6}\,\text{m s}^{-1}$
  ii 10% of the speed of light.
b Determine the frequencies that will be detected on Earth.

10 Galaxies contain billions of stars all orbiting their common centre of mass (the centre of the galaxy).
If we are able to observe the stars in a galaxy ('side-on') suggest how the Doppler effect can be used to determine their rotational speeds around the centre.

LINKING QUESTIONS

- How can the Doppler effect be utilized to measure the rotational speed of extended bodies?
- How can the use of the Doppler effect for light be used to calculate speed? (NOS)

These questions link to understandings in Topics A.4.

TOK

Knowledge and technology

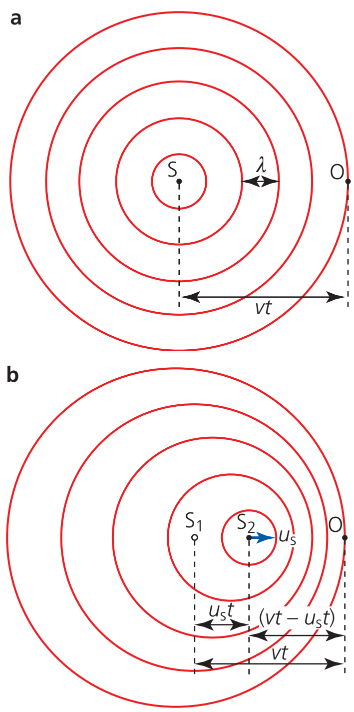
- To what extent are technologies, such as the microscope and telescope, merely extensions to the human senses, or do they introduce radically new ways of seeing the world?

Doppler first identified the effect that bears his name about 180 years ago, but he could not have foreseen the many useful applications that his discovery would lead to. This is mainly because the relatively small changes of frequency involved require a high level of technology.

Equations for use with the Doppler effect for sound (or other mechanical waves)

SYLLABUS CONTENT

- The observed frequency for sound and mechanical waves due to the Doppler effect as given by:
  moving source, $f' = f\left(\frac{v}{v \pm u_s}\right)$, where u_s is the velocity of the source
  moving observer, $f' = f\left(\frac{v \pm u_o}{v}\right)$ where u_o is the velocity of the observer.



■ **Figure C5.16 a** Waves between a stationary source and a stationary observer **b** Waves between a moving source and stationary observer

Figure C5.16a shows waves of frequency f and wavelength λ travelling at a speed v between a stationary source S and a stationary observer O. In the time, t, that it takes the first wavefront emitted from the source to reach the observer, the wave has travelled a distance vt. The number of waves between the source and observer is $\frac{t}{T} = ft$.

The wavelength, λ, equals the total distance divided by the number of waves:

$\lambda = \frac{vt}{ft} = \frac{v}{f}$ as we would expect from Topic C.2.

Figure C5.16b represents exactly the same waves emitted in the same time from a source moving towards a stationary observer with a speed u_s. In time, t, the source has moved from S_1 to S_2. The number of waves is the same as Figure C5.16a, but because the source has moved forwards a distance, $u_s t$, the waves between the source and the observer are now compressed within the length $vt - u_s t$.

This means that the observed (received) wavelength, λ', equals the total distance divided by the number of waves:

$$\lambda' = \frac{vt - u_s t}{ft} = \frac{v - u_s}{f}$$

The observed (received) frequency, f', is given by:

$$f' = \frac{v}{\lambda'} = \frac{vf}{v - u_s}$$

If the source is moving away from the observer, the equation becomes:

$$f' = \frac{vf}{v + u_s}$$

In general, we can write:

DB

$$\boldsymbol{f' = f\left(\frac{v}{(v \pm u_s)}\right)}$$

This is the equation for the Doppler effect from a *moving source* (speed u_s) detected by a stationary observer.

u_s is added when the source is moving away from the observer, and subtracted when the motion is towards the observer.

DB

The equation for the frequency detected by a *moving observer* from a stationary source is:

$$\boldsymbol{f' = f\left(\frac{v \pm u_o}{v}\right)}$$

HL ONLY

u_o is added when the observer is moving towards the source, and subtracted when the motion is away from the source.

These equations assume that the motion involved is in the same direction as a straight line joining the source and the observer.

Remember that these equations cannot be applied to electromagnetic waves ($c >> v$).

WORKED EXAMPLE C5.4

a A source of sound emitting a frequency of 480 Hz is moving directly towards a stationary observer at $50.0\,\text{m}\,\text{s}^{-1}$.

If it is a hot day and the speed of sound is $350\,\text{m}\,\text{s}^{-1}$, calculate the frequency received.

b What frequency would be heard on a cold day when the speed of sound was $330\,\text{m}\,\text{s}^{-1}$?

c Explain why the speed of sound is less on a colder day.

Answer

a $f' = f\left(\frac{v}{v - u_s}\right) = \frac{(480 \times 350)}{(350 - 50)} = 560\,\text{Hz}$

b $f' = \frac{(480 \times 350)}{(330 - 50)} = 566\,\text{Hz}$

c Sound is transferred though air by moving air molecules. On a colder day the molecules will have a lower average speed.

11 Calculate the frequency which will be received by an observer moving with a speed of $24.2\,\text{m}\,\text{s}^{-1}$ directly away from a stationary source of sound waves of frequency 980 Hz. (Assume the speed of sound to be $342\,\text{m}\,\text{s}^{-1}$.)

12 In a Doppler ultrasound measurement, as shown in Figure C5.5, blood was flowing at a rate of $9.7\,\text{cm}\,\text{s}^{-1}$ along the artery.

- **a** If the angle $\theta = 75°$, calculate the speed that the transducer should detect.
- **b** If the transducer emits waves of frequency 5.87400 MHz, and the speed of the ultrasound waves is $1540\,\text{m}\,\text{s}^{-1}$, determine the frequency received by the blood cells.

13 A car is travelling along a straight road at a constant speed of $31\,\text{m}\,\text{s}^{-1}$. The car emits a sound with a constant frequency of 224 Hz. A pedestrian on a footbridge over the motorway watches the car approach, travel directly under them, and then move away from the bridge. (Assume the speed of sound to be $342\,\text{m}\,\text{s}^{-1}$.)

- **a** Determine the frequencies heard by the pedestrian:
  - **i** before the car passes the footbridge
  - **ii** after the car passes the footbridge.
- **b** Another person watches and hears the same car but is many metres away from the side of the road. Describe and explain how the sound reaching this person is different from the sound heard by the pedestrian on the footbridge.

14 An ultrasound wave of frequency 30.0 kHz is directed at an approaching car. The wave reflects off the car and is received back at the stationary emitter with a frequency of 32.7 kHz.

- **a** Using an equation highlighted on page 409, calculate the velocity of the car. (Assume that the speed of sound is $335\,\text{m}\,\text{s}^{-1}$.)
- **b** Compare your answer to the answer obtained by using:

$$\frac{\Delta f}{f} \approx \frac{\text{relative speed of source}}{\text{speed of waves}}$$

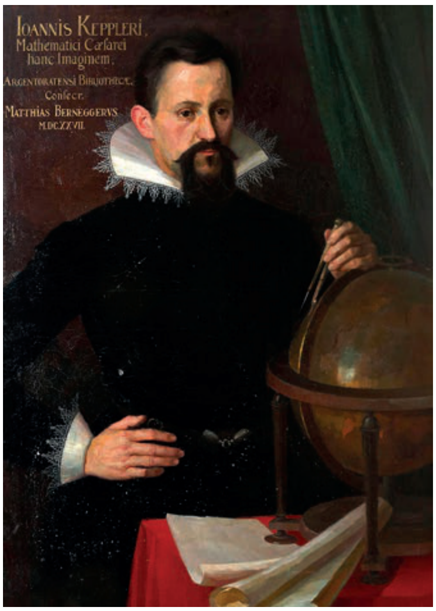
15 A boat is travelling directly towards a jetty at a speed of $37\,\text{cm}\,\text{s}^{-1}$ and creating waves on the water surface of original wavelength 59 cm.

If the water waves travel at a speed of $94\,\text{cm}\,\text{s}^{-1}$, determine the frequency and wavelength of the waves reaching the jetty.

D.1 Gravitational fields

Guiding questions

- How are the properties of a gravitational field quantified?
- How does an understanding of gravitational fields allow for humans to explore the Solar System?



■ **Figure D1.1** Johannes Kepler

The fact that (most) objects tend to fall towards the ground is, of course, a common and unsurprising observation. But a satisfactory explanation was not achieved until the work of Isaac Newton in the seventeenth century. Until then, Aristotle's ideas, from more than two thousand years earlier, were the accepted wisdom: falling objects were just returning to their 'natural' places.

A scientific understanding of gravity requires accurate observations of objects moving large distances where gravitational effects are variable and unaffected by air resistance: the planets and the Moon.

The Danish nobleman Tycho Brahe was renowned for his remarkably accurate astronomical measurements at a time before the invention of the telescope (late sixteenth century). But they were just that: empirical observations, without explanation.

Johannes Kepler (Figure D1.1) worked with Brahe's data and analysed his measurements mathematically, particularly those concerning the planet Mars. Kepler's three laws of planetary motion were a key development in the history of astronomy.

Kepler's laws of planetary motion

SYLLABUS CONTENT

▶ Kepler's three laws of orbital motion.

Kepler's laws were developed to describe the motions of the planets in the Solar System, but they can be applied to any group of bodies orbiting a common centre under the effects of gravity. (To *orbit* means to continuously move, revolving around another, larger, object.)

Kepler's laws were empirical (based on observations) and it was not until 70 years later that Newton was able to provide the underlying explanation.

Tool 3: Mathematics

Construct and use scale diagrams

An **ellipse** is the name that we give to the type of complete curve for which for all points: the sum of the distances from two fixed points is always the same. See Figure D1.2.

The two fixed points, shown as F_1 and F_2 in Figure D1.2, are each called a focus of the ellipse (plural: foci). If F_1 and F_2 are at the same point, in the centre, the ellipse becomes a circle.

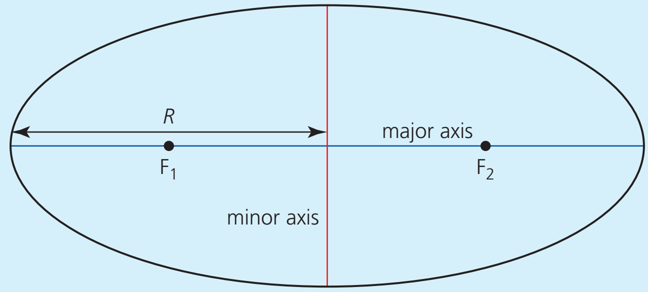


Figure D1.2 An ellipse

◆ **Ellipse** Closed curve consisting of points whose distances from each of two fixed points (**focuses, foci**) always add up to the same value.

Kepler's first law

Kepler showed that the planets of the Solar System move in *elliptical* paths. See Figure D1.3.

The planets orbit in elliptical paths, with the Sun at one of the two foci.

It should be noted that the orbits of most of the planets of our Solar System are nearly circular. Figures D1.2 and D1.3 have exaggerated the shape (*eccentricity*) of the ellipses.

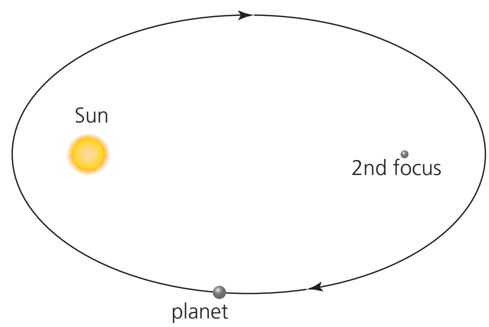


Figure D1.3 Elliptical path of a planet.

Kepler's second law

Kepler's second law expresses the fact that planets move faster when they are closer to the Sun.

A line joining a planet and the Sun sweeps out equal areas in equal times.

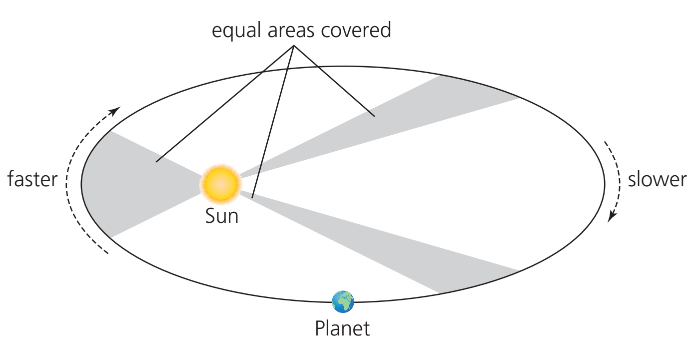


Figure D1.4 Equal areas in equal times

Kepler's third law

In effect, the third law provides a mathematical relationship between half the length of the major axis (R in Figure D1.1) and the planet's speed. In practice, because many orbits are close to being circular, we can usually assume that R is the average distance (orbital radius) to the Sun:

The square of a planet's orbital time period, T, is proportional to the cube of its average orbital radius, R:

$$T^2 \propto R^3$$

or

$$\frac{R^3}{T^2} = \text{constant}$$

The Earth has an average distance from the Sun of $\mathbf{1.50 \times 10^{11}}$ **m**. (This is often called one *astronomical unit*, AU.)

WORKED EXAMPLE D1.1

If the average distance between the Sun and the planet Venus is 1.08×10^{11} m, calculate the time period of Venus's orbit.

Answer

$\frac{T^2}{R^3}$ = constant, the same for Venus as the Earth

If T_V is the period of Venus's orbit:

$$\frac{365.25^2}{(1.496 \times 10^{11})^3} = \frac{T_V{}^2}{(1.08 \times 10^{11})^3}$$

T_V = 224 (Earth) days

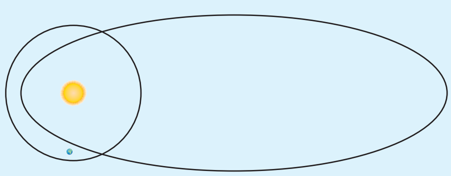
LINKING QUESTION

- How is uniform circular motion like – and unlike – real-life orbits?

This question links to understandings in Topic A.4.

1 The Earth's orbit around the Sun is not perfectly circular.
  a Use the internet to determine their maximum and minimum separation, and when these events occur.
  b Discuss what effect this has on the climate at any particular location (if any).

2 Figure D1.5 shows the orbits of a planet and a comet around the Sun.



■ **Figure D1.5** The orbits of a planet and a comet around the Sun

  a State how you know which is which.
  b At what positions will the comet be travelling:
    i fastest **ii** slowest?

3 Determine the value of T^2/R^3 in SI units for the planets of the Solar system.

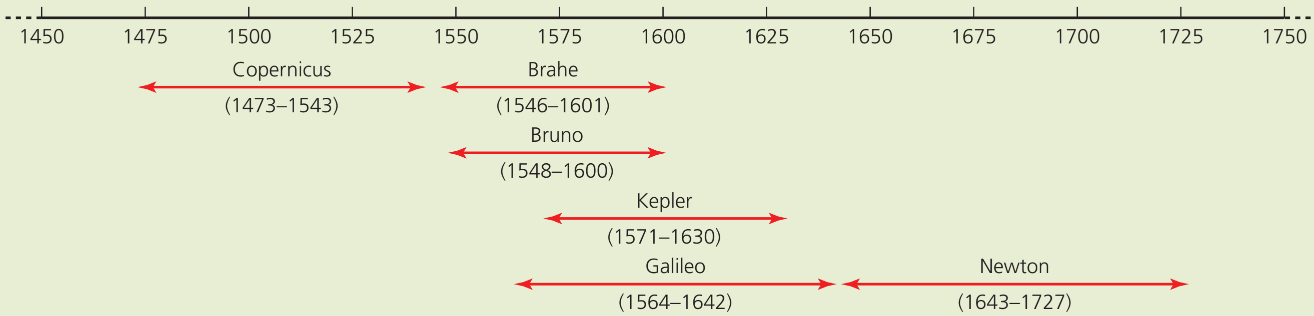
4 Calculate the period of the planet Mars, which has an average distance of 228 million kilometres from the Sun.

5 One of the planets of the Solar System has a period of approximately 84 years.
  a Determine its distance from the Sun.
  b Express your answer to part **a** in astronomical units, AU.
  c Find out which planet it is.

6 **a** Research how long it takes for the Moon to orbit the Earth.
  b The centre of the Moon is an average distance of 384 000 km from the centre of the Earth. Calculate a value of T^2/R^3 from this data.
  c Use Kepler's third law to determine the orbital radius of an Earth satellite which takes exactly one day to compete its orbit. (A satellite with an orbit of this radius can appear to remain 'stationary' above the equator.)

Nature of science: Science as a shared endeavour

'The shoulders of giants'



■ **Figure D1.6** Time lines of some famous early astronomers

Isaac Newton's law of gravitation is central to this topic and the name Newton appears prominently in physics text books. Famously, he is quoted as saying *'If I have seen further, it is by standing on the shoulders of giants'*. Figure D1.6 shows a time line of Newton's most famous predecessors in the study of astronomy in the sixteenth and seventeenth centuries.

■ **Figure D1.7** Isaac Newton

■ **Figure D1.8** Copernicus

Nicolas Copernicus, a Polish astronomer (Figure D1.8), is considered by many to be the founder of modern astronomy. In 1530 he published a famous paper stating that the Sun was the centre of the universe and that the Earth, stars and planets orbited around it (a *heliocentric* model). At that time, and for many years afterwards, these views directly challenged 'scientific', philosophical and religious beliefs. It was then generally believed that the Earth was at the centre of everything (a *geocentric* model). That profound and widespread belief dated all the way back to Ptolemy, Aristotle and others nearly 2000 years earlier. It should be noted, however, that Aristarchus in Ancient Greece is generally credited with being the first well-known person to propose a heliocentric model.

In Italy, the astronomer Giordano Bruno took the heliocentric model further with revolutionary suggestions that the universe was infinite and that the Sun was not at the centre. The Sun was, Bruno suggested (correctly), similar in nature to the other stars. He was burned at the stake in 1600 for these beliefs – at the time, considered by some to be heresy. About 30 years later, one of the greatest scientific thinkers of all time, Galileo Galilei, was placed on trial by the Roman Catholic Church under similar charges. Many years earlier he had used the newly invented telescope to observe the moons of Jupiter and had reasoned that the Earth orbited the Sun in a similar way, as had been proposed by Copernicus. Under pressure, he publicly renounced these beliefs and was allowed to live the rest of his life under house arrest. All this has provided the subject of many books, plays and movies.

About 700 years before the time of Newton, during the Islamic Golden Age, Abd al-Rahman al-sufi and other Muslim astronomers identified stars and constellations with impressive accuracy (building on the work of Ptolemy, centuries earlier). Abd al-Rahman al-sufi's 'Book of fixed stars' has an important place in the history of astronomy.

Newton's universal law of gravitation

SYLLABUS CONTENT

- Conditions under which extended objects can be treated as point masses.
- Newton's universal law of gravitation as given by: $F = G\frac{m_1m_2}{r^2}$ for bodies treated as point masses.

Isaac Newton was the first to realize that if the force of gravity makes objects (like apples) fall to the Earth and also keeps the Moon in orbit around the Earth, then it is reasonable to assume that the force of gravity acts between *all* masses. This is why it is called *universal* gravitation. Newton believed (correctly) that the size of the gravitational force between two masses increased with the sizes of the masses, and decreased with increasing distance between them – following an *inverse square relationship.*

Universal gravitation and the inverse square law

Newton knew that the distance between the Earth and the Moon was equal to 60 Earth radii, and he was able to prove that the centripetal acceleration of the Moon towards the Earth was equal to $g/60^2$ (using $a = v^2/r$ from Topic A.2). See Worked example D1.2 and Figure D1.9.

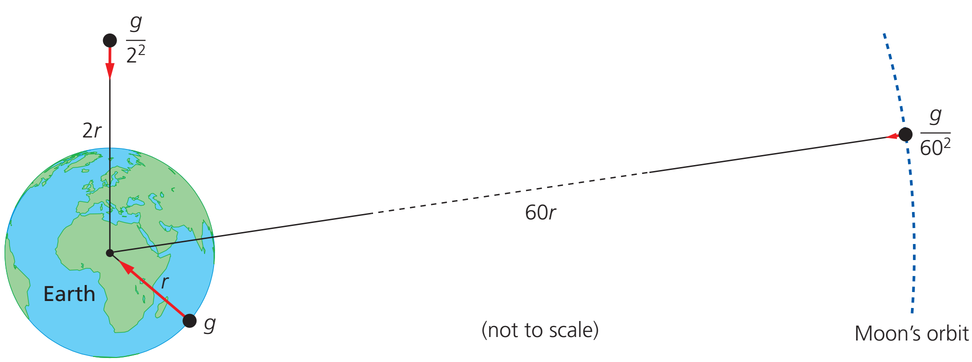


Figure D1.9 How the acceleration due to gravity varies with distance from the Earth

WORKED EXAMPLE D1.2

The average distance between the Earth and the Moon is 384 000 km and the Moon takes 27.3 days to orbit the Earth.

a Calculate the average orbital speed of the Moon. Assume that its orbit is circular.

b Determine the centripetal acceleration of the Moon towards the Earth.

c Compare your answer for **b** to $g/60^2$, with $g = 9.81\,\text{m s}^{-2}$. Comment on the difference.

Answer

a $v = \frac{2\pi r}{T} = \frac{(2 \times \pi \times 3.84 \times 10^8)}{(27.3 \times 24 \times 3600)} = 1.02 \times 10^3\,\text{m s}^{-2}$ (1022.90... seen on calculator display)

b $a = \frac{v^2}{r} = \frac{(1.0229 \times 10^3)^2}{3.84 \times 10^8} = 2.725 \times 10^{-3}\,\text{m s}^{-2}$

c $\frac{9.81}{60^2} = 2.725 \times 10^{-3}\,\text{m s}^{-2}$

The two answers are the same. This is very good evidence that gravitational accelerations (and forces) are represented by inverse square laws.

◆ **Newton's universal law of gravitation** There is a gravitational force between two point masses, m_1 and m_2, given by $F = G\frac{m_1 m_2}{r^2}$, where r is the distance between them and G is the universal gravitation constant.

◆ **Gravitational forces** Fundamental attractive forces that act across space between all masses. Gravitational force reduces with an inverse square law with increasing distance between point masses.

◆ **Universal gravitation constant, G** The constant that occurs in Newton's universal law of gravitation. $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

Newton's law

To simplify the situation, we will consider the forces acting on only two masses. The masses may be of any magnitude but, to begin with, we will assume that they are *point masses.* That is, all their mass is considered to be at a single point.

The forces acting between two point masses (m_1 and m_2) are proportional to the product of the masses and inversely proportional to their separation (r) squared.

$$F \propto (m_1 \times m_2) \quad \text{and} \quad F \propto \frac{1}{r^2}$$

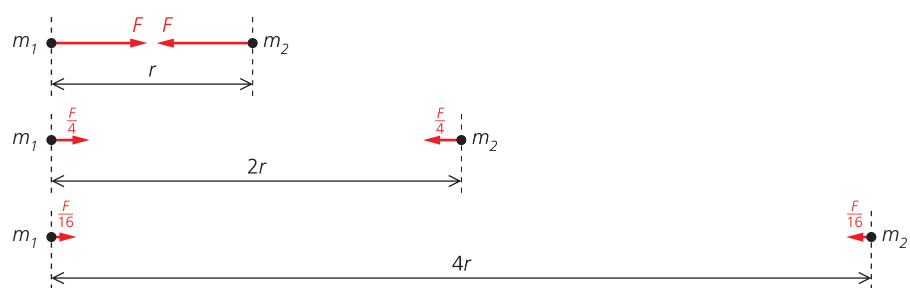
Putting a constant of proportionality into the relationship, we get Newton's universal law of gravitation:

gravitational force between two (point) masses, $\boldsymbol{F = G\frac{m_1 m_2}{r^2}}$

G is known as the **universal gravitation constant**. It has a value of $\mathbf{6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}}$.

The small value of G reflects the fact that gravitational forces are small unless one (or both) of the masses is very large. G is a fundamental constant which, as far as we know, always has exactly the same value everywhere in the universe and for all time. It should not be confused with g, the acceleration due to gravity, which varies with location. The relationship between g and G is covered later in this topic.

The relationship between force and distance is illustrated in Figure D1.10. Note that exactly the same force always acts on *both* masses (but in opposite directions), even if one mass is larger than the other. This is an example of Newton's third law of motion.



■ **Figure D1.10** The gravitational force between point masses m_1 and m_2 decreases with increasing separation (the vectors are not drawn to scale)

Of course, the mass of an object is not all located at one point, but this does not mean that Newton's equation cannot be used for real masses. The forces between two spherical masses of uniform density located a long way apart are the same as if the spheres had all of their masses concentrated at their centre points. The gravitational effects around a planet (assumed to be spherical) are effectively the same as would be produced by a similar mass concentrated at the centre of the planet.

Newton was also able to confirm his law of gravitation by showing that it was consistent with Kepler's third law, as follows:

For circular gravitational orbits the necessary centripetal force (Topic A.2) is provided by gravity. For a relatively small mass, m, orbiting a much larger mass, M:

$$\frac{\cancel{m}v^2}{r} = \frac{GM\cancel{m}}{r^2} \Rightarrow v^2 = \frac{GM}{r}$$

Then, since $v = \frac{2\pi r}{T}$:

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r} \Rightarrow$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} \text{ (a constant)}$$

This is Kepler's third law (as introduced earlier in this topic), showing a value for the constant on the right-hand side of the equation.

WORKED EXAMPLE D1.3

Calculate the gravitational forces acting between the Earth and a 1.0 kg book on the Earth's surface. (The Earth's mass is 6.0×10^{24} kg and its radius is 6.4×10^{6} m.)

Answer

$$F = G\frac{m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11}) \times 1.0 \times (6.0 \times 10^{24})}{(6.4 \times 10^{6})^2} = 9.8\,\text{N}$$

This is the *weight* of a 1.0 kg mass on the Earth's surface. The book attracts the Earth up towards it with an equally sized force which has a negligible effect on the Earth. This is another example of Newton's third law.

LINKING QUESTION

- Physics utilizes a number of constants such as G. What is the purpose of these constants and how are they determined? (NOS)

Nature of science: Measurement

Weighing the Earth

At the time Newton proposed his law of universal gravitation it was not possible to determine an accurate value for the gravitational constant, G. The only gravitational forces that could be measured were those of the weights of given masses on the Earth's surface. The radius of the Earth was known, but that still left two unknowns in the equation $F = Gm_1m_2/r^2$: the gravitational constant and the mass of the Earth. If either of these could be found, then the other could be calculated using Newton's law of gravitation. That is why the determination of an accurate value for G was known as '*weighing the Earth*'.

Certainly, it was possible in the seventeenth century to get an approximate value for the mass of the Earth from its volume and estimated average density (using $m = \rho V$). But density estimates would have been little more than educated guesses. We know now that the Earth's crust has a much lower average density (about 3000 kg m^{-3}) than most of the rest of the Earth. However, it was possible to use an estimate of the Earth's mass to calculate an approximate value for the gravitation constant. The first accurate measurement was made more than 100 years later by Henry Cavendish in an experiment that is famous for its precision and accuracy.

To calculate a value for G without needing to know the mass of the Earth (or the Moon, or another planet) required the direct measurement of the force between two known masses. Cavendish used lead spheres (see Figure D1.11) because of their high density (11.3 g cm^{-3}). The forces involved are very difficult to measure because they are so small, but also because similar-sized forces can arise from various environmental factors. (In fact, Cavendish's main aim was to get a value for the density of the Earth rather than to measure G.)

In an early attempt to estimate the gravitational constant and calculate a value for the mass of the Earth, pendulums were suspended near mountains (see an exaggerated representation in Figure D1.12).

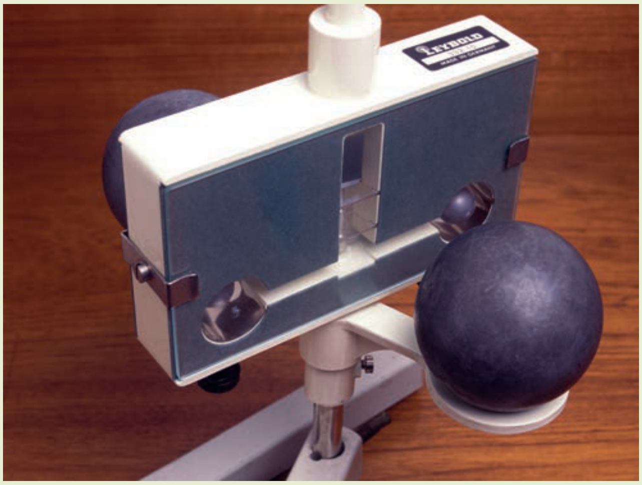


Figure D1.11 A modern version of Cavendish's apparatus

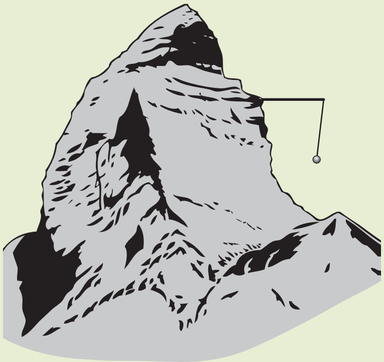


Figure D1.12 A pendulum and a mountain attract each other

7 The gravitational force acting on a satellite orbiting 50 km above the Moon's surface was 840 N. Calculate a value for the force if the height above the surface was ten times greater. Radius of Moon = 1.74×10^6 m

8 Estimate the gravitational force between you and your pen when you are 1 m apart.

9 a Determine the gravitational force between two steel spheres each of radius 45 cm and separated by 10 cm. Density of steel = 7900 kg m^{-3}

b Show that if solid steel spheres with twice the radius were used, with the same separation between their surfaces, the force would increase by a factor of about 20.

10 Calculate the average gravitational force between the Earth and the Sun. (You will need to research the relevant data.)

11 A proton has a mass of 1.7×10^{-27} kg and the mass of an electron is 9.1×10^{-31} kg.

a Estimate the gravitational force between these two particles in a hydrogen atom, assuming that they are 5.3×10^{-11} m apart.

b Compare your answer to the magnitude of the electric force between the same two particles (8.2×10^{-8} N, which is explained in Topic D.2).

c Comment on your answer.

12 Ganymede and Callisto are the two largest moons of Jupiter. Ganymede has an orbital radius of 1.07×10^6 km and orbits every 7.15 Earth days. Callisto has an orbital radius of 1.88×10^6 km and orbits every 16.7 Earth days.

a Determine if this data is consistent with Kepler's third law.

b Calculate the mass of Jupiter.

Gravitational fields

SYLLABUS CONTENT

- Gravitational field strength, g, at a point is the force per unit mass experienced by a small point mass at that point as given by: $g = \frac{F}{m} = \frac{GM}{r^2}$.
- Gravitational field lines.

A region (around a mass) in which another mass would experience a gravitational force is called a gravitational field.

Theoretically, all masses produce gravitational fields around themselves, but in practice we only use the term when discussing the space around very large masses like moons, planets and stars. We all live in the gravitational field of the Earth, while the Earth moves in the gravitational field of the Sun.

TOK

The natural sciences

- What kinds of explanations do natural scientists offer?

Fields

Understanding gravitational, electric and magnetic forces is fundamental knowledge about the universe in which we live. It seems that these forces can act instantaneously across space, even if there is nothing but vacuum (free space) in between. And, in the case of gravity, the forces can act across unbelievably large distances. All this is very difficult to comprehend!

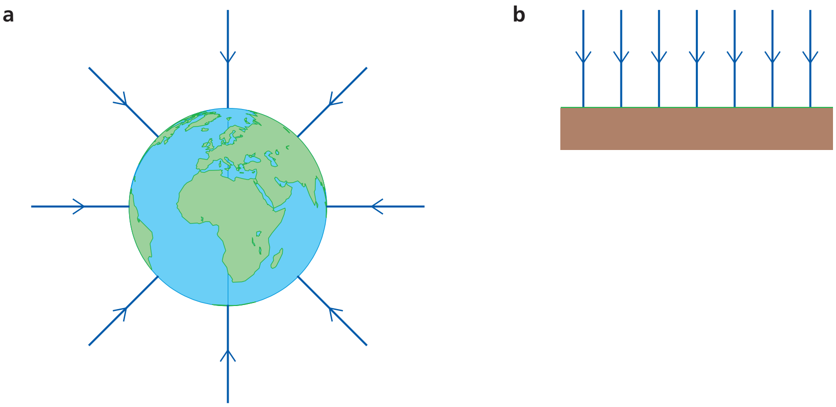
Using the concept of **fields** (gravitational, electric and magnetic) to describe the intermediate spaces may seem to help understanding, but it does not explain the origin of the forces.

Scientists cannot say 'that is just the way it is' and leave it at that. They need a deeper, more fundamental understanding, but that is beyond the requirements of this course.

◆ **Field (gravitational, electric or magnetic)** A region of space in which a mass (or a charge, or a current) experiences a force due to the presence of one or more other masses (charges, or currents – moving charges).

◆ **Field lines and patterns** Representation of fields in drawings by a pattern of lines. Each line shows the direction of force on a mass (in a gravitational field), of force on a positive charge (in an electric field), or on a north pole (in a magnetic field). In any particular drawing, the field is strongest where the lines are closest together.

We often want to represent a gravitational field on paper, or on a screen, and this can be done with gravitational **field lines** as shown in Figure D1.13. The arrows show the direction of the gravitational force that would be experienced by a mass placed at any particular place in the field. Figure D1.13a represents the spreading radial gravitational field lines around the Earth. The lines are closer together nearer to the Earth, which shows that the gravitational field is stronger. Field lines never cross each other; that would mean that gravitational force was acting in two different directions at the same place.



■ **Figure D1.13** Field lines are used to represent gravitational fields on paper or on screen. **a** radial field **b** uniform field

The parallel lines in Figure D1.13b represent a uniform gravitational field, such as in a small region of the Earth's surface where variations in the field are negligible. For example, the room where you are sitting.

Gravitational field strength

We may want to ask the question 'if a mass was put in a particular place, what would be the gravitational force on it?' The answer, of course, depends on the magnitude of the mass, so it is more helpful to generalize and ask 'what would the force be on a unit mass (1 kg)?' If we know this, then we can easily calculate the gravitational force on any other mass.

Gravitational field strength, g, is defined as the force per unit mass that would be experienced by a small test mass placed at that point:

DB

$$g = \frac{F}{m}$$

◆ **Test mass** An object of insignificant mass used in the definition and measurement of gravitational fields.

Reference is made to a 'small **test mass**' because a large mass (compared to the mass, or masses, creating the original field) could have a significant gravitational field of its own. Gravitational field strength is given the symbol g and has the SI unit $\mathrm{N\,kg^{-1}}$. Gravitational field strength is a vector quantity and its direction is shown by the arrows on field lines.

As explained in Topic A.2, in general, we know from Newton's second law of motion, that $a = F/m$, so that gravitational field strength ($g = F/m$) in $\mathrm{N\,kg^{-1}}$ is numerically equal to the acceleration due to gravity in $\mathrm{m\,s^{-2}}$.

Imagine you were on an unknown planet and wanted to find experimentally the gravitational field strength. This can be done easily by hanging a small test mass of 1.0 kg on a force-meter, calibrated in newtons. The reading will be the strength of the gravitational field (in $\mathrm{N\,kg^{-1}}$) and the direction of the field will be the same as the direction of the string – 'downwards' towards the centre of the planet.

WORKED EXAMPLE D1.4

A student measures the time it takes a stone to fall from rest to the ground from a height of 1.18 m to be 0.49 s. Determine the value this gives for the acceleration due to gravity at her location.

Answer

$$s = ut + \frac{1}{2}at^2$$

$$1.18 = 0 + \frac{1}{2} \times a \times 0.49^2$$

$$a = 9.8\,\text{m s}^{-2}$$

This is numerically equal to the gravitational field strength, $g = 9.8\,\text{N kg}^{-1}$

Common mistake

r in the highlighted equation represents distances from the centre of the planet, or moon. It is not the radius (unless we are calculating g on the surface).

Gravitational field strength around a planet

The gravitational field strength around a large mass (a planet for example) can be determined by combining $g = \frac{F}{m}$ with the equation for gravitational force, $F = G\frac{m_1 m_2}{r^2}$ ($m = m_1$ represents the small mass):

$$g = \frac{F}{m} = \frac{G \cancel{m_1} m_2}{\cancel{m_1} r^2}$$

Representing the large mass by M, rather than m_2:

$$\boldsymbol{g = G\frac{M}{r^2}}$$

WORKED EXAMPLE D1.5

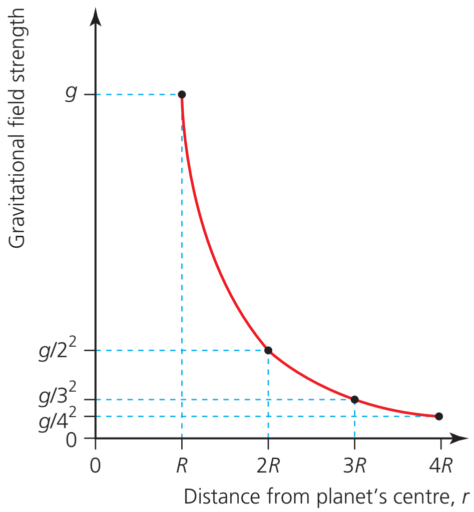
Determine the gravitational field strength on the surface of a planet which has a mass of 4.87×10^{24} kg and a radius of 6.05×10^{6} m.

Answer

$$g = G\frac{M}{r^2} = \frac{(6.67 \times 10^{-11}) \times (4.87 \times 10^{24})}{(6.05 \times 10^{6})^2} = 8.87\,\text{N kg}^{-1}$$

(This planet is Venus.)

Similar to gravitational force, the gravitational field strength around a planet, or a moon, follows an inverse square law. This is represented graphically in Figure D1.14.



The gravitational field strength beneath the surface of a planet cannot be determined from $g = GM/r^2$ because the planet can no longer be considered to be a point mass. At any significant depth the mass above and to the side would also be pulling a 'test mass'. At the centre of a planet the field strength will be assumed to be zero, because the surrounding masses will pull equally in all directions. Moving from the centre to the surface, the gravitational field strength will increase.

■ **Figure D1.14** Variation of gravitational field strength with distance from a planet (or moon) of radius R

Tool 3: Mathematics

Determine the effect of changes to variables on other variables in a relationship

For a planet of radius R, the gravitational field strength on its surface can be determined from:

$$g = G\frac{M}{R^2}$$

It is easy to assume, incorrectly, that g decreases for planets of greater radius. In fact, the opposite is true because the mass, M, of a planet also depends on its radius, as shown below.

To determine how the gravitational field strength on the surface of a planet depends on its radius, R, we need to use these facts:

- the volume of a sphere equals $\frac{4}{3}\pi R^3$
- mass, M, is equal to density, ρ, multiplied by volume, V

So, we can write:

$$M = \frac{4}{3}\rho\pi R^3$$

The density of a planet is not uniform, so the value of ρ used here is an average.

Putting this equation for M back into the equation for g we get:

$$g = \frac{4}{3}G\rho\pi\left(\frac{R^3}{R^2}\right)$$

So that:

$$g = \frac{4}{3}G\rho\pi R$$

This equation (which students are not expected to remember) predicts that the gravitational field strength at the surface of a planet is proportional to its radius. From the equation we would expect bigger planets to have stronger fields, but that is only true if they have equal average densities. (The Earth is the densest planet in our Solar system, with an average density of $5510\,\text{kg}\,\text{m}^{-3}$. Venus and Mercury have similar densities to Earth but the density of Mars is significantly lower. The outer planets are gaseous and have lower densities. Saturn has the lowest average density, at $687\,\text{kg}\,\text{m}^{-3}$.)

WORKED EXAMPLE D1.6

Predict a value for the gravitational field strength on the 'surface' of Saturn (radius = 5.8×10^7 m).

Answer

$$g = \frac{4}{3}G\rho\pi r = \frac{4}{3} \times (6.67 \times 10^{-11}) \times 687 \times \pi \times (5.8 \times 10^7) = 11\,\text{N}\,\text{kg}^{-1}$$

(Accepted value is $10.4\,\text{N}\,\text{kg}^{-1}$)

Combining gravitational field strengths

It is possible that a mass may be in two or more separate and significant gravitational fields. For example, we are in the fields of both the Earth and the Moon. For most purposes the Moon's gravitational field on the Earth's surface can be considered to be negligible compared to the Earth's field. But, if a spacecraft is travelling directly from the Earth to the Moon, the gravitational field due to the Earth will get weaker as the Moon's field gets stronger. There will be a point at which the two fields will be equal in strength, but opposite in direction (shown as P in Figure D1.15).

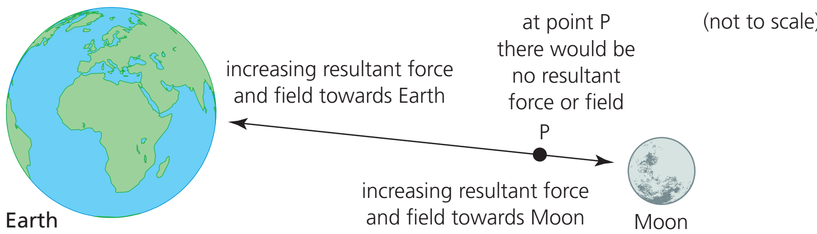


Figure D1.15 Opposing fields cancel at a precise point P between the Earth and the Moon

At P, the total gravitational field strength is zero and there will be no resultant force on the spacecraft because the pulls of the Moon and the Earth are equal and opposite. As the spacecraft travels from the Earth to P there is a resultant force pulling it back to Earth but this is reducing in size. After the spacecraft passes P there will be an increasing resultant force pulling the spacecraft towards the Moon.

In general, if two or more masses are creating gravitational fields at a certain point, then the total field is determined by adding the individual fields, remembering that they are vector quantities.

In this chapter as we will only be concerned with locations somewhere on the line passing through the masses, then the vector addition of the two fields is straightforward, as shown in the following Worked example.

WORKED EXAMPLE D1.7

In Figure D1.16 (which is not drawn to scale), P is a point midway between the centres of the planets A and B. At P the gravitational field strength due to A is 4.0 N kg^{-1} and that due to B is 3.0 N kg^{-1}.

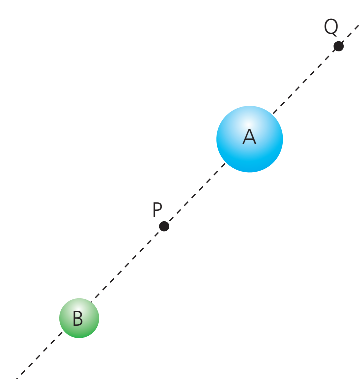


Figure D1.16 Point P between the centres of the planets A and B

a Determine the resultant gravitational field strength at P.

b Calculate the combined gravitational field strength at point Q. P and Q are the same distance from A.

Answer

a Taking the field towards the bottom of the diagram to be positive,

$(-4.0) + (+3.0) = -1.0$

The gravitational field strength is 1.0 N kg^{-1} towards A.

b The size of the field due to A is the same at Q as it is at P, although it is in the opposite direction. The strength of the field due to B at Q is 3^2 times less than at P because it is three times further away, but it is in the same direction.

$(+4.0) + (3.0/9) = 4.3$ N kg^{-1} towards A and B.

13 Make a sketch of the gravitational field in the room where you are sitting.

14 The weight of a 12 kg mass on the surface of Mercury would be 44 N.
Calculate the gravitational field strength on the surface of the planet.

15 a Determine the gravitational field strength at a height of 300 km above the Earth's surface. (The radius of the Earth is 6.37×10^6 m. The mass of the Earth is 5.97×10^{24} kg.) Many satellites orbit at about this height.

b Calculate the percentage this value is of the accepted value for the gravitational field strength on the Earth's surface.

16 The gravitational field strength of a planet is $5.8\,\mathrm{N\,kg^{-1}}$ at a distance of 2.1×10^4 km from its centre. Determine the field strength at a distance 1.4×10^4 km further away.

17 Draw a sketch graph to show how the gravitational field strength varies from the centre of the Earth to a distance of 12.8×10^6 m. (radius of Earth = 6.4×10^6 m)

18 a Calculate the gravitational field strength on the surface of the Moon. The mass of the Moon is 7.35×10^{22} kg and its Radius is 1740 km.

b Calculate the gravitational field strength at a point on the Earth's surface due to the Moon (not the Earth). The distance between the centre of the Moon and the Earth's surface is 3.8×10^8 m.

c State one effect that the Moon's gravitational field has on Earth.

19 Titan is a moon of the planet Saturn. It has an average density of $1900\,\mathrm{kg\,m^{-3}}$. The gravitational field strength on its surface is approximately 14% of that on Earth. Estimate Titan's radius using the equation given above.

20 Consider Figure D1.16 again, but with different data. If planet A has a gravitational field of $15\,\mathrm{N\,kg^{-1}}$ at Q, but the combined field at the same point is $16\,\mathrm{N\,kg^{-1}}$, calculate the combined field at point P.

21 The gravitational fields of the Sun and the Moon cause the tides on the world's oceans. The highest tides occur when the resultant field is greatest (at times of a 'new moon'). Draw a sketch to show the relative positions of the Earth, Sun and Moon when the resultant field on the Earth's surface is:

a greatest

b weakest.

Tool 2: Technology

Use spreadsheets to manipulate data

1 Research the data that will allow you to set up a spreadsheet to calculate the combined gravitational field strengths due to the Earth and the Moon at points along a straight line joining their surfaces.
2 Combine the fields to determine the resultant field and draw a graph of the results.
3 Where does the resultant gravitational field equal zero?

Orbital motion

◆ **Satellite** Object that orbits a much larger mass. Satellites can be **natural** (like the Earth or the Moon), or **artificial** (as used for communication, for example).

◆ **Altitude** Height of an object above the surface of a planet.

The gravitational forces between two masses are equal in size but opposite in direction. However, if one of the masses is very much bigger than the other, we often assume that the force on the larger mass has negligible effect, while the same force acting on the much smaller mass produces a significant acceleration. If the smaller mass is already moving in a suitable direction, then the gravitational force can provide the centripetal force to make it orbit the larger mass. It is then described as a **satellite** of the larger mass. The Earth and the other planets orbiting the Sun, and moons orbiting planets, are all examples of **natural satellites**. In the modern world we are becoming more and more dependent on the **artificial satellites** that orbit around the Earth.

Newton's famous thought experiment was described in Topic A.1: a cannonball fired 'horizontally' from the top of a mountain would move in a parabolic path if there were no air resistance, and hit the ground some distance away, as shown again by path A in Figure D1.17. If the cannonball was travelling fast enough, it could move as shown in path B, and 'escape' from the Earth. Path C shows the path of an object moving with exactly the right speed and direction so that it remains at the same **altitude** (distance above the Earth's surface), that is, it remains in orbit around the Earth: a satellite. Remember that we are assuming that there is no air resistance.

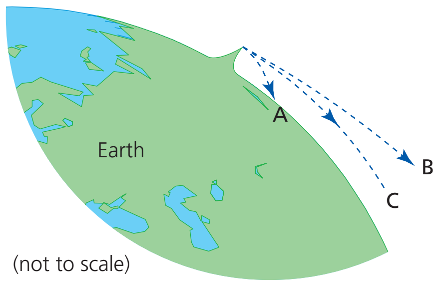


Figure D1.17 The path of objects projected at different speeds from a mountain top

Gravity is the only force acting on the satellite and it is acting continuously and perpendicularly to its instantaneous velocity along path C. As we described in Topic A.2, this is the necessary condition for circular motion. The force of gravity (weight of satellite) is providing the centripetal force.

An actual satellite needs to be at a height which is at least 200 km above the Earth's surface to avoid the effects of air resistance and, at that height, the orbital speed needed is about $8\,\mathrm{km\,s^{-1}}$ (explained below). This was first achieved in 1957 by the Soviet Union with their satellite Sputnik One. Its lowest orbital height was 215 km and it took 96 minutes for each orbit. See Figure D1.18.

Figure D1.19 shows a satellite of mass m in orbit around a much larger mass, M (a planet, for example). In the mathematical treatment of satellites in this course, we will only consider perfectly circular orbits with constant radius r, as shown.

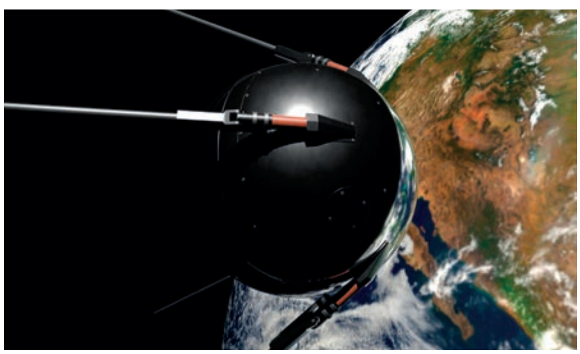


Figure D1.18 Sputnik One

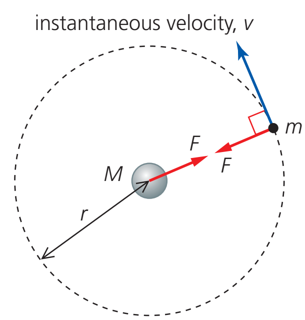


Figure D1.19 A satellite of mass m orbiting a planet of mass M.

Orbital speed and time period of a satellite

Remembering the equation for centripetal acceleration, we can write:

$$\text{centripetal acceleration, } g = \frac{v^2}{r}$$

Or, considering forces:

$$\text{centripetal force, } mg = \frac{mv^2}{r}$$

In order to maintain a satellite in a circular orbit around the Earth (or other planet), it needs to be given the correct speed, v, for its particular radius, r, as given by:

$$\frac{v^2}{r} = g$$

This equation enables us to determine the theoretical speed for a satellite which orbits just above the Earth's surface – as in the cannonball thought experiment (Earth's radius = 6.4×10^6 m):

$v^2 = gr = 9.8 \times (6.4 \times 10^6)$

$v = 7.9 \times 10^3\,\mathrm{m\,s^{-1}}$ $(7.9\,\mathrm{km\,s^{-1}})$

At a more realistic height of 200 km (for example), $g = 9.23\,\mathrm{N\,kg^{-1}}$ calculated from $g = \frac{GM}{r^2}$, so that the necessary speed is reduced to $7.7\,\mathrm{km\,s^{-1}}$. Assuming there is no air resistance, a satellite moving with a speed of $7700\,\mathrm{m\,s^{-1}}$, 200 km above the surface, can orbit the Earth. Knowing the value of g at any particular height enables us to calculate the speed necessary for a circular orbit at that height. The speed does not depend on the mass. All satellites at the same height move with the same speed. If there is no air resistance, a satellite in a circular orbit will continue to orbit the Earth without the need for any engine. The force of gravity acts perpendicularly to motion, so that no work is done by that force.

$$v = \frac{2\pi r}{T}$$

(Topic A.2) can be used to calculate the time period, T, for an orbit.

If the speed of a satellite is greater than the speed necessary for a circular orbit, but less than the *escape speed* (explained later) it will move in an elliptical path. However, for calculations in this course, we will assume that the orbits of planets, moons and satellites are circular.

WORKED EXAMPLE D1.8

Determine the orbital speed and time period of a satellite that orbits the Earth at a distance which is as far above the surface as the centre of the Earth is below.
(Radius of Earth = 6.4×10^6 m)

Answer

At twice the distance from the centre of the Earth the gravitational field strength is reduced to:

$$\frac{9.8}{2^2} = 2.45\,\mathrm{N\,kg^{-1}}$$

Then:

$$g = \frac{v^2}{r}$$

$$2.45 = \frac{v^2}{(2 \times 6.4 \times 10^6)}$$

$$v = 5.6 \times 10^3\,\mathrm{m\,s^{-1}}$$

$$T = \frac{2\pi r}{v} = \frac{2 \times \pi \times (6.4 \times 10^6)}{5.6 \times 10^3}$$

$$= 7.2 \times 10^3\,\mathrm{s}\ \text{(two hours)}$$

22 a Calculate values for the gravitational field strengths at heights above the Earth's surface of 1000 km, 10 000 km and 40 000 km.

b Calculate the necessary speeds for circular orbits at these heights.

c Use a compass to draw a scale diagram of the Earth with these orbits around it.

d Determine the times for complete orbits (time periods), T, at these heights and mark them on your diagram.

23 Two satellites of equal mass orbit the same planet as shown in Figure D1.20. Satellite B is twice as far away from the centre of the planet as satellite A. Copy the table and complete it to show the properties of the orbit of satellite B.

	Satellite A	Satellite B
Distance from planet's centre	r	$2r$
Gravitational field strength	g	
Gravitational force	F	
Circumference of orbit	c	
Speed	v	
Time period	T	

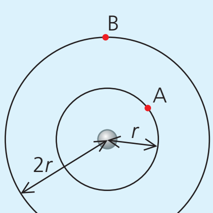


Figure D1.20 Two satellites of equal mass orbiting the same planet

24 The Earth is an average distance of 1.5×10^{11} m from the Sun. Assuming that the orbit is circular,

a calculate the average orbital speed of the Earth around the Sun

b determine the centripetal acceleration of the Earth towards the Sun.

c Use your answers to calculate a value for the mass of the Sun.

25 The gravitational field strength on the surface of Mars is $3.72\,\text{N kg}^{-1}$.

The radius of Mars is 3.4×10^6 m.

a What is the gravitational field strength at a distance of 3.4×10^6 m above the surface?

b Calculate the orbital speed necessary for a satellite orbiting Mars at this height.

c What would be the time period of this satellite?

◆ **Polar satellite orbit** Descriptive of the path of a low-orbit satellite that passes over the poles of the Earth and completes many orbits every day.

◆ **Geosynchronized orbit** Any satellite orbit that has the same period as the Earth spinning on its axis. The orbit must have exactly the correct radius.

◆ **Geostationary orbit** A satellite which appears to remain 'above' the same location on the Earth's surface.

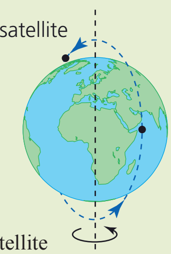
Nature of science: Observations

Uses of satellites

The use of artificial satellites has extended the range and nature of observations that we can make of phenomena both on the Earth and in space. Satellites can be put into orbit at any desired height above the Earth's surface, assuming that they are given the right velocity and are high enough to avoid air resistance. The lower orbits have obvious advantages, especially if the Earth's surface is being monitored for some reason. But, as previously explained, the higher a satellite, the longer its orbital time period. The orientation of the orbit compared with the Earth's axis and whether it is circular or elliptical are also important.

Polar orbits

Many satellites have orbits that pass approximately over both poles of the Earth at heights of up to about 2000 km (Figure D1.21). The **Polar orbits** remain in the same plane as the Earth rotates, so that the satellite passes over different parts of the planet on each orbit. These satellites make many orbits every day.

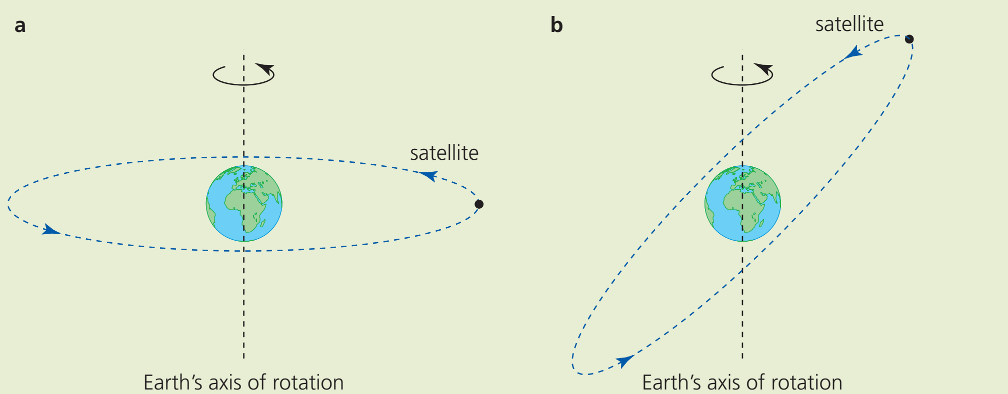


■ **Figure D1.21** Polar-orbiting satellite

Geostationary orbits

A **geostationary** satellite is one that appears to remain in the same position as seen by an observer on the Earth's surface. This is only possible if the period of the satellite is the same as the period of rotation of the Earth (23 h 56 min). As we have seen, this requires that a geostationary satellite is at exactly the right height: 4.2×10^7 m.

Any satellite at this height will have the same period as the rotation of the Earth and they are described as **geosynchronous**. However, to be geostationary the satellites must be made to orbit in the same plane as the equator. Both orbits shown in Figure D1.22 are geosynchronous, but only the orbit in **a** is geostationary.



■ **Figure D1.22** Geostationary orbits must be in the plane of the equator

LINKING QUESTION

- How can the motion of electrons in the atom be modelled on planetary motion and in what ways does this model fail? (NOS)

This question links to understandings in Topics E.1 and E.2.

Gravitational potential energy

SYLLABUS CONTENT

- Gravitational potential energy, E_p, of a system is the work done to assemble the system from infinite separation of the components of the system.
- Gravitational potential energy for a two-body system as given by: $E_p = -G\frac{m_1 m_2}{r}$ where r is the separation between the centres of mass of the two bodies.

◆ **Gravitational potential energy**, E_p, is the work done when bringing all the masses of a system to their present positions from infinity.

We introduced the concept of **gravitational potential energy**, E_p, in Topic A.2. The equation $\Delta E_p = mg\Delta h$ was used to calculate *changes* in gravitational potential energy close to the Earth's surface, where the gravitational field strength, g, can be considered constant ($9.8\,\text{N kg}^{-1}$).

However, a general understanding of gravitational potential energy, which applies *anywhere*, must answer these two questions:

- Is there a true zero of gravitational potential energy and, if so, where is it?
- How can varying values of gravitational field strength be including in gravitational energy calculations?

The zero of gravitational potential energy is chosen to be where the masses are separated by an infinite distance.

TOK

Mathematics and the arts

- Why is mathematics so important in some areas of knowledge, particularly the natural sciences?
- How do mathematicians reconcile the fact that some conclusions seem to conflict with our intuitions?

Infinity

Infinity is not an actual place, but an abstract concept that appears regularly in physics and mathematics.

The idea of an **infinite** quantity (gravitational field, distance, time, number, and so on) is used in physics to suggest a quantity that is limitless (without end). It is greater than any real, measurable quantity. The opposite of infinite is **finite**, which means within limits. The idea of an infinite series of numbers, an infinite time, or even a field that extends for ever (but becomes vanishingly small) may all seem somehow acceptable to the human mind. However, the concept of an infinite universe gives most of us problems.

◆ **Infinite** Without limits.

◆ **Finite** Limited.

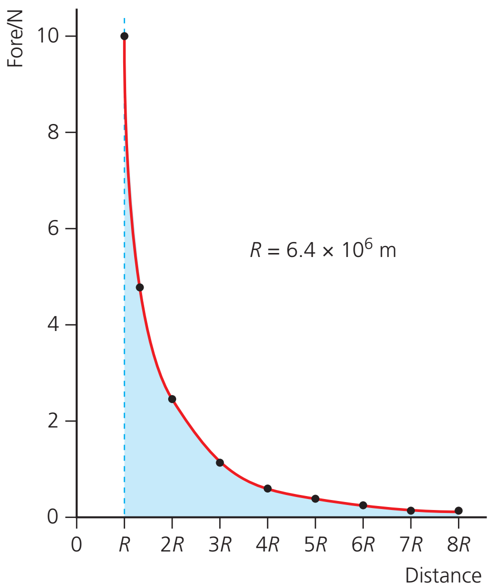
We usually refer to the gravitational potential energy of a single mass, a book on a table for example but, more exactly, the gravitational potential energy is a property of the whole *system* of the book, the table, the rest of the Earth (and the rest of the Universe!). In practice, it is acceptable to talk about the gravitational potential energy of a single object that is very much less massive than the mass creating the gravitational field in which it is situated (a person on the Earth, for example).

Gravitational potential energy is stored between two or more masses because of the gravitational forces between them. In theory, the forces never reduce to zero (consider Newton's law of gravitation), no matter how large the distances.

A 1 kg book placed on a table top which is 0.80 m above the floor has about 8 J ($mg\Delta h$) more gravitational potential energy than if it were placed on the floor. We may consider that the same book on the same table has about 40 J of gravitational potential energy if the room where they are located was on the first floor. If the location was 200 m above sea level, we might say that the book has 2000 J of gravitational potential energy. Defining a zero of gravitational energy which is agreed by everyone (infinity) avoids all these differences and possible misunderstandings.

The total gravitational potential energy of a system, E_p, is defined as the work done when bringing all the masses of the system to their present positions, assuming that they were originally at infinity.

The gravitational potential energy of the book (and the Earth) is the work done in bringing them together from an infinite distance apart. Of course, this is a theoretical exercise, but the determination is straightforward, as follows.



■ **Figure D1.23** Variation of the force on a 1 kg mass with distance from the Earth.

We know (from Topic A.3), work done = force × distance moved in the direction of the force.

The gravitational force between the 1 kg book varies with distance from the Earth, but we know that the work done can be determined from the area under a force–distance graph. See Figure D1.23, in which R represents the radius of the Earth (6.4×10^6 m).

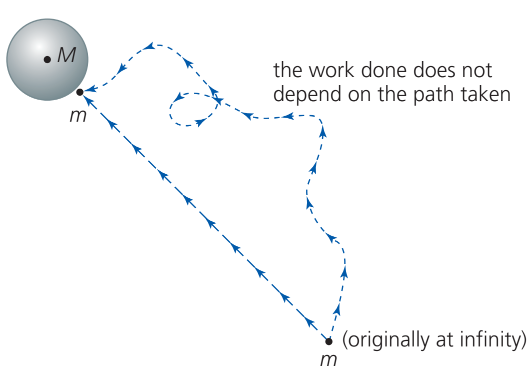
Of course, a separation of 8R is a long way from infinity (!), but we can see from the graph that the force is becoming very small and the gravitational potential energy (= shaded area) is tending to a limit. An accurate determination of the complete area (or the use of calculus) shows that the gravitational potential energy of 1 kg on the Earth's surface is 6.26×10^7 J. However, as explained below, gravitational potential energy is *always* given a negative sign, so that $E_p = -6.26 \times 10^7$ J.

If a mass of 1 kg on the Earth's surface was given $+6.26 \times 10^7$ J of energy, then it would have just the right amount of energy to reach infinity, where it would then have $(-6.26 \times 10^7 \text{ J}) + (6.26 \times 10^7 \text{ J}) = 0$ J of gravitational potential energy.

Top tip!

In this course we show gravitational forces as positive, but it may be considered that gravitational forces between two masses should be shown with negative signs because they are vectors with directions opposite to that of increasing separation. That would then be consistent with gravitational potential energy always being negative.

However, because gravitational forces are always attractive (unlike the electric forces discussed in Topic D.2), we are using only positive signs.

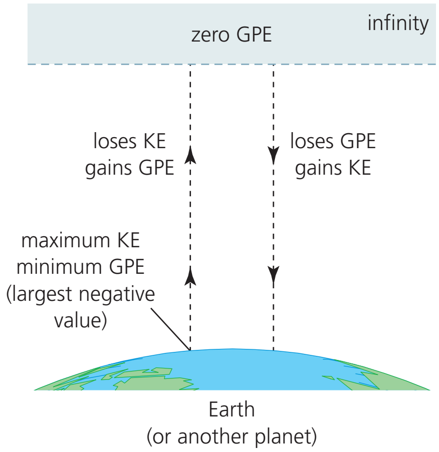


■ **Figure D1.24** Work done when moving in a gravitational field is independent of the path

Gravitational potential energies are *always* given negative values because (positive) energy would have to be supplied to separate the masses to infinity, where a system then has zero gravitational potential energy.

The total work done when moving to, or from, the same locations in a gravitational field does not depend on the path taken. See Figure D1.24.

A ball thrown upwards has been given kinetic energy. As it rises its kinetic energy is transferred to gravitational potential energy. As it falls, the process is reversed. This physics principle is exactly the same for a mass moving large distances, even (in theory) to infinity and back. See Figure D1.25.



■ **Figure D1.25** Changes of energy when a projectile moves between a planet and infinity

WORKED EXAMPLE D1.9

a Use an area under the graph shown in Figure D1.23 to estimate the change in gravitational potential energy if a mass of 1 kg moved 'up' from the Earth's surface (distance = R) to a height of $2R$ above the surface.

b Determine the change in gravitational potential energy if a 5.0 kg mass moved the same distance 'down' towards the Earth's surface.

c Compare your answer to part **a** to the value obtained using $\Delta E_p = mg\Delta h$, with $g = 9.8\,\mathrm{N\,kg^{-1}}$.

Answer

a Area ≈ $4.0 \times (2R - R) = 4.0 \times (6.4 \times 10^6) = +2.6 \times 10^7\,\mathrm{J}$

This is a rough estimate of the amount of energy needed to be given to the mass to increase its height above the Earth's surface, and its gravitational potential energy.

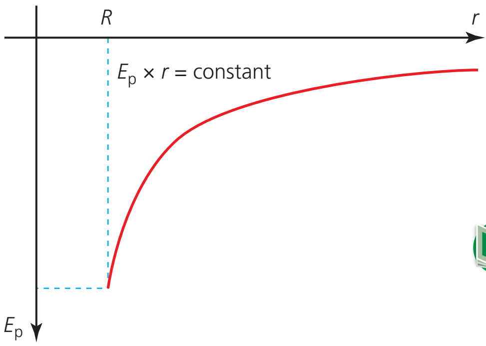
b $5.0 \times (-2.6 \times 10^7) = -1.3 \times 10^8\,\mathrm{J}$

The negative sign arises because it represents the fact that the mass has lost gravitational potential energy as it moved closer to the Earth.

c $\Delta E_p = mg\Delta h = 1 \times 9.8 \times (6.4 \times 10^6) = 6.3 \times 10^7\,\mathrm{J}$

It should be clear that this is a very different, and incorrect, answer.

Equation for gravitational potential energy



■ **Figure D1.26** Variation of the gravitational potential energy of a mass, E_p, with distance, r, from the surface of a planet or moon of radius R

Clearly it would be inconvenient to have to use graphs to determine every gravitational potential energy. We need a direct equation. However, because the gravitational force is not constant, this is not obtained by a straightforward calculation. It requires calculus, which is not needed for this course. The relationship can be stated as:

gravitational potential energy between two masses, $\boldsymbol{E_p = -G\dfrac{m_1 m_2}{r}}$

See Figure D1.26 for a graphical representation of this relationship.

This equation is consistent with work done = force × distance

$$= \frac{Gm_1m_2}{r^2} \times r$$

WORKED EXAMPLE D1.10

Use the equation above to confirm that the gravitational potential energy of 1.0 kg on the Earth's surface is −6.3 × 10^7 J (as stated previously).

Mass of Earth = 6.0 × 10^{24} kg.

Radius of Earth is 6.4 × 10^6 m.

Answer

$$E_p = -G\frac{m_1 m_2}{r}$$

$$= -\frac{(6.67 \times 10^{-11}) \times (6.0 \times 10^{24}) \times 1.0}{6.4 \times 10^6}$$

$$= 6.3 \times 10^7 \text{ J}$$

26 Explain how it is possible for a body to have negative potential energy.

27 A satellite of mass 200 kg was raised from the Earth's surface to a height of 320 km.

a Determine the change in its gravitational potential energy.
Mass of Earth = 6.0 × 10^{24} kg
Radius of Earth = 6.4 × 10^6 m

b What value would the (incorrect) use of using $\Delta E_p = mg\Delta h$ produce?

28 a Determine a value for the gravitational potential energy associated with the Earth–Moon system (only).
Mass of Moon = 7.3 × 10^{22} kg
Separation of centres averages at 3.8 × 10^8 m

b Discuss whether it is acceptable to ignore the effect of the Sun in this calculation

29 a Use the graph shown in Figure D1.23 to estimate the change in gravitational potential energy when 1 kg moves from a distance 4*R* to a distance 5*R* from the centre of the Earth.

b Compare your answer to a value determined by using the equation:

$$E_p = -G\frac{m_1 m_2}{r}$$

30 Calculate the gravitational potential energy of a 200 kg satellite orbiting at a height of 150 km above the surface of the planet Mars.
(The mass of Mars = 6.0 × 10^{23} kg,
radius of Mars = 3.4 × 10^6 m)

Top tip!

The energies of electrons in atoms (and nucleons in nuclei) are also given negative values for the same reason: an attractive force results in a confined system. A negatively charged electron needs to be given energy to free it from the attractive force between it and the positively charged nucleus. After removal, an electron is then considered to have zero electrical potential energy. This is similar to saying that a mass has zero gravitational potential energy when it is a great distance from a planet.

Gravitational potential

SYLLABUS CONTENT

- Gravitational potential, V_g, at a point, is the work done per unit mass in bringing a mass from infinity to that point as given by: $V_g = -G\frac{M}{r}$.
- Equipotential surfaces for gravitational fields.
- The relationship between equipotential surfaces and gravitational field lines.

◆ **Gravitational potential,** V_g Work done in moving a test mass of 1 kg to a specified point from infinity.

When calculating gravitational potential energies, we refer to a particular mass in a particular place. But, if we want to answer questions such as: 'how much energy would be needed to put a mass in a specified location?', it is better to use the more generalized concept of **gravitational potential**, V_g. The concept of gravitational potential is used to describe the space around massive objects such as planets, stars, and so on.

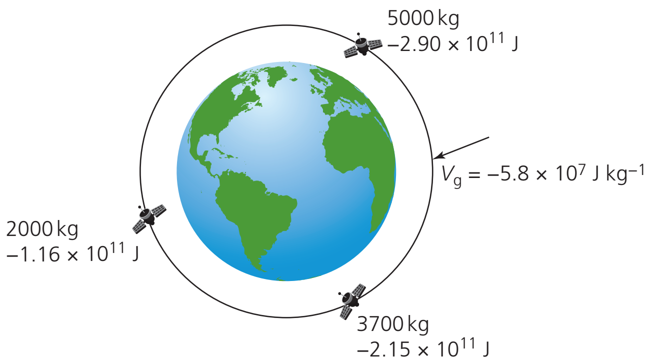
Gravitational potential can be considered as gravitational potential energy per unit mass.

For a relatively small mass, m, in the gravitational field of a very much larger mass, M, (such as a planet), we can make that clear by rewriting:

$$E_p = -\frac{Gm_1m_2}{r}$$

as $E_p = -G\frac{Mm}{r}$

Then, diving by the small mass, m, gives:



■ **Figure D1.27** Three satellites in orbit

gravitational potential, $V_g \left(= \frac{E_p}{m}\right) = -G\frac{M}{r}$

Figure D1.27 may help to explain the usefulness of the concept of potential. It shows three satellites, all at the same height in orbit around the Earth.

Because the satellites have different masses, they have different gravitational potential energies. If we divide their gravitational potential energies by their masses, we get the same result: $-5.8 \times 10^7\,\mathrm{J\,kg^{-1}}$ (the gravitational potential in that particular orbit).

Calculations with any satellite in the same orbit will produce the same result, and we can label that orbit as having a gravitational potential of $-5.8 \times 10^7\,\mathrm{J\,kg^{-1}}$.

A more formal definition of gravitational potential:

The gravitational potential at a point is defined as the work done per unit mass (1 kg) in bringing a small test mass from infinity to that point.

Gravitational potential is a scalar quantity; it has no direction. Like gravitational potential energy, the zero of gravitational potential is defined to be at infinity and all values of gravitational potential energy are negative. The SI unit for gravitational potential is $\mathrm{J\,kg^{-1}}$.

Common mistake

Students frequently get confused between *potential energy* and *potential*. Perhaps because of the similarity in their names. It may be helpful to use a shopping analogy. A shop may have a wide selection of different sized packets of rice (for example), all at different prices. When faced with such a selection, the most useful information to the shopper is not the prices of each packet, but the prices per unit mass (kg). Similarly, energy per unit mass (gravitational potential) is usually more useful information than individual energies.

WORKED EXAMPLE D1.11

Consider Figure D1.27.

a If a satellite of mass 4250 kg was placed in the same orbit, calculate its gravitational potential energy.

b Determine the height of the orbit above the Earth's surface.

(Mass of Earth = 6.0×10^{24} kg.
Radius of Earth = 6.4×10^6 m.)

Answer

a gravitational potential energy = potential × mass

$= -(5.8 \times 10^7) \times 4250$

$= -2.5 \times 10^{11}$ J

b $V_g = -G\frac{M}{r}$

$$-5.8 \times 10^7 = -\frac{(6.67 \times 10^{-11}) \times (6.0 \times 10^{24})}{r}$$

$r = 6.9 \times 10^6$ m

height $= (6.9 \times 10^6) - (6.4 \times 10^6)$

$= 5 \times 10^5$ m (500 km)

HL ONLY

31 Determine the gravitational potential on the Earth's surface.

32 If the gravitational potential on the surface of a planet is $-4.8\ \text{MJ kg}^{-1}$, determine the gravitational potential energy of an 86 kg mass on the planet.

33 a Calculate the gravitational potential at a height of 1000 km above the surface of Mars. (Mass $= 6.4 \times 10^{23}$ kg, radius $= 3.4 \times 10^{6}$ m)

b Determine the change in gravitational potential energy for a mass of 1200 kg moving from the surface of Mars to a height of 1000 km.

c State whether the change in gravitational potential energy is positive or negative.

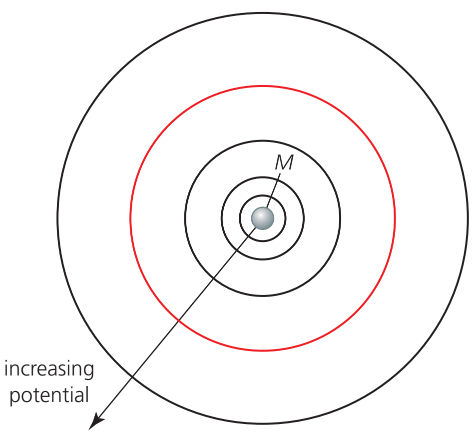
34 The gravitational potential at a distance of 1.4×10^{7} m from a planet is $-1.9 \times 10^{7}\ \text{J kg}^{-1}$. Calculate the gravitational potential at a distance of 3.7×10^{8} m.

◆ **Equipotential line (or surface)** Line (or surface) joining points of equal potential. Equipotential lines are always perpendicular to field lines.

Equipotential surfaces

Drawings of equipotential lines provide useful visualizations of gravitational fields.

An **equipotential surface (or line)** connects places which have the same potential.



■ **Figure D1.28** Equipotential lines around a spherical mass

Figure D1.28 shows typical equipotential lines around a spherical or point mass, *M*. The circular lines are drawn with equal numerical intervals of potential, which means that they must get further and further apart. In other words, increasing separation of equipotential lines indicates a weakening gravitational field. A three-dimensional representation would have spherical *surfaces*.

As the distance from the mass increases, the potential increases, but we know that the potential on the surface of the mass *M* is negative. This means that the increasing potential is indicated by a negative value decreasing in magnitude.

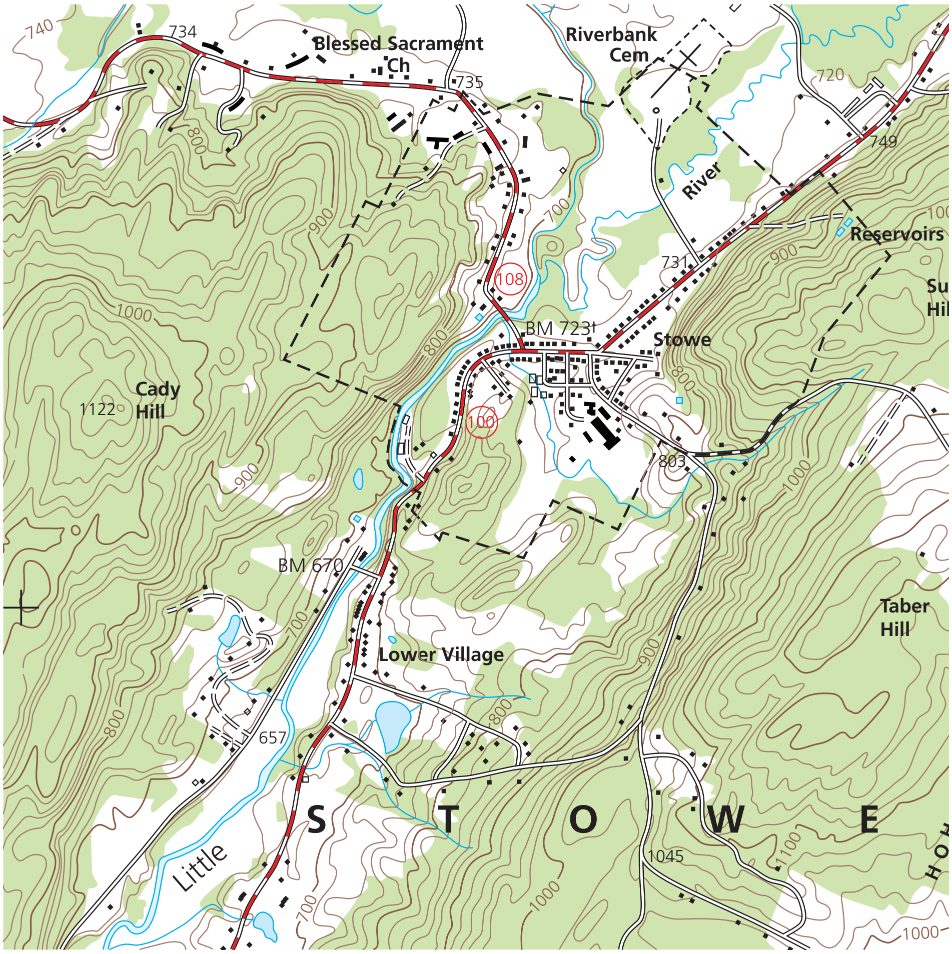
All equipotential lines form closed loops. A satellite of mass *m,* placed anywhere on any particular equipotential (the red line, for example), will have the same gravitational potential energy. Moving from any one location to any other on the same equipotential line, by any path, requires zero *net* energy input.

◆ **Contour lines** Lines on a map joining places of the same altitude.

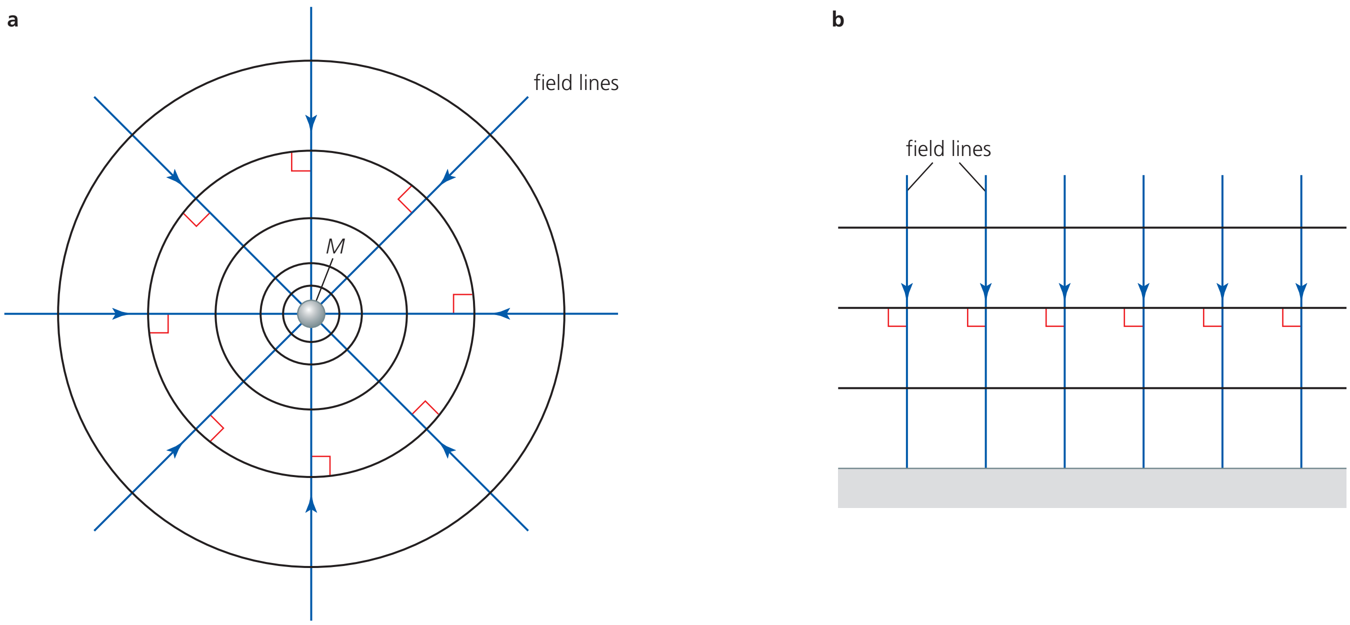
Contour lines on a geographical map (see Figure D1.29) are similar to equipotential lines. The lines join places of equal vertical heights (above sea level), which in effect are equipotential lines. Where the lines are closest, the ground is steepest and anything that is free to move, such as water in a river for example, will move perpendicular to the contours.

Equipotential lines and gravitational field lines (as seen in Figure D1.13) can both be used to provide visualizations of the same gravitational field. Figure D1.30 shows their simple relationship:

Field lines are always perpendicular to equipotential lines. They point from higher potential to lower potential.



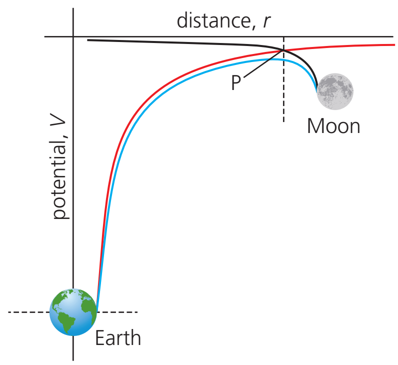
■ **Figure D1.29** Contour map of Stowe, Vermont, USA



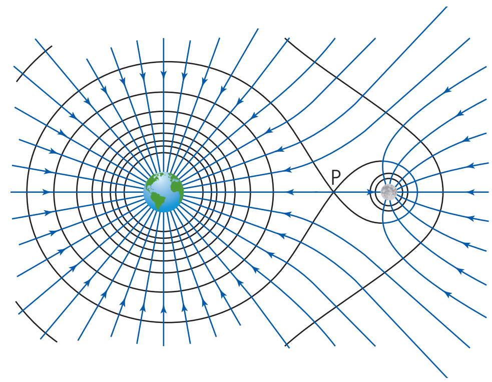
■ **Figure D1.30** Equipotential lines and field lines are perpendicular to each other. **a** radial field; **b** uniform field

Gravitational potential is a scalar quantity, unlike gravitational field strength. This means that, if there are two (or more) large masses creating significant fields, we can determine the potential at any point by simple addition. Figure D1.31 shows an example: the potentials in the Earth–Moon system. The blue line represents the combined potential of the system.

Figure D1.32 shows the equipotential lines (black) and field lines (blue) in the same system.



■ **Figure D1.31** The potentials in the Earth–Moon system



■ **Figure D1.32** Equipotential and field lines around the Earth and Moon.

35 Calculate suitable values of the gravitational potential around the Earth that will enable you to draw a *scale* diagram showing at least three equipotential lines.

36 Discuss the significance of point P seen in Figures D1.31 and D1.32.

37 Explain why contour lines on a geographic map can be considered to be equipotential lines.

Tool 2: Technology

Use spreadsheets to manipulate data

Set up a spreadsheet which will enable you to calculate the combined potentials along a straight line joining the Earth and the Sun. Use the spreadsheet to draw a graph similar to that seen in Figure D1.31.

Gravitational potential difference

SYLLABUS CONTENT

- Work done in moving a mass, m, in a gravitational field as given by: $W = m\Delta V_g$.
- Gravitational field strength, g, as the gravitational potential gradient as given by: $g = -\frac{\Delta V_g}{\Delta r}$.

◆ **Gravitational potential difference** Difference in gravitational potential between two points, which equals the work done when 1 kg is moved between the points.

The central theme of this topic is the movement of masses between different places in gravitational fields, for example, the field around the Earth. This means that the difference in potential – the *potential difference* – between two locations is of particular importance.

Gravitational potential difference, ΔV_g, is the work, W, done on unit mass (1 kg) when it moves between two points in a gravitational field.

$$\Delta V_g = \frac{W}{m} \text{ or}$$

$$W = m\Delta V_g$$

Work has to be done *on* a mass to increase its gravitational potential energy; that is to move it to a greater potential. (For example, away from a planet to a location where the potential has a lesser negative value.) Then W will have a positive value. The mass does the work when it moves to a lesser potential, and W will have a negative value. For example, when a mass falls towards the Earth, gravitational potential energy will be transferred from the mass to kinetic energy.

Top tip!

For comparison, you should remember, from Topic B.5, that in electric circuits, the electric potential difference was the defined as the work done *per unit charge* as given by:

$$V = \frac{W}{q}$$

This concept will be developed further in discussion of *electric* fields in Topic D.2.

HL ONLY

WORKED EXAMPLE D1.12

A satellite of mass 850 kg is in an orbit 7.9×10^6 m from the centre of the Earth.

a Calculate the gravitational potential in this orbit.

b The satellite is to be moved to an orbit where the gravitational potential is $-5.40 \times 10^7\,\text{J kg}^{-1}$. State whether this is a higher, or lower orbit.

c Calculate the work done in this change of orbit.

Answer

a $V_g = -G\frac{M}{r} = -\frac{(6.67 \times 10^{-11}) \times (6.0 \times 10^{24})}{7.9 \times 10^6}$

$= -5.1 \times 10^7\,\text{J kg}^{-1}$

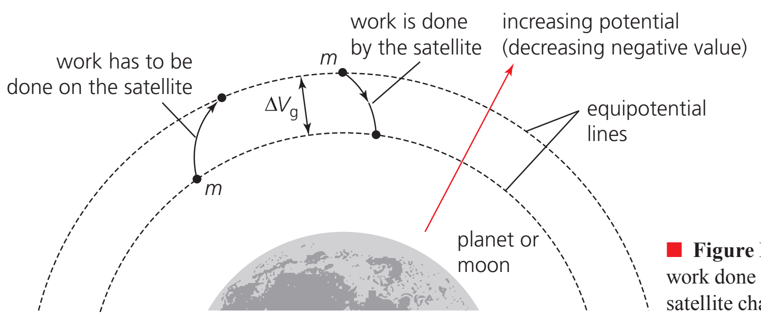
($5.0658.... \times 10^7$ seen on calculator display)

b The potential is lower in the new orbit (greater negative value), so it must be a lower orbit. See Figure D1.33.

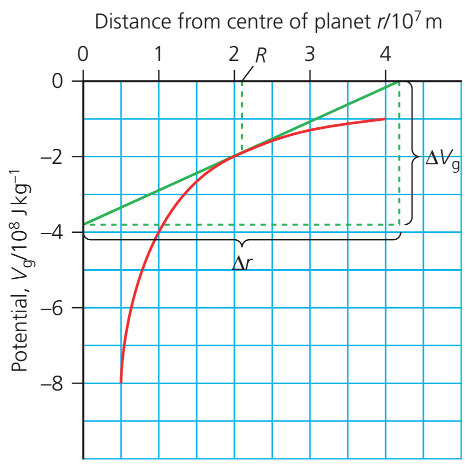
c $W = m\Delta V_g = (850 \times ((-5.40 \times 10^7) - (-5.07 \times 10^7)) = -2.8 \times 10^9\,\text{J}$

The satellite will have to 'lose' this amount of gravitational potential energy to be in the lower orbit.

Note that the answer to **c** is not dependent on the path taken between the two orbits.



■ **Figure D1.33** work done when a satellite changes orbit



■ **Figure D1.34** Graph showing variation of gravitational potential around a planet

◆ **Gravitational potential gradient** Rate of change of potential with distance, equal in magnitude to field strength.

Gravitational potential–distance graphs

Figure D1.34 shows the variation of gravitational potential around a spherical planet. We can use the gradient of the graph at any point ($\Delta V_g/\Delta r$) to determine the strength of the gravitational field, g, which can be explained as follows.

We know that the work done, W, when moving a mass, m, through a potential difference ΔV_g is given by $W = m\Delta V_g$.

We also know that the work can be calculated from force × distance = $mg \times \Delta r$, where Δr is a small enough distance that the value of g does not change significantly. (This assumption would need further explanation or justification at a higher level.)

Hence:

$W = \cancel{m}\,\Delta V_g = \cancel{m}\,g\Delta r$

So that the magnitude of the gravitational field strength $g = \frac{\Delta V_g}{\Delta r}$, called the **gravitational potential gradient**.

Gravitational field strength equals potential gradient: $\boldsymbol{g = -\frac{\Delta V_g}{\Delta r}}$

HL ONLY

The negative sign has been added to the equation to show that the direction of the vector quantity g is opposite to the direction of increasing potential.

WORKED EXAMPLE D1.13

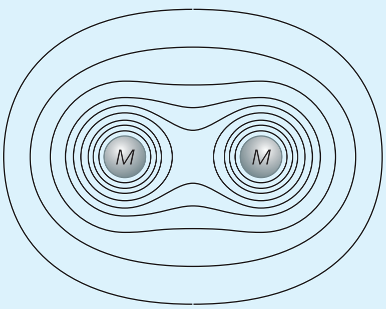
Determine the magnitude of the gravitational field strength at a distance r from the centre of the planet represented in Figure D1.34.

Answer

$$g = \frac{\Delta V_g}{\Delta r} = \frac{(3.8 \times 10^8) - 0}{(4.2 \times 10^7) - 0} = 9.0\,\mathrm{N\,kg^{-1}}$$

38 Calculate the gravitational potential difference when moving up from a 100 m contour line to a 200 m contour line.

39 Figure D1.35 shows equipotential lines around two equal masses. Draw a sketch to represent the gravitational field lines around the same masses.



■ **Figure D1.35** Equipotential lines around two equal masses

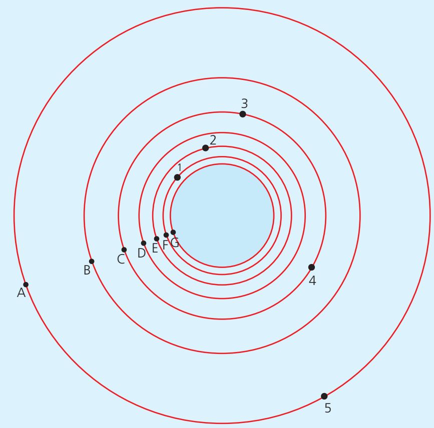
40 Figure D1.36 shows equipotential lines around a planet. Determine the gravitational potential differences associated with the following movements from:

a 1 to 5 **b** 5 to 1 **c** 3 to 4 **d** 2 to 5.

41 a Draw a graph of the potential around the Earth from its surface to a radius of 2.6×10^7 m.

b Use your graph to determine a value of the gravitational field strength at a radius of 1.7×10^7 m.

Line	Potential $\mathrm{J\,kg^{-1}}$
A	-4×10^7
B	-6×10^7
C	-8×10^7
D	-10×10^7
E	-12×10^7
F	-14×10^7
G	-16×10^7

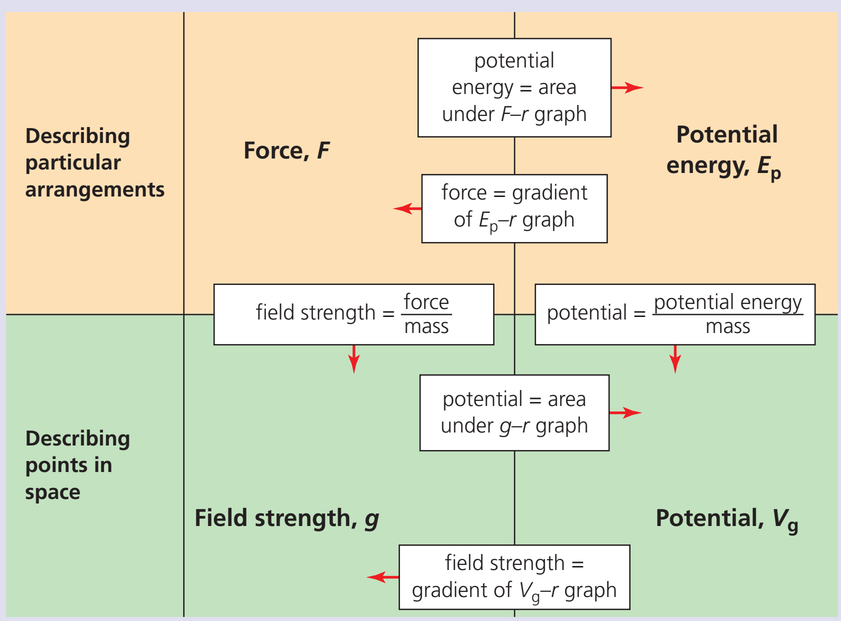


■ **Figure D1.36** Equipotential lines around a planet

ATL D1A: Communication skills

Clearly communicating complex ideas in response to open-ended questions

A useful way to deepen your understanding is to present concepts visually, using charts or other visual organizers to show relationships between concepts. Here are two examples of how the concepts in this topic could be presented in this way:



■ **Figure D1.37** Connections between the magnitudes of the four key concepts

HL ONLY

Force	Potential energy
$F = G\dfrac{m_1m_2}{r^2}$	$E_p = -G\dfrac{m_1m_2}{r}$
Field	**Potential**
$g = G\dfrac{M}{r^2}$	$V_g = -G\dfrac{M}{r}$

■ **Figure D1.38** Equations for radial gravitational fields

Can you think of other ways in which these concepts – and the relationships between them – could be represented?

Speeds and energies of satellites

SYLLABUS CONTENT

- The escape speed v_{esc} at any point in a gravitational field as given by: $v_{esc} = \sqrt{\dfrac{2GM}{r}}$.
- The orbital speed $v_{orbital}$ of a body orbiting a large mass as given by: $v_{orbital} = \sqrt{\dfrac{GM}{r}}$.
- The qualitative effect of a small viscous drag due to the atmosphere on the height and speed of an orbiting body.

In all of this section, for the sake of simplicity, we will assume that the planet from which a satellite is launched is not rotating.

Escape speed

In theory, an object can be *projected* (not powered) upwards in such a way that it could continue to move away from the Earth forever. For this to be possible the object would need to be given a very high speed. To calculate that speed, we need to consider energies.

◆ **Escape speed** Minimum theoretical speed that an object must be given in order to move to an infinite distance away from a planet (or moon, or star): $v_{esc} = \sqrt{\dfrac{2GM}{r}}$.

In general, the initial kinetic energy given to an object will be transferred to gravitational energy, but also dissipated due to air resistance in the Earth's atmosphere. But if we assume that the effects of air resistance are negligible, we can calculate the *minimum* theoretical speed that a projectile of mass *m* needs in order to 'escape' from a planet of mass *M*. This is called its **escape speed**, v_{esc}. It can be calculated as follows:

kinetic energy needed = change in gravitational potential energy between location and infinity

$$\frac{1}{2}mv_{esc}^2 = 0 - \left(\frac{-GMm}{r}\right)$$

which leads to:

speed needed to 'escape' from a planet without air resistance (drag), $v_{esc} = \sqrt{\dfrac{2GM}{r}}$

HL ONLY

Note that:

- This speed is the same, regardless of direction of travel (assuming no air resistance).
- We have assumed that there are no other significant gravitational fields, as might be provided by, for example, a moon.

WORKED EXAMPLE D1.14

a Calculate the escape speed from the Earth's surface.

b Outline why all masses have the same escape speed.

Answer

a $v_{esc} = \sqrt{\dfrac{2GM}{r}} = \sqrt{\dfrac{(2 \times (6.67 \times 10^{-11}) \times (6.0 \times 10^{24}))}{6.4 \times 10^{6}}} = 1.1 \times 10^{4}\,\text{m s}^{-1}$

(about $11\,\text{km s}^{-1}$)

b Double the mass, for example, will need to gain double the gravitational potential energy. So, it needs double the kinetic energy, which it will have with double the mass at the same speed.

LIINKING QUESTION

- How is the amount of fuel required to launch rockets into space determined by considering energy?

Although the equation above can be used for any radius, it assumes that the mass starts with zero kinetic energy. If, for example, we wanted to know the escape speed needed for a satellite already in a steady orbit, we need to take into consideration the initial kinetic energy of the satellite. See below.

Orbital speed

We have already seen that a satellite needs to have the correct **orbital speed** to maintain a circular orbit. From Kepler's third law, we showed that:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Replacing using T with $\dfrac{2\pi r}{v}$ gives us:

$$\frac{r^3 v^2}{4\pi^2 r^2} = \frac{GM}{4\pi^2}$$

which simplifies to (using $v_{orbital}$ instead of v):

◆ **Orbital speed** For a satellite in a circular orbit, its speed must have the correct value for the chosen radius: $v_{orbit} = \sqrt{\dfrac{GM}{r}}$.

DB

speed required to maintain a circular orbit, $\boldsymbol{v_{orbital}} = \sqrt{\dfrac{\boldsymbol{GM}}{\boldsymbol{r}}}$

WORKED EXAMPLE D1.15

What speed is required for a satellite to maintain a circular orbit 100 km above the Moon's surface? (mass of Moon = 7.35×10^{22} kg, radius of Moon = 1737 km)

Answer

$v_{orbital} = \sqrt{\dfrac{GM}{r}}$

$= \sqrt{\dfrac{(6.67 \times 10^{-11}) \times (7.35 \times 10^{22})}{1.837 \times 10^{6}}}$

$= 1.63 \times 10^{3}\,\text{m s}^{-1}$

HL ONLY

WORKED EXAMPLE D1.16

The average distance between the centre of the Moon and the centre of the Earth is 3.84×10^8 m. The Earth has a mass of 6.0×10^{24} kg. Determine a value for the time for each orbit of the Moon around the Earth (assuming that its path is circular).

Answer

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} \Rightarrow \frac{(3.84 \times 10^8)^3}{T^2} = \frac{(6.67 \times 10^{-11}) \times (6.0 \times 10^{24})}{4\pi^2}$$

$T = 2.4 \times 10^6$ s (27 days)

Artificial satellites

To put a satellite into orbit we need to provide enough energy to:

- increase the gravitational potential energy of the satellite
- increase the gravitational potential energy of the launch vehicle fuel, etc.
- give the satellite the required kinetic energy for the required orbit
- overcome fictional forces
- allow for thermal energy dissipation.

The gravitational and kinetic energy given to the orbiting satellite will only be a small percentage of the total energy transferred. This is not a very efficient process!

Once a satellite is in orbit, the energy relationships are less complicated. As we have seen:

gravitational potential energy of a satellite of mass m orbiting a planet, or moon, of mass M at a distance r from the planet's (or moon's) centre is given by:

$$E_p = -G\frac{Mm}{r}$$

For a satellite already in orbit:

$$\text{Total energy, } E_T = E_k + E_p = \frac{1}{2}mv_{orbital}^2 + \left(-G\frac{Mm}{r}\right)$$

But we know that:

$$v_{orbital}^2 = \frac{GM}{r}$$

so that:

$$\text{kinetic energy of a satellite in a circular orbit, } E_k = \frac{1}{2}\frac{GMm}{r}$$

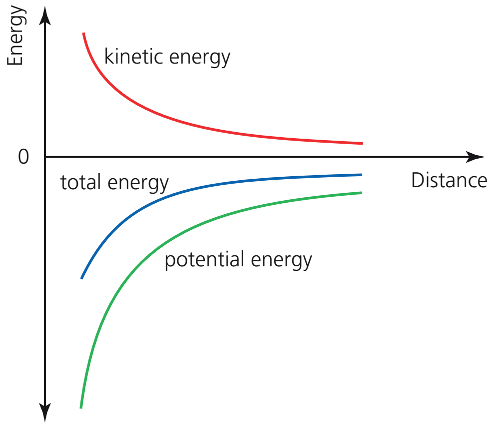
$$\text{Then, total energy, } E_T = \frac{1}{2}\frac{GMm}{r} + \left(-\frac{-GMm}{r}\right)$$

so that:

$$\text{total energy of a satellite in a circular orbit, } E_T = -\frac{1}{2}\frac{GMm}{r}$$

Note that:

$$E_T = \frac{1}{2}E_p = -E_k$$



■ **Figure D1.39** Energies of a satellite

A satellite in a circular orbit does not have enough energy to escape the gravitational field, so that both its potential energy and its total energy are negative. Figure D1.39 shows these relationships graphically.

WORKED EXAMPLE D1.17

A 500 kg satellite is orbiting at a height of 300 km above the surface of the planet Mars.

Mass of Mars = 6.4×10^{23} kg. Radius of Mars = 3.4×10^{6} m.

Determine the satellite's:

a gravitational potential energy

b kinetic energy

c total energy.

Answer

a $E_p = -G\dfrac{Mm}{r} = -\dfrac{(6.67 \times 10^{-11}) \times (6.4 \times 10^{23}) \times 500}{3.4 \times 10^{6}} = -6.3 \times 10^{9}\,\text{J}$

b $E_k = \dfrac{1}{2}G\dfrac{Mm}{r} = +3.1 \times 10^{9}\,\text{J}$

c $E_T = -\dfrac{1}{2}\dfrac{GMm}{r} = -3.1 \times 10^{9}\,\text{J}$

Changing orbits

- Continuing Worked example D1.17, suppose that it was required that a satellite was to be re-directed to an orbit 200 km higher. Then the new values are:
  $E_p = -5.93 \times 10^{9}$ J. This is an increase $\approx 0.35 \times 10^{9}$ J.
  $E_k = +2.96 \times 10^{9}$ J. This is a decrease $\approx 0.17 \times 10^{9}$ J.
  $E_T = -2.96 \times 10^{9}$ J. This is an increase $\approx 0.17 \times 10^{9}$ J.
  The satellite will have less kinetic energy because its necessary orbital speed is less, but there is a greater gain of gravitational potential energy due to the increased height. Overall, energy must be *supplied* for the change.
- If it required that a satellite *already in orbit* (radius r) is to 'escape' the Earth's gravity, we can calculate the extra energy needed from: required gain in gravitational potential energy = existing orbital kinetic energy + extra kinetic energy needed
  extra kinetic energy needed $= \dfrac{GMm}{r} - \dfrac{1}{2}\dfrac{GMm}{r} = \dfrac{1}{2}\dfrac{GMm}{r}$
  This is the same as its existing orbital kinetic energy.
- If a satellite is to be re-directed to a lower orbit, energy must be *removed*. It has to travel faster in its new orbit and gain kinetic energy, but there is an even greater reduction in gravitational potential energy.
- Satellites in low orbits may experience some very slight air resistance (viscous drag). This results in a dissipation of kinetic energy to thermal energy. It would travel more slowly, but as it moves to a lower height it gains an even greater amount of kinetic energy. Overall, its speed increases and the effects of increasing air resistance result in even greater dissipation of thermal energy. An uncontrolled satellite will spiral towards Earth, burn up and disintegrate.

LINKING QUESTION

- How can air resistance be used to alter the motion of a satellite orbiting Earth?

This question links to understandings in Topic A.2.

HL ONLY

42 a Explain what you think 'burn up' means in the paragraph above.

b Research into an occasion when a satellite actually crashed on the Earth's surface and find out what happened.

43 A typical air molecule travels at $450\,m\,s^{-1}$ at room temperature. Explain why the Earth's atmosphere does not spread out into space away from the planet.

44 A satellite of mass 820 kg is orbiting at a height of 320 km above the Earth's surface. Calculate:

a its gravitational potential energy

b its kinetic energy

c its total energy (Earth's radius = 6.4×10^6 m, Earth's mass = 6.0×10^{24} kg)

d the speed it would need to have in order to escape from the Earth.

45 a A 300 kg satellite is in orbit around the Moon at an altitude of 60 km. Calculate how much extra energy it needs to escape from the Moon. (Mass of Moon = 7.3×10^{22} kg, radius of Moon = 1.7×10^6 m)

b State any assumptions you made in answering **a**.

46 a Determine a value for the escape velocity from the surface of the planet Mars. (Mass of Mars = 6.4×10^{23} kg, radius of Mars = 3.4×10^6 m)

b Outline why this escape speed is less than the escape speed from Earth.

47 A satellite in a geosynchronized orbit has a time period of 24 hours.

a Determine the radius of this orbit.

b Calculate the orbital speed of the satellite.

48 What happens to the total energy of a satellite in a circular orbit if it encounters some air resistance, moves to a lower orbit, but gains speed? Explain your answer.

49 a Suggest what effect the spin of the Earth will have on the escape speed.

b Suggest why satellite launch sites are often close to the equator.

50 Draw a Sankey diagram to represent the energy flows as a satellite is launched from the surface of the Earth and then enters an orbit. Assume that the whole process is very inefficient.

51 Titania is a moon of the planet Uranus. It orbits at an average distance of 4×10^8 m from the centre of Uranus. The planet has a mass of 8.7×10^{25} kg.

a Determine the time period (Earth days) of Titania's orbit.

b Calculate its average orbital speed.

c Calculate the strength of the gravitational field of Uranus at the height of Titania.

ATL D1B : Research skills

Using search engines and libraries effectively

Use the internet to find out the latest progress on space launch systems that intend to propel spinning rockets upwards from the Earth's surface by giving them a large amount of kinetic energy. After the rocket has significantly slowed down and reached an altitude of about 60 km, then the engines are ignited for the rest of the trip into orbit.

D.2 Electric and magnetic fields

Guiding questions

- Which experiments provided evidence to determine the nature of the electron?
- How can the properties of fields be understood using both an algebraic approach and a visual representation?
- What are the consequences of interactions between electric and magnetic fields?

Electric charge

SYLLABUS CONTENT

- The direction of forces between the two types of electric charge.
- The conservation of electric charge.

ATL D2A: Communication skills

Clearly communicating complex ideas in response to open-ended questions

The concept of electric charge was introduced in Topic B.5, in which the electric charges of electrons, protons and ions were briefly described. Before beginning this topic, review the beginning of Topic B.5: Electric charge and its conservation.

Make notes on the important concepts you find there. Use a visual organizer and/or diagrams to connect the key concepts.

Charge is measured in *coulombs*, C. One coulomb is a relatively large amount of charge and we often use microcoulombs ($1\ \mu C = 10^{-6}\ C$) and nanocoulombs ($1\ nC = 10^{-9}\ C$).

All protons have a positive charge of $+1.60 \times 10^{-19}\ C$ and all electrons have a negative charge of $-1.60 \times 10^{-19}\ C$.

Any quantity of charge consists of a whole number of these charged particles, each $\pm 1.6 \times 10^{-19}\ C$. Charge is *quantized*.

$1.6 \times 10^{-19}\ C$ is called the elementary charge and it is given the symbol e.

(Note: nuclear particles – protons and neutrons – are themselves composed of smaller particles called *quarks*. Quarks also have quantized charge, but the charge is quantized into multiples of $\pm e/3$. However, quarks cannot be observed as isolated particles. Knowledge of quarks is not required in this course.)

Later in this topic, we will describe Millikan's famous experiment to determine the charge of an electron.

LINKING QUESTION

- Charge is quantized. Which other physical quantities are quantized? (NOS)

This question links to understandings in Topic E.2.

Electrostatic charging and discharging

SYLLABUS CONTENT

- Electric charge can be transferred between bodies using friction, electrostatic induction and by contact, including the role of grounding (earthing).

◆ **Charge (to)** Add or remove electrons, so that an object acquires an overall net charge, for example, by friction. (To 'charge' a battery has a different meaning.)

◆ **Electrostatics** The study of the effects of charges at rest (that is, not electric currents).

Everyday objects contain an enormous number of charged particles and they usually have equal numbers of positive charges (protons) and negative (electrons) charges, so that they have no overall charge and are therefore described as being *neutral*.

Negatively charged electrons are the outermost particles in atoms and some of them are not tightly bound to atoms. If electrons can be added to, or removed from, a neutral object, it will then have an overall charge and we describe the object as being **charged**, which can then result in **electrostatic** effects. Protons, unlike electrons, are located in the nuclei of atoms and cannot be separated or moved from their positions, so they are not involved in producing electrostatic effects.

If a neutral object is given excess electrons, it becomes negatively charged. If the number of electrons is reduced, the object becomes positively charged.

Charging by friction

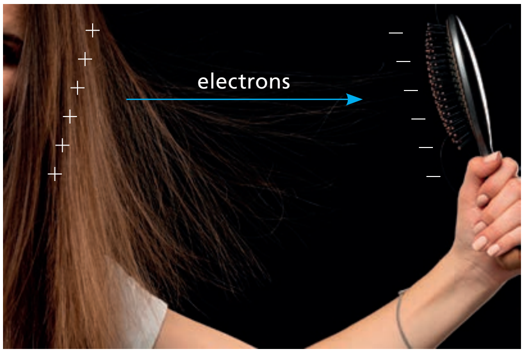


Figure D2.1 When you brush your hair, individual hairs may move apart because of the repulsion between similar charges

One common example is seen in Figure D2.1: when brushing dry hair with a plastic brush, electrons can be transferred in the process, one object (the hair brush) gains electrons, becoming negatively charged, while removing electrons from the other object (the hair), leaving it with a positive charge. The two objects will then attract each other. In this example, individual hairs with similar charge can also be seen to be repelled from each other.

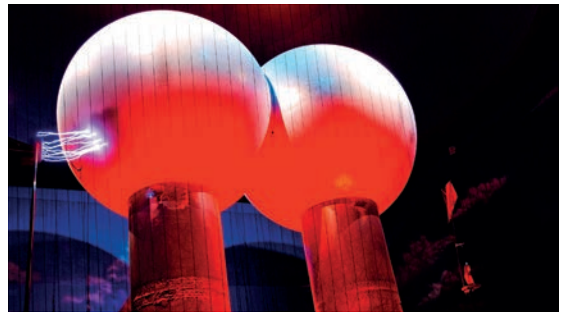
In school experiments, in order to produce electrostatics effects, insulating rods are often rubbed with cloths. Depending on the materials, one will become positively charged and the other negatively charged, as electrons are transferred between the rod and the cloth. These insulating materials have much fewer mobile charge carriers (free electrons) than metals, but if metals were used, the charges would not stay in the same place.

When friction occurs between substances, electrons are often transferred between their surfaces.

Charging by friction is not limited to solids. For example, electrostatic effects can be produced when liquids flow through pipes, or when air flows past fans.

Very high electrostatic voltages can be produced using friction with specially designed (and safe) apparatus. Figure D2.2 shows an example.

■ **Figure D2.2** Classroom electrostatic generator



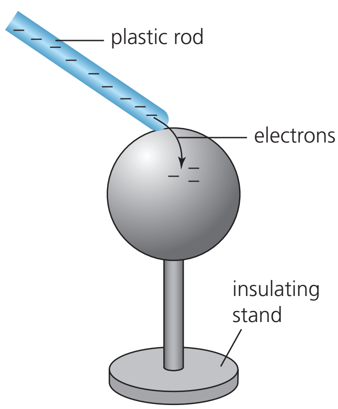
■ **Figure D2.3** The world's largest Van de Graaff frictional electrostatic generator is in the Boston Museum of Science (USA). It has two 4.5 m diameter spheres mounted on 7.6 m insulating poles. It can generate a potential difference of up to seven million volts.

Charging and discharging by contact

If a charged object comes into actual physical contact with another object, it is possible for charges (electrons) to flow between them. Figure D2.4 shows a laboratory example: an isolated metal-coated sphere is touched by a charged plastic rod and some of the excess electrons flow off the rod onto the sphere. When the rod is removed, the excess negative charge will remain on the sphere. If the rod was positively charged, electrons would flow in the opposite direction.

In effect, the charge has been *shared*, but the amount of charge that flows off the original object (a plastic rod in this example) depends on many factors (sizes, shapes, ability to conduct and so on).

If a charged object is touched by a non-insulator which is connected to the ground, electrons will be easily attracted onto, or off, the object, so that it quickly loses its overall charge, although the effect on the ground is insignificant. This is called **discharging**. Charged objects usually tend to become discharged easily because charges can flow through the air, especially if the air is humid (high water content), as in wet weather conditions.



■ **Figure D2.4** Charging a conducting sphere negatively by contact

When objects come in contact with each other, excess electrons, or a deficit of electrons may be shared between them.

■ **Figure D2.5** Symbol for a ground connection

◆ **Discharge** Flow of electrons to or from an object that reduces the overall charge on it.

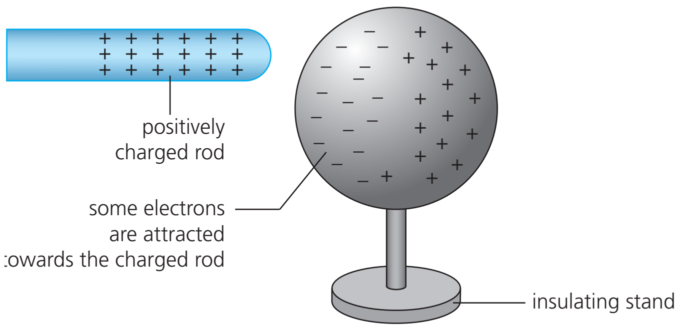
◆ **Earth (ground) connection** A good conductor connected between a point on a piece of apparatus and the ground. This may be part of a safety measure, or to ensure that the point is kept at 0 V.

If we wish to make sure that an object is not charged, a good conducting path (of low resistance) is made with the earth / ground. This is called **earthing** (or **grounding**). In domestic wiring this is done by connecting a thick copper wire to a metal plate in the ground, or by connecting to a metal water pipe. In this way, the frames of metal devices can be kept safe at 0 V – the same as the Earth. In some experiments it may be necessary to keep one point in a circuit at 0 V and this is also done with a connection to earth (ground). The symbol for ground connection is shown in Figure D2.5.

Charged objects will become discharged if they are connected to the ground. If this is done deliberately, it will happen very quickly and is called grounding. The object will then be at 0 V.

Charging by electrostatic induction

If a charged object is brought close to an uncharged object, but *without touching*, forces will be exerted on electrons in the uncharged object. Some electrons in the surface near the charged object will be either repelled or attracted, and this results in some charge separation.
See Figure D2.6, in which a positively charged rod is attracting some electrons towards it, leaving the other side of the sphere positively charged.



■ **Figure D2.6** Charge separation induced without contact

◆ **Electrostatic induction** Movement of charged particles (electrons) caused by the influence of a nearby charged object, but without physical contact.

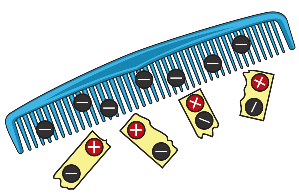
If the rod was negatively charged, some electrons would be repelled to the far side of the sphere, leaving the side closest to the rod with an excess of positive charge.

Charge separation caused, without contact, by a nearby charge is known as **electrostatic induction**.

(In this sense of the word, *induction* is being used to describe something being made to happen without physical contact)

When the charged rod is removed the electrons will redistribute evenly.

Electrostatic induction is needed to explain most of the electrostatic effects we may see in everyday life. For example, Figure D2.7 shows how a comb (which has been previously charged by friction) can attract uncharged small pieces of paper (without contact).



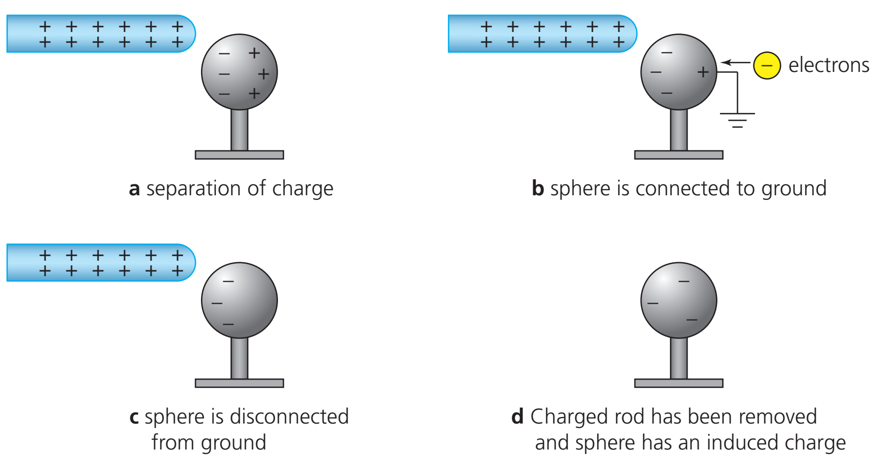
■ **Figure D2.7** An example of electrostatic induction

The excess electrons on the charged comb repel some electrons on the pieces of paper. The comb then attracts the paper because the paper nearest the comb now has a positive charge.

Electrostatic induction can be the best way to charge an object for an experiment, because it does not involve sharing charge. Figure D2.8 shows how. When the sphere is grounded, electrons flow onto the sphere and they will remain there when the connection is removed. Using a negatively charged rod can result in a positively charged sphere.

Common mistake

Do not confuse *electrostatic* induction (described here) with *electromagnetic* induction (Topic D.4).



■ **Figure D2.8** Charging by induction

Dangers of static electricity

Large-scale electrostatic effects can be unwanted and even dangerous. Lightning is an obvious example (see the activity later in this topic). Cars and planes can become charged as they move through the air or along the ground, and this could be a problem when they stop for refuelling – any sparks from a charged vehicle might cause an explosion of the fuel and air. This risk can be prevented by making sure that the vehicle and the fuel supply are well grounded (see question 5).

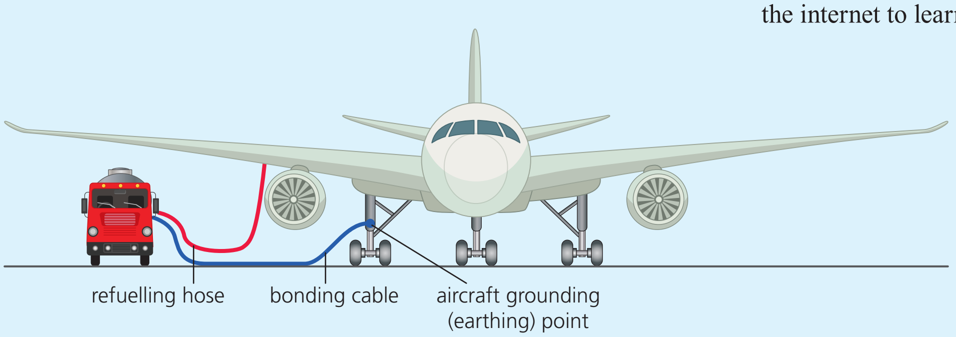
1 Describe the movement of electrons that causes a plastic rod to become positively charged when it is rubbed with a dry cloth.

2 Describe and explain an electrostatic effect that you have seen in your home.

3 Two conducting spheres have charges of −20.0 μC and +6.0 μC. If the spheres are identical and come briefly into contact, determine the charge on each sphere when they are separated.

4 If you were given two conducting spheres on insulating stands (similar to those seen in the previous figures), explain how you could make one positively charged and the other negatively charged by using a negatively charged plastic rod.

5 Explain:
 a how an electrostatic effect could be dangerous when refuelling an aircraft (Figure D2.9)
 b how grounding can prevent the problem arising
 c how sparks could occur at a petrol (gas) station. Use the internet to learn about how dangerous this may be.



■ **Figure D2.9** Refuelling an aircraft

Electric forces: Coulomb's law

SYLLABUS CONTENT

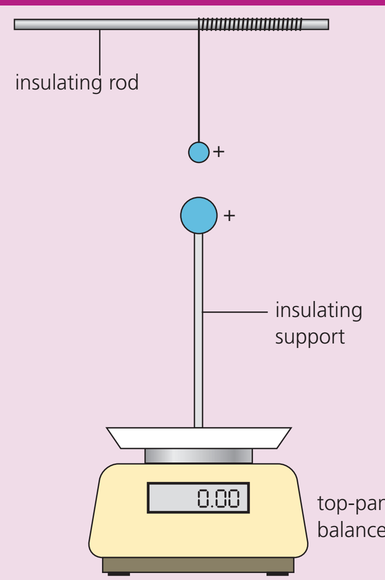
▶ Coulomb's law as given by: $F = k\dfrac{q_1q_2}{r^2}$ for charged bodies treated as point charges where $k = \dfrac{1}{4\pi\varepsilon_0}$

Inquiry 2: Collecting and processing data

Collecting data

Identify issues that might arise when attempting to collect accurate data

Figure D2.10 shows the apparatus that a student plans to use to determine how the force between charges depends on the magnitudes of the charges and their separation. Identify the issues that might arise when the student attempts to collect accurate data.



■ **Figure D2.10** Possible investigation of electric forces

TOK

The natural sciences

- Why are many of the laws in the natural sciences stated using the language of mathematics?

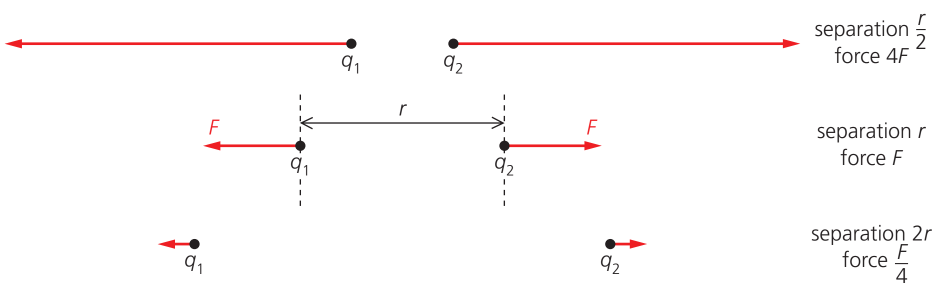
Another useful analogy

Electric forces and fields can be unfamiliar and difficult to visualize. Gravitational fields (Topic D.1) are generally considered easier to understand because they are more familiar. Fortunately, there is a close mathematical *analogy* between these two types of field. A thorough understanding of gravitational fields will be of great help in studying this topic.

The forces between point charges can be represented by an inverse square law: $F \propto 1/r^2$, where r is the distance between the charges (q_1 and q_2). See Figure D2.11, which shows the forces between similar charges (both positive, or both negative). If the charges were opposite (one positive, the other negative) the forces would be attractive.

◆ **Coulomb's law** There is an electric force between two point charges, q_1 and q_2 given by $F = k\frac{q_1 q_2}{r^2}$, where r is the distance between them and k is the Coulomb constant.

◆ **Coulomb constant, k** The constant that occurs in the Coulomb's law equation. $k = 8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$. $k = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is the electrical permittivity of free space.



■ **Figure D2.11** The repulsive force varies with distance between similar charges

Coulomb's law represents the relationship between the forces, F, between two point charges and their separation, r. It has a similar form to Newton's law of gravitation:

The forces between two point charges (q_1 and q_2) separated by a distance r:

$$F = k\frac{q_1 q_2}{r^2}$$

■ **Figure D2.12** French physicist Charles Augustin de Coulomb (1736-1806)

Charged bodies which are spherical can be considered to act as point charges (at the centres of the spheres), so that they also obey Coulomb's law.

The law was first published by Charles Augustin de Coulomb (Figure D2.12) in 1783.

If the two charges are of the same type (positive and positive, or negative and negative), the forces will have positive signs, representing repulsive forces. If the two charges are opposite (positive and negative), the forces will have negative signs, meaning that the forces are attractive.

The constant k is known as the **Coulomb constant**. It has the value **$8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$**.

k is not a fundamental constant (unlike G), because it can be further simplified:

$$\text{Coulomb constant, } k = \frac{1}{4\pi\varepsilon_0}$$

Permittivity

The $1/4\pi$ in the expression for k represents the radial nature of the force and ε_0 represents the electric properties of free space (vacuum).

ε_0 is called the electrical **permittivity of free space** and it has a value of **$8.85 \times 10^{-12}\,C^2\,N^{-1}\,m^{-2}$**.

◆ **Permittivity of free space, ε_0** Fundamental constant that represents the ability of a vacuum to transfer an electric force and field, $\varepsilon_0 = 8.85 \times 10^{-12}\,C^2\,N^{-1}\,m^{-2}$.

◆ **Permittivity (electric) of a medium, ε** Constant that represents the ability of a particular medium to transfer an electric force and field. Often expressed as **relative permittivity**: $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$ (no units), which is also sometimes called **dielectric constant**.

The electrical permittivity of free space, ε_0, is a fundamental constant which represents the ability of free space to transfer an electric force and field.

The permittivities of other substances are all greater than ε_0, although dry air has similar electrical properties to free space. This means that the force between two charges in air would be reduced if the air was replaced by another medium.

The **permittivity of a particular medium, ε,** is divided by the permittivity of free space to give the **relative permittivity, ε_r,** of the medium. Some examples are shown in Table D2.1. (Relative permittivity is sometimes known as the **dielectric constant** of the medium.)

$$\text{relative permittivity} = \frac{\text{permittivity of medium}}{\text{permittivity of free space}}$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Because it is a ratio, relative permittivity does not have a unit. For example, if the permittivity of a certain kind of rubber was $4.83 \times 10^{-11}\,C^2\,N^{-1}\,m^{-2}$, its relative permittivity would be:

$$\frac{4.83 \times 10^{-11}}{8.85 \times 10^{-12}} = 5.46$$

■ **Table D2.1** The approximate relative permittivities of some common insulators.

Free space (a vacuum)	1 (by definition)
dry air	1.0005
polythene	2
paper	4
concrete	4
rubber	6
water	80

WORKED EXAMPLE D2.1

A point charge of 4.5×10^{-8} C is situated in air 3.2 cm from another charge of -1.3×10^{-7} C.

a Determine the electrical force between them.

b If they were separated by polythene, calculate the approximate force between the charges.

Answer

a $F = k\frac{q_1 q_2}{r^2} = \frac{(8.99 \times 10^9) \times (4.5 \times 10^{-8}) \times (-1.3 \times 10^{-7})}{(3.2 \times 10^{-2})^2} = -5.1 \times 10^{-2}\,N$

The negative sign represents an attractive force.

b Polythene has a relative permittivity of about 2, so the force would be divided by 2 (approximately), $F \approx -3 \times 10^{-2}$ N.

6 Given that the electrical permittivity of free space is $8.85 \times 10^{-12}\,C^2\,N^{-1}\,m^{-2}$, show that the Coulomb constant has a value of $8.99 \times 10^9\,N\,m^2\,C^{-2}$.

7 The force between two identical point charges was 5.0 N when they were separated by 7.6 cm.
What were the magnitudes of the charges?

8 The surfaces of two insulated conducting spheres were separated by 14.0 cm. One sphere had a radius of 2.7 cm and had a charge of 3.6×10^{-7} C. The other had a radius of 3.9 cm and had a charge of -4.8×10^{-7} C.

a Determine the force between the spheres in magnitude and direction.

b State any assumption you made when answering **a**.

9 Calculate the force between two point charges of 7.4 C and 2.2 C which are separated by 1.2 m in a non-conducting liquid of relative permittivity 3.1.

10 The force between two point charges was 2.7×10^{-6} N when they were separated by 29 cm.
Predict the force between the same two charges if the separation was increased to 40 cm.

11 Calculate the value of the force between a proton and an electron in a hydrogen atom (separation = 5.3×10^{-11} m).

Nature of science: Patterns and trends

Electric and gravitational forces compared

Two or more charged particles experience *both* electric and gravitational forces between them and it informative to compare the magnitudes of these forces. The forces between an electron and a proton gives perhaps the obvious example.

The electric force between a proton and an electron is about 10^{39} × greater than the gravitational force. This is truly an unimaginably large number! Immediately, we can see that gravitational forces are totally insignificant when discussing atomic particles.

Both types of force follow a similar inverse square law, so why are electrical forces apparently insignificant on the very large scale? For example, gravitation seems to dominate an understanding of the formation and motions of planets and stars. This is because gravitational forces are only attractive and increase with the size of the masses involved, but electric forces are both attractive and repulsive. On the small scale, separate charges result in significant electric forces, but on the very large scale the enormous numbers of positive and negative charges are usually approximately balanced, so that electrostatic effects are insignificant.

LINKING QUESTION

- What are the relative strengths of the four **fundamental forces**?

This question links to understandings in Topics D.1, D2, E.1 and E.3.

◆ **Fundamental forces (interactions)** Strong nuclear, electromagnetic and gravitational forces (and the weak nuclear force) are the four fundamental forces.

◆ **Radial field** Field (electric or gravitational) that spreads out from a point equally in all directions.

Electric fields

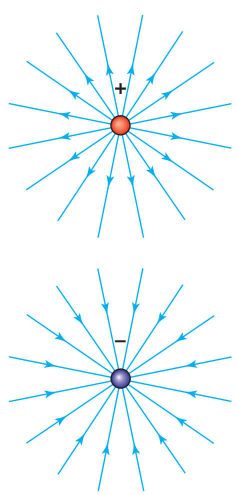
SYLLABUS CONTENT

- Electric field lines.
- The relationship between field line density and field strength.
- The electric field strength as given by: $E = \frac{F}{q}$.
- The electric field strength between parallel plates as given by: $E = \frac{V}{d}$.

A region in which a charge would experience an electric force is called an *electric field*.

Electric fields are represented on paper, or on a screen, with electric field lines. The direction of electric forces depends on the nature of the charges but, by convention we choose that:

electric field lines always point in the directions of the forces on positive charges.



■ **Figure D2.13** Radial electric fields around point charges

Figure D2.13 shows the two most basic *electric fields*: **radial fields** around an isolated point positive charge and around an isolated point negative charge

The field lines can never cross each other and the field is strongest (in a particular diagram) where the field lines are closest together (densest).

Figure D2.14 shows the combined electric field of two equal point charges. If the charges were both positive the field lines would point in the opposite directions. Figure D2.15 shows the field around opposite charges of equal magnitude.

Figure D2.16 shows the electric field around a charged solid conducting sphere (in this example the excess charge is negative). The mobile charges (electrons) repel each other, so that they are evenly distributed on the outer surface of the sphere. The resulting electric field is perpendicular to the surface, but there is no field inside the sphere. The field is identical to that around a point charge at the centre of the sphere which had the same charge as all the excess electrons combined. The same is true for a hollow conducting sphere.

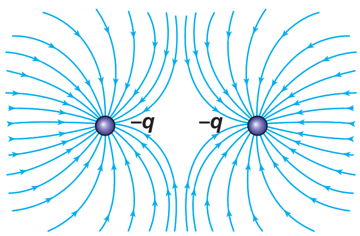


Figure D2.14 Field around two similar point charges

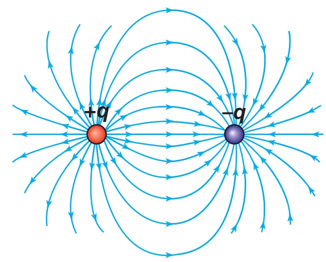


Figure D2.15 Field around two opposite point charges

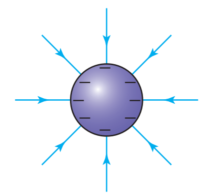


Figure D2.16 Electric field around a charged solid sphere

Electric field lines must be perpendicular to any conducting surface.

If this was not true, there would be a component of the electric force acting on the electrons, so that they would rearrange until the field was perpendicular.

If a surface is flat, the forces between adjacent mobile charges will be parallel to the surface and this results in an even distribution of charges, as seen in Figure D2.17a. But if the surface has a variable curvature, the forces between adjacent charges will not be parallel to the surface and will vary with the amount of curvature. (Figure D2.17b). This results in charge becoming concentrated where the curvature changes most suddenly, that is, near points. See Figure D2.18.

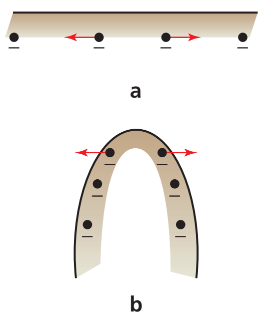


Figure D2.17 Forces between charges near the surface of a conductor

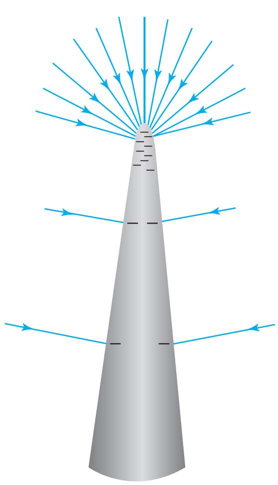


Figure D2.18 Concentration of charge and electric field near points

Figure D2.19 shows an important arrangement: the *uniform electric field* that can be created between parallel metal plates. The positive terminal on the battery attracts electrons, so that the top plate becomes positively charged. The lower plate becomes negatively charged because electrons have been repelled away from the negative terminal of the battery. The field is weaker and non-uniform beyond the edges of the plates. At the points midway between the edges of the two plates, the strength of the field is half of its maximum value.

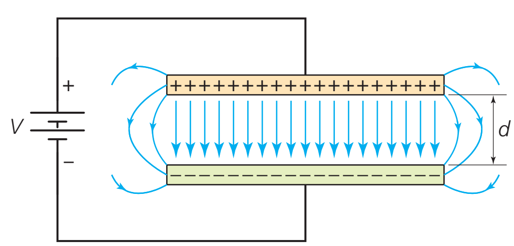


Figure D2.19 Creating a uniform electric field

The arrangement seen in Figure D2.19 can be used with high voltages if strong, uniform fields are needed in experiments.

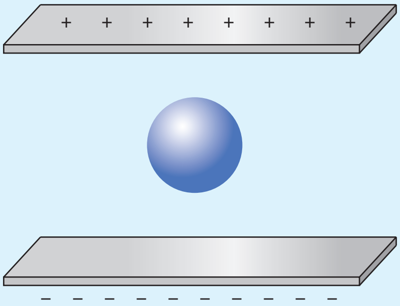
12 By considering components of forces, discuss why field lines must always be perpendicular to conducting surfaces.

13 Suggest why pointed conductors (see Figure D2.20) are a good way of discharging static electricity.

■ **Figure D2.20** Static dischargers on the wing of an aircraft

14 Consider Figure D2.19. Draw electric field lines to represent the electric field if the battery was reversed and the p.d. halved.

15 An uncharged metal sphere is between two charged metal plates as shown in Figure D2.21. Copy the figure, show where charges are induced and add lines to represent the electric field.



■ **Figure D2.21** An uncharged metal sphere between two charged metal plates

Electric field strength

Electric field strength is defined in a similar way to gravitational field strength:

◆ **Electric field strength, *E*** The electric force per unit charge.

Electric field strength, *E*, is defined as the force per unit charge that would be experienced by a small positive test charge placed at that point:

$$E = \frac{F}{q}$$

SI unit: $\mathrm{N\,C^{-1}}$

TOK

The natural sciences

- What is the role of imagination and intuition in the creation of hypotheses in the natural sciences?
- Are gravitational and electric fields real?

The effects of gravitational and electric forces are easily observable, but we have defined these fields in term of such forces, so do the fields really exist if there are no forces acting? For example, is there really an electric field around a stationary charge if there is no other charge present? Or, is the concept of 'field' just an imaginary device constructed to help us understand that forces can act without contact?

WORKED EXAMPLE D2.2

If there is a force of 3.4×10^{-6} N acting on a point charge of 6.7 nC, calculate the magnitude of the electric field strength at that location. ($1\,\mathrm{nC} = 1 \times 10^{-9}\,\mathrm{C}$)

Answer

$$E = \frac{F}{q} = \frac{3.4 \times 10^{-6}}{6.7 \times 10^{-9}} = 5.1 \times 10^{2}\,\mathrm{N\,C^{-1}}$$

There is not enough information in this question to know the direction of the field.

An expression for the strength of the constant electric field between parallel plates (Figure D2.19) can be obtained by considering the work done as a charge, q, moves from one plate to the other:

work done = force × distance = potential difference × charge (from Topics A.3 and B.5)

$W = Fd = Vq$

Rearranging, and remembering that $E = \frac{F}{q}$, gives:

electric field strength between parallel metal plates, $\boldsymbol{E = \frac{V}{d}}$

Expressed in this way we can see that $V\,m^{-1}$ is an alternative to $N\,C^{-1}$ as the units for electric field strength.

(As we shall see later, this is an example of electric field strength equalling the potential gradient.)

Tool 3: Mathematics

Using units, symbols and numerical values

Consider again the situation shown in Figure D2.19. The space between the plates has a vacuum, and a charge, q, is next to the negative plate. The charge will accelerate towards the positive plate and gain kinetic energy. In this process, the work done on the charge is $Fd = Vq$, as explained above.

Consider a numerical example:

An electron (mass 9.110×10^{-31} kg and charge -1.60×10^{-19} C) is accelerated across a distance of 5.0 cm by a potential difference of 3000 V.

Assuming that the electron starts with zero kinetic energy, its final kinetic energy equals the work done on it by the uniform electric field, $W = 3000 \times (1.60 \times 10^{-19}) = 4.80 \times 10^{-16}$ J. (If we want to determine the speed of the electron, we can equate this to $\frac{1}{2}mv^2$, which gives an answer of $3.25 \times 10^7\,m\,s^{-1}$.)

In situations similar to this, rather than using joules as the unit of energy, it is more common and easier to use the **electronvolt**.

One electronvolt is an amount of energy equal to 1.60×10^{-19} J. This the amount of energy gained by a charge of 1.60×10^{-19} C when accelerated by a potential difference of 1.00 V. (work done, $W = qV$).

In the previous example, the work done on the electron by the electric field (= kinetic energy it gains) can be stated as 3000 eV, without the immediate need for any further calculations.

Common mistake

Although it is called an *electron*volt, this unit is widely used for the atomic-scale energies of any particles, or radiation. Some examples: a proton accelerated by 5 kV will gain 5 keV of energy; A doubly charged ion accelerated by 2000 V will gain 4 keV of energy; a gamma ray (Topic E.1) may transfer 5 MeV of energy.

◆ Electronvolt, eV
An amount of energy equivalent to that which is transferred when an electron is accelerated by a potential difference of 1 V. $1\,eV = 1.60 \times 10^{-19}$ J.

WORKED EXAMPLE D2.3

Parallel metal plates are separated by a distance of 0.50 cm. Determine the potential difference needed to create an electric field of one million $V\,m^{-1}$.

Answer

$$E = \frac{V}{d}$$

$$1.0 \times 10^6 = \frac{V}{0.0050}$$

$$V = 5.0 \times 10^3\,V$$

16 Calculate the value of electric field strength that would exert a force of 6.3×10^{-14} N on a proton.

17 a Calculate the electric field strength along a straight wire of length 38 cm if there is a potential difference of 0.0010 V between its ends.

b What force would this field exert on a free electron in the wire?

c Determine the acceleration of the electron.

d Explain why the electron is not accelerated to an extreme speed.

18 a Calculate the electric field strength between parallel metal plates separated by 8.0 cm when a p.d. of 15 kV is connected across them.

b How much energy is gained by an electron accelerated between the plates in:

i eV **ii** joules?

19 If the electric field strength exceeds about 3 kV mm^{-1}, air can begin to conduct electricity. Predict the potential difference needed across 20 cm for this to happen.

Electric field strength around a point charge

The strength of an electric field around a point charge decreases with the square of the distance (an inverse square law relationship):

$$E = \frac{F}{q} = \frac{kq}{r^2}$$

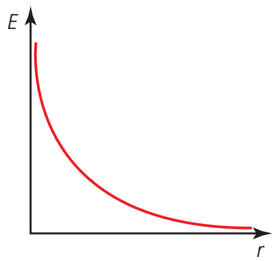


Figure D2.22 Electric field strength around a positive charge

WORKED EXAMPLE D2.4

Determine the electric field strength at a distance of 1.0 m from a charge of $+2.9 \times 10^{-8}$ C.

Answer

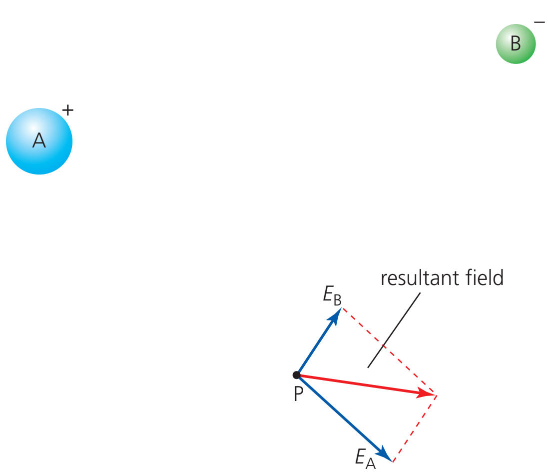
$$E = \frac{kq}{r^2} = \frac{(8.99 \times 10^{9}) \times (+2.9 \times 10^{-8})}{1.0^2}$$

$$= 260\,\text{N C}^{-1}$$

Figure D2.22 shows the variation of electric field strength around a point positive charge. The field strength would be negative if the charge was negative.

Combining electric fields

Electric fields can be combined to determine a resultant by using vector addition.



This is straightforward for places on the line that joins charges, for example: if a field of 420 N C^{-1} to the left and a field of 550 N C^{-1} to the right act at a point; the combined field is 130 N C^{-1} to the right.

More generally, we can use a parallelogram and scale drawing (or trigonometry) as seen in Figure D2.23.

Figure D2.23 Determining a resultant electric field

WORKED EXAMPLE D2.5

Figure D2.24 shows two separate electric fields acting at a point, P. Determine the resultant field.

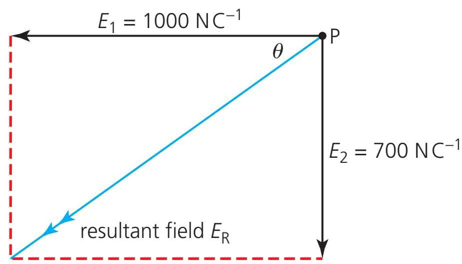


Figure D2.24 Two separate electric fields acting at a point P

Answer

Using Pythagoras, $E_R^{\ 2} = E_1^{\ 2} + E_2^{\ 2} = 1000^2 + 700^2$

$E_R = 1220\,\text{N}\,\text{C}^{-1}$

Electric field is a vector quantity, so the answer must include a direction:

$\tan\theta$ (as shown) $= \dfrac{700}{1000} = 0.700$

$\theta = 35°$

20 Sketch a graph to represent the variation of electric field strength with distance around a negatively charged conducting sphere.

21 Calculate the electric field strength 15 cm from a charge of −8.4 μC when it is in:

a air

b a material of relative permittivity 1.6.

22 The electric field strength 150 cm from a point charge is $2.0 \times 10^5\,\text{N}\,\text{C}^{-1}$.
Determine at what distance the field strength would be one million $\text{N}\,\text{C}^{-1}$.

23 A nucleus of a carbon atom has a charge of $+6e$.
Determine the distance from its centre where the electric field strength has a value of $3.0 \times 10^{10}\,\text{N}\,\text{C}^{-1}$.

24 a Calculate the electric field strength midway between point charges of 26 nC and −10 nC when they are separated by a distance of 50 cm.

b Determine the field strength at a point which is 35 cm from the negative charge and 15 cm from the positive charge.

25 A point charge Q is 8.3 cm from a charge of +56 μC. The electric field strength is zero at a point which is 4.7 cm from Q on a line joining the two charges. Determine the charge of Q.

26 The average electric field strength just above the surface of the Earth is about $150\,\text{N}\,\text{C}^{-1}$, directed downwards.
Estimate the total resultant charge carried by the Earth. (The radius of Earth is 6.4×10^6 m.)

27 Sketch the approximate shape of the electric field lines around two charged spheres of equal radius, R, and separated by $4R$: one with a charge of $+Q$, the other with a charge of $-4Q$.

Millikan's experiment

SYLLABUS CONTENT

- Millikan's experiment as evidence for quantization of electric charge.

A strong, uniform electric field was an essential component of the famous 1909 experiment to determine the charge of an electron. See Figure D2.25.

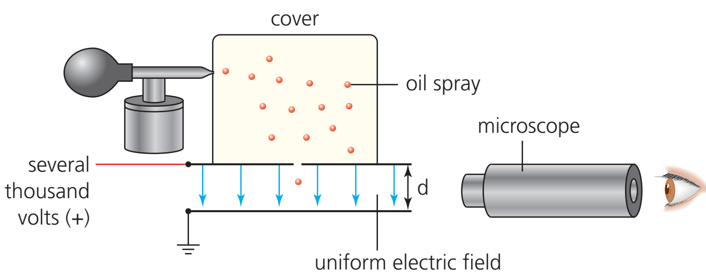


Figure D2.25 Millikan's oil drop experiment

Robert Millikan and Harvey Fletcher used this apparatus to determine the small quantities of charge on oil drops. Details are provided below. The drops can be electrostatically charged because of friction when they are sprayed into the upper compartment. (Additionally, the charge on the drops can be changed using X-rays or a radioactive source, but the details are not needed here.)

The importance of this famous experiment lies in the fact that

> the charges of *all* drops were multiples of the same number (-1.60×10^{-19} C).

So, for example, the following oil drop charges ($\times 10^{-19}$ C) could have been determined: −9.62, −11.20, −4.84, −17.58, −6.43, −8.00. Allowing for experimental uncertainty, statistical analysis informs us that it is highly probable that all these numbers are multiples of −1.60.

A simple analogy may help. Suppose you were given six sealed bags each containing an unknown number of the same coins. The masses of the coins (excluding the bags) were 84 g, 63 g, 98 g, 35 g, 56 g and 42 g. What was the probable mass of one coin? (Answer: 7 g)

When enough measurements reach the same conclusion, it effectively becomes a certainty.

Electric charge is not a continuous quantity. It can only have certain discrete values (multiples of *e*). We say that it is quantized.

> Millikan's experiment confirms that electric charge is a quantized quantity.

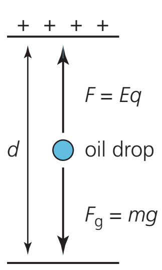


Figure D2.26 Balanced forces on a stationary oil drop

Experimental details – understanding Millikan's experiment

If the potential difference, V, between the plates is varied, then the electric force, F, on a charged drop changes. With the correct potential difference, it is possible for the electrical force and gravitational force to become equal and opposite. The resultant force will then be zero and the drop will be stationary. See Figure D2.26.

$$F = mg = Eq = \frac{Vq}{d}$$

If V, d and m are known, q can be calculated.

WORKED EXAMPLE D2.6

What potential difference across parallel metal plates separated by 2.1 cm is necessary to keep an oil drop of mass 2.7×10^{-14} kg stationary if it has five excess electrons?

Answer

$$mg = \frac{Vq}{d}$$

$$2.7 \times 10^{-14} \times 9.8 = \frac{(V \times 5 \times (1.60 \times 10^{-19}))}{2.1 \times 10^{-2}}$$

$$V = 6.9 \times 10^{3}\,\text{V}$$

28 An oil droplet has a weight of 7.68×10^{-15} N.

a If it is stationary between two horizontal metal plates which are 1.0 cm apart with a voltage of 120 V across them, determine the charge on the oil droplet.

b How many excess electrons are on the droplet?

29 Show, with an approximate calculation, why it may be reasonable to ignore the buoyancy force in the previous calculation.

30 Explain why it is reasonable to assume that the masses of the coin bags described above (84 g, 63 g, 98 g, 35 g, 56 g and 42 g) lead to the conclusion that the mass of each coin is 7 g.

Note that there is also a buoyancy force (see Topic A.2) acting upward on the oil drop. This force is much less than the weight of the oil drop and has been ignored for the sake of simplicity. For very accurate work it would need to be included in the calculation.

The mass of a spherical oil drop can be determined from its dimensions and the density of the oil. More accurately (the drop will not be perfectly spherical), the terminal speed of a drop can be used to determine its mass using Stokes's law (Topic A.2).

Nature of science: Models

Lightning

Figure D2.27 Lightning over Kuala Lumpur

Water and ice droplets of different sizes in clouds are variously affected by convection currents. As they move past each other, friction causes the transfer of electrons (as described earlier in this topic). This typically results in the upper and lower surfaces of the cloud becoming charged differently and charge separation induced on the ground. The result can be a strong electric field between the cloud and the ground.

This can be a complex situation. Scientific explanations often require that reasonable assumptions are made so that a simplified model can be suggested that allows us to understand complicated natural phenomena. In the case of lightning, we could use a simplified model such as that shown in Figure D2.28.

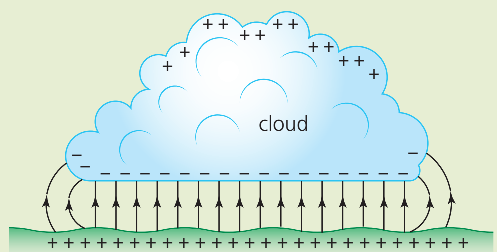


Figure D2.28 Electric field under a storm cloud

When the potential difference between the ground become large enough the (wet) air can dramatically conduct electricity: a lightning strike.

Some approximate numerical values:

p.d., $V = 2.0 \times 10^{6}$ V

separation of ground and cloud, $d = 0.5$ km

$E = \frac{V}{d} = 4000\,\text{V m}^{-1}$

Charge transferred in lightning strike of $\Delta t = 0.2$ s

$\Delta q = 3000$ C

so that current:

$I = \frac{\Delta q}{\Delta t} = 15\,000 A$

Power $= IV = 2.0 \times 10^{6} \times 15\,000 = 3.0 \times 10^{10}$ W

Total energy transferred $= P\Delta t = 3.0 \times 10^{10} \times 0.2 = 6.0 \times 10^{9}$ J

Magnetic fields around permanent magnets

SYLLABUS CONTENT

- Magnetic field lines.

◆ **Magnetic forces** Fundamental forces that act across space between all moving charges, currents and/or permanent magnets.

◆ **Permanent magnet** Magnetized material that creates a significant and persistent magnetic field around itself.

◆ **Ferromagnetic materials** Materials from which permanent magnets are made.

◆ **Magnetic poles (north and south)** Regions in a magnetic material where the field is strongest

Magnetic forces can act across space in a similar way to gravitational and electrical forces, but the equations for magnetic forces and fields are different in form from the other two. Magnetic effects are very closely connected to electrical effects.

A magnetic field exists anywhere that a magnetic force occurs.

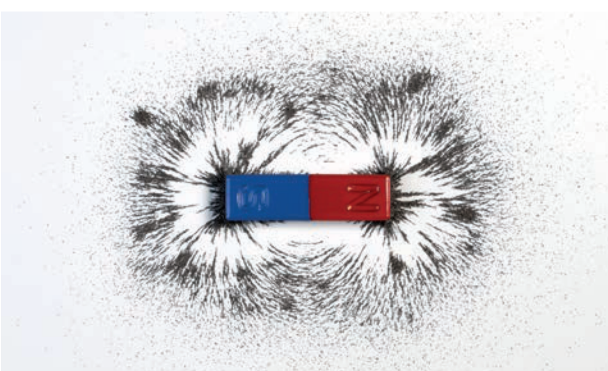
> Magnetic fields are produced around all moving charges (currents).

Any 'stationary' charge has an electric field around it but, if a charge moves, there is also a magnetic field around it, which is perpendicular to the electric field. We frequently refer to the combinations as *electromagnetic* fields.

Before considering the important subject of the production of temporary magnetic fields when currents flow in circuits, we will consider the **permanent magnets** with which we are all familiar, similar to that seen in Figure D2.29.

■ **Figure D2.29** Magnet on a refrigerator door

The motion of electrons within atoms creates tiny magnetic fields, but in most elements these effects cancel each other, so that they have no significant magnetic properties. Iron is a notable exception because it can be magnetized. Iron alloys, nickel, cobalt and some rare Earth metals can also be magnetized. They are known as **ferromagnetic materials**. After a ferromagnetic material has been magnetized, it may lose its magnetism quickly, it is then described as being 'soft' (magnetically). Pure iron is an important example. However, many other ferromagnetic materials, steel for example, are magnetically 'hard' and can retain their magnetism for a long time.



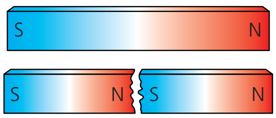
■ **Figure D2.30** Iron filings show the shape of the magnetic field around a bar magnet

The simplest permanent magnets are made in the shape of a straight bar. See Figure D2.30. The shape of the magnetic field around the bar magnet has been shown by sprinkling tiny pieces of iron (iron filings) around the magnet. This is explained later.

The magnetism is effectively concentrated at the ends of the bar and these are called **magnetic poles**. We have seen that there is only one type of mass, but two types of charge (positive and negative) which can be isolated from each other. Magnetism is different: there are only two types of magnetic pole, but they *always* occur in pairs. Magnetic poles are called magnetic north poles and magnetic south poles. Confusingly, these names have no direct link with geography. An explanation is included later.

◆ **Dipole** Two close electric charges (or magnetic poles) of equal magnitude but of opposite sign (or polarity).

One end of the bar magnet seen in Figure D2.30 is a magnetic north pole (N), the other is a magnetic south pole (S). This simple arrangement is called a magnetic **dipole**.



■ **Figure D2.31** Cutting a bar magnet in half

If the magnet was cut in half, we would *not* have separate poles. The result would be two smaller, weaker magnets, but they would still have a magnetic north pole at one end and a magnetic south pole at the other end. See Figure D2.31.

Magnetic field lines

Magnetic field lines are used to represent magnetic fields on paper or screen. As with gravitational and electric fields, we give a direction to magnetic field lines.

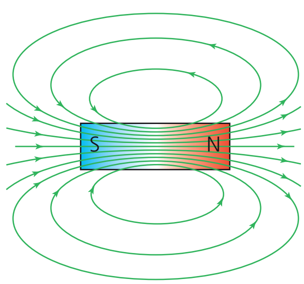
Magnetic field lines always form closed loops. The direction of field lines (around a magnet) is from a magnetic north pole to a magnetic south pole.

Usually we do not show the field lines *inside* magnets but, when we do, their direction is from the south magnetic pole to the north magnetic pole. See Figure D2.32.

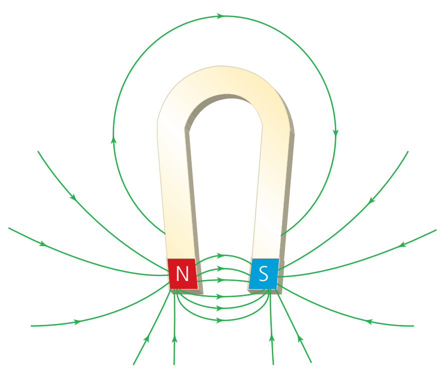
As with other field lines, magnetic field lines can never cross each other and the field is strongest where the lines are closest together (in a specific diagram). Figure D2.32 shows clearly that the field outside the magnet is strongest close to the poles.

Magnets are often made into U-shapes, as seen in Figure D2.33. This strengthens the field near to the poles.

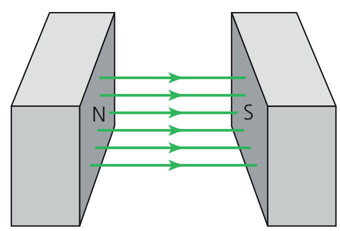
A strong *uniform* magnetic field is often needed for experiments. Figure D2.34 show how this can be achieved with parallel permanent magnets.



■ **Figure D2.32** Field lines in and around a bar magnet



■ **Figure D2.33** The field lines near a U-shaped magnet



■ **Figure D2.34** Creating a strong, uniform magnetic field

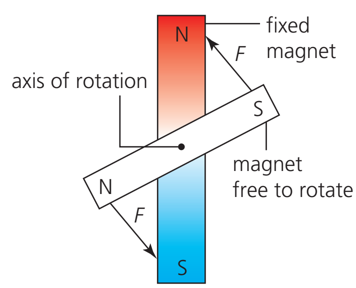
Forces between permanent magnets

Similar magnetic poles repel each other, opposite poles attract.

◆ **Compass** A device for determining direction. Small plotting compasses are used to investigate the shapes of magnetic fields in the laboratory.

If two bar magnets are brought close to each other, the forces between them will cause them to align (if at least one of them is free to move), as shown in Figure D2.35.

Many **compasses** use this effect.



■ **Figure D2.35** Magnetic forces forming a couple (see Topic A.4)

◆ **Induced magnetism** When a ferromagnetic material becomes magnetized because it is in an external magnetic field.

Figure D2.36 Example of induced magnetism

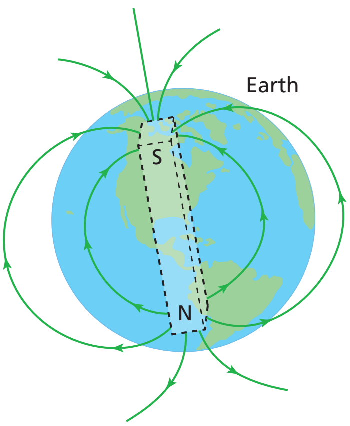


Figure D2.37 The Earth's magnetic field

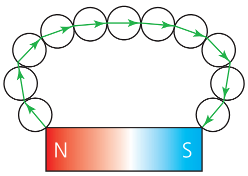


Figure D2.38 Plotting compasses indicating a magnetic field line

Induced magnetism

When ferromagnetic materials are located in magnetic fields, they tend to become magnetized to some extent.

This is called **induced magnetism**. These effects may reduce, or disappear, if the material is removed from the field.

This effect explains why, for example, a permanent magnet can attract unmagnetized nails, as shown in Figure D2.36. Each nail becomes magnetized and can then induce magnetism in other nearby nails.

In a similar way, all the tiny iron filings seen in Figure D2.30 each get magnetically induced and then line up with the field of the magnet.

The Earth's magnetic field

The Earth behaves as a very large, weak bar magnet with a magnetic south pole near the geographic North Pole. See Figure D2.37.

Many compasses detect the Earth's magnetic field in order to indicate direction. The compass needle is a small permanent magnet which is able to rotate freely. It aligns with the Earth's magnetic field, so that the north pole of the compass magnet points towards the south pole of the Earth's magnetic field, which is close to the geographic North Pole.

The magnetic north pole of a magnet is so called because that end of a freely suspended magnet points towards geographic North (where there is a south magnetic pole).

Detecting magnetic fields

As explained above, a compass effectively detects the Earth's magnetic field, and small 'plotting compasses' are widely used to detect magnetic fields around magnets in school laboratories. They point along the magnetic field lines, effectively from magnetic north to magnetic south. See Figure D2.38.

The plotting compasses are also in the Earth's magnetic field, but the Earth's field is weak in comparison to the field close to the bar magnet. Iron filings are also widely used to show the shape of a magnetic field, as seen in Figure D2.30. In recent years tiny *magnetometers* for measuring magnetic fields have become common. They are to be found in most mobile phones.

Magnetic fields around currents in wires

As was stated at the beginning of this topic, magnetic fields are produced by currents, so the best place to start when understanding magnetism is with the simplest situation.

Magnetic field around a steady current in a long straight wire

The magnetic field lines around a current in a long straight wire are circular.

The increasing separation of the field lines indicates that the field strength decreases with increasing distance from the current (see Figure D2.39). This basic magnetic field can be demonstrated using iron filings or compasses, but the field is not strong unless a large current is used. The direction of the field can be predicted by using the **right-hand grip rule**, as seen in Figure D2.40.

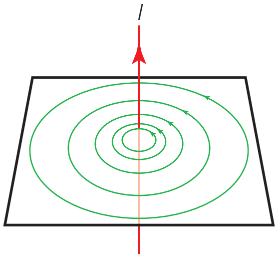


Figure D2.39 Field around a constant current in a long straight wire

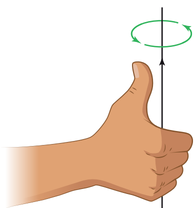


Figure D2.40 Right-hand grip rule

Magnetic fields around steady currents in coils and solenoids

As we would expect, to produce a stronger field requires a greater current in the wire, but there is an easier way: wind insulated wire into a coil or **solenoid**, then each extra turn of wire increases the field strength (with the same current). A solenoid is a coil of insulated wire wrapped regularly so that the turns do not overlap and it is significantly longer than it is wide.

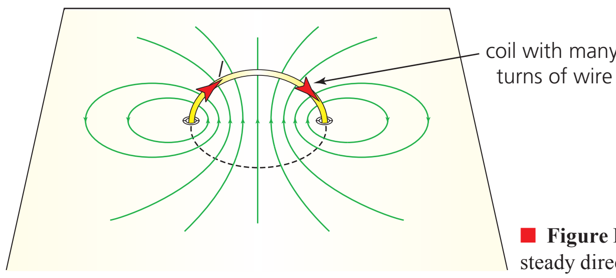


Figure D2.41 Magnetic field around a steady direct current in a circular coil

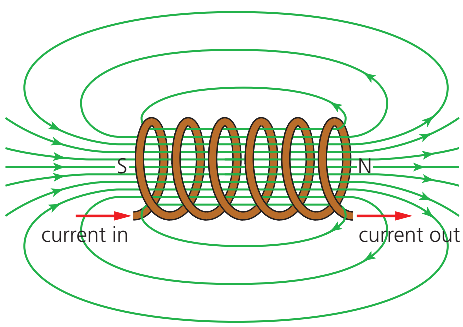


Figure D2.42 Magnetic field due to the current in a solenoid

Figure D2.42 shows the magnetic field in and around a solenoid. (The number of turns shown has been reduced for clarity.) Of especial importance is the strong, uniform field *inside* the solenoid. Comparing Figure D2.42 with Figure D2.32, it is clear that the magnetic field produced by a current in a solenoid is similar in shape to the magnetic field of a bar magnet.

The **polarity** of the magnetic field (which end is a north pole, and which end is a south pole), depends on the direction of the current. It can be predicted using the right-hand grip rule. (Alternatively, when viewed from an end, that end is acting as a south pole if the current is clockwise.)

◆ **Right-hand grip rule** Rule for determining the direction of the magnetic field around a current.

◆ **Solenoid** Long coil of wire with turns that do not overlap (helical).

◆ **Polarity** Separation of opposite electric charges or opposite magnetic poles, which produces uneven effects in a system.

◆ **Electromagnet** Magnet which needs the flow of an electric current in a coil to produce magnetic effects.

◆ **Soft iron** Form of iron (pure or nearly pure) that is easily magnetized and demagnetized.

Electromagnets

Strong **electromagnets** can be made by winding a coil or solenoid on soft iron. A simple example is shown in Figure D2.43.

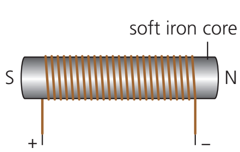


Figure D2.43 Electromagnet

The strength of the magnetic field can be controlled by changing the magnitude of the current and using a soft iron core greatly increases the strength of the field. Importantly, the electromagnet loses its magnetism as soon as the current is turned off. If the core was made of steel, it would retain much of its magnetism when the current was disconnected.

Electromagnets have a very wide range of uses.

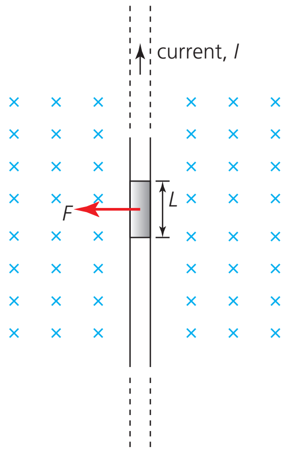


Figure D2.44 Force on current in a magnetic field. (The crosses represent a magnetic field directed perpendicularly into the page.)

◆ **Magnetic field strength, *B*** Defined in terms of the force on a current: The field which produces a force of 1 N on each 1 m length of a conductor carrying a current of 1 A perpendicularly across the field.

◆ **Tesla, T** Unit of magnetic field strength. $1\,T = 1\,N\,A^{-1}\,m^{-1}$

◆ **Permeability (magnetic)** Constant that represents the ability of a particular medium to transfer a magnetic force and field.

◆ **Permeability of free space, μ_0** Fundamental constant that represents the ability of a vacuum to transfer a magnetic force and field, $\mu_0 = 4\pi \times 10^{-7}\,T\,m\,A^{-1}$.

Magnetic field strength

We have seen that gravitational field strength $g = \frac{F}{m}$ and electric field strength $E = \frac{F}{q}$ but magnetic field strength is less easily defined. We will describe the strength of a magnetic field in terms of the magnetic force experienced by a current flowing across the field.

We will see in Topic D.3 that moving charges experience magnetic forces when they move across a magnetic field. The simplest example would be the electrons in a current in a straight wire *which is perpendicular* to a magnetic field, as seen in Figure D2.44.

The magnitude of the magnetic force, F, is proportional to three things: the magnitude of the current, I, the length of conductor in the field, L, and **magnetic field strength**, which is given the symbol B.

$F = BIL$ or, rearranging:

$$\text{magnetic field strength, } B = \frac{F}{IL}$$

The SI units of B are newtons per amp metre. $1\,N\,A^{-1}\,m^{-1}$ is known as 1 **tesla, T**. 1 T corresponds to a very strong magnetic field, so, milliteslas (mT) and microteslas (μT) are in common use.

WORKED EXAMPLE D2.7

Calculate the magnetic field strength that produces a force of 5.0×10^{-4} N on each metre length of a long straight conductor carrying a constant current of 2.0 A. (The field and the current are perpendicular to each other.)

Answer

$$B = \frac{F}{IL} = \frac{5.0 \times 10^{-4}}{1.0 \times 2.0}$$

$$= 2.5 \times 10^{-4}\,T$$

Magnetic field strength around a current in a long straight wire

We have previously described the shape of the magnetic field around a current in a long, straight wire, now we will consider how we can calculate values for the field strength.

The magnetic field is created by the charges moving in the current and it spreads around the wire. The **magnetic permeability** of a medium, or free space, represents its ability to transfer a magnetic field and force. It may be considered analogous to *electric permittivity*, which describes electric properties. The magnetic properties of air are almost identical to the magnetic properties of free space.

The magnetic **permeability of free space** is given the symbol μ_0 and has the value $\mathbf{4\pi \times 10^{-7}\,T\,m\,A^{-1}}$.

The magnetic field strength around a straight current depends on the:

- magnitude of the current, I
- perpendicular distance from the wire, r
- magnetic permeability of the air surrounding the wire (≈ permeability of free space).

$$\text{Magnetic field strength at a distance } r \text{ from a current } I \text{ in a long straight wire in air, } B = \frac{\mu_0 I}{2\pi r}$$

WORKED EXAMPLE D2.8

Determine at what distance from a long straight wire carrying a current of 5.0 A, the resulting magnetic field has a strength of 100 μT.

Answer

$$B = \frac{\mu_0 I}{2\pi r}$$

$$100 \times 10^{-6} = \frac{((4\pi \times 10^{-7}) \times 5.0)}{(2\pi r)}$$

$r = 0.01$ m (1 cm)

For comparison, the Earth's magnetic field strength averages about 50 μT, which is comparable to the field within one or two centimetres of the current in this Worked example.

31 a Sketch the magnetic field pattern around two bar magnets as seen in Figure D2.45.

b Sketch the magnetic field pattern around the two bar magnets if the polarity of one of the magnets was reversed.

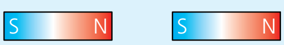


Figure D2.45 Two bar magnets

32 Explain in detail how it is possible for a bar magnet to attract an unmagnetized steel pin.

33 Describe where you would expect the magnetic field of the Earth to be strongest, and in what direction does it act?

34 Consider Figure D2.43.

a Describe how the polarity of the electromagnet can be determined.

b Sketch the magnetic field that this electromagnet would produce.

35 State three applications of electromagnets.

36 The electromagnet seen in Figure D2.43 has been wound on a 'soft' iron core. What difference would it make if the core was made of steel?

37 Draw a graph to represent how the magnetic field strength due to a 2.0 A current in a long straight wire varies with perpendicular distances up to 10 cm from the wire.

38 Show that the SI units of permeability are T m A^{-1}.

LINKING QUESTION

- How can moving charges in magnetic fields help probe the fundamental nature of matter?

This question links to understandings in Topics D.3, E.1 and E.2.

◆ **Electric potential energy** E_p, is the work done when bringing all the charges of a system to their present positions from infinity.

Electric potential energy

SYLLABUS CONTENT

- The electric potential energy, E_p, in terms of work done to assemble the system from infinite separation.
- The electric potential energy for a system of two charged bodies as given by: $E_p = k\frac{q_1 q_2}{r}$.

Electric potential energy is stored in any system of charges because of the forces between them. As with gravitational potential energy, for the same reasons, we chose infinity to be the zero of **electric potential energy**:

The zero of electric potential energy is chosen to be when the charges are separated by an infinite distance.

The total electric potential energy of a system, E_p, is defined as the work done when bringing all the charges of the system to their present positions, assuming that they were originally at infinity.

We saw in Topic D.1 that gravitational potential energy can only ever be negative because the forces are always attractive, meaning that energy has to be supplied to increase their separation.

However, electric potential energies can be negative (if the forces are attractive between opposite charges), or positive (if the forces are repulsive between similar charges). In other words, we need to *supply* energy to separate charges which are attracted to each other, but energy is released (to kinetic energy) as opposite charges are repelled apart from each other.

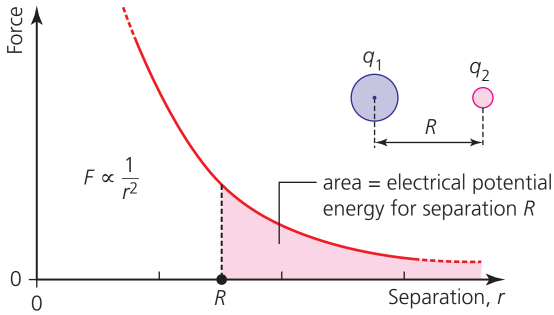


Figure D2.46 Area under graph represents electric potential energy

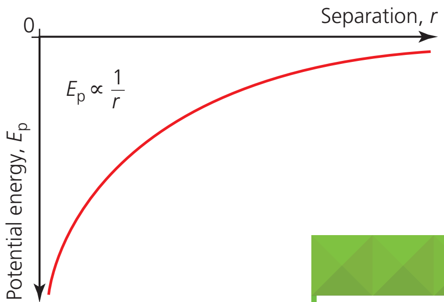


Figure D2.47 Electric potential energy variation with separation between oppositely charged point charges.

Calculating electrical potential energies

Electrical potential energy can be determined from the area under a force–distance graph, as shown in Figure D2.46 for similar charges.

However, for point charges, using the following equation is easier (analogous to the gravitational potential energy equation seen in Topic D.1).

electric potential energy of two point charges, $E_p = k\dfrac{q_1 q_2}{r}$

This equation can be used for two point charges, q_1 and q_2, separated by a distance r. It can also be used with isolated spherical conductors, when r is then the distance between their centres. The sign of the electric potential energy will be dependent on the signs of q_1 and q_2.

Figure D2.47 represents this inverse relationship graphically, for oppositely charged point charges. If the charges both had the same sign, the potential energy would be positive.

WORKED EXAMPLE D2.9

Calculate the electric potential energy that was stored between two isolated spherical conductors: one had a radius of 2.5 cm and charge -4.7×10^{-8} C, the other had a radius of 1.5 cm and charge -6.3×10^{-8} C. Their surfaces were separated by 1.7 cm.

Answer

Separation of centres, $r = 2.5 + 1.5 + 1.7 = 5.7$ cm

$$E_p = k\frac{q_1 q_2}{r} = \frac{(8.99 \times 10^9) \times (-4.7 \times 10^{-8}) \times (-6.3 \times 10^{-8})}{5.7 \times 10^{-2}}$$

$$= +4.7 \times 10^{-4}\ \text{J}$$

The energy is positive because the charges are repelled from each other and they would gain kinetic energy if they were free to move.

Top tip!

Do not confuse the symbols for electric potential energy, E_p, and electric field strength, E.

39 There was a force of -4.7×10^{-7} N between two point charges when they were separated by 40 cm.

a Draw a force–separation graph to show how the force varies over distances of 10 to 100 cm.

b Use your graph to determine the change in electrical potential energy in the system if the separation was increased from 40 cm to 90 cm.

40 The surfaces of two identically charged spheres are separated by 12 cm. If the radius of each sphere is 2.8 cm and the electrical potential energy in the arrangement is 3.6×10^{-4} J, determine the charge on each sphere.

41 Determine how much electrical potential energy is associated with the proton–electron arrangement in a hydrogen atom. (separation = 5.3×10^{-11} m)

Electric potential

SYLLABUS CONTENT

- The electric potential is a scalar quantity with zero defined at infinity.
- The electric potential V_e at a point is the work done per unit charge to bring a test charge from infinity to that point as given by: $V_e = \frac{kQ}{r}$.
- The electric field strength E as the electric potential gradient as given by: $E = -\frac{\Delta V_e}{\Delta r}$.
- The work done in moving a charge q in an electric field as given by: $W = q\Delta V_e$.
- Equipotential surfaces for electric fields.
- The relationship between equipotential surfaces and electric field lines.

◆ **Electric potential** Work done in moving a test charge of +1 C to a specified point from infinity.

The concept of **electric potential**, V_e, is used to describe points in the space around charges (Compare with gravitational potential.)

Electric potential can be considered as electric potential energy per unit charge.

More precisely, it is defined as follows:

The electric potential at a point is defined as the work done per unit charge (1 C) in bringing a small positive test charge from infinity to that point.

The SI unit for electric potential is $\mathrm{J\,C^{-1}}$. This should be familiar from discussing potential difference in Topic B.5: $1\,\mathrm{J\,C^{-1}}$ is called 1 volt (V).

For a relatively small charge in the electric field of a larger charge, we can make that clear by rewriting

$$E_p = k\frac{q_1q_2}{r} \text{ as } E_p = k\frac{Qq}{r}$$

Then, dividing by the small charge, q, gives

DB

Electric potential around a point charge Q:

$$V_e = \frac{kQ}{r}$$

The potential around a negative charge will be negative (as shown by the equation). Increasing the distance, r, from the charge, $-Q$, reduces the magnitude of the negative potential, which is equivalent to an increase in potential. This is similar to gravitational fields.

The potential around a positive charge will be positive (as shown by the equation). Increasing the distance, r, from the charge, $+Q$, reduces the magnitude of the positive potential, which is equivalent to a decrease in potential.

WORKED EXAMPLE D2.10

Calculate the electric potential due to a point charge of -1.00×10^{-8} C at a distance of:

a 1.00 m **b** 2.00 m.

Answer

a $V_e = \frac{kQ}{r} = \frac{(8.99 \times 10^9) \times (-1.00 \times 10^{-8})}{1.00} = -89.9\,\mathrm{V}$

b $V_e = \frac{kQ}{r} = \frac{(8.99 \times 10^9) \times (-1.00 \times 10^{-8})}{2.00} = -45.0\,\mathrm{V}$

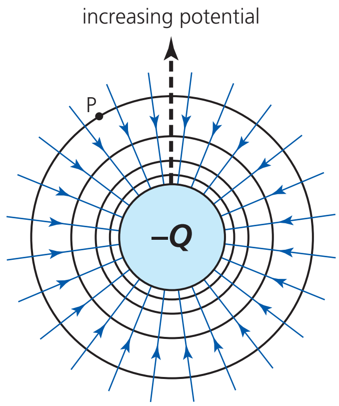
The potential increases by 45.0 V when moving from 1.00 m to 2.00 m from the charge.

Electric equipotential surfaces and lines

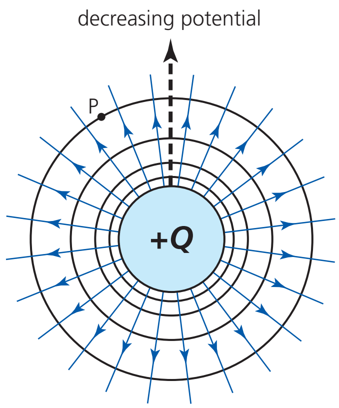
Drawings of equipotential lines provide useful visualizations of electric fields.

An equipotential surface (or line) connects places which have the same electric potential.

Equipotential lines are always perpendicular to electric field lines.



■ **Figure D2.48** Equipotential and field lines around a negative charge



■ **Figure D2.49** Equipotential and field lines around a positive charge

◆ **Electrode** Conductor used to make an electrical connection to a non-metallic part of a circuit.

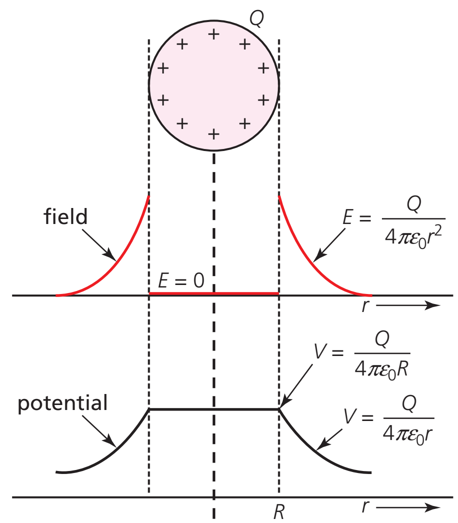
No overall work is done if a charge moves between different positions on the same equipotential line (surface).

Figure D2.48 shows electric equipotential lines and electric field lines around a spherical negative charge, $-Q$. The circular lines are drawn with equal numerical intervals of potential, which means that they must get further and further apart because the field is weakening. A three-dimensional representation would have spherical *surfaces*.

A test positive charge placed at P would be attracted to $-Q$, as shown by the arrows on the field lines. Moving a test charge further away from $-Q$ requires work to be done, so that the electric potential energy and potential must increase.

In Figure D2.49 the central charge is positive, $+Q$. A test positive charge placed at P would be repelled from $+Q$. so that the electric potential energy and potential must decrease if it is able to move.

Figure D2.50 represents the field and potential *inside* and outside of a positively charged conducting sphere (solid or hollow).



■ **Figure D2.50** Electric field and potential of a conducting sphere

ATL D2B: Thinking skills

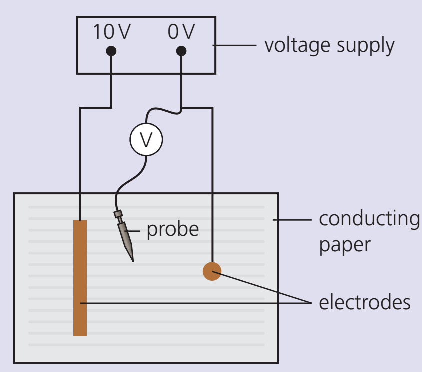
Applying key ideas and facts in new contexts

Mapping electric fields

Figure D2.51 shows a way in which potential and potential difference can be mapped experimentally using conducting paper.

Two or more metal **electrodes** are placed in good electrical contact with a special type of paper which has been coated with carbon so that it conducts electricity, but still has significant resistance. The shape and location of the electrodes can be varied. A p.d. is connected across the electrodes and, typically, one electrode is kept at 0 V. A movable probe is connected between 0 V and a point of interest in the electric field between the electrodes. Lines of equipotential are easily identified.

The voltmeter will display the potential at that point. Alternatively, the voltmeter can be used to determine the p.d. between any two points in the field.



■ **Figure D2.51** mapping potential

Faraday's cage

If a constant electric field is applied to a metal conductor surrounding a space, free electrons will very quickly redistribute themselves on the outer surface depending on the strength and direction of the field, and the shape of the conductor. For a spherical conductor the charge distribution would be the same everywhere on the surface.

The safest place to be in a lightning storm is inside a conductor, like a car or a building (that has a lightning conductor if it is in an exposed location).

Figure D2.52 shows a dramatic example of a 'Faraday' cage.

Electronic equipment can be protected by putting it inside a Faraday cage.

Effective electromagnetic shielding is used widely to protect important equipment from external electromagnetic waves, or to stop the emission of electromagnetic signals.

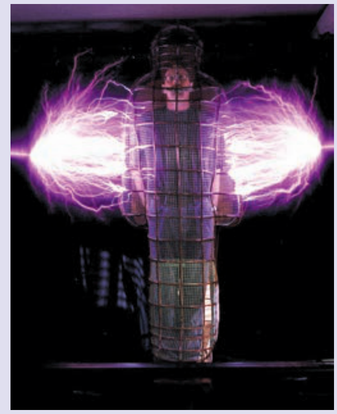


Figure D2.52 A Faraday cage, showing sparks on the outside, but with someone safe inside

In pairs, research Faraday cages and other forms of electromagnetic shielding. Using what you know about electric fields and conductors, explain how they shield objects placed inside from electric fields.

Combining electric potentials

Electric potential is a scalar quantity and potentials can be combined by simple addition.

WORKED EXAMPLE D2.11

Determine the combined potential at a point which is 34 cm from a point charge A of -1.9×10^{-7} C and 45 cm from a point charge B of $+2.3 \times 10^{-7}$ C.

Answer

$$V_e = \left(\frac{kQ}{r}\right)_A + \left(\frac{kQ}{r}\right)_B = \frac{(8.99 \times 10^9) \times (-1.9 \times 10^{-7})}{(34 \times 10^{-2})} + \frac{(8.99 \times 10^9) \times (+2.3 \times 10^{-7})}{(45 \times 10^{-2})} = -4.3 \times 10^2\ \text{V}$$

Figure D2.53 shows an example of potential mapping around three charged spheres. Remember that, if required, field lines can be drawn perpendicular to the equipotential lines.

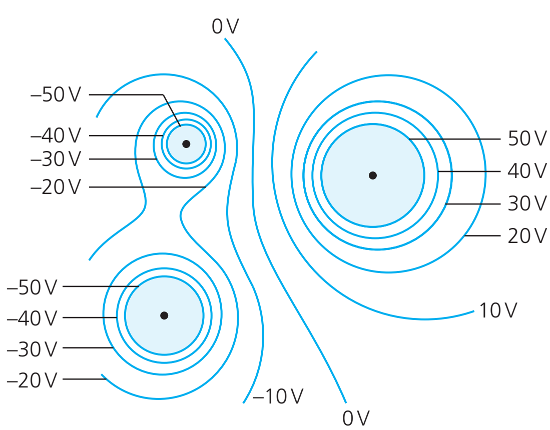


Figure D2.53 Equipotential lines around three charged spheres

42 If 4.95×10^{-5} J of energy were transferred when a charge of 5.1 μC was moved from a certain point to earth (ground), determine the magnitude of the potential at the point.

43 At what distance from an isolated point charge of 4.6×10^{-8} C would the electric potential have a value of −3000 V?

44 Sketch the equipotential and field lines around two point charges of different magnitudes if
  a they have similar signs
  b they have opposite signs.

45 Figure D2.54 shows the electric field lines around an isolated charged conductor. Make a copy of Figure D2.54 and add lines of equipotential.

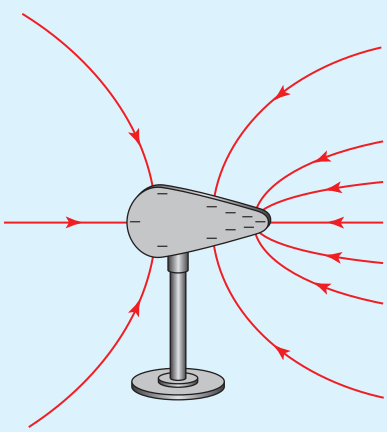


Figure D2.54 Electric field lines around an isolated charged conductor

46 Figure D2.55 shows a kind of *coaxial cable* that is widely used for transferring data, such as signals to televisions. The outer copper mesh is maintained at 0 V and the signal is sent as an electromagnetic wave in the insulator between the central copper wire and the surrounding mesh. This design, with its earthed outer mesh, helps reduce interference to and from other electromagnetic waves.

Make a sketch of a cross-section of the cable and add electric field lines and equipotential lines.

Figure D2.55 Coaxial cable

47 Figure D2.56 represents the variation of potential between two conductors. Sketch the electric field pattern of this arrangement.

48 A hollow conducting sphere has a radius of 6.3 cm. If it is charged with −4.3 nC, determine values for the electric field strength and the electric potential at a distance of:
  a 20.0 cm from the centre of the sphere
  b 3.0 cm from the centre of the sphere.

49 A point charge of -1.9×10^{-7} C is placed 56 cm from another point charge of 5.6×10^{-8} C. Identify the location of one place where the electric potential is zero.

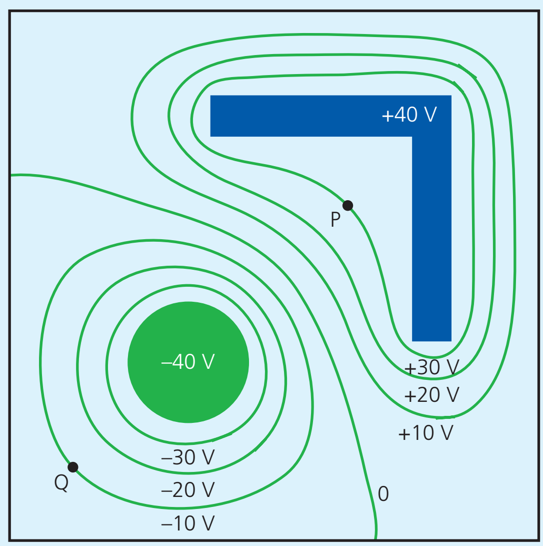


Figure D2.56 The variation of potential between two conductors

Electric potential difference

The central theme of this topic is the movement of charges between different places in electric fields. This means that the difference in potential – the *potential difference* – between two locations is of particular importance.

◆ **Potential difference, p.d. ΔV_e** Difference in electric potential between two points, which equals the work done when a charge of 1 C is moved between the points.

Electric potential difference, ΔV_e, is the work, W, done on unit charge (1 C) when it moves between two points in an electric field.

$$\Delta V_e = \frac{W}{q} \quad \text{or} \quad W = q\Delta V_e$$

The unit for electric potential difference is the same as for potential: the volt (joule per coulomb).

The same concept (electric potential difference) was used frequently in Topic B.5, where it was abbreviated to p.d. or referred to as voltage. In this topic we are applying the term more generally in two and three dimensions, whereas in Topic B.5 our focus was just on electric potential differences across components in electrical circuits.

WORKED EXAMPLE D2.12

Consider Figure D2.57 which shows equipotential lines around a conducting sphere.

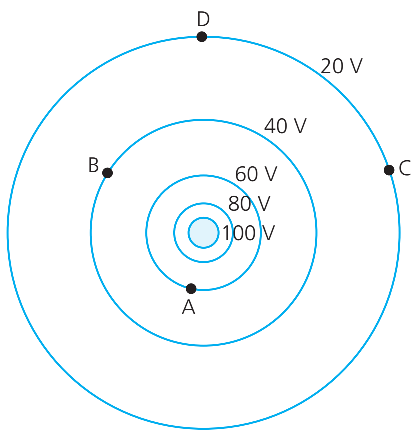


Figure D2.57 Equipotential lines around a conducting sphere

a State whether the sphere is positively or negatively charged.

b Calculate the potential difference when moving from:

i C to A

ii C to D.

c Determine how much work is done when a charge of +2.0 C moves from B to C.

Answer

a Positively charged (potentials are positive)

b **i** (+60) – (+20) = + 40 V

ii (+20) – (+20) = + 0 V

c $W = q\Delta V_e = 2.0 \times [(+20) - (+40)] = -40\,\text{J}$

The negative sign shows that electric potential energy falls. (A freely moving positive charge will be repelled from the positive sphere and gain kinetic energy.)

Electric potential–distance graphs

We know that the work done, W, when moving a charge, q, through a potential difference ΔV_e is given by $W = q\Delta V_e$.

We also know that the work can be calculated from force × distance = $Eq \times \Delta r$, where Δr is a small enough distance that the value of E does not change significantly.

Hence $W = \cancel{q}\Delta V_e = E\cancel{q}\Delta r$, so that:

DB

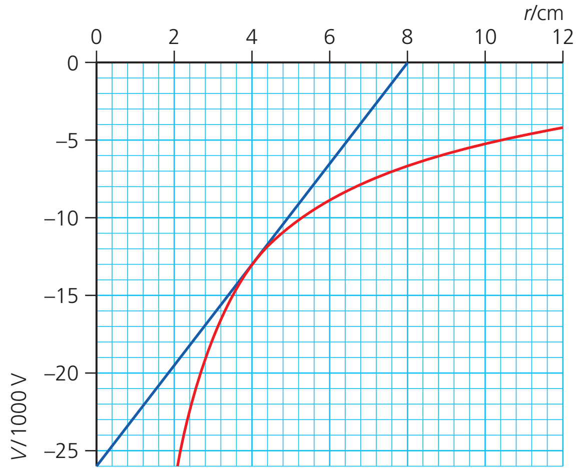
electric field strength, $\boldsymbol{E} = -\dfrac{\boldsymbol{\Delta V_e}}{\boldsymbol{\Delta r}}$

where $\dfrac{\Delta V_e}{\Delta r}$ is called the electric *potential gradient.*

The negative sign has been added to the equation to show that the direction of the vector quantity E is opposite to the direction of increasing potential.

In other words, electric fields exist where electric potential is changing. If potential is constant, then the electric field strength is zero.

Figure D2.58 shows how the value of potential varies around a negative point charge. The tangent to the curve can be used to determine the gradient at any required distance (the example is for $r = 4.0\,\text{cm}$).



■ **Figure D2.58** Variation of potential around a point charge

WORKED EXAMPLE D2.13

Determine:

a the electric field strength at a distance of 4.0 cm from the point charge represented in Figure D2.58

b the value of the point charge involved.

Answer

a $E = -\frac{\Delta V_e}{\Delta r} = -\frac{0 - (-26 \times 10^3)}{(8.0 \times 10^{-2})}$

$= -3.3 \times 10^5\,\mathrm{N\,C^{-1}}$

b $V_e = \frac{kQ}{r}$

$-15 \times 10^3 = \frac{(8.99 \times 10^9) \times Q}{3.5 \times 10^{-2}}$

$Q = -5.8 \times 10^{-8}\,\mathrm{C}$

50 Two points in an electric field have potentials of 12.7 V and 15.3 V. Determine how much energy will be transferred when an electron moves between these points:

a in eV **b** in J.

51 Calculate how much energy is gained by a charge of 2.0 C when it passes through a battery that has a terminal p.d. of 12 V.

52 A charge of +4.5 C is moved from point P to point Q in Figure D2.56.

a Determine how much energy is transferred.

b State whether work is done on the charge, or by the charge.

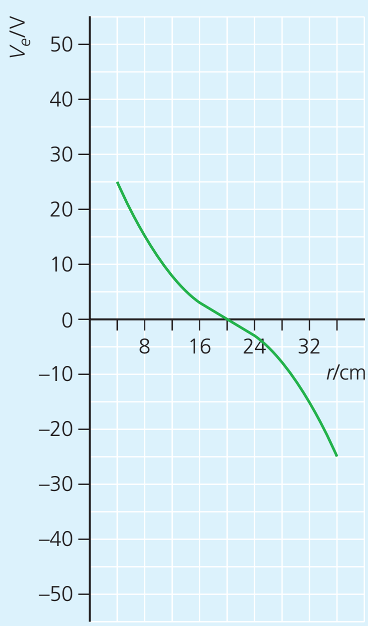
53 Figure D2.59 shows how the electrical potential varies with distance, r, from a certain point, P.

a State where the electric field strength is a minimum.

b Determine the magnitude of the electric field strength:

i 12 cm from P **ii** 32 cm from P.

c Suggest what arrangement of charges might produce this variation in potential.

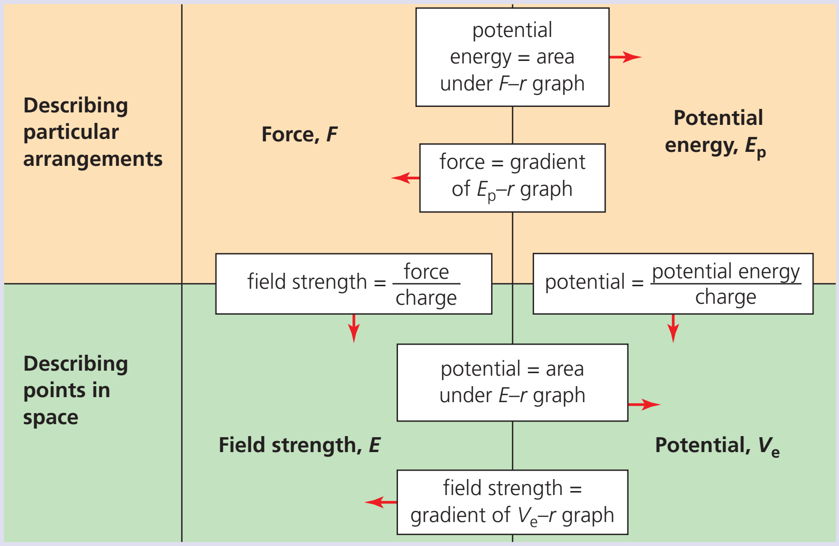


■ **Figure D2.59**

ATL D2C: Communication skills

Clearly communicating complex ideas in response to open-ended questions

As you may have seen in Topic D.1, we can summarize key concepts and their connections using a visual organizer. Here are two visual organizers for the key concepts in this topic.



■ **Figure D2.60** Connections between the four key concepts

Force $F = k\dfrac{q_1q_2}{r^2}$	**Potential energy** $E_p = k\dfrac{q_1q_2}{r^2}$
Field $E = k\dfrac{Q}{r^2}$	**Potential** $V_e = k\dfrac{Q}{r}$

■ **Figure D2.61** Equations for radial electric fields

Can you think of other ways in which you might represent the concepts from this topic and their connections?

LINKING QUESTION

- How are electric and magnetic fields like gravitational fields?

This question links to understandings in Topic D.1.

D.3 Motion in electromagnetic fields

Guiding questions

- How do charged particles move in magnetic fields?
- What can be deduced about the nature of a charged particle from observations of it moving in electric and magnetic fields?

The motion of charged particles in uniform electric fields

SYLLABUS CONTENT

- The motion of a charged particle in a uniform electric field.
- The motion of a charged particle in a perpendicularly orientated uniform electric field.

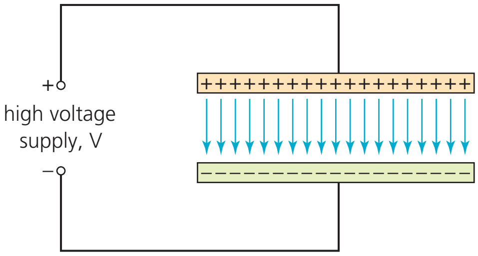


Figure D3.1 Producing a uniform electric field

In this section we will discuss the motion of charged particles (electrons, protons and ions) that are free to move in uniform electric fields. We will assume that the particles are in a vacuum (unless otherwise stated), so that their movements do not involve collisions with other particles (air molecules).

In Topic D.2 we explained that the easiest way of producing a uniform electric field was by connecting a potential difference across parallel metal plates, as shown again in Figure D3.1.

Stationary charged particles

From an understanding of the kinetic theory of matter, it should be appreciated that particles are never truly 'stationary'. However, even a small p.d. can accelerate charged particles to speeds very much greater than their random velocities without the p.d. So, assuming that a particle is stationary to begin with will not result in any significant error when determining its final speed and kinetic energy.

A particle with charge q situated in an electric field will experience a force $F = Eq$ towards the oppositely charged plate (Topic D.2). Figure D3.2 shows two opposite charges in an electric field.

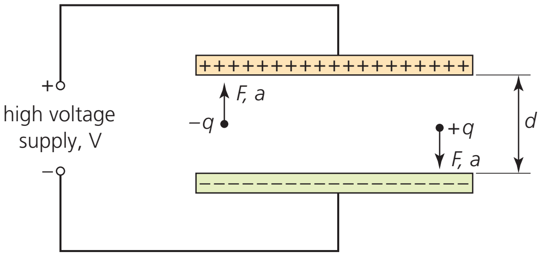


Figure D3.2 Forces and accelerations of opposite charges in a uniform electric field

Since $E = \frac{V}{d}$, $F = \frac{Vq}{d}$.

The forces will cause the charges to accelerate perpendicularly towards the plates. The equations of motion (Topic A.1) can then be used to determine how the charge moves.

'Stationary' mobile charges will accelerate along a field line in a uniform electric field.

WORKED EXAMPLE D3.1

An electron is very close to the negatively charged plate seen in Figure D3.2.

a Determine its speed when it reached the positive plate if there was a p.d. of 2000 V across the metal plates which were separated by 8.0 cm.

b State any assumptions you made when answering part **a**.

Answer

a $F = \frac{Vq}{d} = \frac{2000 \times (1.60 \times 10^{-19})}{0.080} = 4.0 \times 10^{-15}\,\text{N}$

Then, acceleration can be determined using Newton's second law ($F = ma$):

$a = \frac{F}{m} = \frac{4.0 \times 10^{-15}}{9.110 \times 10^{-31}} = 4.4 \times 10^{15}\,\text{m s}^{-2}$

Finally, the final speed can be determined from $v^2 = u^2 + 2as$:

$v^2 = 0^2 + (2 \times 4.4 \times 10^{15} \times 0.080)$

$v = 2.7 \times 10^7\,\text{m s}^{-1}$

b The calculation has assumed that:

- the electron started with speed $u = 0$
- there were no gas molecules between the plates (that would collide with the electrons).

Charged particles moving across a uniform electric field

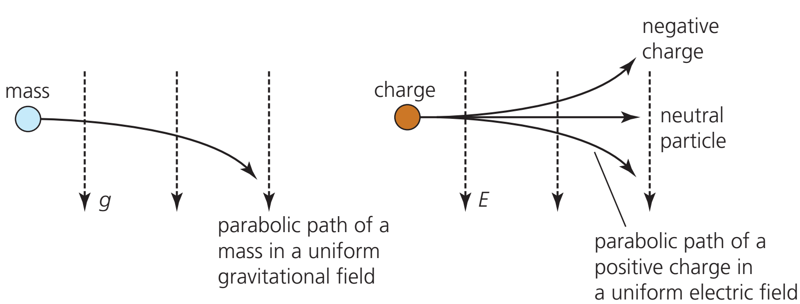


Figure D3.3 Comparing motion in gravitational and electric fields

This situation is analogous to the motion of masses projected in uniform gravitational fields. Both situations involve motion which can be analysed as a constant speed in one direction and an acceleration in a perpendicular direction. See Figure D3.3.

Freely moving charged particles travelling across electric fields will move in parabolic paths.

WORKED EXAMPLE D3.2

An electron travelling horizontally at a constant speed of $5.5 \times 10^7\,\text{m s}^{-1}$ is directed into a uniform electric field of $2.8 \times 10^5\,\text{N C}^{-1}$ acting vertically downwards.

a Determine the force and acceleration of the electron in the field (magnitude and direction).

b If the field extends for a horizontal distance of 10 cm, determine the time that the electron spends in the field.

c Determine the vertical displacement of the electron from its original path as it leaves the field. (Charge on electron is -1.6×10^{-19} C, mass of electron is 9.110×10^{-31} kg.)

Answer

a $F = Eq = (2.8 \times 10^5)(1.60 \times 10^{-19}) = 4.5 \times 10^{-14}\,\text{N}$ upwards

$a = \frac{F}{m} = \frac{4.48 \times 10^{-14}}{9.110 \times 10^{-31}} = 4.9 \times 10^{16}\,\text{m s}^{-2}$ upwards

(4.48×10^{-14} seen on calculator display)

($4.9177... \times 10^{16}$ seen on calculator display)

b Horizontal speed is constant:

$v = \frac{s}{t}$

$5.5 \times 10^7 = \frac{0.10}{t}$

$t = 1.8 \times 10^{-9}\,\text{s}$

($1.81818... \times 10^{-9}$ s seen on calculator display)

c Using the equation of motion (Topic A.1):

$s = ut + \frac{1}{2}at^2 = 0 + \left(\frac{1}{2} \times (4.918 \times 10^{16}) \times (1.818 \times 10^{-9})^2\right)$

$= 0.081\,\text{m}$ (8.1 cm)

Tool 3: Mathematics

Express measurements and processed uncertainties to an appropriate number of significant figures or level of precision

Worked example D3.2 shows the effect of 'rounding off' too early in a multi-step calculation. All three answers are correctly given to 2 significant figures (the same as the data given in the question). However, if those answers were used in part **c**, a different answer (0.079 m) would be obtained.

◆ **Particle beams** Streams (flows) of very fast-moving particles, most commonly charged particles (electrons, protons or ions), moving across a vacuum. Properties of the individual particles can be investigated by observing the behaviour of the beams in electric and/or magnetic fields.

◆ **CERN** European organization for nuclear research.

◆ **Cathode** An electrode out of which (conventional) current flows.

◆ **Thermionic emission** Release of electrons from a very hot metal surface.

◆ **Anode** An electrode into which (conventional) current flows.

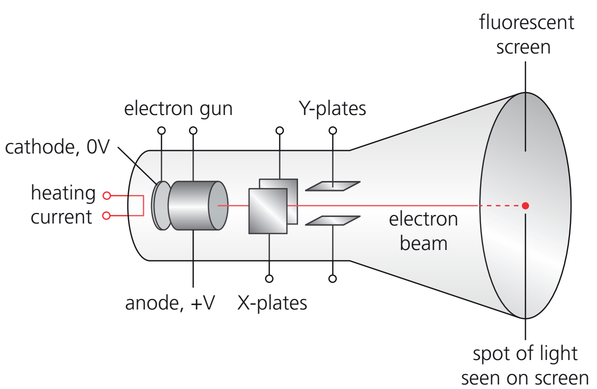
Particle beams

So far, we have been discussing the motion of *individual* charged particles but, in practice,

experiments with charged particles will involve large numbers moving together with the same velocity as a **particle beam**.

Experiments with particle beams have had great historic importance (for example, the discovery of the electron), and they continue to be an essential part of the latest research into nuclear physics (at **CERN**, for example, see Figure D3.12).

We will use the production of an electron beam as an example. Figure D3.4 shows a type of electron beam deflection tube that is commonly used to demonstrate to students the production and properties of electron beams.



■ **Figure D3.4** Electron deflection tube

The heating current is used to heat the metal **cathode** (the terminal connected to 0 V) and the thermal energy supplied increases the kinetic energy of the free electrons in the metal. Some of the electrons have energy to be released (emitted) from the metal's surface. This process is called **thermionic emission**. A large positive voltage is applied to the other terminal (called the **anode**) and this accelerates the electrons into a beam travelling with very high speeds to the right (as shown). The tube contains a vacuum. This arrangement is commonly called an 'electron gun'. When the beam of electrons strikes the fluorescent screen at the end of the tube, some of their kinetic energy is transferred to visible light in the form of a spot that can be easily observed.

If a p.d. is connected across the 'X-plates' the beam (and the spot) is deflected to the left or to the right. If a p.d. is connected across the 'Y-plates' the beam is deflected up or down.

1 **a** Calculate the force exerted on a singly charged positive ion in an electric field of $1200\,\mathrm{N\,C^{-1}}$ acting vertically downwards.

b Determine the mass of the ion if it started to accelerate at $7.4 \times 10^{9}\,\mathrm{m\,s^{-2}}$.

c State in which direction the ion will accelerate.

2 A proton is accelerated by a p.d. of $3.7 \times 10^{4}\,\mathrm{V}$ connected between two parallel metal plates which are 2.8 cm apart.

a Calculate the strength of the electric field.

b Assuming that the electric field is uniform, determine what force the proton will experience.

c Determine the maximum amount of energy that the proton can gain when it has been accelerated in

i eV **ii** J.

3 Show that the final speed of an electron accelerated from rest over a distance of 5.0 cm in a vacuum by a uniform electric field of $9.2 \times 10^{4}\,\mathrm{N\,C^{-1}}$ is about forty million metres per second.

4 An electron beam consisting of electrons travelling at speeds of $1.3 \times 10^7\,\mathrm{m\,s^{-1}}$ is directed horizontally into a uniform electric field of $7.4 \times 10^4\,\mathrm{N\,C^{-1}}$ acting vertically upwards.

- **a** Determine the force and acceleration of the electrons in the field (magnitude and direction).
- **b** If the field extends for a horizontal distance of 8.5 cm, calculate how much time the electrons spend in the field.
- **c** Determine the vertical displacement of the electron from its original path as it leaves the field.
- **d** State the name given to the shape of the electron beam's path.

5 An alpha particle is emitted from the nucleus of a radium atom (Topic E.1) with kinetic energy of 4.8 MeV.

- **a** Show that the initial speed of the alpha particle is between ten million and twenty million metres per second. (Mass of alpha particle = 6.64×10^{-27} kg)
- **b** An alpha particle has a charge of 3.2×10^{-19} C. Determine the force acting on the particle when it is moving perpendicularly across a uniform electric field of strength $4.9 \times 10^4\,\mathrm{N\,C^{-1}}$.
- **c** Explain, by considering your answers to Question 4, but without a detailed calculation, why the particle will not be significantly deflected in the field.

The motion of charged particles in uniform magnetic fields

SYLLABUS CONTENT

- The motion of a charged particle in a uniform magnetic field.
- The motion of a charged particle in a perpendicularly orientated uniform magnetic field.
- The magnitude and direction of the force on a charge moving in a magnetic field as given by: $F = qvB\sin\theta$

A 'stationary' charge in a magnetic field will not experience any magnetic force.

Any charge *moving* in a magnetic field will experience a magnetic force unless its motion is parallel to the magnetic field (that is, if it is moving along a magnetic field line). The magnitude of the force increases with the speed of the particle.

Consider Figure D3.5 which shows a charge q entering a magnetic field of strength B. The constant velocity, v, of the charge makes an angle θ with the direction of the magnetic field.

A moving charge will experience a force which is perpendicular to both the directions of its velocity and the magnetic field.

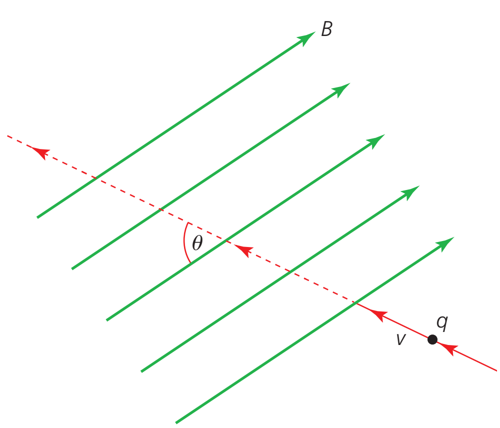


Figure D3.5 An individual charge entering a magnetic field

Since in Figure D3.5 the velocity and the field are both in the plane of the paper, we know that the force on the charge will act perpendicularly into, or out of, the paper. The direction can be predicted using **Fleming's left-hand rule**, as shown in Figure D3.6.

If the moving charge shown in Figure D3.5 was positive, using Fleming's left-hand rule predicts that the force on the particle acts perpendicularly downwards, into the paper. If the particle was negatively charged, the same rule predicts that the force on the particle acts perpendicularly upwards, out of the paper.

Nature of science: Observations

Once we have understood that a charged particle needs to be *moving* across a magnetic field in order to experience a force, we probably should not be surprised that the magnitude of that force increases with the speed of the particle. However, this is an example in which fundamental physics seems to contradict our 'common sense'. An explanation requires a relativistic treatment.

◆ **Left-hand rule (Fleming's)** Rule for predicting the direction of the magnetic force on moving charges, or a current in a wire.

Top tip!

Any moving charge can be considered to be an electric current. Remember that, by convention, the direction of current is *always* shown to be the direction in which positive charges are moving (Topic B.5). If the charges are negative, the conventional current will be shown to be in the opposite direction to their velocity.

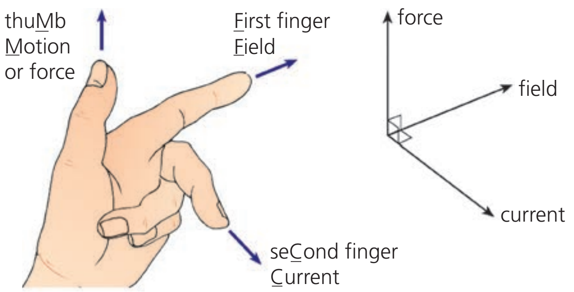


Figure D3.6 Fleming's left-hand rule predicts the direction of the force

Because of the magnetic force, the charged particle will not continue to move in a straight line. Because the force is perpendicular to the velocity, this is the necessary condition for circular motion (Topic B.2) *if* the particle is moving perpendicularly across the field.

A charged particle moving perpendicularly across a uniform magnetic field will follow a circular path.

The charged particle will move along an arc of a circular path as long as it remains in the magnetic field. Figure D3.7 shows the four different possibilities, each for particles travelling with different speeds. Crosses represent fields into the paper/screen and dots represent fields out of the paper/screen.

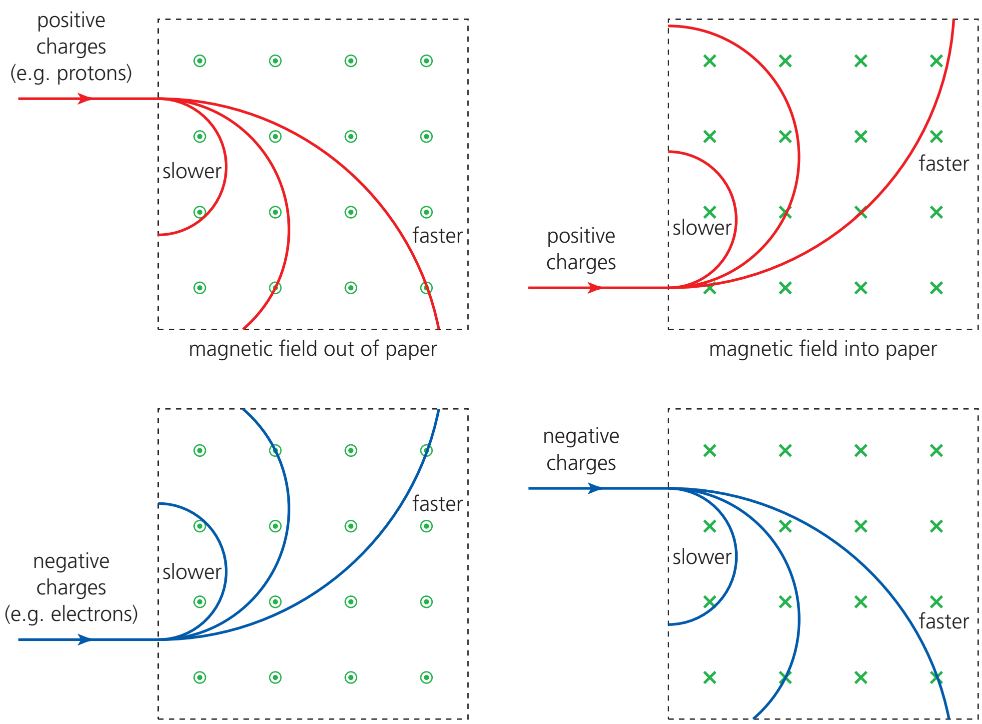


Figure D3.7 Circular paths of charges moving perpendicular to magnetic fields

During its circular motion in a perpendicular magnetic field, the speed and the kinetic energy of the charged particle will remain constant.

TOK

Knowledge and the knower

- *How do we acquire knowledge?*

Mapping

'**Mapping**' is the process of representing information in the form of a diagram, map, or picture. For example, you may choose to *map* the basic ideas in the physics course in order to show the inter-connections between them.

The purpose of mapping is to simplify and make something easier to understand.

The use of lines and patterns to represent fields is accepted by the scientific community as possibly the only way of presenting these difficult ideas simply to the human mind. No one thinks that the lines are 'real' and it might be argued that such a simplification in some ways restricts our understanding, or imagination, about the subject because it channels our thoughts in certain prescribed directions. The mapping of any knowledge is a simplification to aid understanding and one which has obvious appeal but, like all simplifications, has its limitations.

Make a list of the key concepts introduced in Theme D (so far), then display them on a full page annotating the connections between them.

◆ **Mapping** Representing the interrelationships between ideas, knowledge or data by drawing.

LINKING QUESTION

- How are the properties of electric and magnetic fields represented? (NOS)

Equation for the force on a charged particle moving across a magnetic field

The magnitude of the force, F, on a charged particle moving across a uniform magnetic field depends on the:

- charge of the particle, q
- velocity of the particle, v
- strength of the magnetic field, B
- angle between the field and the velocity, θ.

The force is proportional to q, v, B and $\sin\theta$, so that:

the magnitude of the magnetic force on a charge moving in a magnetic field is given by:

$$F = qvB\sin\theta$$

WORKED EXAMPLE D3.3

Determine the force experienced by an electron moving with a speed of $4.7 \times 10^6\,\mathrm{m\,s^{-1}}$ at an angle of 50° across a magnetic field of strength 0.56 T.

Answer

$F = qvB\sin\theta = (1.60 \times 10^{-19}) \times (4.7 \times 10^6) \times 0.56 \times \sin 50° = 3.2 \times 10^{-13}\,\mathrm{N}$

If a particle is moving *perpendicularly* to a uniform magnetic field, $\sin\theta = 1$ so that the equation for the force reduces to $F = qvB$. This magnetic force can be considered as the *centripetal force* causing circular motion:

we know from Topic A.2, centripetal force $F = \dfrac{mv^2}{r}$

So that:

$$qvB = \frac{mv^2}{r}$$

which can be rearranged to show that:

the radius of the circular path of a charged particle moving in a perpendicular magnetic field can be determined from:

$$r = \frac{mv}{qB}$$

WORKED EXAMPLE D3.4

Determine the radius of the path followed by a proton moving with a speed of $1.9 \times 10^7\,\mathrm{m\,s^{-1}}$ perpendicularly across a magnetic field of strength 0.30 T. (Charge on proton is $+1.60 \times 10^{-19}$ C, mass of proton is 1.673×10^{-27} kg.)

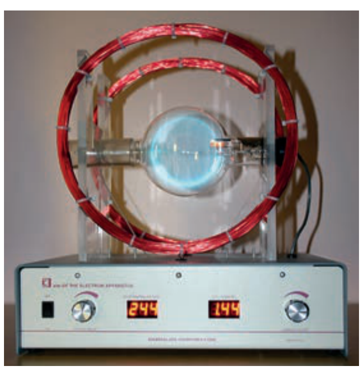
Answer

$$r = \frac{mv}{qB} = \frac{(1.673 \times 10^{-27}) \times (1.9 \times 10^7)}{(1.60 \times 10^{-19}) \times 0.30} = 0.66\,\mathrm{m}$$

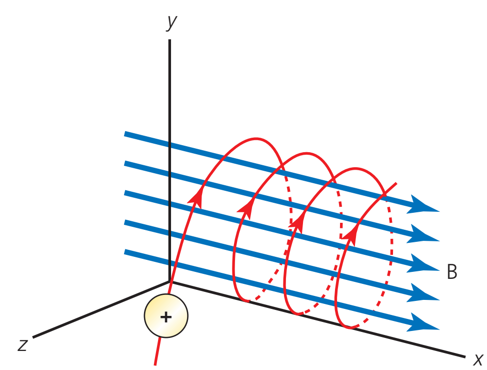
Figure D3.8 shows a school laboratory experiment to demonstrate the circular path of electrons moving perpendicular to a uniform magnetic field.

◆ **Electron gun** Component that fires a beam of electrons across a vacuum.

An electron beam is produced by an **electron gun** arrangement, as described previously (in Figure D3.4 but not visible in D3.8). The electrons are fired into a perpendicular uniform magnetic field, which is produced by steady currents in the coils which can be seen in Figure D3.8, which also shows the resultant circular path of the electron beam. The path of the electrons can be seen because the tube contains a very small amount of an inert gas at very low pressure. The gas molecules gain energy from collisions with the electrons and then emit light.



■ **Figure D3.8** Electrons moving in circular paths



■ **Figure D3.9** Helical path

WORKED EXAMPLE D3.5

The electrons moving in the circular path seen in Figure D3.8 had been accelerated from rest by a voltage of 5000 V.

a Determine their maximum energy in:

i electronvolts **ii** joules.

b Calculate their maximum speed.

c If the strength of the magnetic field was 0.0033 T, what was the radius of the electrons' path?

Answer

a **i** 5000 eV

ii $qV = (1.60 \times 10^{-19}) \times 5000 = 8.00 \times 10^{-16}\,\mathrm{J}$

b $qV = \frac{1}{2}mv^2$

$$8.00 \times 10^{-16} = \frac{1}{2} \times (9.110 \times 10^{-31}) \times v^2$$

$v = 4.19 \times 10^7\,\mathrm{m\,s^{-1}}$

c $$r = \frac{mv}{qB} = \frac{((9.110 \times 10^{-31}) \times (4.19 \times 10^7))}{((1.60 \times 10^{-19}) \times 0.0033)} = 0.072\,\mathrm{m}\ (7.2\,\mathrm{cm})$$

◆ **Helical** In the shape of a spiral.

If a charged particle is moving across a magnetic field but not moving perpendicularly or parallel to the field, its path will be **helical** (like a spiral), as shown in Figure D3.9 for positively charged particles.

6 Determine the magnetic force acting on a proton moving at an angle of 32° across a magnetic field of 5.3×10^{-3} T at a speed of $3.4 \times 10^{5}\,\mathrm{m\,s^{-1}}$.

7 An electron is moving at a speed of $1.6 \times 10^{7}\,\mathrm{m\,s^{-1}}$ perpendicularly to a magnetic field of 1.4×10^{-4} T. Calculate the radius of its path.

8 **a** Outline how it is possible for a charged particle to move through a magnetic field without experiencing a force.
b Discuss whether it is possible for the same particle to move through electric and gravitational fields without experiencing forces.

9 An alpha particle (Topic E.1) has a charge of $+3.2 \times 10^{-19}$ C and a mass of 6.7×10^{-27} kg. . It moves perpendicularly across a magnetic field in a vacuum with a speed of $1.4 \times 10^{7}\,\mathrm{m\,s^{-1}}$
a If it experiences a magnetic force of 4.1×10^{-14} N, determine the strength of the field.
b Describe the path of the alpha particle.
c When a similar alpha particle moved at an angle across the same magnetic field, the force it experienced was 3.3×10^{-14} N. Determine the angle between the field and the particle's velocity.
d Describe the shape of the particle's trajectory.

10 **a** A beam of singly charged ions ($q = 1.6 \times 10^{-19}$ C) is projected perpendicularly across a magnetic field of strength 0.87 T. Determine the speed of each ion if they each experience a magnetic force of 1.1×10^{-12} N.
b The ions move in the arcs of circular paths of radius 2.78 m. Determine their mass.
c If the beam was replaced with doubly charged ions of the same mass, but moving with half the speed, predict the radius of their path.

11 **a** Electrons are accelerated into a beam by a p.d. of 7450 V in a vacuum. Determine the kinetic energy of the electrons in:
i electronvolts **ii** joules.
b Calculate the final speed of the electrons.
c Determine the strength of magnetic field needed to make these electrons move in a circle of radius 14.8 cm.
d If the accelerating voltage is halved, predict by what factor the radius of the electrons' path will change (in the same field).

12 A charge of $+4.8 \times 10^{-19}$ C moving perpendicularly across a magnetic field of 1.9×10^{-2} T experiences a force of 9.5×10^{-14} N.
a Determine the speed of the particle.
b What electric field would be needed to produce the same force on this charge?
c Draw the path of the particle moving across the fields in a direction such that these two forces could be equal and opposite to each other (so that the resultant force on the particle was zero).

ATL D3A: Communication skills

Being curious about the natural world

Find out how the Aurora Borealis is formed (see Figure D3.10)

Figure D3.10 The Aurora Borealis

Motion of charged particles in both an electric field and a magnetic field

SYLLABUS CONTENT

- The motion of a charged particle in perpendicularly orientated uniform electric and magnetic fields.

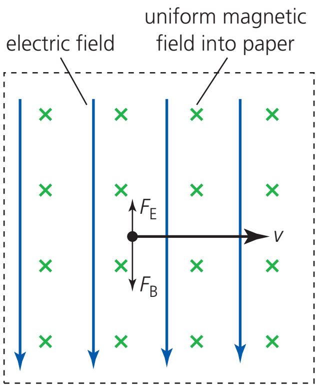


Figure D3.11 Three vectors perpendicular to each other

Any charged particle which is moving across both an electric and a magnetic field will experience two forces, one force parallel to the electric field and one force perpendicular to the magnetic field.

Consider the specific situation in which a charged particle is moving with a velocity v *perpendicularly* across an electric field, E, and both vectors are *perpendicular* to a magnetic field, B. That is, the three vectors are perpendicular to each other as shown in Figure D3.11. The directions of the two forces depend on the nature of the charge. In the example shown (for a negative charge), the forces are in opposite directions because of the relative directions of the fields. This is also true for a positively charged particle moving in this arrangement of fields.

This perpendicular arrangement of fields is of particular importance because, by adjusting the strengths of the two fields, it is possible for the forces on the charged particles to be made equal and opposite. That is, the forces cancel each other out, so that the particle continues its original motion in a straight line with a constant speed.

If the electric force, F_E = magnetic force, F_B: $Eq = Bqv$, so that:

if the motion of a charged particle is unaffected by perpendicular electric and magnetic fields, then, its velocity:

$$v = \frac{E}{B}$$

Tool 3: Mathematics

Check an expression using dimensional analysis of units

The units of both sides of the equation $v = \frac{E}{B}$ can be checked to confirm that they are equivalent to each other.

Units of E divided by units of B are:

$$\frac{\mathrm{N\,C^{-1}}}{\mathrm{N\,A^{-1}m^{-1}}} = \frac{\cancel{\mathrm{N\,A^{-1}}}\,\mathrm{s^{-1}}}{\cancel{\mathrm{N\,A^{-1}}}\,\mathrm{m^{-1}}} = \mathrm{m\,s^{-1}}$$

(the same units as v.)

WORKED EXAMPLE D3.6

An electron with a velocity of $5.9 \times 10^6\,m\,s^{-1}$ is passing between parallel metal plates which are separated by 10 cm. A uniform magnetic field of 42 mT is acting perpendicular to both the plates and the velocity of the electrons.

a Determine what p.d. across the plates will keep the electrons travelling in a straight line.

b If the direction of the electric field is downwards and the electrons are moving to the right, calculate the necessary direction of the magnetic field.

Answer

a $v = \frac{E}{B}$

$5.9 \times 10^6 = \frac{E}{42 \times 10^{-3}}$

$E = 2.5 \times 10^5\,N\,C^{-1}$ (or $V\,m^{-1}$)

Then, since for a parallel plate arrangement $E = \frac{V}{d}$

$V = Ed = 2.5 \times 10^5 \times 0.10 = 2.5 \times 10^4\,V$

b The conventional current is to the left and electric force on a negatively charged electron is in the opposite direction to the electric field: upwards. The magnetic force must be downwards. Using the left-hand rule, the magnetic field must be directed towards the observer.

Charge to mass ratio of particles, *q/m*

◆ **Charge to mass ratio (of a particle)** The ratio *q/m* affects the motion of charged particles in electric and magnetic fields (important when charge and mass are not known separately).

If the exact nature of a particle is unknown, that is, neither the mass nor the charge of a particle is known, then the **charge / mass ratio**, *q/m*, becomes all-important. All particles with the same velocity, *v* and charge / mass ratio will follow paths of the same radius, *r*, when they pass into the same magnetic field, *B*, as shown by $r = \frac{mv}{Bq}$, as seen before:

In other words:

We cannot determine the charge on an unknown particle if we do not know its mass, or we cannot determine the mass of an unknown particle if we do not know its charge.

The charge to mass ratio of electrons was determined by J.J. Thomson (in 1897) before their mass and charge were confirmed separately.

Rearranging the previous equation, we get:

$$\frac{q}{m} = \frac{v}{Br}$$

Experiments such as that shown in Figure D3.8 can determine the radius of the particle beam's path in a known magnetic field. But to determine a mass to charge ratio, the speed of the particles must also be determined. This can be done as explained previously: perpendicular electric and magnetic fields are adjusted until the particles' motions are unaffected. Then $v = \frac{E}{B}$:

For example, if the charges moved with constant velocity when they were moving perpendicular to an electric field of $4.7 \times 10^6\,N\,C^{-1}$ and a magnetic field of 190 mT, then :

$$\text{speed, } v = \frac{E}{B} = \frac{4.7 \times 10^6}{190 \times 10^{-3}} = 2.5 \times 10^7\,m\,s^{-1}$$

If the same particle beam was directed perpendicularly across a separate magnetic field of strength 1.8 mT and the result was movement in the arc of a circle of radius 7.8 cm (similar to that seen in Figure D3.8), then:

$$\frac{q}{m} = \frac{v}{Br} = \frac{2.5 \times 10^7}{(0.0018 \times 0.078)} = 1.8 \times 10^{11}\ \mathrm{C\,kg^{-1}}$$

These values are consistent with a beam of electrons:

$$\text{charge} = 1.60 \times 10^{-19}\ \mathrm{C},\ \text{mass} = 9.110 \times 10^{-31}\ \mathrm{kg},\ \frac{\text{charge}}{\text{mass}} = 1.8 \times 10^{11}\ \mathrm{C\,kg^{-1}}$$

◆ **Particle accelerator** Apparatus designed to produce particle beams.

◆ **Superconducting** Without significant electrical resistance; only occurring at very low temperatures.

LINKING QUESTION

- How can conservation of energy be applied to motion in electromagnetic fields?

This question links to understandings in Topic A.3.

TOK

Knowledge and technology, and The natural sciences

- Why might some people regard science as the supreme form of all knowledge?
- To what extent are technologies merely extensions to the human senses, or do they introduce radically new ways of seeing the world?

CERN

The letters of CERN represent the Conseil Européen pour la Recherche Nucléaire. It has an informative website.

The main activity at CERN is the use of **particle accelerators** to produce the extremely high particle energies needed to investigate the fundamental forces and particles of nature. This is achieved by the use of extremely large and strong magnetic fields to force charged particles to keep moving faster and faster in circular paths. Then, subatomic particles are made to collide together at speeds close to the speed of light.

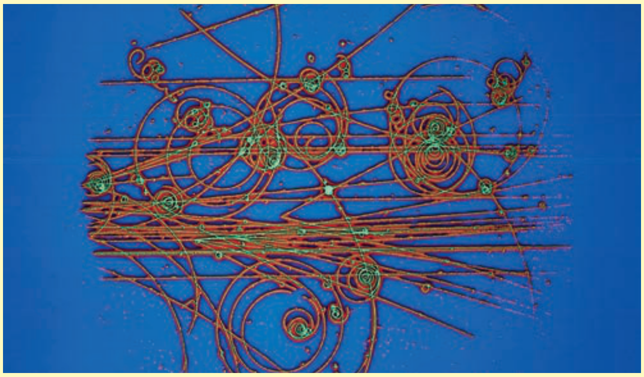
The Large Hadron Collider, shown in Figure D3.12, is the world's largest particle accelerator. *Hadrons* are a class of subatomic particles which includes protons and neutrons. The accelerator has a radius of 4.3 km and a circumference of 27 km with numerous **superconducting** magnets.

Figure D3.13 shows an example of the types of paths that can be produced by subatomic particles (produced following collisions) as they then move through a strong perpendicular magnetic field (in a *bubble chamber*). Measurements made from such images can lead to a determination of particle properties.

The following is a quote from the CERN website: *'The process (of colliding subatomic particles) gives us clues about how the particles interact, and provides insights into the fundamental laws of nature. We want to advance the boundaries of human knowledge by delving into the smallest building blocks of our universe'.*

Would you agree that CERN are aiming to improve the most fundamental form of knowledge?

■ **Figure D3.12** The Large Hadron Collider at CERN is underground and has a radius of about 4 km



■ **Figure D3.13** Curved paths of individual particles in a nuclear physics bubble chamber

13 We have stated that a particle will move in a circular path in a vacuum in a magnetic field which is perpendicular to its velocity. Figure D3.14 shows the path of a negatively charged particle in a container which contains some gas at low pressure.

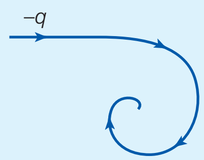


Figure D3.14 The path of a negatively charged particle

a State the direction of the magnetic field.

b Describe how the following quantities are changing:

i radius ii velocity iii kinetic energy.

c Explain why these changes are occurring.

14 A charged particle is travelling parallel to, and mid-way between, two parallel metal plates which are separated by 15 cm and have a p.d. of 12.5 kV across them.

a If the particle has a speed of $5.7 \times 10^6\,m\,s^{-1}$, what strength of magnetic field can be used to keep the particle moving with the same velocity?

b State the direction in which the magnetic field must act.

c Explain why you do not need to know the particle's charge to answer part **a**.

15 Use the internet to learn what a mass spectrometer is used for, and how they use electric and magnetic fields.

LINKING QUESTIONS

- What causes circular motion of charged particles in a field?
- How can the orbital radius of a charged particle moving in a field be used to determine the nature of the particle?
- How are the concepts of energy, forces and fields used to determine the size of an atom?

These questions link to understandings in Topics E.1 and E.2.

Forces on current-carrying conductors

SYLLABUS CONTENT

- The magnitude and direction of the force on a current-carrying conductor in a magnetic field as given by: $F = BIL\sin\theta$
- The force per unit length between parallel wires as given by: $\frac{F}{L} = \mu_0\frac{I_1I_2}{2\pi r}$ where r is the separation between the two wires.

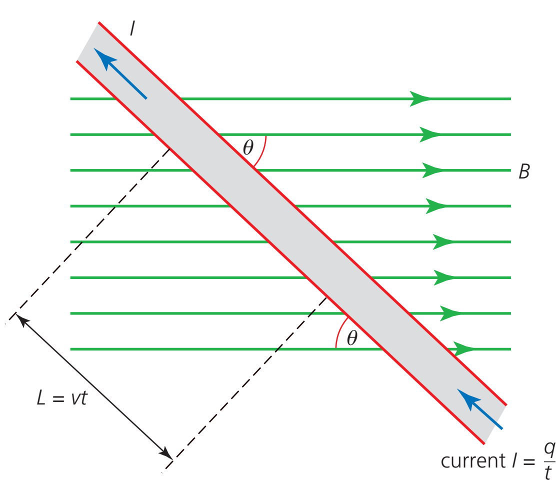


Figure D3.15 A wire carrying a current

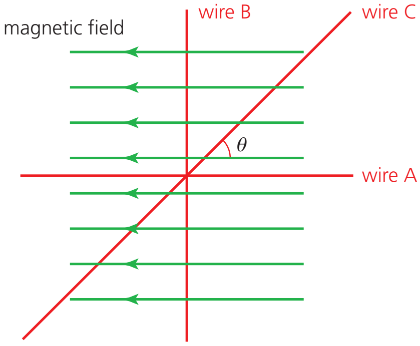
In Topic B.5 we described the motion of *free electrons* constituting electric currents through conductors. Such electrons moving across a magnetic field in a current-carrying conductor will experience the same forces as if they were in an electron beam travelling across a vacuum.

Figure D3.15 shows a wire carrying a current I (= q/t) through a wire which is at an angle θ to a uniform magnetic field, B.

The electrons have an average (drift) speed of v through the wire, so that in time t they travel an average distance $L = vt$.

We can rewrite the equation ($F = qvB\sin\theta$) for the force as:

$$F = \left(\frac{q}{t}\right)(vt)B\sin\theta$$



■ **Figure D3.16** How force varies with the angle of the current to the magnetic field: there will be no force on wire A and the biggest force per unit length will be on wire B. Wire C will experience a force, but the force per unit length of wire C will be smaller than for wire B.

to show that:

the force on a current I, passing through a conductor of length L across a uniform magnetic field B at an angle θ is given by:

$$\boldsymbol{F = BIL \sin\theta}$$

We briefly saw this equation in Topic D.2, where it was used to represent magnetic field strength as:

$$B = \frac{F}{IL}$$

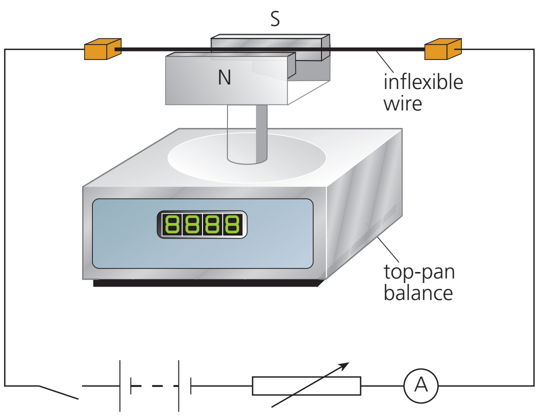
(when $\sin\theta = 1$)

Figure D3.16 illustrates how the magnetic force *per metre* depends on the angle of the wire to the magnetic field.

Note that the *total* force on the current in wire C would be the same as for wire B because there is a longer length in the field.

WORKED EXAMPLE D3.7

In Figure D3.17, a measured current is flowing in a wire across a small, uniform magnetic field.



■ **Figure D3.17** Current flowing in a wire across a small, uniform magnetic field

a State the direction in which the magnetic force is acting on the wire.

b In which direction is the force acting on the balance?

c When the current is flowing, the balance indicates that there is an extra mass of 4.20×10^{-2} g on the balance. Calculate the extra downwards force.

d If the current is 1.64 A and the length of the field is 8.13 cm, determine the strength of the magnetic field.

Answer

a Using the left-hand rule, the force is upwards.

b Using Newton's third law, the force is downwards.

c $F_g = mg = (4.20 \times 10^{-2} \times 10^{-3}) \times 9.81 = 4.12 \times 10^{-4}$ N

d Using $F = BIL \sin\theta$, with $\sin\theta = 1$, gives:

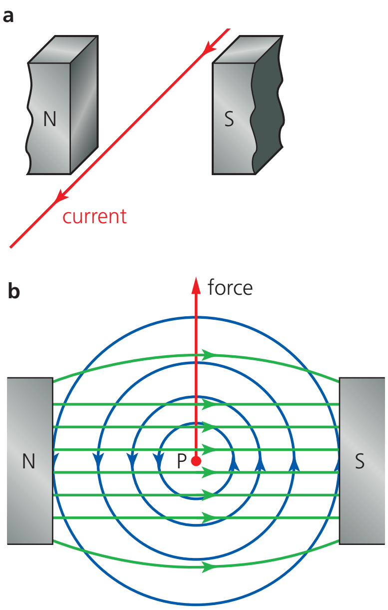
$4.12 \times 10^{-4} = B \times 1.64 \times 0.0813$

$B = 3.09 \times 10^{-3}$ T

Figure D3.18 shows an alternative approach to understanding the magnetic force on a current in a magnetic field.

Figure D3.18a shows a wire carrying an electric current across a magnetic field. The current is perpendicular to the magnetic field from the permanent magnets. In Figure D3.18b the same situation is drawn in two-dimensional cross-section, with the wire represented by the point P and the magnetic fields from the magnets (shown in green) and from the current (shown in blue) included.

The two fields are in the same plane, so it is easy to consider the combined field that they produce. Above the wire, the fields act in opposite directions and they combine to produce a weaker field. Below the wire, the fields combine to give a stronger field. This difference in magnetic field strength on either side of the wire produces an upwards force on the wire, which can make the wire move (if it is not fixed in position).



■ **Figure D3.18** Comparing the directions of current, field and force

◆ **Motor effect** Magnetic force on a current in a magnetic field, as used in electric motors.

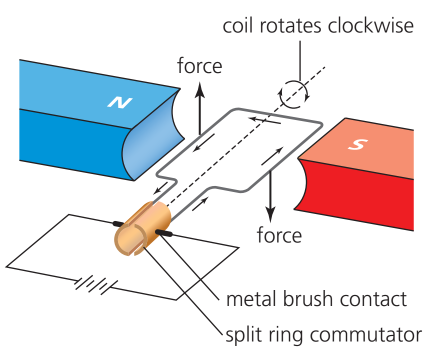
Simple dc motor

The force acting on a wire crossing a magnetic field is commonly called the **motor effect** because it can be used to rotate a loop of wire, as shown in Figure D3.19.

At the moment shown in Figure D3.19, the current on the right-hand side of the loop will experience a force downwards, while the current on the left-hand side will experience an upwards force because the current is flowing in the opposite direction. (Use Fleming's left-hand rule.)

It is not possible for a rotating loop to have fixed, permanent connections to an external power supply. The connection in Figure D3.19 is called a *commutator and brushes*. With this arrangement, the current will always enter the side of the loop on the right-hand side seen in the picture. In this way, the loop will experience the same forces every half rotation, which will keep it moving.

Increasing the current, strength of the magnetic field, or the number of turns in a coil will all make the motor spin faster. Winding the coil on an iron coil will also increase the rate of rotation.



■ **Figure D3.19** Essential parts of a dc motor

Parallel current-carrying wires

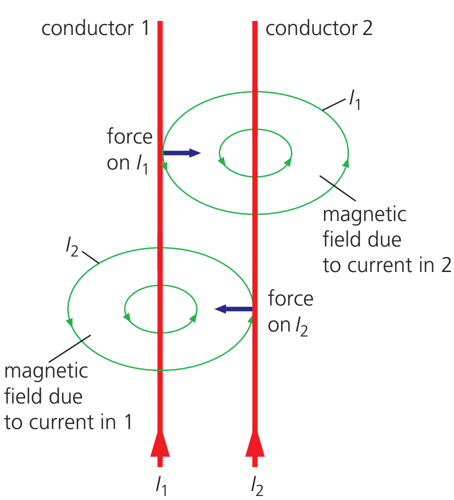
Consider the two parallel wires carrying currents as shown in Figure D3.20.

In Topic D.2 we explained that the direction of the magnetic field created by a current in a single long straight wire can be determined using the *right-hand grip rule*.

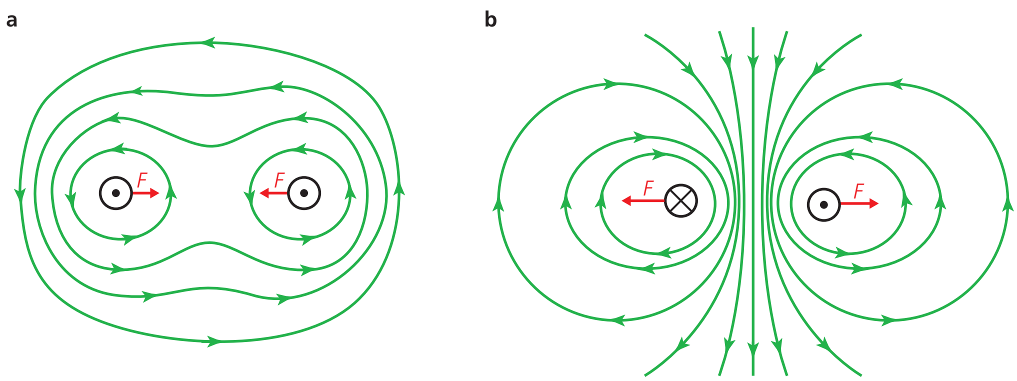
If both wires are carrying a current, then each wire is in the magnetic field created by the current in the other. Both wires will experience a force and, using the left-hand rule, the forces will be attractive between the wires if the currents are in the same direction. The forces are equal and opposite (Newton's third law).

If the currents are in opposite directions, the wires will repel each other.

Figure D3.21 shows the combined magnetic fields produced in the two situations, looking down from above.



■ **Figure D3. 20** Forces between parallel currents



■ **Figure D3.21** Magnetic fields around parallel currents in long wires :
a currents in the same direction, and **b** currents in opposite directions.

The force on current I_1 can be determined from $F = B_2 I_1 L \sin\theta$, but in this case the field is perpendicular to the wire, so that $\sin\theta = 1$, which leads to:

force on unit length of conductor carrying current, $I_1 = \dfrac{F}{L} = B_2 I_1$

We saw in Topic D.2 that the field round a wire at a distance r can be determined from:

$$B = \frac{\mu_0 I}{2\pi r}$$

and in this case:

$$B_2 = \frac{\mu_0 I_2}{2\pi r}$$

so that the force per unit length between parallel currents:

$$\boldsymbol{\frac{F}{L} = \mu_0 \frac{I_1 I_2}{2\pi r}}$$

The same force acts on both currents.

This arrangement was, until recently, used to define the SI unit of current, the ampere. One ampere, 1 A, was defined as the current flowing in two infinitely long, straight, parallel wires that produced a force of exactly $2 \times 10^{-7}\,\mathrm{N\,m^{-1}}$ between the wires if they were exactly 1 m apart in a vacuum.

WORKED EXAMPLE D3.8

Two very long straight wires are 12 cm apart. One carries a current of 3.7 A, the other carries a current of 1.6 A in the opposite direction.

a Calculate the force exerted on 1.0 m of the 3.7 A current (magnitude and direction).

b State the force per metre acting on the other current.

Answer

a $\dfrac{F}{L} = \mu_0 \dfrac{I_1 I_2}{2\pi r} = \dfrac{((4\pi \times 10^{-7}) \times 3.7 \times 1.6)}{(2\pi \times 0.12)}$

$= 9.9 \times 10^{-6}\,\mathrm{N}$ acting in a direction away from the other wire.

b The same: $9.9 \times 10^{-6}\,\mathrm{N}$ acting in a direction away from the other wire. That is, the two forces act in opposite directions.

16 Calculate the magnetic force per metre on a wire carrying a current of 1.2 A through a magnetic field of 7.2 mT if the angle between the wire and the field is:

a 30° **b** 60° **c** 90° **d** 0°.

17 a The Earth's magnetic field strength at a particular location has a horizontal component of 24 μT. Calculate the maximum force per metre that a horizontal cable carrying a direct current of 100 A could experience.

b State the direction in which the current needs to be flowing for this force to be vertically upwards.

c Discuss whether it is possible that such a force could support a cable.

18 A current is flowing in a horizontal wire perpendicularly across a magnetic field of strength 0.36 T. It experiences a force of 0.18 N, also horizontally.

a Draw a diagram to show the relative directions of the force, field and current.

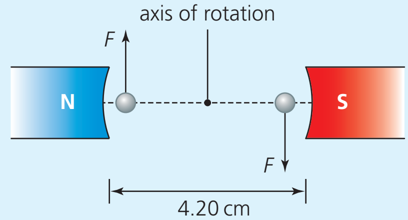
b If the length of wire in the field is 16 cm, calculate the magnitude of the current.

19 a A current of 3.8 A in a long wire experiences a force of 5.7×10^{-3} N when it flows through a magnetic field of strength 25 mT. If the length of wire in the field is 10 cm, determine the angle between the field and the current.

b If the direction of the wire is changed so that it is perpendicular to the field, calculate the new force on the current.

20 Consider Figure D3.19. Figure D3.22 shows a side view of the same situation: the loop of wire and the magnetic poles.

The current in the loop of wire is 0.530 A, the horizontal magnetic field strength is 25.0 mT and the length of the right-hand side of loop in the magnetic field is 3.80 cm.



■ **Figure D3.22** Forces between parallel currents; side view

a Determine the downwards force on the current in right-hand side of the loop.

b Calculate the torque applied to the loop by this force.

c What is the magnitude of the torque provided by the *couple* acting on the loop, and in what 'sense' is it acting?

d How will the magnitude of the torque change as the loop begins to rotate from its horizontal position (as shown)? Explain your answer.

e To make the loop rotate faster, the wire can be wound into a coil of many turns. Predict how many turns are needed to increase the maximum torque to 1.0×10^{-4} Nm.

21 Show that, when a current of 1.0 A flows in two infinitely long, straight, parallel wires, a force of exactly 2.0×10^{-7} N m^{-1} acts between them if they are exactly 1.0 m apart in a vacuum.

22 Two long straight wires are placed parallel to each other and 2.0 cm apart. One wire carries a current of 1.8 A.

a Determine what current in the other wire will result in a force of 4.7×10^{-5} N m^{-1} acting on it.

b State the magnitude of the force per metre on the other wire.

c If the currents are in opposite directions, in which directions will the forces act?

D.4 Induction

HL ONLY

Guiding questions

- What are the effects of relative motion between a conductor and a magnetic field?
- How can the power output of electrical generators be increased?
- How did the discovery of electromagnetic induction effect industrialization?

Electromagnetic induction

As before, the word *induction* is being used to describe something being made to happen without physical contact. Previously, in Topic D.2, we have discussed *electrostatic* induction and *magnetic* induction.

◆ **Electromagnetic induction** Process in which an emf is produced across a conductor that is experiencing a changing magnetic field.

Whenever a conductor moves across a magnetic field, or a magnetic field moves across a conductor, an emf will be induced. This effect is called **electromagnetic induction**.

Reminder from Topic B.5: the *electromotive force* (emf) of a battery, or any other source of electrical energy, is defined as the total energy transferred in the source per unit charge passing through it. In simple terms, it is the potential difference across the source when there is no current flowing.

There are numerous important applications of electromagnetic induction, including:

- generating electricity
- transforming voltages
- using bank cards and smart cards
- metal detecting and security checks
- regenerative braking.

All examples of electromagnetic induction are produced by one of the following.

- A conductor moves across a permanent magnetic field.
- A permanent magnetic field is moved across a conductor.
- A changing current in a circuit produces a changing magnetic field which passes through a *separate* conductor (without any physical movement).
- A changing current in a circuit produces a changing magnetic field which passes through the *same* circuit (without any physical movement).

We will describe each of these in the rest of this topic.

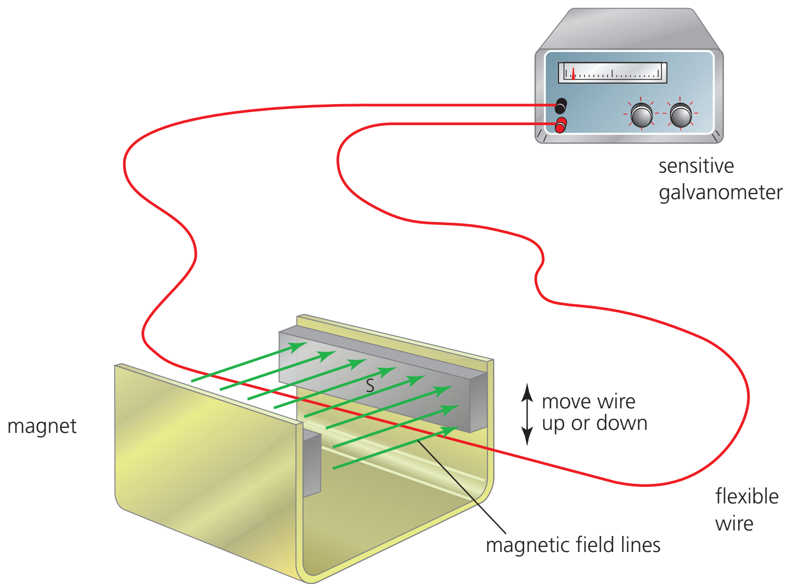
Electromagnetic induction by a conductor moving across a permanent magnetic field

SYLLABUS CONTENT

▶ A uniform magnetic field induces an emf in a straight conductor moving perpendicularly to it, as given by: $\varepsilon = BvL$.

◆ **Galvanometer** Ammeter that measures very small currents.

Figure D4.1 shows an experiment in which an emf is induced when a conductor (a metal wire) is moved across a permanent magnetic field. The induced emf can be detected because it makes a small current flow through a circuit containing a sensitive ammeter, called a **galvanometer**.

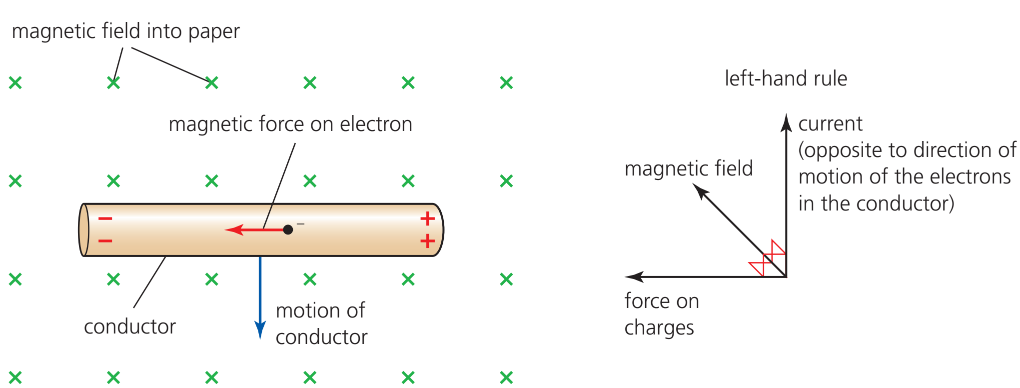


■ **Figure D4.1** Inducing an emf by moving a wire up or down across the magnetic field

The charged particles in the conductor experience forces because they are moving with the wire as it crosses the magnetic field (as discussed in Topic D.3). Because it is a conductor, the wire contains free electrons that can move along the wire under the action of these forces. Other charges (protons and most of the electrons) also experience forces but are not able to move along the conductor.

An emf is induced in a conductor because free electrons experience forces which make them move along the wire as it crosses a magnetic field.

Moving the wire, containing free electrons, is equivalent to a conventional current of positive charge in the opposite direction. We can use Fleming's left-hand rule (Topic D.3) to predict the direction of the forces on the electrons, as shown on the right in Figure D4.2. In this case the magnetic force pushes the electrons to the left, so the left-hand end of the conductor becomes negatively charged, while the other end becomes positively charged (because some electrons have flowed the other way). This charge separation produces a potential difference (emf) across the ends of the conductor.



■ **Figure D4.2** Magnetic force on electrons produces charge separation

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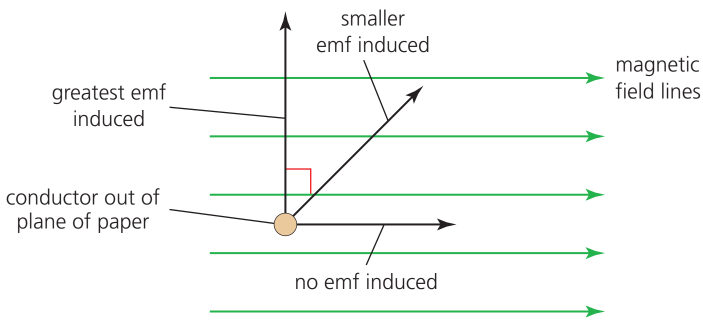
If the motion or the magnetic field is reversed in direction, then the emf is also reversed. If both the motion and the magnetic field are reversed, then the direction of the emf is unchanged. If the conductor and the magnetic field are both moving, but with the same velocity, no emf is induced. For electromagnetic induction to occur there must be *relative* motion between the conductor and the magnetic field.



■ **Figure D4.3** The size of an induced emf depends on the direction of motion

In order to induce an emf, a conductor needs to move *across* a magnetic field. Magnetic fields are represented by field *lines* and the conductor needs to be moving so that it 'cuts' across (through) the field lines. There will be no induced emf if the conductor is moving in a direction that is parallel to the magnetic field lines. Consider Figure D4.3, which shows three possible movements of a straight conductor which remains perpendicular to the plane of the paper. For similar conductors moving at the same speed, the induced emf is highest if the motion is perpendicular to the magnetic field.

If the conductor shown in Figure D4.3 is moved in a direction which is parallel to its own axis, it will not cut field lines and no emf will be induced. If it is rotated in the position shown, emfs will be induced unless the plane of rotation is parallel to the magnetic field.

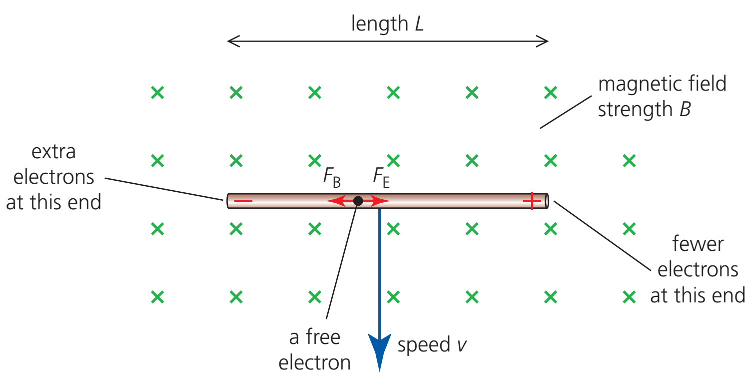
Experiments with the apparatus seen in Figure D4.1 can demonstrate that, for a conductor moving perpendicularly across the field, the emf, ε, induced can be increased by:

- increasing the speed of the movement, v
- using a magnetic field of greater strength, B
- increasing the length of the conductor in the magnetic field, L (which may mean increasing the extent of the magnetic field)
- winding the wire into a coil of many turns, N (with one side of the coil inside the magnetic field).

By considering the forces on free electrons, we can derive an equation for the emf, as follows.

Equation for an induced emf

Figure D4.4 shows a closer look at the situation seen in Figure D4.2. A conductor of length L is moving perpendicularly across a uniform magnetic field of strength B, with speed v. Free electrons in the conductor will each experience a magnetic force, F_B, given by the expression, $F = qvB\sin\theta$ (Topic D.3). In this perpendicular arrangement $\sin\theta = 1$. These forces tend to move free electrons towards the left of the conductor (as shown). As more electrons move along the conductor, the increasing amount of negative charge repels the motion of further electrons to that end. The right-hand end of the conductor, which has lost electrons, will become positively charged and act as an attractive force on the electrons.

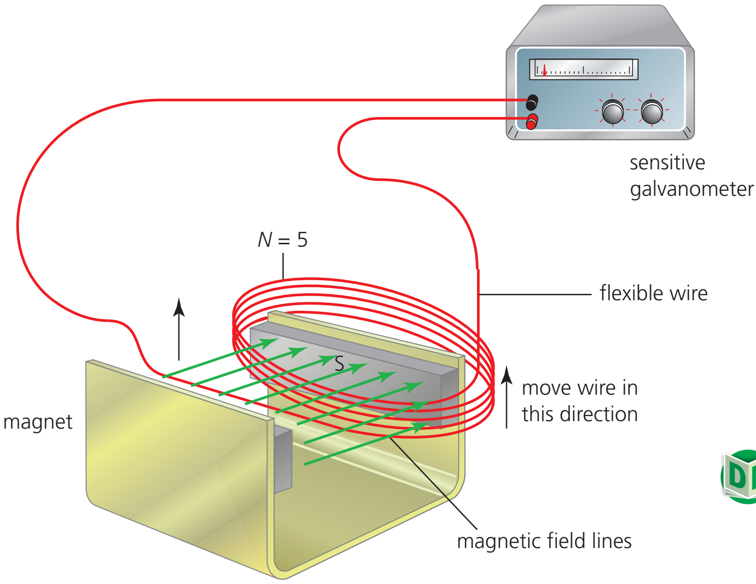


■ **Figure D4.4** Deriving $\varepsilon = BvL$

The charge separation produces an electric field along the conductor:

$$E = \frac{\varepsilon}{L}$$

where ε is the induced emf across the ends of the conductor.



■ **Figure D4.5** Electromagnetic induction with more than one turn

The maximum induced potential difference will occur when the force on each free electron due to the magnetic field, F_B, is equal and opposite to the force on the electron, F_E, due to the electric field.

$$\text{electric force, } F_E = \text{electric field} \times \text{charge} = \frac{\varepsilon q}{L}$$

At equilibrium, $F_E = F_B$

$$\frac{\varepsilon q}{L} = qvB$$

So that:

the induced emf when a straight conductor moves perpendicularly across a uniform magnetic field:

$$\varepsilon = BvL$$

(If the field was not perpendicular to the wire, the component of the field in that direction would have to be used in the calculation.)

If a long length of wire is wound into a loose coil and one side (only) is moved in the magnetic field, as shown in Figure D4.5, each extra loop of wire will add an emf of the same value in series, similar to adding more cells to a battery. If there are N turns in the coil, the induced emf will become $\varepsilon = NBvL$.

The directions of the induced emf and current are important and will be discussed later.

WORKED EXAMPLE D4.1

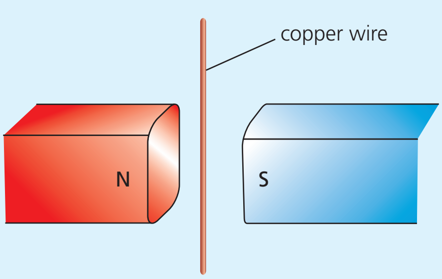
Calculate the induced emf produced across a 23.0 cm long conductor moving at 98.0 cm s^{-1} perpendicularly across a magnetic field of strength 120 μT.

Answer

$\varepsilon = BvL = (120 \times 10^{-6}) \times 0.98 \times 0.23 = 2.7 \times 10^{-5}$ V

1 Explain why no emf is induced across a string made of plastic when it is moved through a magnetic field.

2 Figure D4.6 shows a copper wire between the poles of a permanent magnet. Describe the direction(s) in which the wire should be moved to induce:



■ **Figure D4.6** A copper wire between the poles of a permanent magnet

a the highest emf
b zero emf.
c Explain why no current can be induced in this wire as shown.

3 When a straight conductor of length 90 cm moved perpendicularly across a uniform magnetic field of strength 4.5×10^{-4} T, an emf of 0.14 mV was induced. Calculate the speed of the conductor.

4 Determine the strength of magnetic field needed for a voltage of 0.12 V to be induced when a conductor of length 1.6 m moves perpendicularly across it at a speed of 2.7 m s^{-1}.

5 Show that the units of BvL are the same as for ε.

6 Consider Figure D4.5. Calculate the effective width of the uniform magnetic field if it has a strength of 7.8×10^{-3} T and an emf of 3.8 mV is induced when the side of the coil moves vertically with a speed of 1.8 m s^{-1}.

7 An aircraft is flying horizontally at a speed of $280\,\mathrm{m\,s^{-1}}$ at a place where the vertical component of the Earth's magnetic field is $12\,\mu\mathrm{T}$, as shown in Figure D4.7.

a Calculate the emf induced across its wing tips if its wingspan is 58 m.

b Suggest a possible reason why this voltage would be larger if the aircraft was flying close to the North, or South, Pole.

c Could the induced emf be used to do anything useful on the aircraft? Explain your answer.

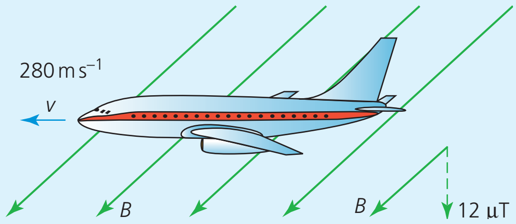


Figure D4.7 An aircraft flying horizontally

Electromagnetic induction by moving a permanent magnetic field across a conductor

Moving a conductor through a permanent magnetic field has a similar effect to keeping the conductor still and moving the field.

Figure D4.8 shows electromagnetic induction by moving a magnetic field (around a permanent magnet) through a conductor in the form of a loop of wire. Again, the induced emf and current will be very small in this basic example, but Figure D4.9 shows how the effects can be increased greatly by winding the conductor into a coil, or solenoid, with many turns. The direction of the induced current around the coil will be reversed if the magnet is reversed, or alternatively, if the motion of the magnet is reversed.

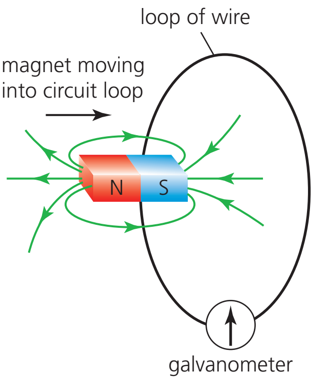


Figure D4.8 Moving a magnet to induce an emf and a current

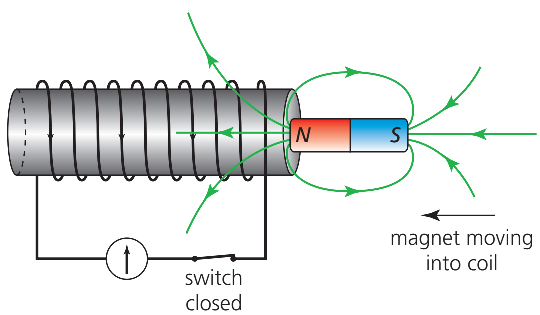


Figure D4.9 Inducing an emf and a current in a coil of wire

Figure D4.10 shows an electromagnetic induction experiment recorded on a data logger and computer. The data logger records the emf being induced at regular time intervals when a magnet is dropped through a coil, and then the data is used to draw a graph.

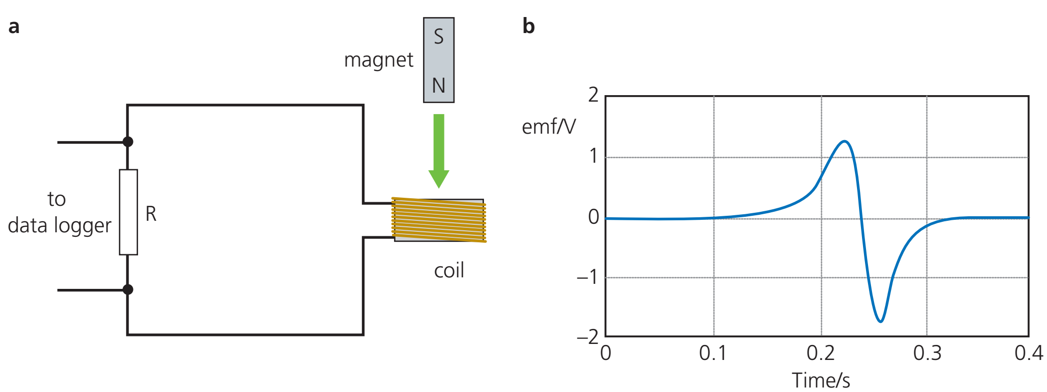


Figure D4.10 Inducing a current by dropping a magnet through a coil

WORKED EXAMPLE D4.2

Consider Figure D4.10. Describe how the graph would change if the:

a polarity of the magnet was reversed

b magnet was dropped from a greater height?

Answer

a The graph would be inverted.

b The peaks would be higher and their durations reduced.

8 In a demonstration of electromagnetic induction similar to that shown in Figure D4.8, the induced current was very small.

a Suggest two ways of increasing the induced current while still using the same single loop of wire.

b State two ways in which the current can be made to flow in the opposite direction around the circuit.

9 Draw a sketch similar to Figure D4.9 to show the current direction when the bar magnet comes out of the coil at the other end.

10 Suggest explanations for the shape of the graph shown in Figure D4.10b.

11 Figure D4.11 shows a magnet oscillating vertically on a spring. As it oscillates, with a frequency of 0.67 Hz, the end of the magnet passes in and out of a coil of wire which is in a circuit with a centre-reading galvanometer and a switch.

a Describe how the pointer on the galvanometer will move while the switch is closed.

b Sketch a graph of the induced current–time for 3.0 s.

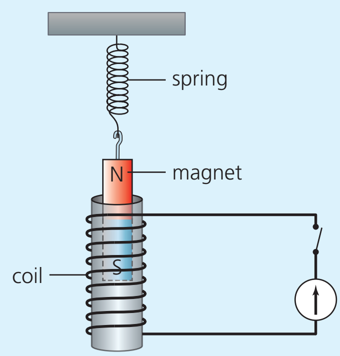


Figure D4.11 A magnet oscillating vertically on a spring

Electromagnetic induction without physical movement

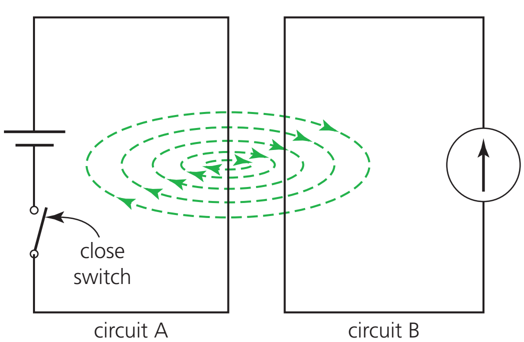


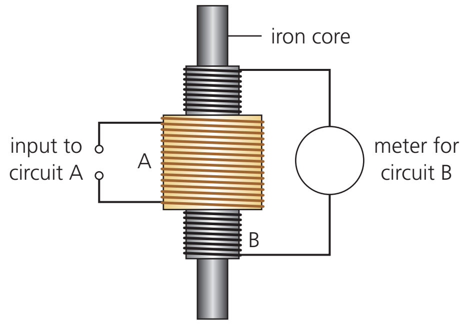
Figure D4.12 When the switch is closed, a magnetic field passes from circuit A to circuit B

Emfs can also be induced, not by movement, but by changes in the current in one circuit affecting another, completely separate, circuit. Figure D4.12 represents the simplest example.

First consider circuit B at a time when the switch in circuit A is open – there is no power source and no changing magnetic field near B, so there is no current shown by the galvanometer. However, at the moment that the switch in circuit A is closed, a current starts to flow around circuit A and this sets up a magnetic field around it. This field spreads out and passes through circuit B.

The sudden *change* of magnetic field induces an emf and a current that is detected by the galvanometer in circuit B. The changing current produces a changing magnetic field in the same way as moving a magnet does.

This induced emf / current only lasts for a moment, while the switch in A is being turned on, because when the current in A is constant there is no *changing* magnetic field. When the switch is turned off, there is an induced emf / current for a moment in the opposite direction. As described so far, this is a very small (but important) effect. However, the induced emf can be increased greatly by winding the conducting wires in both circuits into coils of many turns (to increase the strength of the magnetic field) and placing them on top of each other with an iron core through the middle. This is shown in Figure D4.13. Remember from Topic D.2 that iron has high magnetic permeability and greatly increases the strength of the magnetic field.



■ **Figure D4.13** Making the induced emf larger by using an iron core and coils of many turns

If, when a steady direct current is flowing around circuit A, it is suddenly switched off, the change in the magnetic field through circuit B can be so quick that a very large voltage can be momentarily induced if the coil in circuit B has a large number of turns. Used in this way, an *induction coil* can be both useful and dangerous.

◆ **Transformer** A device that transfers electrical energy from one circuit to another using electromagnetic induction between coils wound on an iron core. Transformers are used widely to transform one alternating voltage to another of different magnitude.

If the voltage source in circuit A is changed from one that provides a *direct* current (dc) of constant value to a source of *alternating* current (ac), then the magnetic field in both circuits will change continuously and an alternating emf will be induced continuously. This has many useful applications, including **transformers**, as discussed below.

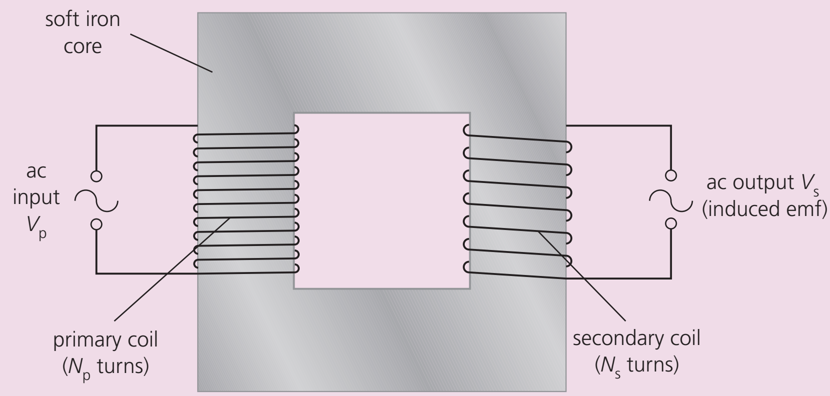
A changing current produces a changing magnetic field which can induce an emf without any physical movement. With alternating currents this effect is continuous.

Inquiry 1: Exploring and designing

Designing

Transformers

Figure D4.14 shows the basic components of a device known as a transformer. It functions in a similar way to the coils seen in Figure D4.13, where the coils are wound together. Transformers are used to change alternating voltages to lower, or higher, levels.



■ **Figure D4.14** Transformer

The alternating current in the *primary coil* creates a constantly changing magnetic field. The field is concentrated in the iron core and passes around to the *secondary coil*, where it induces an emf.

1 List the factors that will affect the value of the induced emf seen on the meter.
2 Design an experiment methodology (using two self-made coils) to investigate how the value of the induced emf depends on one of those variables. Pilot (try out) your design with the test coils that you have constructed.

■ **Figure D4.15** A transformer on a road-side pole

12 Consider Figure D4.12. Suggest two ways in which an emf momentarily induced in circuit B could be increased, without twisting the wires into coils.

13 Use a sketch graph to explain why an alternating current (which varies between the same maximum and minimum values) will induce a greater emf in a surrounding circuit when the frequency is greater.

14 See Figure D4.14. The output voltage, V_s, from a transformer can be calculated from:

$$V_s = V_p \times \left(\frac{N_s}{N_p}\right)$$

(You are not expected to remember this equation.)

- **a** Calculate the output voltage from a transformer which has an input of 230 V (ac), 350 turns on its primary coil and 18 turns on its secondary coil.
- **b** Another transformer is used to ‘step-up’ an alternating voltage from 50 V to 2000 V. If the primary coil has 40 turns, predict how many turns are needed for the secondary coil.

15 Explain why the kind of transformer seen in Figure D4.14 cannot transform steady voltages.

16 Suggest how induction between circuits is used in the operation of bank cards and transport cards. (See Figure D4.16 for an example.)

■ **Figure D4.16** Using a transport card

◆ **Transmission of electrical power** Electrical power is sent (transmitted) from power stations to different places around a country along wires (cables), which are commonly called transmission (or power) lines. These lines are linked together in an overall system called the transmission grid.

ATL D4A: Research skills

Evaluate information sources for accuracy, bias, credibility and relevance; use a single standard method of referencing and citation

A typical power station may produce electricity at a few hundred volts. Research and write a short report explaining the reasons why:

- Transformers are used to greatly increase this voltage before it is **transmitted** around the country.
- The currents are sent through aluminium cables.

Ensure you use reliable sources of information by carrying out credibility checks. In your report, be sure to provide clear references to the sources you used using the referencing and citation standard advocated by your school.

■ **Figure D4.17** Transmission lines transfer electrical power around countries

Magnetic flux and magnetic flux linkage

SYLLABUS CONTENT

▶ Magnetic flux Φ as given by: $\Phi = BA\cos\theta$.

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Magnetic flux

Nature of science: Models

Flux

The concept of *flux* has many applications. In general, the term is used to describe some kind of flow. A non-scientific example could be the (in)flux of people into a particular location, which could be recorded in terms of the number of people in a certain time. In physics we may refer to a flux of thermal energy, or light, or radiation, each of which could be measured in terms of the amount of energy flowing through a given area every second ($W\,m^{-2}$). See Question 20.

Magnetic flux, as explained below, is slightly different, because no movement is implied, although the vector arrows seen, for example, in Figure D4.18, may suggest motion.

◆ **Magnetic flux, Φ** Defined as the product of an area, A, and the component of the magnetic field strength perpendicular to that area.

◆ **Weber, Wb** Unit of magnetic flux. $1\,Wb = 1\,T\,m^2$.

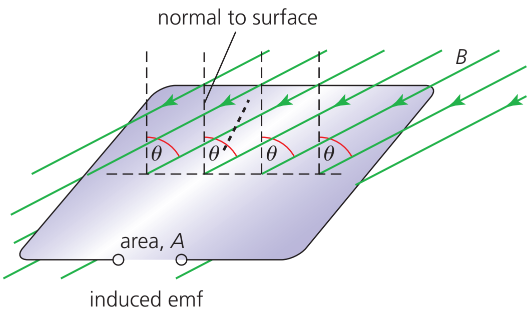
◆ **Magnetic flux density, B** The term more commonly used at Higher Level for magnetic field strength. $B = \frac{\Phi}{A}$.

Electromagnetic induction becomes easier to understand after the concept of **magnetic flux** has been introduced.

Suppose we want to induce an emf across a loop of wire. There are a number of possibilities, including:

- move the loop into, or out of, a permanent magnetic field
- rotate the loop in a permanent magnetic field
- keep the loop still and move a permanent magnetic field into, or out of, the loop
- keep the loop still in the changing magnetic field produced by a changing current in another circuit
- any combination of the above.

We will simplify the geometry of these situations to that shown in Figure D4.18, in which a magnetic field is acting into a loop of wire. To simplify the diagram, only a few field lines are seen, but we will assume that a uniform magnetic field is acting across the whole area of the loop. An emf can be induced by any of the changes listed above.



■ **Figure D4.18** Magnetic flux depends on field strength, area and angle

The size of the induced emf depends not only on the strength of the magnetic field, B, but also on the area, A, of the circuit over which it is acting, and the angle, θ , at which it is passing through the circuit.

Magnetic flux, Φ, (for a uniform magnetic field) is defined as the product of the area, A, and the component of the magnetic field strength which is perpendicular to that area:

$$\Phi = BA\cos\theta$$

DB

If the field is perpendicular to the area, $\cos\theta = 1$ so the equation reduces to: $\Phi = BA$.

Common mistake

Note that the angle θ is the angle between the field and a normal to the surface, not the angle between the field and the surface.

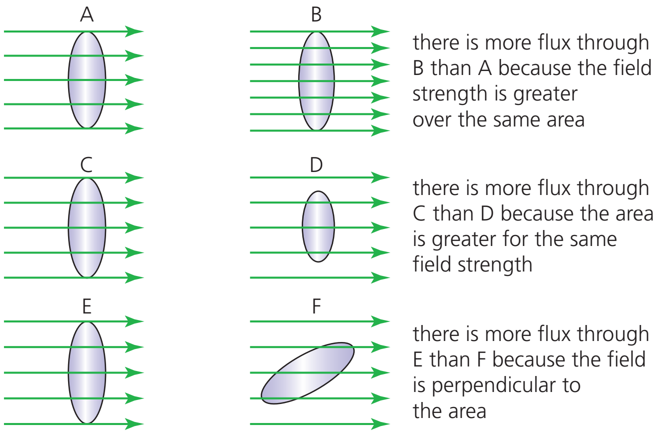
The SI unit of magnetic flux is the **Weber**, Wb. One weber is equal to one tesla multiplied by one metre squared ($1\,Wb = 1\,T\,m^2$).

We can rearrange the equation for flux to give:

$$B = \frac{\Phi}{A}$$

for B perpendicular to A; this shows us why magnetic field strength is widely known as **magnetic flux density** (flux / area). That is, 1 tesla = 1 weber per square metre.

The emf induced across a single loop of wire is proportional to the rate of change of magnetic flux through it (more details later).



■ **Figure D4.19** Magnetic flux explained in terms of field lines

Magnetic flux can be a difficult concept to understand and Figure D4.19 may help. It shows a non-mathematical interpretation of magnetic flux as the number of magnetic field lines that pass through the area.

It may be helpful to consider that the magnitude of an induced emf depends on the rate at which a conductor 'cuts' magnetic field lines (or the rate at which magnetic field lines cut a conductor).

Magnetic flux linkage

So far, we have been discussing electromagnetic induction using single loops of wire. But, if the wire is wound into a coil with N turns, each turn contributes the same emf (in series), so that the overall induced emf is multiplied by N. The concept of **magnetic flux linkage** takes this into account:

◆ **Magnetic flux linkage, $N\Phi$** The product of magnetic flux and the number of turns in a circuit (unit: Wb).

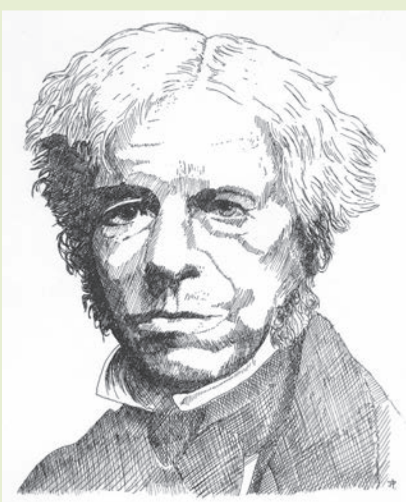
Magnetic flux linkage is defined as the product of magnetic flux and the number of turns in the coil. It does not have a widely used standard symbol:

$$\text{magnetic flux linkage} = N\Phi$$

The units of flux linkage are the same as flux (Wb), although sometimes Wb-turns is used.

Nature of science: Observations

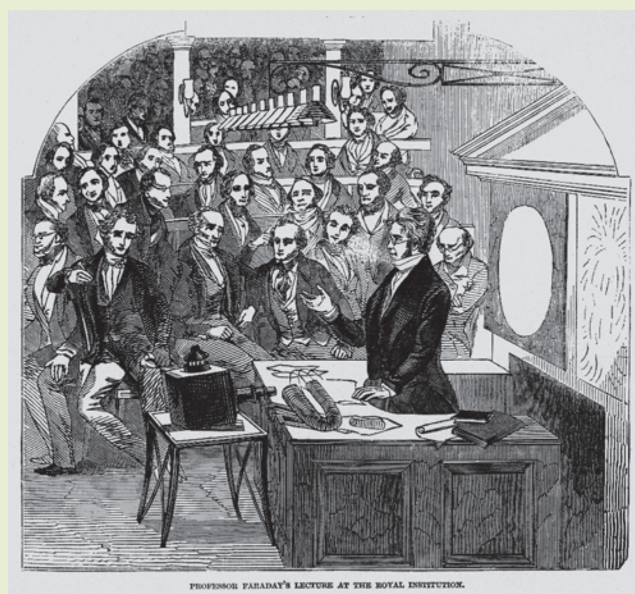
Unexpected or unplanned observations



■ **Figure D4.20** Michael Faraday (1791–1867) is considered to be one of the greatest scientists

In 1831 Michael Faraday (Figure D4.20) became the first person to demonstrate electromagnetic induction. See Figure D4.21.

The equipment available at the time made this a difficult phenomenon to observe, and some observers may have doubted its importance at the time, but its far-reaching consequences are now undeniable. In a similar way, the first transmission of radio waves more than 50 years later (by Heinrich Hertz) may have seemed trivial at the time, but both discoveries ultimately changed the world for ever.



■ **Figure D4.21** Michael Faraday giving a lecture at the Royal Institution in London

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WORKED EXAMPLE D4.3

a Calculate the magnetic flux in a square loop of wire of sides 6.2 cm when it is placed at 45° to a magnetic flux density of 4.3×10^{-4} T.

b Calculate how many turns would be needed on a coil of the same dimensions to create a flux linkage of 8.4×10^{-4} Wb.

c Determine the magnetic flux linkage in the coil if only half of it was in the magnetic field.

Answer

a $\Phi = BA\cos\theta = (4.3 \times 10^{-4}) \times (6.2 \times 10^{-2})^2 \times \cos 45° = 1.2 \times 10^{-6}$ Wb

($1.16879\ldots \times 10^{-6}$ seen on the calculator display)

b $N\Phi = 8.4 \times 10^{-4}$

$$N = \frac{8.4 \times 10^{-4}}{1.16879 \times 10^{-6}} = 7.2 \times 10^2$$

c The magnetic flux linkage would be reduced to half (4.2×10^{-4} Wb) because the area used in the calculation is the area of the coil in the magnetic field, not the total area of the coil.

17 Calculate the magnetic flux in a flat coil of area 48 cm^2 placed in a field of magnetic flux density 5.3×10^{-3} T if the field is at an angle of 30° to the plane of the coil.

18 A magnetic field of strength 3.4×10^{-2} T passes perpendicularly through a flat coil of 480 turns and area 4.4×10^{-5} m^2.
Determine the flux linkage.

19 A flat coil of 600 turns and area 8.7 cm^2 is placed where the magnetic flux density is 9.1×10^{-3} T. The axis of the coil was originally parallel to the magnetic field, but it was then rotated by 25°.
Calculate the *change* of flux linkage through the coil.

20 This question provides a solar radiation analogy to help understanding of the concept of flux. The 'solar flux density' arriving perpendicularly at the Earth's upper atmosphere is 1360 W m^{-2}. (This was called the Solar constant in Topic B.2.)
Suppose that near the Earth's surface this value has reduced to 800 W m^{-2}.
Calculate the power arriving at a horizontal solar panel of area 4.0 m^2 if the radiation arrives at an angle of 40° to the vertical (see Figure D4.22).

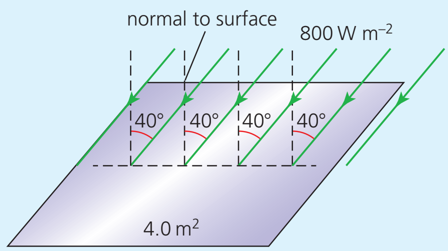


Figure D4.22 Solar flux

Faraday's law of electromagnetic induction

SYLLABUS CONTENT

- A time-changing magnetic flux induces an emf ε as given by Faraday's law of induction: $\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$.

We can now write down an equation which can be used to determine the value of an emf induced under *any* circumstances:

If a coil with N turns experiences a magnetic flux which changes by $\Delta\Phi$ in time Δt, the induced emf:

$$\boldsymbol{\varepsilon = -N\frac{\Delta\Phi}{\Delta t}}$$

◆ **Faraday's law of electromagnetic induction** The magnitude of an induced emf is equal to the rate of change of magnetic flux linkage, $\varepsilon = \frac{-N\Delta\Phi}{\Delta t}$.
For an explanation of the negative sign, see Lenz's law.

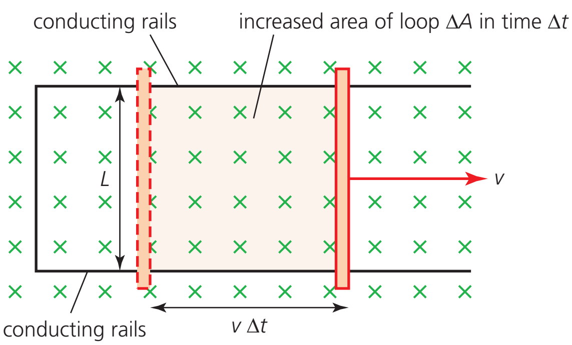
The negative sign here is important and it is explained later in this topic.

We will now explain how **Faraday's law** can be applied to three different situations.

Induction because of motion of a conductor across a uniform magnetic field

Figure D4.23 shows a simple visualization of one type of electromagnetic induction: a metal rod is lying across two fixed parallel conducting rails. The rod is able to move horizontally which is perpendicularly across the magnetic field, but as it does so, it continues to complete an electrical circuit, as shown in the figure.

In this example of electromagnetic induction, the magnetic field is constant but the area of the circuit changes.



■ **Figure D4.23** Inducing an emf with a moving conductor

Because there is only one loop, Faraday's law for the magnitude of the induced emf reduces to:

$$\varepsilon = \frac{\Delta\Phi}{\Delta t} = \frac{\Delta(BA)}{\Delta t} = B\left(\frac{\Delta A}{\Delta t}\right)$$

for a uniform magnetic field.

Suppose that the rod, of length L, moves to the right with a constant speed v. In time Δt it will move a distance, $v\Delta t$ perpendicularly across a uniform magnetic field of strength B.

The rate of change of area:

$$\frac{\Delta A}{\Delta t} = \frac{Lv\Delta t}{\Delta t} = vL$$

vL is often described as the 'area swept out' by the moving rod in time Δt; so that:

$\varepsilon = B\left(\frac{\Delta A}{\Delta t}\right)$ becomes:

$\varepsilon = BvL$ which is the same equation as we have seen earlier.

WORKED EXAMPLE D4.4

Two parallel and horizontal conducting rails, which are 44 cm apart, are placed in a uniform magnetic flux density of 8.7×10^{-4} T which is acting vertically downwards, as shown in Figure D4.23. The rod moves to the right with a speed of 48 cm s^{-1}.

a Determine the value of the emf induced across the loop.

b State the rate of change of magnetic flux.

c Determine how much extra magnetic flux passes through the circuit when the rod moves 25 cm.

Answer

a $\varepsilon = BvL = (8.7 \times 10^{-4}) \times 0.48 \times 0.44 = 1.8 \times 10^{-4}$ V

(1.83744×10^{-4} V seen on calculator display)

b 1.8×10^{-4} Wb s^{-1}

c Increase in area in $0.25 \times 0.44 = 0.11$ m^2

increase in magnetic flux = increase in area × magnetic flux density

$= 0.11 \times (8.7 \times 10^{-4}) = 9.6 \times 10^{-5}$ Wb

Alternatively:

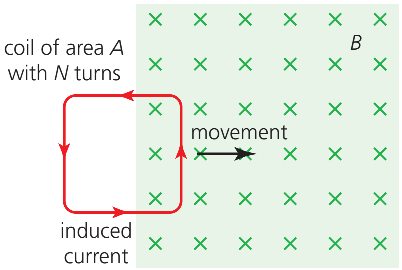
increase in magnetic flux = rate of change of magnetic flux × time

$= (1.83744 \times 10^{-4}) \times \left(\frac{0.25}{0.48}\right) = 9.6 \times 10^{-5}$ Wb

Tool 3: Mathematics

Interpret areas under graphs

Consider again Figure D4.10b. What do the two areas between the curve and the horizontal axis represent? Explain why they are equal in magnitude.



■ **Figure D4.24** Coil moving into a magnetic field

Induction because of motion of a coil into and out of a uniform magnetic field

Figure D4.24 shows a coil of wire being moved into a perpendicular magnetic field. As the right-hand side of the coil enters the field, an emf is induced and a current flows around the coil as shown. When all of the coil is moving inside the field, there will be no changing magnetic flux and so there is no induced emf or current. When the coil moves out of the right-hand side of the field, the emf and current are reversed from their directions when entering the field.

In this example of electromagnetic induction, the area (of the coil) is constant but the magnetic field through the coil changes. The same effect can also be produced by keeping the coil stationary and moving the magnetic field. Then, the magnitude of the induced emf given by

$$\varepsilon = N\frac{\Delta\Phi}{\Delta t}$$

becomes:

$$\varepsilon = NA \times \frac{\Delta B}{\Delta t}$$

WORKED EXAMPLE D4.5

Consider Figure D4.24.

a Determine the average magnitude of the induced emf when a coil of 40 turns and area 5.0 cm^2 is moved from completely outside to completely inside a uniform magnetic field of strength 0.34 T in 0.56 s.

b The coil is then turned upside-down (rotated 180°) in the same magnetic field, in a time of 0.29 s. Calculate the magnitude of the induced emf.

Answer

a $\varepsilon = N\frac{\Delta\Phi}{\Delta t} = NA \times \frac{\Delta B}{\Delta t}$

$= 40 \times (5.0 \times 10^{-4}) \times \left(\frac{0.34}{0.56}\right)$

$= 1.2 \times 10^{-2}$ V

b The field changes from 0.34 T in one direction through the coil to 0.34 T in the opposite direction. Which is an overall change of 0.68 T.

$\varepsilon = N\frac{\Delta\Phi}{\Delta t} = NA \times \frac{\Delta B}{\Delta t}$

$= 40 \times (5.0 \times 10^{-4}) \times \left(\frac{0.68}{0.29}\right)$

$= 4.7 \times 10^{-2}$ V

21 A train is travelling with a speed of $38\,m\,s^{-1}$ through a region where the Earth's magnetic field is $42\,\mu T$, acting at $50°$ to the horizontal. An axle on the train has a length of 1.43 m.

- **a** Calculate the area swept out by the axle every second.
- **b** Determine the component of the magnetic field acting perpendicular to the axle.
- **c** Calculate the rate of change of magnetic flux experienced by the axle.
- **d** What was the magnitude of the induced emf across the axle?

22 The magnetic flux through a coil of 1200 turns increases from zero to 4.8×10^{-5} Wb in 2.7 ms. Calculate the magnitude of the average induced emf during this time.

23 A coil of area $4.7\,cm^2$ and 480 turns is in a magnetic field of strength 3.9×10^{-2} T.

- **a** Calculate the maximum possible magnetic flux linkage through the coil.
- **b** Determine the average induced emf (mV) when the coil is moved to a place where the perpendicular magnetic field strength is 9.3×10^{-3} T in a time of 0.22 s.

Induction between circuits

◆ **Mutual induction** Electromagnetic induction between separate circuits.

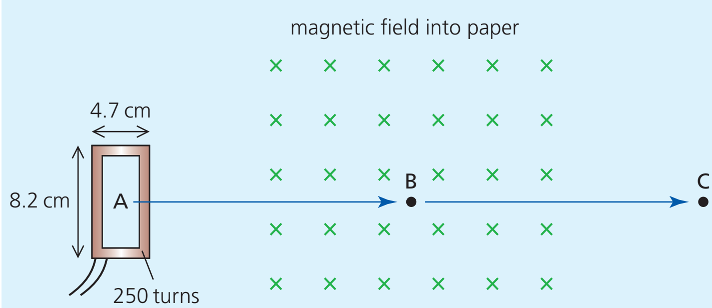
Reconsider Figure D4.13. Induction between circuits is called **mutual induction**. The changing magnetic flux passing through coil B depends on the rate at which the current, I, is changing in coil A.

For mutual induction in a fixed arrangement $\frac{\Delta\Phi}{\Delta t}$ and the induced emf, ε, are proportional to $\frac{\Delta I}{\Delta t}$.

You will *not* be expected to answer detailed quantitative questions on mutual induction.

24 Figure D4.25 shows a coil of 250 turns moving from position A, outside a strong uniform magnetic field of strength 0.12 T, to position B at the centre of the magnetic field in a time of 1.4 s.

- **a** Calculate the change of magnetic flux in the coil when it is moved.
- **b** State any assumption that you made in answering **a**.
- **c** Determine the change of magnetic flux linkage.
- **d** Calculate the average induced emf.
- **e** Sketch a graph to show how the induced emf changes as the coil is moved at constant speed from A to C (no values needed).



■ **Figure D4.25** A moving coil of 250 turns

25 Imagine you are holding a flat coil of wire in the Earth's magnetic field.

- **a** Draw a sketch to show how you would hold the coil so that there is no magnetic flux through it.
- **b** At a place where the magnitude of the Earth's magnetic field strength is $48\,\mu T$, what emf would be induced by moving a coil of 550 turns and area $17\,cm^2$ from being parallel to being perpendicular to the magnetic field in 0.50 s?

26 A small coil of area $1.2\,cm^2$ is placed in the centre of a long solenoid with a large cross-sectional area. A steady current of 0.50 A in the solenoid produces a magnetic field of strength $8.8 \times 10^{-4}\,T$.

a Determine how many turns would be needed on the coil if an induced emf of 2.4 mV was required when the current in the solenoid was increased to 2.0 A in a time of 0.10 s.

b Describe the position in which the coil needs to be placed.

ATL D4B: Thinking skills

Applying key ideas and facts in new contexts

Induction cookers

In an induction cooker, like that shown in Figure D4.26, there are coils of wire below the flat top surface. When a high-frequency current is passed through a coil in the cooker, a strong, rapidly oscillating magnetic field is created that will pass through anything placed on or near the cooker's surface, like a cooking pot. If the material of the pot is a conductor, emfs and currents will be induced in it. Currents circulating within solid conductors, rather than around wire circuits, are known as **eddy currents**.

The electrical energy in the eddy currents will be transferred to internal energy and the pan will get hotter. As in any circuit, the power generated can be calculated using $P = I^2R$. Thermal energy is then transferred by conduction to any food that is in the pan.

Apply your understanding of thermal physics and electromagnetic induction to explain the following:

- why the choice of metal for the pan is important
- why the cooker itself does not get noticeably hot (except for any thermal energy transferred from the pan)
- why induction cookers are more efficient at cooking food than conventional electric hot plates.

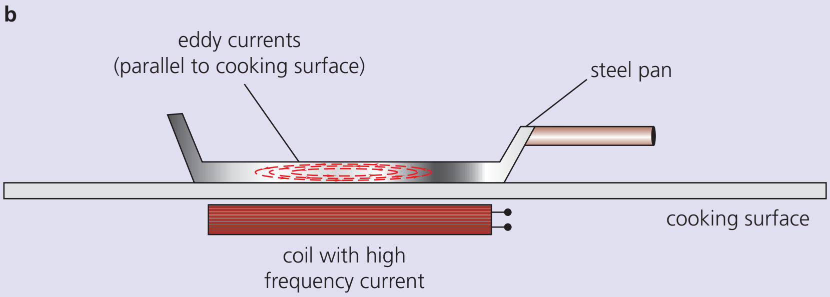


Figure D4.26 a A steel pan on an induction cooker and **b** How an induction cooker works

◆ **Eddy currents** Circulating currents induced in solid pieces of metal when changing magnetic fields pass through them.

Energy transfers during electromagnetic induction

SYLLABUS CONTENT

▶ The direction of an induced emf is determined by Lenz's law and is a consequence of energy conservation.

We will now explain why a negative sign appears in Faraday's law of electromagnetic induction.

If a current is generated from motion by electromagnetic induction, then energy must have been transferred from outside the circuit. We know this from an understanding of the *law of conservation of energy*. The origin of this energy is often the kinetic energy of the moving conductor or moving magnet. The moving object must therefore slow down as it loses some of its kinetic energy (unless there is an external force keeping it moving).

An induced electric current has had energy transferred to it from the process that induced it, for example from kinetic energy of motion.

Consider again Figure D4.11. To begin the experiment, a student pulled the magnet downwards, stretching the spring. The student supplied the energy. The magnet then oscillates vertically, interchanging elastic potential energy and kinetic energy (assuming that changes of gravitational potential energy are not significant). If there is no coil, or if the switch is open, the oscillations will continue for some time, although there will be a little energy dissipation (damping), as discussed in Topic C.2.

However, if the magnet oscillates into and out of a conducting coil, an emf will be induced across the coil because of the changing magnetic flux in it. A current will flow *if* the switch is closed and energy will be transferred in the coil. Energy is transferred to the current from the kinetic energy of the magnet, which means that the magnet must move more and more slowly.

Consider again Figure D4.9. A current is induced in the coil as the magnet moves towards it. That current makes the coil behave as an electromagnet, with one end a south magnetic pole and the other a north magnetic pole (Topic D.2). The induced emf across the coil is in the direction such that the induced current makes the right-hand side of the coil (as shown) a north pole. In this way there is a repulsive force between the magnet and the coil because two north poles are close together. Work has to be done to overcome this force and move the magnet into the coil. When this is done, the energy is transferred to the current in the coil.

If the motion of the magnet is reversed, an attractive force will be created and, again, work has to be done to move the magnet and induce a current in the coil.

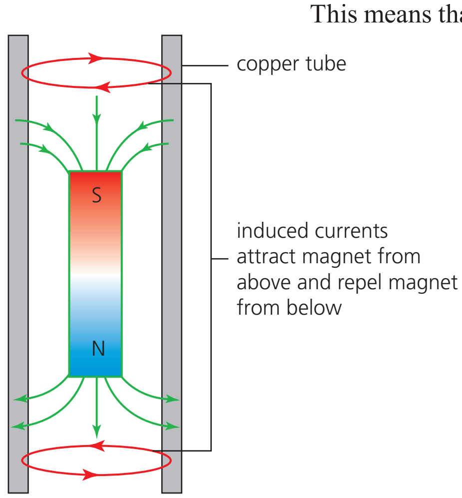
◆ **Lenz's law (of electromagnetic induction)** The direction of an induced emf is such that it will oppose the change that produced it. This is represented mathematically by the negative sign in the equation representing Faraday's law.

If the magnet was already in the coil and then removed from the left-hand side, the induced current would set up a magnetic field to oppose that motion.

Whenever the magnet is moved in any way (in Figure D4.9) a current will be induced and the magnetic field of that current will tend to stop the movement. This application of the law of conservation of energy is known as **Lenz's law** and it is the reason why there is a negative sign in Faraday's law:

$$\varepsilon = -N\frac{\Delta \Phi}{\Delta t}$$

If the switch in Figure D4.9 is opened, there will still be an induced emf, but no current can flow. This means that there will be no magnetic field created and no force from the coil.



■ **Figure D4.27** Magnet falling through a copper tube

Lenz's law

Lenz's law states that the direction of any induced emf (and current) is always such that it will oppose the change that produced it.

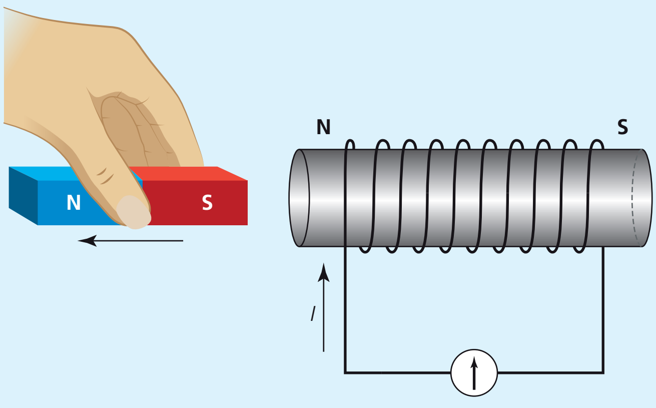
That is, an induced emf will be in such a direction that any induced current will set up a magnetic field that resists the change.

Dropping a bar magnet through a vertical copper tube makes an interesting demonstration of Lenz's law. See Figure D4.27. The physics involved in this demonstration is similar to that shown in Figure D4.10.

As the magnetic field surrounding the falling magnet cuts through the copper tube, eddy currents are created. These currents produce their own magnetic fields which oppose the motion of the falling magnet. As a result, the magnet takes a surprisingly long time to reach the bottom of the tube.

27 Explain what positive and negative values of emfs represent.

28 Figure D4.28 shows what happens when a bar magnet is being moved away from a solenoid connected in a complete circuit. A galvanometer shows that a current *I* is flowing at that moment.



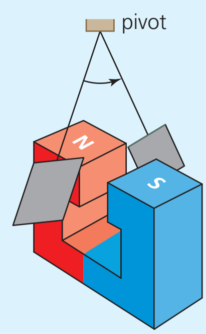
■ **Figure D4.28** What happens when a bar magnet is being moved away from a solenoid connected in a complete circuit?

a Draw a similar diagram to represent what happens when the magnet is moved towards the solenoid.

b Under these circumstances, the coil is acting like an electromagnet. Make up rules to help another student predict how the polarity of the electromagnet is related to the motion of the magnet and the direction of the current.

29 Consider again a magnet falling through a metal tube (Figure D4.27).
Describe and explain what difference it would make if the copper tube was replaced by an aluminium tube of similar dimensions. (Aluminium has a higher resistivity than copper.)

30 Figure D4.29 shows an aluminium plate swinging as a pendulum through a magnetic field.
Sketch an appropriate graph to represent three complete oscillations.



■ **Figure D4.29** An aluminium plate swinging as a pendulum

HL ONLY

ATL D4C RESEARCH SKILLS

Research into the uses and advantages of electromagnetic induction in 'regenerative braking'.

Top tip!

In Question 30 there is no quantitative data provided, so there is no need to include any numbers on the sketch. The quantities being represented should be shown on the axes, preferably in words, although standard symbols are acceptable. Any important features of the graph should be labelled.

TOK

The natural sciences

- Does the precision of the language used in the natural sciences successfully eliminate all ambiguity?

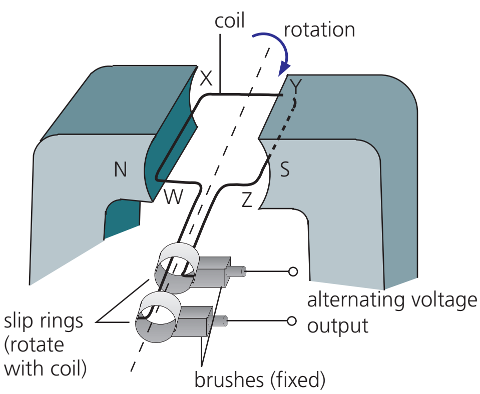
The terminology used in physics can often confuse people who have not studied the subject. The theory of electromagnetic induction is a good example. Does the use of specialized terminology make communicating scientific concepts to the public more difficult?

Faraday's law states that 'an induced emf is equal to the rate of change of magnetic flux linkage'. You should appreciate that this is an elegant and precise way of expressing a very important concept. However, to many non-scientists it may seem like a foreign language. Scientists aim to express ideas as briefly and as succinctly as possible, especially when communicating with other scientists. This involves the use of precise scientific terminology, including the introduction of new words for new ideas, or perhaps the use of common words in precise scientific ways.

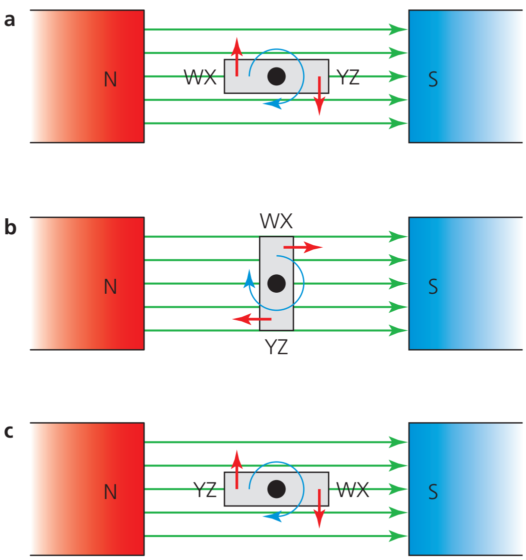
It is certainly possible to write an explanation of Faraday's law without using the phrases induced emf, rate of change and magnetic flux linkage, but it might require several pages instead of one line.

LINKING QUESTION

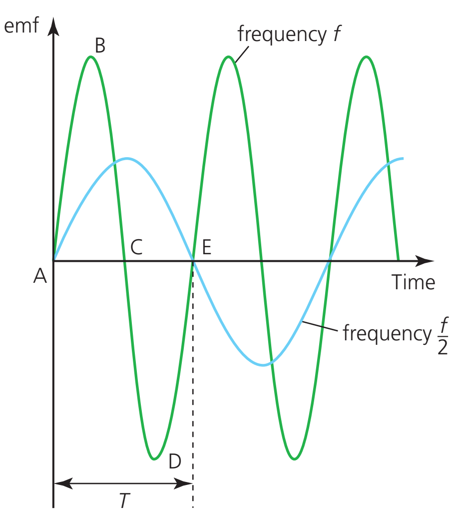
- Faraday's law of induction includes a rate of change. Which other areas of physics relate to rates of change? (NOS)



■ **Figure D4.30** A simple ac generator



■ **Figure D4.31** The sides of the coil cut through the magnetic field at different angles as they rotate, alternating the emf produced



■ **Figure D4.32** Comparing induced emfs at different frequencies

◆ **Slip rings and brushes** In an ac generator these are used for connecting the rotating coil to the external circuit.

Electromagnetic induction in rotating coils

SYLLABUS CONTENT

- A uniform magnetic field induces a sinusoidal varying emf in a coil rotating within it.
- The effect on induced emf caused by changing the frequency of rotation.

Electromagnetic induction is used to generate most of the world's electrical energy.

Alternating current (ac) generators

Consider Figure D4.30, which shows a coil of wire between the poles of a magnet. For simplicity, only one loop is shown, but in practice there will be a large number of turns on the coil(s) of any practical ac generators.

If the coil is rotating, there will be a changing magnetic flux passing through it and a changing emf will be induced. As side WX moves upwards, the induced emf will make a current flow into the page, if it is connected in a circuit. At the same time any induced current in YZ will flow out of the page, because it is moving in the opposite direction. In this way, the current flows continuously around the coil.

The emfs induced in opposite sides of a coil rotating in a magnetic field act in series to drive a current around the coil.

The connection between the coil and the external circuit cannot be fixed and permanent because the wires would become twisted as the coil rotated. Therefore carbon 'brushes' are used to make the electric contact with **slip rings** which rotate with the coil, so that the induced current can flow into an external circuit.

Figure D4.31 shows three views of the rotating coil from the side. In Figure D4.31a the plane of the coil is parallel to the magnetic field and, at that moment, the sides WX and YZ are cutting across the magnetic field at the fastest rate, so this is when the maximum emf is induced. In Figure D4.31b the sides WX and YZ are moving parallel to the magnetic field, so no emf is induced at that moment. In Figure D4.31c the induced emf is a maximum again, but the direction is reversed because the sides are moving in the opposite direction to Figure D4.31a.

The overall result, if the coil rotates at a constant speed in a uniform magnetic field, is to induce an emf that varies *sinusoidally*. This is shown by the green line in Figure D4.32.

In positions B and D the plane of the coil is parallel to the magnetic field. At A, C and E the plane of the coil is perpendicular to the field. One complete revolution occurs in time T. Frequency, f, equals $1/T$. If the coil rotates at a slower frequency (fewer rotations every second), then there will be a smaller rate of change of magnetic flux through it and a smaller emf will be induced. For example, halving the frequency will halve the rate of change of magnetic flux linkage and therefore halve the induced emf. The time period is doubled. This is represented by the blue line in Figure D4.32. You should watch a computer simulation of an ac generator as the coil(s) rotates slowly to help your understanding.

Throughout the world, electrical energy is generated in this way using **turbines** with ac generators. A turbine is a device which transfers the kinetic energy of a moving fluid into useful rotation. Turbine blades can be made to rotate by, for example, forces from the wind, or from high-pressure steam produced from burning fossil fuels or nuclear reactions, or from falling water (hydroelectricity). See Figures D4.33 and D4.34.

◆ **Turbine** Device that transfers the energy from a moving fluid to do mechanical work and cause (or maintain) rotation.

◆ **Alternator** ac electrical generator.

In order to generate electricity, turbine blades are attached to the coils inside an ac generator. The coils – with many turns and cores with high magnetic permeability – are rotated in strong magnetic fields by the action of the turbine blades. Electricity can also be generated using the same principle, but with the magnetic field rotating inside the circuit, rather than the other way around. Such devices are commonly used in cars and they are often called **alternators**. Note that dc generators can be similar to ac generators in basic design, but the connections to the external circuit need to be modified.

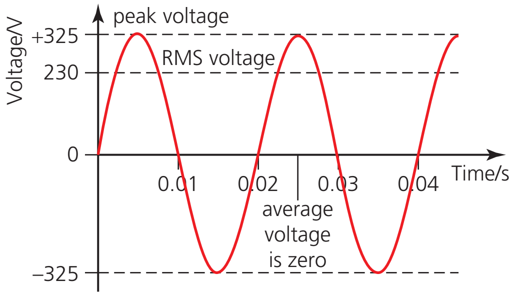
Mains electricity (also known as *utility power*) is the name given to the electrical power supply that is delivered from large power stations to homes and businesses. Figure D4.35 shows the symbol for an ac power supply.

In most of the world the electricity power supply is rated at 230 V ac 50 Hz. This is shown in Figure D4.36. 120 V 60 Hz is common in North America. There are a range of different designs of sockets and plugs used in different countries.

■ **Figure D4.33** An engineer working on a steam turbine

■ **Figure D4.34** Wind turbine blades are set in motion by the flow of wind past them

■ **Figure D4.35** Symbol for an ac power supply



■ **Figure D4.36** Mains voltage rated at 230 V

An ac electrical supply which is rated at 230 V actually varies between peaks of +325 V and −325 V. The average value is zero, but that is not really useful information. The supply is rated at 230 V because it delivers the same power as a steady 230 V dc would in the same circuit. (The effective voltage is described as the *RMS voltage* – root mean squared voltage. RMS voltage = peak voltage divided by $\sqrt{2}$. But you are *not* expected to remember this.)

Two wires are required to make a connection from the mains and deliver electrical energy to a circuit. One connection, called the *neutral wire*, is always kept at 0 V, while a varying voltage, as seen in Figure D4.36, drives a current backwards and forwards. This connection is commonly called the *live wire*.

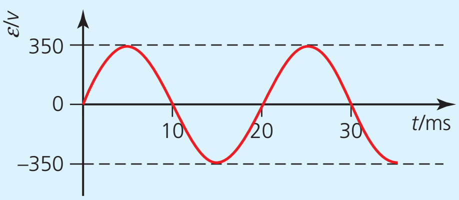
LINKING QUESTION

- How is the efficiency of electricity generation dependent on the source of energy?

This question links to understandings in Topic B.4.

31 Figure D4.37 shows the output of an ac generator.

- **a** Determine the frequency of the output.
- **b** Estimate the approximate voltage rating of the supply.
- **c** Make a copy of Figure D4.37 and add a curve to represent the output if the frequency of the generator was reduced to 25 Hz.



■ **Figure D4.37** Output of an a.c. generator

32 a Sketch a voltage–time graph, with numerical values, for a 120 V 60 Hz rated power supply.

- **b** Estimate the maximum voltage in each cycle of the supply.

33 A rectangular coil of copper wire has 500 turns and sides of length 5.2 cm and 8.7 cm. The coil rotates at a constant frequency of 24 Hz about an axis that passes centrally through the shorter sides. A uniform magnetic field of 0.58 T acts on the coil in a direction perpendicular to the axis.

- **a** Draw a labelled sketch of this arrangement.
- **b** Calculate the linear speed of the sides of the coil.
- **c** Use $\varepsilon = NBvL$ to determine the emf that is induced across one of the longer sides at the instant that it is moving perpendicularly across the field.
- **d** What are the maximum and minimum induced emfs as the coil rotates?

34 What are the voltages on the live and neutral wires in your home?

35 Outline why some electrical sockets have three connections (rather than two).

Self-induction

When a current in any circuit changes, the magnetic flux associated with that current must also change. This means that an emf will be induced. So far, we have discussed induction in a *separate* circuit, but induction also occurs within the *same* circuit and then the induced emf opposes the change of current (Lenz's law), so that it acts in the reverse direction to the original emf producing the current. It is often called a **back-emf**.

◆ **Back-emf** An induced potential difference that opposes a change of current in the same circuit.

◆ **Self-induction** Electromagnetic induction within a single circuit.

In most simple circuits this effect will not be noticeable or important, but if a many-turned coil is involved, especially if it is wound on a core of high magnetic permeability, the effect will become significant when dealing with alternating currents. It is called **self-induction**.

Self-induction is the effect in which a change in the current in a circuit tends to produce an induced emf which opposes the change of current *in the same circuit*.

■ **Figure D4.38** Treasure hunting

Self-induction becomes more important at higher frequencies, which usually involve greater values of $\Delta I/\Delta t$, and so greater rates of changing magnetic flux.

The magnitude of self-induction (or mutual induction) effects will change with the magnetic properties of different surrounding materials (especially metals). This has some interesting applications, including:

- The presence of cars waiting at traffic lights can be detected by changes to the self-induction of coils under the road surface.
- Metal detectors (for example at airport security checks) can use changes in self-inductance to identify the presence of metals. See Figure D4.38 for a similar application.

E.1 Structure of the atom

Guiding questions

- What is the current understanding of the nature of an atom?
- What is the role of evidence in the development of models of the atom?
- In what ways are previous models of the atom still valid despite recent advances in understanding?

The nuclear model of the atom

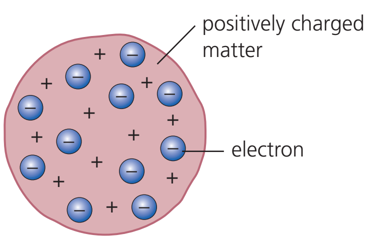
We have already briefly described the nuclear model of the atom, at the beginning of Topic B.5. As a reminder, Figure E1.9 shows another visualization of this model.

We now want to use knowledge about electric forces (Topic D.2) to explain how and why the *nuclear* model of the atom was first developed.

◆ **Subatomic particle** Any particle contained within an atom.

At the end of the nineteenth century, it had become clear that the atom was not an elementary, indivisible particle. However, only one **subatomic particle**, the negatively charged electron, had been identified, although it was not known for sure how many electrons were in each atom. Since atoms were not charged overall, it was clear that there must also be some part of atoms which were positively charged. This led to a model of the atom often described as the 'plum pudding model' (J.J. Thomson in 1904), see Figure E1.1.

Today, we may be more inclined to use a blueberry muffin visualization (Figure E1.2): an unknown number of individual blueberries (negative electrons) in a muffin of spread-out positive charge.



■ **Figure E1.1** The 'plum pudding model' of the atom

■ **Figure E1.2** Blueberry muffin

This model of the atom was far from being satisfactory and it raised many questions, but it was to be about seven years before it was improved, following the famous experiments of Geiger, Marsden and Rutherford.

TOK

The natural sciences

'What is everything made of?' is one of the most basic questions we can ask and there are records going back about 2500 years, to Greek philosophers (Democritus and others), asking just that. It was at that time that the concept of the 'atom' was first introduced: as tiny, solid spheres.

Moving forward about 2200 years, scientists of that time still had much the same ideas. The following is a quote from the famous physicist Isaac Newton in 1704.

'All these things being considered, it seems probable to me that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, moveable Particles, of such Sizes and Figures, and with such other Properties, and in such Proportion to Space, as most conduced to the End for which he form'd them; and that these primitive Particles being Solids, are incomparably harder than any porous Bodies compounded of them; even so very hard, as never to wear or break in pieces; no ordinary Power being able to divide what God himself made one in the first Creation.'

A lot has changed in the following three centuries. Chemical reactions were explained by the elements having different atoms which could be combined to form molecules (Dalton and others). However, the concept of the indivisible atom remained until the discovery (1897) of a constituent particle: the electron.

Rutherford's proposal of a nuclear atom (and the existence of protons and neutrons), as explained below, was another paradigm shift in models of the atom, but not the last. The discovery of the wave properties of electrons (1924) meant that the model had to be significantly changed again.

◆ **Geiger–Marsden–Rutherford experiment** The scattering of alpha particles by a thin sheet of gold foil, which demonstrated that atoms consist of mostly empty space with a very dense positively charged core (the nucleus).

◆ **Alpha particle** A positively charged particle emitted by a radioactive nucleus.

◆ **Radioactive source** Radioactive substance used for the nuclear radiation it emits.

Geiger–Marsden–Rutherford experiment

SYLLABUS CONTENT

▶ The Geiger–Marsden–Rutherford experiment and the discovery of the nucleus.

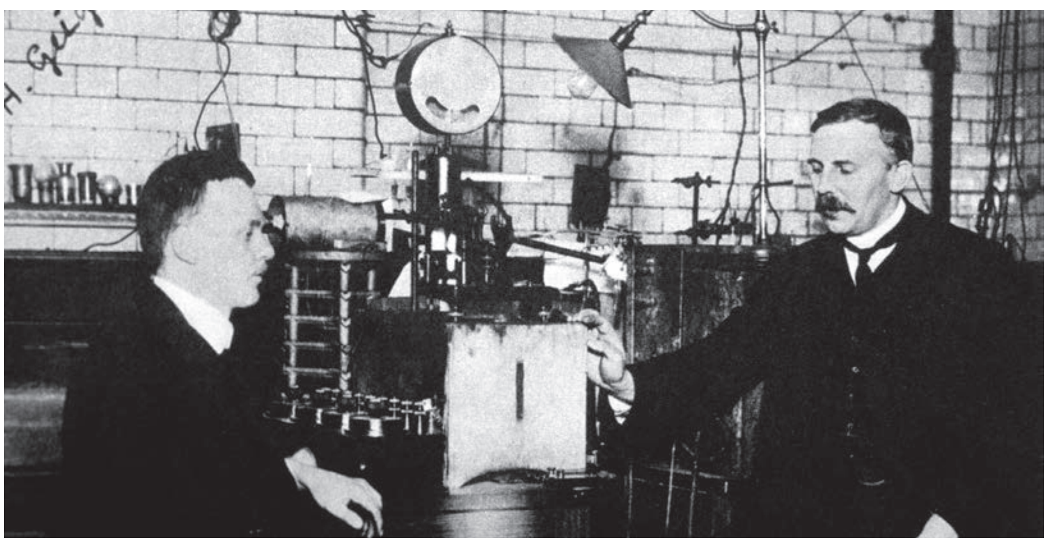


Figure E1.3 Geiger and Rutherford (right)

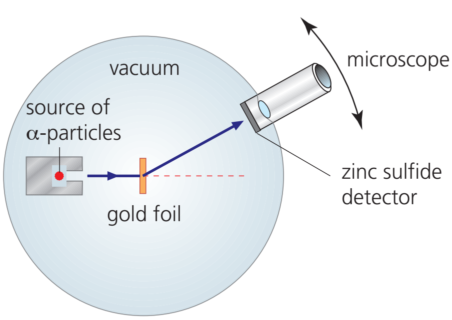


Figure E1.4 The alpha particle scattering experiment

In 1909, Ernest Rutherford and two of his research students, Hans Geiger and Ernest Marsden, working at the University of Manchester, UK, directed a narrow beam of positively charged **alpha particles** (see below) from a **radioactive source** at very thin gold foil. A zinc sulfide detector was moved in a circle around the foil to determine the directions in which alpha particles travelled after striking the foil (Figure E1.4). The alpha particles had enough energy individually to be detected by a tiny flash of light when they were stopped by the zinc sulfide.

Inquiry 1: Exploring and Designing

Exploring

What are alpha particles?

The nuclei of some atoms are unstable and they can emit particles and electromagnetic radiation. This is called **radioactivity** and the subject is covered in Topic E.3. Alpha particles are a common product of radioactivity and they are often used in school demonstrations. Research using a variety of relevant sources to find out:

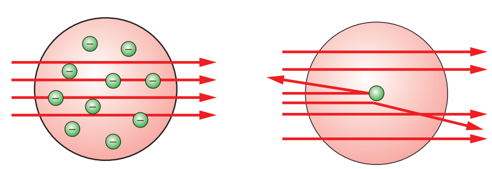
1 What subatomic particles are alpha particles made from?
2 What is the overall electrical charge of an alpha particle?

◆ **Radioactive decay (radioactivity)** Spontaneous emission of particles and / or radiation from unstable nuclei.

Alpha particles carry a *very* large amount of energy relative to their small size and, at that time (1909), it was expected that the alpha particles would not be affected much by passing through such thin gold. Gold foil can be made very thin (less than 10^{-6} m) and the foil may then only have about 6000 layers of atoms. Although alpha particles only travel about 4 cm in air, they would encounter many more molecules travelling that distance in air than passing through gold atoms in very thin foil.

But Geiger and Marsden's results were surprising. Rutherford published the results in 1911. He reported that:

- Most of the alpha particles passed through the foil with very little or no deviation from their original path (as was expected).
- A small number of particles (about 1 in 1800) were deviated through an angle of more than about 10° (see Figure E1.5).
- An extremely small number of particles (about 1 in 10 000) were deflected through an angle larger than 90°. Some particles were even deflected by 180°, returning in the direction from which they came.



■ **Figure E1.5** Alpha scattering experiments produced unexpected results

The importance of this last point is emphasized by Rutherford's famous quote: *'It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell [large bullet] at a piece of tissue paper and it came back and hit you.'*

From alpha particle scattering experimental results Rutherford drew the following conclusions:

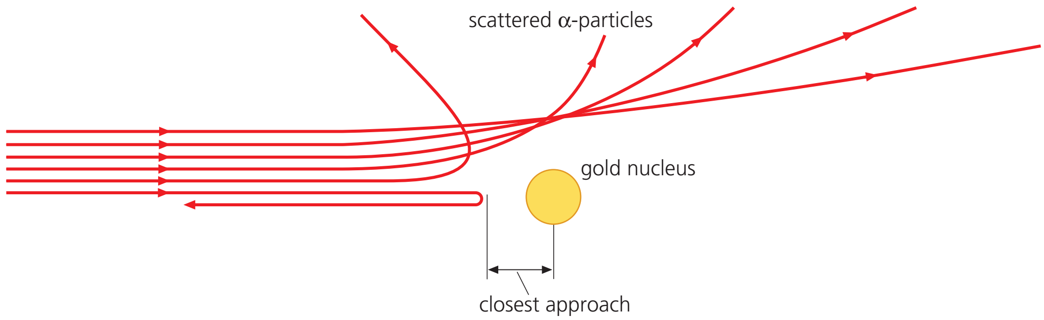
Most of the mass of an atom is concentrated in a very small volume at the centre of the atom. Most alpha articles would therefore pass through the foil undeviated (continuing in a straight line) because most of the atom was empty space.

The centre of an atom (he called it the nucleus) must be positively charged in order to repel the positively charged alpha particles. Alpha particles that pass close to a nucleus will experience a strong electrostatic repulsive force, causing them to change direction.

Only alpha particles that pass very close to the nucleus, striking, or almost striking it directly, will experience electrostatic repulsion large enough to cause them to deviate through large angles. The fact that so few particles did so, confirms that the nucleus is very small, and that most of the atom is empty space.

Figure E1.6 shows some of the possible trajectories (paths) of the alpha particles. Rutherford used his new nuclear model of the atom and Coulomb's inverse square law (covered in Topic D.2) to explain the repulsive force between the positively charged particles. He used the magnitudes of the forces to calculate the fraction of alpha particles expected to be deviated through various angles.

Rutherford's calculations agreed very closely with the results from the experiment, supporting his proposal of a nuclear model of the atom.



■ **Figure E1.6** Alpha particle trajectories in the gold foil experiment

■ **Figure E1.7** Alpha particle scattering analogue

Alpha particle scattering can be modelled using simple apparatus such as that shown in Figure E1.7, in which a small ball rolls down a wooden ramp onto a specially shaped metal 'hill'. The shape of the hill is made so that, when viewed from above, the ball moves as if it was being repelled from the centre of the hill by an inverse square law of repulsion. In other words, gravitational forces are used to model electric forces. Using this apparatus, it is possible to investigate how the direction in which a ball travels after leaving the hill (the scattering angle) depends on its initial direction ('aiming error') and/or its energy. In Geiger and Marsden's experiment it was not possible to observe the scattering paths of individual alpha particles, but the observed scattering pattern of large numbers of alpha particles in a beam is found to be in very close agreement with modelling based on individual balls rolling on hills.

From his results, Rutherford calculated that the diameter of the gold nucleus was of the order of 10^{-14} m, compared to the diameter of the whole atom, which was known to be of the order of 10^{-10} m.

LINKING QUESTION

- How have observations led to developments in the model of the atom? (NOS)

1 Explain in your own words (less than 100), without using a diagram, why Rutherford concluded that atoms contain a small, positively charged central nucleus.

2 Sketch the path of an alpha particle being scattering by a large angle by a positive nucleus. Label the 'aiming error' and the 'scattering angle'.

3 'Gold foil can be made very thin (less than 1×10^{-6} m) and the foil may then only have about 6000 layers of atoms.'
   a Use this information to determine an approximate radius of a gold atom.
   b State any assumptions that you made in answering part **a**.

4 a Calculate the forces acting between an alpha particle and a nucleus of charge $+81e$ when they are at their closest, separated by a distance of 2.0×10^{-14} m.
   b If the alpha particle was scattered by an angle of about 40°, sketch its path, showing the forces that you calculated in part **a**.

5 Suggest what would have happened if neutrons had been used in Rutherford's experiment instead of alpha particles. Explain your answer.

WORKED EXAMPLE E1.1

a Make a sketch of an alpha particle being deflected through about 120° by a gold nucleus.

b On the same sketch draw the path of an alpha particle (approaching along the same path as before) being scattered by a copper nucleus (which has a much smaller charge).

c Show how an alpha particle of higher energy could be affected if it approached a gold nucleus along the same path.

Answer

a

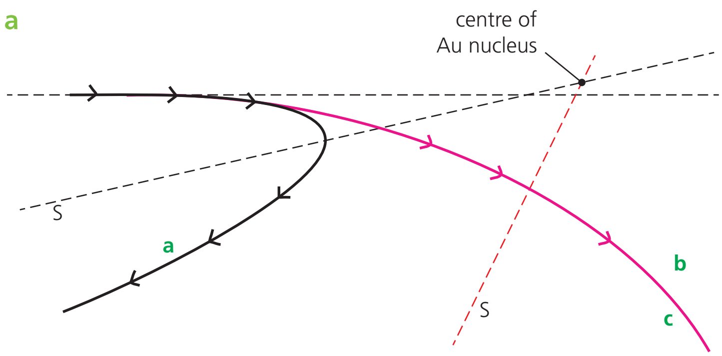


Figure E1.8 Answer to Worked example E1.1

The repulsive force from a copper nucleus is less than from a gold nucleus, so the alpha particle is scattered less. An alpha particle of greater energy will be scattered through a smaller angle than an alpha particle of less energy. For convenience, **b** and **c** have been shown with similar deflection, but that is unlikely. All paths are symmetrical about the dashed lines labelled S.

Composition of the nucleus

In the years that followed Rutherford's famous experiment, it was confirmed that a nucleus consists of *separate subatomic particles: protons and neutrons*, which contain almost all of the mass of the atom. The protons are positively charged and the neutrons are electrically neutral. The electrons are negatively charged but have very little mass in comparison to protons and neutrons. Atoms are electrically neutral because there are equal numbers of protons and electrons.

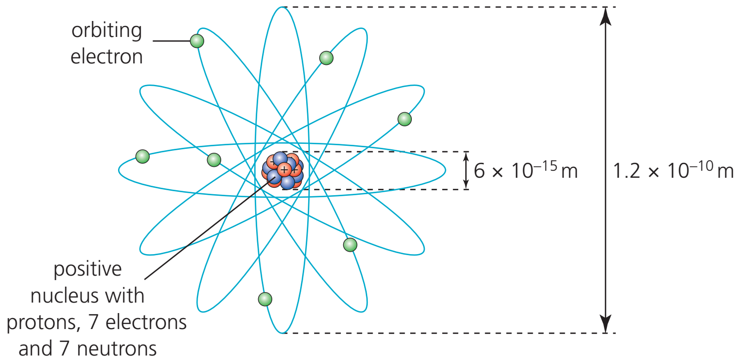
In this model of the atom, the electrons orbit the nucleus because of the centripetal force provided by the electrical attraction between opposite charges. If the electrons were not in circular motion, the electrostatic force of attraction would accelerate them towards the nucleus.

The vast majority of an atom is empty space, a vacuum. The properties of protons, neutrons and electrons are summarized in Table E1.1.

Table E1.1 Properties of subatomic particles

Name of particle	Approximate relative mass	Relative charge
proton	1	+1
neutron	1	0
electron	1/1840	−1

Figure E1.9 shows the features of the nuclear model of an atom as visualized in the years immediately following Rutherford's discovery. The example shown is a nitrogen atom. Although this model has been changed in important ways, some of which are explained below, it persists in popular culture and it continues to be a very useful starting point in the study of atomic structure in elementary science lessons.



■ **Figure E1.9** The orbital model of a nitrogen atom (not to scale)

Nature of science: Models

Power of visual models

The latest models of the atom cannot be drawn on paper or a computer screen. Instead, we need complex mathematical models (equations) to describe what we cannot see. (These are *not* included in the IB course.) It is unlikely that scientists will ever produce an accurate model of the atom that can be visualized, but we should have no expectation that the atomic-scale world behaves in any way similar to the world we see around us.

The visual model seen in Figure E1.9 can be understood to some extent by many people, but the complex mathematical modelling needed to explain the latest theories about the structure of matter will continue to be inaccessible to most people.

What holds the particles in the nucleus together?

After it was proposed that the nucleus was composed of separate particles (protons and neutrons), there was an obvious question to ask: what forces are there between these particles that holds them so closely together? In particular, it was known that there is a very large *repulsive* force between protons, as the following approximation demonstrates:

$$F = k\frac{q_1 q_2}{r^2}$$

(from Topic D.2)

$$F \approx \frac{(8.99 \times 10^{9}) \times (1.60 \times 10^{-19})^2}{(6 \times 10^{-15})^2} \approx 10\,\text{N (to an order of magnitude)}$$

which, on the atomic scale, is an extremely large force.

To oppose this repulsive force, we now know that there is a very *short-range* **strong nuclear force** (attractive) between the nuclear particles. However, detailed knowledge of this force is not required in the IB course.

The term **nucleon** is used to describe a particle in the nucleus of an atom which is either a proton or a neutron.

◆ **Strong nuclear force** Fundamental force that is responsible for attracting nucleons together. It is a short-range attractive force (the range is about 10^{-15} m), but for smaller distances it is repulsive, and hence it also prevents a nucleus from collapsing.

◆ **Nucleon** A particle in a nucleus, either a neutron or proton.

ATL E1A: Research skills

Use search engines and libraries effectively

For many years (until the 1960s) it was believed that protons and neutrons were **elementary particles**, meaning that it was thought that they were not composed of smaller particles. Scientists now know that protons and neutrons are *composite particles*, each consisting of three *quarks* with the strong nuclear force holding them together. You are not expected to have knowledge of quarks for examinations.

There are 17 known elementary particles, which can be arranged into four groups. Use the internet to find out the names of these particles. Do you find it surprising that there are 17? Explain your answer.

◆ **Elementary particles** Particles that have no internal structure. They are not composed of other particles. For example, electrons.

◆ **Proton number, *Z*** The number of protons in a nucleus.

◆ **Nucleon number, *A*** The total number of protons and neutrons in a nucleus.

◆ **Neutron number, *N*** The number of neutrons in a nucleus.

The number of protons in the nucleus of an atom determines which element it is. So, atoms of a particular element are identified by their **proton number** (sometimes called *atomic number*), which is given the symbol Z. The *periodic table* of the elements arranges the elements in order of increasing proton number.

The proton number, Z, is the number of protons in the nucleus of an atom.

Because atoms are electrically neutral, the number of protons must be equal to the number of electrons in the space around the nucleus. If electrons are added or removed from an atom, it is then described as an *ion* (of the same element).

The **nucleon number**, A, is defined as the total number of protons and neutrons in a nucleus.

The nucleon number represents the mass of an atom, because the mass of the electrons is (almost) negligible. (Nucleon number is sometimes referred to as the *mass number*.)

The difference between the nucleon number and the proton number gives the number of neutrons in the nucleus: $N = A - Z$.

The **neutron number**, N, is defined as the number of neutrons in a nucleus.

The number of neutrons in a nucleus is similar to the number of protons, although the ratio of the number of neutrons / number of protons generally increases with increasing proton number. (See Figure E3.22 in Topic E.3). As we will see, this ratio is an important factor when considering the stability of nuclei.

Nuclear notation

SYLLABUS CONTENT

▶ Nuclear notation $^{A}_{Z}X$ where A is the nucleon number Z is the proton number and X is the chemical symbol.

◆ **Nuclide** Term used to identify one particular species (type) of atom, as defined by the structure of its nucleus. A *radionuclide* is unstable and will emit radiation.

The term **nuclide** is used to specify one particular species (type) of atom, as defined by the structure of its nucleus.

All atoms with the same nucleon number and the same proton number are described as the same nuclide.

There is a standard notation used to represent a nuclide by identifying its proton number and nucleon number, as shown in Figure E1.10, which uses C-12 as an example.

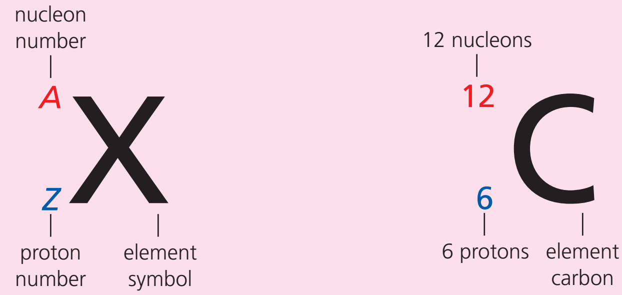


Figure E1.10 Standard notation for specifying a nuclide

Isotopes

◆ **Isotope** One of two or more atoms of the same element with different numbers of neutrons (and therefore different masses). A *radioisotope* is unstable and will emit radiation.

Two or more nuclides with the same proton number may have different numbers of neutrons. The atoms are of the same element and have identical chemical properties, but they have different nucleon numbers. These atoms are called **isotopes**.

An isotope is one of two or more different nuclides of the same element (which have the same number of protons, but different nucleon numbers).

Some elements have many isotopes, but others have few or even one. For example, the most common isotope of hydrogen is hydrogen-1, ${}^{1}_{1}H$. Its nucleus is a single proton.

Hydrogen-2, ${}^{2}_{1}H$, is called deuterium; its nucleus contains one proton and one neutron.

Hydrogen-3, ${}^{3}_{1}H$, with one proton and two neutrons, is called tritium.

Hydrogen isotopes (Figure E1.11) are involved in nuclear fusion reactions (see Topic E.5).

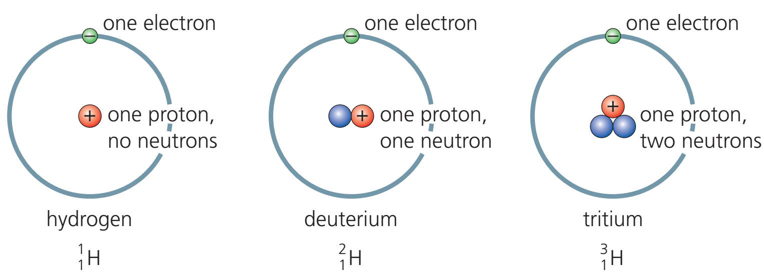


Figure E1.11 The three isotopes of hydrogen

As a further example, the following nuclides are three isotopes of carbon:

- ${}^{12}_{6}C$ (six protons, six neutrons)
- ${}^{13}_{6}C$ (six protons, seven neutrons)
- ${}^{14}_{6}C$ (six protons, eight neutrons).

◆ **Diffusion** Random movement of particles from a place of high concentration to places of lower concentration.

Samples of elements are often mixtures of isotopes. Isotopes cannot be separated by chemical means. Separation can only be achieved by processes that depend on the difference in masses of the isotopes, for example the **diffusion** rate of gaseous compounds.

The notation for describing nuclides can also be applied to the nucleons. For example, a proton can be written as ${}^{1}_{1}p$ and a neutron as ${}^{1}_{0}n$. An electron's charge is −1 compared to the +1 charge on a proton, so an electron can be represented by ${}^{0}_{-1}e$, remembering that the mass (number) of the electron is effectively zero compared to the proton and neutron.

WORKED EXAMPLE E1.2

A certain element has the proton number 17.

a Research which element this is.

b Suggest the nucleon numbers of two possible isotopes of this element.

c Chemists may say that the atomic mass (weight) of this element is 35.5.
Explain what this number represents.

Answer

a Chlorine

b Chlorine has a large number of isotopes, each with a different nucleon number. We know that the number of neutrons is approximately equal to the number of protons, so $A = 33, 34, 35, 36, 37$ and so on are all reasonable guesses. (In fact, the two most common are chlorine-35 and chlorine-37.)

c 35.5 represents the *average* number of a large number of nucleons in a sample of chlorine (with a mixture of isotopes).

6 Explain the differences between an atom, a nuclide and an isotope.

7 The nuclides $^{129}_{53}I$, $^{137}_{55}Cs$ and $^{90}_{38}Sr$ were all formed during atomic weapons testing more than 40 years ago. State the number of neutrons, protons and electrons in the atoms of these nuclides.

8 State the electric charge of the nucleus $^{4}_{2}He$.

9 The number of electrons, protons and neutrons in an ion of sulfur, S, are equal to 18, 16 and 16, respectively. What is the correct nuclide symbol for this sulfur ion?

10 State the number of nucleons in one carbon-13 atom, $^{13}_{6}C$.

11 Chlorine, Cl, is an element that has 17 protons in its nucleus. The two most common isotopes of chlorine are chlorine-35 and chlorine-37. Write down the nuclide symbols for these two isotopes.

12 U-238 and U-235 are the two most common isotopes found in uranium ore. The more massive isotope has 146 neutrons in its nucleus.

a Write down the nuclide symbols for these two isotopes.

b Explain why it is difficult to separate these isotopes from each other.

Energy levels within atoms

SYLLABUS CONTENT

- Emission and absorption spectra provide evidence for discrete atomic energy levels.
- Emission and absorption spectra provide information on the chemical composition.

The total energy of an atom, such as the nitrogen atom represented in Figure E1.9, may be considered to be the sum of the kinetic energies of the electrons plus the electric potential energy of the system of negatively charged electrons moving around the positively charged nucleus (assumed to be stationary). Energies *within* the nucleus are for another discussion and they are not included in this topic.

Comparing this simplified model of an atom to a gravitational model of satellites orbiting a planet (Topic D.1), we define our zero of electric potential energy in the same way: at infinity.

The energies of electrons within atoms are given negative values because we would have to supply energy to remove them from the atom (to infinity), where they would then be considered to have zero electric potential energy.

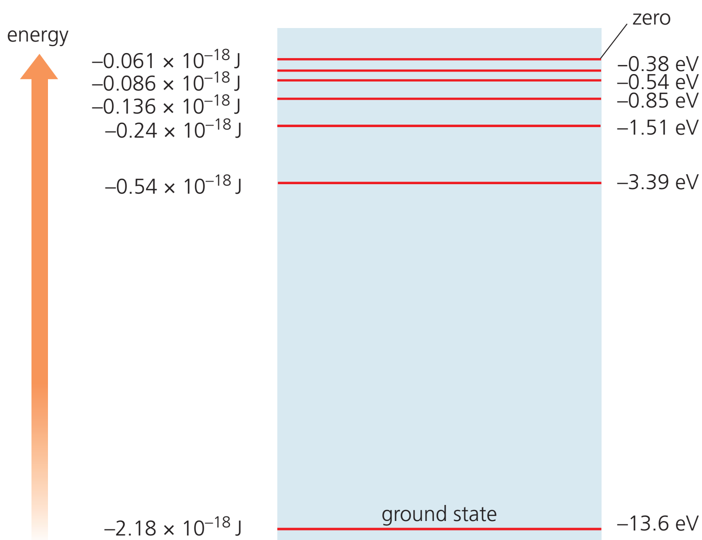
◆ **Atomic energy level** One of a series of possible discrete (separate) energy levels of an electron within an atom.

◆ **Ground state** The lowest energy state of an atom / electron (or nucleus).

◆ **Ionization energy** Amount of energy needed to remove an electron from an atom or molecule.

The orbiting electrons model of an atom has its uses, but is a long way from the whole truth, as we shall explain. Most significantly, orbiting satellites can, in principle, orbit at *any* height, with a *continuous* range of possible energies, but electron energies are very different: they can only have one of a range of very precise values. That is, their possible energies are *discrete*, *quantized*. We refer to the possible energies as **atomic energy levels**.

Figure E1.12 shows the simplest example: possible energy levels (of an electron) within the simplest atom, hydrogen. These levels will be discussed in detail later.



■ **Figure E1.12** The energy levels of the hydrogen atom

The following points should be noted about this important diagram:

- The energy levels are drawn to scale vertically, but the shape of this diagram has no physical meaning.
- The **ground state** is the lowest possible energy level. An electron in the ground state of any atom is the most difficult to remove. Atoms are usually in their ground states.
- *All* energies are negative (as explained above).
- The highest energy level shown is equivalent to removing the electron from the atom (to infinity, where it would then have zero energy – if it were not moving). That is, the **ionization energy** of hydrogen atoms is 13.6 eV (2.18×10^{-18} J).
- The energy levels have been given in both joules and electronvolts. Electron volts are widely used for atomic-scale energies.

Top tip!

A reminder: 1 eV (electronvolt) is the amount of energy transferred when unit charge e is accelerated, or decelerated, by a potential difference of one volt. ($W = qV$; $1\,\text{eV} = 1.6 \times 10^{-19}$ J) The electronvolt is a common unit of energy used throughout atomic physics, not just for accelerated charges.

13 Explain what it means if we say that the (first) ionization energy of an atom is 4.0×10^{-18} J.

14 Show that an energy of -0.136×10^{-18} J is equivalent to −0.85 eV (as shown in Figure E1.12).

15 110 eV is required to ionize an atom in its ground state. The four lowest energy levels above the ground state are −70 eV, −40 eV, −20 eV and −10 eV.

  a Draw an energy level diagram for this atom.

  b Determine the number of different transitions possible between these five levels.

We need to *explain* how physicists discovered that atoms had discrete energy levels. The evidence came from examining in detail the light that is emitted from (or absorbed by) atoms.

Evidence for energy levels within atoms

Emission and absorption spectra

Light is electromagnetic radiation that has been emitted from atoms. We can learn a lot about the energy inside atoms by examining the radiation (spectra) that atoms emit.

When elements (in the form of gases) are **excited** – given enough energy (by heating, or by an electrical current at high voltage), the spectra of the light that they *emit* are seen as a series of bright lines on black backgrounds– called *line spectra*. See Figure E1.13. Each line corresponds to a precise frequency. (In this Figure, a prism has been used to disperse the light. Alternatively, a diffraction grating could be used, as discussed in Topic C.3 for HL students.)

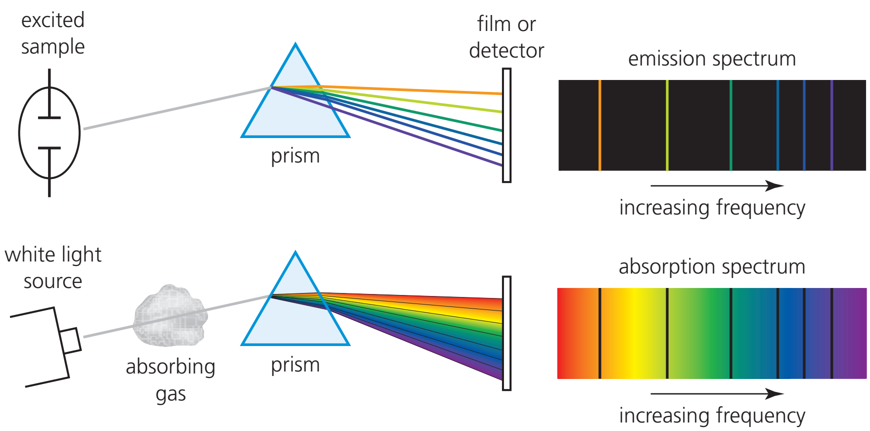


Figure E1.13 Emission and absorption spectra of the same element

When a continuous spectrum passes through a gas, the atoms in the gas will *absorb* the same frequencies as they would emit when given energy. This results in a spectrum with black absorption lines, also as seen in Figure E1.13 (lower diagram.) The atoms re-emit the energy, but in random directions.

◆ **Excitation** The addition of energy to a particle, changing it from its ground state to an excited state.

◆ **Emission spectrum** Line spectrum associated with the emission of electromagnetic radiation by atoms, resulting from electron transitions from higher to lower energy states.

◆ **Absorption spectrum** A series of dark lines across a continuous spectrum produced when white light passes through a gas at low pressure.

Each line on an *emission spectrum* is explained by electrons moving to a lower energy level within the atom. Each line on an *absorption spectrum* is explained by electrons moving to a higher energy level within the atom.

Since the energy levels of atoms of different elements are different, emission and absorption spectra can be used to identify the elements involved.

Inquiry 2: Collecting and processing data

Collecting data

The study of spectra is called (optical) **spectroscopy**, and instruments used to measure the wavelengths of spectra are called **spectrometers** (see Figure E1.14).

Ask your teacher, or otherwise find out, how a spectrometer is used to measure wavelengths of line spectra. Identify what precautions and methods must be taken to ensure accurate readings.

Figure E1.14 A spectrometer

LINKING QUESTION

- How can emission spectra allow for the properties of stars to be deduced?

This question links to understandings in Topic E.5.

Photons

SYLLABUS CONTENT

- Photons are emitted and absorbed during atomic transitions.
- The frequency of the photon released during an atomic translation depends on the difference in energy level as given by: $E = hf$.

◆ **Transition (between energy levels)** A photon is emitted when an atom (or nucleus) makes a transition to a lower energy level. The energy of the photon is equal to the difference in energy of the levels involved.

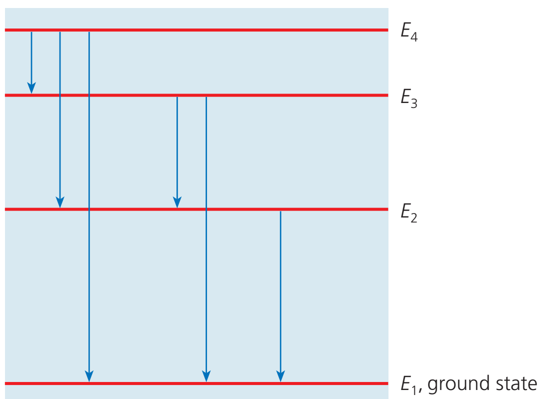
◆ **Photon** A quantum of electromagnetic radiation, with an energy given by $E = hf$.

◆ **Quantum** The minimum amount of a physical quantity that is quantized. Plural: quanta..

◆ **Planck's constant, *h*,** Fundamental constant of quantum physics which connects the energy and frequency of a photon.

The emission, transmission and absorption of light are not continuous processes. They are a very large number of separate events.

When an (excited) atom moves to a lower energy level, it emits an amount of energy equivalent to the *difference* in energy levels. Figure E1.15 shows a simplified example: an atom with four different energy levels that could have six possible energy **transitions between energy levels**.



■ **Figure E1.15** Energy transitions between four energy levels in an atom

If an atom receives electromagnetic energy equal to the difference between its energy level and a higher level, it can move to the higher level when the energy is absorbed. Typically, the energy is then quickly re-emitted as the atom returns to a lower energy level.

The 'bundles' of emitted electromagnetic energy are called **photons**. More generally, the term **quanta** (singular: **quantum**) is used to describe the smallest possible quantity of any entity that can only have discrete values. We can say that light is quantized.

The energy, E, carried by one photon of electromagnetic radiation depends only on its frequency, f, as follows:

$$\boldsymbol{E = hf}$$

h is a very important fundamental constant that controls the properties of electromagnetic radiations.

h is called **Planck's constant**. It has a value of $\mathbf{6.63 \times 10^{-34}\,Js}$.

Since we know from Topic C.2 that $c = f\lambda$, this equation is often rewritten as $E = \dfrac{hc}{\lambda}$.
(It is often convenient to know that $\boldsymbol{hc} = \mathbf{1.99 \times 10^{-25}\,Jm = 1.24 \times 10^{-6}\,eV\,m}$)

Nature of science: Measurement

Fundamental constants

Fundamental constants are the numbers that appear in the equations physicists use to describe the properties of force, mass and energy in the Universe around us (as in, for example, h in $E = hf$). They are believed to have exactly the same value at all places and for all time. Fundamental constants are determined experimentally and are not theoretical.

If any of the values of these constants were different, then the Universe would be very different.

In the IB Physics course, the list of fundamental constants used includes:

- gravitational constant, G
- speed of electromagnetic radiation in free space, c
- electric permittivity of free space, ε_0
- magnetic permeability of free space, μ_0 (connected to ε_0 and c)
- Planck's constant, h
- elementary charge, e

WORKED EXAMPLE E1.3

Calculate the energy carried by one photon of microwaves of wavelength 10 cm (as might be used in a mobile phone):

a in J

b in eV.

Answer

a $E = \frac{hc}{\lambda} = \frac{((6.63 \times 10^{-34}) \times (3.00 \times 10^{8}))}{0.10}$

$E = 2.0 \times 10^{-24}$ J

b $\frac{(1.989 \times 10^{-24})}{(1.60 \times 10^{-19})} = 1.3 \times 10^{-5}$ eV

Figure E1.16 shows the visible emission line spectrum of hydrogen.

Figure E1.16 Lines on the hydrogen spectrum

Taking one line as an example: 434.2 nm

We can use $E = \frac{hc}{\lambda}$ to determine the energy of a photon with this wavelength:

$$E = \frac{hc}{\lambda} = \frac{((6.63 \times 10^{-34}) \times (3.00 \times 10^{8}))}{(434.2 \times 10^{-9})} = 4.58 \times 10^{-19} \text{ J (equal to 2.86 eV)}$$

Referring back to Figure E1.12, we can see that this amount of energy is equivalent to the difference between the energy levels of −0.54 eV and −3.39 eV.

Here, we are using one line of the hydrogen spectrum as an example but, similarly, all spectral lines can be directly related to specific transitions between the discrete energy levels of the atoms of different elements. In practice, the reverse is also true:

spectral lines were used to determine atomic energy levels.

Common mistake

In examination questions when you are asked to *show* that a given value is valid, you should *show* the result of your calculation to more significant figures than given in the question.

WORKED EXAMPLE E1.4

a Show that the frequency of a photon emitted by a transition in the hydrogen atom between its two lowest energy levels is approximately 2×10^{15} Hz.

b State in which part of the electromagnetic spectrum this radiation occurs.

Answer

a Consider Figure E1.12. The transition is from -0.54×10^{-18} J down to -2.18×10^{-18} J = -1.64×10^{-18} J

$E = hf$

$1.64 \times 10 = 6.63 \times 10^{-34} \times f$

$f = 2.5 \times 10^{15}$ Hz

b This frequency is in the ultraviolet part of the spectrum.

16 a Show that when an electron in an energy level of -1.36×10^{-18} J moved to a level of -0.74×10^{-18} J, a photon of energy approximately 4 eV was involved.

b Was the photon emitted or absorbed?

17 a Determine the frequency of electromagnetic radiation which has photons of energy 1.0×10^5 eV.

b State the name we give to that kind of radiation.

18 a A microwave oven uses electromagnetic photons of energy 1.6×10^{-24} J. What is the wavelength of this radiation?

b Use the internet to find out why this wavelength is used.

19 One of the electromagnetic frequencies absorbed by the greenhouse gas carbon dioxide is 1.4×10^{14} Hz.

a Calculate how much energy is carried by the absorbed photons.

b In what part of the electromagnetic spectrum is this radiation?

20 A particular visible line in the spectrum of oxygen has a wavelength of 5.13×10^{-7} m. Determine the energy (eV) transferred by one photon of this radiation.

21 Light has a typical wavelength of 5×10^{-7} m, and X-rays have a typical wavelength of 5×10^{-11} m.

a Draw a small square of sides 2 mm to represent the energy carried by a light photon.

b Assuming that photon energy is represented by the area, draw another square to represent the energy carried by an X-ray photon.

c Suggest why X-rays are more dangerous than light.

22 A light bulb emits light of power 7.0 W. Estimate the number of photons emitted every second.

23 An atom has six energy levels. What is the maximum possible number of transitions between these levels?

24 Consider Figure E1.17, which shows some of the energy levels in a mercury atom.

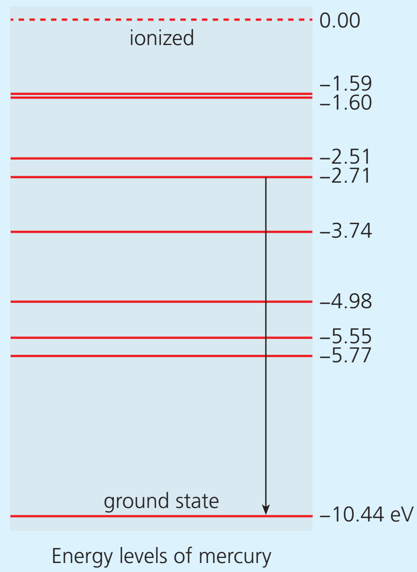


Figure E1.17 Some of the energy levels of mercury

a Determine the wavelength of radiation emitted by the transition shown.

b State in which part of the electromagnetic spectrum this radiation occurs.

c When radiation of frequency 1.18×10^{15} Hz passes through cool mercury vapour it is absorbed. Identify the transition involved in this process.

d Determine the longest wavelength of radiation that could be emitted by a transition between the levels shown.

25 When the spectrum emitted by the Sun is observed closely using a spectrometer, by looking at a white surface – **not** the Sun directly, it is found that light of certain frequencies is missing and, in their place, are dark lines.

a Explain how the cooler outer gaseous atmosphere of the Sun is responsible for the absence of these frequencies.

b Suggest how an analysis of the solar absorption spectrum could be used to determine which elements are present in the Sun's atmosphere.

A mathematical understanding of the Geiger–Marsden–Rutherford experiment

LINKING QUESTION

- How is the distance of closest approach calculated using conservation of energy?

This question links to understandings in Topics A.3 and D.2.

SYLLABUS CONTENT

- The distance of closest approach in head-on scattering experiments.
- The relationship between the radius and the nucleon number for a nucleus as given by: $R = R_0 A^{\frac{1}{3}}$ and the implications for nuclear densities.

Refer again to Figure E1.6. As an alpha particle approaches a positive nucleus, it loses kinetic energy, E_k, because it is being repelled, but the same amount of energy is transferred to electric potential energy, E_p, as shown in more detail in Figure E1.18.

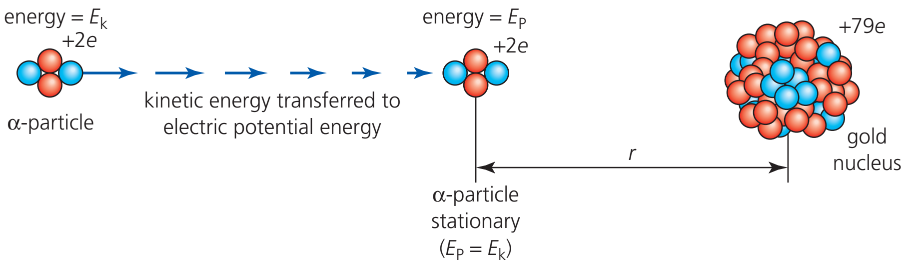


Figure E1.18 As an alpha particle approaches a positive nucleus, energy is transferred to electric potential energy, E_p

An alpha particle moving *directly* towards a nucleus will have lost *all* of its initial kinetic energy at the moment it is stationary, before returning back in the opposite direction. Then, assuming that the gold nucleus does not gain any significant kinetic energy:

initial kinetic energy of the alpha particle = the maximum electric potential energy momentarily stored in the system when the alpha particle is at its closest to the nucleus, with a separation of *r*. ($E_p = kq_\alpha q_n/r$)

WORKED EXAMPLE E1.5

Determine the closest distance from a gold nucleus that is possible for an alpha particle with kinetic energy of 5.0 MeV. Charge on alpha particle = ($2 \times 1.60 \times 10^{-19}$ C), charge on gold nucleus = ($79 \times 1.60 \times 10^{-19}$ C)

Answer

Kinetic energy of alpha particle, $E_k = k\dfrac{q_\alpha q_n}{r}$

$$(5.0 \times 10^6) \times (1.60 \times 10^{-19}) = \frac{((8.99 \times 10^9) \times (2 \times 1.60 \times 10^{-19}) \times (79 \times 1.60 \times 10^{-19}))}{r}$$

$r = 4.5 \times 10^{-14}$ m

Considering that the alpha particle has a very large amount of kinetic energy (for its small mass) and would therefore be expected to be able to get close to a nucleus, this type of calculation was the first to provide some evidence for the possible size of a nucleus. However, a gold nucleus is smaller than the value shown above. (The actual radius of a gold nucleus is about 0.7×10^{-14} m.)

◆ **Nuclear radius, R** R is proportional to the cube root of the nucleon number. $R = R_0A^{\frac{1}{3}}$, where R_0 is called the **Fermi radius**.

◆ **Rutherford scattering** Sometimes called Coulomb scattering. The scattering of alpha particles by nuclei, which can only be explained by the action of an inverse square law of electric repulsion. When high-energy particles are used they might enter the nucleus, so that strong nuclear forces are also involved and then the scattering will no longer follow the same pattern.

◆ **Nuclear density** All nuclear densities are similar in magnitude and are extremely large.

Nuclear radii

Rutherford scattering experiments (and similar) have shown that the radius, R, of any nucleus is proportional to the cube root of its nucleon number, A:

radius of nucleus $R = R_0A^{\frac{1}{3}}$

DB

This has implications for nuclear density, as seen below.

The constant $R_0 = 1.20 \times 10^{-15}$ **m** is called the **Fermi radius**, which is the assumed radius of a nucleus with only one proton ($A = 1$).

DB

WORKED EXAMPLE E1.6

Show that the radius of a $^{197}_{79}$Au (gold) nucleus is 'about 7.0×10^{-15} m'.

Answer

$R = R_0A^{1/3} = (1.20 \times 10^{-15}) \times 197^{1/3} = 6.98 \times 10^{-15}$ m

Nuclear density

We can use $\rho = m/V$ to estimate **nuclear density**. The mass of a nucleus will be approximately equal to Au, where u can be considered as the average mass of a nucleon, 1.661×10^{-27} kg. (u is called the *atomic mass unit*. It is explained in Topic E.3.)

$$\rho = \frac{Au}{\frac{4}{3}\pi\left(R_0A^{\frac{1}{3}}\right)^3} = \frac{3u}{4\pi R_0^{\,3}}$$

Importantly, this result shows us that nuclear densities do *not* depend on the radius of the nucleus, or the number of nucleons.

The densities of all nuclei are approximately the same.

$$\rho = \frac{(3 \times (1.661 \times 10^{-27}))}{(4\pi \times (1.20 \times 10^{-15})^3)} = 2.3 \times 10^{17}\,\text{kg}\,\text{m}^{-3}$$

This is an extremely large density! If the electrons in an atom are considered to have negligible mass compared to the nucleons, and the radius of an atom is typically 10^5 times larger than a nucleus, then an order of magnitude density for atoms would be:

$$\frac{10^{17}}{(10^5)^3} \approx 10^2\,\text{kg}\,\text{m}^{-3}$$

which is comparable to everyday observations of the density of matter, as expected. The only macroscopic objects with densities comparable to nuclear densities are collapsed massive stars, known as neutron stars and black holes.

26 Determine the closest distance that an alpha particle of energy 1.37 MeV could approach to:

a a gold nucleus

b a copper nucleus.

Copper has a proton number of 29.

27 An alpha particle nearly collides with a gold nucleus and returns along the same path.
Sketch a graph showing how the electric potential energy and kinetic energy possessed by the alpha particle vary with the distance of the alpha particle from the gold nucleus (assumed to be stationary).

28 a Calculate the velocity at which an alpha particle (mass of 6.64×10^{-27} kg) should travel directly towards the nucleus of a gold atom (charge $+79e$) in order to get within 2.7×10^{-14} m of it. Assume the gold nucleus remains stationary.

b Calculate the energy (MeV) of an alpha particle with this velocity.

29 Explain why alpha particles were used in the Geiger–Marsden–Rutherford experiments.

30 Discuss whether it is reasonable to assume that when an alpha particle approaches a gold atom in thin foil:

a the repulsive force from the alpha particle is the only force acting on the gold nucleus

b the gold atom remains stationary.

31 a Estimate the radius of:

i a gold nucleus (A = 197)

ii an oxygen nucleus (A = 16).

b The measured radius of a gold-197 nucleus is 6.87×10^{-15} m.
How does this compare with the value calculated in part **a**?

32 Show that the largest possible nuclear radius (of a naturally occurring element) is only about six times the smallest.

33 Gold is considered to be a dense element. Estimate what fraction of the volume of a gold ring is actually occupied by the subatomic particles. Assume that the radius of a gold atom is 1.5×10^{-10} m. State any other assumptions that you make.

34 The mass of the Sun is 2.0×10^{30} kg and its radius is 7.0×10^{8} m.

a Estimate the radius of a neutron star that has twice the mass of the Sun. Assume that a neutron star has the same density as a nucleus.

b Compare your answer to the radius of the Sun.

The strong nuclear force

SYLLABUS CONTENT

- Deviations from Rutherford scattering at high energies.

Up to this point, we have assumed that the only force acting between the alpha particle and a nucleus is a repulsive electric force between positive charges. Rutherford scattering is sometimes called Coulomb scattering because Coulomb's law can be used to describe it.

However, there is another field force acting between individual nucleons when they are close together: the strong nuclear force. This is the attractive force that overcomes the repulsive forces between positively charged protons as mentioned earlier in this topic.

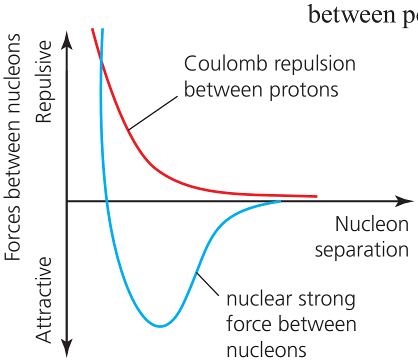


Figure E1.19 How the strong nuclear force varies with distance between two protons

A very energetic alpha particle can get close enough to the nucleons that it is affected by the attractive strong nuclear force as well as the repulsive electric force.

If this happens, the scattering can no longer be explained simply by Coulomb's law. Figure E1.19 approximately compares the strong nuclear force to the electric force between two protons. The electric repulsion force dominates for separations greater than about 3×10^{-15} m, whereas the strong nuclear force is 'short range' and only becomes significant for separations less than about 1.5×10^{-15} m. At that separation, the strong force attracts the protons together, but if they get much closer the force becomes repulsive.

The Bohr model of the hydrogen atom

SYLLABUS CONTENT

- The discrete energy levels in the Bohr model for hydrogen as given by: $E = \frac{-13.6}{n^2}$ eV.
- The existence of quantized energy and orbits arise from the quantization of angular momentum in the Bohr model for hydrogen as given by: $mvr = \frac{nh}{2\pi}$.

HL ONLY

◆ **Energy levels of hydrogen** Because hydrogen is the atom with the simplest structure, scientists were very interested in determining the energy levels of the electron within the atom by examining hydrogen's line spectrum. They were able to show that the energy levels could be predicted by the empirical equation $E = 13.6n^2$.

◆ **Principal quantum number, *n*** Number used to describe the energy level of an atom. The lowest energy level is called the ground state, with $n = 1$, the next level has $n = 2$, and so on.

◆ **Bohr model** A theory of atomic structure that explains the spectrum of hydrogen atoms.

The hydrogen atom is the simplest atom and, as such, it was at the centre of attention when physicists were beginning to understand atomic structure. The ideas in this section can be expanded to include other atoms, but we will restrict discussion to hydrogen.

Figure E1.20 shows the **energy levels of hydrogen** again, this time with all the possible energy transitions, arranged into three groups.

All the possible levels are shown and they are numbered, beginning with the lowest level, the ground state as $n = 1$. n is called the **principal quantum number**.

The energy levels in hydrogen atoms get closer together as n increases. This enables the possible transitions to be grouped as shown. All transitions down to the ground state ($n = 1$) are larger than all transitions down to the level $n = 2$, but note that Figure E1.20 is not drawn to scale. All transitions down to level $n = 2$ are larger than all transitions down to the level $n = 3$. (You do not need to remember the names of these three series. The transitions of the Lyman series all produce photons of ultraviolet radiation, the transitions of the Balmer series all produce photons of visible light, the transitions of the Paschen series all produce photons of infrared radiation.)

It is easily confirmed that all the discrete energy levels of hydrogen can be predicted by:

energy levels of hydrogen $E = \dfrac{-13.6}{n^2}$ eV (electronvolts)

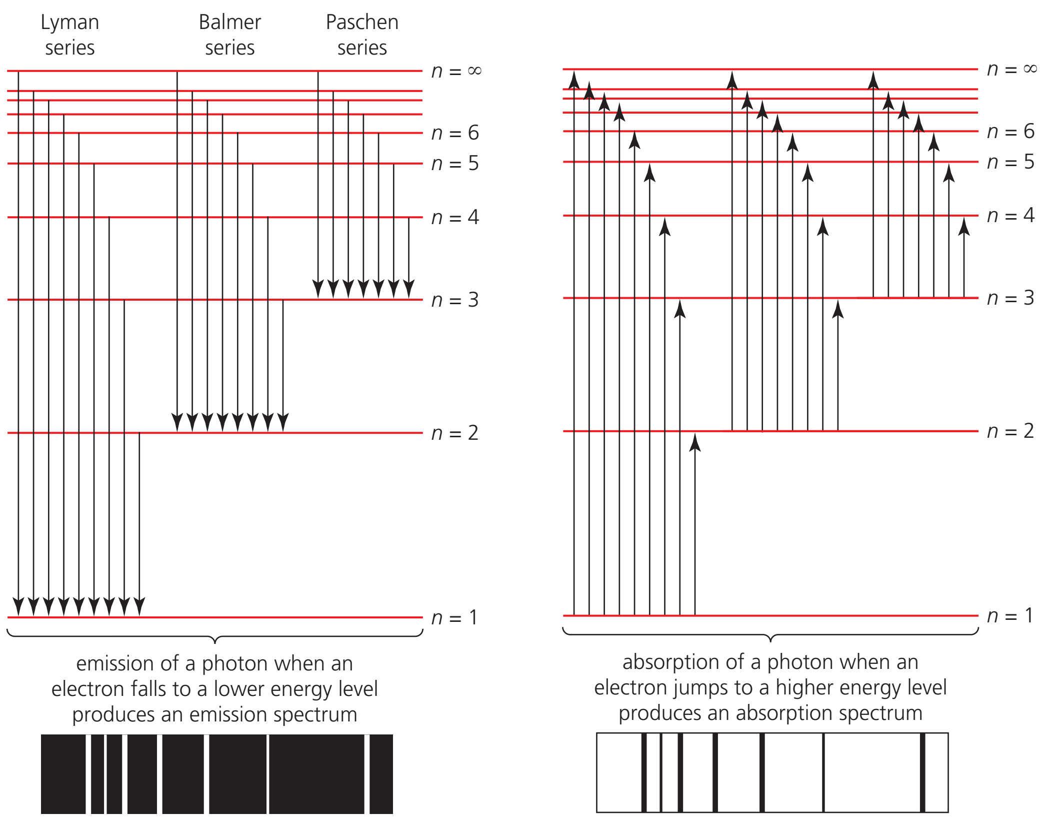


Figure E1.20 The Lyman, Balmer and Paschen series of the hydrogen atom

WORKED EXAMPLE E1.7

Calculate a value for the fifth energy level of the hydrogen atom ($n = 5$) in:

a electronvolts **b** joules.

Answer

a $E = \frac{-13.6}{n^2}$ eV $= \frac{-13.6}{5^2} = -0.544$ eV

b $-0.544 \times 1.60 \times 10^{-19} = -8.70 \times 10^{-20}$ J

Figure E1.21 Niels Bohr

The equation highlighted above was empirical, not based on any theory, when it was first discovered by the Danish physicist Neils Bohr (Figure E1.21). Of course, scientists wanted an explanation of *why* atoms had discrete energy levels and why the energy levels of hydrogen were predicted by a simple equation.

The **Bohr model** of the atom, first proposed in 1913, has electrons orbiting around the nucleus due to the centripetal force provided by electric attraction between opposite charges. But the essential feature of the Bohr model was that it restricted the orbits to only certain distances from the nucleus and, most importantly, because they remained in that orbit, they did not emit electromagnetic radiation, lose energy and spiral inwards. It was known that accelerated charges emit electromagnetic radiation, remembering that moving in a circle involves a centripetal acceleration.

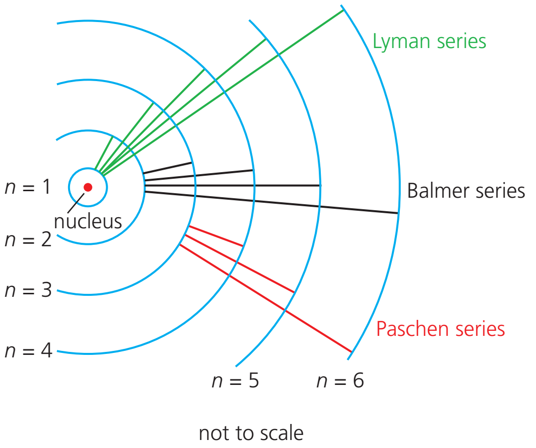


Figure E1.22 The Bohr model explains the spectrum of Hydrogen using possible orbits of different radii for the electron

In the Bohr model, each electron orbit had a definite and precise energy, and intermediate energies were not possible. Photons were emitted or absorbed when electrons moved between these energy levels. See Figure E1.22, in which the radii of the orbits are not drawn to scale.

Bohr showed that the quantized radii, r, of possible orbits of an electron of mass, m, moving with speed, v, in a hydrogen atom can be calculated from the equation:

$$mvr = \frac{nh}{2\pi}$$

The product of linear momentum and radius, mvr, is the *angular momentum, L*, of the electron about the nucleus (see Topic A.4). h, as before, is Planck's constant.

WORKED EXAMPLE E1.8

Calculate three possible values for the angular momentum of an electron in a hydrogen atom.

Answer

Angular momentum, $L = \frac{nh}{2\pi}$

For $n = 1$, $L = 1 \times \frac{(6.63 \times 10^{-34})}{(2 \times 3.14)} = 1.06 \times 10^{-34}\,\text{kg}\,\text{m}^2\,\text{s}^{-1}$

For $n = 2$, $L = 2.11 \times 10^{-34}\,\text{kg}\,\text{m}^2\,\text{s}^{-1}$

For $n = 3$, $L = 3.17 \times 10^{-34}\,\text{kg}\,\text{m}^2\,\text{s}^{-1}$

HL ONLY

As the following mathematics shows, the Bohr model combined the classical physics of circular motion and the force of electric attraction with quantum concepts in order to predict the radii of orbits and the energy levels in hydrogen atoms.

Equating the centripetal force on the electron to the electric attraction between it and the proton in a hydrogen atom, remembering that both the electron and the proton have charges of the same magnitude, e, we get:

$$\frac{mv^2}{r} = \frac{kee}{r^2}$$

(the signs of the charges are not relevant here); which leads to:

$$v = \sqrt{\left(\frac{ke^2}{mr}\right)}$$

Putting this expression for v in the equation for angular momentum (highlighted above), we get:

$$\sqrt{ke^2mr} = \frac{nh}{2\pi}$$

and rearranging enables us to obtain an expression for r (which need not be remembered):

$$r = \frac{n^2h^2}{4\pi^2ke^2m}$$

Putting in various values for n enables us to correctly predict the radii of possible electron orbits within the hydrogen atom.

WORKED EXAMPLE E1.9

Determine a value for the radius of the electron's orbit when the hydrogen atom is in its ground state ($n = 1$).

Answer

$$r = \frac{n^2h^2}{4\pi^2ke^2m} = \frac{(1^2 \times (6.63 \times 10^{-34})^2)}{(4 \times \pi^2 \times (8.99 \times 10^9) \times (1.60 \times 10^{-19})^2 \times (9.110 \times 10^{-31}))}$$

$$= 5.3 \times 10^{-11}\ \text{m}$$

Top tip!

The calculation on the right has determined a value for the ground state energy level of the hydrogen *atom*. It has clearly involved the electric potential energy of the electron–proton system. Nevertheless, it is common to refer to the energy levels of *electrons* within hydrogen (and other) atoms. This is understandable because we commonly visualize electrons moving between orbits when an atom receives or emits energy.

Then, using equations for electrical potential energy and kinetic energy, the total energy associated with the ground state of the atom can be calculated as follows.

The total energy of the hydrogen–electron system, a hydrogen atom, can be determined as follows:

$$E_{total} = E_k \text{ of electron} + E_k \text{ of proton} + \text{electric potential energy}$$

assuming that the electron (charge $-e$, mass m) is orbiting at speed v in a circular orbit of radius r around a proton (charge $+e$) which is effectively stationary:

$$\text{total energy of a hydrogen atom, } E_{total} = \frac{1}{2}mv^2 + \left(-\frac{ke^2}{r}\right)$$

But, we know that the magnitude of the centripetal force:

$$\frac{mv^2}{r} = \frac{ke^2}{r^2} \text{ (see above), or } mv^2 = \frac{ke^2}{r}$$

Leading to:

$$E_{\text{total}} = \frac{1}{2}\frac{ke^2}{r} + \left(-\frac{ke^2}{r}\right) = -\frac{1}{2}\frac{ke^2}{r}$$

For the ground state, $r = 5.3 \times 10^{-11}$ m (as above), so that:

$$E_{\text{total}} = -\frac{1}{2}\frac{ke^2}{r} = -2.2 \times 10^{-18}\text{ J}$$

So, Bohr's quantization of angular momentum equation leads directly to a correct calculation of the hydrogen atom ground state (and other energy levels).

35 Consider Figure E1.20.

- **a** Calculate the lowest frequency of the Balmer series.
- **b** In which part of the electromagnetic spectrum is this radiation?

36 Determine the energy (J) of the level with $n = 8$ in the hydrogen atom.

37 Determine the angular momentum of an electron in a hydrogen atom if it has a principal quantum number of four:

- **a** in terms of h/π
- **b** in SI units.

38 Calculate the radius of the first electron orbit above the ground state of a hydrogen atom.

39 Determine the total energy of a hydrogen atom if its electron is in the energy level which has a principal quantum number of three.

LINKING QUESTION

- Under what circumstances does the Bohr model fail? (NOS)

Although Bohr's quantized model of the atom was a big step forward in understanding the structure of the atom, and it was very accurate in predicting the energy levels of one-electron atoms such as a hydrogen atom or a helium ion, it was less successful with atoms containing more electrons. Furthermore, the reasons for the existence of energy levels were still not understood. The Bohr model still remains an important initial step for students learning about quantization in atoms, but the discovery of the wave properties of electrons (Topic E.2) quickly led to dramatic changes in physicists' understanding of the atom, but there are not included in the IB course.

◆ **Quantum mechanics** The mathematical aspects of quantum physics

Quantum mechanics is the name given to the important branch of physics that deals mathematically with events on the atomic and subatomic scales that involve quantities that can only have discrete (quantized) values. In the quantum world the laws of classical physics are often of little use; trying to apply knowledge and intuition gained from observing the macroscopic world around us often only leads to confusion.

HL ONLY

E.2 Quantum physics

Guiding questions

- How can light be used to create an electric current?
- What is meant by wave–particle duality?

Nature of science: Theories

What is quantum physics?

A *quantum* is the general term used to describe the minimum amount of any physical quantity that can only exist in discrete quantities (which are all basic multiples of one quantum).

On the subatomic scale, we have seen that charge is quantized, and in Topic E.1 we discussed the quantization of atomic energy levels, angular momentum and electromagnetic radiation. In a more general sense, matter itself could be considered as quantized because it is made of discrete particles, rather than being continuous.

Quantum ideas are so fundamental to understanding the behaviour of subatomic particles and waves (which, of course, also affects everything in our macroscopic world), that this branch of science has become generally known as quantum physics, and its detailed mathematic treatment is called quantum mechanics.

A famous quote from Neils Bohr: *'Everything that we call real is made of things that cannot be regarded as real. If quantum mechanics has not profoundly shocked you, you haven't understood it yet.'*

The photoelectric effect

SYLLABUS CONTENT

- The photoelectric effect as evidence of the particle nature of light.
- Photons of a certain frequency, known as the threshold frequency, are required to release photoelectrons from the metal.
- Einstein's explanation using the work function and the maximum kinetic energy of the photoelectrons as given by: $E_{max} = hf - \Phi$, where Φ is the work function of the metal.

◆ **Photoelectric effect** Ejection of electrons from a substance by incident electromagnetic radiation.

◆ **Photoelectrons** Electrons ejected in the process of the photoelectric effect.

The **photoelectric effect** (described below) was first discovered by Hertz in 1887. Eighteen years later, in 1905, Einstein expanded ideas about quantized energy ($E = hf$) proposed by Max Planck five years earlier, to propose that light and other electromagnetic radiation consisted of individual bundles of energy, thereby explaining the photoelectric effect. These events marked the beginnings of quantum physics. We will start by giving details of the photoelectric effect.

When electromagnetic radiation is directed onto a clean surface of some metals, electrons may be ejected. This is called the **photoelectric effect** and the ejected electrons are known as **photoelectrons**. The principle is shown in Figure E2.1. The importance of this effect lies in understanding why photoelectrons are emitted under some circumstances, but not others.

Under suitable circumstances, the photoelectric effect can occur with visible light, X-rays or gamma rays, but it is most often demonstrated with ultraviolet radiation and zinc. A possible arrangement is shown in Figure E2.2.

Ultraviolet radiation is shone onto a zinc plate attached to a coulombmeter (an instrument which measures very small quantities of charge). The ultraviolet radiation causes the zinc plate to become positively charged because some negatively charged electrons on the (previously neutral) zinc plate have gained enough kinetic energy to escape from the surface.

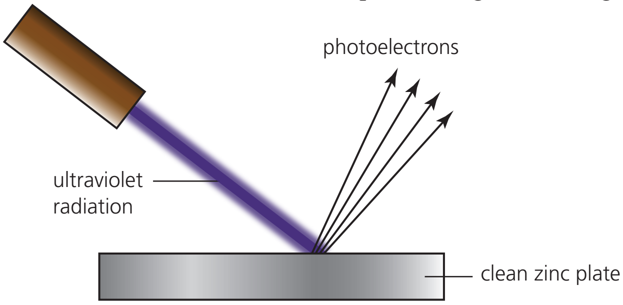


Figure E2.1 The photoelectric effect – a stream of photoelectrons is emitted from a metal surface illuminated with ultraviolet radiation

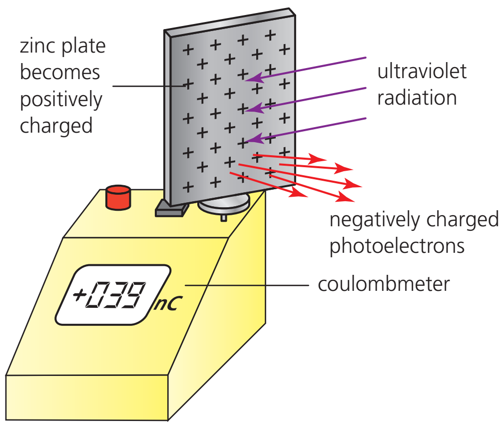


Figure E2.2 Demonstration of the photoelectric effect

◆ **Threshold frequency, f_0** The minimum frequency of a photon that can eject a photoelectron from the surface of a metal.

Investigations of the photoelectric effect show a number of important observations.

- If the intensity of the radiation is increased, the charge on the plate increases more quickly because more photoelectrons are being released every second.
- There is no time delay between the radiation reaching the metal surface and the emission of photoelectrons. The release of photoelectrons from the surface appears to be instantaneous.
- The photoelectric effect can only occur if the frequency of the radiation is above a certain minimum value. The lowest frequency for emission is called the **threshold frequency, f_0**. (Alternatively, we could say that there is maximum wavelength above which the effect will not occur.) If the frequency used is lower than the threshold frequency, the effect will not occur even if the intensity of the radiation is greatly increased. The threshold frequency of zinc, for example, is 1.04×10^{15} Hz, which is in the ultraviolet part of the spectrum. Visible light will not release photoelectrons from zinc (or other common metals).
- For a given incident frequency, the photoelectric effect occurs with some metals but not with others. This is because different metals have different threshold frequencies.
- The photoelectrons emitted from a particular metal by monochromatic radiation of a known frequency have a range of kinetic energies, up to a well-defined maximum.

Explaining the photoelectric effect: the Einstein model

If we tried to use the wave theory of radiation to make predictions about the photoelectric effect, we would expect the following. (1) Radiation of any frequency will cause the photoelectric effect if the intensity is made high enough. (2) There may be a delay before the effect begins because it needs time for enough energy to be provided (similar to heating water until it boils).

These predictions are wrong, so an alternative theory is needed. Einstein realized that we cannot explain the photoelectric effect without first understanding the quantum nature of radiation.

The Einstein model explains the photoelectric effect using the concept of photons.

When a photon in the incident radiation interacts with an electron in the metal surface, it transfers all of its energy to that electron. It should be stressed that a *single* photon interacts with a *single* electron and that this transfer of energy is instantaneous; there is no need to wait for a build-up of energy. If a photoelectric effect is occurring, increasing the intensity of the radiation only increases the number of photons and photoelectrons, not their individual energies.

HL ONLY

Einstein realized that some of the energy carried by the photon was used to overcome the attractive forces that normally keep an electron within the metal surface. The remaining energy is transferred to the kinetic energy of the newly released photoelectron. Using the principle of conservation of energy, we can write:

energy carried by photon
= work done in removing the electron from the surface + kinetic energy of photoelectron

But the energy required to remove different electrons from the same surface is not always the same. It will vary with the position of the electron with respect to the surface. Electrons closer to the surface will require less energy to remove them. However:

there is a well-defined minimum amount of energy needed to remove an electron from the surface of any particular metal and this is called the **work function**, Φ, of the metal.

◆ **Work function, Φ** The minimum amount of energy required to free an electron from the attraction of ions in a metal's surface.

◆ **Photoelectric equation** The maximum kinetic energy of an emitted photoelectron is the difference between the incident photon's energy and the work function, Φ: $E_{max} = hf - \Phi$.

Different metals have different values for their work functions. For example, the work function of a clean zinc surface is 4.3 eV. This means that at least 4.3 eV (= 6.9×10^{-19} J) of work must be done to remove an electron from zinc.

To understand the photoelectric effect, we need to compare the photon's energy, hf, to the work function, Φ, of the metal:

If $hf < \Phi$

If an incident photon has less energy than the work function of the metal, the photoelectric effect cannot occur. Radiation that may cause the photoelectric effect with one metal may not have the same effect with another (which has a different work function).

If $hf\,(= hf_0) = \Phi$

At the threshold frequency, f_0, the incident photon has exactly the same energy as the work function of the metal. We may assume that the photoelectric effect occurs, but any released photoelectrons will have zero kinetic energy.

If $hf > \Phi$

If an incident photon has more energy than the work function of the metal, the photoelectric effect occurs and a photoelectron will be released. Photoelectrons produced by different photons (of the same frequency) will have a range of different kinetic energies because different amounts of work will have been done to release them.

It is important to consider the situation in which the minimum amount of work is done to remove an electron (equal to the work function):

energy carried by photon = work function + *maximum* kinetic energy of photoelectrons

Or in symbols: $hf = \Phi + E_{max}$.

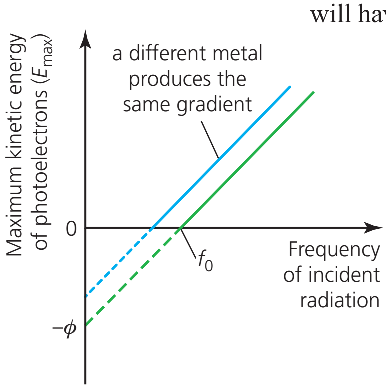
Or:

$$E_{max} = hf - \Phi$$

This equation is often called Einstein's **photoelectric equation**.

Because $hf_0 = \Phi$, we can also write this as: $E_{max} = hf - hf_0$.

Figure E2.3 shows a graphical representation of how the maximum kinetic energy of the emitted photons varies with the frequency of the incident photons. The equation of the line is $E_{max} = hf - \Phi$, as above.



■ **Figure E2.3** Theoretical variation of maximum kinetic energy of photoelectrons with incident frequency (for two different metals)

Tool 3: Mathematics

Interpret features of graphs including gradient, intercepts

Any straight line on an x–y graph can be represented by the equation $y = mx + c$, where m is the gradient ($\Delta y/\Delta x$) and c is the intercept on the y-axis ($y = c$, when $x = 0$). See Figure E2.4. The value of the intercept on the x-axis: $x = -c/m$ when $y = 0$.

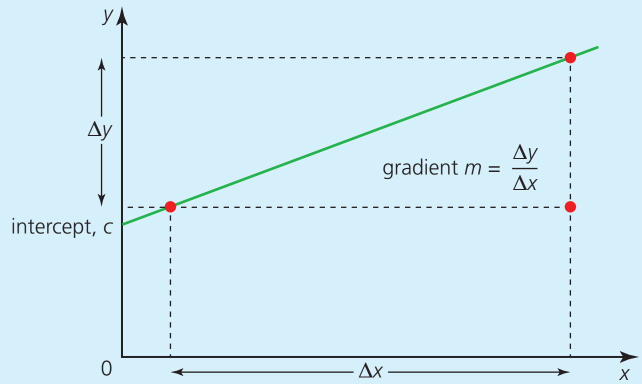


Figure E2.4 The line $y = mx + c$

HL ONLY

We can take the following measurements from graphs of the form seen in Figure E2.3:

- The gradient of the line is equal to Planck's constant, h. The gradient will be the same for all circumstances because it does not depend on photon frequencies, or the metal used.
- The intercept on the frequency axis gives us the value of the threshold frequency, f_0.
- A value for the work function can be determined from: when $E_{max} = 0$, $\Phi = hf_0$; or when $f = 0$, $\Phi = -E_{max}$.

WORKED EXAMPLE E2.1

Radiation of wavelength 5.59×10^{-8} m was incident on a metal surface that had a work function of 2.70 eV.

a Calculate the frequency of the radiation.

b Determine how much energy is carried by one photon of the radiation.

c Calculate the value of the work function expressed in joules.

d Explain whether the photoelectric effect occurs under these circumstances.

e Determine the maximum kinetic energy of the photoelectrons.

f Calculate the threshold frequency for this metal.

g Sketch a fully labelled graph to show how the maximum kinetic energy of the photoelectrons would change if the frequency of the incident radiation was varied.

Answer

a $f = c/\lambda = \dfrac{(3.00 \times 10^{8})}{(5.59 \times 10^{-8})} = 5.37 \times 10^{15}\,\text{Hz}$

b $E = hf = (6.63 \times 10^{-34}) \times (5.37 \times 10^{15}) = 3.56 \times 10^{-18}\,\text{J}$

c $2.70 \times (1.60 \times 10^{-19}) = 4.32 \times 10^{-19}\,\text{J}$

d Yes, because the energy of each photon is greater than the work function.

e $E_{max} = hf - \Phi = (3.56 \times 10^{-18}) - (4.32 \times 10^{-19}) = 3.13 \times 10^{-18}\,\text{J}$

f $\Phi = hf_0$

$f_0 = \dfrac{4.32 \times 10^{-19}}{6.63 \times 10^{-34}} = 6.52 \times 10^{-14}\,\text{Hz}$

g The graph should be similar to Figure E2.3, with numerical values provided for the intercepts.

HL ONLY

1 Repeat Worked example E2.1 but for radiation of wavelength 6.11×10^{-7} m incident on a metal with a work function of 2.21 eV. Omit part **e**.

2 a Outline how Einstein used the concept of photons to explain the photoelectric effect.
  b Explain why a wave model of electromagnetic radiation is unable to explain the photoelectric effect.

3 The threshold frequency of a metal is 7.0×10^{14} Hz. Calculate the maximum kinetic energy of the electrons emitted when the frequency of the radiation incident on the metal is 1.0×10^{15} Hz.

4 a The longest wavelength that emits photoelectrons from potassium is 550 nm. Calculate the work function (in joules).
  b Determine the threshold wavelength for potassium. What is the name given to this kind of radiation?
  c State one colour of visible light that will not produce the photoelectric effect with potassium.

5 When electromagnetic radiation of frequency 2.90×10^{15} Hz is incident on a metal surface, the emitted photoelectrons have a maximum kinetic energy of 9.70×10^{-19} J.
Calculate the threshold frequency of the metal.

Experiments to test the Einstein model

Investigating stopping voltages (potential differences)

To test Einstein's equation (model) for the photoelectric effect, it is necessary to determine the maximum kinetic energy of the photoelectrons emitted under a variety of different circumstances. In order to do this the kinetic energy must be transferred to another (measurable) form of energy.

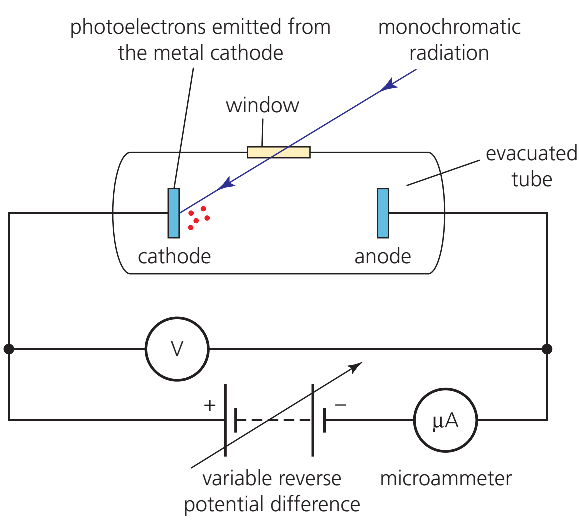


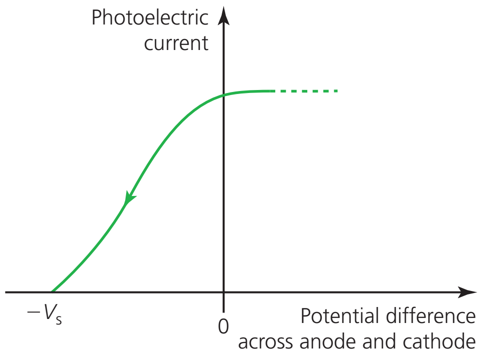
Figure E2.5 Experiment to test Einstein's model of photoelectricity

The kinetic energy of the photoelectrons can be transferred to electrical potential energy if they are repelled by a negative voltage. This experiment was first performed by the American physicist Robert Millikan and a simplified version is shown in Figure E2.5.

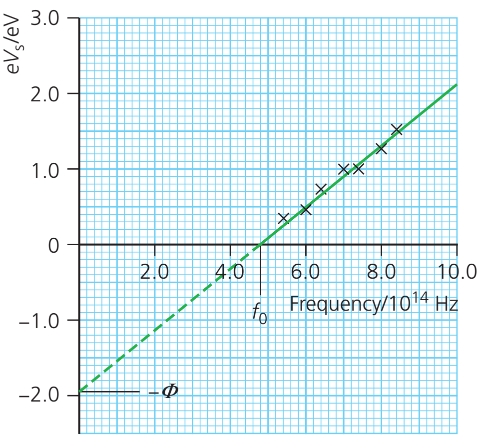
Ideally *monochromatic* radiation should be used, but it is also possible to use a narrow range of frequencies such as those obtained by using coloured filters with white light.

When radiation is incident on a suitable emitting surface, photoelectrons will be released with a range of different energies, as explained previously. Because it is emitting negative charge, this surface can be described as a *cathode* (the direction of conventional current flow will be out of a cathode and around the circuit). Any photoelectrons that have enough kinetic energy will be able to move across the tube and reach the other electrode, the *anode*. The tube is evacuated (the air is removed to create a vacuum) so that the electrons do not collide with air molecules during their movement across the tube.

The most important thing to note about this circuit is that the (variable) source of potential difference is connected the 'wrong way around'. We say that it is supplying a *reverse* potential difference across the tube. This means that there is a negative voltage on the anode that will repel the photoelectrons. Photoelectrons moving towards the anode will have their kinetic energy reduced as it is transferred to electrical potential energy. (Measurements for positive voltages can be made by reconnecting the battery the 'correct' way.)



■ **Figure E2.6** Increasing the reverse potential difference decreases the photoelectric current



■ **Figure E2.7** Experimental results showing variation of maximum energy (eV_s) of photoelectrons with incident frequency

Any flow of charge across the tube and around the circuit can be measured by a sensitive microammeter. When the reverse voltage on the anode is increased from zero, more and more photoelectrons will be prevented from reaching the anode and this will decrease the current. (Remember that the photoelectrons have a range of different energies.) Eventually the reverse potential difference will be large enough to stop even the most energetic of photoelectrons, and the current will fall to zero (Figure E2.6).

The potential difference across the tube needed to *just* stop all photoelectrons reaching it is called the **stopping voltage** (p.d.), V_s.

Because, by definition, potential difference = energy transferred / charge, after measuring V_s we can use the following equation to calculate values for the maximum kinetic energy of photoelectrons under a range of different circumstances: $E_{max} = eV_s$.

For convenience, it is common to quote all energies associated with the photoelectric effect in electronvolts (eV). In which case, the maximum kinetic energy of the photoelectrons is numerically equal to the stopping voltage. That is, if the stopping voltage is, say, 3 V, then E_{max} = 3 eV.

Einstein's equation ($E_{max} = hf - \Phi$) can be rewritten as: $eV_s = hf - \Phi$.

By experimentally determining the stopping voltage for a range of different frequencies, the theoretical graph shown previously in Figure E2.3 can now be confirmed by plotting a graph from actual data, as shown in Figure E2.7.

◆ **Stopping voltage** The minimum voltage required to reduce a photoelectric current to zero.

WORKED EXAMPLE E2.2

Use Figure E2.7 to determine:

a the threshold frequency

b the work function

c a value for Planck's constant.

Answer

a f_0 can be determined from the intercept on the frequency axis: $f_0 = 4.8 \times 10^{14}$ Hz

b from the intercept on the eV_s axis: 1.9 eV

c h can be determined from the gradient (remembering to convert electronvolts to joules):

$$h = \frac{(2.1 \times 1.60 \times 10^{-19})}{((10 - 4.8) \times 10^{14})} = 6.5 \times 10^{-34}\text{ Js}$$

Common mistake

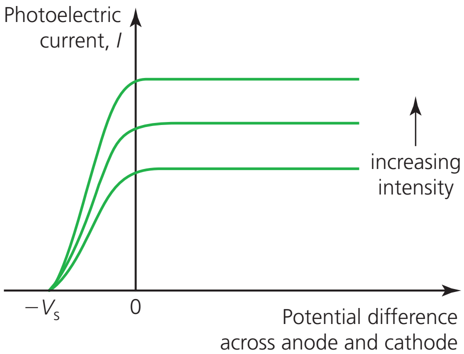
If two sources of monochromatic radiation have the same intensity, but different frequencies, the photons from the source with the lower frequency will carry less energy, so there must be more photons emitted every second from that source.

Investigating photoelectric currents

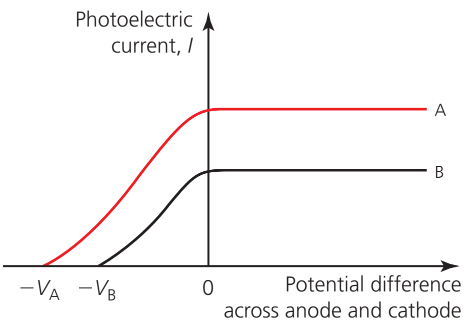
Using apparatus similar to that shown in Figure E2.5, it is also possible to investigate quantitatively the effects on the photoelectric current of changing the:

- intensity
- frequency
- metal used in the cathode.

HL ONLY



■ **Figure E2.8** Variation of photoelectric current with potential difference for radiation of three different intensities (same frequency)



■ **Figure E2.9** Variation of photoelectric current with potential difference for radiation of two different frequencies

Intensity

Figure E2.8 shows the photoelectric currents produced by monochromatic radiation of the same frequency with three different intensities.

For positive p.d.s, each of the photoelectric currents remain constant because the photoelectrons are reaching the anode at the same rate as they are being produced at the cathode, and this does not depend on the size of the positive voltage on the anode. Greater intensities (of the same frequency) produce higher photoelectric currents because there are more photons releasing more photoelectrons (with the same range of energies). Because the maximum kinetic energy of photons depends only on frequency and and work function, but not intensity, all these graphs have the same value for stopping voltage, V_s.

Frequency

Figure E2.9 shows the photoelectric currents produced by radiation from two monochromatic sources of different frequencies, A and B, incident on the same metal.

The individual photons in radiation A must have more energy (than B) and produce photoelectrons with a higher maximum kinetic energy. We know this because a greater reverse voltage is needed to stop the more energetic photoelectrons produced by A. No conclusion can be drawn from the fact that the current for A has been drawn higher than for B, because the intensities of the two radiations are not known. In the unlikely circumstances that the two intensities were equal, the maximum current for B would have to be higher than for A because the radiation from B must have more photons, because each photon has less energy than in A.

Metal used in the cathode

Experiments confirm that when different metals are tested using the same frequency, the photoelectric effect is observed with some metals but not with others (those metals for which their work function is higher than the energy of the photons).

6 Calculate the maximum kinetic energy of photoelectrons emitted from a metal if the stopping voltage was 2.4 V. Give your answer in electronvolts and in joules.

7 Make a copy of Figure E2.6 and sketch lines to show the results that would be obtained with:
   a the same radiation, but with a metal of higher work function (assume that the photoelectric effect still occurs)
   b the original metal and the same frequency of radiation but using radiation with a greater intensity.

8 In an experiment using monochromatic radiation of frequency 7.93×10^{14} Hz with a metal that had a threshold frequency of 6.11×10^{14} Hz, it was found that the stopping voltage was 0.775 V.
Calculate a value for Planck's constant from these results.

9 Make a copy of Figure E2.6 and sketch the results that would be obtained using radiation of a higher intensity (of the same frequency) incident on a metal that has a smaller work function.

10 Make a copy of Figure E2.6. Add to it a line showing the results that would be obtained with radiation of a higher frequency but with same number of photons every second incident on the same metal.

11 a Select **five** different metallic elements and then use the internet to research their work functions.
   b Calculate the threshold frequencies of the five metals.

HL ONLY

Inquiry 2: Collecting and processing data

Processing data

Light emitting diodes (LEDs) can be used to determine an approximate value for Planck's constant. Each LED emits photons of a precise frequency. The energy of each photon ($E = hf$) is transferred when an individual electron is accelerated by the voltage across the LED. That is, $eV = hf$.

If the voltage across the LED that *just* results in the LED emitting light is measured, then this equation can be used to determine a value for h if the frequency of the radiation is known. However, it is better to draw a graph.

1 Draw a voltage–frequency graph of the results shown in Table E2.1 and use the gradient to determine a value for Planck's constant.

Table E2.1

Colour	Frequency / 10^{14} Hz	Voltage / V
red	4.54	1.91
amber	5.01	2.06
yellow	5.10	2.12
green	5.37	2.21
blue	6.37	2.65

The wave nature of matter

SYLLABUS CONTENT

- Diffraction of particles as evidence of the wave nature of matter.
- The de Broglie wavelength for particles as given by: $\lambda = \frac{h}{p}$.

The fact that light and other electromagnetic waves could behave as particles (photons) raises an obvious question: can particles behave like waves?

In 1924 the French physicist Louis de Broglie proposed that electrons, which were thought of as particles, might also have a wave-like character. He later generalized his hypothesis to suggest that:

All moving particles have wave-like properties.

◆ **de Broglie's hypothesis** All particles exhibit wave-like properties, with a de Broglie wavelength, $\lambda = \frac{h}{p}$

◆ **Hypothesis** A suggested explanation of a phenomenon (but not proven).

According to **de Broglie's hypothesis**, the wavelength, λ, of a moving particle was inversely proportional its momentum, p, as represented by:

wavelength of a moving particle $\boldsymbol{\lambda = \frac{h}{p}}$

Once again, we can see the importance of Planck's constant, h, in predicting the size of quantum phenomena. The very small value of Planck's constant shows us that wave properties of particles are only significant for those with tiny momenta, as Worked example E2.3 illustrates.

Nature of science: hypotheses and theories

A **hypothesis** uses limited information to suggest a possible outcome or explanation. A hypothesis is not assumed to be true until it has been tested. For example, student investigations will usually begin with a hypothesis about what they *think* will happen in their experiments. Many scientific advances are preceded by hypotheses.

An explanation that has been confirmed from much-repeated experiments / observations is usually described as a *theory*.

It is reported that Isaac Newton was strongly opposed to the use of hypotheses, and that he believed that theories should be inferred directly from observations. But experimentation was not such a major feature of science at that time.

Despite the fact that it has been confirmed that all moving particles have wave properties, de Broglie's work is still usually described as a 'hypothesis' rather than a theory. Suggest why.

ATL E2A: Social skills

Working collaboratively to achieve a common goal

Working in small groups, discuss one of the following, and then summarize your conclusions for the rest of your class.

- Are there circumstances under which it may not be reasonable to require a hypothesis before a student's investigation?
- Are there examples of major scientific advances which were not preceded by hypotheses?
- Should governments provide funds for research projects which are not aimed at producing a useful result?
- Will the research of private science-based companies always be aimed at financial profit and, if so, what are the implications?

WORKED EXAMPLE E2.3

a Calculate the momentum of a moving particle that has a de Broglie wavelength of 200 pm (1 pm = 1×10^{-12} m).

b Determine the wavelength associated with an electron moving with a speed of five million metres per second.

c If we were to suppose that de Broglie's hypothesis extends to macroscopic objects, estimate the wavelength of a moving tennis ball.

Answer

a $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{200 \times 10^{-12}} = 3.32 \times 10^{-24}\,\text{kg m s}^{-1}$

b $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{(9.110 \times 10^{-31}) \times (5.0 \times 10^{6})} = 1.5 \times 10^{-10}\,\text{m}$

c Estimate momentum, $p = mv = 0.06 \times 20 \approx 1\,\text{kg m s}^{-1}$

$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1} \approx 7 \times 10^{-34}\,\text{m}$

If a ball had this wavelength, it would be too small to measure.

12 A neutron has a de Broglie wavelength of 80 pm.
Calculate the velocity of the neutron.

13 Show that a potential difference of about 4000 V is needed to accelerate an electron from rest so that it has a de Broglie wavelength of 2.0×10^{-11} m.

14 Which is associated with a de Broglie wavelength of longer wavelength – a proton or an electron travelling at the same velocity? Explain your answer.

15 Explain why a moving car has no detectable wave properties.

LINKING QUESTION

- What are the defining features and behaviours of waves?

This question links to understandings in Topics C.2 and C.3.

Evidence for the wave nature of matter

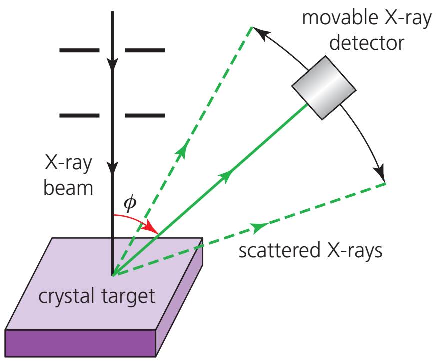
Superposition (interference and diffraction) is behaviour that is characteristic of waves, but not particles.

In order to verify de Broglie's hypothesis, it was necessary to observe and measure the diffraction of a beam of particles (electrons).

◆ **Davisson–Germer experiment** Experiment that verified the wave properties of matter by showing that a beam of electrons is diffracted by a crystal (at an angle dependent upon the velocity of the electrons).

A reminder: we have seen in Topic C.3 that the diffraction of light by a diffraction grating can be represented by the equation $n\lambda = d\sin\theta$. Knowledge of d (the spacing of lines on the grating) and the measurement of the diffraction angle θ (for $n = 1$), can lead to a determination of an unknown wavelength, λ. An important example of this is the determination of spectral wavelengths and frequencies (Topic E.1).

In principle, other electromagnetic waves can be similarly diffracted into patterns. The diffraction / scattering of X-rays is of especial importance because a typical X-ray wavelength is comparable to the separation of atoms / ions (approximately 10^{-10} m). This is the necessary condition for significant diffraction effects.



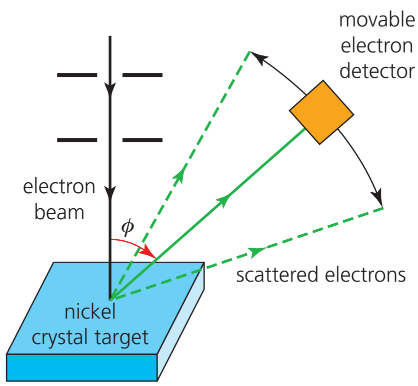
■ **Figure E2.10** Diffraction/ scattering of X-rays by a crystal

The regular three-dimensional arrangement of atoms in a crystal has similarities with the regular two-dimensional arrangement of lines on a diffraction grating. Figure E2.10 shows a much-simplified arrangement. (Knowledge of X-ray diffraction is *not* required for the IB Physics examination.)

When X-rays are diffracted / scattered by parallel layers of atoms / ions in a crystal, knowledge of n, λ and θ can lead to a determination of d, the separation of the layers.

De Broglie's hypothesis proposed that electrons can have wavelengths similar to X-rays and atomic separations ($\approx 10^{-10}$ m) as shown in Worked example E2.3, so it was anticipated that electrons would also be diffracted / scattered by regular arrangements of atoms / ions.

The diffraction of an electron beam was first achieved in the **Davisson–Germer experiment** as seen in Figure E2.11, using a nickel crystal target.

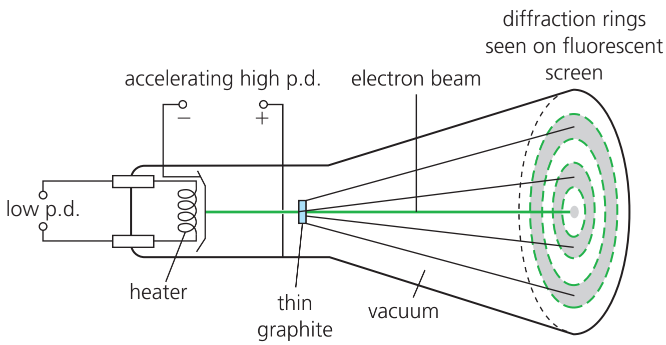


■ **Figure E2.11** Davisson-Germer experiment

A scattered beam of electrons was detected at an angle which confirmed that diffraction was occurring of electron waves with a wavelength consistent with de Broglie's hypothesis.

When the electrons were accelerated to greater speeds (using a larger voltage), they were diffracted through a smaller angle because the wavelength was less.

Figure E2.12 shows the type of modern apparatus used in schools to demonstrate the diffraction of electrons. Two prominent diffraction rings are seen, representing constructive interference of electron waves from two different sets of layers of carbon atoms in the graphite target. (The graphite has many sets of layers with different orientations.)



■ **Figure E2.12** An electron diffraction apparatus

If the voltage accelerating the electrons is increased, they will have greater speed and momentum when they arrive at the graphite. This means that the electron wavelength and diffraction angle will be reduced.

Tool 1: Experimental techniques

Recognize and address safety, ethical or environmental issues in an investigation

What safety precautions are necessary when using the apparatus shown in Figure E2.12? Make a list of safety precautions for other experimenters to follow.

◆ **X-ray diffraction (crystallography)** Investigating the arrangements of atoms and molecules in matter by detecting how X-rays are diffracted by crystalline materials.

WORKED EXAMPLE E2.4

Using apparatus similar to that seen in Figure E2.11, electrons were accelerated by 5.0 kV. Calculate the:

a kinetic energy of the electrons (J)

b speed of the electrons

c momentum of the electrons

d wavelength of the electrons using de Broglie's hypothesis

e diffraction angle for these electrons, assuming that:

- layers causing the diffraction had a separation of 1.4×10^{-10} m
- diffraction can be modelled by the equation $\lambda = 2d\sin\theta$.

Answer

a $W = qV = (1.60 \times 10^{-19}) \times 5000 = 8.0 \times 10^{-16}\,\text{J}$ (= 5000 eV)

b $E_k = \frac{1}{2}mv^2$

$8.0 \times 10^{-16} = \frac{1}{2} \times (9.110 \times 10^{-31}) \times v^2$

$v = 4.2 \times 10^{7}\,\text{m s}^{-1}$

c $p = mv = (9.110 \times 10^{-31}) \times (4.2 \times 10^{7}) = 3.8 \times 10^{-23}\,\text{kg m s}^{-1}$

d $\lambda = \dfrac{h}{p} = \dfrac{6.63 \times 10^{-34}}{3.8 \times 10^{-23}} = 1.7 \times 10^{-11}\,\text{m}$

e $\lambda = 2d\sin\theta$

$(1.7 \times 10^{-11}) = 2 \times 1.4 \times 10^{-10} \times \sin\theta$

$\theta = 3.6°$

ATL E2B: Social skills

Appreciate the diverse talents of others

Ask an IB chemistry student (or teacher) to give a short presentation to your class about X-ray crystallography (diffraction), as it is explained in the chemistry course. Find out if electron diffraction is used similarly.

16 If the accelerating voltage in Figure E2.12 was doubled, determine by what factor the following would change:

a electron kinetic energy

b momentum of electrons

c wavelength of electrons

d sine of the diffraction angle?

17 In Figures E2.10 and E2.11, the beams can be seen to pass between two slits. Suggest a reason why the slits are needed.

18 Outline the differences and similarities between electrons and X-rays.

Wave–particle duality

SYLLABUS CONTENT

▶ Matter exhibits wave–particle duality.

◆ **Wave–particle duality** Theory that all particles have wave properties and that all electromagnetic waves have particle properties.

In principle, all particles have wave properties and all electromagnetic waves have particle (photon) properties. This is widely known as **wave–particle duality**.

Nature of science: Theories

Two theories for the same thing

We may choose to believe that light, for example, is composed of waves (because it interferes and diffracts), or we may choose to think of light as a stream of particles (photoelectric and Compton effects), but we have difficulty believing the confusing truth that light can behave as both waves and particles, depending on the circumstances.

The following two famous quotations reflect the situation:

'It seems as though we must use sometimes the one theory and sometimes the other, while at times we may use either. We are faced with a new kind of difficulty. We have two contradictory pictures of reality; separately neither of them fully explains the phenomena of light, but together they do.' (Albert Einstein)

'God runs electromagnetics by wave theory on Monday, Wednesday, and Friday, and the Devil runs them by quantum theory on Tuesday, Thursday, and Saturday.' (William Lawrence Bragg)

Maybe we are tempted to believe that there is some, yet unknown, discovery which will solve this paradox, but it is much more likely that we have to accept, once again, that the world of quantum physics does not match the 'reality' we observe in everyday life.

Adding to the confusion, we use the frequency of a wave to determine the energy carried by an individual particle ($E = hf$).

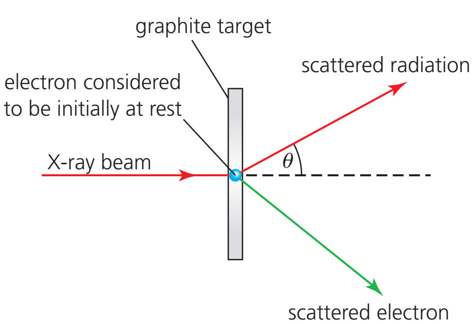
HL ONLY

◆ **Compton effect (scattering)** The increase in wavelength (decrease in energy) of high-frequency photons when they interact (collide) with electrons. Important evidence for the particle nature of electromagnetic radiation.

Compton scattering

SYLLABUS CONTENT

- Compton scattering of high-frequency photons by electrons as additional evidence of the particle nature of light.
- Photons scatter off electrons with increased wavelength.
- The shift in photon wavelength after scattering off an electron as given by: $\lambda_f - \lambda_i = \Delta\lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$.



■ **Figure E2.13** Compton scattering

As well as the photoelectric effect, the interaction of electromagnetic radiation with matter was also investigated in the **Compton effect (scattering)**, named after Arthur Compton, the American physicist who won the Nobel prize in 1927 for his work. The effect is most significant with shorter wavelength radiation, such as X-rays and gamma rays. (Compton used wavelengths typically smaller than those used in X-ray diffraction experiments.)

The experiment examined the interaction of monochromatic X-rays with electrons. The electrons were the outer electrons in carbon atoms in a small graphite target placed in the X-ray beam. See Figure E2.13.

The essential feature of Compton scattering is that it cannot be explained by considering the incident X-rays to be waves. A 'particle model' is needed: an individual photon (with momentum, $p = \dfrac{h}{\lambda}$) 'collides' with an individual electron.

The laws of conservation of energy and momentum can be applied to the scattering. The electron gains kinetic energy, so the photon must lose energy. Since for the X-ray photon, $E = hf$, if it loses energy, it must change to a smaller frequency (equivalent to a change to a greater wavelength: λ_i to λ_f). This was confirmed by experiment, although the change in wavelength was relatively small.

Applying the laws of conservation of momentum and energy leads to the following equation, which accurately predicts the change in wavelength, $\Delta\lambda$, of the photons. (You do *not* need to know how this equation is derived.)

Compton scattering equation:

$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

θ is the scattering angle, as shown in Figure E2.13 and m_e is the mass of the electron, which is assumed to be effectively stationary (at rest) before being scattered.

WORKED EXAMPLE E2.5

Determine the change in wavelength for photons scattered at an angle of 30° in the Compton effect.

$\left(\frac{h}{m_e c}\right.$ is a constant, called the Compton wavelength, and it has a value of 2.426×10^{-12} m.$\left.\right)$

Answer

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta) = (2.426 \times 10^{-12}) \times (1 - \cos 30°) = 3.25 \times 10^{-13}\,\text{m}$$

This is a small change of wavelength and requires excellent experimental techniques and equipment to measure.

Photons scattered at different angles will have different wavelengths, as is represented by the equation.

LINKING QUESTIONS

- How can particles diffract?
- Why is Compton scattering more convincing evidence for the particle nature of light than that from the photoelectric effect? (NOS)

These questions link to understandings in Topic C.3.

Interestingly, the change of wavelength in Compton scattering depends only on the scattering angle, it does *not* depend on the energy, or wavelength of the incident photon. It is also relatively small: for example, if the incident X-ray photon had a wavelength of 2.25×10^{-10} m, a change of 3.25×10^{-13} m would only be 0.14%.

19 Arthur Compton's scattering experiment is considered to be one of the classic physics investigations of the early twentieth century. Explain why.

20 Calculate the change in photon frequency that occurred in Worked example E2.5.

21 Compton's original experiment used X-rays of wavelength 0.0709 nm.
Calculate the change of wavelength of photons scattered through an angle of 45°.

22 At what scattering angle would you expect to observe the largest change in photon wavelength? Explain your answer.

23 **a** Calculate a typical energy (eV) of:
  i a photon of visible light
  ii a photon of X-rays.
b It requires about 10 eV to remove an electron from a carbon atom. Suggest why this was ignored in the previous discussion of the Compton effect.
c State **two** reasons why Compton scattering of visible light is not observed.

24 An X-ray photon with initial wavelength of 4.700×10^{-11} m was scattered through 34°.
a Determine the change in photon:
  i wavelength **ii** energy.
b State the resulting kinetic energy of the electron with which it collided.
c State any assumption(s) you made in answering part **b**.

25 Outline the main evidence for:
a the wave nature of electromagnetic radiation
b the particle nature of electromagnetic radiation.

E.3 Radioactive decay

Guiding questions

- Why are some isotopes more stable than others?
- In what ways can a nucleus undergo change?
- How do large, unstable nuclei become more stable?
- How can the random nature of radioactive decay allow for predictions to be made?

What is radioactivity?

Isotopes

The nuclei of some atoms are unstable. Spontaneous changes within an unstable nucleus can result in the emission of a particle and/or a high-energy photon. This process is called **radioactivity**. When particles are emitted, the proton number of the atom will change, so that it becomes a different element. This is called **transmutation** or radioactive decay.

A material involved in the process of radioactivity is described as being **radioactive**, while an atom with an unstable nucleus may be referred to as a **radioisotope** or **radionuclide**.

The term isotope was explained in Topic E.1. As a reminder: an *isotope* is one of two or more different nuclides of the same element (which have the same proton numbers, but different nucleon numbers).

Radioactive decay should not be confused with chemical or biological decay. The decay of a radioactive material will not usually involve any obvious change in appearance.

Most of this topic is concerned with explaining radioactivity, but a straightforward example now will help you to begin to understand all these terms, which will become more familiar as your understanding develops.

Atoms of $^{235}_{92}U$ have unstable nuclei, so we can describe the material as being *radioactive*. The element uranium has several *isotopes* which are all unstable / radioactive. They can all be described as *radioisotopes*. In the last two sentences, we can replace 'isotope' with 'nuclide' if we wish to stress that we are discussing nuclei.

At some (uncertain) time in the future, any $^{235}_{92}U$ nucleus may emit an alpha particle, $^{4}_{2}He$, and when this happens, we say that the nucleus has *decayed* or *transmuted*. In this example $^{231}_{90}Th$ (the element thorium) is formed and it may be called the *decay product* or the **daughter product**.

◆ **Radioactivity** Spontaneous transmutation of an unstable nucleus, accompanied by the emission of ionizing radiation in the form of alpha particles, beta particles or gamma rays.

◆ **Transmutation** When a nuclide changes to form a different element after emitting a particle.

◆ **Radioactive** Describes a substance which contains unstable nuclei which will emit radiation.

◆ **Radioisotope** or **radionuclide** Isotope / nuclide with an unstable nucleus which emits radiation.

◆ **Daughter product** The resulting nuclide after a radionuclide ('parent') emits a particle.

TOK

The natural sciences

- Does the precision of the language used in the natural sciences successfully eliminate all ambiguity?

New terminology

The last section illustrates a recurring theme in science education: so many new words to learn! When acquiring new scientific knowledge, is the introduction of new terms unavoidable? Does it help, or discourage, a student? Is science different from other areas of knowledge in this respect?

Radioactivity experiments

SYLLABUS CONTENT

- Effect of background radiation on count rate.

Figure E3.1 shows a typical experimental arrangement for investigating radioactivity in a school laboratory.

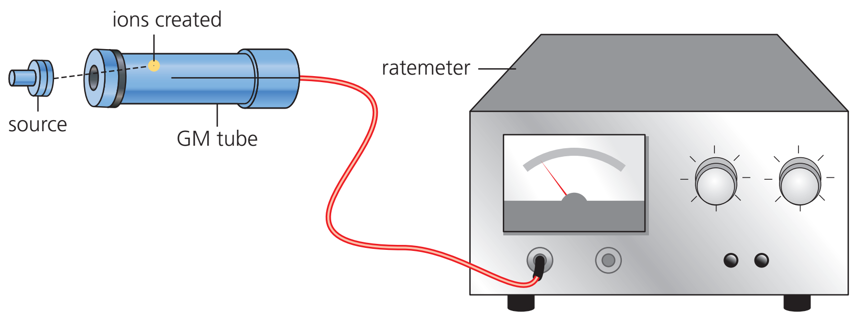


Figure E3.1 Basic components of a radioactivity experiment

◆ **Geiger–Muller tube** Apparatus used with a counter or ratemeter to measure the radiation from a radioactive source.

◆ **Count rate (radioactivity)** The number of nuclear radiation events detected in a given time (per minute or per second).

◆ **Ratemeter** Meter which is connected to a Geiger–Muller tube (or similar) to measure the rate at which radiation is detected.

A tiny amount of a radioactive nuclide is contained in the 'source'. When nuclear radiation emitted by the source enters the **GM (Geiger–Muller) tube** through the end 'window', it causes ionization of the gas inside and a sudden tiny burst of current. These events are 'counted' by an electronic 'counter', or **ratemeter**, and the results are expressed as a *radioactive count,* or a count per second, or per minute. (The tube and counter together are often described as a Geiger counter.) For example, if over a period of five minutes a total count of 7200 was detected, this would probably be recorded as a **count rate** of $1440\,\text{min}^{-1}$ or $24\,\text{s}^{-1}$. Many radiation detectors display a count rate directly, as seen in Figure E3.1.

Top tip!

If a radioactive count is repeated, it will probably *not* give the same result. This is because of the random nature of radioactive decays and not because of uncertainty in the measurement. For example, if a repeated count had an average of 9, it probably varied between 6 and 12, which means that a single measurement could have been unreliable. Larger counts are better. For example, if a repeated count had an average of 900, it probably varied between 870 and 930.

Tool 1: Experimental techniques

Recognize and address safety, ethical or environmental issues in an investigation

Nuclear radiation can be hazardous to humans and animals. Any experiment with radioactive materials must follow safety precautions, which include the following.

- The radioactive sources must be well marked and stored securely in lead-lined boxes. They should be used for as short a time as possible.
- All experiments should be done by, or supervised by, a teacher experienced with the appropriate procedures.
- Sources should be handled with tongs and never pointed directed towards anybody.
- Students watching a demonstration should be a safe distance away. (Nuclear radiation from a point source will be absorbed to some extent in air (depending on the type of radiation) and it will also spread out.)

However, radiation sources used in schools emit very low levels of radiation.

Figure E3.2 Radiation hazard sign

There are a number of things that could be investigated with the apparatus seen in Figure E3.1, including:

- How does the count rate vary when the distance between the GM tube and the source is changed?
- How is the count rate affected by placing various materials between the source and the GM tube?
- Does the count rate change with time?
- Is any count detected if the source is removed?
- Are the radiations affected by passing through electric or magnetic fields?
- How much radiation is emitted by the source every second?

◆ **Background radiation** Radiation from radioactive materials in rocks, soil and building materials, as well as cosmic radiation from space and any radiation escaping from artificial sources.

◆ **Background count** Measure of background radiation.

It is important to understand that there are tiny amounts of radioactive materials in almost everything around us (and in our bodies). These materials emit very low amounts of nuclear radiation which we are all *unavoidably* exposed to everyday. Under most circumstances, this **background radiation** is low enough to be considered completely harmless.

Because of background radiation, a GM tube and ratemeter, such as seen in Figure E3.1 will record a **background count**, even when there is no obvious source of radiation present. A typical count might be 0.25 to 0.5 s^{-1}. If an experiment is measuring low counts, the effect of this background count is significant and it should be deducted from all readings before they are processed.

WORKED EXAMPLE E3.1

In a radioactivity experiment, a count of 42 was recorded from a source in one minute. If the background count rate in that location was 0.44 s^{-1}, what was the value of the count from the source after it had been adjusted for background radiation?

Answer

$42 - (0.44 \times 60) = 16\,\text{min}^{-1}$

1 Give **two** reasons why it is better to use larger count rates in radioactivity experiments.

2 a A ratemeter recorded an average 400 counts per minute from repeated measurements. Use information from the previous 'Top tip' box to predict the range of the count rates detected.

  b Discuss which is better:
  - determine a count rate over ten minutes, or
  - calculate an average of ten one-minute measurements.

3 Research on the internet to find possible sources of background radiation.

4 In 15 minutes, a count of 5486 was measured when the GM tube was directed towards a radioactive source. It was known that the background count at that location was 18 per minute. Calculate the average count rate, per second, due to nuclear radiation coming directly from the source.

5 At a location where the background count was 22 min^{-1}, in separate experiments, count rates of 50 min^{-1} and 5000 min^{-1} were measured.
Compare the significance of the background counts in these experiments.

Alpha particles, beta particles and gamma rays

SYLLABUS CONTENT

- The penetration and ionizing ability of alpha particles, beta particles and gamma rays.
- The changes in the state of the nucleus following alpha, beta and gamma radioactive decay.
- The radioactive decay equations involving α, β^-, β^+, γ.
- The existence of neutrinos v and antineutrinos $\bar{v}$.

◆ **Beta particle** A high-speed electron that is released from a nucleus during beta negative decay, or a high-speed positron released during beta positive decay.

In a school laboratory we can detect three different kinds of radiation emitted from radionuclides:

- alpha particles: fast-moving helium-4 nucleus (2 protons and 2 neutrons tightly bound together), released from a nucleus during alpha decay
- **beta particles**
- gamma rays (usually associated with alpha or beta emission).

Atoms of the same radionuclide always emit the same types of radiation

ATL E3A: Communication skills

Using terminology, symbols and communication conventions consistently and correctly

Nuclear equations

The particle(s) before the reaction are shown on the left and the products shown on the right.

Nuclear equations must balance: the sum of the nucleon numbers and proton numbers must be equal on both sides of the equation.

Alpha particles

The composition of an alpha particle is the same as a helium-4 nucleus: the combination of two protons and two neutrons, which is very stable. It has a nucleon number of 4 and a proton number of +2. Alpha particles can be represented by the symbols ${}^{4}_{2}\alpha$ or ${}^{4}_{2}\mathrm{He}$.

Clearly the emission of an alpha particle results in the loss of two protons and two neutrons from a nucleus, so that the proton number of the nuclide decreases by two and a new element is formed (transmutation). This is represented in a generalized **radioactive decay equation** as follows:

$${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\alpha$$

parent nucleus $\longrightarrow$ daughter nucleus + alpha particle

◆ **Nuclear equation** An equation representing a nuclear reaction. The sum of nucleon numbers (A) on the left-hand side of the nuclear equation must equal the sum of the nucleon numbers on the right-hand side of the equation. Similarly with proton numbers (Z).

◆ **Radioactive decay equation** Balanced nuclear equation which shows a radionuclide and its decay products.

As an example, the decay of radium-226 results in the emission of an alpha particle:

$${}^{226}_{88}\mathrm{Ra} \longrightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\alpha$$

The change to a more stable nucleus is equivalent to a decrease in nuclear potential energy. This energy is transferred to the kinetic energy of the alpha particle (and a lesser amount to the daughter nucleus).

All alpha particles from the decay of radium-226 have exactly the same (kinetic) energy: 4.7 MeV, or 7.5×10^{-13} J. (Some radionuclides emit alpha particles with different, but discrete, energies. This is explained later in this topic for HL students.)

Assuming that there are only two particles after the decay, they must move (recoil) in exactly opposite directions. This is because of the law of conservation of momentum, which also predicts that the alpha particle will have more kinetic energy and a much faster speed, because it is the less massive particle.

One mole of radium (226 g) would release a total energy of: $6.02 \times 10^{23} \times 4.71 = 2.83 \times 10^{24}$ MeV. This is a lot of energy (4.53×10^{11} J) from a relatively small mass, but the energy will be released over a very long time (because the *half-life* of radium-226 is about 1600 years – the concept of half-life is explained later).

Radionuclides are not generally used to transfer large amounts of energy because they are both low power and expensive, but they can provide energy for a long period of time. Alpha sources