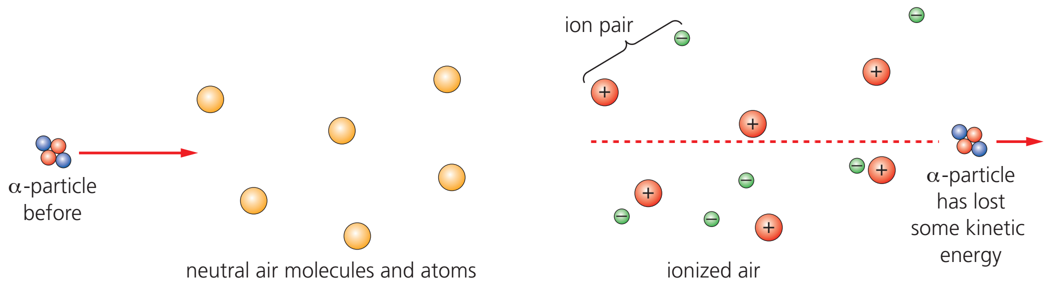
can be used to generate small amounts of electrical energy, or in places that are difficult to access, such that replacing a power source would be problematic. This includes some uses on satellites and space probes. But, as we will see later, another kind of nuclear reaction is capable of transferring large amounts of energy quickly – nuclear fission.

◆ **Penetrating power** The penetrating power of nuclear radiation depends upon the ionizing power of the radiation. The radiation continues to penetrate matter until it has lost (nearly) all of its energy. The greater the ionization per cm, the less penetrating power it will possess.

◆ **Ionizing ability** A measure of how much ionization is caused when a particular type of radiation passes through a material.

Penetrating power and ionizing ability

We have already noted in discussing the Geiger–Muller–Rutherford experiment (Topic E.2), that alpha particles have considerable kinetic energy (for a subatomic particle) however they have limited penetration of matter (**penetrating power**). This is because they transfer significant amounts of energy in collisions with other atoms / molecules. (Kinetic energy transfer is greatest when the colliding particles have comparable masses, as discussed in Topic A.3.) See Figure E3.3.



■ **Figure E3.3** Formation of ion pairs by alpha particles from molecules in the air

The collisions transfer the energy needed to ionize a large number of atoms / molecules in the material through which the alpha particles are passing. After most of their kinetic energy has been transferred, the alpha particles are effectively absorbed (as tiny amounts of helium). See Question 14 for a numerical example.

Typically, all the alpha particles emitted from a source will be absorbed by a few centimetres of air, or a sheet of paper (although they will mostly pass through much thinner gold foil). Alpha particles would be absorbed in the outer layers of skin, so that radioactive sources that only emit alpha particles are not considered to be dangerous outside of the human body. However, sources of alpha radiation that have been taken into the body (by eating, drinking or breathing) are a significant health hazard.

◆ **Antiparticle** Every particle has an antiparticle with opposite physical properties.

◆ **Antimatter** Matter consisting of antiparticles.

◆ **Antineutrino** Low-mass, uncharged and very weakly interacting particle emitted during beta-negative decay. Antiparticle of neutrino.

◆ **Annihilation** When a particle and its antiparticle interact, their mass is totally converted to electromagnetic energy.

◆ **Neutrino** Low-mass, and very weakly interacting uncharged particle emitted during beta-positive decay. Antiparticle of antineutrino.

Beta-negative particles

In an unstable nucleus it is possible for an uncharged neutron to be converted into a positive proton and a negative electron. This also involves the creation of another particle called an **antineutrino**, $\overline{v}$. An antineutrino is an example of an **antiparticle** (**antimatter**).

$$^{1}_{0}n \longrightarrow {}^{1}_{1}p + {}^{0}_{-1}e + \overline{v}$$

ATL E3B: Communication skills

Clearly communicating complex ideas in response to open-ended questions

Find out what physicists mean by 'antimatter'.

When particles of matter and antimatter collide, they destroy (**annihilate**) each other, with an enormous release of energy. Since we live in a universe which is made of matter, if any antimatter is created, it very quickly annihilates.

Antineutrinos (and **neutrinos**) are very small particles with no charge, which travel at speeds close to the speed of light, so they are *very* penetrating and *very* difficult to detect. (They cannot be detected in a school experiment.)

After the nuclear reaction shown in the equation above occurs, it is not possible for the newly formed electron to remain within the nucleus and it is ejected from the atom at a very high speed (close to the speed of light). It is then called a beta-negative particle and it is represented by the symbol ${}^{0}_{-1}\beta^{-}$ or ${}^{0}_{-1}e$.

When **beta-negative decay** occurs, the number of nucleons in the nucleus remains the same, but the number of protons increases by one, so that a new element is formed. This can be represented in a radioactive decay equation of the general form:

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\beta^{-} + \overline{\nu}$$

A typical example is the decay of a strontium-90 nuclide:

$$^{90}_{38}\mathrm{Sr} \rightarrow {}^{90}_{39}\mathrm{Y} + {}^{0}_{-1}\beta^{-} + \overline{\nu}$$

The beta particles in this decay have a range of energies up to 0.55 MeV.

Beta particles (unlike alpha particles) are emitted with a continuous range of different energies, but there is a well-defined maximum energy from any particular source, typically about 1 MeV.

Penetrating power and ionizing ability

Beta particles are considerably less massive than alpha particles, which means that they transfer less energy in ionizing collisions with atoms and molecules. Therefore, they travel further before they lose their kinetic energy and become absorbed.

Beta particles travel typically about 30 cm in air, although more energetic particles may go as far as one metre. They will mostly pass through a sheet of paper easily and their absorption in other materials is usually characterized by saying that a sheet of aluminium of thickness 3 mm will just absorb them all.

Sources of beta radiation can be dangerous if they enter the body, but they should also be considered as a possible health hazard if they are outside the body.

Top tip!

You will *not* be expected to remember the names of elements from their proton numbers.

◆ Beta-negative decay Radioactive decay resulting in the emission of an electron (and an antineutrino)

◆ Beta-positive decay Radioactive decay resulting in the emission of a positron (and a neutrino).

◆ Positron Antiparticle of the electron; released during beta-positive decay.

◆ Gamma radiation / ray Electromagnetic radiation (photons) emitted from some radionuclides and having an extremely short wavelength.

Beta-positive particles

In a similar process to beta-negative decay, called **beta-positive decay**, a proton in a nucleus can be converted into neutron and a positively charged electron, called a **positron** (another example of antimatter), which is then ejected from the atom, after which it is called a beta-positive particle and it is represented by the symbol ${}^{0}_{1}\beta^{+}$ or ${}^{0}_{1}e$.

A neutrino, ν, is created at the same time.

$$^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{1}\beta^{+} + \nu$$

The following equation represents a typical beta-positive decay:

$$^{23}_{12}\mathrm{Mg} \rightarrow {}^{23}_{11}\mathrm{Na} + {}^{0}_{1}\beta^{+} + \nu$$

Gamma rays

Gamma rays are high-frequency, high-energy electromagnetic radiation (photons) released from unstable nuclei. A typical wavelength is about 10^{-12} m. This corresponds to an energy of about 1 MeV (use $E = hc/\lambda$). Gamma rays are usually emitted after an unstable nucleus has emitted an alpha or beta particle. Gamma rays are represented by the symbol ${}^{0}_{0}\gamma$.

For example, when a thorium-234 nucleus is formed from a uranium-238 nucleus by alpha decay, the thorium nucleus contains excess energy and is said to be in an *excited state*. The excited thorium nucleus (shown by the symbol * in an equation) returns to a more stable state by emitting a gamma ray:

$$^{234}_{90}\mathrm{Th}^{*} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{0}_{0}\gamma$$

Because gamma rays have no mass or charge, the composition of the emitting nucleus does not change. There is no transmutation.

Penetrating power and ionizing ability

Gamma rays cause less ionization, so that they have much greater penetrating power than alpha particles or beta particles.

We usually assume that gamma rays are not significantly absorbed in air, but if a beam is spreading out, its intensity falls with distance, following an inverse square law (assuming that they come from a point source). At least a two centimetres thickness of solid lead is needed to absorb most gamma rays. Because they are so penetrating, gamma rays are less easy to detect than alpha particles and beta particles rays.

However, because all of their energy can be transferred in one interaction, gamma rays can cause significant chemical and biological changes when absorbed in the human body. Because they are so penetrating, sources outside the body can be as dangerous as sources inside the body.

LINKING QUESTION

- Are there differences between the photons emitted as a result of atomic versus nuclear transitions?

Summary of the properties of alpha, beta and gamma nuclear radiations

Table E3.1 Summary of properties of alpha, beta and gamma radiations

Property	Alpha (α)	Beta negative (β^-)	Beta positive (β^+)	Gamma (γ)
relative charge	+2	−1	+1	0
relative mass	4	1/1840	1/1840	0
typical range in air	4 cm	30 cm	very quickly annihilates	very little absorption in air
composition	helium nucleus	electron	positron	electromagnetic wave / photon
typical speed	$\approx 10^7\,\text{m s}^{-1} = 0.1c$	$\approx 2.5 \times 10^8\,\text{m s}^{-1} \approx 0.9c$	$\approx 2.5 \times 10^8\,\text{m s}^{-1} \approx 0.9c$	$3.00 \times 10^8\,\text{m s}^{-1} = c$
notation	${}^4_2\text{He}$ or ${}^4_2\alpha$	${}^{0}_{-1}e$ or ${}^{0}_{-1}\beta^-$	${}^{0}_{+1}e$ or ${}^{0}_{+1}\beta^+$	γ or ${}^0_0\gamma$
ionizing ability	very high	low	very quickly annihilates	very low
absorbed by	thick piece of paper	3 mm aluminium	very quickly annihilates	intensity halved by about 2 cm lead

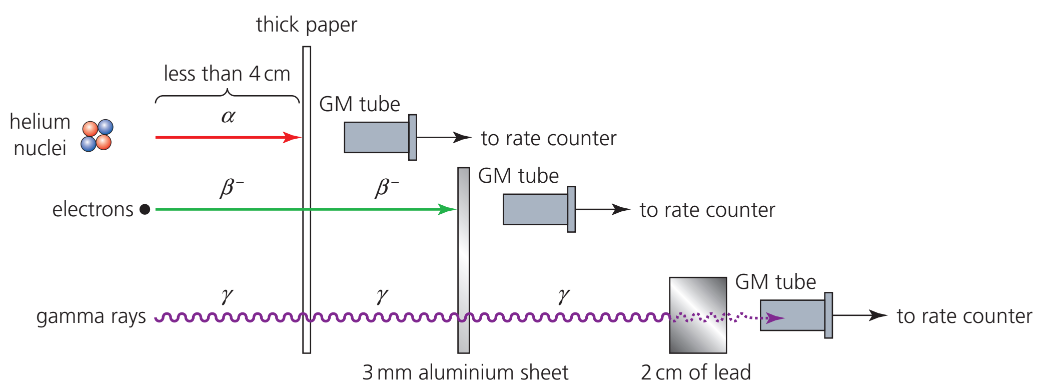


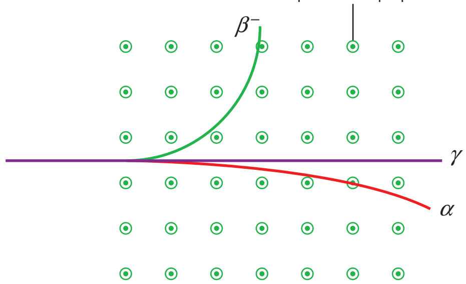
Figure E3.4 Absorption of ionizing radiations

Deflection of nuclear radiations in electric and magnetic fields

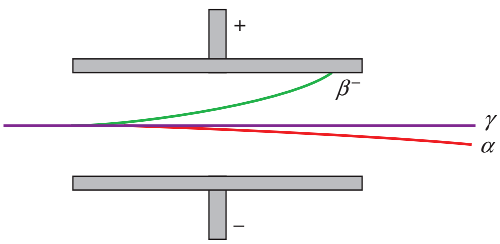
Alpha and beta radiation will be emitted in random directions from their sources, but they can be formed into narrow beams (collimated) by passing the radiation through slits.

Because a beam of alpha particles, or beta particles, is a flow of charge they will be deflected if they pass across a an electric field or magnetic field (as discussed in Theme D).

Gamma radiation is uncharged, so it cannot be deflected in this way.



■ **Figure E3.5** Behaviour of ionizing radiations in a magnetic field



■ **Figure E3.6** Behaviour of ionizing radiations in an electric field

Figure E3.5 shows the passage of the three types of ionizing radiation perpendicularly across a strong magnetic field. Fleming's left-hand rule can be applied to confirm the deflection of the alpha and beta particles into circular paths, the magnetic force providing the centripetal force. The radius of the path of a charged particle moving perpendicularly across a magnetic field can be calculated from:

$$r = \frac{mv}{qB} \text{ (Topic D.3).}$$

An alpha particle has twice the magnitude of charge and about 8000 times the mass of a beta particle, although a typical beta particle may be moving ten times faster. Taking all three factors into consideration, we can predict that the radius of an alpha particle's path may be about 400 times the radius of a beta particle in the same magnetic field: it is deflected much less. (Note that observation of the deflection of alpha particles will require a vacuum.)

Alpha and beta radiation can also be deflected by electric fields, as shown in Figure E3.6. Alpha particles are attracted to the negative plate; beta particles are attracted to the positive plate. The combination of constant speed in one direction, with a constant perpendicular force and acceleration, produces a parabolic trajectory. This is similar to the projectile movement discussed in Topic A.1. The deflection of the alpha particles is small in comparison to beta particles, due to the same factors as discussed for magnetic deflection.

6 **a** Assuming they have energies of 1.0 MeV, calculate the speeds of alpha particles (mass = 6.64×10^{-27} kg).

b What potential difference would be needed to accelerate doubly charged helium ions to the same energy from rest?

7 Explain why the distance before the thick paper in Figure E3.4 is labelled as 'less than 4 cm'.

8 Alpha particles usually carry more energy than beta particles, or gamma rays but, paradoxically, they are less penetrating. Explain why.

9 Explain why a source of alpha radiation outside the human body may be considered to be very low risk (for example, they are used in smoke detectors), but a source inside the body is considered dangerous.

10 Explain how a beam of beta particles can be distinguished experimentally from alpha particles and gamma rays.

11 Explain why gamma rays are considered to be particularly dangerous.

12 Calculate the amount of energy carried by a gamma ray photon of wavelength 2.6×10^{-12} m in **i** J and **ii** eV.

13 An adjusted count rate of 45 min^{-1} was detected from a gamma ray source when the GM tube was 20 cm from the source.
Predict what average count rate would be detected at a distance of:
a 40 cm **b** 100 cm **c** 10 m.

14 Alpha particles lose about 2.2×10^{-18} J of kinetic energy in each collision with an atom or molecule in the air.
An alpha particle travelling through air makes 7×10^4 ionizing collisions with molecules or atoms in the air for each centimetre of travel.
Calculate the approximate range of an alpha particle if the particle begins with an energy of 7.0×10^{-13} J.

15 Represent in a drawing a magnetic field acting perpendicularly into the paper.
Then draw a straight line down the page to represent the original direction of a beam containing alpha particles, beta particles and gamma rays passing through the field.
Finally, show in your drawing, what happens to the three different types of radiation as they pass through the magnetic field.

16 Discuss why beta particles are usually affected more than alpha particles as they pass through electric and magnetic fields.

17 Some beta-negative particles in a beam have an speed of $2.2 \times 10^8\,\mathrm{m\,s^{-1}}$. The beam passes perpendicularly across a magnetic field of strength 6.5 mT in a vacuum. Using an equation from Topic D.3, determine the radius of the arc of their circular path.

18 From Topic A.3, we know that kinetic energy, $E_k = \dfrac{p^2}{2m}$.

a Use the law of conservation of momentum to show that, after a stationary nucleus X emits an alpha particle, the kinetic energy of the alpha particle = $\dfrac{m_X}{m_\alpha} \times$ kinetic energy of X.

b Earlier in this section it was stated that the alpha particle emitted in the decay of radium-226 had a (kinetic) energy of 4.7 MeV.
Show that the total energy released in the decay is about 4.8 MeV.

Chart of the nuclides and decay series

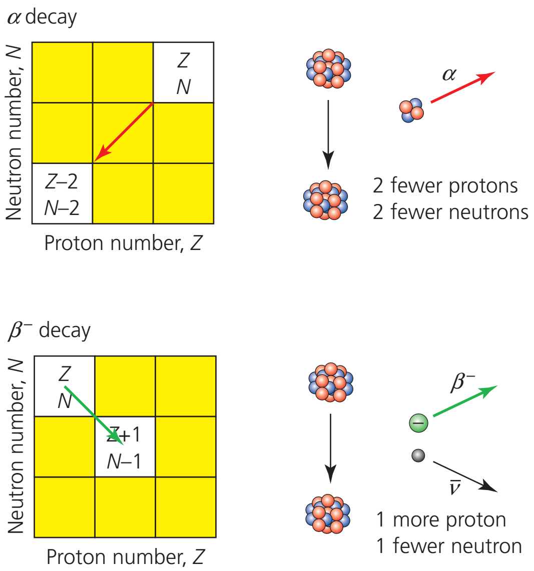
◆ **Chart of nuclides** A chart which displays every possible nuclide on axes of proton number and neutron number.

Every nuclide can be placed on a **chart of nuclides**, which has a square for every possible combination of proton number and neutron number.

This is a large chart, as can be seen in Figure E3.7, but the start of it can be seen in Figure E3.21 (it is discussed in greater detail later in this topic for HL students).

■ **Figure E3.7** A full chart of the nuclides contains a lot of data and requires a large wall!

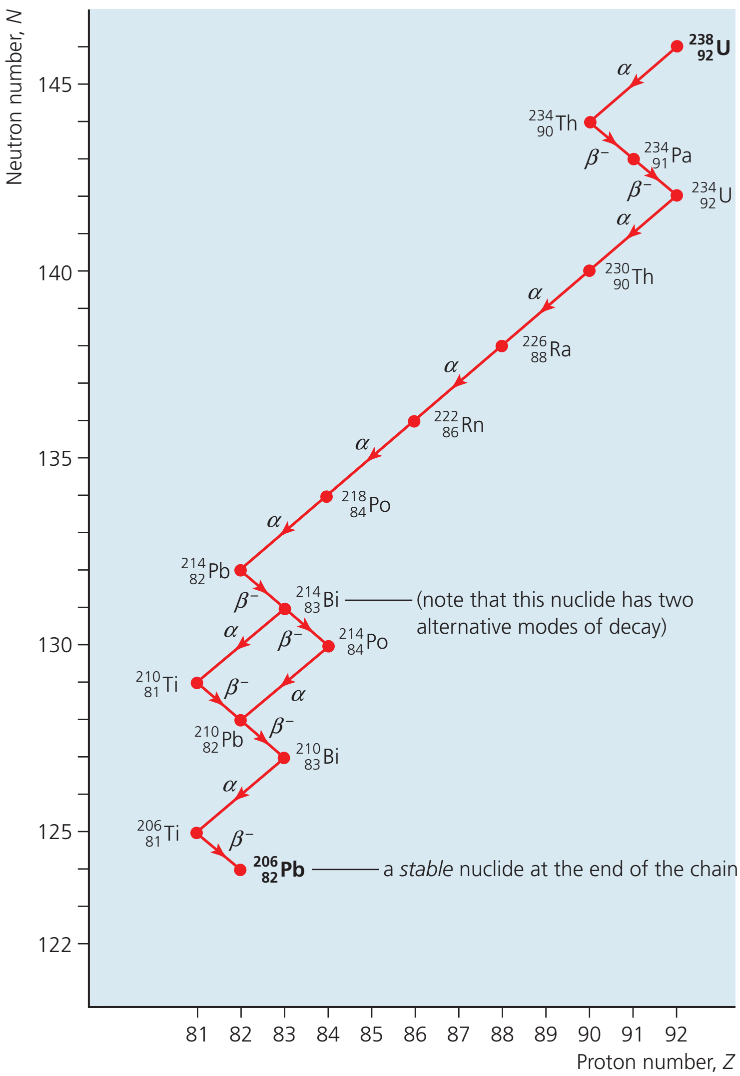
When a nucleus emits an alpha particle or beta particle, we know that it transmutes to a different nuclide. These changes can be tracked on a chart of the nuclides, as shown in Figure E3.8.



■ **Figure E3.8** Transmutation on a chart of the nuclides

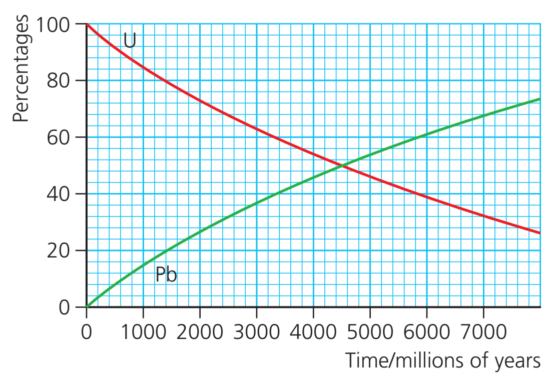
◆ **Decay series** A series of nuclides linked in a chain by radioactive decay. Each nuclide in the chain decays to the next until a stable nuclide is reached.

Heavy radioactive nuclides, such as radium-226 and uranium-238, cannot become stable by emitting just one particle. They undergo a radioactive **decay series**, producing either an alpha or a beta particle and maybe gamma radiation during each step, until a stable nuclide is formed. For example (not to be remembered!), uranium-238 undergoes the decay series seen in Figure E3.9 to eventually form the stable nuclide lead-206. Each decay will have its own particular half-life. (For the sake of clarity, the individual squares have not been included in the figure.)



■ **Figure E3.9** An example of a decay series (uranium-238) on a chart of nuclides

If it was possible to have a source of pure uranium-238, for example, it would immediately start decaying into other nuclides and, after some time, all the nuclides in the decay series would be present in the sample. The relative proportions of different nuclides depend on their half-lives. After a *very* long time most of the source will have turned into lead. Figure E3.10 is a rough indication of how the proportions of U-238 and Pb-206 change over billions of years.



■ **Figure E3.10** Uranium transmuting into lead

We should expect that most radioactive sources (of the heavier elements particularly) to contain a range of different nuclides.

19 Make a sketch, similar to those seen in Figure E3.8, to represent the transmutation that occurs as a result of beta-plus emission.

20 $^{222}_{86}$Rn, decays to Po-218. This radionuclide then emits an alpha particle to create an isotope of lead. Next in the decay series is Bi-214. Write out the full decay equations for these three nuclear reactions.

21 Use the internet to find out how the nuclide carbon-14 decays and then represent the process in a similar way to that shown in Figure E3.8.

Patterns of radioactive decay

SYLLABUS CONTENT

- Random and spontaneous nature of radioactive decay.
- Activity, count rate and half-life in radioactive decay.
- Changes in activity and count rate during radioactive decay using integral values of half-life.

Radioactivity comes from unstable nuclei, but when any particular nucleus will decay and emit a particle or radiation, is completely unpredictable and uncontrollable. At some point in time an unstable nucleus will decay, but there is no way that the process can be controlled by scientists. Temperature, for example, cannot be used to control nuclear reactions (unlike chemical reactions).

Imagine that we could observe the decay of a number of unstable nuclei (another 'thought experiment'):

◆ **Random** Without pattern or predictability.

◆ **Spontaneous** (decay, for example) Without any cause, cannot be controlled.

Individual nuclei do not decay in any pattern (the decays are **random**) and each decay occurs without any obvious cause, (the decays are **spontaneous**).

Paradoxically, such randomness and unpredictability on the scale of individual nuclei, results in predictability when we consider very large numbers of nuclei.

Nature of science: Patterns and trends

Randomness

This is not the first time that the random behaviour of particles has been discussed in this course. Our understanding of the physical properties of gases developed from an appreciation of the random motions of gas molecules. Although the individual motions of gas particles are random and unpredictable, over large numbers of particles (in bulk) we can observe patterns and trends in the properties of the gas.

In everyday life, the toss of a single coin or the throw of a single dice (die) are used to make an event random and unpredictable. However, if we toss a coin enough times, we can be sure that, to a close approximation, 50% will be 'heads' and 50% will be 'tails'. Similarly, if a six-sided dice is thrown, for example 100 times, then any particular number can be expected to occur about once in every six throws (about 17 times in 100 throws). The greater the number of events, the more precisely the outcome can be predicted.

The same principle can be applied to random nuclear decays: we might say, for example, that 50% of the nuclei of a particular nuclide in a source will decay during the next year.

Activity of a radioactive source

◆ **Activity, *A* (of a radioactive source)** The number of nuclei which decay in a given time (second).

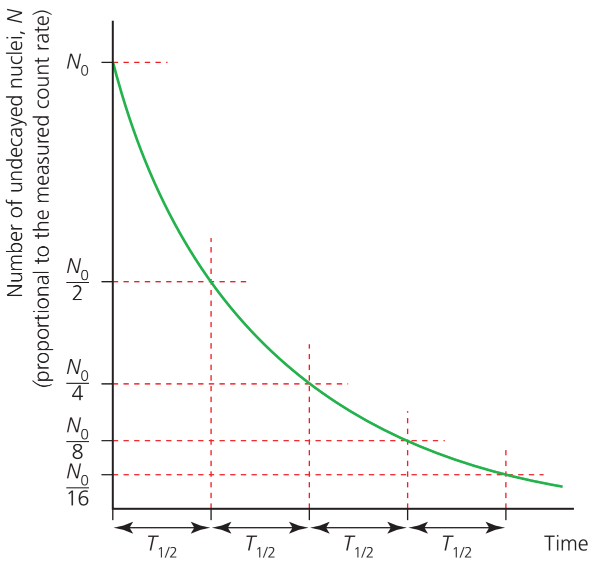
◆ **Becquerel, Bq** The SI unit for (radio) activity. 1 Bq = one nuclear decay every second.

The **activity**, A, of a radioactive source is the total number of nuclei decaying every second.

Activity may also be described as the rate of decay. We usually assume that the activity of a source is equal to the number of particles emitted every second. The SI unit of (radio) activity is the **becquerel**, Bq.

1 Bq is equivalent to one decay every second and is considered to be a very low activity.

The activity of a source is proportional to the number of undecayed atoms it contains.



■ **Figure E3.11** Radioactive decay curve

It should be noted that a *count rate* from a source (as being measured in Figure E3.1, for example) is *not* the same as its activity. This is because the GM tube is certainly not detecting all the radiation emitted. However, it is often assumed that a count rate is proportional to the activity.

The activity of all radioactive sources decreases with time. This is because the number of nuclei decaying every second (the activity) depends on the number of nuclei in the source which have not yet decayed. As more nuclei decay, the number remaining undecayed decreases, so the activity decreases. This reducing activity and count rate (adjusted for background count) are represented in Figure E3.11.

As explained above, half of the nuclei of any particular nuclide in a source will decay during a well-defined period of time. This is called the **half-life**, $T_{½}$, of the nuclide.

The half-life, $T_{½}$, of a radionuclide is the time it takes for half of its undecayed nuclei to decay. It is also the time taken for the activity (or count rate) to halve.

■ **Table E3.2** Half-life examples

Radionuclide	Half-life
uranium-238	4.5×10^9 years
radium-226	1.6×10^3 years
radon-222	3.8 days
francium-221	4.8 minutes
astatine-217	0.03 seconds

Half-lives of different radionuclides can be as short as fractions of a second, or as long as millions of years, or anything in between. See Table E3.2 for some diverse examples.

The graph seen in Figure E3.11 represents an **exponential decrease**: in equal intervals of time (shown clearly on the time axis) the count rate falls by the same fraction (one half): starting at N_0, then $N_0/2$, then $N_0/4$ and so on. In theory, for an exponential decrease, the count rate will never reduce to zero.

◆ **Half-life (radioactive)** The time taken for the activity, or count rate, from a pure source to be reduced to half. Also, equals the time taken for the number of radioactive atoms in a pure source to be reduced to half.

◆ **Exponential change** A change which occurs when the rate of change of a quantity at any time is proportional to the actual quantity at that moment. Can be an *increase* or a *decrease*.

Tool 3: Mathematics

Carry out calculations involving logarithmic and exponential functions

Any exponential decrease can be recognized by the fact that a quantity decreases to the same fraction in equal intervals of time. We usually refer to a quantity falling to *half* of its value at the end of each equal time interval, but the same behaviour also falls by *any* other chosen fraction in different, but equal, time intervals.

LINKING QUESTION

- Which areas of physics involve exponential change? (NOS)

Common mistake

Many people wrongly believe that the term ‘exponential’ is only used to describe rapid increases. However, exponential changes can also be decreases and they are just as likely to be slow: consider, for example, that uranium-238 has a half-life of about 4.5 billion years.

Figure E3.12 shows a visualization that may help understanding. The same information is displayed in Table E3.3. The radionuclide americium-242 has a half-life of 16 hours.

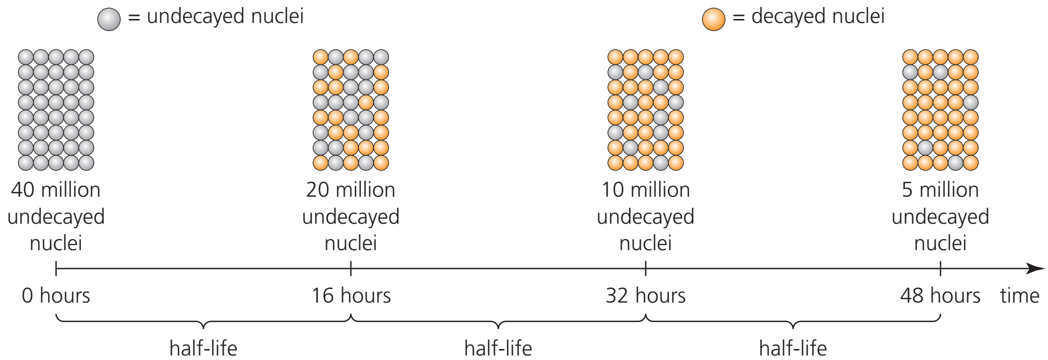


Figure E3.12 The radioactive decay of a sample of americium-242

Table E3.3

Number of undecayed nuclei	Fraction of original undecayed nuclei remaining	Number of decayed nuclei	Number of half-lives elapsed	Number of hours elapsed
40×10^6	1	0	0	0
20×10^6	$\frac{1}{2}$	20×10^6	1	16
10×10^6	$\frac{1}{4}$	30×10^6	2	32
5.0×10^6	$\frac{1}{8}$	35×10^6	3	48
2.5×10^6	$\frac{1}{16}$	37.5×10^6	4	64

WORKED EXAMPLE E3.2

Radium-226 has a half-life of 1620 years. A source which has a total mass of 0.010 g contains 30% of Ra-226 and no other radionuclides.

a Calculate the mass of Ra-226 that will remain in the source after 3240 years.

b Determine how many Ra-226 nuclei will have decayed in this time.

Answer

a After two half-lives, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of the unstable nuclei will remain ($\frac{3}{4}$ has decayed).

Mass of Ra-226 remaining $= \frac{1}{4} \times 0.30 \times 0.010 = 7.5 \times 10^{-4}$ g

b Mass of Ra-226 decayed $= \frac{3}{4} \times 0.30 \times 0.010 = 22.5 \times 10^{-4}$ g

226 g of radium-226 contain 6.02×10^{23} atoms (Avogadro constant)

$$\frac{22.5 \times 10^{-4}}{226} \times 6.02 \times 10^{23} = 5.99 \times 10^{18} \text{ nuclei.}$$

Experimental determination of half-life

In principle, the half-life of any radionuclide can be determined from a graph of count rate against time (as in Figure E3.11).

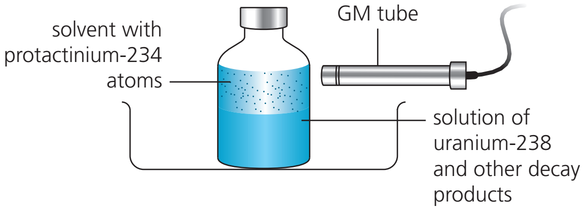


Figure E3.13 Measuring the half-life of protactinium-234

However, this can be difficult to obtain, for two reasons:

- Half-lives will usually be too short or too long for convenient measurement. For example, for a school experiment a half-life between a few minutes and few hours may be considered ideal, but there are not many obtainable radioisotopes that fit that description.
- When a nuclide decays it is probable that its daughter product will also be radioactive. This means that there will often be two (or more) radioisotopes with different half-lives in the same source.

The decay of protactinium-234 is widely used in schools as a demonstration of a half-life determination. (Details need not be remembered.) A compound of uranium-238 dissolved in water is contained in a very securely sealed plastic bottle. A separate layer contains a chemical which reacts with protactinium. U-238 decays to thorium-234 by emitting an alpha particle. The thorium then decays to protactinium-234 by beta-negative emission. Protactinium then decays to uranium-234 when beta-negative particles are emitted. These decays are shown below. This decay series should be understood, but not remembered.

$$^{238}_{92}\mathrm{U} \xrightarrow{\alpha} {}^{234}_{90}\mathrm{Th} \xrightarrow{\beta^-} {}^{234}_{91}\mathrm{Pa} \xrightarrow{\beta^-} {}^{234}_{92}\mathrm{Pa}$$

Of all the radionuclides present in the bottle, only Pa-234 has a suitable half-life for measurement. The protactinium compound can be separated chemically when the contents of the bottle are shaken up. The protactinium moves into the upper layer. See Figure E3.13.

Inquiry 2: Collecting and Processing data

Processing data

Table E3.4 shows the variation with time, *t*, of the count rate of a sample of a radioactive nuclide X. The average background count during the experiment was 36 $\mathrm{min^{-1}}$.

Table E3.4 Variation with time of the count rate of a sample of radioactive nuclide *X*

t/hour	0	1	2	3	4	5	6	7	8	9	10
Count rate/min^{-1}	854	752	688	576	544	486	448	396	362	334	284

Plot a graph to show the variation with time of the corrected count rate and use the graph to determine the half-life of nuclide *X*.

22 One hundred dice were thrown at the same time and all the dice that showed 6 were then removed. The remaining dice were thrown again and, again, all the 6s were removed. The process was repeated another five times.

a Draw a bar chart to represent the results you would expect.

b Explain why the shape of your chart should be similar to Figure E3.11.

23 Count rates detected every five minutes ($\mathrm{s^{-1}}$) were as follows: 75, 60, 48, 38, 31, 25. Assuming that these readings were adjusted for background count, do they represent an exponential decrease? Justify your answer.

24 The initial count rate from a sample of a radioactive nuclide is 560 $\mathrm{s^{-1}}$ (adjusted for background count). The half-life of the nuclide is 5 minutes. Sketch a graph to show how the activity of the sample changes over a time interval of 25 minutes.

25 Explain why it would be difficult for a laboratory to provide a school with a pure radioisotope with a half-life of about of about one hour.

26 A radioactive isotope has a half-life of eight days and the initial count rate is 996 $\mathrm{min^{-1}}$. If the average background count was 20 $\mathrm{min^{-1}}$, predict the count rate after 32 days.

27 a The half-life of francium-221 is 4.8 minutes. Calculate the fraction of a sample of francium-221 remaining undecayed after a time of 24.0 minutes.

b The half-life radon-222 is 3.8 days. Calculate the fraction of a sample of radon-222 that has decayed after 22.8 days.

c Cobalt-60 is used in many applications in which gamma radiation is required. The half-life of cobalt-60 is 5.26 years. A cobalt-60 source has an initial activity of 2.00×10^{15} Bq. Calculate its activity after 26.30 years.

d A radioactive element has a half-life of 80 minutes. Determine how long will it take for the count rate to decrease to 250 per minute if the initial count rate is 1000 per minute.

e The half-life of radium-226 is 1620 years. For an initial sample:

i calculate what fraction has decayed after 4860 years

ii what fraction remains undecayed after 6480 years?

28 Technetium-99 is a radioactive waste product from nuclear power stations. It has a half-life of about 212 000 years.

a *Estimate* the percentage of technetium that is still radioactive after one million years.

b *Approximately* how many years are needed for the activity from the technetium to fall to 1% of its original value?

◆ **Carbon dating** Using the radioactive decay of carbon-14 to estimate the age of once-living material.

◆ **Tracer (radioactive)** Radioisotope introduced into a system (for example, a human body) to track where it goes by detecting the radiation that it emits

Practical uses of radionuclides

Radioactive substances have a wide range of uses including:

- diagnosis of illness
- treatment of cancer
- food preservation and sterilization of medical equipment (see Question 35)
- determining the age of rocks (see Question 34)
- locating faults in metal structures, such as pipes (see Question 32)
- **carbon dating** (See Question 30)
- determining thicknesses (see Question 33)
- smoke detectors (see Question 31).

The choice of a suitable radionuclide for each of these applications requires careful consideration of the health risks involved, a suitable half-life and the penetrating power of the emitted radiation.

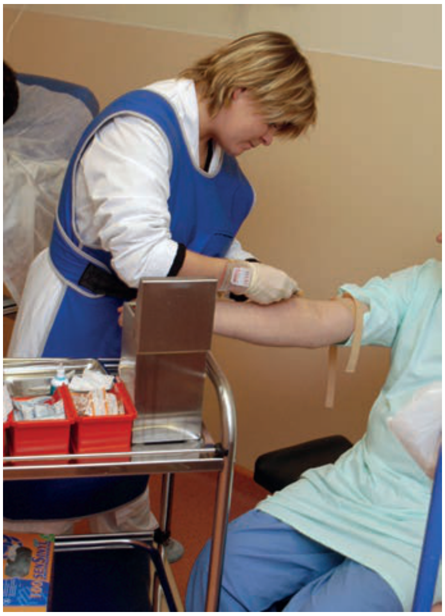


Figure E3.14 Injecting a radioactive tracer

We will look at one application in detail: medical tracers.

Medical tracers

Substances introduced into the body for the purpose of checking the functioning of particular organs are called medical **tracers**. They may be injected (see Figure E3.14) or swallowed. The radioactive substance most commonly used is technetium-99m. This is an excited nuclide produced from molybdenium-99 by beta decay. The decay of the Tc-99m has a half-life of six hours and the gamma ray photons emitted have an energy of 0.14 MeV. Both of these properties make technetium-99m a good choice for tracer studies. The amount of energy carried by the gamma photons makes them easy to detect outside of the body using a gamma 'camera'. The doctor will be able to use the pattern of gamma rays detected to compare the patient's organ to the pattern received from a fully healthy organ.

The half-life of 6.0 hours is a good compromise between activity lasting long enough to be useful and the need to expose the patient to ionizing radiation for as short a time period as possible. Atoms in a chemical compound which is preferentially taken up by the organ are replaced by radioactive substitutes. This process is often called 'labelling'.

29 **a** Determine the percentage of technetium-99m that will remain in a patient 24 hours after they have been injected with a tracer.

b Discuss why gamma ray sources are needed for medical tests similar to that seen in Figure E3.14.

30 Neutrons are continually created in the Earth's atmosphere by cosmic rays. The following nuclear reaction can then occur:

$^{14}_{7}N + ^{1}_{0}n \rightarrow ^{14}_{6}C + ^{1}_{1}H$

a Describe what this equation represents.

b Carbon-14 is radioactive and decays by beta-negative emission.
Write an equation for this decay.

c All living plants and animals contain many carbon atoms. A very small fraction of those atoms are C-14. This fraction remains constant while the plant or animal is alive (12 atoms of C-14 in every 1×10^{13} atoms of C-12).
Explain why this percentage will decrease after the plant or animal dies.

d If 12% of the atoms in a human body are carbon, and a body has about 7×10^{27} atoms, estimate how many radioactive carbon-14 atoms are in the body.

e C-14 has a half-life of approximately 5700 years. Predict how many years after death will the fraction of C-14 have fallen to an average of 0.15 atoms of C-14 in every 10^{13} atoms of C-12.

f **i** Explain how scientists can 'date' the age of once-living material using the presence of C-14.

ii Suggest one reason why the results of this process may have a large uncertainty.

31 Many smoke detectors (see Figure E3.15) contain a tiny amount (about one quarter of a microgram) of the radionuclide Am-241. An alarm automatically sounds if smoke comes between the Am-241 and a small radiation detector inside the unit.

■ **Figure E3.15** Smoke detector

a Suggest what kind of radiation is being used, and why.

b Discuss what would be an ideal half-life for the source.

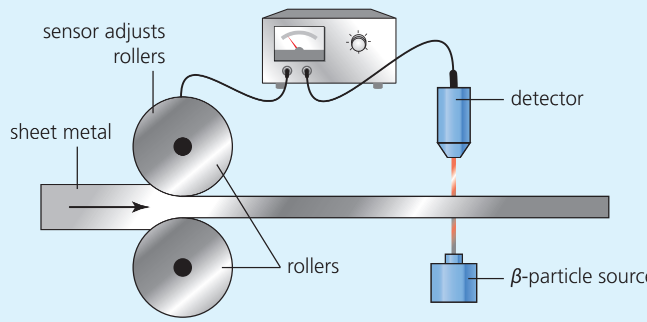
c Use the internet to determine the half-life of Am-241.

32 A leak from a pipe can be traced by using a radionuclide. Suggest how this can be done, including your choice for the type of radiation to be used and its half-life.

33 Figure E3.16 shows how the relative absorption of nuclear radiation in a metal sheet can be used to control its thickness during manufacture.

a Explain why beta particles are being used.

b Discuss whether the same source would be suitable for monitoring the production of rolls of paper or thin plastic.



■ **Figure E3.16** Using a radionuclide to control the manufacture of sheet metal.

34 Uranium-238 is a naturally occurring radionuclide with a half-life comparable to the age of the Earth.
It is widespread in the rocks of the Earth, but in relatively small quantities.

a Discuss whether U-238 in the rocks around us is a significant health hazard.

b Determine the approximate percentage of U-238 that remains since the creation of planet Earth.

c U-238 is the start of a long decay series. What stable nuclide is at the end of that series?

d Explain how the decay of U-238 can be used to obtain a value for the age of the Earth.

35 Explain why gamma rays can be used in hospitals to treat cancerous growths.

The energy inside a nucleus

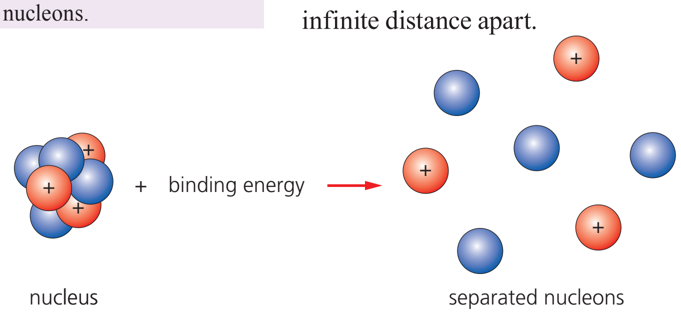
SYLLABUS CONTENT

- Existence of the strong nuclear force, a short-range attractive force between nucleons.
- Nuclear binding energy and mass defect.
- Variation of the binding energy per nucleon with nucleon number.
- The mass–energy equivalence as given by $E = mc^2$ in nuclear reactions.

◆ **Binding energy** The energy released when a nucleus is formed from its constituent nucleons. Alternatively, it is equal to the work required to completely separate the nucleons.

We have already seen (Topic E.1) that it is the *strong nuclear force* that holds nucleons together in a nucleus. Because of these forces, we consider that all nuclei have *nuclear potential energy*. Importantly, the magnitude of that energy is enormous, considering the small size of nuclei.

If we wanted to completely separate the nucleons of a nucleus (another thought experiment which is impossible in practice), we would need to supply energy. After the nucleons were separated, they would then effectively have zero nuclear potential energy. As with gravitational or electric forces, the zero of nuclear potential energy is defined as occurring when the particles concerned are an infinite distance apart.



■ **Figure E3.17** Binding energy is needed to separate nucleons; this example is lithium-7

The energy that would be needed to completely separate all the nucleons of a nucleus is called its nuclear **binding energy**.

Figure E3.17 shows an example.

Alternatively, we can consider that binding energy of a nucleus is the energy that would be *released* when the nucleus was formed from separate nucleons.

Nature of science: Models

Energy in bound systems

Consider any system in which there are attractive forces between the particles it contains. For example, between masses, between opposite charges, or between nucleons.

When the particles move, work will be done (provided the movement is not perpendicular to the force) and energy is transferred. We describe this as a change in the potential energy of the system.

If we wish to compare different systems, we need to agree on a common zero for potential energy: for this we choose the situation when the particles are a long way apart from each other (infinity), where the forces are zero.

Stationary separated particles which are free to move will be attracted closer together and gain kinetic energy and lose potential energy, so that their total energy remains the same. This implies that all potential energies in systems like these must be negative and that, if possible, any such systems will change to lower potential energy, when they could then be described as being more stable. In effect, this means that the potential energy of the system will change to a larger negative value as it becomes more stable.

If we wish to separate particles which are attracted together, for example nucleons in a nucleus, we need to do work (supply energy) so as to increase the potential energy of the system which, in effect, means to decrease the magnitude of negative potential energy. This can be confusing!

Nuclear binding energy is equivalent to nuclear potential energy in magnitude, but the concept of nuclear binding energy looks at the same situation in a different, perhaps less confusing, way. Binding energy is the energy that an *external agent* would need to supply to separate the nucleons (rather than being a property of the nucleus). Binding energy is always positive and using positive numbers is more intuitive, also larger positive binding energies corresponds well with greater stability.

A nucleus which had a potential energy of −50 MeV would be more stable than the same nucleus if it had a potential energy of −40 MeV. In terms of the binding energy for the same situations, we would say that a nucleus which had a binding energy of +50 MeV would be more stable than the same nucleus if it had a binding energy of +40 MeV.

The more nucleons in a nucleus (greater nucleon number, A), the greater the total binding energy will be.

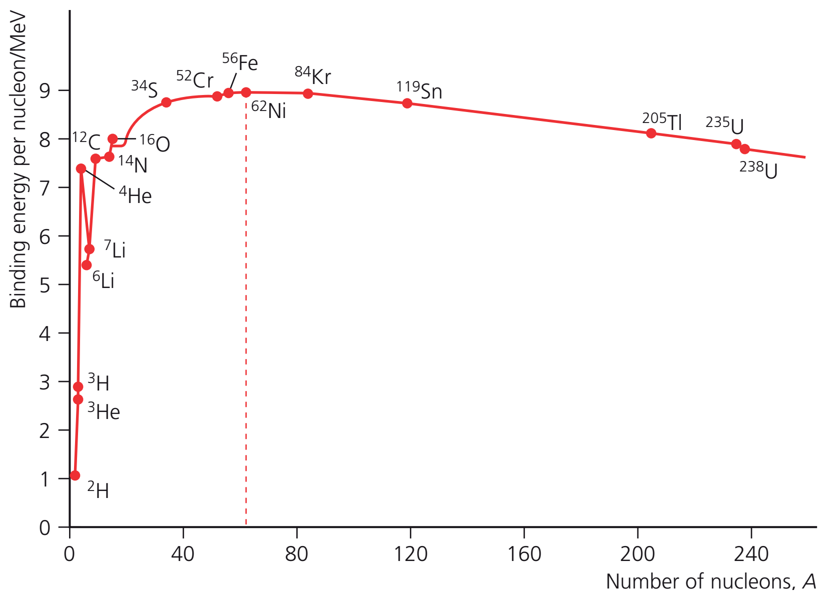
Binding energy is central to understanding nuclear events because when a nucleus changes in any way, the nuclear potential energy (binding energy) will also change. So, changes in binding energy can be more important than total binding energy, and in particular, the concept of average **binding energy per nucleon** is widely used.

◆ **Binding energy per nucleon (average)** Binding energy of a nucleus divided by the number of nucleons it contains. It is a measure of the stability of a nucleus.

$$\text{binding energy per nucleon} = \frac{\text{total binding energy}}{\text{number of nucleons in the nucleus}}$$

Binding energy per nucleon is a guide to a nucleus's stability.

Figure E3.18 shows how average binding energy per nucleon varies with nucleon number. A typical value is about 8 MeV per nucleon, but the variations seen in Figure E3.18 have important consequences, as we will explain later.



■ **Figure E3.18** A plot of binding energy per nucleon against number of nucleons

WORKED EXAMPLE E3.3

a Use Figure E3.18 to estimate the average binding energy per nucleon of the nuclide carbon-12.

b Calculate the total binding energy of the carbon-12 nuclide in:

i MeV **ii** joules.

c One mole of carbon atoms has a mass of 12 g.

Determine the total binding energy of that 12 g.

Answer

a 7.7 MeV/nucleon

b **i** $12 \times 7.7 = 92$ MeV (92.4 seen on calculator display)

ii $(92.4 \times 10^6) \times (1.60 \times 10^{-19}) = 1.5 \times 10^{-11}$ J (1.4784×10^{-11} J seen on calculator display)

c $(1.4784 \times 10^{-11}) \times (6.02 \times 10^{23}) = 8.9 \times 10^{12}$ J

This answer illustrates the truly enormous amount of nuclear energy associated with even a relatively small amount of matter. However, this energy is not usually accessible to us.

36 a Use Figure E3.18 to determine which nuclide is the most stable.

b Estimate the binding energy per nucleon of that nuclide.

37 a Use Figure E3.18 to estimate the binding energy per nucleon of oxygen-16.

b What is the total binding energy of the same nuclide?

38 If a nucleus of uranium-238 could be split in half to make two nuclei each of nucleon number 119 (this fission does not occur), estimate the change in overall binding energy.

39 If two hydrogen-2 nuclei could be combined to make one helium-4 nucleus, estimate the change in overall binding energy.

Nuclear fission and fusion

The last two questions illustrate two important types of nuclear reaction. Both of which can, under certain circumstances, release large amounts of energy from within a nucleus.

◆ **Nuclear fission** A nuclear reaction in which a massive nucleus splits into more stable smaller nuclei whose total binding energy is greater than the binding energy of the initial nucleus, with the release of energy.

◆ **Nuclear fusion** Nuclear reaction in which two low mass nuclei combine to form a more stable and more massive nucleus whose binding energy is greater than the combined binding energies of the initial nuclei, with the release of energy.

◆ **Energy–mass equivalence** Any mass is equivalent to a certain amount of energy, according to the equation $E = mc^2$.

Nuclear fission is the splitting of a massive nucleus into two smaller nuclei.

Topic E.4 is about nuclear fission.

Nuclear fusion is the combination of two small nuclei to produce a more massive single nucleus.

Topic E.5 discusses the process of nuclear fusion in stars.

Mass–energy equivalence

One of the consequences of Einstein's theory of special relativity was that particles (at rest) have intrinsic energy and that the amount of that energy, E, could be calculated from his famous equation:

$$E = mc^2$$

Where m is the mass of the particle in kilogrammes and c is the speed of light in m s^{-1}.

c^2 is a constant, and mass and energy are **equivalent** to each other. As Einstein said (in a film he made in 1948 called 'Atomic Physics'):

'Mass and energy are both but different manifestations of the same thing.'

If energy is added to, or removed from, a system by any means (examples: in chemical reactions, by heating or cooling, by changing speed, by moving up or down, and so on) then there will be a corresponding change in mass. Mass should not be considered to be an absolute, unchanging property of particles.

Top tip!

The equation $E = mc^2$ is being quoted here in its famous form. However, it may be more informative to express it as: $\Delta E = \Delta mc^2$, in order to stress that a *change* of energy, ΔE, is equivalent to a *change* in mass, Δm.

WORKED EXAMPLE E3.4

1.0 kg of water was raised in temperature by 10 °C. Calculate the corresponding change in mass. (Specific heat capacity of water is 4180 J °C^{-1} kg^{-1})

Answer

$Q = mc\Delta T = 1.0 \times 4180 \times 10 = 4.18 \times 10^4$ J. This amount of energy has increased the kinetic energy of the water molecules.

Then, $Q = E = mc^2$

$4.18 \times 10^4 = m \times (3.00 \times 10^8)^2$

Increase in mass $= 4.64 \times 10^{-13}$ kg

This is effectively unmeasurable and shows us that increases in masses involved in everyday activities are negligible. However, as we shall see, the changes in mass during nuclear reactions *are* significant.

Units of measurement for masses of atomic particles

◆ **Rest mass** Mass of an isolated particle that is at rest relative to the observer.

The **rest mass** of a particle is its mass as would be measured by an observer who is moving with the same velocity as the particle. That is, the particle would seem to be at rest as seen by the observer. This is the same as our usual understanding of mass.

The SI unit for mass is the kilogramme, but this may be considered to be an inconveniently large unit when quoting the values of the masses of atomic particles, as the following examples show:

- The rest mass of an isolated electron is $\mathbf{9.110 \times 10^{-31}}$ **kg**.
- The rest mass of an isolated proton is $\mathbf{1.673 \times 10^{-27}}$ **kg**.
- The rest mass of an isolated neutron is $\mathbf{1.675 \times 10^{-27}}$ **kg**.

◆ **Atomic mass unit (amu), u** Unit of mass widely used in atomic physics. Approximately equal to the mass of a single nucleon. Defined to be exactly $1.660\,539\,066\,60 \times 10^{-27}$ kg.

As an alternative to the kilogramme, the masses of nuclides and subatomic particles are more usually quoted in terms of the equivalent number of nucleons. The (unified) **atomic mass unit**, u, (amu) is intended to represent the mass of a proton or a neutron (which are very similar, as can be seen above), so that the oxygen-16 nuclide, which has 16 nucleons, would have a mass of 16 u. However, this is not accurate enough for most nuclear physics calculations, so that a more precise definition is needed, and the carbon–12 atom was chosen as the standard as follows. (The mass of oxygen-16 is then 15.994914 u, rather than 16.)

The atomic mass unit, u, is defined to be exactly one twelfth of the mass of an isolated carbon-12 atom, that is:

1 u = $\mathbf{1.661 \times 10^{-27}}$ **kg**

We can now restate the masses of electrons, protons and neutrons (using a large number of significant figures):

- The rest mass of an isolated electron is **0.000 549 u**.
- The rest mass of an isolated proton is **1.007 276 u**.
- The rest mass of an isolated neutron is **1.008 665 u**.

It is useful to know how much energy corresponds to a mass of 1 u:

$E = mc^2 = 1\text{ u} \times c^2 = (1.6605 \times 10^{-27}) \times (2.9979 \times 10^8)^2 = 1.4924 \times 10^{-10}$ J (using 5 significant figures)

Converting to eV: $\dfrac{1.4924 \times 10^{-10}}{1.6022 \times 10^{-19}} = 9.315 \times 10^8$ eV, or 931.5 MeV

Summarizing, $1\,u \times c^2 = 931.5\,MeV$, or:

$1\,u = 931.5\,MeV\,c^{-2}$

We now have a third way of expressing the masses of electrons, protons and neutrons:

- The rest mass of an isolated electron is **$0.511\,MeV\,c^{-2}$** ($0.000549\,u \times 931.5\,MeV\,c^{-2}$).
- The rest mass of an isolated proton is **$938\,MeV\,c^{-2}$**.
- The rest mass of an isolated neutron is **$940\,MeV\,c^{-2}$**

WORKED EXAMPLE E3.5

The rest mass of an alpha particle is 6.644657×10^{-27} kg. Express this in:

a atomic mass units

b $MeV\,c^{-2}$.

Answer

a $\dfrac{6.644657 \times 10^{-27}}{1.661 \times 10^{-27}} = 4.000\,u$

b $4.000 \times 931.5 = 3726\,MeV\,c^{-2}$

Nuclear reactions involve changes of mass

We have seen that any change of energy of a system is accompanied by an equivalent change in mass, but such changes of mass are immeasurably small in the events of everyday life. However, because the strong nuclear forces between nucleons are relatively large and involve small masses, all nuclear reactions will involve a significant change of mass.

If a nucleon system becomes more stable (which is the usual course of events), energy is released, total mass decreases and the total binding energy increases (because the nucleons have become harder to separate).

WORKED EXAMPLE E3.6

Consider the spontaneous decay of radium-226.

$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}\alpha$

After the decay, the nucleus has become more stable, so that the total binding energy has increased, and the two particles have gained kinetic energy.

Consider the masses on both sides of this equation:

- Rest mass of radium = 226.0254 u
- Rest mass of radon = 222.0176 u
- Rest mass of alpha particle = 4.0026 u

Calculate:

a the change of mass that occurs during this decay

b the total kinetic energy of the resulting particles (MeV).

Answer

a $222.0176 + 4.0026 - 226.0254 = -0.0052\,u$

b $0.0052 \times 931.5 = 4.84\,MeV$

Most of this energy is carried by the smaller mass, the alpha particle.

Mass defect

In our thought experiment, we have seen that energy (binding energy) would have to be supplied to completely separate all the nucleons in a nucleus. In total, the separated nucleons have more potential energy than when they were together in the nucleus. The equivalence of mass and energy informs us that the total mass of the separated nucleons must be more than their total mass when they were combined in the nucleus. In other words:

◆ **Mass defect** The difference in mass between a nucleus and the total mass of its nucleons if they were separated. Equal to nuclear binding energy.

The **mass defect** of a nucleus is the reduction in mass that occurs when separated nucleons combine together to form a nucleus. The mass defect is equivalent to the binding energy.

Common mistake

The term *mass defect* should only be used for the change in mass when *all* the nucleons are separated (equivalent to binding energy). For example, the decrease in mass which occurs during radioactive decay (see Worked example E3.6) should *not* be described as a mass defect.

WORKED EXAMPLE E3.7

Calculate the mass defect (in electronvolts) and binding energy of a helium-4 atom (mass 4.00260 u). It consists of two protons (each of mass 1.007276 u), two neutrons (each of mass 1.008665u) and two electrons (each of mass 0.000549 u).

Answer

The total mass of the separate particles =

$(2 \times 1.007\,276) + (2 \times 1.008\,665) + (2 \times 0.000\,549) = 4.032\,98\,\text{u}$

Mass defect $= 4.032\,98 - 4.002\,60 = 0.030\,38\,\text{u}$

$\Delta E = 0.030\,38 \times 931.5 = 28.30\,\text{MeV}$

WORKED EXAMPLE E3.8

A particular nucleus has a mass defect of 0.369 u.

a Calculate its binding energy in MeV.

b If it contains 40 nucleons, determine the average binding energy per nucleon.

Answer

a $0.369 \times 931.5 = 344\,\text{MeV}$ (343.72... seen on calculator display)

b $\dfrac{343.72}{40} = 8.59\,\text{MeV}$

40 The mass of a lithium-7 nucleus is 7.01600 u. Express this in:

a kilogrammes **b** MeV c^{-2}.

41 An aluminium-27 nucleus has a mass of 26.9815 u. Determine:

a its total binding energy

b its binding energy per nucleon.

42 A nuclide of $^{197}_{79}\text{Au}$ has a mass of 196.9665 u. Determine its mass defect.

43 Thorium-232 decays to form radium-228.

a What particle is emitted?

b Thorium-232 has a nuclear mass of 232.0381 u and radium-228 has a nuclear mass of 228.03107 u. Determine the energy released in this decay. (Alpha particle mass = 6.6447×10^{-27} kg)

44 A plutonium-239 nucleus can be split into smaller nuclei by interaction with a neutron as follows:

$^{239}_{94}\text{Pu} + {}^{1}_{0}n \rightarrow {}^{134}_{54}\text{Xe} + {}^{103}_{40}\text{Zr} + 3\,{}^{1}_{0}n$

a Determine the energy released in this reaction in MeV (mass of Pu-239 nucleus = 239.0522 u, mass of Xe-134 nucleus = 133.9054 u, mass of Zr-103 nucleus = 102.9266 u).

b State the form of this released energy.

45 The following equation represents one kind of nuclear reaction that occurs in stars.

$4\,{}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + 2\,{}^{0}_{1}e + 2\nu + {}^{0}_{0}\gamma$

Describe what is happening in this reaction.

The strong nuclear force and nuclear stability

◆ **Mesons** Unstable subatomic particles involved with the strong nuclear force.

SYLLABUS CONTENT

- Evidence for the strong nuclear force.
- Role of the ratio of neutrons to protons for the stability of nuclides.
- Approximate constancy of binding energy curve above a nucleon number of 60.

■ **Figure E3.19** Japanese physicist Hideki Yukawa in 1951

We have seen that the existence of an attractive *strong nuclear force* is needed to explain why the protons in a nucleus are not forced apart by the repulsion that occurs between similar charges.

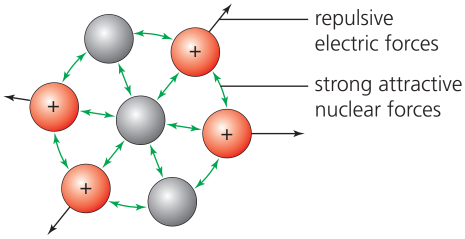
The Japanese physicist Hideki Yukawa (Figure E3.19) in 1935 proposed that the exchange of (as yet undiscovered) subatomic particles (called **mesons**) between nucleons was the cause of a *strong nuclear force* holding the nucleons together in a nucleus. His hypothesis was effectively proven when mesons were discovered in 1947.

In a stable nucleus, we can consider that the very short-range attractive strong nuclear forces are balanced by the longer range repulsive electric forces.

This is illustrated in Figure E3.20 with a nucleus which includes, as an example, four protons and three neutrons randomly arranged.

As can be seen in Figure E3.21, $^{7}_{4}\mathrm{Li}$ is a stable nuclide.

The stability of a nuclide depends on the ratio of neutrons to protons (*N/Z*) in its nucleus.



■ **Figure E3.20** Attractive and repulsive forces balance in a stable nucleus

Figure E3.21 shows a small part (the beginning) of a *chart of nuclides*. Each box on the chart corresponds to a particular nuclide. The boxes highlighted in blue show stable nuclides. The others have been produced artificially. The boxes highlighted in green show well-known radionuclides. A full chart of nuclides may show the relative abundances of the nuclides and the way in which unstable nuclides decay. (These charts are sometimes drawn with the axes reversed from that shown in Figure E3.21.)

Common mistake

Radioactivity is usually associated with the more massive nuclides, but Figure E3.21 illustrates the fact that *all* elements have radioisotopes.

Neutron number, *N*										
11					B-16	C-17	N-18	O-19	F-20	Ne-21
10				Be-14	B-15	C-16	N-17	O-18	F-19	Ne-20
9			Li-12		B-14	C-15	N-16	O-17	F-18	Ne-19
8		He-10	Li-11	Be-12	B-13	C-14	N-15	O-16	F-17	Ne-18
7		He-9	Li-10	Be-11	B-12	C-13	N-14	O-15	F-16	Ne-17
6		He-8	Li-9	Be-10	B-11	C-12	N-13	O-14	F-15	Ne-16
5		He-7	Li-8	Be-9	B-10	C-11	N-12	O-13	F-14	Ne-15
4		He-6	Li-7	Be-8	B-9	C-10	N-11	O-12		
3		He-5	Li-6	Be-7	B-8	C-9	N-10			
2	H-3	He-4	Li-5	Be-6	B-7	C-8				
1	H-2	He-3	Li-4							
0	H-1									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	Proton number, *Z*									

■ **Figure E3.21** A chart of the nuclides for $Z \leq 10$

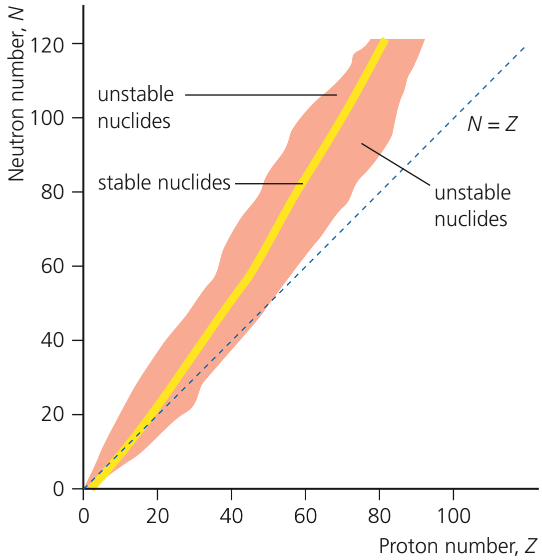


Figure E3.22 Stable nuclei shown on a chart of the nuclides

Figure E3.22 shows the overall pattern seen on a full chart of nuclides.

It is possible to identify several trends within the full chart of nuclides. Most importantly, from Figure E3.22, we can see that the neutron / proton (N/Z) ratio of a nuclide is a rough guide to its possible stability. $^{208}_{82}Pb$ is the nuclide with the largest nucleon number which is stable.

For nuclides with $Z < 20$, stable nuclides have $N/Z \approx 1$; For larger nuclides N/Z gradually increases to a maximum of about 1.5.

The reason for the increasing N/Z ratio of stable nuclei is as follows. In smaller nuclei with fewer nucleons, the *short-range* attractive nuclear force from any particular nucleon will have some effect on all the other surrounding nucleons. However, in larger nuclei some nucleons will be far enough apart from each other that the nuclear forces between them become less significant. The *longer range* repulsive coulomb force between protons could then make the nucleus unstable. The addition of extra neutrons (affected by the attractive nuclear force, but not the repulsive coulomb force) results in stability.

Variations in binding energy per nucleon

Adding more nucleons to a nucleus clearly increases its *total* binding energy, but we need to have some understanding of why the *average* binding energy per nucleon (and nuclear stability) varies as seen in Figure E3.22.

Imagine adding a nucleon to a nucleus *which only has a few nucleons*: the strong nuclear force will act between the additional nucleon and its closest 'neighbour', but also the surrounding nucleons. The total binding energy will increase. Then, imagine another nucleon is added: the total binding energy will increase by more than the previous amount because the strong nuclear force is affecting more nucleons close to the new nucleon. In this way, the average binding energy per nucleon increases.

However, because of the short range of the strong nuclear force, its effect on nucleons that are *not* relatively close together is insignificant. This explains why:

adding neutrons to larger nuclei with $A >$ about 60 increases the total binding energy, but has little effect on the binding energy per nucleon.

46 Calculate the N/Z ratios for the following stable nuclides:
a $^{12}_{6}C$ **b** $^{107}_{47}Ag$ **c** $^{208}_{82}Pb$.

47 Estimate the number of nucleons in stable isotopes of the following elements:
a boron ($Z = 5$)
b bromine ($Z = 35$)
c mercury ($Z = 80$).

48 Refer to Figure E3.21.
Suggest why the following nuclides are unstable:
a carbon-14
b nitrogen-12.

49 Explain why a more massive nucleus needs more neutrons per proton (than a less massive nucleus) in order to be stable.

50 Describe the variation of binding energy per nucleon shown in Figure E3.18.

51 **a** Use Figure E3.18 to estimate the binding energy per nucleon for nuclides with nucleon numbers 100, 150 and 200.
b Do you agree that your answers are 'approximately constant'?
c Suggest why binding energy per nucleon shows much greater variation for smaller nuclei.

HL ONLY

What can we learn from the spectra of alpha, beta and gamma radiations?

SYLLABUS CONTENT

- The spectrum of alpha and gamma radiations provides evidence for discrete nuclear energy levels.
- The continuous spectrum of beta decay as evidence for the neutrino.

Alpha particle spectrum

Many radionuclides which emit alpha particles, emit them with only *one* precise energy. However, some radionuclides can emit alpha particles with different energies as displayed in an **alpha particle spectrum**. This provides physicists with important information about energy levels within nuclei.

The *spectrum* of nuclear radiations emitted from an unstable nucleus describes the relative numbers of particles emitted with different energies.

◆ **Alpha particle spectrum** The range of discrete energies possessed by alpha particles emitted from the same radionuclide.

◆ **Excited state** When a particle is at a higher energy level than its ground state.

For example, nuclei of americium-241 emit alpha particles in the process of decaying to nuclei of neptunium-237.

$$^{241}_{95}\mathrm{Am} \rightarrow {}^{237}_{93}\mathrm{Np} + {}^{4}_{2}\alpha$$

An alpha particle emitted from americium-241 will have one of the following energies:

- 5.389 MeV (1.0%)
- 5.443 MeV (12.5%)
- 5.486 MeV (86.0%)
- 5.512 MeV (0.2%)
- 5.545 MeV (0.3%).

The spectrum is shown in Figure E3.23.

It is important to realize that the energy of any emitted alpha particle can only have one of these energies. The energies are *discrete* and the spectrum is not continuous. All alpha particles have discrete energies and all alpha particles from a particular radionuclide will have the same energies.

Alpha particles with different energies are possible because the nucleus of the daughter product (neptunium-237) can be left in its ground state, or in one of several discrete **excited states** (an energy state above the ground state). Figure E3.24 shows the emission of alpha particles of five different energies from americium-241 to various nuclear energy levels of neptunium-237. The total energy released from the Am-241 nucleus is always 5.545 MeV.

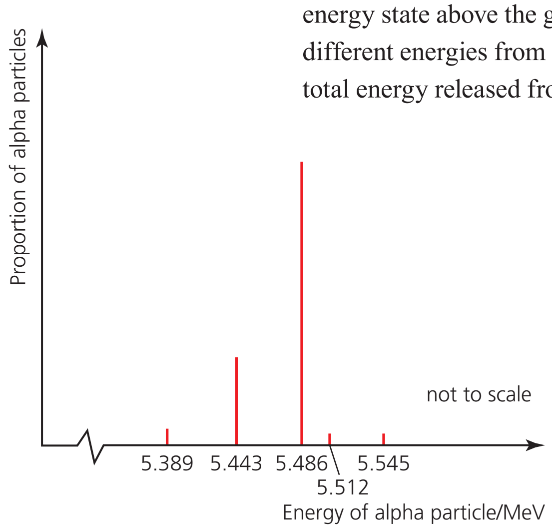


Figure E3.23 Spectrum of alpha particles

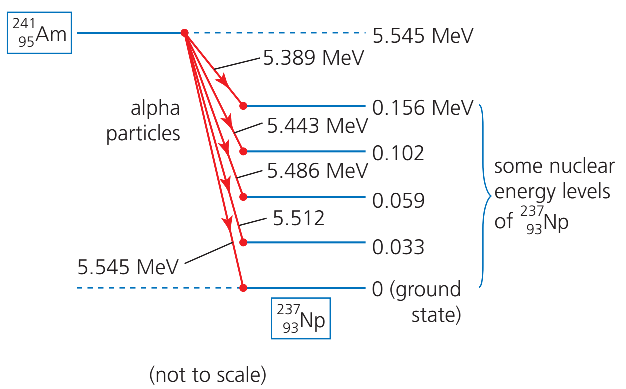
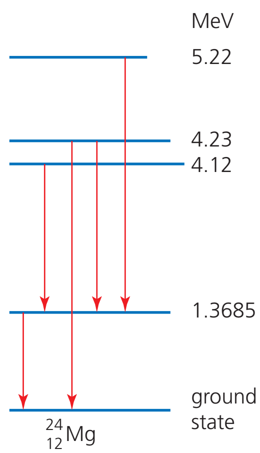


Figure E3.24 Energies of alpha particles emitted from americium-241

◆ **Gamma ray spectrum** Range of discrete photon energies that may be emitted from a single radionuclide.



■ **Figure E3.25** Excited energy levels within the magnesium-24 nuclide

LINKING QUESTION

- How did conservation lead to experimental evidence of the neutrino? (NOS)

This question links to understandings in Topics A.2 and A.3.

Gamma ray spectrum

Consider Figure E3.24 again. After the alpha decays, four excited states of the neptunium-237 nuclide can be seen. Afterwards, when a nucleus changes from an excited state to a lower energy level, a gamma ray photon will be emitted. The nucleus cannot emit a continuous range of gamma rays. The discrete energies of the photons (as seen in a **gamma ray spectrum**) again provide evidence for discrete energy levels within nuclei.

Figure E3.25 shows another example, nuclear energy levels within the magnesium-24 nuclide after beta-negative decay from sodium 24 left the nucleus in an excited state. Five prominent transitions to lower energy levels have been shown.

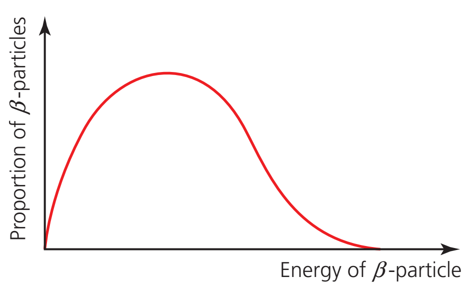
The discrete energy levels of nuclei are the reason why alpha particles and gamma rays are emitted with discrete energies.

Beta particle spectrum

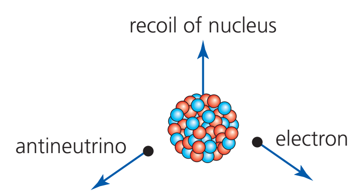
The **spectrum of beta particles** emitted from a particular radionuclide is very different from alpha particles. See Figure E3.26 and compare it to Figure E3.23.

This range of beta particle energies was puzzling for physicists: why were beta particles different from alpha particles, which all have the same energy(s) from the same source? Without further information, it appeared that beta particle emission could break the laws of conservation of energy and momentum.

We now know that when scientists first detected the emission of beta-negative particles (1899), they were unaware of the undetected antineutrinos which were also emitted. It was more than thirty years, in 1930, before the Austrian physicist Wolfgang Pauli hypothesized the involvement of unknown particles in beta decay. See Figure E3.27, which shows possible relative motions of the three particles involved after the beta-negative decay of cobalt-60. The conservation of energy and momentum can only be explained by the emissions of *two* particles from the nucleus at varying angles. The energies of the beta particles will vary with the angles involved.



■ **Figure E3.26** Typical energy spectrum for beta decay



■ **Figure E3.27** Beta-negative decay

The still undiscovered particles were named neutrinos (and antineutrinos) because it was believed that they must be uncharged and have a very small mass – the reasons why they are very hard to detect. Their existence was not confirmed until 1956.

◆ **Beta particle spectra** The continuous range of different energies possessed by beta particles emitted from the same radionuclide.

The emission of a beta particle involves another particle (an undetected neutrino or antineutrino). The particles may travel in random directions, so that a continuous range of beta particle energies is possible.

ATL E3C: Research skills

Using search engines and libraries effectively

It is believed that neutrinos are the second most common type of particle in the Universe (after photons). It is estimated that over 10^{12} neutrinos pass through a fingernail every second. They usually pass through the entire Earth without being affected or detected.

Use the internet to learn about the Long Base-line Neutrino Experiment / DUNE, which is yet to be completed in the USA.

Figure E3.28 The building at the top of the 'ice cube' neutrino detector, which is deep underground near the South Pole

TOK

Knowledge and the knower

Of course, scientists involved in research (for example, into subatomic particles such as the neutrino) will claim to be open-minded and receptive to new ideas. Undoubtedly, this is a primary aim of all scientists, but to some extent we are all inevitably influenced and restricted by our previous experiences, and the culture of the society in which we live.

Expensive projects need to have specific aims (or governments would not provide funding), and these will focus and direct the thinking of the scientists involved. However, it is clear from the development of science over the centuries that it is in the nature of science and scientists to use their imagination to formulate new and original ideas. It is possible that no 'single truth' about neutrinos (in the sense of certain knowledge for all time) will emerge from the latest research, although it is to be hoped that our understanding of these elusive fundamental particles will expand.

52 Figure E3.29 shows four possible alpha particle decays for a plutonium-238 nuclide.

a If the most energetic alpha particle has an energy of 5.510 MeV, determine the energies of the other three.

b After an alpha decay, the daughter nuclide (uranium-234) is in an excited state.
Calculate the highest frequency of gamma ray that could then be emitted.

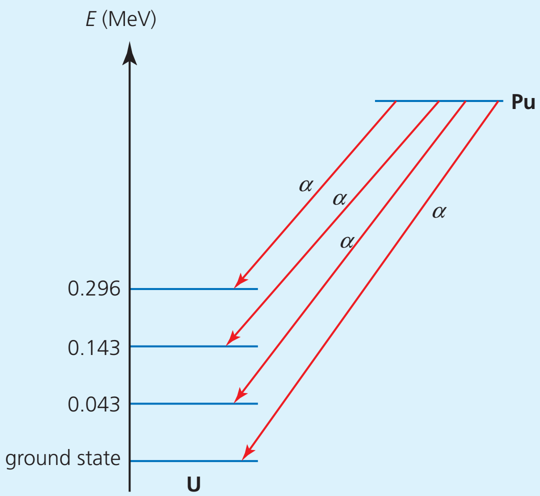


Figure E3.29 Four possible alpha particle decays for a plutonium-238 nuclide to uranium-234

53 a If an isolated neutron at rest decayed into a proton and an electron (an antineutrino is also emitted), estimate the energy (MeV) that would be released.

b Explain why most of this energy will be transferred to the kinetic energy of the electron (beta-negative particle).

54 Compare the magnitude of typical nuclear energy levels with typical electron energy levels.

55 The radionuclide ${}^{15}_{6}C$ undergoes beta-negative decay. The excited state of the daughter product then emits a gamma ray as it moves down to its ground state.
Sketch an energy level diagram to represent these changes.

56 Suppose that it is known that a particular nuclide emits gamma ray photons of energies 0.58 MeV, 0.39 MeV and 0.31 MeV when transitioning to the ground state.
Determine the energies of another three different photons that might be emitted from the same nucleus.

Radioactive decay in more mathematical detail

SYLLABUS CONTENT

- The decay constant λ and the radioactive decay law as given by: $N = N_0 e^{-\lambda t}$.
- The decay constant approximates the probability of decay in unit time only in the limit of sufficiently small λt.
- Activity as the rate of decay as given by: $A = \lambda N = \lambda N_0 e^{-\lambda t}$.
- The relationship between half-life and the decay constant as given by $T_{½} = \dfrac{\ln 2}{\lambda}$.

Earlier in this topic the concept of *half-life* was introduced as a way of representing the pattern of decreasing activity, or count rate, from a radioactive source. We now want to develop that idea so that we can determine values of activity, or count rate, at *any time,* not just for times which are whole number multiples of the half-life. For example, we may wish to know what the activity of a source, which has a half-life of 5.3 years, will be in a year's time.

If a quantity, N, changes by an amount ΔN in a time Δt, the rate of change is $\Delta N/\Delta t$. There are many examples in science, and in everyday life, where a rate of change at any time depends on the quantity at that time. If a rate of change is always proportional to the quantity, it is described as an *exponential change*:

$$\frac{\Delta N}{\Delta t} \propto N$$

Putting in a constant, we get:

$$\frac{\Delta N}{\Delta t} = \lambda N$$

For radioactive decay (an exponential decrease), ΔN will be negative, so that:

◆ **Decay constant, λ,** The probability of decay of an unstable nucleus per unit time: $\lambda = (-\Delta N/N)/\Delta t$ (SI unit: s^{-1}).

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

where λ (which is positive) is called the **decay constant**.

Decay constant, $\lambda = -\dfrac{\Delta N/N}{\Delta t}$ = probability of a nucleus decaying in unit time

SI unit: s^{-1}

For example, if $N = 200$ and in time $\Delta t = 1$ s, N decreases by 5, $\Delta N = -5$, then:

$$\lambda = -\frac{\Delta N/N}{\Delta t} = -\frac{-5/200}{1} = 0.025\ \text{s}^{-1}$$

However, we need to be careful when making calculations like this, because if $\dfrac{\Delta N}{N}$ ($= -\lambda \Delta t$) is too large, it will vary significantly during time Δt.

Larger values of a decay constant correspond to quicker decreases and shorter half-lives.

We saw earlier in this topic that the activity, A, of a radioactive source is the number of nuclei decaying every second (unit: Bq). We can write this as:

$$A = \frac{\Delta N}{\Delta t}$$

so that:

DB

activity, $\boldsymbol{A = \lambda N}$

WORKED EXAMPLE E3.9

The activity of a radioactive sample is 2.5×10^5 Bq. The sample has a decay constant of $1.8 \times 10^{-6}\ s^{-1}$. Determine the number of undecayed nuclei remaining in the sample at that time.

Answer

$A = \lambda N$

$2.5 \times 10^5 = (1.8 \times 10^{-6}) \times N$

$$N = \frac{2.5 \times 10^5}{1.8 \times 10^{-6}}$$

$= 1.4 \times 10^{11}$ undecayed nuclei.

WORKED EXAMPLE E3.10

The radionuclide Tc-99 has a decay constant of $0.115\ h^{-1}$.

Calculate the percentage of the nuclei that decay every:

a hour

b minute (assume the activity is constant over the hour).

Answer

a $$\lambda = -\frac{\Delta N/N}{\Delta t}$$

$$0.115 = -\frac{\Delta N/N}{1.0}$$

$$-\left(\frac{\Delta N}{N}\right) = 0.115$$

which is equivalent to 11.5 %

b $$\frac{11.5}{60} = 0.192\ \%$$

◆ **Law of radioactive decay** The rate of decay is proportional to the number of undecayed nuclei, $\Delta N/\Delta t = -\lambda N$.
Exponential decay equations $N = N_0 e^{-\lambda t}$. N_0 is the number of undecayed nuclei at the start of time t and N is the number remaining at the end of time t. Alternatively, equations of the same form can be used with activity, A, or the count rate. Activity is linked to the initial number of atoms by the equation $A = \lambda N_0 e^{-\lambda t}$.

Solutions to the decay equation

Although the equation $\frac{\Delta N}{\Delta t} = -\lambda N$ (the **law of radioactive decay**) defines the mathematics of radioactive decay, it does not directly provide us with what we are most likely to want to know – the value of N, A, or a count rate at *any* time, t. For that we can use the following equations, which may be described as solutions to the previous equation. You are *not* expected to understand the origin of these **exponential decay equations**.

DB

number of undecayed nuclei, $\mathbf{N = N_0 e^{-\lambda t}}$

In this equation, N_0 represents the number of undecayed nuclei in a source at the beginning of a time t, and N represents the number of undecayed nuclei at the end of time t. Similarly:

activity, $A = A_0 e^{-\lambda t}$

where A_0 represents the activity from a source at the beginning of a time t, and A represents the activity at the end of time t.

Since $A = \lambda N$, the equation can also be expressed as:

DB

activity, $\mathbf{A = \lambda N_0 e^{-\lambda t}}$

The activity of a source is not easily determined, as is explained below. The count rate, C, *(a non-standard symbol)* measured by a radiation detector in a laboratory is not directly measuring the activity of the source, but it will usually be proportional to the activity, so that we can also write:

$C = C_0 e^{-\lambda t}$

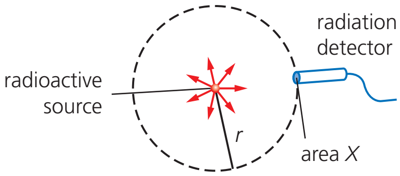


Figure E3.30 A radiation detector receives only a fraction of the radiation emitted from a source

Figure E3.30 shows that a detector can only receive some of the total radiation from a source.

We could use the ratio $X/4\pi r^2$ to determine a value for the fraction of the radiation emitted that arrives at the detector, but that would assume that:

- No radiation was absorbed between the source and the detector.
- The radiation was emitted equally in all directions.

Determining the activity of the source would also be difficult because not all of the radiation passing into the detector will be counted.

Tool 3: Mathematics

Carry out calculations involving logarithmic and exponential functions

$N = N_0 e^{-\lambda t}$

Suppose we are given N_0 and λ and wish to find a value for N at a known time, t, later:

Take natural logarithms of both sides:

$\ln N = \ln N_0 - \lambda t$

Suppose we are given N_0 and N at a known time, t, later, and wish to find a value for λ.

Divide both sides by N_0:

$$\frac{N}{N_0} = e^{-\lambda t}$$

Take natural logarithms of both sides:

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t \quad \text{or,} \quad \ln\left(\frac{N_0}{N}\right) = +\lambda t$$

WORKED EXAMPLE E3.11

The number of radioactive atoms in a source decays by 7/8 in 12 days. Predict the fraction of radioactive atoms remaining after 24 days.

Answer

This can be done in a straightforward way without using exponentials: the source passes through three half-lives to reduce to 1/8, so the half-life is four days. In 24 days, there are six half-lives, so the fraction reduces to 1/64 ($1/2^6$).

We could get the same answer using the exponential equation $N = N_0 e^{-\lambda t}$.

After 12 days: $N/N_0 = 1/8 = e^{-\lambda(12)}$

$8 = e^{12\lambda}$

$\ln 8 = 12\lambda$

$\lambda = \ln 8/12 = 0.173$

After 24 days: $N/N_0 = e^{-0.173(24)} = 0.0157 = 1/64$

WORKED EXAMPLE E3.12

The decay constant for a radioisotope is $0.054\,\text{y}^{-1}$. If the activity at the start of year 2022 was 470 Bq, calculate the activity at the start of year 2026.

Answer

$A = A_0 e^{-\lambda t}$

$A_{2026} = 470 \times e^{-0.054 \times 4}$

$\ln A_{2026} = \ln 470 - (0.054 \times 4)$

$A_{2026} = 379\,\text{Bq}$

57 A sample of radium-226 contains 6.64×10^{23} radioactive atoms. It emits alpha particles and has a decay constant of $1.38 \times 10^{-11}\,s^{-1}$. Determine how many atoms of radium-226 are left after 1000 years.

58 A radioactive nuclide has a decay constant of $0.0126\,s^{-1}$. Initially a sample of the nuclide contains 1.0×10^{10} nuclei.

a Calculate the initial activity of the sample.

b Predict how many nuclei remain undecayed after 200 s.

59 a The count rate from a radioactive source is measured to be $673\,s^{-1}$ (adjusted for background count). If exactly three hours later the count rate has reduced to $668\,s^{-1}$, determine a value for **i** the decay constant and **ii** the half-life.

b Explain why the value may be unreliable.

60 A source has an activity of 4.7×10^4 Bq. Assuming that the background count is negligible and the experiment is carried out in a vacuum, predict the count rate that could be recorded by a detector that has an effective receiving area of $0.85\,cm^2$ if it was placed:

a 50 cm from the source

b 5 cm from the source.

61 The activity from a radioactive source is 8.7×10^5 Bq. If its decay constant is $6.3 \times 10^{-6}\,s^{-1}$, calculate how many days will pass before the activity falls to 1.0×10^4 Bq.

62 A radioactive source of gamma rays, cobalt-60, is commonly used in school demonstrations. If the maximum allowable activity is 200 kBq, calculate the maximum mass of cobalt-60 in a school source. (Decay constant for cobalt-60 is $0.131\,y^{-1}$. Molar mass of Co-60 is 59.93 g)

Decay constant and half-life

The concept of the half-life of a radioactive nuclide was introduced earlier in this topic. Using the equation $N = N_0 e^{-\lambda t}$, it is straightforward to derive an equation that relates the half-life, $T_{½}$, to the decay constant, λ.

For any radioactive nuclide, the number of undecayed nuclei after one half-life is, by the definition of half-life, equal to $N_0/2$, where N_0 represents the original number of undecayed nuclei. Substituting this value for N in the radioactive decay equation at time $t = T_{½}$ we get:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{½}}$$

Dividing each side of the equation by N_0:

$$\frac{1}{2} = e^{-\lambda T_{½}} \quad \text{or,} \quad 2 = e^{\lambda T_{½}}$$

Taking natural logarithms

$$\ln 2 = \lambda T_{½}$$

So that:

Connection between half-life and decay constant:

$$\boldsymbol{T_{½} = \frac{\ln 2}{\lambda}}$$

Alternatively, inserting a value for ln 2:

$$T_{½} = \frac{0.693}{\lambda}$$

WORKED EXAMPLE E3.13

A radioactive source gives a count rate of $100\,s^{-1}$ at a certain instant of time. After 100 s the count rate drops to $20\,s^{-1}$. The average background count rate is measured to be $1.5\,s^{-1}$.

Calculate the half-life of the source. Assume that the count rate is a measure of the activity.

Answer

Initial count rate due to source = 100 – 1.5 = $98.5\,s^{-1}$.

Count rate due to source after 100 s = 20 – 1.5 = $18.5\,s^{-1}$.

$C = C_0 e^{-\lambda t}$

$18.5 = 98.5 \times e^{-100\lambda}$

$100\lambda = \ln\left(\frac{98.5}{18.5}\right)$

$\lambda = 0.016\,72$

$T_{\frac{1}{2}} = 0.693/0.016\,72 = 41$ s

63 Radioactive carbon-14 in a leather sample decays with a half-life of 5730 years.

- **a** Determine the decay constant of C-14.
- **b** Calculate the percentage of radioactive carbon remaining after 10 000 years.

64 At a certain time, a pure source contained 3.8×10^{15} radioactive atoms. Exactly one week later the number of radioactive atoms had reduced to 2.8×10^{14}.
Calculate the half-life of this source.

65 The activity of an americium-241 source used in a school laboratory was 1.6×10^5 Bq. Am-241 has a half-life of 432 years.

- **a** Show that the source contained about 3×10^{15} americium-241 atoms.
- **b** Determine the mass of this number of americium atoms.

66 The half-life of strontium-90 is 28.8 years.
Calculate how long it will take for the count rate from a sample to fall by:

- **a** 1%
- **b** 99%.

67 A cobalt-60 source used in radiotherapy in a hospital needs to be replaced when its activity has reduced to about 40% of its value when it was purchased.
If it was purchased December 2015 and replaced in November 2022, estimate its half-life.

68 The (adjusted) count rate from a radioactive source is $472\,s^{-1}$.
If its half-life is 13.71 minutes, predict the count rate exactly 1 hour later.

69 Uranium-238 is the most common isotope of uranium. It has a half-life of 4.47 billion years. The age of the Earth is 4.54 billion years.
Calculate the percentage (to 1 decimal place) of the original uranium-238 still present in the Earth's crust.

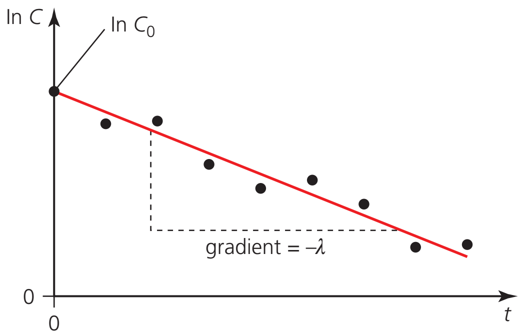
70 The radionuclide ${}^{40}_{19}K$ has a half-life of 1.3×10^9 y. It decays into the stable nuclide ${}^{40}_{18}Ar$. Rocks from the Moon were found to contain a ratio of potassium to argon atoms of approximately 1:7.

- **a** Estimate the age of these rocks.
- **b** Discuss whether the answer to part **a** is also a reasonable approximation for the age of the Moon. Explain your answer.

Experimental determination of half-life

If the count rate from a radioactive source is measured for more than one half-life, it is straightforward to determine a value for that half-life (as discussed earlier in this topic). We will now explain:

- how the calculation can be made more accurate
- how to determine the half-life of a radionuclide which is so long that no change in count rate can be detected.



■ **Figure E3.31** A plot of the natural logarithm of the count-rate against time

HL ONLY

Radioisotopes with relatively short half-lives

This method is suitable if the count rate from a radionuclide decreases measurably over the time available for the experiment, but it is not necessary that the half-life is less than duration of the experiment.

The decay constant can be determined directly from the equation $C = C_0e^{-\lambda t}$, but a more accurate method involves plotting a graph of the natural logarithm of the count rate, C, against time, t. This should give a straight best-fit line (Figure E3.31).

Taking natural logarithms of $C = C_0e^{-\lambda t}$: $\ln C = \ln C_0 - \lambda t$. This equation can be compared to the equation for a straight line ($y = mx + c$), showing that the gradient is equal to $-\lambda$, from which the half-life can be calculated.

Inquiry 2: Collecting and Processing data

Processing data

Determination of half-life

Table E3.4 shows the results from a 2.5 h experiment to determine the half-life of radionuclide. The average background count was 0.20 every second.

Draw a graph of the natural logarithm of the adjusted count rate against time. Use the graph to determine a value for the half-life of the radionuclide. Discuss the accuracy of this experiment.

■ **Table E3.4** Results of half-life experiment

Time	Count rate/min^{-1}
9.30 am	96
10.00 am	88
10.30 am	81
11.00 am	75
11.30 am	69
12.00 pm	64

Radioisotopes with relatively long half-lives

If the count rate does not change during the course of an experiment (other than random variations), we cannot use the equation $C = C_0e^{-\lambda t}$ to determine a decay constant (and half-life). Instead,

To determine the decay constant of radionuclides with long half-lives, we use the equation $A = \lambda N$

We would need to measure the activity, A, (not the count rate) and the number of undecayed nuclei, N, in a sample. However, this is difficult and it will not be possible in a school laboratory.

The activity can be determined from the count rate and the geometry of the apparatus (see Figure E3.30). The number of undecayed atoms in a *pure* sample, N, can be determined from its mass, m.

$$N = \frac{mN_A}{\text{relative atomic mass}}$$ (see Topic B.3)

WORKED EXAMPLE E3.14

A radioactive source of cobalt-60 (only) is known to produce an activity of 4.37×10^8 Bq. Estimate (i) the decay constant for this radionuclide if its mass is 10.5 µg, and (ii) the half-life of cobalt-60.

Answer

(i) First, we need to determine the number of atoms of cobalt-60:

$$N = \frac{mN_A}{\text{relative atomic mass}} = \frac{(10.5 \times 10^{-6}) \times (6.02 \times 10^{23})}{60} = 1.05 \times 10^{17}$$

Then use $A = \lambda N$

$4.37 \times 10^8 = \lambda \times (1.05 \times 10^{17})$

$\lambda = 4.1 \times 10^{-9}\,\text{s}^{-1}$ ($4.1619... \times 10^{-9}$ seen on calculator display)

(ii) $T_{\frac{1}{2}} = \ln 2/\lambda = 1.67 \times 10^8$ s or 5.3 years

71 The cadmium-109 radioisotope has a half-life of 463 days.
Predict the activity you would expect from a mass of 4.3×10^{-4} g of pure cadmium-109.

72 A pure radioisotope has a half-life of 49 hours and is producing a count rate of 81 every minute. The background count averages at 18 every minute.
Discuss, using appropriate calculations, whether it would be possible to accurately confirm the half-life experimentally over the course of a morning in the laboratory.

73 A small rock of mass 214 g is known to contain 8.2% uranium-238 (by mass). U-238 has a half-life of 4.5 billion years. There are no significant amounts of any other radioisotope.
What level of radioactivity will be emitted in this rock?

E.4 Fission

Guiding questions

- In which form is energy stored within the nucleus of the atom?
- How can the energy released from the nucleus be harnessed?

Nuclear fission

SYLLABUS CONTENT

▶ Energy is released in spontaneous and neutron-induced fission.

Nuclear fission was introduced briefly in Topic E.4, we will now look at it in more detail. Fission is the splitting of a massive nucleus into two smaller nuclei.

◆ **Fission fragments** The nuclei produced in a fission reaction.

After nuclear fission occurs, the resulting nuclei are called **fission fragments**. Typically, neutrons and gamma rays are also released during the fission.

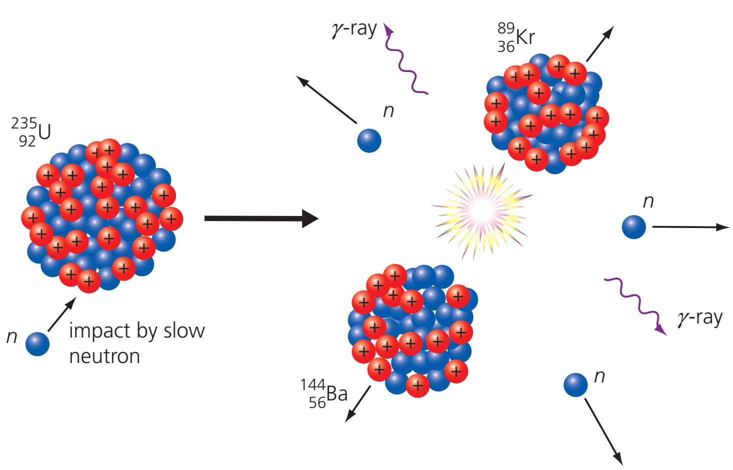
Nuclear fission can sometimes be a *spontaneous* (random, without cause) form of radioactive decay of very massive nuclei. The following fission / decay of californium-252 (a nuclide which does not occur naturally) is an example.

$$^{252}_{98}\text{Cf} \longrightarrow {}^{142}_{56}\text{Ba} + {}^{106}_{42}\text{Mo} + 4^{1}_{0}n$$

As we saw in Topic E.3, a detailed analysis of the masses involved in nuclear reactions can lead to a determination of the energy released.

More important than spontaneous fission, fission can be deliberately *induced* in nuclei which would not otherwise readily split into fragments. Most commonly, nuclear fission is induced by using *slow-moving* neutrons.

See Figure E4.1, which shows the well-known and important induced fission of uranium-235.



■ **Figure E4.1** Fission of uranium-235

◆ **Fissile** Capable of sustaining a nuclear fission chain reaction.

Uranium-235 is the world's only *naturally occurring* nuclide that is capable of sustained nuclear fission (it is said to be **fissile**). There are other fissile nuclides (plutonium-239, for example) but they are not found naturally: they need to be produced in nuclear reactors.

$${}^{1}_{0}n + {}^{235}_{92}\text{U} \rightarrow \left[{}^{236}_{92}\text{U}\right] \rightarrow {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3{}^{1}_{0}n \text{ (+ gamma ray photons and antineutrinos)} + \text{about 180 MeV of energy}$$

You need to understand this equation, but you do *not* need to remember it.

LINKING QUESTION

- In which form is energy released as a result of nuclear fission?

In this example, the fission fragments are barium and krypton, but these fragments can vary. The number of neutrons released can also vary.

The neutron is absorbed by the uranium-235 nucleus, becoming uranium-236 which very quickly fissions as shown.

Three further neutrons are produced, which means that further fission may be possible (see later).

The amount of energy released is considerable (≈ 180 MeV for each fission reaction).

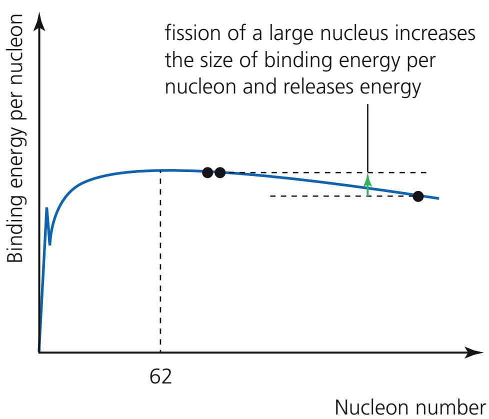
The majority of the energy released is the form of kinetic energy of the fragments. Neutrons and gamma rays also transfer significant energy.

More energy is released later as the fragments decay radioactively and the neutrons and gamma rays are absorbed. A total of over 200 MeV will be transferred to internal energy in the material containing the uranium-235. This is the energy that is utilized in nuclear reactors which generate electrical energy.

Changes of binding energy and mass during nuclear fission

For nuclear fission to occur, the fission fragments must be more stable than the original nucleus. That is, the fission fragments must have higher values of binding energy per nucleon than the original nucleus undergoing fission.

Looking again at Figure E3.18 we can see that fission will only be theoretically possible for nuclides which can split into two nuclides which have a greater nucleon number than (approximately) nickel-62. This is also represented in Figure E4.2.



■ **Figure E4.2** Fission increases binding energy per nucleon

WORKED EXAMPLE E4.1

Returning to the fission of uranium-235, The average binding energies (MeV) per nucleon involved are uranium 235: 7.59; krypton 89: 8.72; barium 144: 8.27.

Determine the change in total binding energy in this fission process.

Answer

$(89 \times 8.72) + (144 \times 8.27) - (235 \times 7.59) \approx 183$ MeV (increase)

The increase in binding energy calculated in the Worked example E4.1 is accompanied by the release of the same amount of energy in the fission process, as shown in the equation above.

The release of energy from the nuclei must result in a decrease in mass (see mass–energy equivalence in Topic E.3), as follows for the fission of uranium-235:

- Rest mass of each neutron = 1.008 665 u.
- Rest mass of uranium-235 = 235.043 930 u.
- Rest mass of barium-144 = 143.922 955 u.
- Rest mass of krypton-89 = 88.917 836 u.
- Rest mass on left-hand side of the equation = 236.052 595 u.
- Rest mass on right-hand side of the equation = 235.866 786 u.
- Mass difference = 0.185 809 u.

This mass difference is equivalent to 173 MeV (0.185809 × 931.5), which is in reasonable agreement with the 180 MeV quoted above.

1 Consider the spontaneous fission:
$^{252}_{98}\text{Cf} \rightarrow {}^{142}_{56}\text{Ba} + {}^{106}_{42}\text{Mo} + 4^{1}_{0}n$
Use the internet to determine the relevant masses, and hence calculate a value for the total energy released in this fission.

2 Explain why a nuclide such as bromine-79 cannot undergo nuclear fission.

3 One possible induced fission of $^{235}_{92}\text{U}$ produces Xe-140 (proton number = 54) and Sr-94, after the uranium nucleus captures a neutron.
  a Write a nuclear equation for this process.
  b Determine the energy released. (mass of uranium-235 = 235.044 u, mass of xenon-140 = 139.922 u, mass of strontium-94 = 93.915 u).
  c The mass defect of the xenon-140 nucleus is 1.2461 u. What is its binding energy per nucleon (MeV)?
  d Compare you answer to part **c** with the binding energy per nucleon of uranium-235 (use the internet for data).

4 The following is a possible fission of a plutonium nuclide.
$^{1}_{0}n + {}^{239}_{94}\text{Pu} \rightarrow {}^{?}_{?}? \rightarrow {}^{134}_{54}\text{Xe} + {}^{?}_{?}\text{Zr} + 3^{1}_{0}n$
Identify the 5 question marks.

Controlled release of nuclear energy in chain reactions

SYLLABUS CONTENT

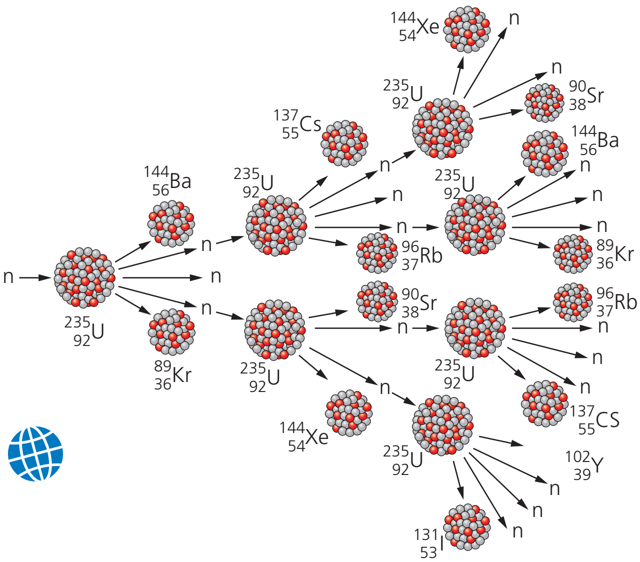
- The role of chain reactions in nuclear fission reactions.
- The role of control rods, moderators, heat exchangers and shielding in a nuclear power plant.

We have seen that a neutron is usually needed to induce nuclear fission, but then that fission produces further neutrons. These neutrons may be able to induce further fission in other nuclei, so that there is the possibility of continued fissions and the continual release of energy, as is needed in a nuclear power plant (station). (A relatively small number of neutrons are emitted randomly in any sample of uranium-235.)

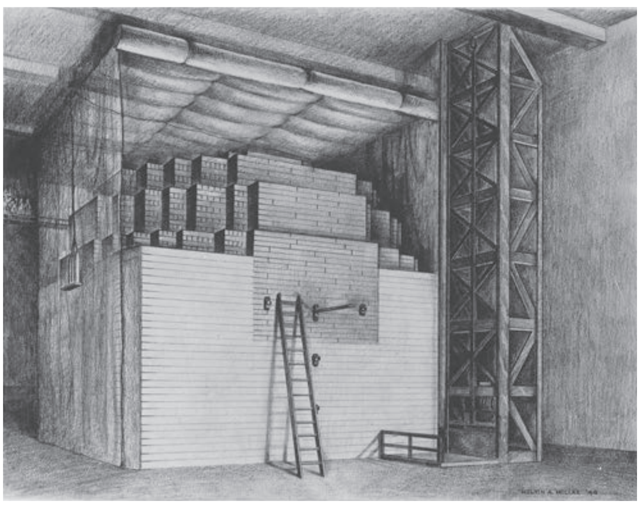
◆ **Chain reaction (nuclear)** Self-sustaining nuclear fission because each fission causes further fission.

If, on average, the neutrons produced in each nuclear fission produce at least one further fission, the process continues, and it is known as a **chain reaction**.

After each U-235 fission, two or three neutrons are created and then they may cause further fission, but, as we will explain, sustained fission is impossible unless we arrange for suitable circumstances, as described below. Figure E4.3 shows an example of a chain reaction involving uranium-235, in which a variety of different fission fragments can be seen.



■ **Figure E4.3** The principle of a chain reaction



■ **Figure E4.4** First self-sustaining nuclear chain reaction

Chain reactions will *not* normally occur. To understand why not, we need to consider the following inter-related factors.

- Uranium-235 atoms will only be a relatively small percentage of all the uranium atoms in the material being used.
- Neutrons are penetrating particles and many will simply pass out of the material without interacting with any nuclei.
- In order to cause fission, the neutrons need to be travelling relatively slowly.
- Because neutrons are uncharged, some of them will be absorbed into uranium-235 and (especially) uranium-238 nuclei without causing fission. We say they are 'captured'. (The newly formed nuclides will probably emit gamma rays.)

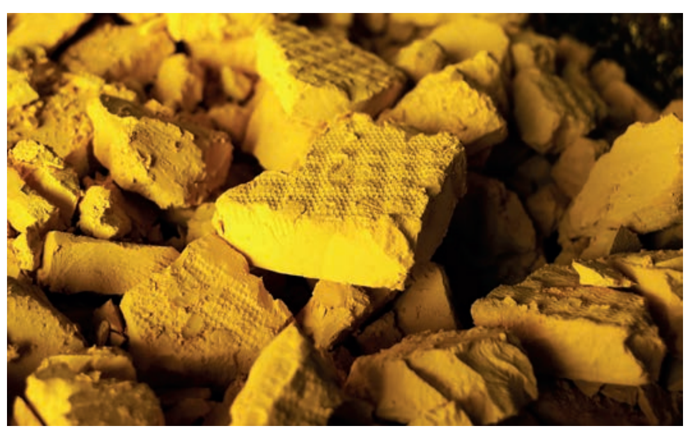
It is technically *very* difficult to produce the necessary conditions for sustained and controlled chain reactions, and the continual release of energy. The following sections explain how it is done. It was famously first achieved in 1942 at the University of Chicago. See Figure E4.4.

Fuel enrichment

The metal uranium occurs throughout the Earth's crust and is said to be a lot more common than gold (for example). The rocks from a uranium mine (see Figure E4.5) typically contain less than 0.1% uranium. The isotopes in uranium ore that is extracted from the ground are approximately in the ratio 99.3% uranium-238 and 0.7% uranium-235 (with traces of other isotopes). All the isotopes of uranium are radioactive, but the half-life of uranium-238 is very long (4.5×10^9 years), similar to the age of the Earth, whereas uranium-235 has a half-life of 7.0×10^8 years.

After the ore has been purified its appearance is as shown in Figure E4.6. It is often known as 'yellowcake'. The isotope proportions are the same as before.

■ **Figure E4.5** McClean Lake uranium mine in Saskatchewan, Canada



■ **Figure E4.6** Yellowcake

◆ **Fuel enrichment** Increasing the percentage of 235-U in uranium fuel in order to make it of use in a nuclear power station or for a nuclear weapon.

For a chain reaction and power generation, the percentage of uranium-235 has to be increased to around 3% to 5%, although higher percentages are needed for specialized reactors. (Nuclear weapons require a *much* higher percentage.) This process is called **fuel enrichment**. Figure E4.7 shows a photograph of unused enriched uranium in the form of *fuel rods*. Remember that naturally occurring radionuclides of solid uranium have very long half-lives and are not usually a significant health hazard if handled correctly.

■ **Figure E4.7** Uranium fuel rods

Uranium-238 nuclei can absorb / capture neutrons without causing fission, so too much uranium-238 will also discourage a chain reaction. Enrichment cannot be done chemically because isotopes of the same element have identical chemical properties, so physical processes need to be involved (for example, using the diffusion of gaseous uranium hexafluoride) but these are difficult and expensive technologies. The remaining uranium is called *depleted* uranium; it has physical properties, especially its high density, which have made it useful in military engineering but this has been controversial (because it is slightly radioactive).

Critical mass

◆ **Critical mass** The minimum mass needed for a self-sustaining nuclear chain reaction.

◆ **Moderator** Material used in a nuclear reactor to slow down neutrons to low energies and enable nuclear fission.

The ratio of volume to surface area of a solid increases as it gets larger (consider solid cubes of different sizes). This means that the more massive a material is, the smaller the percentage of neutrons that will reach the surface and escape. That is, a higher percentage may cause fission. The **critical mass** of a material is the minimum mass needed for a self-sustaining chain reaction. (Uranium that contains 20% of uranium-235 has a critical mass of over 400 kg, which is equivalent to a sphere of radius 17 cm.)

The critical mass can be reduced by surrounding the material with neutron reflectors.

Moderator

The neutrons released in nuclear fission have typical energies of more than 1 MeV, which means that they travel very fast. This is usually too fast to initiate another fission reaction. The slower a neutron travels, the higher the probability it has of causing fission. Therefore, before a chain reaction can occur the neutrons need to be slowed down to energies of less than 1 eV. They are often then described as *thermal neutrons*, meaning that they have average kinetic energies similar to that of the surrounding particles at the same temperature.

Reducing the speed of neutrons is called *moderating* and the material used is called a **moderator**.

In order for the fast neutrons to lose so much of their kinetic energy, they need to collide many times with the nuclei of atoms. In general, when particles collide there is a higher rate of transfer of kinetic energy between them if they have approximately the same mass (see Topic A.2). The mass of a neutron is always lighter than the mass of a whole nucleus, but the difference is less for nuclei with low mass. This is why atoms with nuclei of small mass are preferable for this process of moderation, but it is also important that the nuclei do not absorb neutrons. Commonly, the hydrogen atoms in water molecules, or graphite (carbon) is used as a moderator.

Essential features of a nuclear reactor

◆ **Heat exchanger** Apparatus designed to efficiently transfer energy from one system to another.

Figure E4.8 shows the essential features of one type of nuclear reactor. In this type of reactor, water is the moderator, but the same water is used to transfer thermal energy from the reactor vessel to another, separate, water system, using a **heat exchanger**. The water that cools the fuel rods is in a tightly sealed system and none of it leaves the concrete containment.

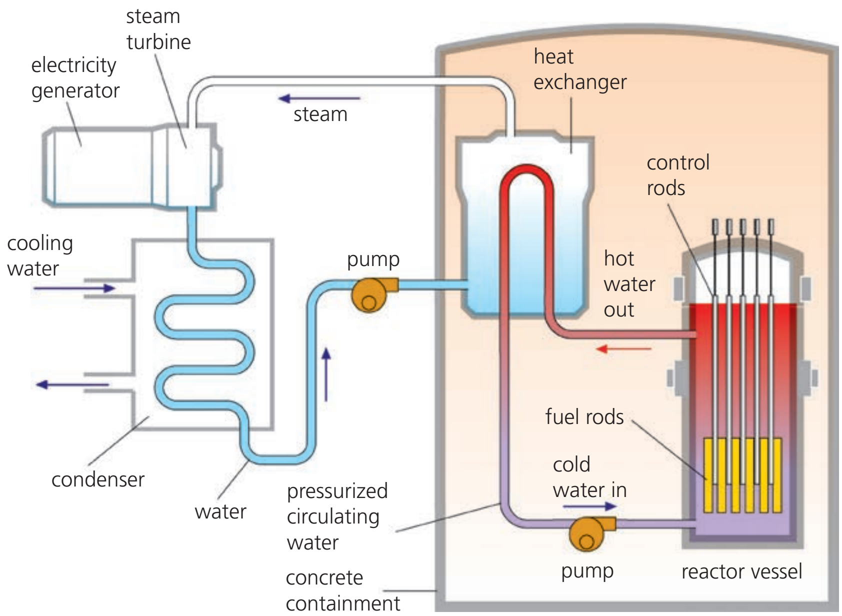


Figure E4.8 A pressurized water-cooled reactor

Steam is generated by the thermal energy and this is used to drive a turbine and generate electricity, as shown in the energy flow diagram of Figure E4.9.

A heat exchanger transfers thermal energy from the reactor to the water and steam used in the separate turbine system.

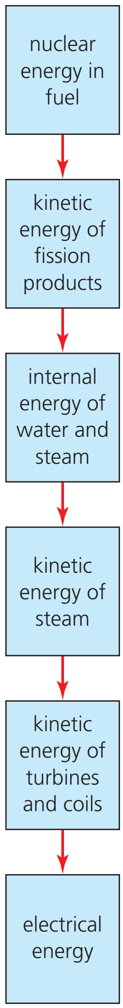


Figure E4.9 Energy flow in a nuclear power station

Controlling the rate of fission

In a controlled, sustained chain reaction, on average each fission will result in one further fission.

Control rods are used for adjusting the rate of the fission reactions by absorbing neutrons.

This is done by moving the rods up and down, into or out of the system as necessary. The control rods are made of a material, boron for example, which is excellent at absorbing / capturing neutrons.

As a safety measure (in the case of an electricity supply failure, for example) the control rods will fall under gravity all the way into the core and quickly shut down the reactor.

Shielding

The contents of the nuclear reactor emit large quantities of alpha, beta and gamma rays, as well as neutrons. These could be significant health hazards and people in the surroundings need to be protected:

- during normal operation
- in the event of an accident.

Shielding is used to protect people from the dangers of nuclear radiation.

The air-tight containment building is typically made of very thick concrete (with steel reinforcements) to absorb nuclear radiations and prevent any high-pressure leaks if a fault occurred in the system inside. The building should also be strong enough to withstand impact from aircraft, bombs or earthquakes.

LINKING QUESTION

- How is binding energy used to determine the rate of energy production in a nuclear power plant?

This question links to understandings in Topic E.3.

◆ **Control rods** Used for adjusting the rate of fission reactions in nuclear reactors by absorbing more, or fewer, neutrons.

◆ **Shielding** Protective barrier around a nuclear reactor designed to absorb and reflect dangerous radiations.

■ **Figure E4.10** Fukushima nuclear reactor after explosions in the wake of the tsunami (2011)

Safety issues

It is fair to say that the risks of nuclear power are very well understood and, as a result, safety standards are very high. But, for many people, this is not reassuring enough, because no matter how careful nuclear engineers are, accidents and natural disasters can happen. Safety standards do vary from country to country, but the consequences of a nuclear accident anywhere could be really disastrous.

The world's worst nuclear accidents, at Chernobyl in Ukraine in 1986 and Fukushima (see Figure E4.10) in Japan in March 2011, remain as vivid warnings about the possible risks of nuclear power.

Since the Chenobyl disaster, modern designs and safety procedures have been vastly improved but may still be insufficient in the face of an extreme natural disaster. The radiation leaks and explosions at Fukushima followed the damage caused by a tsunami. Hundreds of thousands of people had to be evacuated because of these incidents, and the number of long-term illnesses and deaths caused by them will take many years to be confirmed. A thermal *meltdown* is probably the most serious possible consequence of a nuclear accident. If, for some reason (for example, the loss of coolant or the control rods failing to work properly), the core of the reactor gets too hot or even melts, the reactor vessel may be badly damaged. Fires and explosions may happen as extremely hot materials are suddenly exposed to the air. Highly concentrated and dangerous radioactive materials may then be released into the ground, water or air so that they are spread over large distances by geographic and weather conditions.

◆ **Nuclear waste** Radioactive materials associated with the production of nuclear power that are no longer useful, and which may have to be stored safely for a long period of time.

When considering the dangers of nuclear power, it should always be remembered that the uses of other kinds of energy resources also have their various risks. In particular, coal mining has been responsible for an extremely high number of serious injuries, long-term health problems and deaths over the past 200 years.

Nature of science: Science as a shared endeavour

Nuclear power plants are operating in 32 different countries, producing about 10% of the world's electricity. It is believed that nine countries have nuclear weapons. The aim of the International Atomic Energy Agency (IAEA), based in Vienna, Austria, is 'to promote the safe, secure and peaceful use of nuclear technologies'. IAEA is reported to have more than 2000 multi disciplinary professional and support staff from over 100 different countries.

Waste materials from nuclear reactors

SYLLABUS CONTENT

▶ Properties of the products of nuclear fission and their management.

After fission has occurred, the various fission fragments are radioactive, and maybe the nuclides in any decay series from them are too. These 'waste' radionuclides can have a wide range of different concentrations and half-lives, some very long. This makes the safe storage of **nuclear waste** for the foreseeable future very important. The first step is usually storage on site in water ponds to allow time for the initial decrease in radioactivity and the dissipation of thermal energy. See Figure E4.11.

■ **Figure E4.11** Storage of nuclear waste

High level nuclear waste is often secured underground in strong and secure containers.

Some countries prefer to store hazardous waste securely in shielded containers on the same site as the reactor.

It is also possible to recycle nuclear fuel after the percentage of fissile material decreases, but these processes are not included in this course.

5 Calculate the volume to surface area ratios for solid cubes with sides of 10 cm, 20 cm and x cm.

6 The critical mass of a pure uranium-235 sphere is reported to be about 50 kg.
  a Explain what this means.
  b Why is the critical mass of the uranium used in a reactor larger than 50 kg?

7 Explain why isotopes of uranium are difficult to separate from each other.

8 Calculate the speed of a neutron which has kinetic energy of 1.0 MeV.

9 a Write down the equation (from Topic B.1) which relates the average kinetic energy of particles to the temperature.
  b Calculate the average kinetic energy of particles (J) at 300 °C (the approximate temperature inside a nuclear reactor).
  c Determine a typical value for the speed of thermal neutrons at this temperature.

10 Explain why a heat exchanger is needed in a nuclear reactor.

11 Represent the energy transfers seen in Figure E4.9 in a Sankey diagram by making rough estimates for the efficiency of each transfer.

12 Outline the differences between the purposes of control rods and moderators in a nuclear reactor.

13 Discuss whether it is reasonable to claim that the longer the half-life of a radionuclide, the less dangerous it is.

14 a Use the internet to determine which countries of the world have the greatest percentage of their electricity generated by nuclear fission.
  b Suggest possible reasons for the popularity of nuclear power in those countries.

15 Radon, $^{222}_{86}\text{Rn}$, is gas produced naturally from $^{226}_{88}\text{Ra}$ in the uranium decay series.
  a Write an equation for this decay.
  b Explain why radon is a health hazard of particular concern, including for workers in uranium mines.

16 Discuss the advantages and disadvantages of storing nuclear waste i deep underground and ii on the site of the nuclear power plant.

Energy density of nuclear fuels

In Topic A.3 we met the concept of *energy density* of fuel sources (energy from unit volume, J m^{-3}).

It is one of the major advantages of nuclear power that the fuels have exceptionally high-energy density, as in shown by Worked example E4.2.

Common mistake

Energy density (energy transferred/volume) is often confused with *specific energy* (energy transferred/mass). Both concepts are in common use.

WORKED EXAMPLE E4.2

We have seen that a typical fission of one uranium-235 nucleus releases about 200 MeV of energy.

a Determine the total amount of energy (J) that could theoretically be released from 1.00 kg of pure uranium-235.

b The density of uranium-235 is very high: 19.1 g cm^{-3}. Show that your answer to part **a** corresponds to an energy density of about $10^{18}\ \text{J m}^{-3}$.

c The energy available from natural gas is 54.0 MJ kg^{-1}. Compare this to your answer to part **a**.

Answer

a 235 g of uranium-235 contains 6.02×10^{23} atoms (one mole)

1000 g of uranium-235 contains $\left(\frac{1000}{235}\right) \times (6.02 \times 10^{23}) = 2.5617 \times 10^{24}$ atoms

2.5617×10^{24} atoms $\times$ 200 MeV per atom $= 5.1234 \times 10^{26}$ MeV

$(5.1234 \times 10^{26}) \times (10^6 \times 1.60 \times 10^{-19}) = 8.20 \times 10^{13}$ J

($8.1974... \times 10^{13}$ seen on calculator display)

b $(8.1974 \times 10^{13}) \times (19.1 \times 10^3) = 1.57 \times 10^{18}\ \text{J m}^{-3}$

c $\frac{8.1974 \times 10^{13}}{54.0 \times 10^6} = 1.52 \times 10^6$

The energy per kilogramme available from uranium-235 is more than a million times greater than from natural gas.

17 Using data from Worked example E4.2:

a Calculate the mass of uranium-235 that would be used every year in a nuclear power station which has an output power of 0.85 GW and operates at an overall efficiency of 33%.

b Compare your answer to part **a** with a natural gas power station operating at the same output power, but with an overall efficiency of 52%.

18 Using data from Question 17a, determine how many fissions of U-235 are occurring in every kg of U-235 every second.

19 a If an individual uses electrical energy at an average rate of 1 kW, predict their annual energy consumption.

b Calculate the mass of uranium-235 atoms that has to undergo fission to provide the energy needed for a year. (Assume 35% efficiency.)

Advantages and disadvantages of nuclear power

Table E4.1 Some advantages and disadvantages of nuclear power

Advantages	Disadvantages
extremely high-energy density no greenhouse gases emitted during routine operation (some scientists think that nuclear power may be the only realistic solution to global warming) no chemical pollution during operation reasonably large amount of nuclear fuels are still available despite a few serious incidents, statistically over the last 50 years, nuclear power has overall proven to be a reasonably safe energy technology	dangerous and very long-lasting radioactive waste products expensive efficiency is not high when the whole process is taken into account threat of serious accidents possible target for terrorists linked with nuclear weapons not a renewable source

LINKING QUESTION

- To what extent is there a role for fission in addressing climate change? (NOS)

This question links to understandings in Topic B.2.

ATL E4A: Communication skills

Practice active listening skills

Debating

In groups of three or four, prepare arguments in advance either in favour of, or against, the following statement.

'Nuclear power has an important role to play in providing energy for electricity generation for the foreseeable future of planet Earth.'

Then have a thirty-minute debate, with students from other subjects invited to attend.
At the end take a vote.

TOK

The natural sciences

- Should scientific research be subject to ethical constraints or is the pursuit of all scientific knowledge intrinsically worthwhile?

Many (most?) people probably think that the world would be a better place without nuclear power plants and nuclear weapons. However, even if they were all dismantled, the knowledge needed to construct them would still exist. Fission cannot be 'undiscovered'.

Are there any scientific discoveries, or technological advances, that you wish had never happened?

Can it ever be possible to stop the ever-expanding scientific knowledge, especially into areas which may (later) be considered undesirable? Would we want to?

E.5 Fusion and stars

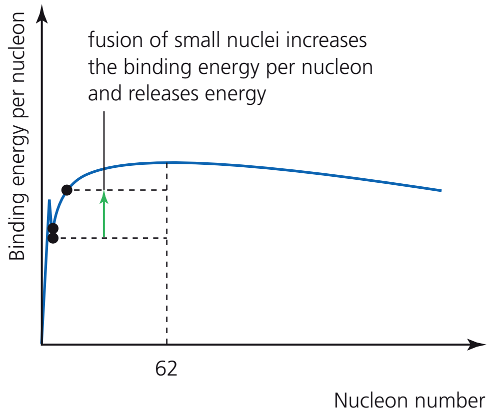


Figure E5.1 Fusion increases average binding energy per nucleon

Guiding questions

- How are elements created?
- What physical processes lead to the evolution of stars?
- Can observations of the present state of the Universe predict the future outcome of the Universe?

Nuclear fusion

Nuclear fusion was introduced briefly in Topic E.3, we will now look at it in more detail. Nuclear fusion is the combination of two small nuclei to produce a more massive single nucleus.

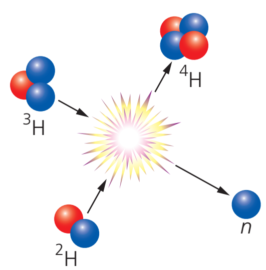


Figure E5.2 Fusion of two hydrogen nuclei

Nuclear fusion involves changing to a more stable system, with an increase in binding energy and the release of energy, mostly in the form of kinetic energy.

Consider Figure E5.1, which is similar to E3.18. In principle, the creation of a new nucleus by fusion may be possible for nuclei that have neutron numbers less than approximately 62.

An example of fusion is between a hydrogen-2 nucleus and a hydrogen-3 nucleus. (Hydrogen-2 is known as deuterium, hydrogen-3 is known as tritium.) See Figure E5.2.

$$^2_1\text{H} + ^3_1\text{H} \Rightarrow ^4_2\text{He} + ^1_0n + \text{energy}$$

We can use changes in binding energy to determine a value for the energy released in this nuclear reaction, as shown in Worked example E5.1.

ATL E5A : Research skills

Evaluating information sources for accuracy, bias, credibility and relevance

Use the internet to find out the latest developments in nuclear fusion research. Be sure to evaluate the sources you refer to – do they have a particular standpoint on nuclear energy research?

WORKED EXAMPLE E5.1

The binding energy per nucleon of hydrogen-2 is 1.11 MeV. The binding energy per nucleon of hydrogen-3 is 2.83 MeV. The binding energy per nucleon of helium-4 is much higher: 7.07 MeV.

Calculate:

a the energy released in the fusion reaction shown in the equation above

b the change in total mass of the nucleons.

Answer

a Binding energy of hydrogen-2 nucleus = 2 × 1.11 = 2.22 MeV

Binding energy of hydrogen-3 nucleus = 3 × 2.83 = 8.49 MeV

Binding energy of one helium-4 nucleus = 4 × 7.07 = 28.28 MeV

The neutron has zero binding energy because it is a single particle.

Difference in binding energies = energy released = 28.28 – 2.22 – 8.49 = 17.57 MeV.

b The mass of the system will reduce by as much as the energy that was released:

$$\frac{17.57}{931.5} = 0.019\text{ u}$$

For nuclear fusion to occur the nuclei must initially have sufficient kinetic energy to overcome the repulsive forces between the positive charges. This requires extremely high temperatures, more than 10^7 K.

If the nuclei can overcome the electric repulsion between positive charges, they can get very close to each other, and then the attractive nuclear forces pull them together and fusion may occur.

LINKING QUESTION

- How is fusion like – and unlike – fission?

This question links to understandings in Topic E.4.

The principles are well understood and *if* nuclear fusion could be sustained, it would release enormous amounts of energy for electricity generation from plentiful raw materials, without significantly contributing to climate change, and without producing large quantities of radioactive waste (as occurs with nuclear fission). The prospect of plentiful energy from nuclear fusion has pre-occupied and excited scientists for much more than fifty years, but the technical problems seem as large as ever.

1 Calculate the repulsive electric force between two protons that are 1.0×10^{-15} m apart.

2 Use data from Worked example E5.1.
  a Determine how much energy (J) would be released from the fusion of one kilogramme of helium-4.
  b How would you describe the energy density of this process?

3 Calculate the kinetic energy (eV) of a proton (hydrogen nucleus) at a temperature of 1.0×10^7 K.

4 The following is the simplest possible example of fusion. It occurs in stars.
$${}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + {}^{0}_{1}e^{+} + {}^{0}_{0}\nu$$
  a Using data (with a suitable number of significant figures) from the internet, determine:
    i the change in mass that occurs in this fusion
    ii the energy released (MeV) in this reaction.
  b Explain why the fusion of each helium-4 nucleus releases much more energy than the fusion of hydrogen-2.

5 Explain why nuclear fusion is only possible with two nuclei which have relatively small numbers of nucleons.

Although nuclear fusion is a rare event here on Earth, we will now turn our attention to where it dominates: the rest of the Universe.

◆ **Interstellar matter** Matter that exists in the space between stars (usually at very low density).

◆ **Nebula** (plural: **nebulae**) Identifiable, diffuse ‘cloud’ of interstellar matter; mainly gases (mostly hydrogen and helium) and dust.

Formation of stars

SYLLABUS CONTENT

- The stability of stars relies on an equilibrium between outward radiation pressure and inward gravitational forces.
- Fusion is a source of energy in stars.
- The conditions leading to fusion in stars in terms of density and temperature.

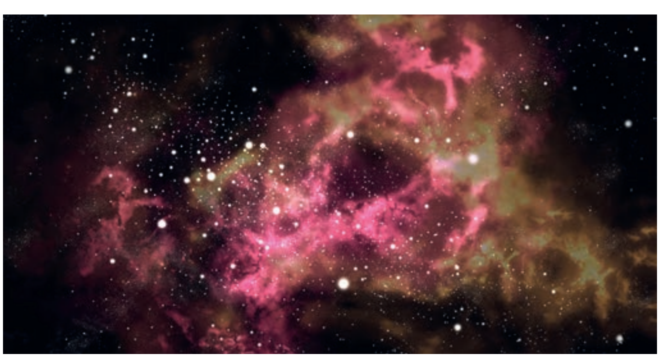


Figure E5.3 The Orion nebula

The space between stars has evolved over billions of years to contain very low concentrations of particles, which are collectively known as **interstellar matter**, and commonly described as ‘dust and gas’. Hydrogen is, on average, about 70% of all interstellar matter (by mass), helium has 28% and the remaining 2% is other elements (mainly remnants of exploded older stars). Depending on the circumstances, the gas particles may be molecules, atoms or ions. A **nebula** (Figure E5.3) is the name given to a distinct and identifiable giant ‘cloud’ of dust and gas in space.

In places where there is an increased density of material (for whatever reason), gravitational forces will (very slowly) pull the particles closer together. This results in increasing kinetic energies of particles, which is equivalent to increasing temperature.

◆ **Main sequence stars** Stable stars which are fusing hydrogen into helium in their cores.

Eventually, the temperature and density of hydrogen will be great enough for nuclear fusion of hydrogen into helium to occur. Temperatures of at least 10^7 K are needed. At this temperature the hydrogen nuclei (protons) have enough kinetic energy to overcome the Coulomb repulsion between them. This is the dominant energy transfer occurring in all **main sequence stars**. ('Main sequence' stars are explained below. Most stars are main sequence stars.) There are two principal ways in which fusion can happen in main sequence stars, as explained below. At these temperatures, hydrogen exists simply as protons.

Only a very small fraction of interactions between protons results in fusion. The closer the protons are to each other (on average), the greater the frequency of interactions and the rate of fusion. In other words, a high density is needed to sustain nuclear fusion in stars.

Nuclear fusion at high temperatures and densities in main sequence stars combines four protons to form helium-4.

Proton–proton cycle

◆ **Proton–proton cycle** The simplest nuclear fusion process which converts hydrogen into helium, releasing large amounts of energy in medium and smaller-sized main sequence stars.

The **proton–proton cycle** is also called the proton–proton chain.

This is the principal nuclear fusion process in main sequence stars which have a *mass similar to the Sun, or less.*

This is a 3-step process, which can be summarized as follows but the details do not need to be remembered:

$$4{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2{}^{0}_{1}e + \text{neutrinos} + \text{photons}$$

The three separate reactions are:

1 Two protons fuse to make a ${}^{2}_{1}\mathrm{H}$ (deuterium) nucleus. In this process, one of the protons converts into a neutron in a beta-plus decay, also forming a positron and a neutrino.

$${}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{He} \rightarrow {}^{2}_{1}\mathrm{H} + {}^{0}_{1}e^{+} + \nu$$

2 The deuterium nucleus fuses with another proton to make helium-3. In this process, a gamma ray photon is also emitted.

$${}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{He} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma$$

3 Two helium-3 nuclei combine to make helium-4. Two protons are released in this reaction.

$${}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + 2{}^{1}_{1}\mathrm{H}$$

These three stages are illustrated in Figure E5.4.

The energy released in each 3-stage cycle is 27 MeV. This is transferred to the kinetic energy and electromagnetic energy of the products.

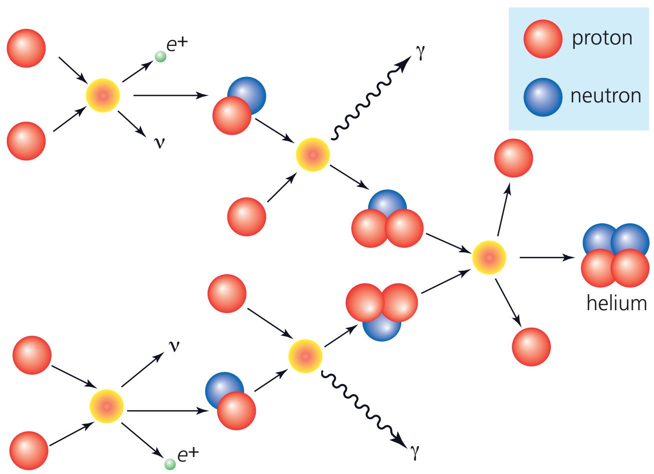


Figure E5.4 The proton–proton cycle

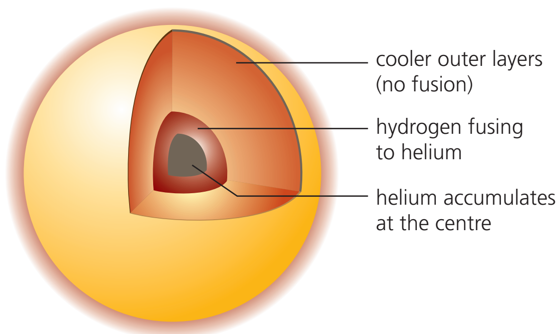


Figure E5.5 Internal structure of a main sequence star

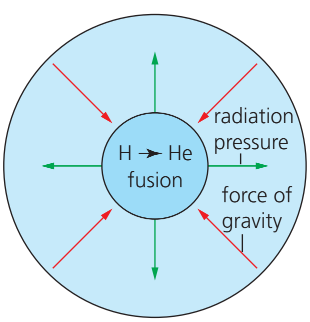


Figure E5.6 Equilibrium in a main sequence star

CNO cycle

This is the principal nuclear fusion process in main sequence stars *which have a mass significantly greater than the Sun*. This is because of the higher temperatures in the cores of the more massive stars.

CNO represents carbon, nitrogen, oxygen. These elements must be present in the core, but they are not changed by the process. (They act as *catalysts*.)

The **CNO cycle** is a complicated 6-step process (no details are needed), but in effect it still involves the fusion of four protons to one helium nucleus (similar to the proton–proton cycle) and releases about the same amount of energy (27 MeV).

Stellar equilibrium

Energy is transferred from the *core* of the star, where it is hottest and where the fusion is happening, by radiation, conduction and convection to the surface. See Figure E5.5. Helium will accumulate at the centre of the core because of its greater density. From the surface, enormous amounts of energy are radiated into space in all directions. These fusion processes will continue as long as there is enough hydrogen in the core, at a high enough temperature.

Main sequence stars can remain in equilibrium for millions, or billions of years, under the following condition (see also Figure E5.6):

In **stellar equilibrium**, the outward radiation (thermal) pressure is balanced by inwards gravitational forces.

This is a self-correcting (feedback) process: if the fusion rate was to increase slightly, the temperature would increase so that the outwards pressure rises. This would result in an expansion of the star, so that density and temperature decrease, and the fusion rate falls.

◆ **CNO cycle** Nuclear fusion process in larger main sequence stars which forms helium from hydrogen.

◆ **Stellar equilibrium** Main sequence stars are in equilibrium under the balanced effects of radiation pressure acting outwards against gravitational forces acting inwards.

◆ **Feedback** Occurs when a response within a system influences the continuing behaviour of the same system.

Nature of science: Patterns and trends

Feedback processes

Feedback is the name used to describe any response within an on-going system which affects the future behaviour of the same system. For example, if you are told that you are doing well in your physics studies, you may be encouraged, and then work harder. This would be called *positive* feedback.

Negative feedback can often help to stabilize a system, as in the example of stellar equilibrium.

Tool 3: Mathematics

Use of units whenever appropriate

In calculations related to the properties of stars, it becomes convenient to compare a star to our Sun: $M_{\odot}$ = mass of Sun, $L_{\odot}$ = luminosity of Sun, $R_{\odot}$ = radius of Sun.

Lifetimes of main sequence stars

There will come a time when most of the hydrogen in the *core* of a star becomes depleted (already used, in fusion), so that the rate of fusion decreases to such an extent that it is no longer possible to maintain the equilibrium described above. This will be the end of the **lifetime** of the main sequence star and it will then change in ways which are described later in this topic. There will still be a significant amount of hydrogen *outside* of the core.

◆ **Lifetime (of main sequence star)** The duration for which a star is fusing hydrogen into helium, emitting radiation and maintaining stellar equilibrium.

Main sequence stars will come to the end of their lifetimes when most of the hydrogen in their cores has been converted to helium.

It might be expected that more massive stars would have longer lifetimes because they contain more hydrogen, but the opposite is true.

The rate of fusion in more massive (hotter) stars is so much greater, that they have significantly *shorter* lifetimes.

This can be seen in Table E5.1 in the next section.

We can make a *rough estimate* for the future lifetime of a main sequence star as follows (using data for the Sun as an example). *This lengthy and detailed calculation need not be remembered.*

mass, $M_\odot = 1.99 \times 10^{30}\,\text{kg}$

luminosity, $L_\odot = 3.85 \times 10^{26}\,\text{W}$

Each proton–proton cycle releases 27 MeV ($= 4.3 \times 10^{-12}$ J) of energy.

We will assume that, when they are first formed, main sequence stars consist of *approximately* 70% hydrogen, and that the Sun will end its main sequence lifetime when about 15% of its total hydrogen remains (mostly in the outer layers).

We can calculate an approximate value for its main sequence lifetime as follows:

amount of hydrogen that will be fused ('burned') during the main sequence lifetime = (100 − 15)% of 70 % of $1.99 \times 10^{30}\,\text{kg} = 1.18 \times 10^{30}\,\text{kg}$

mass of hydrogen involved with each proton–proton cycle $= 4 \times (1.673 \times 10^{-27}) = 6.688 \times 10^{-27}\,\text{kg}$

number of proton–proton cycles during the main sequence lifetime $= \dfrac{1.18 \times 10^{30}}{6.688 \times 10^{-27}} = 1.76 \times 10^{56}$

rate of proton–proton cycles $= \dfrac{3.85 \times 10^{26}}{4.3 \times 10^{-12}} = 8.95 \times 10^{37}\,\text{s}^{-1}$ (average for the main sequence lifetime)

main sequence lifetime of our Sun $= \dfrac{1.77 \times 10^{36}}{8.95 \times 10^{37}} = 2.0 \times 10^{18}\,\text{s}$ ($\approx 6 \times 10^{10}$ years)

This is just an approximation, but it is in broad agreement with the accepted value for the future lifetime of the Sun ($\approx 10^{10}$ years).

We can also estimate the decrease in the mass of the Sun due to nuclear fusion reactions from $E = mc^2$ (Topic E.3):

energy transferred during main sequence lifetime,
$E = \text{power} \times \text{time} = (3.85 \times 10^{26}) \times (2.0 \times 10^{18}) = 7.7 \times 10^{44}\,\text{J}$

$E = mc^2$

$7.7 \times 10^{44} = m \times (3.0 \times 10^{8})^2$

$m = 8.6 \times 10^{27}\,\text{kg}$

This is equivalent to about four billion kilogrammes every second!

TOK

Knowledge and the knower

Too big or too small to comprehend?

More than any other area of knowledge, physics involves an appreciation of numerical values over enormous ranges: from subatomic particles to the size of the observable Universe.

However, the larger a number, the worse we are at really understanding what it represents. Similarly, very small numbers are difficult to comprehend, for example 10^{-15} m: the approximate radius of a nucleus. Clearly, 'four billion kilogrammes' (from last section), is a large number, but to begin to appreciate just how large, comparisons are usually helpful. We might say that 4×10^{9} kg is about equal to the total mass carried by every person on Earth if they each had 0.5 kg. Or 4×10^{9} kg is approximately equal to the mass of water in a reservoir of dimensions 1 km × 1 km × 4 m. However, such comparisons are less useful when much larger numbers are involved.

Suggest a comparison you could use to make 10^{-10} m (an approximate size of an atom) understandable for a 10 year-old child.

6 Discuss whether you would expect that most hydrogen in interstellar matter was in the form of ions, atoms or molecules.

7 Calculate the acceleration due to gravity of two protons separated by one metre.

8 Determine a value for the energy released (MeV) in the third stage of the proton–proton cycle, as described above. (Use the internet to determine relevant data.)

9 The CNO cycle needs carbon, nitrogen and oxygen in the core. Suggest where these elements have come from.

10 Explain what will happen if, for some reason, the fusion rate in a main sequence star was to decrease slightly. Assume that there is plenty of hydrogen available in the core.

11 a Using the same method and assumptions as shown in the calculation of the lifetime of the Sun (above), determine the lifetime of a main sequence star which has a mass ten times greater than the Sun's and a luminosity three thousand times greater than that of the Sun.

b The relationship between the luminosity and mass of a main sequence star is:

$$L \approx L_{\odot}\left(\frac{M}{M_{\odot}}\right)^{3.5}$$

(You do not need to remember this.)
Show that this relationship confirms the approximate luminosity of the more massive star given in part **a**.

Comparing main sequence stars

SYLLABUS CONTENT

- The main regions of the Hertzsprung–Russell (HR) diagram and how to describe the main properties of stars in that region.
- How to determine stellar radii.

When we observe stars from Earth, they all appear as point sources of light with different *apparent brightnesses* and slightly different colours. Direct observation provides no more information, except their positions relative to each other, which enables star maps to be drawn. Although stars move at high speeds, the distances between them are so great that no changes in these positions are apparent to observers on Earth, even over hundreds of years. (But note that some stars, which are relatively close to the Earth, show *small* repeated movements (on a star map) during the course of every year. This is called *stellar parallax*, and it is explained later.)

The same nuclear fusion processes are occurring in all main sequence stars, but astronomers have calculated that these stars have different masses, radii and temperatures (core and surface). As a rough guide, our Sun may be considered to be an 'average' main sequence star, other stars have radii which are up to 1000 × greater or smaller than the Sun's, with an even wider range of luminosities: from 10 000 × less to 1 000 000 × greater.

The differences between main sequence stars are not random. There is a very clear pattern (see Table E5.1), because the rate of fusion depends on the masses of the stars.

Main sequence stars of greater mass have greater radii, greater rates of fusion, higher temperatures, greater luminosities and shorter lifetimes.

This pattern is apparent in the data shown in Table E5.1.

Table E5.1 Properties of main sequence stars (figures are approximate)

Mass/$M_\odot$	Luminosity/$L_\odot$	Effective temperature/K	Radius/$R_\odot$	Lifetime/y
0.10	3×10^3	2900	0.16	2×10^{12}
0.50	0.03	3800	0.6	2×10^{11}
1.0	1	5800	1.0	1×10^{10}
3	60	11 000	2.5	2×10^8
5	600	17 000	3.8	7×10^7
10	10 000	22 000	5.6	2×10^7
25	80 000	35 000	8.7	7×10^6
60	790 000	44 500	15	3×10^6

The 'effective temperature' is the name given to the surface temperature calculated assuming the star behaves as a perfect black body.

Clearly, there is no need to remember any of the data seen in Table E5.1, but the trends are important and are probably easier to understand as shown graphically in Figure E5.7, Figure E5.8 and Figure E5.11.

Mass has a considerable effect on luminosity. For example, if star A has twice the mass of star B it will have *approximately* ten times the luminosity. This general trend is indicated in Figure E5.7.

This pattern is shown in more detail in Figure E5.8, which is known as a **Hertzsprung–Russell (HR) diagram**.

◆ **Hertzsprung–Russell (HR) diagram** Diagram that displays order in the apparent diversity of stars by plotting the luminosity of stars against their surface temperatures.

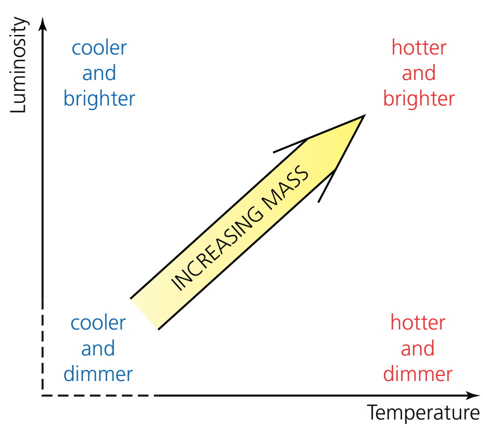


Figure E5.7 Linking mass, temperature and luminosity for main sequence stars

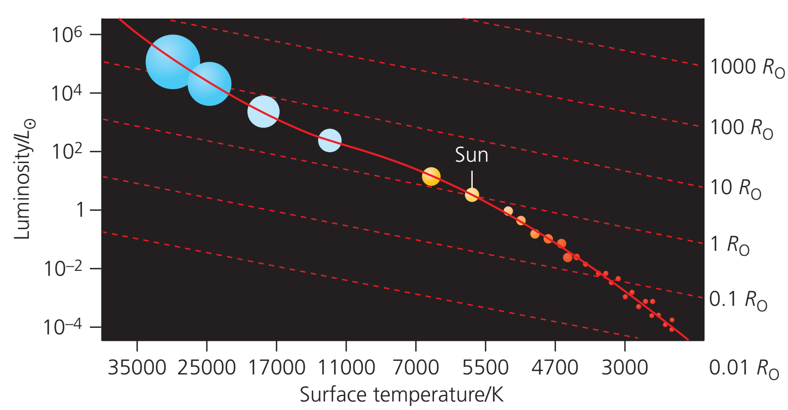


Figure E5.8 Hertzsprung–Russell diagram *(incomplete)* with colours enhanced

LINKING QUESTIONS

- How can the understanding of black-body radiation help determine the properties of stars?
- How do emission spectra provide information about observations of the cosmos?

These questions link to understandings in Topics B.1 and E.2.

Figure E5.8 is an important diagram and it should be well understood. Note, in particular:

- For historical reasons, the temperature scale is reversed, with lower temperatures on the right.
- The differences in stars' luminosities and temperatures are so great that logarithmic scales are used.
- The luminosity scale is shown relative to the Sun's luminosity, $L_{\odot}$ (= 3.8×10^{26} W).
- The dotted lines represent constant radius. $R_{\odot}$ = radius of Sun (7.0×10^{8} m).

The connection between surface temperature and colour was explained in Topic B.1.

Other types of stars (non-main sequence) will be added to the HR diagram after they have been explained.

Top tip!

Reminders from Topic B.1:

Stars can be considered to be perfect black bodies and the radius of a star can be determined from $L = \sigma A T^4$, by using surface area, $A = 4\pi r^2$. The temperature of a star's surface is sometimes called the star's effective temperature. (This is because a value for T is often calculated from knowledge of L and A, assuming that the star is acting as a perfect black body.)

The surface temperature, T, of a star can be determined from $\lambda_{max}T = 2.9 \times 10^{-3}$ m K, where λ_{max} is the peak wavelength of the black-body spectrum emitted.

WORKED EXAMPLE E5.2

The large, bright star Canopus has a luminosity = $10\,700\,L_{\odot}$ and a surface temperature of 7400 K.

Determine a value for its radius.

Answer

$L = \sigma A T^4 = \sigma(4\pi r^2)T^4$

$10\,700 \times (3.85 \times 10^{26}\text{ W})$

$= (5.67 \times 10^{-8}) \times 4 \times \pi r^2 \times 7400^4$

$r = 4.39 \times 10^{10}$ m

WORKED EXAMPLE E5.3

The main sequence star Vega has a surface temperature of 9600 K.

a Determine the peak wavelength in its spectrum.

b Estimate the luminosity of this star.

Answer

a $\lambda_{max}T = 2.9 \times 10^{-3}$

$$\lambda_{max} = \frac{2.9 \times 10^{-3}}{9600} = 3.0 \times 10^{-7}\text{ m}$$

b From Figure E5.8, $L \approx 50\,L_{\odot}$

12 A main sequence star has a radius of approximately $10R_0$ and a luminosity of $(2 \times 10^4)L_\odot$.

a Use Figure E5.8 to estimate its surface temperature.

b State the colour you would expect this star to be.

13 Estimate the luminosity of a main sequence star which has a surface temperature of 15 000 K.

14 A main sequence star is observed to be slightly blue in colour. Suggest possible values for its luminosity, radius and surface temperature.

15 A star has a radius of $100\,R_0$ and a luminosity of $10\,000\,L_\odot$.

a Explain why you can be sure that this is not a main sequence star.

b Estimate its surface temperature.

16 The main sequence star Altair has a radius which is approximately twice the radius of the Sun and surface temperature of about 7500 K.

a Use Figure E5.8 to estimate its luminosity:

i as a multiple of $L_\odot$ **ii** in W.

b Use an equation for luminosity to determine a more accurate value for Altair's luminosity.

17 a Use Figure E5.8 to estimate how much greater the luminosity of a star of radius $100R_0$ is compared to a star of radius $10R_0$.

b Explain why the larger star has a *much* greater luminosity.

18 A main sequence star has a peak wavelength of 9.7×10^{-8} m.

a Determine its surface temperature.

b Estimate its luminosity.

Evolution of stars

SYLLABUS CONTENT

- The effect of stellar mass on the evolution of a star.
- The main regions of the Hertzsprung–Russell (HR) diagram and how to describe the main properties of stars in these regions.

◆ **Evolution (stellar)** Describes the changes that occur in a star during its 'lifetime'.

◆ **Giant (and super giant) stars** Usually relatively cool stars, so they are yellow / red in colour; their luminosity is high because of their large size. Most stars will become **red giants** (or red **super giants**) at the end of their time on the main sequence.

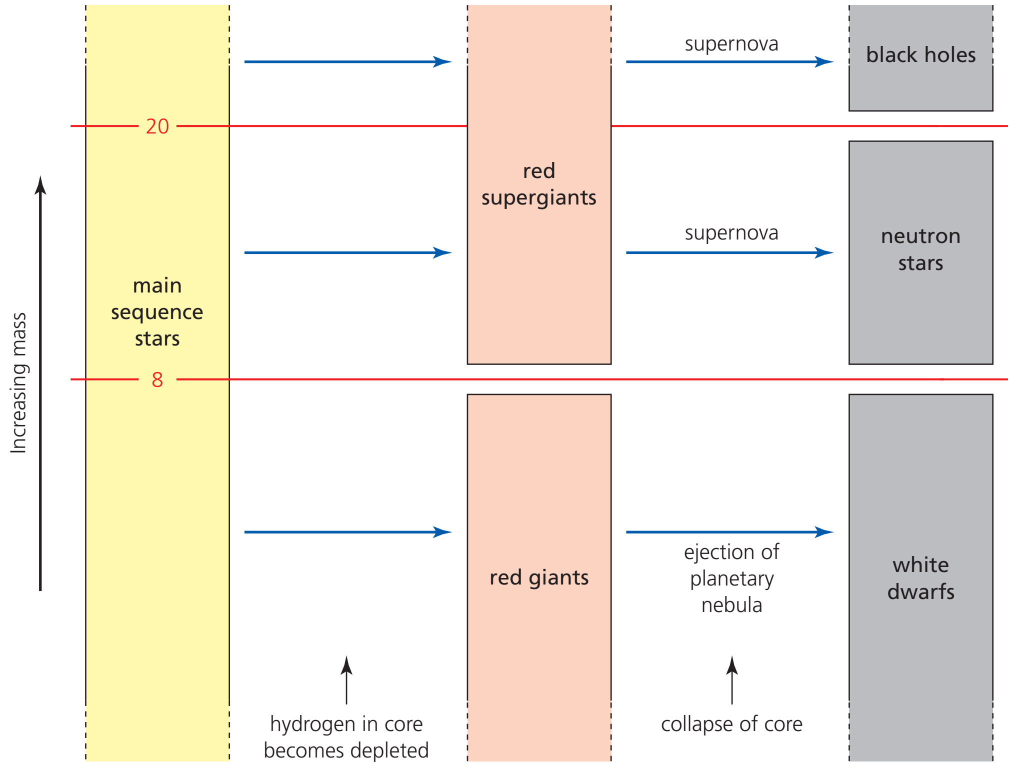
The term **stellar evolution** is being used here to describe what happens to stars after the depletion of hydrogen in their cores. That is, what happens to them after the end of their main sequence lifetimes. The core begins to contract because, once the rate of fusion is reduced, the inward gravitational forces are greater than the outward radiation pressure. Gravitational potential energy is then again transferred to kinetic energy of the nuclei in the core, so that the temperature rises significantly. This causes fusion of hydrogen outside of the core (in a 'shell' around the core).

The rate of fusion in the shell is greater than in the core when it was a main sequence star. So, a star will spend more time on the main sequence than afterwards.

The resulting increased radiation pressure produces a significant expansion of the star. The radius could be as much as one thousand times greater, which would result in a one million times greater surface area. The overall effect is a *reduction* in surface temperature and therefore a change in colour to become redder. These changes are represented by the name of the type of star formed: a **red giant**. Or, if sufficiently large, a red **super giant**. Only a small percentage of main sequence stars will evolve to become red super giants.

Red giant stars (and super giants) are formed by the increased rate of nuclear fusion that occurs because of the greater temperatures created in the collapse of main sequence stars at the end of their lifetimes.

All but the very smallest main sequence stars will become red giants or super giants after the hydrogen in their core has been depleted. What happens after that depends (again) on their masses, as seen in Figure E5.9.



■ **Figure E5.9** Evolution of stars of different masses (the numbers shown represent the approximate mass limits of the stars as multiples of the current mass of the Sun)

Common mistake

Planetary nebula is a misleading term. It has nothing to do with planets.

◆ **Planetary nebula** Material emitted from the outer layers of a red giant star at the end of its lifetime. The core then becomes a white dwarf star.

◆ **White dwarf stars** Relatively hot stars, so that they are blue / white in colour, but their luminosity is low because of their small size. They are formed after the end of the lifetime of smaller main sequence stars.

◆ **Electron degeneracy** Process occurring within white dwarf stars that keeps them stable and stops them collapsing.

◆ **Supernova** Sudden and very luminous explosion of a massive star, resulting in a neutron star or black hole.

◆ **Neutron stars** Very dense stars formed after a supernova.

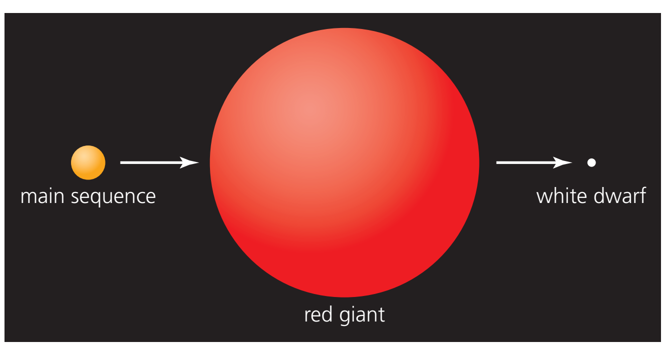
◆ **Black hole** Extremely dense remnant of a giant star formed after a supernova. Gravitational forces are so great that even light cannot escape.

After all fusion has finished in a red giant, gravitational forces will cause the star to collapse inwards. The outer layers are ejected to form a relatively short-lived diffuse cloud of ionized gas called a **planetary nebula**, leaving behind a very dense core. Red super giants evolve differently, as discussed below.

The remaining core has enough internal energy to continue to emit radiation at low luminosity, for a very long time and its surface temperature is high enough that it appears white: this explains the name of this type of star: **white dwarf**. See Figure E5.10.

A white dwarf star is formed from a red giant star after all nuclear fusion has stopped.

A white dwarf star can remain stable for a long time because of a process called **electron degeneracy** (no details required).



■ **Figure E5.10** Evolution of most main sequence stars (our Sun, for example)

Red super giants do not evolve into white dwarfs. The electron degeneracy pressure is insufficient to resist the gravitational forces and the gravitational potential energy released is so high that there are dramatic changes in the core that result in an enormous explosion called a **supernova**. Here again, the result depends on the mass involved. If the original mass of the star was between 8 and 20 solar masses, the remaining core after the supernova will form a **neutron star**. If the mass was greater, a **black hole** is formed. (Further details are not required.)

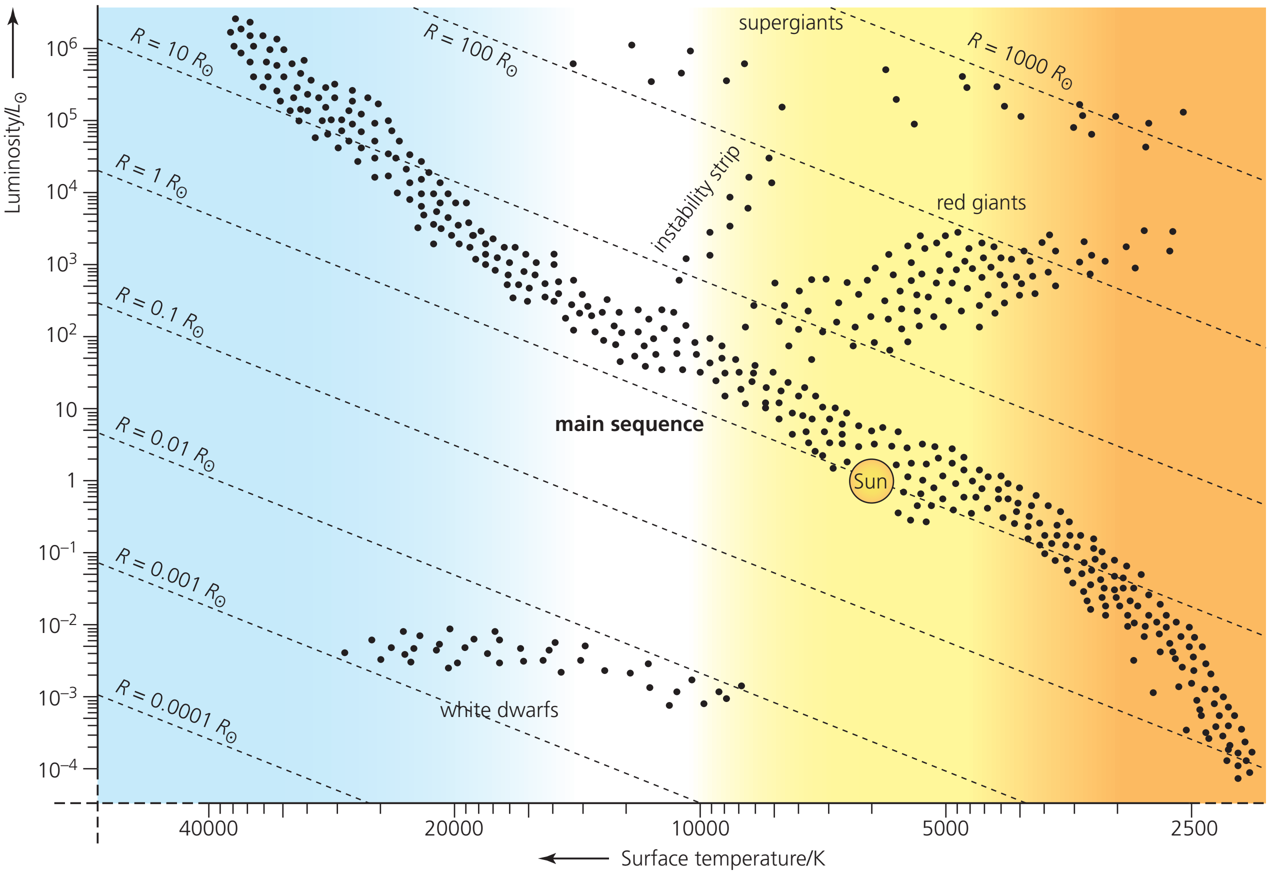
◆ **Evolutionary path** The evolution of a star as drawn on a Hertzsprung–Russell diagram.

◆ **Instability strip** A region of the Hertzsprung–Russell diagram containing pulsating, variable stars.

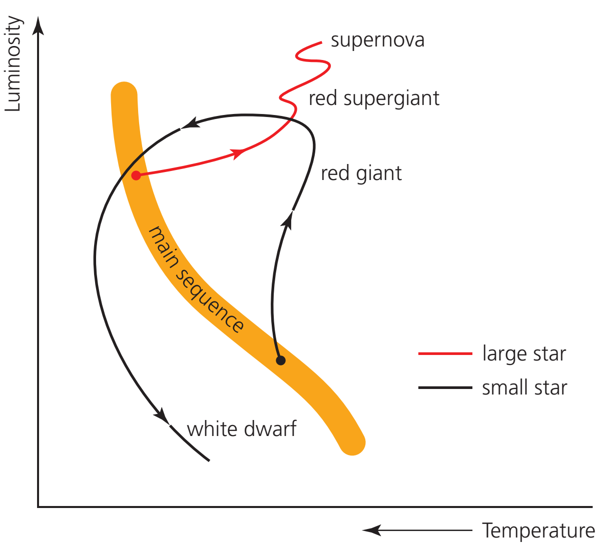
A supernova is an 'explosion' of a red super giant, creating a neutron star or a black hole.

Evolution on a HR diagram

Figure E5.11 shows a more detailed HR diagram with other types of stars included (not just main sequence stars).



■ **Figure E5.11** Detailed HR diagram



■ **Figure E5.12** Evolutionary path of stars after they leave the main sequence

When a main sequence star expands to a red giant, or a red super giant, its luminosity and surface temperature change and this, and subsequent changes, can be tracked on an HR diagram. It is known as a star's **evolutionary path**. Typical evolutionary paths of low-mass and high-mass main sequence stars are shown in Figure E5.12.

The **instability strip** on the HR diagram (seen in Figure E5.11) contains stars at an intermediate stage between being main sequence stars and being super giants or more massive red giants. These stars will be in the instability strip for relatively short times. They pulsate because their outer layers are unstable: their luminosities vary periodically.

The changes to stars which happen after the end of their main sequence lifetimes can be tracked on an HR diagram.

19 Outline the process by which a main sequence star can evolve into a red giant star.

20 Explain the difference between a red giant star and a red super giant star.

21 A red giant has a higher rate of fusion than the main sequence star from which it was formed.
Explain why it has a lower surface temperature.

22 Outline what is meant by the term 'planetary nebula'.

23 a Explain what the colour of a white dwarf star tells us about its surface temperature.
b Explain why, despite their temperatures, white dwarfs can be difficult to observe.

24 Sketch the axes of a HR diagram and add a line to indicate main sequence stars.
a Draw the future evolutionary path of the Sun.
b Draw the evolutionary path of a massive main sequence star that eventually forms a red super giant, after spending time in the instability strip.

25 Use the internet to learn about supernovas that have been detected on Earth.

LINKING QUESTION

- HR diagrams have been helpful in the classification of stars by finding patterns in their properties. Which other areas of physics use classification to help our understanding? (NOS)

Distances from Earth to stars

SYLLABUS CONTENT

▶ The use of stellar parallax as a method to determine the distance, d, to celestial bodies as given by:

$$d(\text{parsec}) = \frac{1}{p(\text{arc-second})}$$

In Topic B.1 it was explained that, *if* we know the luminosity, L, of any star, the equation $b = \frac{L}{4\pi d^2}$ can be used to determine its distance from Earth, d, if we measure its apparent brightness b.

The problem is that, for most stars, we have no direct way of knowing their luminosities.

However, importantly, astronomers have identified a few 'standard candles'. These are stars which have known luminosities wherever they are located (including a type of supernova and some stars in the instability strip). You are not expected to have knowledge of these methods.

The HR diagram can also be used to obtain an approximate distance to a main sequence star, as shown in the following example.

WORKED EXAMPLE E5.4

The surface temperature of a main sequence star was determined to be 17 000 K.

a Explain how this value was calculated using information from the star's spectrum.

b Use the HR diagram to estimate the luminosity of the star (W).

c Determine an approximate distance between this star and Earth if its apparent brightness was measured on Earth to be $3.1 \times 10^{-9}\,\text{W m}^{-2}$ in:

i metres ii light years.

Answer

a By using Wien's law:
$\lambda_{max} T = 2.9 \times 10^{-3}\,\text{m K}$

b $1200 L_{\odot} = 1200 \times 3.8 \times 10^{26} \approx 4.6 \times 10^{29}\,\text{W}$

c i $b = \frac{L}{4\pi d^2}$

$$3.1 \times 10^{-9} = \frac{4.6 \times 10^{29}}{4 \times \pi \times d^2}$$

$d = 3.4 \times 10^{18}\,\text{m} \approx 3 \times 10^{18}\,\text{m}$

ii In light years this is:

$$\frac{3.4 \times 10^{18}}{9.46 \times 10^{15}} \approx 4 \times 10^{2}\,\text{ly}$$

◆ **Stellar parallax** Method of determining the distance to a nearby star from measurement of its *parallax angle*.

◆ **Parallax** The displacement in the apparent position of an object (compared to its background) viewed along two different lines of sight.

Distances to 'nearby' stars

We are able to calculate the distance to 'nearby' stars using simple geometry.

Although stars are moving very quickly, we can draw *star maps* to show the positions of stars (relative to other stars) because their relative positions show no significant changes over very long periods of time. This is because of the enormous distance between the stars (and Earth). However:

if very precise measurements are made of 'nearby' stars, their positions (relative to other stars) are observed to move very slightly (forwards and backwards) during the course of a year. This is called **stellar parallax**.

This repeated apparent movement is represented in Figure 5.13, but the the movement has been exaggerated for clarity.

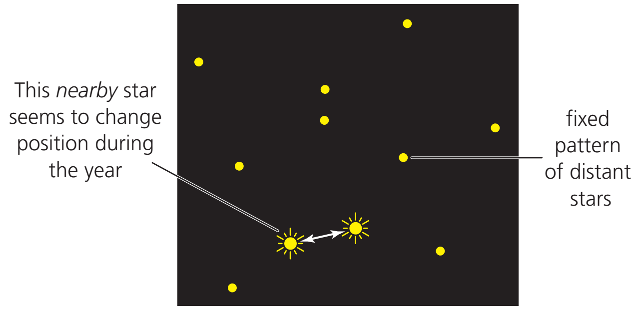


Figure E5.13 A nearby star's apparent movement due to parallax

Inquiry 3: Concluding and evaluating

Evaluating

Parallax

Parallax is the difference in the apparent position of an object (when compared to other objects in the background behind it) when an object is viewed along two different lines of sight. This is shown in Figure E5.14. The closer the object is to the observer, the greater the parallax. The simplest everyday example is seen when observing your finger held in front of your face, first with one eye, then the other.

A numerical value for parallax can be represented by half the angle between the two dotted lines in Figure E5.14.

Identify how **parallax error** can lead to misreading the scales of some measuring instruments.

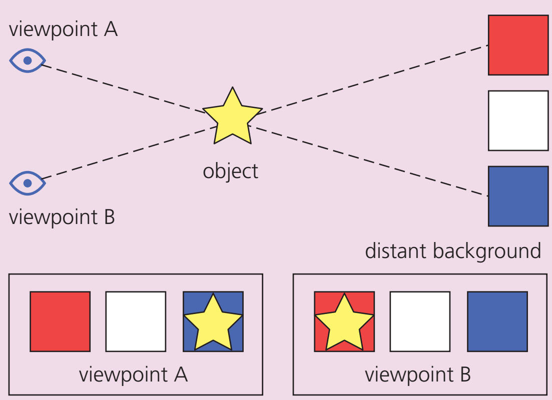
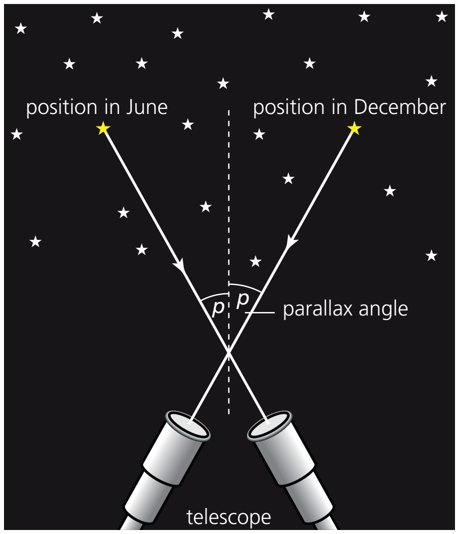


Figure E5.14 Parallax

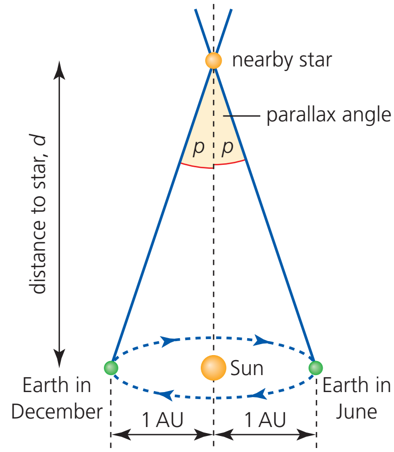
◆ **Parallax error** Occurs when reading a scale from the wrong position.

Because of the limits to measuring very small angles accurately, stellar parallax can be only detected with stars which are less than about 300 ly from Earth. These 'nearby' stars are all within our own galaxy (The Milky Way, which has a diameter of about 10^5 ly.) and include some stars which are visible from Earth without a telescope. In Figure E5.15 and Figure E5.16 the parallax angles have been greatly exaggerated for the sake of clarity.

Using telescopes, astronomers measure the parallax angle, *p*, between, for example, observations of the star made in December and June.



■ **Figure E5.15** Parallax angle six months apart



■ **Figure E5.16** The geometry of the parallax angle

Tool 3: Mathematics

Use of units whenever appropriate

In Figure E5.16, the distance between the Earth and the Sun has been labelled as 1 AU. The Earth's orbit around the Sun is almost circular and the radius varies only by about 3%. The AU, astronomical unit is used in calculations concerning our Solar system. It is *defined* to be exactly $1.495\,978\,707\,00 \times 10^{11}$ m, but we will use 1.50×10^{11} m in calculations.

◆ **Arc-second** An angle which is 1/3600 of one degree.

The stellar parallax of even the closest stars is very small because of the long distances involved and this means that the parallax angles are so tiny that they are measured in **arc-seconds**. (There are 3600 arc-seconds in a degree.)

Once the parallax angle has been measured, simple geometry can be used to calculate the distance to the star (Figure E5.16):

$$\text{parallax angle, } p \text{ (rad)} = \frac{1.50 \times 10^{11}}{d \text{ (m)}}$$

Note that the distance from the Earth to the star and the distance from the Sun to the star can be considered to be equal for such very small angles. That is, we can assume that $p \text{ (rad)} = \sin p = \tan p$.

WORKED EXAMPLE E5.5

Calculate the distance, *d,* to a star if its parallax angle is 0.240 arc-seconds. (Reminder: there are 57.3 degrees in one radian.)

Answer

$$0.240 \text{ arc-seconds} = \left(\frac{0.240}{3600}\right) \times \left(\frac{1}{57.3}\right) = 1.16 \times 10^{-6} \text{ rad}$$

$$p(\text{rad}) = \frac{1.50 \times 10^{11}}{d(\text{m})}$$

$$d = \frac{1.50 \times 10^{11}}{1.16 \times 10^{-6}} = 1.29 \times 10^{17} \text{ m } (= 13.7 \text{ ly})$$

Tool 3: Mathematics

Use of units whenever appropriate

Calculations similar to Worked example E5.5 are common, but it is much easier to use the angle directly as a measure of distance rather than making calculations in SI units. However, this is an inverse relationship – larger parallax angles mean smaller distances.

The **parsec** (pc) – short for parallax of one arc-second – is another unit of distance used by astronomers. Its use is not restricted to stars which exhibit parallax, and it is the most widely used unit of distance in astronomy. A *parsec* is defined as the distance from the Sun (or Earth) to an object that has a parallax angle of one arc-second.

◆ **Parsec, pc** Unit of distance used by astronomers; equal to the distance to a star that has a parallax angle of one arc-second.

DB

$$\text{distance, } \boldsymbol{d}\ \textbf{(parsec)} = \frac{\mathbf{1}}{\boldsymbol{p}\textbf{(arc-second)}}$$

For example, a star with a parallax angle, *p*, of 0.25 arc-seconds will be 1/0.25 = 4 pc distant from Earth.

Table E5.2 shows the relationship between parallax angle and distance.

■ **Table E5.2**

Parallax angle/arc-seconds	Distance away/pc
0.10	10.0
0.25	4.0
0.50	2.0
1.00	1.0

Summary of the non-SI units used in astronomy

DB

1 parsec (pc) = 3.26 ly

■ **Table E5.3** Summary of distance units commonly used in astronomy

Unit	Metres/m	Astronomical units/AU	Light years/ly
1 AU	1.50×10^{11}	–	–
1 ly	9.46×10^{15}	6.32×10^{4}	–
1 pc	3.09×10^{16}	2.06×10^{5}	3.26

WORKED EXAMPLE E5.6

The parallax angle for the star Alpha Centauri is 0.751 arc-seconds.

Calculate its distance from Earth in:

a parsec **b** metre **c** light years **d** astronomical units.

Answer

a distance in parsec = 1/*p*(arc-second) = 1/0.751 = 1.33 pc

b $1.33 \times 3.09 \times 10^{16} = 4.11 \times 10^{16}$ m

c $1.33 \times 3.26 = 4.34$ ly

d $1.33 \times 2.06 \times 10^{5} = 2.67 \times 10^{5}$ AU

26 a Use the HR diagram to estimate the luminosity of a main sequence star which has a surface temperature of 3000 K.

b Estimate the radius of this star from the same diagram.

c Compare your answer to a value of the radius determined from using $L = \sigma AT^4$.

d If the apparent brightness of this star is $4.2 \times 10^{-11}\,\mathrm{W\,m^{-2}}$, estimate its distance from Earth.

27 Explain why stars can be shown in fixed positions on star maps, even though they are moving very fast.

28 a Calculate the total distance travelled in ten years by the Earth as it orbits the Sun. Give your answer in AU.

b The distance between the Sun and Pluto is approximately six billion kilometres. Express this in AU.

29 Convert an angle of 1 arc-second to:

a degrees

b radians.

30 The parallax angle for Barnard's star is measured to be 0.55 arc-seconds. How far away is it from Earth in:

a pc **b** m **c** ly?

31 Calculate the parallax angles for three stars at the following distances from Earth:

a 2.47×10^{15} km

b 7.90 ly

c 2.67 pc.

32 If the upper limit to parallax measurements is for stars which are 300 ly away, calculate the smallest parallax angle that can be measured accurately.

33 Star A is a distance x pc from Earth and has a parallax angle of θ.
Determine the parallax angle for a star B which is $x/2$ from Earth.

34 A star is 50 pc from Earth.

a What is this distance in light years?

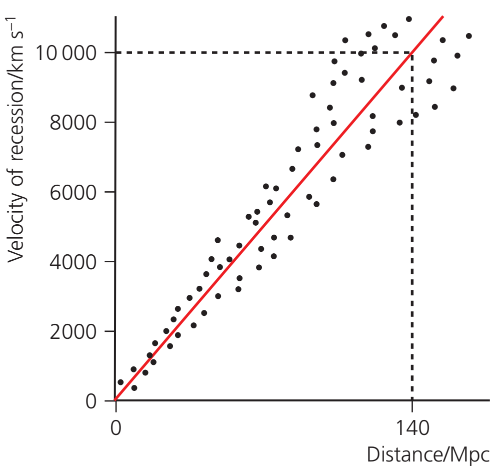
b Will astronomers be able to detect stellar parallax with this star? Explain your answer.

LINKING QUESTION

- In which ways has technology helped to collect data from observations of distant stars? (NOS)

The age of an expanding Universe

We saw in Topic C.5 that the speed of stars and galaxies away from Earth (*recession speeds*) can be determined from the *Doppler shifts* of radiation received from them on Earth. Combining that information with the latest data about their distances from Earth leads us the important graph shown in Figure E5.17. (A simpler version was shown in Topic C.5.)



■ **Figure E5.17** Variation of recession speeds of galaxies with their distances from Earth

Assuming that all the stars began moving at the time of the *Big Bang*, an estimate for the age of the Universe can be determined from the gradient of this graph, by a straightforward use of velocity $= \dfrac{\text{distance}}{\text{time}}$:

$$1 \times 10^4\,\mathrm{km\,s^{-1}} = \frac{140\,\mathrm{Mpc}}{\text{time}}$$

Converting to SI units:

$$10^7 = \frac{140 \times (3.09 \times 10^{22})}{t}$$

$$t = 4.3 \times 10^{17}\,\mathrm{s}\ (1.4 \times 10^{10}\,\mathrm{y})$$

The creation of different elements

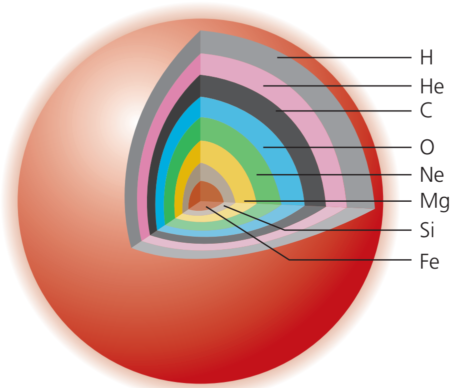
◆ **Nucleosynthesis** Creation of new nuclides (elements) from existing, less massive, nuclei.

◆ **Neutron capture** Nuclear reaction in which a neutron is absorbed by a nucleus to form a more massive nucleus.

We have explained the fusion of hydrogen to form helium, now we will briefly outline how other chemical elements can be created in fusion processes called **nucleosynthesis**. (Details need *not* be remembered.) Basically:

Higher temperatures (particles with greater kinetic energy) are needed for nucleosynthesis, and these exist in the cores of red giants and super giants.

- For red giant stars formed from main sequence stars of mass less than 4 $M_\odot$, the core temperature can reach 10^8 K and this is large enough for the nucleosynthesis of carbon and oxygen. (Helium is still produced in an outer layer.)
- For larger red giant stars (formed from main sequence stars with masses between 4 $M_\odot$ and 8 $M_\odot$), the core temperature exceeds 10^9 K and this is large enough for the nucleosynthesis of neon and magnesium. (Helium, carbon and oxygen are still produced in the outer layers.)
- For red super giant stars (formed from main sequence stars with masses greater than 8 $M_\odot$), the core temperature is large enough for the nucleosynthesis of elements as heavy as silicon and iron. (The lighter elements are still produced in the outer layers.) See Figure E5.18.



■ **Figure E5.18** The layers of a red super giant

From Topic E.3 we know that the nucleus of iron is one of the most stable (it has one of the highest binding energies per nucleon). This means that there would have to be an energy input to create heavier nuclides. Heavier elements are created by **neutron capture**, but that process is not included in the IB Physics course.

We saw in Topic E.1 that the elements present in a star can be identified from measurements made of line spectra.

35 Explain how an element in the outer layers of a star can be identified from the spectrum received from the star.

36 Explain:
  a why very high temperatures are needed to create the more massive nuclides
  b why those higher temperatures are found in more massive stars.

37 Outline why the interior of a red super giant star is composed of different layers.

TOK

Knowledge and the knower

Stardust

All the particles in our body existed for billions of years before we were born. They will continue to exist for billions of years after we die. They originated in nuclear reactions in stars and, ultimately, they will be scattered throughout space.

We are all made of stardust.

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