FOR THE
IB DIPLOMA
PROGRAMME

IN COOPERATION WITH

THIRD EDITION

Physics

John Allum
Paul Morris

Boost

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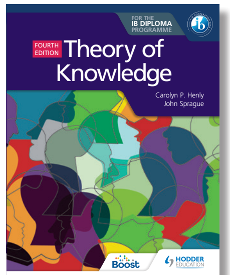

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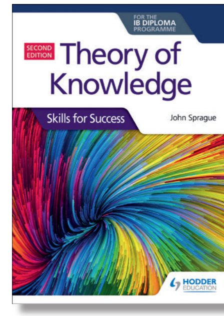

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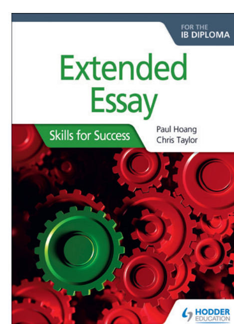

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E Nuclear and quantum

Free online content

Go to our website www.hoddereducation.com/ib-extras for free access to the following:

- Practice exam-style questions for each chapter
- Glossary
- Answers to self-assessment questions and practice exam-style questions
- Tools and Inquiries reference guide
- Internal Assessment – the scientific investigation

Introduction

Welcome to *Physics for the IB Diploma Third Edition*, updated and designed to meet the criteria of the new International Baccalaureate (IB) Diploma Programme Physics Guide. This coursebook provides complete coverage of the new IB Physics Diploma syllabus, with first teaching from 2023. Differentiated content for SL and HL students is clearly identified throughout.

The aim of this syllabus is to integrate concepts, topic content and the nature of science through inquiry. Approaches to learning in the study of physics are integrated with the topics, along with key scientific inquiry skills. This book comprises five main themes:

- **Theme A**: Space, time and motion
- **Theme B**: The particulate nature of matter
- **Theme C**: Wave behaviour
- **Theme D**: Fields
- **Theme E**: Nuclear and quantum physics

Each theme is divided into syllabus topics.

The book has been written with a sympathetic understanding that English is not the first language of many students.

No prior knowledge of physics by students has been assumed, although many will have taken an earlier course (and they will find some useful reminders in the content).

In keeping with the IB philosophy, a wide variety of approaches to teaching and learning has been included in the book (not just the core physics syllabus). The intention is to stimulate interest and motivate beyond the confines of the basic physics content. However, it is very important students know what is the essential knowledge they have to take into the examination room. This is provided by the Key information boxes. If this information is well understood, and plenty of self-assessment questions have been done (and answers checked), then a student will be well-prepared for their IB Physics examination.

The online Glossary is another useful resource. It's aim is to list and explain basic terminology used in physics, but it is not intended as a list of essential information for students. Many of the terms in the Glossary are highlighted in the book as 'Key terms' and also emphasized in the nearby margins.

The 'In cooperation with IB' logo signifies that this coursebook has been rigorously reviewed by the IB to ensure it fully aligns with the current IB curriculum and offers high-quality guidance and support for IB teaching and learning.

How to use this book

The following features will help you consolidate and develop your understanding of physics, through concept-based learning.

Guiding questions

- There are guiding questions at the start of every chapter, as signposts for inquiry.
- These questions will help you to view the content of the syllabus through the conceptual lenses of the themes.

SYLLABUS CONTENT

- This coursebook follows the order of the contents of the IB Physics Diploma syllabus.
- Syllabus understandings are introduced naturally throughout each topic.

Key information

Throughout the book, you will find some content in pink boxes like this one. These highlight the essential Physics knowledge you will need to know when you come to the examination. Included in these boxes are the key equations and constants that are also listed in the IBDP Physics data booklet for the course.

Tools

The Tools features explore the skills and techniques that you require and are integrated into the physics content to be practiced in context. These skills can be assessed through internal and external assessment.

Inquiry process

The application and development of the Inquiry process is supported in close association with the Tools.

Nature of science

Nature of science (NOS) explores conceptual understandings related to the purpose, features and impact of scientific knowledge. It can be examined in Physics papers. NOS explores the scientific process itself, and how science is represented and understood by the general public. NOS covers 11 aspects: Observations, Patterns and trends, Hypotheses, Experiments, Measurements, Models, Evidence, Theories, Falsification, Science as a shared endeavour, and Global impact of science. It also examines the way in which science is the basis for technological developments and how these modern technologies, in turn, drive developments in science.

Key terms

◆ Definitions appear throughout the margins to provide context and help you understand the language of physics. There is also a glossary of all key terms online.

Common mistake

These detail some common misunderstandings and typical errors made by students, so that you can avoid making the same mistakes yourself.

Content from the IBDP Physics data booklet is indicated with this icon and shown in bold. The data booklet contains electrical symbols, equations and constants that you need to familiarize yourself with as you progress through the course. You will have access to a copy of the data booklet during your examination.

ATL ACTIVITY

Approaches to learning (ATL) activities, including learning through inquiry, are integral to IB pedagogy. These activities are contextualized through real-world applications of physics.

Top tip!

This feature includes advice relating to the content being discussed and tips to help you retain the knowledge you need.

WORKED EXAMPLE

These provide a step-by-step guide showing you how to answer the kind of quantitative and other questions that you might encounter in your studies and in the assessment.

International mindedness is indicated with this icon. It explores how the exchange of information and ideas across national boundaries has been essential to the progress of science and illustrates the international aspects of physics.

Self-assessment questions appear throughout the chapters, phrased to assist comprehension and recall, but also to help familiarize you with the assessment implications of the command terms. These command terms are defined in the online glossary. Practice exam-style questions and their answers, together with answers to most self-assessment questions are on the accompanying website, IB Extras: **www.hoddereducation.com/ib-extras**

The IB learner profile icon indicates material that is particularly useful to help you towards developing in the following attributes: to be inquirers, knowledgeable, thinkers, communicators, principled, open-minded, caring, risk-takers, balanced and reflective. When you see the icon, think about what learner profile attribute you might be demonstrating – it could be more than one.

LINKING QUESTIONS

These questions are introduced throughout each topic. They are to strengthen your understanding by making connections across the themes. The linking questions encourage you to apply broad, integrating and discipline-specific concepts from one topic to another, ideally networking your knowledge. Practise answering the linking questions first, on your own or in groups. The links in this coursebook are not exhaustive, you may also encounter other connections between concepts, leading you to create your own linking questions.

TOK

Links to Theory of Knowledge (TOK) allow you to develop critical thinking skills and deepen scientific understanding by bringing discussions about the subject beyond the scope of the content of the curriculum.

About the author

John Allum taught physics to pre-university level in international schools for more than thirty years (as a head of department). He has now retired from teaching, but lives a busy life in a mountainside village in South East Asia. He has also been an IB examiner for many years.

Adviser, writer and reader

Paul Morris is Deputy Principal and IB Diploma Coordinator at the International School of London. He has taught IB Physics and IB Theory of Knowledge for over 20 years, has led teacher workshops internationally and has examined Theory of Knowledge. As an enthusiast for the IB concept-based continuum, Paul designed and developed Hodder Education's 'MYP by Concept' series and was author and co-author of the Physics and Sciences titles in the series. He has also advised on publishing projects for the national sciences education programmes for Singapore and Qatar.

Tools and Inquiry

Skills in the study of physics

The skills and techniques you must experience through the course are encompassed within the tools. These support the application and development of the inquiry process in the delivery of the physics course.

Tools

- **Tool 1:** Experimental techniques
- **Tool 2:** Technology
- **Tool 3:** Mathematics

Inquiry process

- **Inquiry 1:** Exploring and designing
- **Inquiry 2:** Collecting and processing data
- **Inquiry 3:** Concluding and evaluating

Throughout the programme, you will be given opportunities to encounter and practise the skills; and instead of stand-alone topics, they will be integrated into the teaching of the syllabus when they are relevant to the syllabus topics being covered.

You can see what the Tools and Inquiry boxes look like in the *How to use this book* section on page vi.

The skills in the study of physics can be assessed through internal and external assessment. The Approaches to learning provide the framework for the development of these skills.

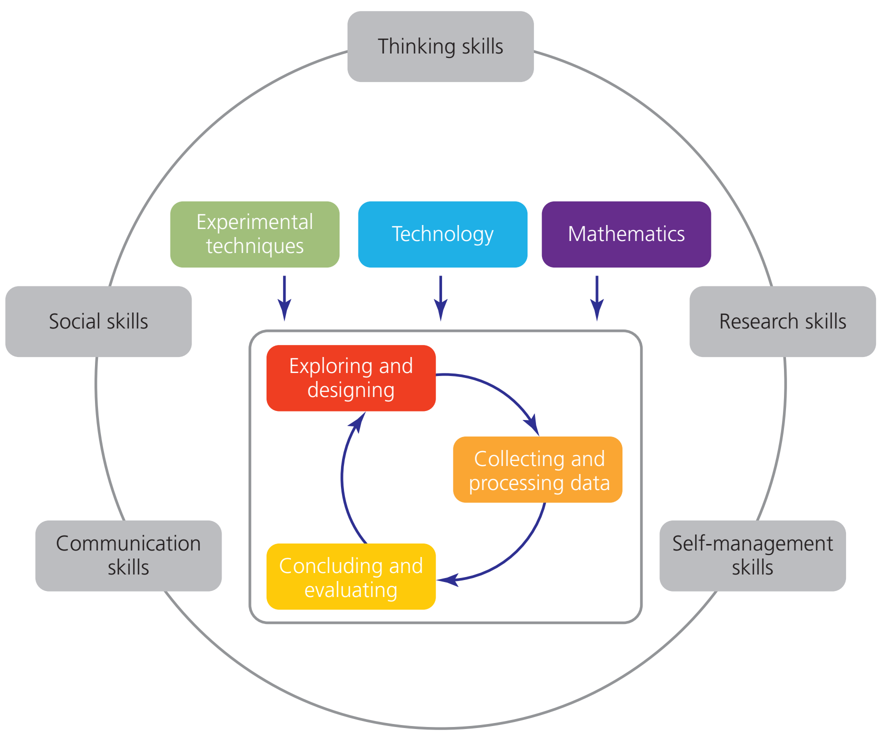


Figure 0.01 Tools for physics

Visit the link in the QR code or this website to view the Tools and Inquiry reference guide: **www.hoddereducation.com/ib-extras**

Tools

Tool 1: Experimental techniques

Skill	Description
Addressing safety of self, others and the environment	• Recognize and address relevant safety, ethical or environmental issues in an investigation.
Measuring variables	Understand how to accurately measure the following to an appropriate level of precision: • Mass • Time • Length • Volume • Temperature • Force • Electric current • Electric potential difference • Angle • Sound and light intensity

Tool 2: Technology

Skill	Description
Applying technology to collect data	• Use sensors. • Identify and extract data from databases. • Generate data from models and simulations. • Carry out image analysis and video analysis of motion.
Applying technology to process data	• Use spreadsheets to manipulate data. • Represent data in a graphical form. • Use computer modelling.

Tool 3: Mathematics

Skill	Description
Applying general mathematics	• Use basic arithmetic and algebraic calculations to solve problems. • Calculate areas and volumes for simple shapes. • Carry out calculations involving decimals, fractions, percentages, ratios, reciprocals, exponents and trigonometric ratios. • Carry out calculations involving logarithmic and exponential functions. • Determine rates of change. • Calculate mean and range. • Use and interpret scientific notation (for example, 3.5×10^6). • Select and manipulate equations. • Derive relationships algebraically. • Use approximation and estimation. • Appreciate when some effects can be neglected and why this is useful. • Compare and quote ratios, values and approximations to the nearest order of magnitude. • Distinguish between continuous and discrete variables.

Skill	Description
	• Understand direct and inverse proportionality, as well as positive and negative relationships or correlations between variables. • Determine the effect of changes to variables on other variables in a relationship. • Calculate and interpret percentage change and percentage difference. • Calculate and interpret percentage error and percentage uncertainty. • Construct and use scale diagrams. • Identify a quantity as a scalar or vector. • Draw and label vectors including magnitude, point of application and direction. • Draw and interpret free-body diagrams showing forces at point of application or centre of mass as required. • Add and subtract vectors in the same plane (limited to three vectors). • Multiply vectors by a scalar. • Resolve vectors (limited to two perpendicular components).
Using units, symbols and numerical values	• Apply and use SI prefixes and units. • Identify and use symbols stated in the guide and the data booklet. • Work with fundamental units. • Use of units (for example, eV, eVc^{-2}, ly, pc, h, day, year) whenever appropriate. • Express derived units in terms of SI units. • Check an expression using dimensional analysis of units (the formal process of dimensional analysis will not be assessed). • Express quantities and uncertainties to an appropriate number of significant figures or decimal places.
Processing uncertainties	• Understand the significance of uncertainties in raw and processed data. • Record uncertainties in measurements as a range (±) to an appropriate precision. • Propagate uncertainties in processed data in calculations involving addition, subtraction, multiplication, division and raising to a power. • Express measurement and processed uncertainties—absolute, fractional (relative) and percentage—to an appropriate number of significant figures or level of precision.
Graphing	• Sketch graphs, with labelled but unscaled axes, to qualitatively describe trends. • Construct and interpret tables, charts and graphs for raw and processed data including bar charts, histograms, scatter graphs and line and curve graphs. • Construct and interpret graphs using logarithmic scales. • Plot linear and non-linear graphs showing the relationship between two variables with appropriate scales and axes. • Draw lines or curves of best fit. • Draw and interpret uncertainty bars. • Extrapolate and interpolate graphs. • Linearize graphs (only where appropriate). • On a best-fit linear graph, construct lines of maximum and minimum gradients with relative accuracy (by eye) considering all uncertainty bars. • Determining the uncertainty in gradients and intercepts. • Interpret features of graphs including gradient, changes in gradient, intercepts, maxima and minima, and areas under the graph.

Inquiry process

Inquiry 1: Exploring and designing

Skill	Description
Exploring	• Demonstrate independent thinking, initiative and insight. • Consult a variety of sources. • Select sufficient and relevant sources of information. • Formulate research questions and hypotheses. • State and explain predictions using scientific understanding.
Designing	• Demonstrate creativity in the designing, implementation and presentation of the investigation. • Develop investigations that involve hands-on laboratory experiments, databases, simulations and modelling. • Identify and justify the choice of dependent, independent and control variables. • Justify the range and quantity of measurements. • Design and explain a valid methodology. • Pilot methodologies.
Controlling variables	Appreciate when and how to: • calibrate measuring apparatus, including sensors • maintain constant environmental conditions of systems • insulate against heat loss or gain • reduce friction • reduce electrical resistance • take background radiation into account.

Inquiry 2: Collecting and processing data

Skill	Description
Collecting data	• Identify and record relevant qualitative observations. • Collect and record sufficient relevant quantitative data. • Identify and address issues that arise during data collection.
Processing data	• Carry out relevant and accurate data processing.
Interpreting results	• Interpret qualitative and quantitative data. • Interpret diagrams, graphs and charts. • Identify, describe and explain patterns, trends and relationships. • Identify and justify the removal or inclusion of outliers in data (no mathematical processing is required). • Assess accuracy, precision, reliability and validity.

Inquiry 3: Concluding and evaluating

Skill	Description
Concluding	• Interpret processed data and analysis to draw and justify conclusions. • Compare the outcomes of an investigation to the accepted scientific context. • Relate the outcomes of an investigation to the stated research question or hypothesis. • Discuss the impact of uncertainties on the conclusions.
Evaluating	• Evaluate hypotheses. • Identify and discuss sources and impacts of random and systematic errors. • Evaluate the implications of methodological weaknesses, limitations and assumptions on conclusions. • Explain realistic and relevant improvements to an investigation.

A.1 Kinematics

◆ **Kinematics** Study of motion.

◆ **Classical physics** Physics theories that pre-dated the paradigm shifts introduced by quantum physics and relativity.

◆ **Uniform** Unchanging.

◆ **Magnitude** Size.

◆ **Scalars** Quantities that have only magnitude (no direction).

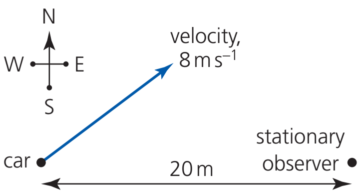
◆ **Vector** A quantity that has both magnitude and direction.

Guiding questions

- How can the motion of a body be described quantitatively and qualitatively?
- How can the position of a body in space and time be predicted?
- How can the analysis of motion in one and two dimensions be used to solve real-life problems?

Kinematics is the study of moving objects. In this topic we will describe motion by using graphs and equations, but the causes of motion (forces) will be covered in the next topic, A.2 Forces and Momentum. The ideas of **classical physics** presented in this chapter can be applied to the movement of all masses, from the very small (freely moving atomic particles) to the very large (stars).

To completely describe the motion of an object at any one moment we need to state its position, how fast it is moving, the direction in which it is moving and whether its motion is changing. For example, we might observe that a car is 20 m to the west of an observer and moving northeast with a constant (**uniform**) velocity of $8\,m\,s^{-1}$. See Figure A1.1.



■ **Figure A1.1** Describing the position and motion of a car

Of course, any or all, of these quantities might be changing. In real life the movement of many objects can be complicated; they do not often move in straight lines and they might even rotate or have different parts moving in different directions.

In this chapter we will develop an understanding of the basic principles of kinematics by dealing first with objects moving in straight lines, and calculations will be confined to those objects that have a uniform (unchanging) motion.

Tool 3: Mathematics

Identify a quantity as a scalar or a vector

Everything that we measure has a magnitude and a unit. For example, we might measure the mass of a book to be 640 g. Here, 640 g is the **magnitude** (size) of the measurement, but mass has no direction.

Quantities that have only magnitude, and no direction, are called **scalars**.

All physical quantities can be described as scalars or **vectors**.

Quantities that have both magnitude and direction are called vectors.

For example, force is a vector quantity because the direction in which a force acts is important.

Most quantities are scalars. Some common examples of scalars used in physics are mass, length, time, energy, temperature and speed.

However, when using the following quantities, we need to know both the magnitude and the direction in which they are acting, so they are vectors:

- displacement (distance in a specified direction)
- velocity (speed in a given direction)
- force (including weight)
- acceleration
- momentum and impulse
- field strength (gravitational, electric and magnetic).

In diagrams, all vectors are shown with straight arrows, pointing in a certain direction from the correct point of application.

The lengths of the arrows are proportional to the magnitudes of the vectors.

Distance and displacement

SYLLABUS CONTENT

- The motion of bodies through space and time can be described and analysed in terms of position, velocity and acceleration.
- The change in position is the displacement.
- The difference between distance and displacement.

◆ **Distance** Total length travelled, without consideration of directions.

◆ **Displacement, linear** Distance in a straight line from a fixed reference point in a specified direction.

◆ **Metre, m** SI unit of length (fundamental).

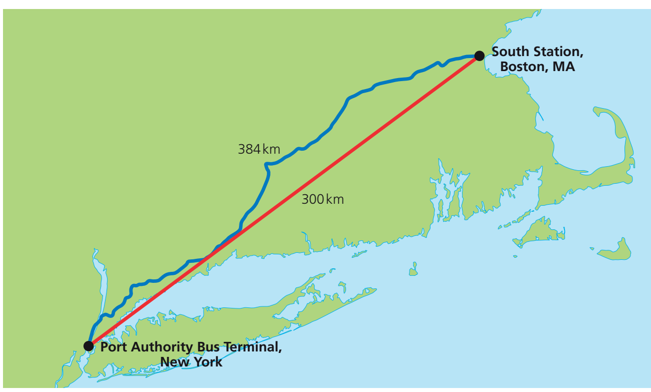
The term **distance** can be used in different ways, for example we might say that the distance between New York City and Boston is 300 km, meaning that a straight line between the two cities has a length of 300 km. Or, we might say that the (travel) distance was 348 km, meaning the length of the road between the cities.

We will define distance as follows:

Distance (of travel) is the total length of a specified path between two points. SI unit: **metre, m**

In physics, **displacement** (change of position) is often more important than distance:

The displacement of an object is the distance in a straight line from a fixed reference point in a specified direction.

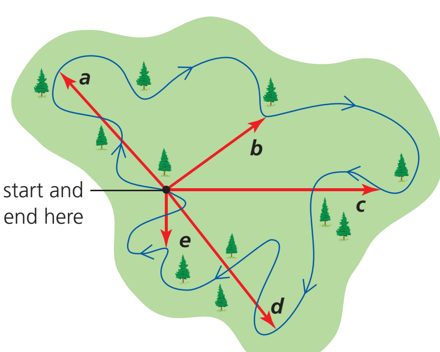


■ **Figure A1.2** Boston is a travel distance of 384 km and a displacement of 300 km northeast from New York City

Continuing the example given above, if a girl travels from New York to Boston, her displacement will be 300 km to the northeast (see Figure A1.2).

Both distance and displacement are given the symbol s and the SI unit metres, m. Kilometres, km, centimetres, cm, and millimetres, mm, are also in widespread use. We often use the symbol h for heights and x for small displacements.

Figure A1.3 shows the route of some people walking around a park. The total distance walked was 4 km, but the displacement from the reference point varied and is shown every few minutes by the vector arrows (***a***–***e***). The final displacement was zero because the walkers returned to their starting place.



■ **Figure A1.3** A walk in the park

Speed and velocity

SYLLABUS CONTENT

- Velocity is the rate of change of position.
- The difference between instantaneous and average values of velocity, speed and acceleration, and how to determine them.

Speed

The displacement of Wellington from Auckland, New Zealand, is 494 km south (Figure A1.4). The road distance is 642 km and it is predicted that a car journey between the two cities will take 9.0 hours.

■ **Figure A1.4** Distance and displacement from Auckland to Wellington

If we divide the total distance by the total time (642 / 9.0) we determine a speed of 71 km h^{-1}. In this example it should be obvious that the speed will have changed during the journey and the calculated result is just an **average speed** for the whole trip. The value seen on the speedometer of the car is the speed at any particular moment, called the **instantaneous speed**.

◆ **Speed, *v*** Average speed = distance travelled/time taken. Instantaneous speed is determined over a very short time interval, during which it is assumed that the speed does not change.

◆ **Reaction time** The time delay between an event occurring and a response. For example, the delay that occurs when using a stopwatch.

◆ **Sensor** An electrical component that responds to a change in a physical property with a corresponding change in an electrical property (usually resistance). Also called a transducer.

◆ **Light gate** Electronic sensor used to detect motion when an object interrupts a beam of light.

Tool 1: Experimental techniques

Understand how to accurately measure quantities to an appropriate level of precision: time

Accurate time measuring instruments are common, but the problem with obtaining accurate measurements of time is starting and stopping the timers at exactly the right moments.

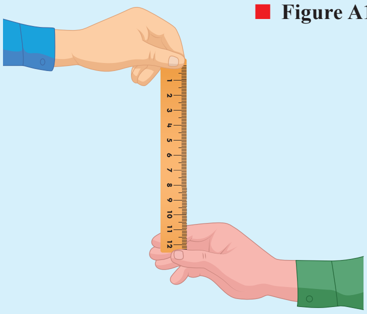
Whenever we use stopwatches or timers operated by hand, the results will have an unavoidable and variable uncertainty because of the delays between seeing an event and pressing a button to start or stop the timer. The delay between seeing something happen and responding with some kind of action is known as **reaction time**. For example, for car drivers it is usually assumed that a driver takes about 0.7 s to press the brake pedal after they have seen a problem. (But some drivers will be able to react quicker than this.) A car will travel about 14 m in this time if it is moving at 50 km h^{-1}. Reaction times will increase if the driver is distracted, tired, or under the influence of any type of drug, such as alcohol.

A simple way of determining a person's reaction time is by measuring how far a metre ruler falls before it can be caught between thumb and finger (see Figure A1.5). The time, t, can then be calculated using the equation for distance, $s = 5t^2$ (explained later in this topic).

If the distance the ruler falls $s = 0.30$

Rearranging for t, $t = \sqrt{\frac{s}{5}} = \sqrt{\frac{0.30}{5}}$

So, reaction time $t = 0.25$ s.



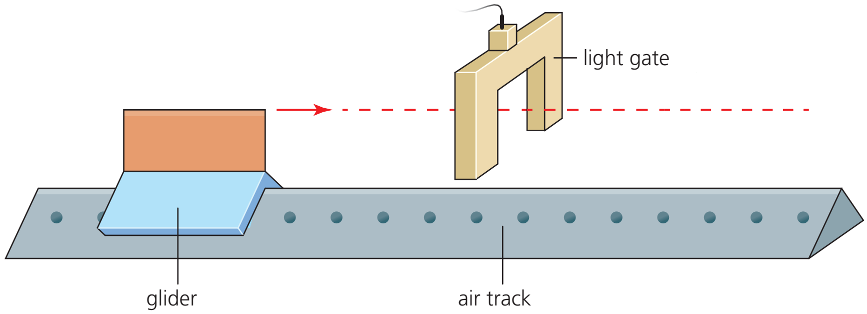
■ **Figure A1.5** Determining reaction time

Under these conditions a typical reaction time is about 0.25 s, but it can vary considerably depending on the conditions involved. The measurement can be repeated with the person tested being blindfolded to see if the reaction time changes if the stimulus (to catch the ruler) is either sound or touch, rather than sight.

In science experiments it is sensible to make time measurements as long as possible to decrease the effect of this problem. (This reduces the percentage uncertainty.) Repeating measurements and calculating an average will also reduce the effect of random uncertainties. If a stopwatch is started late because of the user's reaction time, it may be offset by also stopping the stopwatch late for the same reason.

Electronics **sensors**, such as **light gates**, are very useful in obtaining accurate time measurements. See below.

There are a number of different methods in which speed can be measured in a school or college laboratory. Figure A1.6 shows one possibility, in which a glider is moving on a frictionless air track at a constant velocity. The time taken for a card of known length (on the glider) to pass through the light gate is measured and its speed can be calculated from length of card / time taken.



■ **Figure A1.6** Measuring speed in a laboratory

Tool 2: Technology

Use sensors

An electronic sensor is an electronic device used to convert a physical quantity into an electrical signal. The most common sensors respond to changes in light level, sound level, temperature or pressure.

A light gate contains a source of light that produces a narrow beam of light directed towards a sensor on the other side of a gap. When an object passes across the light beam, the unit behaves as a switch which turns a timer on or off very quickly.

Tool 3: Mathematics

Determine rates of change

The Greek capital letter delta, Δ, is widely used in physics and mathematics to represent a change in the value of a quantity.

For example, $\Delta x = (x_2 - x_1)$, where x_2 and x_1 are two different values of the variable x.

The change involved is often considered to be relatively small.

◆ **Second, s** SI unit of time (fundamental).

Most methods of determining speed involve measuring the small amount of time (Δt) taken to travel a certain distance (Δs). The SI unit for time is the **second**, s.

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}} \quad (\text{SI unit m s}^{-1})$$

This calculation determines an average speed during time Δt, but if Δt is small enough, we may assume that the calculated value is a good approximation to an instantaneous speed.

Speed is a scalar quantity. Speed is given the same symbol, (v), as velocity.

■ **Figure A1.7** The peregrine falcon is reported to be the world's fastest animal (speeds measured up to 390 km h^{-1})

◆ **At rest** Stays stationary in the same position.

◆ **Milky Way** The galaxy in which our Solar System is located.

Nature of science: Observations

Objects at rest

It is common in physics for people to refer to an object being **at rest**, meaning that it is not moving. But this is not as simple as it may seem. A stone may be at rest on the ground, meaning that it is not moving when compared with the ground: it appears to us to have no velocity and no acceleration. However, when the same stone is thrown upwards, at the top of its path its instantaneous speed may be zero, but it has an acceleration downwards.

We cannot assume that an object which is at rest has no acceleration; its velocity may be changing – either in magnitude, in direction, or both.

We may prefer to refer to an object being *stationary*, suggesting that an object is not moving over a period of time.

Of course, the surface of the Earth is moving, the Earth is orbiting the Sun, which orbits the centre of the **Milky Way** galaxy, which itself exists in an expanding universe. So, at a deeper level, we must understand that *all* motion is relative and nowhere is truly stationary. This is the starting point for the study of Relativity (Topic A.5).

Velocity

Velocity, v, is the rate of change of position. It may be considered to be speed in a specified direction.

◆ **Velocity, v** Rate of change of position.

$$\text{velocity, } v = \frac{\text{displacement}}{\text{time taken}} = \frac{\Delta s}{\Delta t} \quad (\text{SI unit m s}^{-1})$$

The symbol Δs represents a change of position (displacement).

Velocity is a vector quantity. $12\,\text{m s}^{-1}$ is a speed. $12\,\text{m s}^{-1}$ to the south is a velocity. We use positive and negative signs to represent velocities in opposite directions. For example, $+12\,\text{m s}^{-1}$ may represent a velocity upwards, while $-12\,\text{m s}^{-1}$ represents the same speed downwards, but we may choose to reverse the signs used.

Speed and velocity are both represented by the same symbol (v) and their magnitudes are calculated in the same way $\left(v = \frac{\Delta s}{\Delta t}\right)$ with the same units. It is not surprising that these two terms are sometimes used interchangeably and this can cause confusion. For this reason, it may be better to define these two quantities in words, rather than symbols.

As with speed, we may need to distinguish between average velocity over a time interval, or instantaneous velocity at a particular moment. As we shall see, the value of an instantaneous velocity can be determined from the gradient of a displacement–time graph.

Top tip!

When a direction of motion is clearly stated (such as ‘up’, ‘to the north’, ‘to the right’ and so on), it is very clear that a velocity is being discussed. However, we may commonly refer to the ‘velocity’ of a car, for example, without stating a direction. Although this is casual, it is usually acceptable because an unchanging direction is implied, even if it is not specified. For example, we may assume that the direction of the car is along a straight road.

WORKED EXAMPLE A1.1

A satellite moves in circles along the same path around the Earth at a constant distance of 6.7×10^3 km from the Earth's centre. Each **orbit** takes a time of 90 minutes.

a Calculate the average speed of the satellite.

b Describe the instantaneous velocity of the satellite.

c Determine its displacement from the centre of the Earth after

i 360 minutes

ii 405 minutes.

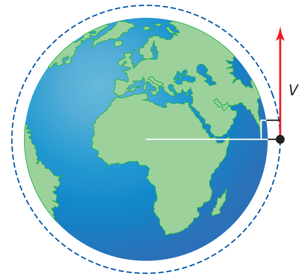


Figure A1.8 Satellite's instantaneous velocity

◆ **Orbit** The curved path (may be circular) of a mass around a larger central mass.

◆ **Tangent** Line which touches a given curve at a single point.

Answer

a $v = \dfrac{\text{circumference}}{\text{time for orbit}} = \dfrac{2\pi r}{\Delta t}$

$= \dfrac{(2 \times \pi \times 6.7 \times 10^6)}{(90 \times 60)}$

$= 7.8 \times 10^3\,\text{m s}^{-1}$

b The velocity also has a constant magnitude of $7.8 \times 10^3\,\text{m s}^{-1}$, but its direction is continuously changing. Its instantaneous velocity is always directed along a **tangent** to its circular orbit. See Figure A1.8.

c i 360 minutes is the time for four complete orbits. The satellite will have returned to the same place. Its displacement from the centre of the Earth compared to 360 minutes earlier will be the same. (But the Earth will have rotated.)

ii In the extra 45 minutes the satellite will have travelled half of its orbit. It will be on the opposite side of the Earth's centre, but at the same distance. We could represent this as -6.7×10^3 km from the Earth's centre.

1 Calculate the average speed (m s^{-1}) of an athlete who can run a marathon (42.2 km) in 2 hours, 1 minute and 9 seconds. (The men's world record at the time of writing.)

Figure A1.9 Eliud Kipchoge, world record holder for the men's marathon

2 A small ball dropped from a height of 2.0 m takes 0.72 s to reach the ground.

a Calculate $\dfrac{2.0}{0.72}$

b What does your answer represent?

c The speed of the ball just before it hits the ground is $5.3\,\text{m s}^{-1}$. This is an instantaneous speed. Distinguish between an instantaneous value and an average value.

d State the instantaneous velocity of the ball just before it hits the ground.

e After bouncing, the ball only rises to a lower height. Give a rough estimate of the instantaneous velocity of the ball as it leaves the ground.

3 A magnetic field surrounds the Earth and it can be detected by a compass. State whether it is a scalar or a vector quantity. Explain your answer.

4 On a flight from Rome to London, a figure of $900\,\text{km h}^{-1}$ is displayed on the screen.

a State whether this is a speed or a velocity.

b Is it an average or instantaneous value?

c Convert the value to m s^{-1}.

d Calculate how long it will take the aircraft to travel a distance of 100 m.

Acceleration

SYLLABUS CONTENT

- Acceleration is the rate of change of velocity.
- Motion with uniform and non-uniform acceleration.

◆ **Acceleration, *a*** Rate of change of velocity with time. Acceleration is a vector quantity.

◆ **Deceleration** Term commonly used to describe a decreasing speed.

Any variation from moving at a constant speed in a straight line is described as an **acceleration**.

Going faster, going slower and/or changing direction are all different kinds of acceleration (changing velocities).

When the velocity (or speed) of an object changes during a certain time, the symbol u is used for the initial velocity and the symbol v is used for the final velocity. These velocities are not necessarily the beginning and end of the entire motion, just the velocities at the start and end of the period of time that is being considered.

Acceleration, a, is defined as the rate of change of velocity with time:

$$a = \frac{\Delta v}{\Delta t} = \frac{(v - u)}{t} \quad \text{(SI unit m s}^{-2}\text{)}$$

One way to determine an acceleration is to measure two velocities and the time between the measurements. Figure A1.10 shows an example.

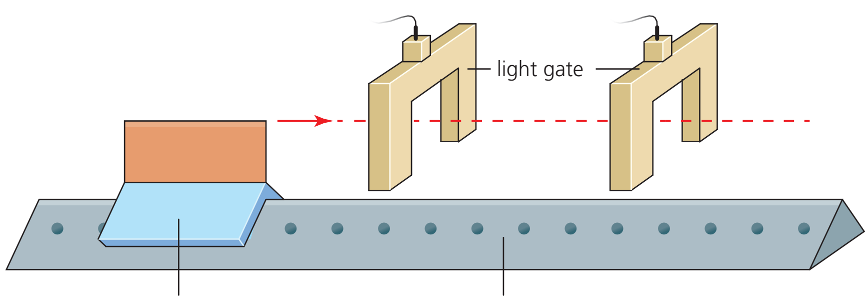


Figure A1.10 Measuring two velocities to determine an acceleration

Acceleration is a vector quantity. For a typical motion in which displacement and velocity are both given positive values, a positive acceleration means increasing speed in the same direction ($+\Delta v$), while a negative acceleration means decreasing speed in the same direction ($-\Delta v$). In everyday speech, a reducing speed is often called a **deceleration**.

For a motion in which displacement and velocity are given negative values, a positive acceleration means a decreasing speed. For example, a velocity change from $-6\,\text{m s}^{-1}$ to $-4\,\text{m s}^{-1}$ in 0.5 s corresponds to an acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{([-4]-[-6])}{0.5} = +\,4\,\text{ms}^{-2}$$

As with speed and velocity, we may need to distinguish between average acceleration over a time interval, or instantaneous acceleration at a particular moment.

WORKED EXAMPLE A1.2

A high-speed train travelling with a velocity of $84\,\mathrm{m\,s^{-1}}$ needs to slow down and stop in a time of one minute.

a Determine the necessary average acceleration.

b Calculate the distance that the train will travel in this time assuming that the acceleration is uniform.

Answer

a $a = \dfrac{\Delta v}{\Delta t} = \dfrac{(0 - 84)}{60} = -1.4\,\mathrm{m\,s^{-2}}$

The acceleration is negative. The negative sign shows that the velocity is decreasing.

b average speed $= \dfrac{(84 - 0)}{2} = 42\,\mathrm{m\,s^{-1}}$

distance = average speed × time = $42 \times 60 = 2.5 \times 10^3\,\mathrm{m}$

5 A car moving at $12.5\,\mathrm{m\,s^{-1}}$ accelerates uniformly on a straight road at a rate of $0.850\,\mathrm{m\,s^{-2}}$.

  a Calculate its velocity after 4.60 s.

  b What uniform rate of acceleration will reduce the speed to $5.0\,\mathrm{m\,s^{-1}}$ in a further 12 s?

6 An athlete accelerates uniformly from rest at the start of a race at a rate of $4.3\,\mathrm{m\,s^{-2}}$. How much time is needed before her speed has reached $8.0\,\mathrm{m\,s^{-1}}$?

7 A trolley takes 3.62 s to accelerate from rest uniformly down a slope at a rate of $0.16\,\mathrm{m\,s^{-2}}$. A light gate at the bottom of the slope records a velocity of $0.58\,\mathrm{m\,s^{-1}}$. What was the speed about halfway down the slope, 1.2 s earlier?

Inquiry 1: Exploring and designing

Designing

Suppose that the Principal of your school or college is worried about safety from traffic on the nearby road. He has asked your physics class to collect evidence that he can take to the police. He is concerned that the traffic travels too fast and that the vehicles do not slow down as they approach the school.

1. Using a team of students, working over a period of one week, with tape measures and stop watches, develop an investigation which will produce sufficient and accurate data that can be given in a report to the Principal. Explain how you would ensure that the investigation was carried out safely.
2. What is the best way of presenting a summary of this data?

Tool 3: Mathematics

Interpret features of graphs

In order to analyse and predict motions we have two methods: graphical and algebraic. Firstly, we will look at how motion can be represented graphically.

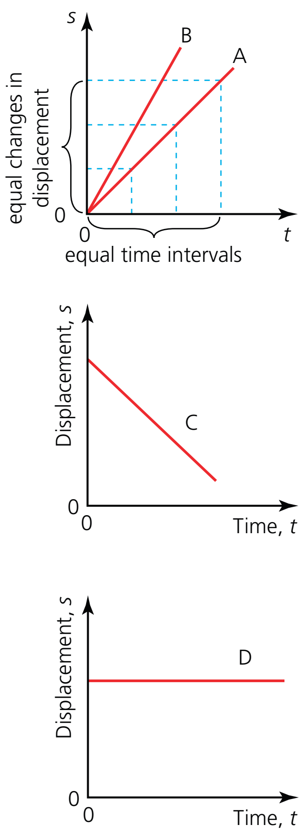
Graphs can be drawn to represent any motion and they provide extra understanding and insight (at a glance) that very few of us can get from written descriptions or equations. Furthermore, the gradients of graphs and the areas under graphs often provide additional useful information.

Displacement–time graphs and distance–time graphs

◆ **Linear relationship** One which produces a straight line graph.

Displacement–time graphs, similar to those shown in Figure A1.11, show how the displacements of objects from a known reference point vary with time. All the examples shown in Figure A1.11 are straight lines and are representing **linear relationships** and constant velocities.

- Line A represents an object moving away from the reference point (zero displacement) such that equal displacements occur in equal times. That is, the object has a constant velocity.



■ **Figure A1.11** Constant velocities on displacement–time graphs

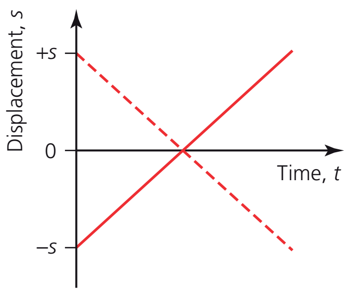
Any linear displacement–time graph represents a constant velocity (it does not need to start or end at the origin).

- Line B represents an object moving with a greater velocity than A.
- Line C represents an object that is moving back towards the reference point.
- Line D represents an object that is stationary (at rest). It has zero velocity and stays at the same distance from the reference point.

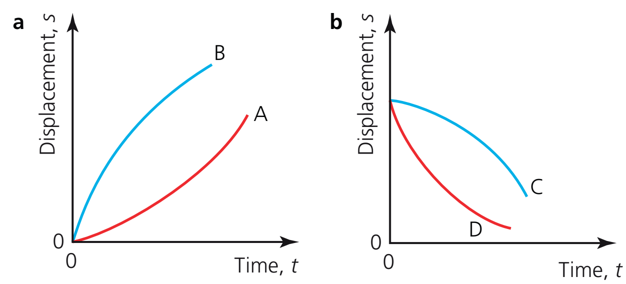
Figure A1.12 shows how we can represent displacements in opposite directions from the same reference point.

The solid line represents the motion of an object moving with a constant (positive) velocity. The object moves towards a reference point (where the displacement is zero), passes it, and then moves away from the reference point with the same velocity. The dotted line represents an identical speed in the opposite direction (or it could also represent the original motion if the directions chosen to be positive and negative were reversed).

Any curved (non-linear) line on a displacement–time graph represents a changing velocity, in other words, an acceleration. This is illustrated in Figure A1.13.



■ **Figure A1.12** Motion in opposite directions represented on a displacement–time graph

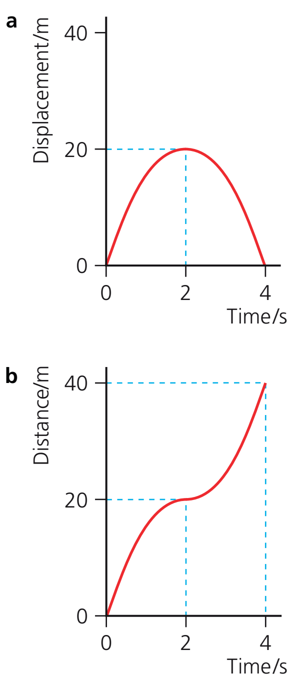


■ **Figure A1.13** Accelerations on displacement–time graphs

Figure A1.13a shows motion away from a reference point. Line A represents an object accelerating. Line B represents an object decelerating. Figure A1.13b shows motion towards a reference point. Line C represents an object accelerating. Line D represents an object decelerating. The values of the accelerations represented by these graphs may, or may not, be constant. (This cannot be determined without a more detailed analysis.)

In physics, we are usually more concerned with displacement–time graphs than distance–time graphs. In order to explain the difference, consider Figure A1.14.

Figure A1.14a shows a displacement–time graph for an object thrown vertically upwards with an initial speed of $20\,\mathrm{m\,s^{-1}}$ (without air resistance). It takes 2 s to reach a maximum height of 20 m. At that point it has an instantaneous velocity of zero, before returning to where it began after 4 s and regaining its initial speed. Figure A1.14b is a less commonly used graph showing how the same motion would appear on an overall distance–time graph.



■ **Figure A1.14** **a** Displacement–time and **b** distance–time graphs for an object moving up and then down

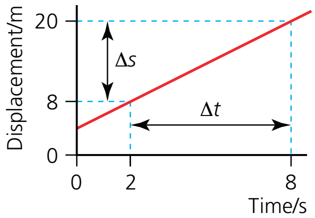
Tool 3: Mathematics

Interpret features of graphs: gradient

In this topic we will need to repeatedly use the following information:

- The gradient of a displacement–time graph equals velocity.
- The gradient of a velocity–time graph equals acceleration.

In the following section we will explore how to measure and interpret gradients.



■ **Figure A1.15** Finding a constant velocity from a displacement–time graph

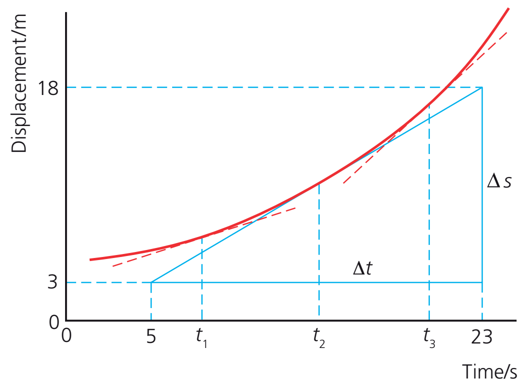
Gradients of displacement–time graphs

Consider the motion at *constant* velocity represented by Figure A1.15.

The **gradient** of the graph $= \dfrac{\Delta s}{\Delta t}$, which is the velocity of the object. A downwards sloping graph would have a negative gradient (velocity).

In this example,

$$\text{constant velocity, } v = \frac{\Delta s}{\Delta t} = \frac{(20 - 8.0)}{(8.0 - 2.0)} = 2.0\,\text{m}\,\text{s}^{-1}$$



■ **Figure A1.16** Finding an instantaneous velocity from a curved displacement–time graph

Figure A1.16 represents the motion of an object with a *changing* velocity, that is, an accelerating object.

The gradient of this graph varies, but at any point it is still equal to the velocity of the object at that moment, that is, the instantaneous velocity.

The gradient (velocity) can be determined at any time by drawing a tangent to the curve, as shown.

The triangle used to calculate the gradient should be large, in order to make this process as accurate as possible. In this example:

$$\text{velocity at time } t_2 = \frac{(18 - 3.0)}{(23 - 5.0)} = 0.83\,\text{m}\,\text{s}^{-1}$$

A tangent drawn at time t_1 would have a smaller gradient and represent a smaller velocity. A tangent drawn at time t_3 would represent a larger velocity.

◆ **Gradient** The rate at which one physical quantity changes in response to changes in another physical quantity. Commonly, for an y–x graph, gradient $= \dfrac{\Delta y}{\Delta x}$.

We have been referring to the object's displacement and velocity, although no direction has been stated. This is acceptable because that information would be included when the origin of the graph was explained. If information was presented in the form of a distance–time graph, the gradient would represent the speed.

In summary:

The gradient of a displacement–time graph represents velocity.

The gradient of a distance–time graph represents speed.

WORKED EXAMPLE A1.3

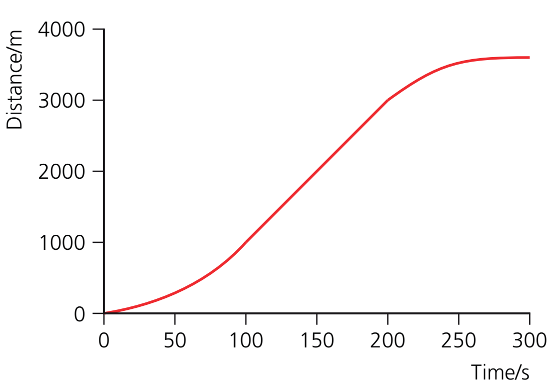
Figure A1.17 represents the motion of a train on a straight track between two stations.

a Describe the motion.

b State the distance between the two stations.

c Calculate the maximum speed of the train.

d Determine the average speed of the train.



■ **Figure A1.17** Distance–time graph for train on a straight track

Answer

a The train started from rest. For the first 90 s the train was accelerating. It then travelled with a constant speed until a time of 200 s. After that, its speed decreased to become zero after 280 s.

b 3500 m

c From the steepest, straight section of the graph:

$$v = \frac{\Delta s}{\Delta t} = \frac{(3000 - 800)}{(200 - 90)} = 20\,\text{m}\,\text{s}^{-1}$$

d $\text{average speed} = \dfrac{\text{total distance travelled}}{\text{time taken}} = \dfrac{3500}{300} = 11.7\,\text{m}\,\text{s}^{-1}$

8 Draw a displacement–time graph for a swimmer swimming a total distance of 100 m at a constant speed of 1.0 m s^{-1} in a swimming pool of length 50 m.

9 Describe the motion of a runner as shown by the graph in Figure A1.18.

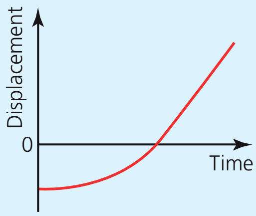


Figure A1.18 Displacement–time graph for a runner

10 Sketch a displacement–time graph for the following motion: a stationary car is 25 m away; 2 s later it starts to move further away in a straight line from you with a constant acceleration of 1.5 m s^{-2} for 4 s; then it continues with a constant velocity for another 8 s.

11 Figure A1.19 is a displacement–time graph for an object.

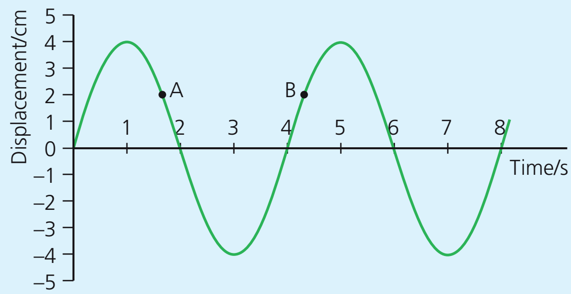


Figure A1.19 A displacement–time graph for an object

- **a** Describe the motion represented by the graph in Figure A1.19.
- **b** Compare the velocities at points A and B.
- **c** When is the object moving with its maximum and minimum velocities?
- **d** Estimate values for the maximum and minimum velocities.
- **e** Suggest what kind of object could move in this way.

Velocity–time graphs and speed–time graphs

Figure A1.20, shows how the velocity of four objects changed with time. Any straight (linear) line on any velocity–time graph shows that equal changes of velocity occur in equal times – that is, it represents *constant* acceleration.

- Line A shows an object that has a constant positive acceleration.
- Line B represents an object moving with a greater positive acceleration than A.
- Line C represents an object that has a negative acceleration.
- Line D represents an object moving with a constant velocity – that is, it has zero acceleration.

Curved lines on velocity–time graphs represent *changing* accelerations.

Velocities in opposite directions are represented by positive and negative values.

We will return to the example shown in Figure A1.14 to illustrate the difference between velocity–time and speed–time graphs. Figure A1.21a shows how the speed of an object changes as it is thrown up in the air (without air resistance), reaches its highest point, where its speed has reduced to zero, and then returns downwards. Figure A1.21b shows the same information in terms of velocity. Positive velocity represents motion upwards, negative velocity represents motion downwards. In most cases, the velocity graph is preferred to the speed graph.

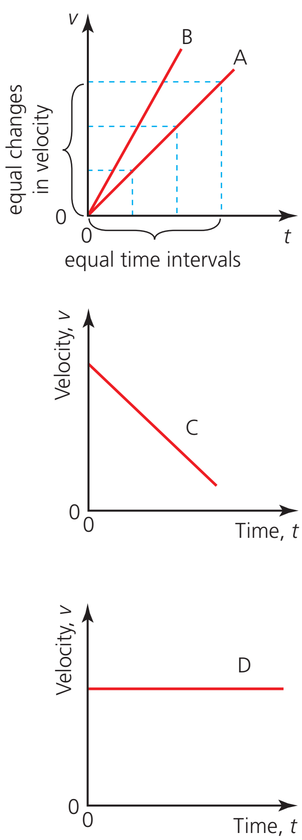


Figure A1.20 Constant accelerations on velocity–time graphs

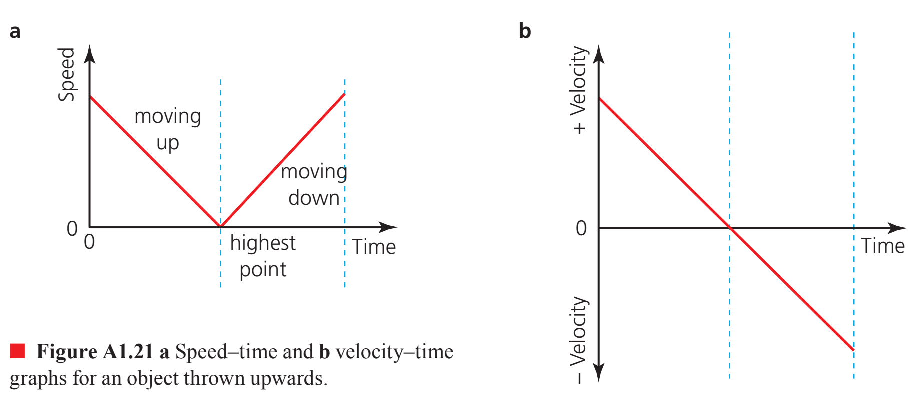


Figure A1.21 a Speed–time and **b** velocity–time graphs for an object thrown upwards.

Gradients of velocity–time graphs

Consider the motion at constant acceleration shown by the straight line in Figure A1.22.

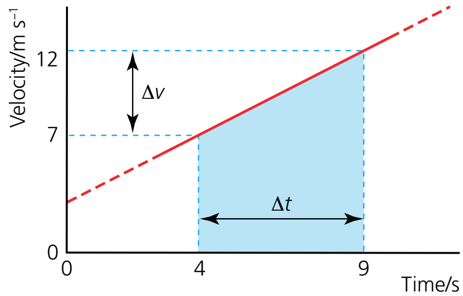


Figure A1.22 Finding the gradient of a velocity–time graph

The gradient of the graph $= \dfrac{\Delta v}{\Delta t}$, which is equal to the acceleration of the object.

In this example, the constant acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{(12.0 - 7.0)}{(9.0 - 4.0)} = +\,1.0\,\mathrm{m\,s^{-2}}$$

The acceleration of an object is equal to the gradient of the velocity–time graph.

A *changing* acceleration will appear as a curved line on a velocity–time graph. A numerical value for the acceleration at any time can be determined from the gradient of the graph at that moment. See Worked example A1.4.

WORKED EXAMPLE A1.4

The red line in Figure A1.23 shows an object decelerating (a decreasing negative acceleration). Use the graph to determine the instantaneous acceleration at a time of 10.0 s.

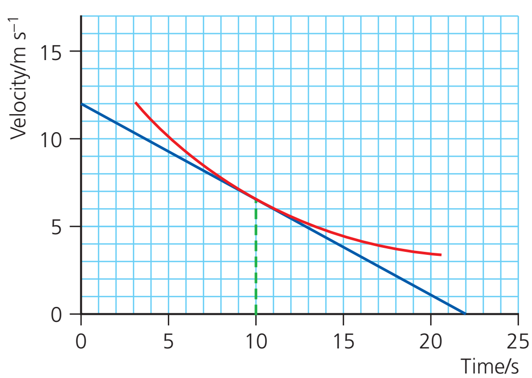


Figure A1.23 Finding an instantaneous acceleration from a velocity–time graph

Answer

Using a tangent to the curve drawn at $t = 10$ s.

$$\text{Acceleration, } a = \frac{\Delta v}{\Delta t} = \frac{(0 - 12)}{(22 - 0)} = -0.55\,\mathrm{m\,s^{-2}}$$

The negative sign indicates a deceleration. In this example the large triangle used to determine the gradient accurately was drawn by extending the tangent to the axes for convenience.

Tool 3: Mathematics

Interpret features of graphs: areas under the graph

The area under many graphs has a physical meaning. As an example, consider Figure A1.24a, which shows part of a speed–time graph for a vehicle moving with constant acceleration. The area under the graph (the shaded area) can be calculated from the average speed, given by $\frac{(v_1 + v_2)}{2}$, multiplied by the time, Δt.

The area under the graph is therefore equal to the distance travelled in time Δt. In Figure A1.24b a vehicle is moving with a changing (decreasing) acceleration, so that the graph is curved, but the same rule applies – the area under the graph (shaded) represents the distance travelled in time Δt.

The area in Figure A1.24b can be estimated in a number of different ways, for example by counting small squares, or by drawing a rectangle that appears (as judged by eye) to have the same area. (If the equation of the line is known, it can be calculated using the process of **integration**, but this is *not* required in the IB course.)

In the following section, we will show how a change in displacement can be calculated from a velocity–time graph.

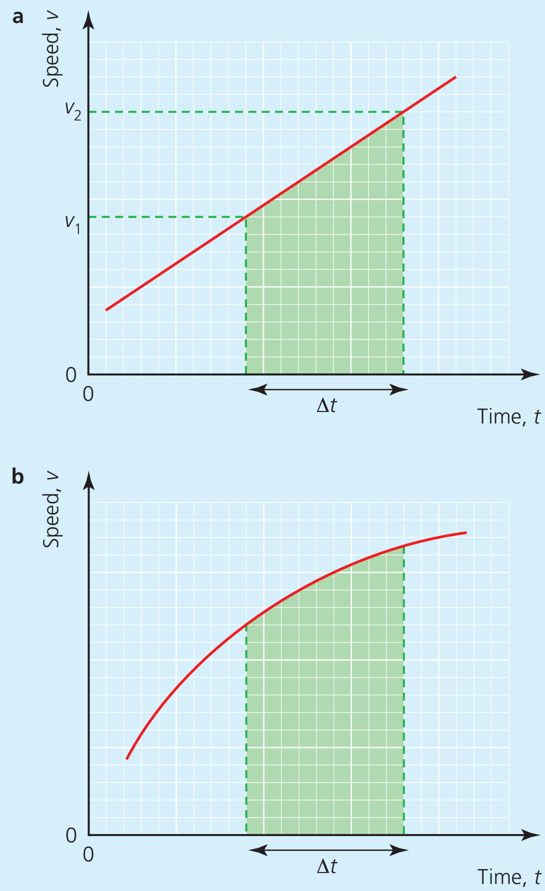


Figure A1.24 Area under a speed–time graph for **a** constant acceleration and **b** changing acceleration

◆ **Integration** Mathematical process used to determine the area under a graph.

Areas under velocity–time and speed–time graphs

As an example, consider again Figure A1.22. The change of displacement, Δs, between the fourth and ninth seconds can be found from (average velocity) × time.

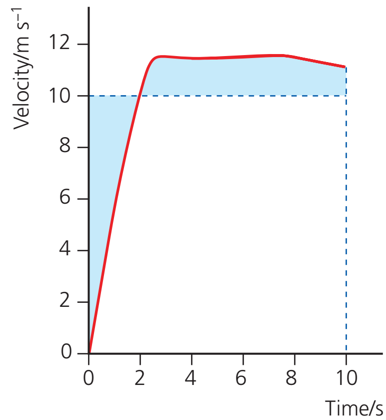
$$\Delta s = \frac{(12.0 + 7.0)}{2} \times (9.0 - 4.0) = 47.5\,\text{m}$$

This is numerically equal to the area under the line between $t = 4.0\,\text{s}$ and $t = 9.0\,\text{s}$ (as shaded in Figure A1.22). This is always true, whatever the shape of the line.

The area under a velocity–time graph is always equal to the change of displacement.

The area under a speed–time graph is always equal to the distance travelled.

As an example, consider Figure A1.21a. The two areas under the speed–time graph are equal and they are both positive. Each area equals the vertical height travelled by the object. The total area = total distance = twice the height. Each area under the velocity graph also represents the height, but the total area is zero because the areas above and below the time axis are equal, indicating that the final displacement is zero – the object has returned to where it started.



■ **Figure A1.25** Velocity–time graph for an athlete running 100 m

Figure A1.25 shows a velocity–time graph for an athlete running 100 m in 10.0 s. The area under the curve is equal to 100 m and it equals the area under the dotted line. (The two shaded areas are judged by sight to be equal.) The initial acceleration of the athlete is very important, and in this example, it is about 5 m s^{-2}.

■ **Figure A1.26** Elaine Thompson-Herah (Jamaica) won the women's 100 m in the Tokyo Olympics in 2021 in a time of 10.54 s

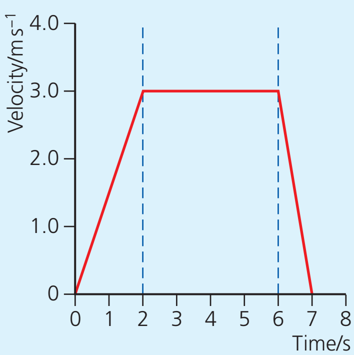
12 Look at the graph in Figure A1.27.

a Describe the straight-line motion represented by the graph.

b Calculate accelerations for the three parts of the journey.

c What was the total distance travelled?

d What was the average velocity?



■ **Figure A1.27** Velocity–time graph

13 The velocity of a car was read from its speedometer at the moment it started and every 2 s afterwards. The successive values (converted to m s^{-1}) were: 0, 1.1, 2.4, 6.9, 12.2, 18.0, 19.9, 21.3 and 21.9.

a Draw a graph of these readings.

b Use the graph to estimate

i the maximum acceleration

ii the distance covered in 16 s.

14 Look at the graph in Figure A1.28.

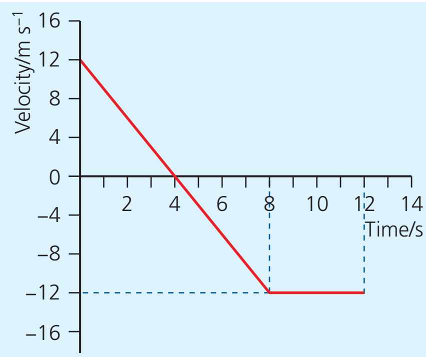
a Describe the straight-line motion of the object represented by the graph.

b Calculate the acceleration during the first 8 s.

c What was the total distance travelled in 12 s?

d What was the total displacement after 12 s?

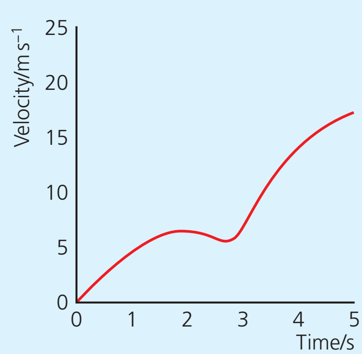
e What was the average velocity during the 12 s interval?



■ **Figure A1.28** Velocity–time graph

15 Sketch a velocity–time graph of the following motion: a car is 100 m away and travelling along a straight road towards you at a constant velocity of 25 m s^{-1}. Two seconds after passing you, the driver decelerates uniformly and the car stops 62.5 m away from you.

16 Figure A1.29 shows how the velocity of a car, moving in a straight line, changed in the first 5 s after starting. Use the area under the graph to show that the distance travelled was about 40 m.



■ **Figure A1.29** Determining the displacement of a car during acceleration

Tool 2: Technology

Use spreadsheets to manipulate data

Figure A1.30 represents how the velocities of two identical cars changed from the moment that their drivers saw danger in front of them and tried to stop their cars as quickly as possible. It has been assumed that both drivers have the same reaction time (0.7 s) and both cars decelerate at the same rate ($-5.0\,\mathrm{m\,s^{-2}}$).

The distance travelled at constant velocity before the driver reacts and depresses the brake pedal is known as the 'thinking distance'. The distance travelled while decelerating is called the 'braking distance'. The total stopping distance is the sum of these two distances.

Car B, travelling at twice the velocity of car A, has twice the thinking distance. That is, the thinking distance is proportional to the velocity of the car. The distance travelled when braking, however, is proportional to the velocity squared. This can be confirmed from the areas under the v–t graphs. The area under graph B is four times the area under graph A (during the deceleration). This has important implications for road safety and most countries make sure that people learning to drive must understand how stopping distances change with the vehicle's velocity. Some countries measure the reaction times of people before they are given a driving licence.

Set up a **spreadsheet** that will calculate the total stopping distance for cars travelling at initial speeds, u, between 0 and $40\,\mathrm{m\,s^{-1}}$ with a deceleration of $-6.5\,\mathrm{m\,s^{-2}}$. (Make calculations every $2\,\mathrm{m\,s^{-1}}$.) The thinking distance can be calculated from $s_t = 0.7u$ (reaction time 0.7 s).

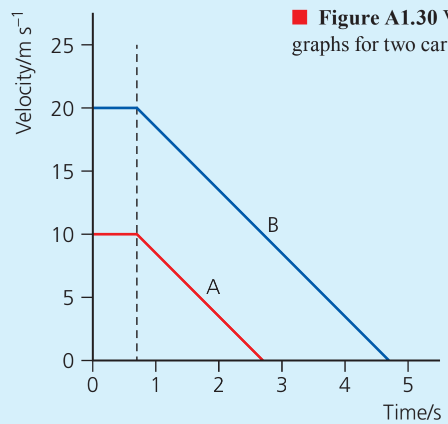
In this example the braking time can be calculated from:

$$t_b = \frac{u}{6.5}$$

and the braking distance can be calculated from:

$$s_b = \left(\frac{u}{2}\right)t_b$$

Use the data produced to plot a computer-generated graph of stopping distance (y-axis) against initial speed (x-axis).

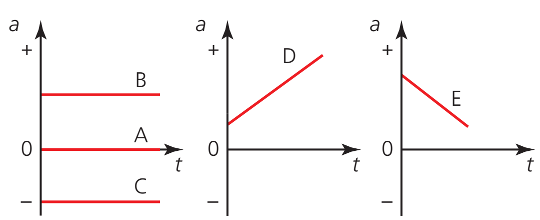


■ **Figure A1.30** Velocity–time graphs for two cars braking

◆ **Spreadsheet (computer)** Electronic document in which data is arranged in the rows and columns of a grid, and can be manipulated and used in calculations.

Acceleration–time graphs

In this topic, we are mostly concerned with constant accelerations. The graphs in Figure A1.31 show five straight lines representing *constant* accelerations. A *changing* acceleration would be represented by a curved line on the graph.



■ **Figure A1.31** Graphs of constant acceleration

- Line A shows zero acceleration, constant velocity.
- Line B shows a constant positive acceleration (uniformly increasing velocity).
- Line C shows the constant negative acceleration (deceleration) of an object that is slowing down at a uniform rate.
- Line D shows a (linearly) increasing positive acceleration.
- Line E shows an object that is accelerating positively, but at a (linearly) decreasing rate.

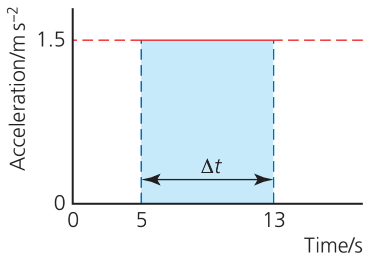
Areas under acceleration–time graphs

Figure A1.32 shows the constant acceleration of a moving car.

Using $a = \frac{\Delta v}{\Delta t}$, between the fifth and thirteenth seconds, the velocity of the car increased by:

$$\Delta v = a\Delta t = 1.5 \times (13.0 - 5.0) = 12\,\text{m s}^{-1}$$

The change in velocity is numerically equal to the area under the line between $t = 5$ s and $t = 13$ s (the shaded area in Figure A1.32). This is always true, whatever the shape of the line.



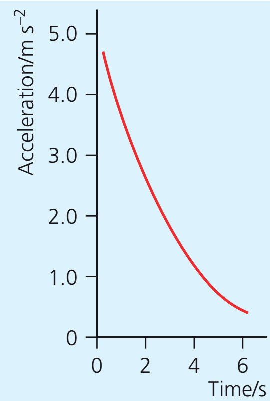
■ **Figure A1.32** Calculating change of velocity from an acceleration–time graph

The area under an acceleration–time graph is equal to the change of velocity.

17 Draw an acceleration–time graph for a car that starts from rest, accelerates at $2\,\text{m s}^{-2}$ for 5 s, then travels at constant velocity for 8 s, before decelerating uniformly to rest again in a further 2 s.

18 Figure A1.33 shows how the acceleration of a car changed during a 6 s interval. If the car was travelling at $2\,\text{m s}^{-1}$ after 1 s, estimate a suitable area under the graph and use it to determine the approximate speed of the car after another 5 s.

19 Sketch displacement–time, velocity–time and acceleration–time graphs for a bouncing ball that was dropped from rest.
Continue the sketches until the third time that the ball contacts the ground.



■ **Figure A1.33** Acceleration–time graph for an accelerating car

◆ **Calculus** Branch of mathematics which deals with continuous change.

◆ **Differentiate** Mathematically determine an equation for a rate of change.

TOK

Mathematics and the arts

- Why is mathematics so important in some areas of knowledge, particularly the natural sciences?

If you study Mathematics: Analysis and Approaches (SL or HL) or Mathematics: Applications and Interpretations (HL) you will explore how **calculus** is used to mathematically describe changing functions. The gradient of a function is found using the process of **differentiation** and the area under a curve is found using the process of integration. The mathematical procedures for calculus were developed by Isaac Newton and he first published his 'method of fluxions' as an appendix to his book *Opticks* in 1704. Newton is usually therefore credited with the 'invention' of calculus – although historians of science point to the earlier work of Gottfried Wilhelm Leibniz, published in 1684. Newton accused Leibniz of plagiarism, even though Leibniz's work was published first! In fact, it is Leibniz's notation that we still use today. So, who invented calculus?

◆ **Equations of motion**
Equations that can be used to make calculations about objects that are moving with uniform acceleration.

Equations of motion for uniformly accelerated motion

SYLLABUS CONTENT

- The **equations of motion** for solving problems with uniformly accelerated motion as given by:

$$s = \frac{(u + v)}{2}t$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

The five quantities u, v, a, s and t are all that is needed to fully describe the motion of an object that is moving with *uniform* acceleration.

- u = velocity (speed) at the start of time t
- v = velocity (speed) at the end of time t
- a = acceleration (constant)
- s = displacement occurring in time t
- t = time taken for velocity (speed) to change from u to v and to travel a distance s.

If any three of the quantities are known, the other two can be calculated using the first two equations highlighted below.

If we know the initial velocity u and the uniform acceleration a of an object, then we can determine its final velocity v after a time t by rearranging the equation used to define acceleration:

$$a = \frac{(v - u)}{t}$$

This gives:

$$\boldsymbol{v = u + at}$$

If an object moving with velocity u accelerates uniformly to a velocity v, then its average velocity is:

$$\frac{(u + v)}{2}$$

Then, since distance = average velocity × time:

$$\boldsymbol{s = \frac{(u + v)}{2}t}$$

These two equations can be combined mathematically to give two further equations, shown below. These very useful equations do not involve any further physics theory, they just express the same physics principles in a different way.

$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$

$$\boldsymbol{v^2 = u^2 + 2as}$$

LINKING QUESTION

- How are the equations for rotational motion related to those for linear motion?

This question links to understandings in Topic A.4.

WORKED EXAMPLE A1.5

A Formula One racing car (see Figure A1.34) accelerates from rest at 18 m s^{-2}.

Figure A1.34 Formula One racing cars at the starting grid

a Calculate its speed after 3.0 s.

b Calculate how far it travels in this time.

c If it continues to accelerate at the same rate, determine its velocity after it has travelled 200 m from the start.

Answer

a $v = u + at = 0 + (18 \times 3.0) = 54\,\text{m s}^{-1}$

b $s = \frac{(u + v)}{2}t = \frac{(0 + 54)}{2} \times 30 = 81\,\text{m}$

But note that the distance can be calculated directly, without first calculating the final velocity, as follows:

$s = ut + \frac{1}{2}at^2 = (0 \times 3.0) + (0.5 \times 18 \times 3.0^2) = 81\,\text{m}$

c $v^2 = u^2 + 2as = 0^2 + (2 \times 18 \times 200) = 7200$

$v = 85\,\text{m s}^{-1}$

WORKED EXAMPLE A1.6

A train travelling at 50 m s^{-1} (180 km h^{-1}) needs to decelerate uniformly so that it stops at a station 2.0 kilometres away.

a Determine the necessary deceleration.

b Calculate the time needed to stop the train.

Answer

a $v^2 = u^2 + 2as$

$0^2 = 50^2 + (2 \times a \times 2000)$

$a = -0.63\,\text{m s}^{-2}$

b $v = u + at$

$0 = 50 + (-0.63) \times t$

$t = 80\,\text{s}$

Alternatively, you could use $s = \frac{(u + v)}{2}t$

In the following questions, assume that all accelerations are uniform.

20 A ball rolling down a slope passes a point P with a velocity of 1.2 m s^{-1}. A short time later it passes point Q with a velocity of 2.6 m s^{-1}.

- **a** What was its average velocity between P and Q?
- **b** If it took 1.4 s to go from P to Q, determine the distance PQ.
- **c** Calculate the acceleration of the ball.

21 An aircraft accelerates from rest along a runway and takes off with a velocity of 86.0 m s^{-1}. Its acceleration during this time is 2.40 m s^{-2}.

- **a** Calculate the distance along the runway that the aircraft needs to travel before take-off.
- **b** Predict how long after starting its acceleration the aircraft takes off.

22 An ocean-going cruiser can decelerate no quicker than $0.0032\,\mathrm{m\,s^{-2}}$.

Figure A1.35 Ocean-going cruise liner

a Determine the minimum distance needed to stop if the ship is travelling at 10 knots. (1 knot = $0.514\,\mathrm{m\,s^{-1}}$)

b How much time does this deceleration require?

23 An advertisement for a new car states that it can travel 100 m from rest in 8.2 s.

a Discuss why the car manufacturers express the acceleration in this way (or the time needed to reach a certain speed).

b Calculate the average acceleration.

c Calculate the velocity of the car after this time.

24 A car travelling at a constant velocity of $21\,\mathrm{m\,s^{-1}}$ (faster than the speed limit of $50\,\mathrm{km\,h^{-1}}$) passes a stationary police car. The police car accelerates after the other car at $4.0\,\mathrm{m\,s^{-2}}$ for 8.0 s and then continues with the same velocity until it overtakes the other car.

a When did the two cars have the same velocity?

b Determine if the police car has overtaken the other car after 10 s.

c By equating two equations for the same distance at the same time, determine exactly when the police car overtakes the other car.

25 A car brakes suddenly and stops 2.4 s later, after travelling a distance of 38 m.

a Calculate its deceleration.

b What was the velocity of the car before braking?

26 A spacecraft travelling at $8.00\,\mathrm{km\,s^{-1}}$ accelerates at $2.00 \times 10^{-3}\,\mathrm{m\,s^{-2}}$ for 100 hours.

a How far does it travel during this acceleration?

b What is its final velocity?

27 Combine the first two equations of motion (given on page 17) to derive the second two equations:

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

Acceleration due to gravity

The motions of objects through the air are common events and deserve special attention.

At the start, we will consider only objects that are moving vertically up, or down, under the effects of gravity only. That is, we will assume (to begin with) that **air resistance** has no significant effect.

When an object held up in the air is released from rest, it will accelerate downwards because of the force of gravity. Figure A1.36 shows a possible experimental arrangement that could be used to determine a value for this acceleration.

◆ **Air resistance** Resistive force opposing the motion of an object through air. A type of drag force.

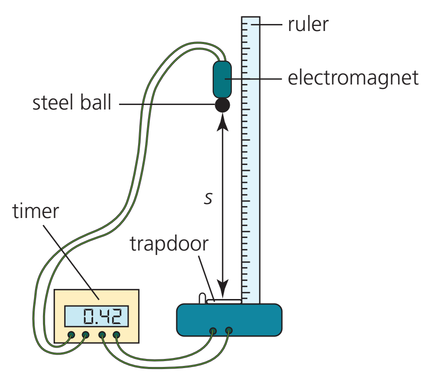


Figure A1.36 An experiment to measure the acceleration due to gravity

Inquiry 2: Collecting and processing data

Collecting data

Figure A1.36 shows how the time for a steel ball to fall a certain distance can be determined experimentally.

Describe how this apparatus can be used to collect and record sufficient, relevant quantitative data which will enable an accurate value for the acceleration of free fall to be determined from a suitable graph.

In the absence of air resistance, all objects (close to the Earth's surface) fall towards the Earth with the same acceleration, $\boldsymbol{g} = \mathbf{9.8\,m\,s^{-2}}$

DB

g is known as the **acceleration of free fall** due to gravity (sometimes called acceleration due to **free fall**).

◆ **Acceleration due to gravity, *g*** Acceleration of a mass falling freely towards Earth. On, or near the Earth's surface, $g = 9.8 \approx 10\,m\,s^{-2}$. Also called **acceleration of free fall**.

◆ **Free fall** Motion through the air under the effects of gravity but without air resistance.

◆ **Negligible** Too small to be significant.

g is not a true constant. Its value varies very slightly at different locations around the world. Although, to 2 significant figures (9.8) it has the same value everywhere on the Earth's surface. A convenient value of $g = 10\,m\,s^{-2}$ is commonly used in introductory physics courses.

The acceleration of free fall (g) reduces with distance from the Earth. (For example, at a height of 100 km above the Earth's surface the value of g is $9.5\,m\,s^{-2}$.) We will return to this subject in Topic D.1.

WORKED EXAMPLE A1.7

A ball is dropped vertically from a height of 18.3 m. Assuming that the acceleration of free fall is $9.81\,m\,s^{-2}$ and air resistance is **negligible**, calculate:

a its velocity after 1.70 s

b its height after 1.70 s

c its velocity when it hits the ground

d the time for the ball to reach the ground.

Answer

a $v = u + at = 0 + (9.81 \times 1.70) = 16.7\,m\,s^{-1}$

b $s = ut + \frac{1}{2}at^2 = 0 + \left(\frac{1}{2} \times 9.81 \times 1.70^2\right) = 14.2\,m$

So, height above ground = (18.3 – 14.2) = 4.1 m

c $v^2 = u^2 + 2as = 0^2 + (2 \times 9.81 \times 18.3) = 359$

$v = 18.9\,m\,s^{-1}$

d $v = u + at$

$18.9 = 0 + (9.81 \times t)$

$t = 1.93\,s$

Tool 3: Mathematics

Appreciate when some effects can be neglected and why this is useful

When studying physics, you may be advised to make assumptions when answering numerical questions. For example: 'assume that air resistance is **negligible** / is insignificant'. It is possible that this is a true statement, for example, air resistance will have no noticeable effect on a solid rubber ball falling 50 cm to the ground. However, the usual reason for advising you to ignore an effect is to make the calculation simpler, and not go beyond what is required in your course.

Calculating the time for a table-tennis ball dropped 50 cm to the ground will result in an underestimate if air resistance is ignored, but the answer can be interpreted as a lower limit to the time taken, and you may be questioned on your understanding of that.

Other examples will be found in all topics. Examples include: assuming friction between surfaces is negligible (Topic A.2); assuming thermal energy losses are negligible (Topic B.1); assuming the internal resistance of a battery is negligible (Topic B.5).

Moving up and down

If gravity is the only force acting, all objects close to the Earth's surface have the same acceleration ($9.8\,m\,s^{-2}$ downwards), whatever their mass and whether they are moving down, moving up or moving sideways.

The velocity of an object moving freely vertically downwards will increase by $9.8\,m\,s^{-1}$ every second. The velocity of an object moving freely vertically upwards will decrease by $9.8\,m\,s^{-1}$ every second.

Top tip!

Displacement, velocity and acceleration are all vector quantities and the signs used for motions up and down can be confusing.

If displacement measured up from the ground is considered to be positive, then the acceleration due to gravity is always negative. Velocity upwards is positive, while velocity downwards is negative.

If displacement measured down from the highest point is considered to be positive, then the acceleration due to gravity is always positive. Velocity upwards is negative, while velocity downwards is positive.

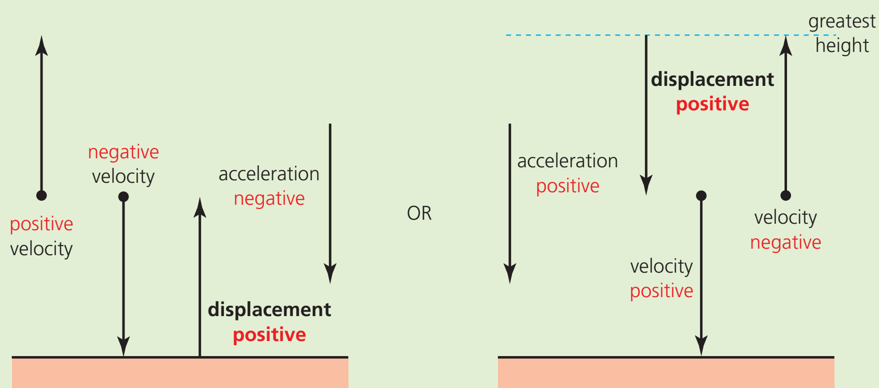


Figure A1.37 Directions of vectors

WORKED EXAMPLE A1.8

A ball is thrown vertically upwards and reaches a maximum height of 21.4 m. For the following questions, assume that $g = 9.81\,m\,s^{-2}$.

a Calculate the speed with which the ball was released.

b State any assumption that you made in answering **a**.

c Determine where the ball will be 3.05 s after it was released.

d Calculate its velocity at this time.

Answer

a $v^2 = u^2 + 2as$

$0^2 = u^2 + (2 \times [-9.81] \times 21.4)$

$u^2 = 419.9$

$u = 20.5\,m\,s^{-1}$

In this example, the vector quantities directed upwards (u, v, s) are considered positive and the quantity directed downwards (a) is negative. The same answer would be obtained by reversing all the signs.

b It was assumed that there was no air resistance.

c $s = ut + \frac{1}{2}at^2 = (20.5 \times 3.05) + \left(\frac{1}{2} \times [-9.81] \times 3.05^2\right)$

$s = +16.9\,m$ (above the ground)

d $v = u + at = 20.5 + (-9.81 \times 3.05)$

$= -9.42\,m\,s^{-1}$ (moving downwards)

In the following questions, ignore the possible effects of air resistance.

Use $g = 9.81\,\mathrm{m\,s^{-2}}$.

28 Discuss possible reasons why the acceleration due to gravity is not exactly the same everywhere on or near the Earth's surface.

29 **a** How long does it take a stone dropped from rest from a height of 2.1 m to reach the ground?
   b If the stone was thrown downwards with an initial velocity of $4.4\,\mathrm{m\,s^{-1}}$, calculate the speed with which it hits the ground.
   c If the stone was thrown vertically upwards with an initial velocity of $4.4\,\mathrm{m\,s^{-1}}$, with what speed would it hit the ground?

30 A small rock is thrown vertically upwards with an initial velocity of $22\,\mathrm{m\,s^{-1}}$.
   a Calculate when its velocity will be $10\,\mathrm{m\,s^{-1}}$.
   b Explain why there are two possible answers to **a**.

31 A falling ball has a velocity of $12.7\,\mathrm{m\,s^{-1}}$ as it passes a window 4.81 m above the ground.
Predict when the ball will hit the ground.

32 A ball is thrown vertically upwards with a velocity of $18.5\,\mathrm{m\,s^{-1}}$ from a window that is 12.5 m above the ground.
   a Determine when it will pass the same point moving down.
   b With what velocity will it hit the ground?
   c Calculate how far above the ground the ball was after exactly 2.00 s.

33 Two balls are dropped from rest from the same height. If the second ball is released 0.750 s after the first, and assuming they do not hit the ground, calculate the distance between the balls:
   a 3.00 s after the second ball was dropped
   b 2.00 s later.

34 A stone is dropped from rest from a height of 34 m. Another stone is thrown downwards 0.5 s later.
If they both hit the ground at the same time, show that the second stone was thrown with a velocity of $5.5\,\mathrm{m\,s^{-1}}$.

Projectile motion

SYLLABUS CONTENT

- The behaviour of projectiles in the absence of fluid resistance, and the application of the equations of motion resolved into vertical and horizontal components.
- The qualitative effect of fluid resistance on projectiles, including time of flight, trajectory, velocity, acceleration, range and terminal speed.

◆ **Projectile** An object that has been projected through the air and which then moves only under the action of the forces of gravity and air resistance.

◆ **Resolve (a vector)** To express a single vector as components (usually two components which are perpendicular to each other).

In our discussion of objects moving through the air, we have so far only considered motion vertically up or down. Now we will extend that work to cover objects moving in any direction. A **projectile** is an object that has been projected through the air (for example: fired, launched, thrown, kicked or hit) and which then moves only under the action of the force of gravity (and air resistance, if significant). A projectile has no ability to power or control its own motion.

Tool 3: Mathematics

Resolve vectors

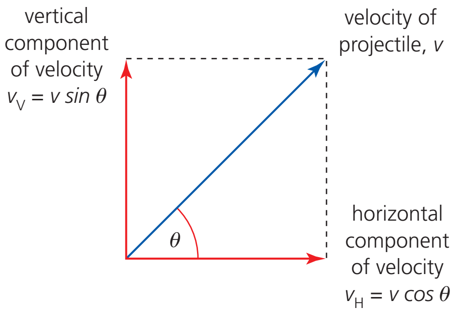
This process occurs in several places during the course, but the most prominent examples are **resolving** velocities (as below) and forces.

Components of a projectile's velocity

The instantaneous velocity of a projectile at any time can conveniently be resolved into vertical and horizontal components, v_V and v_H, as shown in Figure A1.38.

Common mistake

When using these equations make sure that the angle θ is the angle between the velocity and the horizontal.



■ **Figure A1.38** Vertical and horizontal components of velocity

Vertical and horizontal components of velocity, v:

$$v_V = v \sin\theta$$

$$v_H = v \cos\theta$$

WORKED EXAMPLE A1.9

A tennis player strikes the ball so that it leaves the racket with a velocity of $64.0\,\mathrm{m\,s^{-1}}$ at an angle of 6.0° below the horizontal. Calculate the vertical and horizontal components of this velocity.

■ **Figure A1.39** A tennis player serving a ball

Answer

$v_H = v \cos\theta = 64.0 \times \cos 6.0 = 64\,\mathrm{m\,s^{-1}}$ (63.649... seen on calculator display)

$v_V = v \sin\theta = 64.0 \times \sin 6.0 = 6.7\,\mathrm{m\,s^{-1}}$ downwards

◆ **Stroboscope** Apparatus used for observing rapid motions. It produces regular flashes of light at an appropriate frequency chosen by the user.

◆ **Trajectory** Path followed by a projectile.

◆ **Parabolic** In the shape of a parabola. The trajectory of a projectile is parabolic in a gravitational field if air resistance is negligible.

◆ **Range (of a projectile)** Horizontal distance travelled before impact with the ground.

Components perpendicular to each other can be analysed separately

The vertical and horizontal components of velocity can be treated separately (independently) in calculations.

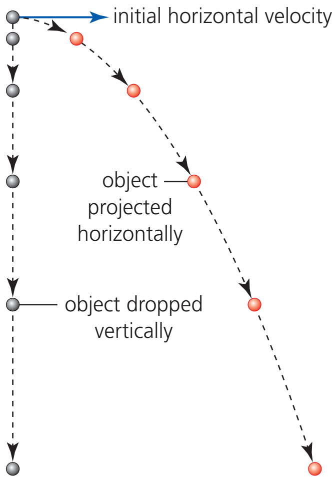
- Earlier in this topic, we stated that any object (close to the Earth's surface) which is affected only by gravity (no air resistance) will accelerate towards the Earth with an acceleration of $9.8\,\mathrm{m\,s^{-2}}$. This remains true even if the object is projected sideways (so that its velocity has a horizontal component).
- If there is no air resistance, the horizontal component of a projectile's velocity will remain constant (until it comes into contact with something else).

■ **Figure A1.40** Parabolic trajectory of a bouncing ball

Figure A1.40 shows a **stroboscopic** picture of a bouncing ball. The time intervals between each image of the ball are all the same.

The horizontal separations of successive images of the ball are all the same because the horizontal component of velocity is constant. The vertical separations of successive images of the ball increase as the ball accelerates as it falls, and the separations decrease as the ball decelerates as it moves upwards after bouncing on the ground.

The path followed by a projectile (as seen in Figure A1.40) is called its **trajectory**. The typical shape of a freely moving projectile is **parabolic**. The horizontal distance covered is called the **range** of the projectile.



■ **Figure A1.41** The parabolic trajectory of an object projected horizontally compared with an object dropped vertically

Figure A1.41 compares the trajectory of an object dropped vertically to the trajectory of an object projected horizontally at the same time. Note that both objects fall equal distances in the same time. This is true whatever the horizontal component of velocity (assuming negligible air resistance)

WORKED EXAMPLE A1.10

Object projected horizontally

A bullet was fired horizontally with a speed of $524\,\mathrm{m\,s^{-1}}$ from a height of 22.0 m above the ground. Calculate where it hit the ground. Assume that air resistance was negligible.

Answer

First, we need to calculate how long the bullet is in the air. We can do this by finding the time that the same bullet would have taken to fall to the ground if it had been dropped vertically from rest (so $u = 0$):

$$s = ut + \frac{1}{2}at^2$$

$$22.0 = 0 + (0.5 \times 9.81 \times t^2)$$

$$t = 2.12\,\mathrm{s}$$

Without air resistance the bullet will continue to travel with the same horizontal component of velocity ($524\,\mathrm{m\,s^{-1}}$) until it hits the ground 2.12 s later. Therefore:

horizontal distance travelled = horizontal velocity × time

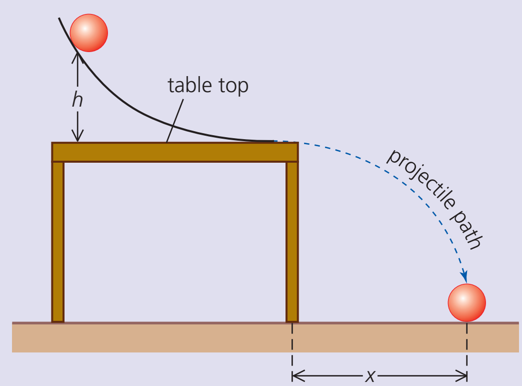
horizontal distance = $524 \times 2.12 = 1.11 \times 10^3\,\mathrm{m}$ (1.11 km)

ATL A1A: Thinking skills

Providing a reasoned argument to support conclusions

Figure A1.42 shows an experimental arrangement in which a steel ball can be projected horizontally from a table top.

Sketch a graph to show the pattern of results that you would expect to see when the range x was measured for different heights, h. Explain your reasoning.



■ **Figure A1.42** Investigating range, x, travelled by a projectile

WORKED EXAMPLE A1.11

Object projected at an angle to the horizontal

A stone was thrown upwards from a height 1.60 m above the ground with a speed of $18.0\,\mathrm{m\,s^{-1}}$ at an angle of 52.0° to the horizontal. Assuming that air resistance is negligible, calculate:

a its maximum height
b the vertical component of velocity when it hits the ground
c the time taken to reach the ground
d the horizontal distance to the point where it hits the ground
e the velocity of **impact**.

◆ **Impact** Collision involving relatively large forces over a short time.

Top tip!

If we know the velocity and position of a projectile, we can always use its vertical component of velocity to determine:

- the time taken before it reaches its maximum height, and the time before it hits the ground
- the maximum height reached (assuming its velocity has an upwards component).

The horizontal component can then be used to determine the range.

Answer

First, we need to know the two components of the initial velocity:

$v_V = v \sin\theta = 18.0 \sin 52.0° = 14.2\,\mathrm{m\,s^{-1}}$

$v_H = v \cos\theta = 18.0 \cos 52.0° = 11.1\,\mathrm{m\,s^{-1}}$

a Using $v^2 = u^2 + 2as$ for the upwards vertical motion (with directions upwards considered to be positive), and remembering that at the maximum height $v = 0$, we get:

$0 = 14.2^2 + [2 \times (-9.81) \times s]$

$s = +10.3\,\mathrm{m}$ above the point from which it was released; a total height of 11.9 m.

b Using $v^2 = u^2 + 2as$ for the complete motion gives:

$v^2 = 14.2^2 + [2 \times (-9.81) \times (-1.60)]$

$v = 15.27 = 15.3\,\mathrm{m\,s^{-1}}$ downwards

c Using $v = u + at$ gives:

$-15.27 = 14.2 + (-9.81)t$

$t = 3.00\,\mathrm{s}$

d Using $s = vt$ with the horizontal component of velocity gives:

$s = 11.1 \times 3.00 = 33.3\,\mathrm{m}$

e Figure A1.43 illustrates the information we have so far, and the unknown angle, θ, and velocity, v_i.

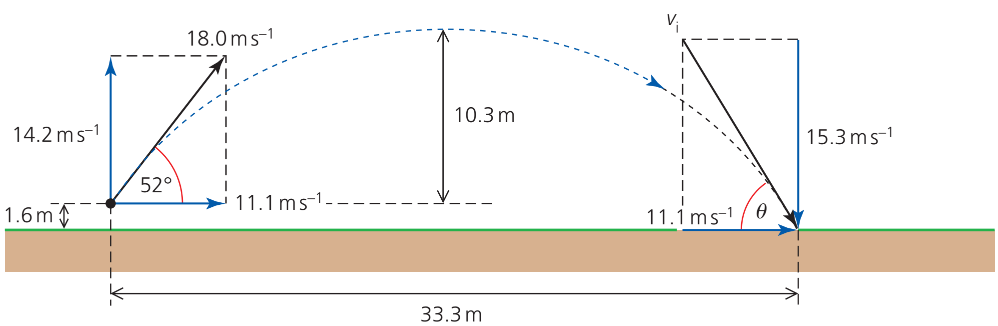


Figure A1.43 Object projected at an angle to the horizontal

From looking at the diagram (Figure A1.43), we can use Pythagoras's theorem to calculate the velocity of impact.

(velocity of impact)^2 = (horizontal component)^2 + (vertical component)^2

$v_i^2 = 11.1^2 + 15.3^2$

$v_i = 18.9\,\mathrm{m\,s^{-1}}$

The angle of impact with the horizontal, θ, can be found using trigonometry:

$$\tan\theta = \frac{15.3}{11.1}$$

$\theta = 54.0°$

◆ **Imagination** Formation of new ideas that are not related to direct sense perception or experimental results.

◆ **Intuition** Immediate understanding, without reasoning.

◆ **Inspiration** Stimulation (usually to be creative).

TOK

The natural sciences

- What is the role of **imagination** and **intuition** in the creation of hypotheses in the natural sciences?

The independence of horizontal and vertical motion in projectile motion may seem unexpected and counterintuitive. It requires imagination (some would say genius) to propose ideas and theories which are contrary to accepted wisdom and 'common sense'. This is especially true in understanding the worlds of relativity and quantum physics, where relying on everyday experiences for **inspiration** is of little or no use.

It is worth remembering that many of the well-established concepts and theories of classical physics that are taught now in introductory physics lessons would have seemed improbable to many people at the time they were first proposed. For example, many people would say (incorrectly) that a force is needed to keep an object moving at constant speed (see Topic A.2).

Fluid resistance and terminal speed

So far, we have only considered projectile motion in which air resistance is negligible. We will now broaden the discussion.

◆ **Drag** Force(s) opposing motion through a fluid; sometimes called fluid resistance.

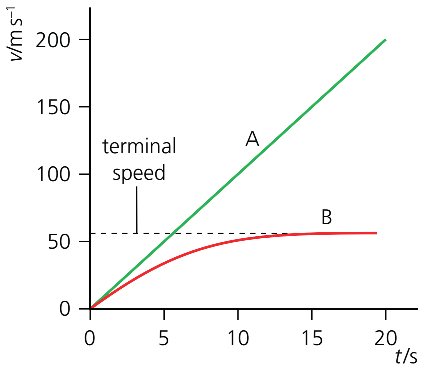
◆ **Fluid** Liquid or gas.

◆ **Fluid resistance (friction)** Force(s) opposing motion through a fluid; sometimes called drag.

As any object moves through air, the air is forced to move out of the path of the object. This causes a force opposing the motion called air resistance, also known as **drag**. Drag forces will oppose the motion of an object moving in any direction through any gas or liquid. (Gases and liquids are both described as **fluids** because they can flow.) Such forces opposing motion are generally described as **fluid resistance**.

Figure A1.44 gives a visual impression of air resistance. It shows the movement of air (marked by streamers) past a model of a car. (The picture was taken in a wind tunnel, in which moving air was directed towards the vehicle.)

■ **Figure A1.44** Air flow over a clay aerodynamic model of a high-performance sports vehicle



■ **Figure A1.45** An example of a graph of velocity against time for an object falling under the effect of gravity, with (B) and without (A) air resistance

Figure A1.45 represents the motion of an object falling towards Earth.

Line A shows the motion without air resistance and with a constant acceleration of $9.8\,m\,s^{-2}$ (≈ 10). Line B shows the motion more realistically, with air resistance.

When any object first starts to fall, there is no air resistance. As the object falls faster, the air resistance increases, so that the rate of increase in velocity becomes less. This is shown in the Figure A1.45 by line B becoming less steep. Eventually the object reaches a constant, maximum speed known as the **terminal speed** or **terminal velocity** ('terminal' means final).

Objects falling through fluids (such as air) have a maximum speed, called terminal speed, which occurs when their acceleration has reduced to zero because of increasing fluid resistance (as their velocity increases).

◆ **Terminal speed (velocity)** The greatest downwards speed of a falling object that is experiencing resistive forces (for example, air resistance). It occurs when the object's weight is equal to the sum of resistive forces (+ upthrust).

The value of an object's terminal speed will depend on its cross-sectional area, shape and weight, as discussed in Topic A.2. The terminal speed of skydivers (Figure A1.46) is usually quoted at about $200\,km\,h^{-1}$ ($56\,m\,s^{-1}$).

Terminal speed also depends on the density of the air. In October 2012 Felix Baumgartner (Figure A1.47), an Austrian skydiver, reached a world record speed of $1358\,km\,h^{-1}$ by starting his jump from a height of about 39 km above the Earth's surface, where the density of air is about 250 times less than near the Earth's surface. In 2014 Alan Eustace completed a jump from greater altitude, but at $1323\,km\,h^{-1}$ he did not break Baumgartner's speed record.

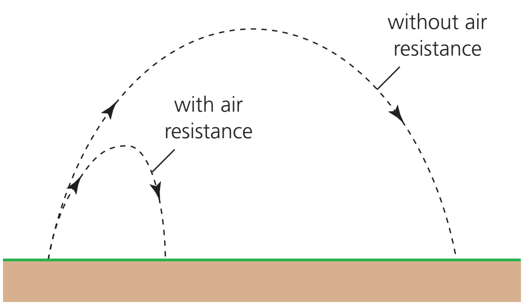
Top tip!

The concept of a top (terminal) speed can also be applied to the horizontal motion of vehicles, like trains, cars and aircraft. As they travel faster, increasing air resistance reduces their acceleration to zero.

■ **Figure A1.46** Skydivers at their terminal speed

■ **Figure A1.47** Felix Baumgartner about to jump from a height of 39 km

Effect of fluid (air) resistance on projectiles



■ **Figure A1.48** Effect of air resistance on the trajectory of a projectile

Without air resistance we assume that the horizontal component of a projectile's velocity is constant, but with air resistance it decreases. Without air resistance the vertical motion always has a downwards acceleration of $9.8\,m\,s^{-2}$, but with air resistance the acceleration will be reduced for falling objects and the deceleration increased for objects moving upwards.

Figure A1.48 shows typical trajectories with and without air resistance (for the same initial velocity).

Air resistance reduces the range of a projectile and its trajectory will not be parabolic.

Tool 2: Technology

Carry out image analysis and video analysis of motion

Video-capture technology is used in sports, such as tennis and soccer. Capturing the trajectory of a projectile on video allows us to analyse its motion frame-by-frame. For example, the cameras used in VAR in football usually capture 50 frames per second, so the motion of the projectile (the ball) can be observed at time intervals of 0.02 s.

Explain how you could use **video analysis** of motion to investigate the motion of a shuttlecock in a game of badminton.

◆ **Video analysis** Analysis of motion by freeze-frame or slow-motion video replay.

■ **Figure A1.49** Consider how video analysis could be used to investigate the motion of a badminton shuttlecock.

In the following questions, ignore the possible effects of air resistance. Use $g = 9.81\,\mathrm{m\,s^{-2}}$.

35 At an indoor rifle range, a bullet was fired horizontally at the centre of a target 36 m away. If the speed of the bullet was $310\,\mathrm{m\,s^{-1}}$, predict where the bullet will strike the target.

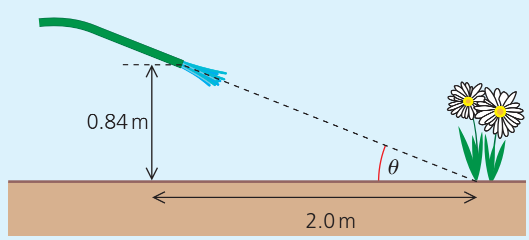
36 Repeat Worked example A1.11 for a stone thrown with a velocity of $26\,\mathrm{m\,s^{-1}}$ at an angle of 38° to the horizontal from a cliff top. The point of release was 33 m vertically above the sea.

37 It can be shown that the maximum theoretical range of a projectile occurs when it is projected at an angle of 45° to the ground (once again, ignoring the effects of air resistance). Calculate the maximum distance a golf ball will travel before hitting the ground if its initial velocity is $72\,\mathrm{m\,s^{-1}}$. (Because you need to assume that there is no air resistance, your answer should be much higher than the actual ranges achieved by top-class golfers. Research to determine the actual ranges achieved in competition golf.)

38 A jet of water from a hose is aimed directly at the base of a flower, as shown in Figure A1.50. The water emerges from the hose with a speed of $3.8\,\mathrm{m\,s^{-1}}$.

a Calculate the vertical and horizontal components of the initial velocity of the water.

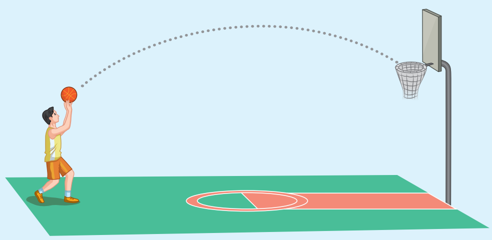
b How far away from the base of the plant does the water hit the ground?



■ **Figure A1.50** Water from a hose aimed at the base of a flower

39 If the maximum distance a man can throw a ball is 78 m, what is the minimum speed of release of the ball? (Assume that the ball lands at the same height from which it was thrown and that the greatest range for a given speed is when the angle is 45°.)

40 Figure A1.51 shows a player making a basketball shot.



■ **Figure A1.51** Basketball player making a shot

a In practice, air resistance can be considered negligible for a basketball. Suggest a reason why.

b Make a copy of the figure and add to it two other possible trajectories which will result in the ball arriving at the basket.

c Suggest which trajectory is best and explain your reasoning.

d Add to your drawing a possible trajectory that would enable a light-weight sponge ball to reach the basket.

Nature of science: Models

The motions of all projectiles are affected – often considerably – by air resistance. But the mathematics we have used to make predictions has assumed that air resistance is negligible. This is a recurring theme in physics: when theories are first developed, or when you are first introduced to a topic, the ideas are simplified. A ‘complete’ understanding of projectile motion may be expected at university level, but the topic is important enough that you should be introduced to the basic ideas at an earlier age.

In Worked example A1.10, the calculated answer predicts that a bullet will travel 1.1 km before striking the ground, although we should stress that this ‘assumes that there is no air resistance’. In reality, it should be well understood that air resistance cannot be ignored, and the bullet will not travel as far as calculated. This should not suggest that the calculation was not useful.

As your knowledge and experience increase, mathematical theories of projectile motion can be expanded to include the effects of air resistance – but this is beyond the limits of the IB Course. Similar comments can be applied to all areas of physics. This simplifying approach to gaining knowledge is not unique to physics but it is, perhaps, most obvious in the sciences.

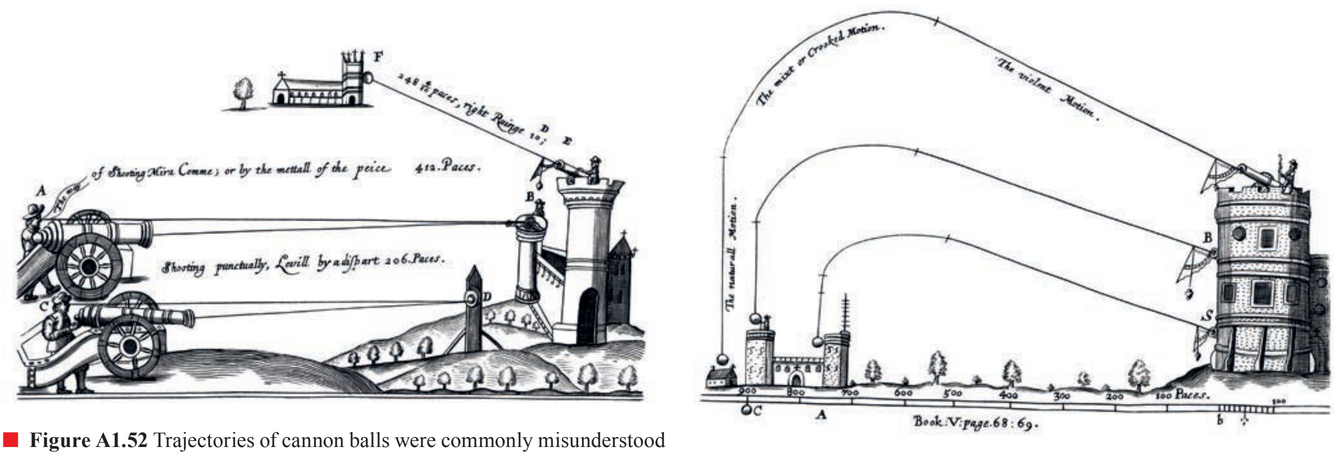
LINKING QUESTION

- How does the motion of an object change within a gravitational field?

This question links to understandings in Topics A.3 and D.1.

Ballistics

The study of the use of projectiles is known as ballistics. Because of its close links to hunting and fighting, this is an area of science with a long history, going all the way back to spears, and bows and arrows. Figure A1.52 shows a common medieval misconception about the motion of cannon balls: they were thought to travel straight until they ran out of energy.

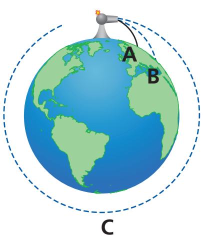


■ **Figure A1.52** Trajectories of cannon balls were commonly misunderstood

Photographs taken in quick succession became useful in analysing many types of motion in the nineteenth century, but the trajectories of very rapidly moving projectiles were difficult to determine until they could be filmed or illuminated by lights flashing very quickly (stroboscopes). The photograph of the bullet from a gun shown in Figure A1.53 required high technology, such as a very high-speed flash and very sensitive image recorders, in order to 'freeze' the projectile (bullet) in its rapid motion (more than $500\,m\,s^{-1}$).

◆ **Thought experiment** An experiment that is carried out in the mind, rather than actually being done, normally because it is otherwise impossible.

■ **Figure A1.53** A bullet 'frozen' by high-speed photography



■ **Figure A1.54** Newton's cannonball thought experiment

'Newton's cannonball' is a famous **thought experiment** concerning projectiles, in which Newton imagined what would happen to a cannonball fired (projected) horizontally at various very high speeds from the top of a very high mountain (in the absence of air resistance). See Figure A1.54.

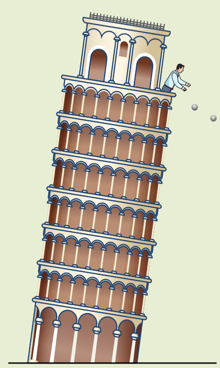
The balls labelled A and B will follow parabolic paths to the Earth's surface. B has a greater range than A because it was fired with greater velocity. Cannonball C has exactly the correct velocity that it never falls back to the Earth's surface and never moves further away from the Earth. (The required velocity would be about $7\,km\,s^{-1}$, but remember that we are assuming that there is no air resistance.) These ideas are developed further in Topic D.1.

Nature of science: Models

In a thought experiment, we use our imagination to answer scientific 'what if…?' type questions. Known principles or a possible theory are applied to a precise scenario, and the consequences thought through in detail. Usually, but not always, it would not be possible to actually carry out the experiment.

At the time of 'Newton's cannonball' thought experiment (published in 1728) it would have been impossible to make any object move at $7\,km\,s^{-1}$ and, even if that had been possible, air resistance would have quickly reduced its speed. Nevertheless, the thought processes involved advanced understanding and led to ideas of satellite motion. The first satellite to orbit the Earth was the Russian Sputnik 1 in 1957, which had a maximum speed of about $8\,km\,s^{-1}$ and avoided air resistance by being above most of the Earth's atmosphere.

Another (possible) thought experiment connected to this topic, and involving an assumption of no air resistance, is the dropping of two spheres of different masses from the same height on the Tower of Pisa. See Figure A1.55. Most historians doubt if there was an actual experiment at the Tower of Pisa that confirmed Galileo's theory that both masses would fall at the same rate.



■ **Figure A1.55** Galileo's famous experiment to demonstrate acceleration due to gravity

Two further famous thought experiments in physics are *Maxwell's demon* and *Schrödinger's cat*. Research online to find out how these thought experiments prompted new hypotheses and theories in physics.

A.2 Forces and momentum

Guiding questions

- How can we represent the forces acting on a system both visually and algebraically?
- How can Newton's laws be modelled mathematically?
- How can knowledge of forces and momentum be used to predict the behaviour of interacting bodies?

The nature of force

◆ **Interaction** Any event in which two or more objects exert forces on each other.

◆ **Newton, N** Derived SI unit of force. $1\,N = 1\,kg\,m\,s^{-2}$.

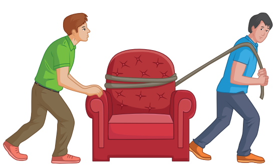
SYLLABUS CONTENT

▶ Forces as interactions between bodies.

In everyday life we may describe a force as a push or a pull but, more generally, a force can be considered to be *any* type of **interaction** / influence on an object which will tend to make it start moving or change its motion if it is already moving (assuming that the force is unopposed). Many forces do not cause changes of motion, simply because the objects on which they are acting are not able to move freely. Forces also change the shapes of objects.

Scientists refer to forces 'acting' on objects, 'exerting' forces on objects and 'applying' forces to objects. If objects 'interact', this means there are forces between them.

The size of a force is measured in the SI unit **newton**, **N**. The direction in which a force acts on an object is important:

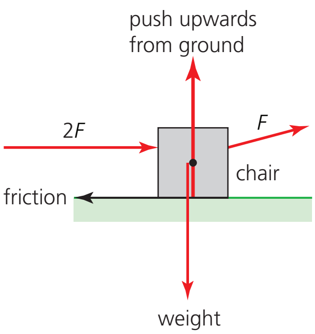


■ **Figure A2.1** Pushing and pulling a chair

Forces, F, are vector quantities and are represented in drawings by arrows of scaled length, direction and point of application. All forces should be labelled with commonly accepted symbols, or names.

(The vectors displacement, velocity and acceleration were introduced in Topic A.1.)

Most situations, such as the two boys moving a chair in Figure A2.1, involve several forces, not just the obvious forces arising from the boys' actions.



■ **Figure A2.2** Representing the forces in Figure A2.1

Figure A2.2 shows all the forces acting on the chair. These include the weight of the chair, the friction opposing its movement and the push upwards from the floor which is supporting the chair. The boy on the left is pushing the chair with a force which is twice the size of the force, F, that the boy on the right is using.

We will return to force diagrams later, but first we need to identify and explain different types of force.

Different types of force

In general, we can classify all forces as one of two kinds.

- Forces that involve physical contact. Examples include everyday pushes and pulls, friction and air resistance.

◆ **Mass** A measure of an object's resistance to a change of motion (inertia).

◆ **Kilogramme, kg** SI unit of mass (fundamental).

◆ **Weight, F_g** Gravitational force acting on a mass. $F_g = mg$.

◆ **Gravitational field strength, g** The gravitational force per unit mass (that would be experienced by a small test mass placed at that point).

$g = F_g/m$ (SI unit: $\mathrm{N\,kg^{-1}}$).

Numerically equal to the acceleration due to gravity.

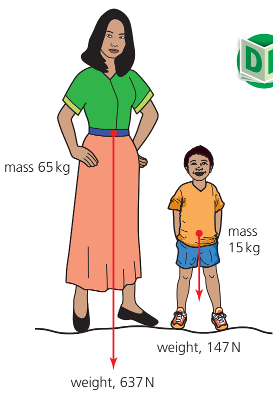


Figure A2.3 Weight acts downwards from the centre of mass

◆ **Centre of mass** Average position of all the mass of an object. The mass of an object is distributed evenly either side of any plane through its centre of mass.

- Forces that act 'at a distance' across empty space. Examples include magnetic forces and the force of gravity. These forces are more difficult to understand and can be described as 'field forces'.

We will now explore some important types of force in greater detail.

Weight

SYLLABUS CONTENT

▶ Gravitational force F_g as the weight of the body and calculated as given by: $F_g = mg$

The **mass** of an object may be considered to be a measure of the quantity of matter it contains. Mass has the SI unit **kilogramme**, kg. Mass does not change with location. This definition may seem rather vague, but this is because mass is such a fundamental concept it is difficult to explain in terms of other things. However, later in this topic we will provide an improved definition.

The **weight**, F_g, of a mass, m, is the gravitational force that pulls it towards the centre of the Earth (or any other planet). Weight and mass are connected by the simple relationship:

weight, $F_g = mg$

Where g is the weight : mass ratio, which is called the **gravitational field strength**. It has the units $\mathrm{N\,kg^{-1}}$.

g is numerically equal to the acceleration due to gravity (see Topic A.1). An explanation is given later in this topic.

Clearly, in principle, the weight of an object is not constant, but varies with location (where the value of g changes). The value of g varies with a planet's or a moon's mass and radius, and with distance from the planet's centre of mass. For example, it has a value of $9.8\,\mathrm{N\,kg^{-1}}$ on the Earth's surface, $1.6\,\mathrm{N\,kg^{-1}}$ on the surface of the Moon and $3.7\,\mathrm{N\,kg^{-1}}$ on Mars.

Weight is represented in a diagram by a vector arrow vertically downwards from the **centre of mass** of the object. See Figure A2.3. When an object is subjected to forces, it will behave as if all of its mass was at a single point: its centre of mass. (In a gravitational field, the same point is sometimes called its 'centre of gravity'.)

WORKED EXAMPLE A2.1

An astronaut has a mass of 58.6 kg. Calculate her weight using data from the preceding paragraphs:

a on the Earth's surface
b in a satellite 250 km above the surface ($g = 9.1\,\mathrm{N\,kg^{-1}}$)
c on the surface of the Moon
d on the surface of Mars
e in 'deep space', a very long way from any planet or star.

Answer

a $F_g = mg = 58.6 \times 9.8 = 5.7 \times 10^2\,\mathrm{N}$
b $F_g = mg = 58.6 \times 9.1 = 5.3 \times 10^2\,\mathrm{N}$, which is only 7% lower than on the Earth's surface
c $F_g = mg = 58.6 \times 1.6 = 94\,\mathrm{N}$
d $F_g = mg = 58.6 \times 3.7 = 217\,\mathrm{N}$
e 0 N, truly weightless

1 Calculate the weight of the following objects on the surface of the Earth:
   a a car of mass 1250 kg
   b a new-born baby of mass 3240 g
   c one pin in a pile of 500 pins that has a total mass of 124 g.

2 It is said that 'an A380 airplane has a maximum take-off weight of 570 tonnes' (Figure A2.4). A tonne is the name given to a mass of 1000 kg.
   a What is the maximum weight of the aircraft (in newtons) during take off?
   b The aircraft can take a maximum of about 850 passengers. Estimate the total mass of all the passengers and crew.
   c What percentage is this of the total mass of the airplane on take off?
   d The maximum landing weight is '390 tonnes'. Suggest a reason why the aircraft needs to be less massive when landing than when taking off.
   e Calculate the difference in mass and explain where the 'missing' mass has gone.

■ **Figure A2.4** The Airbus A380 is the largest passenger airplane in the world

3 A mass of 50 kg would have a weight of 445 N on the planet Venus. What is the strength of the gravitational field there? Compare it with the value of *g* on Earth.

4 Consider two solid spheres made of the same metal. Sphere A has twice the radius of sphere B. Calculate the ratio of the two spheres' circumferences, surface areas, volumes, masses and weights.

◆ **Force meter** Instrument used to measure forces. Also sometimes called a newton meter or a spring balance.

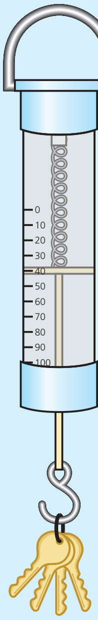
◆ **Calibrate** Put numbered divisions on a scale.

◆ **Weigh** Determine the weight of an object. In everyday use the word 'weighing' usually means quoting the result as the equivalent mass: 'my weight is 60 kg' actually means I have the weight of a 60 kg mass (about 590 N).

Tool 1: Experimental techniques

Understand how to accurately measure quantities to an appropriate level of precision: force, weight and mass

Forces are easily measured by the changes in length they produce when they squash or stretch a spring (or something similar). Such instruments are called **force meters** (also called *newton meters* or *spring balances*) – see Figure A2.5. In this type of instrument, the spring usually has a change of length proportional to the applied force. The length of the spring is shown on a linear scale, which can be **calibrated** (marked in newtons). The spring goes back to its original shape after it has measured the force.



■ **Figure A2.5** A spring balance force meter

Such instruments can be used for measuring forces acting in any direction, but they are also widely used for the measurement of the downwards force of weight. The other common way of measuring weight is with some kind of '*balance*' (scales). In an equal-arm balance, as shown in Figure A2.6, the beam will only balance if the two weights are equal. That is, the unknown weight equals the known weight. (Larger weights can be measured by positioning the pivot closer to the unknown weight and using the 'principle of moments' – mentioned in Topic A.4.)

Either of these methods can be used to determine (**weigh**) an unknown weight (N) and they rely on the force of gravity to do this, but such instruments may be calibrated to indicate mass (in kg or g) rather than weight. This is because we are usually more concerned with the quantity of something, rather than the effects of gravity on it. We usually assume that:

$$\text{mass (kg)} = \frac{\text{weight (N)}}{9.8}$$

anywhere on Earth because any variations in the acceleration due to gravity, *g*, are insignificant for most, but not all, purposes.

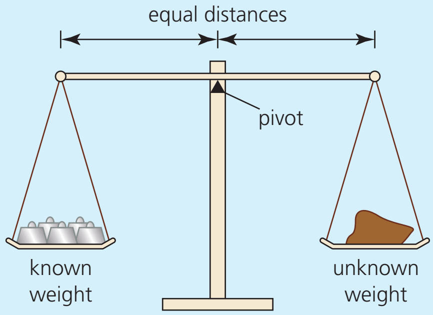


Figure A2.6 An equal-arm balance

Determining a mass without using its weight (gravity) is not so easy. Two ways we can do this are:

- If it is a solid and all the same material, its volume can be measured, then mass = volume × density (assuming that its density is known.)
- As we will see in Topic A.2, resultant force, mass and acceleration are connected by the equation $F = ma$, so that, if the acceleration produced by a known force can be measured, then the mass can be calculated.

Nature of science: Science as a shared endeavour

Science is a collaborative activity – scientists work together across the world to confirm (or dispute) findings by repeating experiments. Scientists review each other's work (**peer review**) to make sure that it is reliable before it is published. Communication is an essential part of science, and precision in communication is very important. Scientists must agree to use specific *terminology*, which is why scientific terminology sometimes differs from everyday use of the same words.

◆ **Peer review** Evaluation of scientific work by experts in the same field of study.

◆ **Tension (force)** Force that tries to stretch an object or material.

◆ **Compression (force)** Force that tries to squash an object or material.

◆ **Deformation** Change of shape.

◆ **Normal** Perpendicular to a surface.

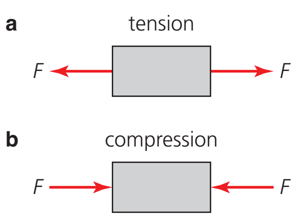


Figure A2.7 Object under **a** tension and **b** compression

Contact forces

Apart from obvious everyday pushes and pulls, the following terms should be understood:

Tension: pulling forces are acting tending to cause stretching.

Compression: forces are pushing inwards on an object (See Figure A2.7).

Both of these types of force will tend to change the shape of an object (**deformation**).

In the following sections we will discuss the following contact forces in more detail: normal forces, buoyancy forces, elastic restoring forces, surface friction and fluid friction.

Normal forces

SYLLABUS CONTENT

▶ Normal force F_N is the component of the contact force acting perpendicular to the surface that counteracts the body.

When two objects come in contact, they will exert forces on each other. This is because the particles in the surfaces resist getting closer together. A simple example is a book on a table, as shown in Figure A2.8. The book presses down on the table with its weight, and the table pushes up on the book with an equal force (so that the book is stationary). This force from the table is called a **normal** force, F_N. 'Normal' in this sense means that it is perpendicular to the surface.

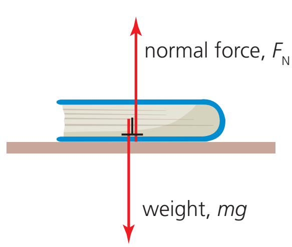


Figure A2.8 Normal force acting upwards on a book

If a force acts on a surface, the surface pushes back. The component of that force which is perpendicular to the surface is called a normal force.

Top tip!

Many students find the idea that solid and hard objects like walls, tables and floors can exert forces, difficult to comprehend, whereas forces from cushions, or trampolines, are easier to visualize and understand. Remember that solid materials will resist any deformation and push back, even if the change of shape is very, very small and not noticeable.

A normal force does not need to be vertical, nor equal to weight, as the two examples in Figure A2.9 illustrate.

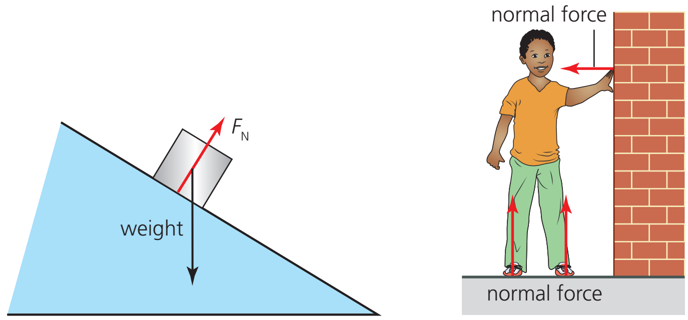


Figure A2.9 Other examples of normal forces

Buoyancy forces

SYLLABUS CONTENT

- Buoyancy force, F_b, acting on a body due to the displacement of the fluid as given by: $F_b = \rho V g$, where V is the volume of fluid displaced.

We have discussed the normal contact forces which act upwards on objects placed on solid horizontal surfaces. Liquids also provide vertical forces upwards on objects placed in, or on them. Gases, too, provide some support, although it is often insignificant.

◆ **Buoyancy force** Vertical upwards force on an object placed in or on a fluid. Sometimes called **upthrust**.

◆ **Density** $\frac{\text{mass}}{\text{volume}}$.

Buoyancy is the ability of any fluid (liquid or gas) to provide a vertical upwards force on an object placed in, or on it. This force is sometimes called **upthrust**. (Buoyancy can be explained by considering the difference in fluid pressures on the upper and lower surfaces of the object. Pressure is explained in Topic B.3.)

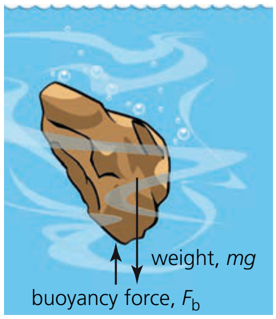
The magnitude of an upthrust will be greater in fluids of greater **density**.

Density is a concept with which you may be familiar, although it is not introduced in this course until Topic B.1.

$$\text{density (SI unit: kg m}^{-3}\text{)} = \frac{\text{mass}}{\text{volume}} \qquad \rho = \frac{m}{V}$$

g cm^{-3} is also widely used as a unit for density. A density of $1000\,\text{kg m}^{-3}$ (the density of pure water at 0 °C) is equivalent to $1.000\,\text{g cm}^{-3}$. It is also useful to know that one litre (1l) of water has a volume of $1000\,\text{cm}^3$ and has a mass of 1.00 kg.

Figure A2.10 shows two forces acting on a rock immersed in water. Its weight is greater than the buoyancy force, so it is sinking.



■ **Figure A2.10** Forces on an object immersed in a fluid

◆ **Archimedes' principle** When an object is wholly or partially immersed in a fluid, it experiences buoyancy force equal to the weight of the fluid displaced.

This area of classical physics was first studied more than 2250 years ago in Syracuse, Italy by **Archimedes** (from Greece, who identified the following principle, which still bears his name):

When an object is wholly or partially immersed in a fluid, it experiences a buoyancy force, F_b, equal to the weight of the fluid displaced. Since weight = mg, and density, $\rho = \frac{m}{V}$:

$$\boldsymbol{F_b = \rho V g}$$

TOK

The natural sciences

- What is the role of imagination in the natural sciences?

Myths, stories and science

The story of Archimedes' discovery of the principle of displacement is well known. The story is that Archimedes was asked by the king of Syracuse, Hiero, to check whether his goldsmith was trying to cheat him by mixing cheaper metals with the gold of a wreath in honour of the gods. Archimedes accepted the challenge, although was uncertain how to establish the true composition of the wreath crown. Reputedly, the idea came to him while sitting in the bath: if the wreath contained other metals, it would be less dense than gold, and as such would need to have a greater volume to achieve the same weight. Archimedes saw that he could test the composition of the wreath by measuring how much water was displaced by it, so measuring its volume and so allowing him to compare its density to that of gold. As the story relates, when Archimedes discovered this he shouted 'I have found it!' or 'Eureka!' in Greek and ran naked through the streets of Syracuse to give Hiero the news!

In fact, this story was never recorded by Archimedes himself and is found in the writings of a Roman architect from much later in the first century BCE called Vitruvius. Many who heard the story doubted it – including Galileo Galilei, who pointed out in his work 'The Little Balance' that Archimedes could have achieved a more precise result using a balance and the law of buoyancy he already knew. But the story persists, perhaps because it is a great way to visualize and so understand the concepts of displacement and density.

■ **Figure A2.11** A statue of Archimedes in a bathtub demonstrates the principle of the buoyant force. Located at Madatech, Israel's National Museum of Science, Technology and Space in Haifa

Consider: In what ways does the story of Archimedes resemble a thought experiment (see Topic A.1)? Do myths and stories serve always to obscure or confuse scientific truths? Can they sometimes enlighten us, too?

WORKED EXAMPLE A2.2

A piece of wood has a volume of 34 cm^3 and a mass of 29 g.

a Calculate its weight.

b Determine the volume of water that it will displace if it is completely under water.

c What buoyancy force will it experience while under water? (Assume density of water = 1000 kg m^{-3}.)

d What resultant force will act on the wood?

e State what will happen to the wood if it is free to move.

f Repeat a–e for the same piece of wood when it is surrounded by air (density 1.3 kg m^{-3}).

Answer

a weight = mg = $(29 \times 10^{-3}) \times 9.8 = 0.28$ N downwards

b 34 cm^3

c Weight of water displaced = $mg = V\rho g = (34 \times 10^{-6}) \times 1000 \times 9.8 = 0.33$ N upwards

d $0.33 - 0.28 = 0.05$ N upwards

e It will move (accelerate) up to the surface, where it will float.

f (see a–e below)

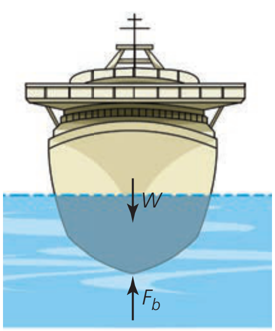
a Weight = 0.28 N downwards, as before

b 34 cm^3 as before

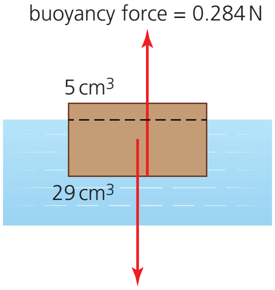
c Weight of air displaced = $mg = V\rho g = (34 \times 10^{-6}) \times 1.3 \times 9.8 = 4.3 \times 10^{-4}$ N upwards. Which is very small!

d $0.28 - (4.3 \times 10^{-4}) \approx 0.28$ N downwards. The buoyancy force in air has an insignificant effect on the wood.

e It will move (accelerate) down towards the Earth.



■ **Figure A2.12** A floating object



■ **Figure A2.13** Floating wood

Floating

An object placed on the surface of water (or any other liquid) will move lower until it displaces its own weight of water. See Figure A2.12. Then there will be no overall force acting on it, because the buoyancy force upwards (upthrust) will be equal to its weight downwards. If that is not possible, it will sink.

Continuing the numerical Worked example A2.2:

The wood has a weight of 0.28 N, so when floating it will displace water of this weight. Density of pure water = 1000 kg m^{-3}.

Weight = $0.28 = V\rho g = V \times 1000 \times 9.8$

$V = 2.9 \times 10^{-5}$ m^3. That is, 29 cm^3. The wood will float with 29 cm^3 below the water surface and 5 cm^3 above the surface, as shown in Figure A2.13.

5 a Calculate the buoyancy force acting on a boy of mass 60 kg and volume 0.0590 m^3 (use $g = 9.81\ \mathrm{N\,kg^{-1}}$)
 i in water of density $1000\ \mathrm{kg\,m^{-3}}$
 ii in air of density $1.29\ \mathrm{kg\,m^{-3}}$.
b Will the boy sink or float in water? Explain your answer.
c Suggest why he would float easily if he was in the Dead Sea. See Figure A2.14.

■ **Figure A2.14** Floating in the Dead Sea

d Calculate a value for the ratio: boy's weight / buoyancy force in air.

6 A wooden cube with a density of $880\ \mathrm{kg\,m^{-3}}$ is floating on water (density $1000\ \mathrm{kg\,m^{-3}}$). If the sides of the cube are 5.5 cm long and the cube is floating with a surface parallel to the water's surface, show that the depth of wood below the surface is 4.8 cm.

7 After the rock shown in Figure A2.10 begins to move downwards (sink) another force will act on it. State the name of that force.

8 Outline the reasons why a balloon filled with helium will rise (in air), while a balloon filled with air will fall.

9 Learning to scuba dive involves being able to remain 'neutrally buoyant', so that the diver stays at the same level under water. Explain why breathing in and out affects the buoyancy of a diver.

■ **Figure A2.15** How much of an iceberg is submerged?

10 It is commonly said that about 10% of an iceberg is above the surface of the sea (Figure A2.15). Use this figure to estimate a value for the density of sea ice. Assume the density of sea water is $1025\ \mathrm{kg\,m^{-3}}$.

Elastic restoring forces

SYLLABUS CONTENT

▶ Elastic restoring force, F_H, following Hooke's law as given by: $F_H = -kx$, where k is the spring constant.

◆ **Elastic behaviour** A material regains its original shape after a force causing deformation has been removed.

◆ **Elastic limit** The maximum force and/or extension that a material, or spring, can sustain before it becomes permanently deformed.

When a force acts on an object it can change its shape: then we say that there is a deformation. Sometimes the deformation will be obvious, such as when someone sits on a sofa; sometimes the deformation will be too small to be seen, such as when we stand on the floor.

If an object returns to its original shape after the force has been removed, we say that the deformation was **elastic**. We hope and expect that most of the objects we use in everyday life behave elastically, because after we use them, we want them to return to the same condition as before their use. If they do not, we say that they have passed their **elastic limit**.

Common mistake

Rubber bands behave elastically and they are useful because they can stretch a lot and exert inwards forces on the objects they are wrapped around. Because of this behaviour, the word 'elastic' in common usage has also come to mean 'easy to stretch' – which is different from its true meaning in science. Most people would be surprised to learn that steel usually behaves elastically.

How deformation depends on force

How any object, or material, responds when forces act on them is obviously very important information when considering their use in practical situations.

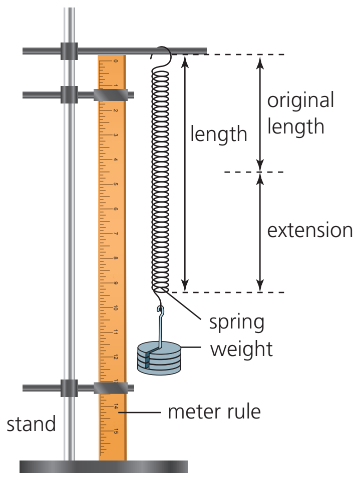
The deformation of a steel spring is a common starting investigation because it is easy to measure and it will usually stretch regularly and elastically (unless over-stretched). See Figure A2.16.

◆ **Extension** Displacement of the end of an object that is being stretched.

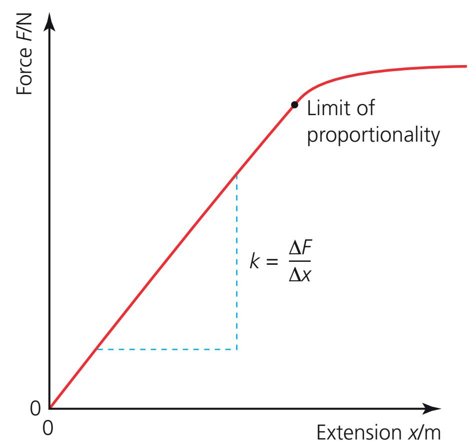
Figure A2.17 shows typical results. The weights provide the downwards force, *F*. In this case the deformation is called the **extension** of the spring, *x*, and it is usually plotted on the horizontal axis of graphs.

Top tip!

No material will behave elastically under all conditions. They all have their limits: *elastic limits*. For this reason, it is probably sensible not to describe a material as being 'elastic'. It is better to say that it behaved elastically under the conditions at that time.



■ **Figure A2.16** Steel spring investigation



■ **Figure A2.17** Results of stretching a steel spring

Most of the graph is a straight line passing through the origin. (The coils of the spring should not be touching each other at the beginning.) The conclusion is that the force, *F*, and the extension, *x*, are proportional to each other, up to a limit (as shown on the graph). The graph also shows that the spring gets easier to stretch after the limit of proportionality has been passed. For the linear part of the graph, starting at the origin: $F \propto x$.

The constant of proportionality is given the symbol *k*: $F = kx$.

◆ **Spring constant, *k*** The constant seen in **Hooke's law** that represents the stiffness of a spring (or other material).

k is a measure of the 'stiffness' of the spring and is commonly called the **spring constant** (or the *force constant*). It can be determined from the gradient of the graph:

$$k = \frac{\Delta F}{\Delta x}$$

k has the SI units $\mathrm{N\,m^{-1}}$. ($\mathrm{N\,cm^{-1}}$ is also widely used.)

Hooke's law

In the seventeenth century, Robert Hooke was famously the first to publish a quantitative study of springs. The physics law that describes his results is still used widely and bears his name:

DB

Hooke's law for elastic stretching: restoring force, $\boldsymbol{F_H = -kx}$

◆ **Restoring force** Force acting in the opposite direction to displacement, returning an object to its equilibrium position.

This is essentially the same as the equation $F = kx$, but the symbol F_H has been used for the force (to show that it is Hooke's law stretching), and the force refers to the **restoring force** within the spring, tending to return it to its original shape – this force is equal in size but opposite in direction to the externally applied force from the weights. The negative sign has been included to indicate that the restoring force acts in the opposite direction to increasing extension.

LINKING QUESTION

- How does the application of a restoring force acting on a particle result in simple harmonic motion?

This question links to understandings in Topic C.1.

Nature of science: Models

Obeying the law

Sometimes, everyday language differs from scientific terminology (for example, when speaking about 'weight'). So, what are 'laws' in science? If the extension of a stretched material is proportional to the force, we describe it as 'obeying' Hooke's law. In what way is that similar / different to 'obeying' a legal law?

Archimedes' description of buoyancy forces is described as a 'principle'. How are scientific 'principles' different from scientific 'laws'?

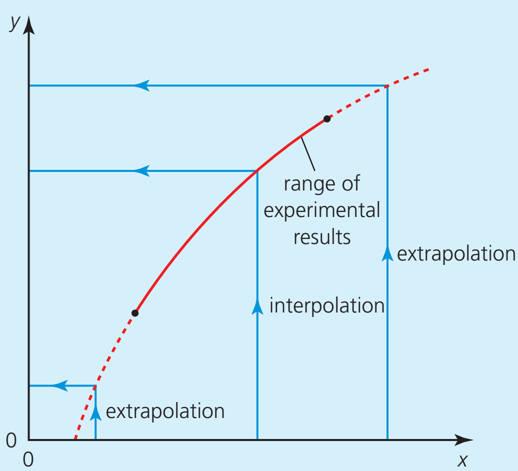
Research this online using search terms such as 'difference scientific principle and law'.

How might these concepts relate to theories and models in science?

Tool 3: Mathematics

Extrapolate and interpolate graphs

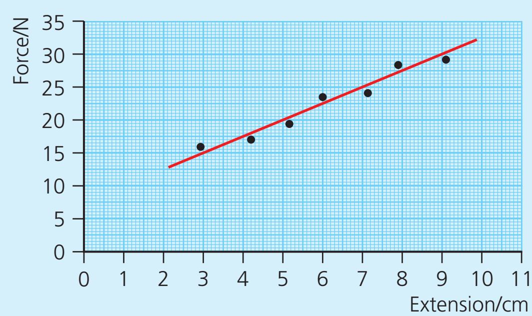
A curve of best fit is usually drawn to cover a specific range of measurements recorded in an experiment, as shown in Figure A2.18. The diagram indicates how values for y can be determined for a chosen values of x. If we want to predict other values within that range, we can usually do that with confidence. This is called **interpolation**.



■ **Figure A2.18** Interpolating and extrapolating to find values on the y-axis

If we want to predict what would happen outside the range of measurements (**extrapolation**) we need to extend the line of best fit. Lines are often extrapolated to see if they pass through the origin, or to find an intercept, as shown in Figure A2.18.

Predictions made by extrapolation should be treated with care, because it may be wrong to assume that the behaviour seen within the range of measurements also applies outside that range.



■ **Figure A2.19** F–x graph for stretching a spring

Force–extension graphs, such as seen in Figure A2.19, are an interesting example.

a Use the graph to determine values for extensions when the force was 25 N, 10 N and 35 N.

b Use the graph to determine a possible value for the intercept on the force axis, and explain what it represents.

c Comment on your answers.

◆ **Interpolate** Estimate a value within a known data range.

◆ **Extrapolate** Predict behaviour that it outside of the range of available data.

WORKED EXAMPLE A2.3

When a weight of 12.7 N was applied to a spring its length was 15.1 cm. When the force increased to 18.3 N, the length increased to 18.1 cm because the extension was proportional to the force.

a Determine the spring constant.

b Calculate the length of the spring when the force was 15.0 N.

c Explain why it is impossible to be sure what the length of the spring would be if the force was 25 N.

Answer

a $k = \frac{\Delta F}{\Delta x} = \frac{(18.3 - 12.7)}{(18.1 - 15.1)} = 1.87\,\mathrm{N\,cm^{-1}}$. Which is the same as $187\,\mathrm{N\,m^{-1}}$.

b Consider the extension from a length of 15.1 cm:

$$\Delta x = \frac{\Delta F}{k} = \frac{(15.0 - 12.7)}{1.87} = 1.23\ \mathrm{cm}$$

So that, length = 15.1 + 1.23 = 16.3 cm

c The spring may have passed its limit of proportionality.

The results shown in Figure A2.19 were probably taken as the spring was *loaded* (as the weight was increased). If the extension is measured as the weight is *reduced* the results will be similar, but only if the elastic limit has not been exceeded.

The elastic limit of the spring is not shown on the graph, but it is often assumed to be close to, or the same as, the limit of proportionality. In other words, when a spring stretches, such as its extension is proportional to the force, we assume that it is behaving elastically. That may or may not be true for other materials.

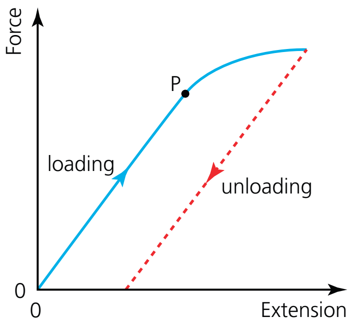


Figure A2.20 Stretching a metal wire

Force–extension graphs and the concepts of elastic limits and 'spring constants' are not restricted to describing springs. They are widely used to represent the behaviour of many materials. Figure A2.20 shows a typical graph obtained when a metal wire is stretched and then the load is removed.

The force is proportional to the extension up until point P. During this time the particles in the metal are being pulled slightly further apart and we may assume that the metal is behaving elastically. But when the force is increased further, the wire begins to stretch more easily, the elastic limit is passed and a permanent deformation occurs. When the wire is unloaded the atoms move back closer together, so that the gradient of the graph is the same as for the loading graph, but the wire has a permanent deformation after all force has been removed.

11 A spring has a spring constant of $125\,\mathrm{N\,m^{-1}}$ and will become permanently deformed if its extension is greater than 20 cm.

a Assuming that it behaves elastically, what extension results from a tensile force of 18.0 N?

b What is the maximum force that should be used with this spring?

12 When a mass of 200 g was hung on a spring its length increased from 4.7 cm to 5.3 cm.

a Assuming that it obeyed Hooke's law, what was its spring constant?

b The spring behaves elastically if the force does not exceed 10 N. What is the length of the spring with that force?

13 Figure A2.21 shows a force–extension graph for a piece of rubber which was first loaded, then unloaded.

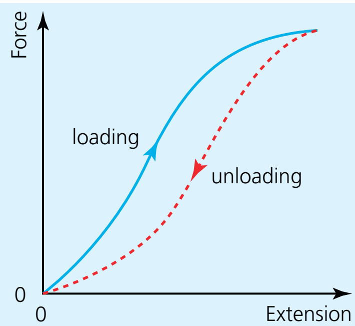


Figure A2.21 Stretching rubber

a Does the rubber behave elastically? Explain your answer.

b Does the rubber obey Hooke's law under the circumstances shown by the graph? Explain your answer.

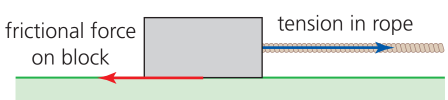
Surface friction

SYLLABUS CONTENT

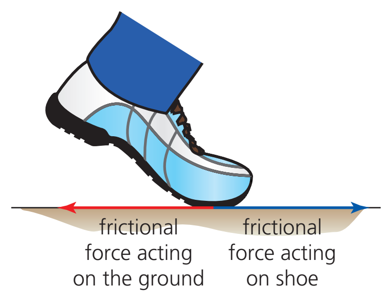
- Surface frictional force, F_f, acting in a direction parallel to the plane of contact between a body and a surface, on a stationary body as given by: $F_f \leqslant \mu_s F_N$, or a body in motion as given by: $F_f = \mu_d F_N$, where μ_s and μ_d are the coefficients of static and dynamic friction respectively.

◆ **Friction** Resistive forces opposing relative motion. Occurs between solid surfaces, but also with fluids. **Static friction** prevents movement, whereas **dynamic friction** occurs when there is already motion.

When we move an object over another surface (or try to move it), forces parallel to the surfaces will resist the movement. Collectively, these forces are known as surface **friction**. The causes of friction can be various, and it is well known that friction can often be difficult to analyse or predict. Figure A2.22 shows a typical simple frictional force diagram. (The frictional force acting on the ground is not shown.) The block is moving to the right and the frictional force is acting to the left.



■ **Figure A2.22** Frictional force on a block opposing its motion to the right

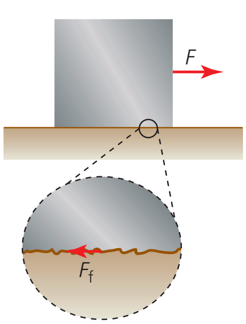


■ **Figure A2.23** We need friction to walk

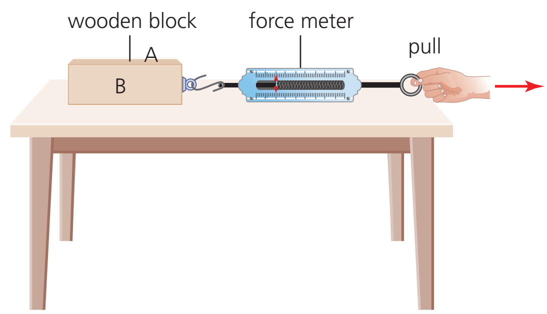
Friction is very useful: without friction we would not be able to walk. Similarly, a car's wheels would just spin on the same spot if there was no friction. Figure A2.23 explains why (the vertical forces are not shown). Because of friction, the shoe is able to push backwards, to the left, parallel to the ground, at the same time an equal frictional force pushes the shoe forward, to the right. (This is an example of Newton's third law of motion, which is discussed later in this topic.)

The roughness of both surfaces (see Figure A2.24) is certainly an important factor in producing friction: rougher surfaces generally increase friction, but this is not always true. For example, there may be considerable friction between very flat and smooth surfaces, like two sheets of glass. Friction can often be reduced by placing a lubricant, such as oil or water, between the surfaces.

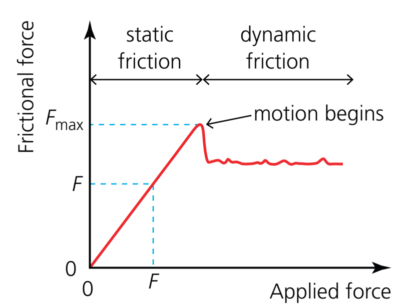
Figure A2.25 shows a basic laboratory investigation of the frictional forces between a wooden block and a horizontal table top.



■ **Figure A2.24** Even smooth surfaces have irregularities



■ **Figure A2.25** A simple experiment to measure frictional forces

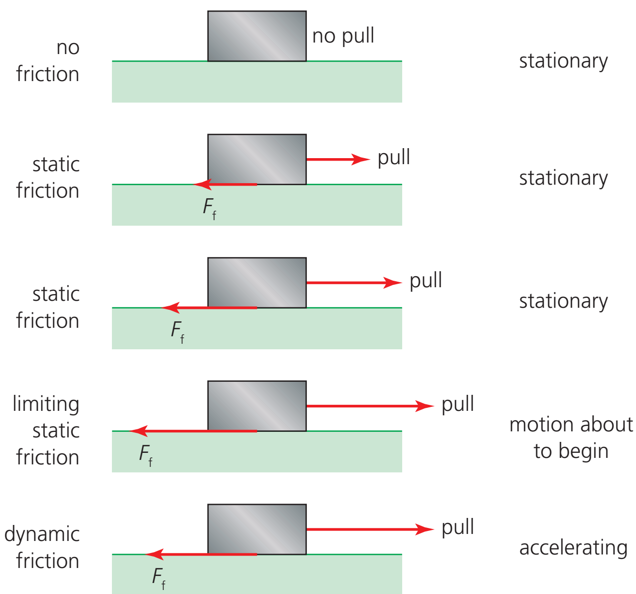


■ **Figure A2.26** Variation of friction with applied force

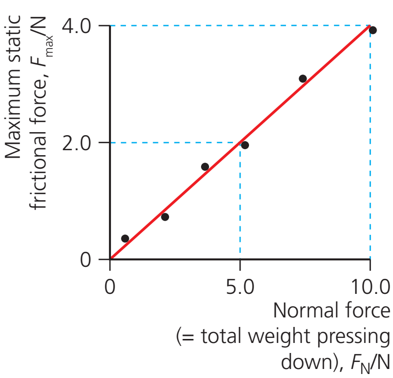
As the applied force (pull) is increased, the block will remain stationary until the force reaches a certain value, F_{max}. The block then starts to move, but after that, a steady force, which is less than F_{max}, will maintain a motion at constant speed. See Figure A2.26.

While the block is stationary (static) the force of friction adjusts, keeping equal to any applied force, but in the opposite direction. Under these circumstances the friction is called **static friction**. The size of the static friction force can increase from zero to a maximum value, F_{max}. Once an object is moving, the reduced friction is called **dynamic friction**, and its value is approximately constant at different speeds.

Figure A2.27 illustrates how frictional forces can change as a pulling force is increased.



■ **Figure A2.27** How frictional forces change as the force applied increases



■ **Figure A2.28** Typical variation of maximum static frictional force with normal force (a similar pattern of results will be obtained for dynamic friction)

The arrangement shown in Figure A2.25 can also be used to investigate how the *maximum* value of static friction depends on the force pushing the surfaces together: weights can be added on top of the block to increase the normal contact force, F_N. Figure A2.28 shows some typical results.

The graph shows that there is more static friction when there is a greater force pushing the surfaces together. In fact, frictional forces, F_f, are proportional to the normal contact forces, F_N. ($F_f \propto F_N$) The constant of proportionality equals the gradient of the graph and is called the **coefficient of friction**, μ (no units)

Just before motion begins: $F_f = F_{max} = \mu_s F_N$, where μ_s is the coefficient of static friction.

When there is no movement, static frictional force: $\boldsymbol{F_f \leq \mu_s F_N}$.

Table A2.1 shows some typical values for the coefficient of static friction between different materials.

◆ **Coefficient of friction, μ**
Constants used to represent the amount of friction between two different surfaces. Maybe static or dynamic.

■ **Table A2.1** Approximate values for coefficients of static friction

Materials		**Approximate coefficients of static friction, μ_s**
steel	ice	0.03
ski	dry snow	0.04
Teflon™	steel	0.05
graphite	steel	0.1
wood	concrete	0.3
wood	metal	0.4
rubber tyre	grass	0.4
rubber tyre	road surface (wet)	0.5
glass	metal	0.6
rubber tyre	road surface (dry)	0.8
steel	steel	0.8
glass	glass	0.9
skin	metal	0.9

When there is movement, dynamic frictional force, $\boldsymbol{F_f = \mu_d F_N}$, where μ_d is the coefficient of dynamic friction.

◆ **Constant** A number which is assumed to have the same numerical value under a specified range of circumstances.

◆ **Fundamental constants** Numbers which are assumed to have exactly the same numerical values under all circumstances and all times.

◆ **Coefficient** A multiplying constant placed before a variable, indicating a physical property.

Tool 3: Mathematics

Applying general mathematics: constants

A number which is assumed to be **constant** always has the same value under the specified circumstances. For example, the spring constant described earlier in this topic represents the properties of a spring, but only up to its limit of proportionality. In Topic A.1, the acceleration due to gravity was assumed to be constant at $9.8\,\text{m}\,\text{s}^{-2}$, but only if we limit precision to 2 significant figures and only apply it to situations close to the Earth's surface.

However, there are a few constants which are believed to have exactly the same value in all locations and for all time. They are called the **fundamental constants**, or universal constants. Two examples are the speed of light and the charge on an electron.

In general, a **coefficient** is a number (usually a constant) placed before a variable in an algebraic expression. For example, in the expression $5a - 2 = 8$, the number 5 is described as a coefficient. In physics, a coefficient is used to characterize a physical process under certain specified conditions.

We have seen that: dynamic frictional force, F_f = *coefficient* of dynamic friction × F_N

Another example (which is not in the IB course): when a metal rod is heated it expands so that increase in length for each 1 °C temperature rise
= *coefficient* of thermal expansion × original length.

Objects also experience friction when they move through liquids and gases (fluids). This is discussed in the next section.

ATL A2A: Research skills

Using search engines and libraries effectively

Tyres and road safety

Much of road safety is dependent on the nature of road surfaces and the tyres on vehicles. Friction between the road and a vehicle provides the forces needed for any change of velocity – speeding up, slowing down, and changing direction. Smooth tyres will usually have the most friction in dry conditions, but when the roads are wet, ridges and grooves in the tyres are needed to disperse the water (Figure A2.29).

To make sure that road surfaces produce enough friction, they cannot be allowed to become too smooth and they may need to be resurfaced every few years. This is especially important on sharp corners and hills. Anything that gets between the tyres and the road surface – for example, oil, water, soil, ice and snow – is likely to affect friction and may have a significant effect on road safety. Increasing the area of tyres on a vehicle will change the pressure underneath them and this may alter the nature of the contact between the surfaces. For example, a farm tractor may have a problem about sinking into soft ground, and such a situation is more complicated than simple friction between two surfaces. Vehicles that travel over soft ground need tyres with large areas to help avoid this problem.

Using a search engine, research online to find what materials are used in the construction of tyres and road surfaces to produce high coefficients of friction. Organize your data in a table, making sure to credit your sources using a recognized, standard method of referencing and citation.

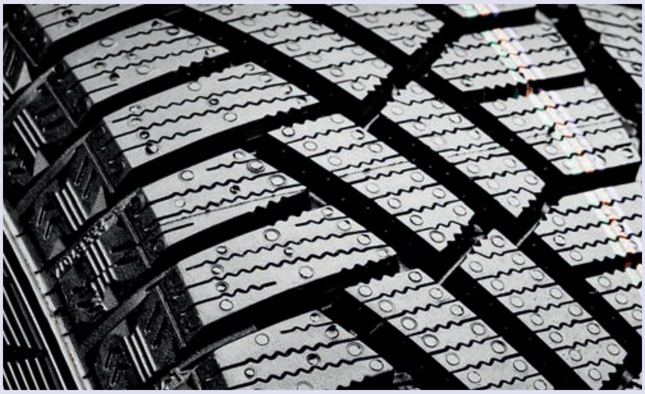


Figure A2.29 Tread on a car tyre

WORKED EXAMPLE A2.4

a Determine the coefficient of friction for the two surfaces represented in the graph shown in Figure A2.28.

b Assuming the results were obtained for apparatus like that shown in Figure A2.25, calculate the minimum force that would be needed to move a block of total mass:

  i 200 g

  ii 2000 g.

  iii Suggest why the answer to part **ii** is unreliable.

c Estimate a value for the dynamic frictional force acting on a mass of 200 g with the same apparatus:

  i for movement at $1.0\,\text{m}\,\text{s}^{-1}$

  ii for movement at $2.0\,\text{m}\,\text{s}^{-1}$.

Answer

a $\mu_s = \dfrac{F_{max}}{F_N} = \dfrac{4.0}{10.0} = 0.40$ (This is equal to the gradient of the graph.)

b **i** $F_f = \mu_s F_N = \mu_s mg = 0.40 \times 0.200 \times 9.8 = 0.78\,\text{N}$

  ii $0.40 \times 2.000 \times 9.8 = 7.8\,\text{N}$

  iii Because the answer is extrapolated from well outside the range of experimental results shown on the graph.

c **i** We would expect the dynamic frictional force to be a little less than the static frictional force, say about 0.6 N instead of 0.78 N.

  ii The dynamic frictional force is usually assumed to be independent of speed, so the force would still be about 0.6 N at the greater speed.

Common mistake

Many students expect that, if the block in Figure A2.25 was rotated so that side B was in contact with the table (instead of the side parallel to A), there would be more friction because of the greater area of contact. However, the frictional force will remain (approximately) the same, because if, for example, the area doubles, the force acting down on each cm^2 will halve.

Use data from Table A2.1 where necessary.

14 If dynamic friction is 85% of the maximum static friction, estimate the frictional force acting on the steel skates of a 47 kg ice-skater moving across the ice.

15 A 54 kg wooden box is on a horizontal concrete floor.

a Estimate the minimum force required to start it sliding sideways.

b Suggest why your answer to part **a** may not be reliable.

c If a force of 120 N keeps the box moving at a constant speed, what is the coefficient of dynamic friction?

d What will happen to the box if the applied force increases above 120 N?

16 a Predict the maximum frictional force possible between a dry road surface and each tyre of a stationary, 1500 kg four-wheeled family car.

b Why will the force be less if the road is wet or icy?

c Discuss how roads can be made safer under icy conditions.

17 Figure A2.30 shows the front of a Formula One racing car. Suggest how this design helps to increase the friction between the tyre and the race track.

Figure A2.30 Front of a Formula One racing car

18 A book of mass 720 g is being held in place next to a vertical wall as shown in Figure A2.31.

a State the weight of the book.

b Suggest an approximate value for the coefficient of static friction between the book and the wall.

c Use your answer to part **b** to estimate the minimum force needed to keep the book stationary against the wall.

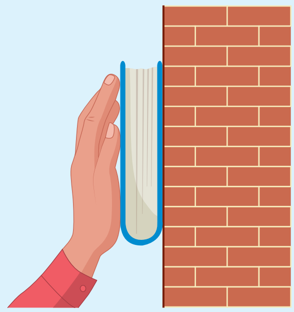


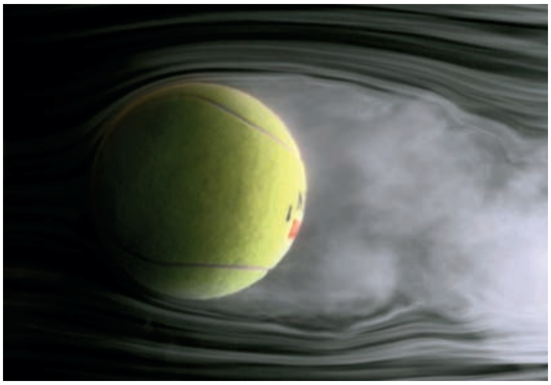
Figure A2.31 Book being held next to a vertical wall

Friction of objects with air and liquids

SYLLABUS COVERAGE

- Viscous drag force, F_d, acting on a small sphere opposing its motion through a fluid as given by: $F_d = 6\pi\eta rv$, where η is the fluid viscosity, r is the radius of the sphere and v is the velocity of the sphere through the fluid.

Air resistance was briefly discussed in Topic A.1. The word *drag* is widely used to describe friction in air and liquids. We will use the symbol F_d for this type of force.



■ **Figure A2.32** Flow of air past a tennis ball in a wind tunnel.

There are a great number of applications of this subject, including moving vehicles, sports and falling objects. Wind tunnels are useful in the study of drag: the object is kept stationary while the speed of air flowing past it is varied. The flow of the air can be marked as shown in Figure A2.32.

Drag can be a complicated subject because the amount of drag experienced by an object moving through air, or a liquid, depends on many factors, including the object's size and shape, the nature of its surface, its speed v, and the nature of the fluid. Drag will also depend on the cross-sectional area of the object (perpendicular to its movement).

Typically, for small objects moving slowly $F_d \propto v$.

But for larger objects, moving more quickly, $F_d \propto v^2$.

Viscosity and Stokes's law

◆ **Viscosity** Resistance of a fluid to movement.

◆ **Viscous drag** The drag force acting on a moving object due to the viscosity of the fluid through which it is moving.

◆ **Turbulence** Flow of a fluid which is erratic and unpredictable.

◆ **Stokes's law** Equation for the viscous drag acting on a smooth, spherical object undergoing non-turbulent motion.

When an object moves through a fluid it has to push the fluid out of its path. A fluid's resistance to such movement is called its **viscosity**. Clearly, greater viscosity will tend to increase drag, and when this is the dominant factor, we refer to **viscous drag**.

Viscosity is given the symbol η (eta) and has the SI unit of Pa s (kg m^{-1} s^{-1}). Some typical values at 20 °C are given in Table A2.2. Viscosities of liquids can be very dependent on temperature.

■ **Table A2.2** Viscosities of some fluids

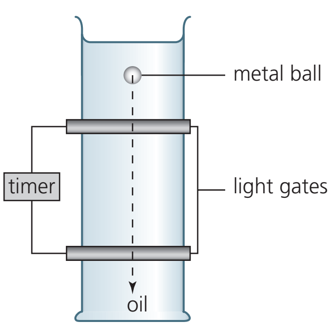
Fluid	**Viscosity η/Pa s**
'heavy' oil	0.7
'light' oil	0.1
water	1×10^{-3}
human blood	4×10^{-3}
gasoline (petrol)	6×10^{-4}
air	1.8×10^{-5}

In order to understand this further, we start by simplifying the situation, as is common in physics: by considering a smooth spherical object, of radius r, moving at a speed v, which is not great enough to cause **turbulence** (irregular movements) in the fluid.

Under these circumstances, the viscous drag, F_d, can be determined from the following equation (known as **Stokes's law**):

DB

viscous drag $F_d = 6\pi\eta r v$



■ **Figure A2.33** Experiment to determine the viscosity of a liquid

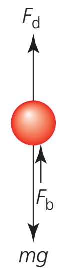
Dropping small spheres through fluids is a widely used method for determining their viscosities and how they may depend on temperature. A method is shown in Figure A2.33, in which an electronic timer is started and stopped as the metal ball passes through the two light gates.

Inquiry 1: Exploring and designing

Designing

Look at the apparatus setup in Figure A2.33. Apply what you know about terminal speed (Topic A.1) and viscous fluid flow to design and explain a valid methodology for an experiment to obtain a single set of measurements. Include an explanation of:

1. why the metal ball is released such that it passes through some oil before reaching the first timing gate
2. why the tube should be as wide as possible.



■ **Figure A2.34** Forces on a sphere falling with terminal speed

If a sphere of mass m and radius r is moving with a constant terminal speed, v_t, then the upwards and downwards forces on it are balanced, as shown in Figure A2.34.

viscous drag, F_d + buoyancy force, F_b = weight, mg:

$$6\pi\eta rv + \rho Vg = mg$$

but:

$$V = \frac{4}{3}\pi r^3$$

so:

$$6\pi\eta rv + \frac{4}{3}\rho g\pi r^3 = mg$$

If the mass and radius of the sphere are measured and the terminal speed determined as shown in Figure A2.33, then this equation can be used to determine a value for the viscosity of the liquid, assuming that its density is known.

Inquiry 3: Concluding and evaluating

Evaluating

The experimental determination of a viscosity discussed above involved just one set of measurements and a calculation.

Explain improvements to increase the accuracy of the determination of the viscosity of a liquid by collecting sufficient data to enable a graph of the results to be drawn.

WORKED EXAMPLE A2.5

Calculate the force of viscous drag on a sphere of radius 1.0 mm moving at 1.0 cm s^{-1} through ‘heavy’ oil.

Answer

$F_d = 6\pi \times \eta \times r \times v = 6 \times 3.14 \times 0.7 \times (1.0 \times 10^{-3}) \times (1.0 \times 10^{-2}) = 1.3 \times 10^{-4}$ N

19 The air resistance acting on a car moving at 5.0 m s^{-1} was 120 N.
Assuming that this force was proportional to the speed squared, what was the air resistance when the car's speed increased to:
a 10 m s^{-1} **b** 15 m s^{-1}?

20 Show that the units of viscosity are Pa s.

21 Calculate the viscous drag force acting on a small metal sphere of radius 1.3 mm falling through oil of viscosity 0.43 Pa s at a speed of 7.6 cm s^{-1}.

22 A drop of water in a cloud had a mass of 0.52 g and radius of 0.50 mm (and volume of 0.52 mm^3).
a Assuming that the density of the surrounding air is 1.3 kg m^{-3}, calculate and compare the size of the three forces acting on the drop if it has just started to fall with a speed of 5.0 cm s^{-1}.
b Draw an annotated diagram to display your answers.
c Determine the subsequent movement of the drop.

23 In an experiment similar to that shown in Figure A2.33, a sphere of radius 8.9 mm and mass 3.1 g reached a terminal speed of 7.6 cm s^{-1} when falling through an oil of density 842 kg m^{-3}.
Determine a value for the viscosity of the liquid.

24 Use the internet to find out how the design of golf balls reduces drag forces in flight. Write a 100 word summary of your findings.

ATL A2B: Thinking skills

Evaluating and defending ethical positions

Air travel

Aircraft use a lot of fuel moving passengers and goods from place to place quickly, but we are all becoming more aware of the effects of planes on global warming and air pollution. Some people think that governments should put higher taxes on the use of planes to discourage people from using them too much. Improving railway systems, especially by operating trains at higher speeds, will also attract some passengers away from air travel. Of course, engineers try to make planes more efficient so that they use less fuel, but the laws of physics cannot be broken and jet engines, like all other *heat engines*, cannot be made much more efficient than they are already.

Planes will use a lower fuel if there is a lower air resistance acting on them. This can be achieved by designing planes with **streamlined** shapes, and also by flying at greater heights where the air is less dense. Flying more slowly than their maximum speed can also reduce the amount of fuel used for a particular trip, as it does with cars, but people generally want to spend as little time travelling as possible.

The pressure of the air outside an aircraft at its typical cruising height is far too low for the comfort and health of the passengers and crew, so the air pressure has to be increased inside the airplane, but this is still much lower than the air pressure near the Earth's surface. The difference in air pressure between the inside and outside of the aircraft would cause problems if the airplane had not been designed to withstand the extra forces.

Aircraft generally carry a large mass of fuel, and the weight of an aircraft decreases during a journey as the fuel is used up. The upwards force supporting the weight of an aircraft in flight comes from the air that it is flying through and will vary with the speed of the airplane and the density of the air. When the aircraft is lighter towards the end of its journey it can travel higher, where it will experience less air resistance.

Debate the issue in class. Break into groups. One group can represent the airline operators, another group can represent passengers, a third group can represent an environmental campaign group, while a fourth group could represent the government. In your groups, allocate roles for researchers and a spokesperson. Using the information above and your understanding of air resistance prepare a proposal from the point of view of your assigned group detailing different ways in which we can reduce the environmental impact of air travel.

To help your research and calculations, refer to the following guiding questions:

- How do airlines hope that in the future they can become 'carbon neutral'. What is 'SAF'?
- Find out how much fuel is used on a long-haul flight of, say, 12 hours.
- Compare your answer with the capacity of the fuel tank on an average sized car.
- On a short-haul flight it is often claimed that as much of 50% of an aircraft's fuel might be used for taxiing, taking off, climbing and landing, but on longer flights this can reduce to under 15%. Explain the difference.

◆ **Streamlined** Having a shape that reduces the drag forces acting on an object that is moving through a fluid.

◆ **Field (gravitational, electric or magnetic)** A region of space in which a mass (or a charge, or a current) experiences a force due to the presence of one or more other masses (charges, or currents – moving charges).

LINKING QUESTION

- How can knowledge of electrical and magnetic forces allow the prediction of changes to the motion of charged particles?

This question links to understandings in Topic D.3.

Field forces

SYLLABUS CONTENT

- The nature and use of the following field forces.
  - Gravitational force, F_g, as the weight of the body and calculated as given by: $F_g = mg$
  - Electric force F_e
  - Magnetic force F_m

These three forces are very important in the study of physics but, apart from the gravitational force of weight, knowledge about them is not required in *this* topic.

These forces can act across empty space, without the need for any material in between. This can be difficult for the human mind to accept. One way of increasing our understanding is to develop the concept of force **fields** surrounding masses (gravitational fields), charges (electric fields) and magnets / electric currents (magnetic fields). Using this concept, we can give numerical values to points in space, for example, by stating that the gravitational field strength at the height of a particular satellite's orbit is $8.86\,\mathrm{N\,kg^{-1}}$.

Free-body diagrams

SYLLABUS CONTENT

- Forces acting on a body can be represented in a free-body diagram.

Even the simplest of force diagrams can get confusing if all the forces are shown. To make the diagrams simpler we usually draw only one object and show only the forces acting on that one object. These drawings are called **free-body diagrams**. (Physicists use the words 'body' and 'object' interchangeably.) Some simple examples are shown in Figure A2.35.

◆ **Free-body diagram** Diagram showing all the forces acting on a single object, and no other forces.

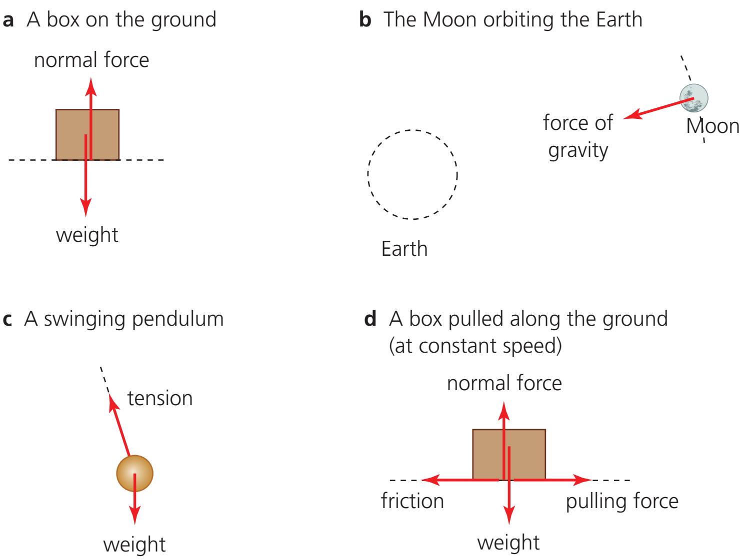


Figure A2.35 Free-body diagrams; the object has a solid outline and the forces are shown in red

◆ **Point particle, mass or charge** Theoretical concept used to simplify the discussion of forces acting on objects (especially in gravitational and electric fields).

The diagrams are often further simplified by representing the object as a small square, or circle, and considering it to be a **point particle / mass**.

Nature of science: Models

Point objects, particles and masses

A point particle is an idealized, simplified representation of any object, whatever its actual size and shape. As the name suggests, a point particle does not have any dimensions, or occupy any space. Typically, the 'point' will be located at the centre of mass of the object.

When the concept is used, we do not need to consider the complications and variations that are involved with extended objects. For example, if we consider an object as a point particle, all forces act through the same point and analysis can ignore any possible rotational effects caused by the forces acting on it.

Resultant forces and components

SYLLABUS CONTENT

- Free-body diagrams can be analysed to find the resultant force on a system.

Tool 3: Mathematics

Add and subtract vectors in the same plane

Vector addition is an important mathematical skill that occurs in several places in the IB Physics course, but the addition of forces is the most common application. Figure A2.36 shows an example of how to find the resultant of two force vectors.

A **resultant force** is represented in size and direction by the diagonal of the parallelogram (or rectangle) which has the two original force vectors as adjacent sides.

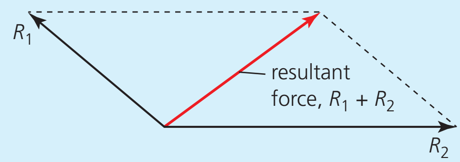


Figure A2.36 Adding two forces to determine a resultant

◆ **Resultant force** The vector sum of the forces acting on an object, sometimes called the unbalanced or net force.

◆ **Resultant** The single vector that has the same effect as the combination of two or more separate vectors.

◆ **Components (of a vector)** Any single vector can be considered as having the same effect as two parts (components) perpendicular to each other.

◆ **Inclined plane** Flat surface at an angle to the horizontal (but not perpendicular). A simple device that can be used to reduce the force needed to raise a load; sometimes called a ramp.

Tool 3: Mathematics

Resolve vectors

As we have seen, two forces can be combined to determine a single **resultant**. The 'opposite' process is very useful: a single force, F, can be considered as being equivalent to two smaller forces at right angles to each other. The two separate forces are called **components**.

This process is called resolving a force into two components. It can be used when the original force is not acting in a direction which is convenient for analysis. Because the two components are perpendicular to each other their effects can be considered separately. Figure A2.37 shows how a force can be resolved into two perpendicular components.

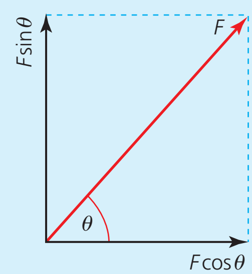


Figure A2.37 Force, F, resolved into two components

Any force, F, can be resolved into two independent components which are perpendicular to each other:

$F\sin\theta$ and $F\cos\theta$

WORKED EXAMPLE A2.6

a Draw a free-body diagram for an object which is stationary on a slope (**inclined plane**) which makes an angle of 35° with the horizontal.

b The object has a mass of 12.7 kg and just begins to slide down the slope if the angle is 35°. Using $g = 9.81\ \mathrm{N\,kg^{-1}}$, calculate the component of the weight for this angle:

  i down the slope

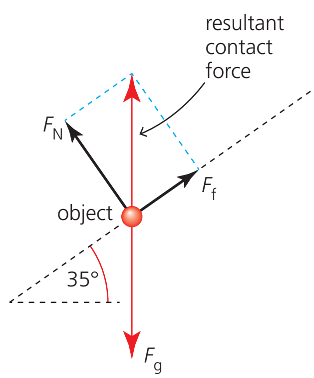
  ii perpendicular into the slope.

c State values of the frictional force and the normal force acting on the object.

d Determine the coefficient of static friction in this situation.

Answer

a See Figure A2.38, which represents the object as a point. The resultant contact force from the slope on the object must be equal and opposite to the weight, F_g. The contact force can be considered as the combination of two perpendicular components: F_N perpendicular to the slope, and F_f, the frictional force stopping the object from sliding down the slope.



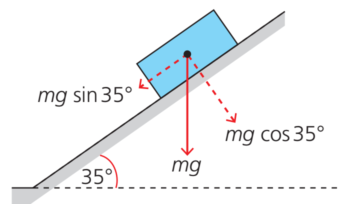
■ **Figure A2.38** Free-body diagram for an object on a slope

Sometimes it is preferred to represent the object as more than just a point. See Figure A2.39 for an example. However, this may cause confusion about exactly where the forces act.

b See Figure A2.39.

Component down slope $mg \sin 35° = 12.7 \times 9.81 \times 0.574 = 71.5\ \mathrm{N}$

Component into slope $= mg \cos 35° = 12.7 \times 9.81 \times 0.819 = 102\ \mathrm{N}$



■ **Figure A2.39** Components of weight

c Frictional force equals component down the slope, but in the opposite direction = 71.5 N up the slope.

Normal force equals component into the slope, but in the opposite direction = 102 N upwards.

d $F_f = \mu_s F_N$

$$\mu_s = \frac{F_f}{F_N} = \frac{71.5}{102} = 0.70 \text{ (which is equal to } \tan\theta)$$

25 Draw fully labelled free-body diagrams for:

  a a car moving horizontally with a constant velocity

  b an aircraft moving horizontally at constant velocity

  c a boat decelerating after the engine has been switched off

  d a car accelerating up a hill.

26 A wooden block of mass 2.7 kg rests on a slope which is inclined at 22° to the horizontal.

  a Make calculations which will enable you to draw a free-body diagram, similar to Figure A2.38, but giving numerical values for the forces.

  b If the angle is increased, the block will slide down the slope. Calculate the coefficient of friction.

  c State whether your answer to part **b** is for static or kinetic friction.

27 A pendulum on the end of a string has a mass of 158 g.

  a Draw a free-body diagram representing the situation when the string is making an angle of 20° to the vertical.

  b By adding components of weight to your diagram, show that the tension in the string is 1.5 N.

  c What effect does the other component ($mg \sin\theta$) have on the pendulum?

  d Discuss how the tension in the string changes while the pendulum is swinging from side to side.

28 Parallel forces of 1 N, 2 N and 3 N can act on an object at the same time. State the values of all the possible resultant forces.

29 Calculate the resultant force (size and direction) of 4.7 N and 5.9 N which are perpendicular to each other and acting away from a point mass.

30 Show that a mass on an inclined plane will just begin to slip down the slope when the tangent of the angle to the horizontal equals the coefficient of static friction.

31 Determine by scale drawing or calculation the size and direction of the resultant force acting on the hook shown in Figure A2.40.

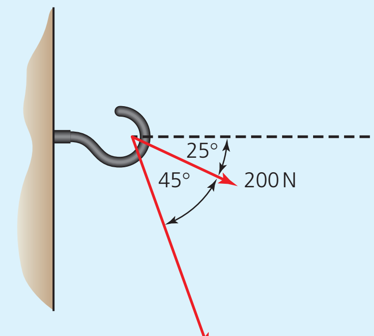


Figure A2.40

Newton's laws of motion

SYLLABUS CONTENT

- Newton's three laws of motion.

Newton's three laws of motion are among the most famous in classical physics. They describe the relationships between force and motion. Although they were first stated more than three hundred years ago, they are equally important today and are essential for an understanding of all motion (except when a speed of motion is close to the speed of light, as discussed in Topic A.5).

◆ **Newton's laws of motion** **First law**: an object will remain at rest, or continue to move in a straight line at a constant speed, unless a resultant force acts on it; **Second law**: acceleration is proportional to resultant force; **Third law**: whenever one body exerts a force on another body, the second body exerts exactly same force on the first body, but in the opposite direction.

◆ **Balanced forces** If an object is in mechanical equilibrium, we describe the forces acting on it as 'balanced'.

◆ **Equilibrium** An object is in equilibrium if it is unchanging under the action of two or more influences (e.g. forces). Different types of equilibrium include **translational**, **rotational** and **thermal**.

◆ **Translational** Changing position.

Newton's first law of motion

Newton's first law of motion states that an object will remain at rest or continue to move in a straight line at a constant speed, unless a resultant force acts on it.

In other words, a resultant force will produce an acceleration (change in velocity).

When the influences on any system are **balanced**, so that the system does not change, we describe it as being in **equilibrium**. (As another example, if an object stays at the same temperature, we say that it is in **thermal equilibrium**.)

When there is no resultant force on an object, we say that it is in **translational equilibrium**.

The term **translational** refers to movement from place to place. An object is in translational equilibrium if it remains at rest or continues to move with a constant velocity (in a straight line at a constant speed), as described by Newton's first law.

In passing, it should be noted that, if equal forces act in opposite directions, an object will be in translational equilibrium, but if the forces are not aligned (see Figure A2.41) then the object may start to rotate, so it will not be in **rotational equilibrium**. The subject of rotational dynamics is covered in Topic A.4.

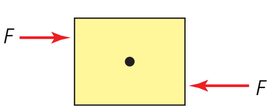


Figure A2.41 The object is in translational equilibrium, but not in rotational equilibrium

Nature of science: Observations

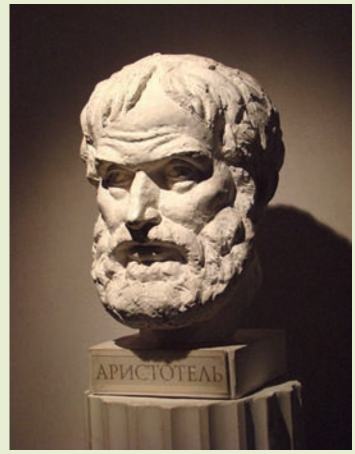
Natural philosophy

'Forces are needed to keep an object moving and, without those forces, movement will stop.' This accepted 'fact' is not true, but it is still widely believed. It was the basis of theories of motion from the time of Aristotle (about 2350 years ago) until the seventeenth century, when scientists began to understand that the forces of friction were responsible for stopping movement.

Aristotle is one of the most respected figures in the early development of human thought. He appreciated the need for wide-ranging explanations of natural phenomena but the 'science' of that time – called natural philosophy – did not involve careful observations, measurements, mathematics or experiments.

Aristotle believed that everything in the world was made of a combination of the four elements of earth, fire, air and water. The Earth was the centre of everything and each of the four Earthly elements had its natural place. When something was not in its natural place, then it would tend to return – in this way he explained why rain falls, and why flames and bubbles rise, for example.

Modern science (characterized by experimentation and the development of unbiased, testable theories) began in the seventeenth century. It includes the work of famous physicists mentioned in this topic: Hooke, Galileo and Newton.

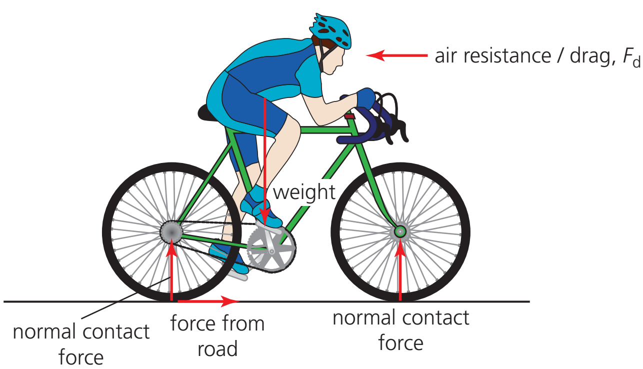


■ **Figure A2.42** A representation of Aristotle

◆ **Natural philosophy** The name used to describe the (philosophical) study of nature and the universe before modern science.

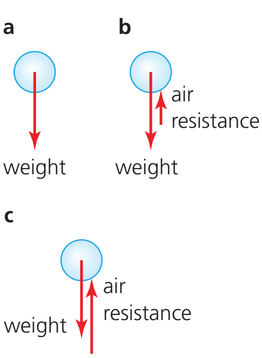
Examples of translational equilibrium

Because all objects on Earth have weight, it is not possible for an object to be in equilibrium because there are no forces acting on it. So, all translational equilibrium arises when two or more forces are balanced.



■ **Figure A2.43** A cyclist moving at constant speed in translational equilibrium

- A book on a horizontal table (Figure A2.8) is in equilibrium because its downwards weight is balanced by the upwards normal contact force.
- A stationary block on a slope (Figures A2.38 and A2.39) is in equilibrium because the component of its weight down the slope is balanced by surface friction up the slope and the component of its weight into the slope is balanced by the normal component of the contact force.
- A cyclist moving with constant speed (Figure A2.43) is in equilibrium because their weight is balanced by the sum of the two normal contact forces and the frictional force from the road is balanced by the drag.



■ **Figure A2.44** The resultant force on a falling object changes as it gains speed

Falling through the air at terminal speed

Figure A2.44 shows three positions of a falling ball. In part **a** the ball is just starting to move and there is no air resistance / drag. In part **b** the ball has accelerated and has some air resistance acting against its motion, but there is still a resultant force and an acceleration downwards. In part **c** the speed of the falling ball has increased to the point where the increasing air resistance has become equal and opposite to the weight. There is then no resultant force and the ball is in translational equilibrium, falling with a constant velocity called its terminal velocity or terminal speed. (Any buoyancy forces are considered to be negligible under these circumstances.) Terminal speed was introduced in Topic A.1.

Three forces in equilibrium

If two forces are acting on an object such that it is not in equilibrium, then to produce equilibrium a third force can be added that is equal in size to the resultant of the other two, but in the opposite direction. All three forces must act through the same point. For example, Figure A2.45 shows a free-body diagram of a ball on the end of a piece of string kept in equilibrium by a sideways pull that is equal in magnitude to the resultant of the weight and the tension in the string.

The translational equilibrium of three forces can be investigated in the laboratory simply by connecting three force meters together with string just above a horizontal surface, as shown in Figure A2.46. The three forces and the angles between them can be measured for a wide variety of different values, each of which maintains the system stationary.

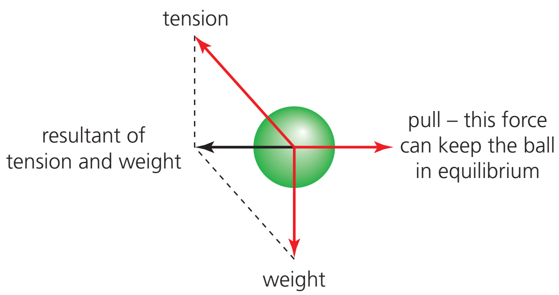


Figure A2.45 Three forces keeping a suspended ball in equilibrium

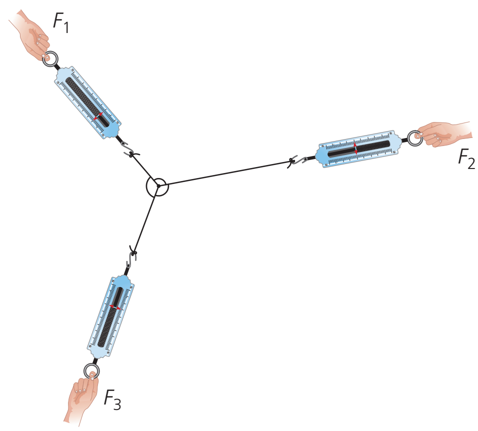


Figure A2.46 Investigating three forces in equilibrium

WORKED EXAMPLE A2.7

A ladder is leaning against a wall, as shown in Figure A2.47. Friction at point P is stopping the ladder from slipping, but there is no need for any friction acting at point Q.

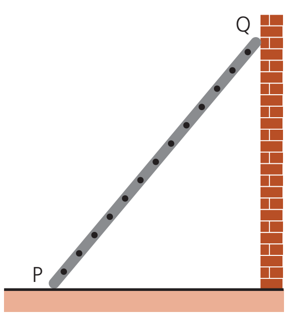


Figure A2.47 A ladder leaning against a wall

a Draw a free-body diagram of the ladder, including its weight and the normal force from the wall.

b The resultant force on the ladder from the ground must be directed at the point where the lines of action of the other two forces intersect. Add this line to your diagram.

c Complete the diagram by adding the two perpendicular components of the force from the ground on the ladder.

Answer

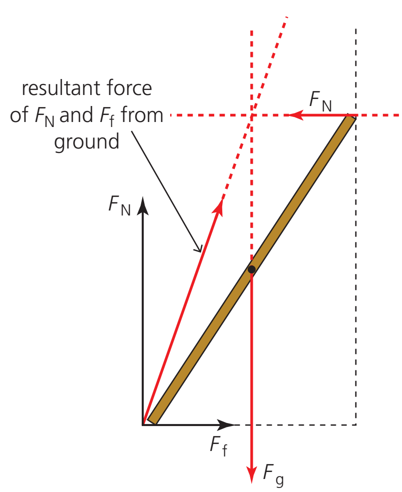


Figure A2.48

32 Under what circumstances will a moving car be in translational equilibrium?

33 If you are in an elevator (lift) without windows discuss whether it is possible to know if you are moving up, moving down or stationary.

34 Figure A2.49 shows a mountain climber who, at that moment, is stationary.

- **a** Draw a free-body diagram that shows that he is in equilibrium.
- **b** Outline the features of your diagram which show that the climber is in equilibrium.

35 Can the Moon be described as being in translational equilibrium? Explain your answer.

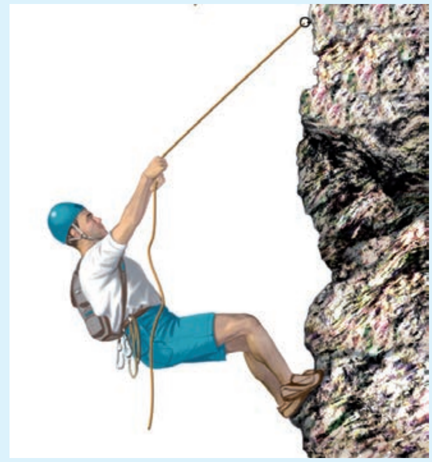


Figure A2.49

Newton's second law of motion

We have seen that Newton's first law establishes that there is a connection between resultant force and acceleration. Newton's second law takes this further and states the mathematical connection: when a resultant force acts on a (constant) mass, the acceleration is proportional to the resultant force: $a \propto F$.

Both force and acceleration are vector quantities and the acceleration is in the same direction as the force.

Investigating the effects of different forces and different masses on the accelerations that they produce is an important part of most physics courses, although reducing the effects of friction is essential for consistent results.

Inquiry 1: Exploring and designing

Exploring

Aristotle's understanding of motion was formed through making observations of the behaviour of objects in motion, but without any deep understanding of the concept of force he was unable to account for the effects of friction or air resistance. What methods are available for reducing friction in investigations into the effects of different forces and masses on an object's acceleration?

In groups, brainstorm how experiments can be designed to reduce or to cancel the effects of frictional forces. Decide on a selection of search terms or phrases that can be used by individual students for internet research. Use your research to formulate a research question and hypothesis.

Such experiments also show that when the same resultant force is applied to different masses, the acceleration produced is inversely proportional to the mass, m: $a \propto 1/m$

Combining these results, we see that acceleration, $a \propto \dfrac{F}{m}$.

Newton's second law can be written as: $F \propto ma$

If we define the SI unit of force, the newton, to be the force that accelerates 1 kg by 1 m s^{-2}, then we can write: force (N) = mass (kg) × acceleration (m s^{-2})

Newton's second law of motion: resultant force, $F = ma$

◆ **Proportional relationship** Two variables are (directly) proportional to each other if they always have the same ratio.

◆ **Uncertainty bars** Vertical and horizontal lines drawn through data points on a graph to represent the uncertainties in the two values.

This version of Newton's second law assumes that the mass of the object is constant. We will see later in this topic that there is an alternative version which allows for changing mass.

When discussing a gravitational force, weight, we have used the symbol F_g and the acceleration involved is g, the acceleration of free fall.

So, the equation $F = ma$ becomes the familiar:

$$F_g = mg$$

Tool 3: Mathematics

On a best-fit linear graph, construct lines of maximum and minimum gradients with relative accuracy (by eye) considering all uncertainty bars

Many basic physics experiments are aimed at investigating if there is a **proportional relationship** between two variables, and this is usually best checked by drawing a graph.

If two variables are (directly) proportional, then their graph will be a straight line passing through the origin

Figure A2.50 represents a proportional relationship. It is important to stress that a linear graph that does not pass through the origin does *not* represent proportionality (Figure A2.51).

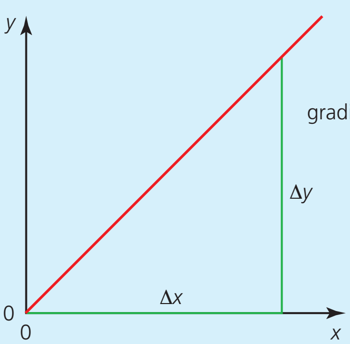


Figure A2.50 A proportional relationship

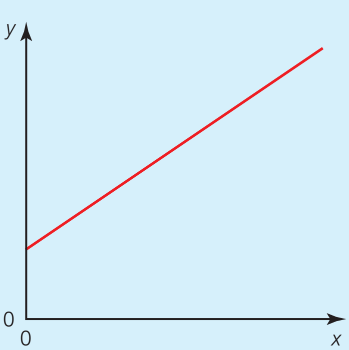


Figure A2.51 A linear relationship that is not proportional does not pass through the origin. **See also Tool 3: Mathematics (Understand direct and inverse proportionality) on page 129.**

Uncertainty in gradients and intercepts

It is often possible to draw a range of different straight lines, all of which pass through the **uncertainty bars** representing experimental data.

We usually assume that the line of best fit is midway between the lines of maximum possible gradient and minimum possible gradient. Figure A2.52 shows an example (for simplicity, only the first and last error bars are shown, but in practice all the error bars need to be considered when drawing the lines).

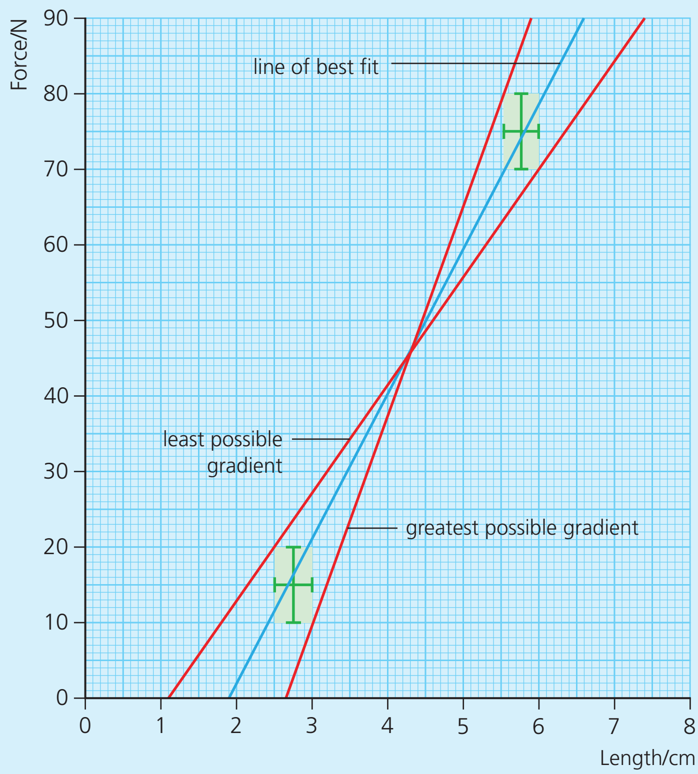


Figure A2.52 Finding maximum and minimum gradients for a spring-stretching experiment

Figure A2.52 shows how the length of a metal spring changed as the force applied was increased. We know that the measurements were not very precise because the uncertainty bars are large. The line of best fit has been drawn midway between the other two. This is a linear graph (a straight line) and it is known that the gradient of the graph represents the force constant (stiffness) of the spring and the horizontal intercept represents the original length of the spring. Taking measurements from the line of best fit, we can make the following calculations:

$$\text{force constant} = \text{gradient} = \frac{(90 - 0)}{(6.6 - 1.9)} = 19\,\text{N}\,\text{cm}^{-1}$$

$$\text{original length} = \text{horizontal intercept} = 1.9\,\text{cm}$$

To determine the uncertainty in the calculations of gradient and intercept, we need only consider the range of straight lines that could be drawn through the first and last error bars. The uncertainty will be the maximum difference between these extreme values obtained from graphs of maximum and minimum possible gradients and the value calculated from the line of best fit. In this example it can be shown that: force constant is between $14\,\text{N}\,\text{cm}^{-1}$ and $28\,\text{N}\,\text{cm}^{-1}$, original length is between 1.1 cm and 2.6 cm.

The final result can be quoted as:
force constant = $19 \pm 9\,\text{N}\,\text{cm}^{-1}$, original length = 1.9 ± 0.8 cm. Clearly, the large uncertainties in these results confirm that the experiment lacked precision.

Table A2.3 shows the results that a student obtained when investigating the effects of a resultant force on a constant mass. Plot a graph of these readings, including uncertainty bars. Then draw lines of maximum and minimum gradients through the error bars. Finally, use your graph to determine the mass that the student used in the experiment and the uncertainty in your answer.

Table A2.3

Resultant force, N, ± 0.5 N	Acceleration, $\text{m}\,\text{s}^{-2}$, ± $0.2\,\text{m}\,\text{s}^{-2}$
1.0	0.7
2.0	1.3
3.0	2.0
4.0	2.8
5.0	3.3
6.0	4.1

Common mistake

Many students believe that the force involved when an object hits the ground is its weight. In reality, the force will depend on the nature of the impact. The longer the duration of the impact, the smaller the force, as explained below.

Non-mathematical applications of Newton's second law

We can use Newton's second law to explain why, for example, a glass will break when dropped on the floor, but may survive being dropped onto a sofa. A collision with the floor will be for a much shorter duration, which means the deceleration will be greater and (using $F = ma$) the force will be greater, and probably more destructive. Similar arguments can be used to explain how forces can be reduced in road accidents.

WORKED EXAMPLE A2.8

A car of mass 1450 kg is accelerated from rest by an initial resultant force of 3800 N.

a Calculate the acceleration of the car.

b If the force and acceleration are constant, what will its speed be after 4.0 s?

c Determine how far it will have travelled in this time.

d After 4.0 s the resistive forces acting on the car are 1800 N. Show that the new force required to maintain the same acceleration is approximately 5.5 kN.

Answer

a $a = \frac{F}{m} = \frac{3800}{1450} = 2.62\,\text{m}\,\text{s}^{-2}$

b $v = u + at = 0 + (2.62 \times 4.0) = 10.5\,\text{m}\,\text{s}^{-1}$

c $s = \frac{(u + v)}{2} \times t = \frac{(0 + 10.5)}{2} \times 4.0 = 21.0\,\text{m}$

d $3800 + 1800 = 5600\,\text{N} \approx 5500\,\text{N} = 5.5\,\text{kN}$

WORKED EXAMPLE A2.9

Figure A2.53 shows two masses attached by a string which passes over a fixed pulley. Assuming that there is no friction in the system and that the string has negligible mass, determine:

a the acceleration of the system

b the tension in the string.

■ **Figure A2.53** Two masses attached by a string which passes over a fixed pulley

Answer

a The resultant force on the system of two masses = weight of the 500 g mass − weight of 100 g mass = $(0.500 - 0.100) \times 9.8 = 3.9\,\text{N}$

$$a = \frac{F}{m} = \frac{3.9}{(0.500 + 0.100)} = 6.5\,\text{m s}^{-2}$$

The 500 g mass will accelerate down while the 100 g mass accelerates up at the same rate.

b Consider the 100 g mass: the resultant force acting = tension, T, in the string upwards – weight acting downwards = $T - (0.100 \times 9.8) = T - 0.98$

$F = ma$

$(T - 0.98) = 0.100 \times 6.5$

$T = 1.6\,\text{N}$

Equally, we could consider the 500 g mass:
the resultant force acting =
weight acting downwards − tension, T, in the string upwards =
$(0.500 \times 9.8) - T = 4.9 - T$

$F = ma$

$(4.9 - T) = 0.500 \times 6.5$

$T = 1.6\,\text{N}$

36 A laboratory trolley accelerated at $80\,\text{cm s}^{-2}$ when a resultant force of 1.7 N was applied to it. What was its mass?

37 When a force of 6.4 N was applied to a mass of 2.1 kg on a horizontal surface, it accelerated by $1.9\,\text{m s}^{-2}$. Determine the average frictional force acting on the mass.

38 When a hollow rubber ball of mass 120 g was dropped on a concrete floor the velocity of impact was $8.0\,\text{m s}^{-1}$ and it reduced to zero in 0.44 s (before bouncing back).

a Calculate:

i the ball's average deceleration

ii the average force exerted on the ball.

b Repeat the calculations for a solid steel ball of the same size, 10 times the mass, but with the same impact velocity. Assume that its speed reduced to zero in 0.080 s.

c Outline why the steel ball can do more damage to a floor than the rubber ball.

39 A small aircraft of mass 520 kg needs to take off with a speed of $30\,\text{m s}^{-1}$ from a runway in a distance of 200 m.

a Show that the aircraft needs to have an average acceleration of $2.3\,\text{m s}^{-2}$.

b What average resultant force is needed during the take off?

40 Discuss why the forces on the long-jumper shown in Figure A2.54 are reduced because he is landing in sand.

■ **Figure A2.54** Impact in a sand-pit reduces force

41 a What resultant force is needed to accelerate a train of total mass 2.78×10^6 kg from rest to $20\,\mathrm{m\,s^{-1}}$ in 60 s?

b If the same train was on a sloping track which had an angle of 5.0° to the horizontal, what is the component of its weight parallel to the track?

c Suggest why railway designers try to avoid hills.

42 Calculate the average force needed to bring a 2160 kg car travelling at $21\,\mathrm{m\,s^{-1}}$ to rest in 68 m.

43 Use Newton's second law to explain why it will hurt you more if you are struck by a hard ball than by a soft ball of the same mass and speed.

44 A trolley containing sand is pulled across a frictionless horizontal surface with a small but constant resultant force. Describe and explain the motion of the trolley if sand can fall through a hole in the bottom of the trolley.

45 A man of mass 82.5 kg is standing still in an elevator that is accelerating upwards at $1.50\,\mathrm{m\,s^{-2}}$.

a What is the resultant force acting on the man?

b What is the normal contact force acting upwards on him from the floor?

46 Figure A2.55 shows two masses connected by a light string passing over a pulley.

a Assuming there is no friction, calculate the acceleration of the two blocks.

b What resultant force is needed to accelerate the 2.0 kg mass by this amount?

c Draw a fully labelled free-body diagram for the 2 kg mass, showing the size and direction of all forces.

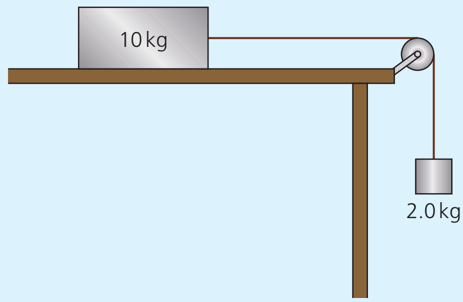


Figure A2.55 Two masses connected by a light string passing over a pulley

47 Outline how air bags (and/or seat belts) reduce the injuries to drivers and passengers in car accidents.

◆ **Inertia** Resistance to a change of motion. Depends on the mass of the object.

Newton's second law offers us a different way of understanding mass: larger masses accelerate less than smaller masses under the action of the same resultant force. So, mass can be considered as a measure of an object's resistance to acceleration. Physicists use the term **inertia** to describe an object's resistance to a change of motion.

Mass is a measure of inertia.

Newton's third law of motion

Whenever any two objects come in contact with each other, or otherwise interact, they exert forces on each other (Figure A2.56). Newton's third law compares these two forces.

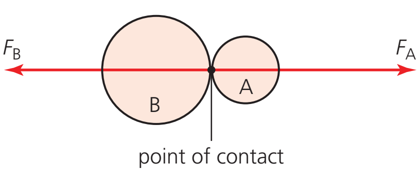


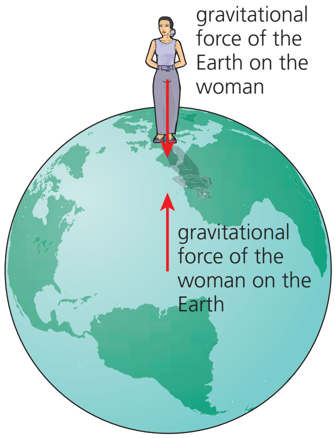
Figure A2.56 When two bodies interact, $F_A = -F_B$

Newton's third law of motion states that whenever one body exerts a force on another body, the second body exerts a force of the same magnitude on the first body, but in the opposite direction.

Essentially this law means that forces must always occur in equal pairs, although it is important to realize that the two forces must act on different bodies and in opposite directions, so that only one of each force pair can be seen in any free-body diagram. The two forces are always of the same type, for example gravity/gravity or friction/friction. Sometimes the law is quoted in the form used by Newton: 'to every action there is an equal and opposite reaction'. In everyday terms, it is simply not possible to push something that does not push back on you. Here are some examples:

- If you pull a rope, the rope pulls you.
- If the Earth pulls a person, the person pulls the Earth (Figure A2.57).
- If a fist hits a cheek, the cheek hits the fist (Figure A2.58).

- If you push on the ground, the ground pushes on you.
- If a boat pushes down on the water, the water pushes up on the boat.
- If the Sun attracts the Earth, the Earth attracts the Sun.
- If an aircraft pushes down on the air, the air pushes up on the aircraft.



■ **Figure A2.57** The force on the woman is equal and opposite to the force on the Earth

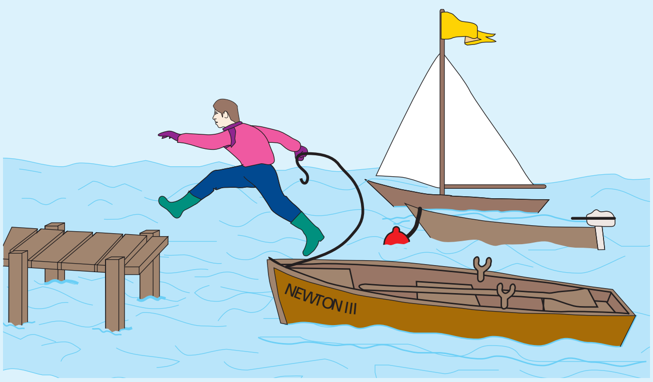
■ **Figure A2.58** The force on the glove is equal and opposite to the force on the cheek

48 A book has a weight of 2 N and is at rest on a table. The table exerts a normal contact force of 2 N upwards on the book.
Explain why these two forces are *not* an example of Newton's third law.

49 Seven examples of pairs of Newton's third law forces are provided above. Give three more examples. Try to use different types of force.

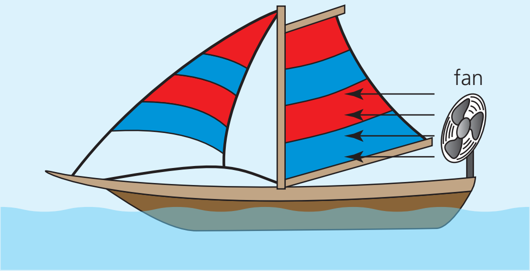
50 Consider Figure A2.58. Outline reasons why forces of equal magnitude, for example on a face and on a fist, can have very different effects.

51 Discuss why the person shown in Figure A2.59 could end up in the water.



■ **Figure A2.59**

52 Figure A2.60 shows a suggestion to make a sailing boat move when there is no wind.
Discuss how effective this method could be.



■ **Figure A2.60** Sailing boat

53 A large cage with a small bird sitting on a perch is placed on weighing scales.
Discuss what happens to the weight shown on the scales when the bird is flying around the cage (compared with when it is sitting still).

We have seen (Figure A2.56) that, when any two objects, A and B, interact, $F_A = -F_B$.

Using Newton's second law $F = ma$ we can write: $[ma]_A = -[ma]_B$.

Remembering that:

$$a = \frac{(v - u)}{t}$$

we can write:

$$\left[\frac{m(v - u)}{t}\right]_A = \left[\frac{m(v - u)}{t}\right]_B$$

The time of the interaction is the same for both, so: $[m(v - u)]_A = [m(v - u)]_B$.

Putting this into words: (mass × change of velocity) for A = −(mass × change of velocity) for B.

◆ **Momentum (linear), *p*** Mass times velocity, a vector quantity.

(mass × velocity) is an important concept in physics. It is called **momentum**. The momentum gained by object A = momentum lost by object B. Always. This assumes that there are no external forces. This is covered in more detail in the next section.

TOK

The natural sciences

- What kinds of explanations do natural scientists offer?

A clockwork universe?

Everything is made of particles and it has been suggested that, if we could know everything about the present state of all the particles in a system (their positions, energies, movements and so on), then maybe we could use the laws of classical physics to predict what will happen to them in the future. The Universe would then behave like a mechanical clock. If these ideas could be expanded to include everything, then the future of the Universe would already be decided and predetermined, and the many apparently unpredictable events of everyday life and human behaviour (like you reading these words at this moment) would just be the laws of physics in disguise.

However, we now know that the laws of physics (as imagined by humans) are not always so precisely defined, nor as fully understood as physicists of earlier years may have believed. The principles of quantum physics and relativity in particular contrast with the laws of classical physics. Furthermore, in a practical sense, it is totally inconceivable that we could ever know enough about the present state of everything in the Universe in order to use that data to make detailed future predictions.

Momentum

SYLLABUS CONTENT

- Linear momentum as given by: $p = mv$ remains constant unless the system is acted upon by a resultant external force.
- Newton's second law in the form $F = ma$ assumes mass is constant whereas $F = \frac{\Delta p}{\Delta t}$ allows for situations where mass is changing.

DB

linear momentum (SI unit: kg m s^{-1}) = mass × velocity, $\boldsymbol{p = mv}$

Momentum is a vector quantity and its direction is always important.

◆ **System** The object(s) being considered (and nothing else). An **isolated system** describes a system into which matter and energy cannot flow in, or out.

The explanation at the end of the last section shows that when two objects interact, with forces between them, the change of momentum for one object is equal and opposite to the change in momentum of the other: one object gains momentum while the other object loses an equal amount of momentum. This means that the total amount of momentum is unchanged, although this is only true if no resultant external force is acting on the objects. (We describe this as an **isolated system**.). This very important principle, which is a consequence of Newton's third law, can be stated as follows:

The law of conservation of momentum: the total momentum of any system is constant, provided that there is no resultant external force acting on it.

This law of physics is always true. There are no exceptions. It is very useful in helping to predict the results of interactions like collisions. See below.

Nature of science: Models

Systems and the environment

◆ **Surroundings** Everything apart from the system that is being considered; similar to the 'environment'.

We use the term *system* to describe and limit the collection of objects we are considering. You may think of this as 'drawing a line around' an object together with all of the surrounding objects with which there are significant interactions. This is especially important when using conservation laws. Objects outside of the 'system' are usually referred to as the environment, or the **surroundings**.

In practice, any situation can be complicated and we often have to decide which objects we can ignore (assume to be outside of the system) because their effect is minimal.

Take a collision between two cars as an example. Commonly, we calculate an outcome by considering the system to be just the two cars. This will give us a quick, reasonably accurate and useful prediction for what happens immediately after impact. Such a calculation has chosen not to include the air and the road in the system. If they were included, the situation would be much more complex, but the immediate consequences of any collision may be similar.

We know that, for uniform acceleration:

$$F = ma = \frac{m(v-u)}{t} = \frac{mv - mu}{t}$$

This demonstrates an alternative, more generalized, interpretation of Newton's second law ($F = ma$) in terms of a *change* of momentum, Δp ($= mv - mu$) that occurs in time Δt.

DB

force = rate of change of momentum: $\boldsymbol{F} = \dfrac{\boldsymbol{\Delta p}}{\boldsymbol{\Delta t}}$

This equation allows for the possibility of a changing mass, whereas the use of $F = ma$ assumes a constant mass. An application of this is given later in the section on explosions and propulsion.

Inquiry 2: Collecting and processing data

Interpreting results

Significant figures

An answer should not have more significant figures than the least precise of the data used in the calculation.

The more precise a measurement is, the greater the number of **significant figures** (digits) that can be used to represent it. For example, a mass stated to be 4.20 g (as distinct from 4.19 g or 4.21 g) suggests a greater precision than a mass stated to be 4.2 g.

Significant figures are all the digits used in data to carry meaning, whether they are before or after a decimal point, and this includes zeros.

But sometimes zeros are used without thought or meaning, and this can lead to confusion. For example, if you are told that it is 100 km to the nearest airport, you might be unsure whether it is approximately 100 km, or 'exactly' 100 km. This is a good example of why **scientific notation** is useful. Using 1.00×10^2 km makes it clear that there are 3 significant figures. 1×10^2 km represents much less precision. When making calculations, the result cannot be more precise than the data used to produce it. As a general and simplified rule, when answering questions or processing experimental data, the result should have the same number of significant figures as the data used. If the number of significant figures is not the same for all pieces of data, then the number of significant figures in the answer should be the same as the least precise of the data (which has the fewest significant figures).

You may assume that all the digits seen in the data shown in this book are significant. For example 100 km should be interpreted as three significant figures.

For example, if a mass of 583 g changed velocity by 15 m s^{-1} in two seconds, then the resultant force acting was:

$$F = \frac{\Delta p}{\Delta t} = \frac{(0.583 \times 15)}{2} = 4.3725\,\text{N}$$

(showing all figures seen on calculator display).

But, since the time was only given to one significant figure, then the answer should have the same: $F = 4$ N.

Maybe this can seem unsatisfactory, but remember that when the time is quoted as 2 s, it simply means that it was more than 1.5 s and less than 2.5 s. (A time of 1.5 s would give an answer of 5.8 N, a time of 2.5 s would give an answer of 3.5 N.) If the time had been 2.0 s, then the quoted answer for the force should be 4.4 N. If the time had been 2.00 s, then the quoted answer for the force should still be 4.4 N, because the velocity was only given to 2 significant figures.

◆ **Scientific notation** Every number is expressed in the following form: $a \times 10^b$, where a is a decimal number larger than 1 and less than 10 and b is an exponent (integer).

◆ **Significant figures (digits)** All the digits used in data to carry meaning, whether they are before or after a decimal point.

◆ **Collision** Two (or more) objects coming together and exerting forces on each other for a relatively short time.

◆ **Explosion** Term used in physics to describe when two or more masses, which were initially at rest, are propelled apart from each other.

Conservation of momentum in collisions and explosions

SYLLABUS CONTENT

- Elastic and inelastic collisions of two bodies.
- Energy considerations in elastic collisions, inelastic collisions, and explosions.

We will use the word '**collision**' to describe any event in which two, or more, objects move towards each other and exert forces on each other. In physics this term is not limited to its typical everyday use to describe accidental events, often involving large forces. The term '**explosion**' will be used to describe any event in which internal forces within a stationary system result in separate parts of the system moving apart. Everyday usage of the term is much more dramatic.

■ **Figure A2.61** Newton's cradle is a demonstration of collisions

Collisions

We can *always* use the law of conservation of momentum to help predict what will happen immediately after a collision, but we must also have some other information. For example, the results of two tennis balls colliding will be very different from two cushions colliding and neither can be predicted by using *only* the law of conservation of momentum. Consider the following worked example.

WORKED EXAMPLE A2.10

An object A of mass of 2.1 kg was moving to the left with a velocity of $0.76\,\mathrm{m\,s^{-1}}$. At the same time, object B of mass 1.2 kg was moving in the opposite direction with a velocity of $3.3\,\mathrm{m\,s^{-1}}$. Discuss what happens after the collision.

Momentum of A = $2.1 \times 0.76 = 1.60\,\mathrm{kg\,m\,s^{-1}}$. But this is a magnitude only. We have not considered direction:

If we choose that velocity to the left is positive, momentum of A = $2.1 \times (+0.76) = +1.60\,\mathrm{kg\,m\,s^{-1}}$ (to the left).

(Alternatively, if we prefer to say velocity to the right is positive, then we get: momentum of A = $2.1 \times (-0.76) = -1.60\,\mathrm{kg\,m\,s^{-1}}$ (to the left)

Using velocity and momentum to the left to be positive:

momentum of B = $1.2 \times (-3.3) = -3.96\,\mathrm{kg\,m\,s^{-1}}$ (to the right)

The combined momentum of A and B before the collision = $1.60 + (-3.96) = -2.36\,\mathrm{kg\,m\,s^{-1}}$ (to the right)

The law of conservation of momentum tells us that after the collision, this momentum will be the same (assuming there is no resultant external force). But we need further information to determine exactly what happened. That information may come in the form of identifying the type of collision (see below), or telling us what happened to one of the objects, so that we can calculate what happened to the other, as follows:

If, after the collision, object A moved to the left with a velocity of $0.87\,\mathrm{m\,s^{-1}}$, what happened to object B?

After the collision, momentum of A + momentum of B = $-2.36\,\mathrm{kg\,m\,s^{-1}}$

$[2.1 \times (-0.87)] + (1.2 \times v_B) = -2.36\,\mathrm{kg\,m\,s^{-1}}$

$v_B = -0.44\,\mathrm{m\,s^{-1}}$ (to the left)

All of this has been explained in detail to help understanding. More directly it can be represented by: momentum before collision = momentum after collision

$[2.1 \times (+0.76)] + [1.2 \times (-3.3)] = [2.1 \times (-0.87)] + (1.2 \times v_B)$

$v_B = -0.44\,\mathrm{m\,s^{-1}}$ (to the left)

Top tip!

In Topic A.3, we will introduce the law of conservation of energy and the concept of kinetic energy, which is the energy of moving masses, calculated by $E_k = \frac{1}{2}mv^2$. That knowledge is needed in order to understand the rest of this section on collisions.

You may prefer to delay the rest of this topic on collisions and explosions until Topic A.3 has been covered in detail.

Kinetic energy in collisions

We need to consider the transfer of energy in a collision. Any moving object has kinetic energy and during a collision some, or all, of this energy will be transferred between the colliding objects. Typically, some energy, perhaps most of the energy, will be transferred to the surroundings as thermal energy and maybe some sound. We can identify the extreme cases:

◆ **Collisions** In an **elastic collision** the total kinetic energy before and after the collision is the same. In any **inelastic collision** the total kinetic energy is reduced after the collision. If the objects stick together it is described as a **totally inelastic collision**.

◆ **Macroscopic** Can be observed without the need for a microscope.

◆ **Microscopic** Describes anything that is too small to be seen with the unaided eye.

A collision in which the *total* kinetic energy before and after the collision is the same is called an **elastic collision.**

All other collisions can be described as **inelastic collisions**, meaning that kinetic energy has not been conserved. In everyday, **macroscopic** events, elastic collisions are a theoretical ideal and they do not happen perfectly. However, elastic collisions are common for **microscopic** particle collisions.

A collision after which the colliding objects stick together is called a **totally inelastic collision**.

In a totally inelastic collision, the maximum possible amount of kinetic energy is transferred from the moving objects to the environment.

Figure A2.62 An inelastic collision

Nature of science: Models

Macroscopic and microscopic

In general, physicists use the terms:

- **macroscopic** to describe events that can be observed with the unaided eye
- **microscopic** to describe events on the molecular, atomic, or subatomic scale.

It was not until scientists began to realize that matter consisted of atoms and molecules (that could not be seen), that many observations of the world around us could be explained.

Perhaps the best example of an (almost) elastic macroscopic collision is that between steel spheres. If a stationary sphere is struck by an identical moving sphere, the moving sphere stops and the other sphere continues with the velocity of the first. ‘Newton’s cradle’, as seen in Figure A2.61 is a famous demonstration of this.

It is easy to find examples where *all* kinetic energy appears to have been lost, for example, when a student jumps down to the floor. The student has had a totally inelastic collision with the Earth, but the change to the motion of the Earth is insignificant and unobservable.

WORKED EXAMPLE A2.11

A 2.1 kg trolley moving at 0.82 m s^{-1} collides with a 1.7 kg trolley moving at 0.98 m s^{-1} in the opposite direction. After the collision, the 1.7 kg trolley reverses direction and moves at 0.43 m s^{-1}.

a Discuss what happened to the other trolley.

b Without making any calculations, comment on the difference in total kinetic energy before and after the collision.

c If the collision had been totally inelastic, what would have happened after the collision?

Answer

a Total momentum before = total momentum after

$(2.1 \times 0.82) + (1.7 \times -0.98) = (2.1 \times v) + (1.7 \times 0.43)$

Velocities in the original direction have been given a + sign and velocities in the opposite direction are given a − sign (or it could be the other way around).

$0.056 = 2.1v + 0.731$

$v = -0.32$ m s^{-1}. The – sign shows us that the trolley reverses its direction of motion.

b Both velocities have been reduced, so the total kinetic energy is significantly less.

c The trolleys will stick together if the collision is totally inelastic.
Total momentum before = total momentum after

$(2.1 \times 0.82) + (1.7 \times -0.98) = (2.1 + 1.7) \times v$

$0.056 = 3.8v$

$v = +0.015$ m s^{-1}. The + sign shows us that they move in the original direction of the 2.1 kg trolley.

The combined trolleys are moving slowly, so there has been a considerable loss of kinetic energy.

LINKING QUESTION

- In which way is conservation of momentum relevant to the workings of a nuclear power station?

This question links to understandings in Topic E.4

54 An object of mass 4.1 kg travelling to the right with a velocity of 1.9 m s^{-1} has a totally inelastic collision with a stationary object of mass 5.6 kg. Determine how they move immediately after the collision.

55 A bus of mass 4900 kg travelling at 22 m s^{-1} collides with the back of a 1300 kg car travelling at 16 m s^{-1}. If the car is pushed forward with a velocity of 20 m s^{-1}, calculate the velocity of the bus immediately after the collision.

56 In an experiment to find the speed of a 2.40 g bullet, it was fired into a 650 g block of wood at rest on a friction-free surface. If the block (and bullet) moved off with an initial speed of 96.0 cm s^{-1}. Calculate the speed of the bullet.

57 A ball thrown vertically upwards decelerates and its momentum decreases, although the law of conservation of momentum states that total momentum cannot change. Explain this observation.

58 Figure A2.63 shows two trolleys on a friction-free surface joined together by a thin rubber cord under tension. When the trolleys are released, they accelerate towards each other and the cord quickly becomes loose.

a Show that the two trolleys collide at the 20 cm mark.

b Predict what happens after they collide if they stick together.

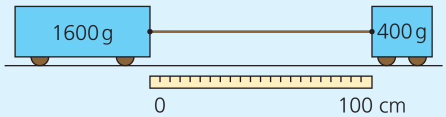


Figure A2.63 Two trolleys on a friction-free surface joined together by a thin rubber cord

59 Two toy cars travel in straight lines towards each other on a friction-free track. Car A has a mass of 432 g and a speed of 83.2 cm s^{-1}. Car B has a mass of 287 g and speed of 68.2 cm s^{-1}. If they stick together after impact, predict their combined velocity.

60 A steel ball of mass 1.2 kg moving at 2.7 m s^{-1} collides head-on with another steel ball of mass 0.54 kg moving in the opposite direction at 3.9 m s^{-1}. The balls bounce off each other, each returning back in the direction it came from on a horizontal surface.

a If the smaller ball had a speed after the collision of 6.0 m s^{-1}, use the law of conservation of momentum to predict the speed of the larger ball.

b In fact, the situation described in part **a** is not possible. Discuss possible explanations of why not.

ATL A2C: Thinking skills

Reflecting on the credibility of results

A student carried out an experiment into the momentum of colliding trolleys on a horizontal runway. A trolley of mass 2.0 kg and speed 80 cm s^{-1} collided with a trolley of mass 1.0 kg and speed 220 cm s^{-1} travelling in the opposite direction. After the collision, both the trolleys reversed their directions and the student measured the speeds of both trolleys to be 60 cm s^{-1}.

Explain why the student must have made a mistake.

Explosions

Figure A2.64 shows a possible laboratory investigation into a one-dimensional 'explosion'. A blow from the hammer releases springs which push the previously stationary trolleys apart. If the trolleys are identical, they will move apart with equal speeds. If the mass on one side is doubled, as shown in the third drawing, the speeds will be in the ratio 2:1, the more massive trolley will move more slowly

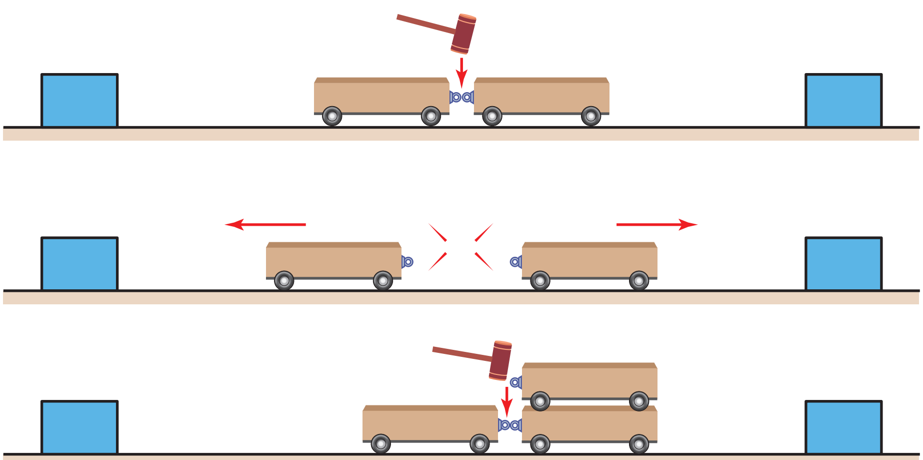


Figure A2.64 A simple 'explosion'

Figure A2.65 Firing a cannon

◆ **Recoil** When a bullet is fired from a gun (or similar), the gun must gain equal momentum in the opposite direction.

Firing a gun, or a cannon, is a more dramatic example. See Figure A2.65. Since there is zero momentum to begin with, the momentum of the bullet / cannon ball must be equal and opposite to the momentum of the gun itself (or cannon), so that the total momentum after firing is also zero. The word **recoil** is used to describe this 'backwards' motion.

WORKED EXAMPLE A2.12

A rifle of mass 1.54 kg fires a bullet of mass 22 g at a speed of $250\,\mathrm{m\,s^{-1}}$.
Calculate the recoil speed of the rifle.

Answer

Total momentum before = total momentum after

$0 = (1.54 \times v) + (0.022 \times 250)$

$v = -3.6\,\mathrm{m\,s^{-1}}$.

The '−' sign shows us that the gun moves in the opposite direction to the bullet's velocity.

61 In an experiment similar to that shown in the first drawing of Figure A2.64, after being released one trolley with a mass of 980 g recoiled with a velocity of $0.27\,\mathrm{m\,s^{-1}}$. What was the speed of the other trolley, which had a mass of 645 g?

62 An isolated and 'stationary' astronaut of mass 65 kg accidentally pushes a 2.3 kg hammer away from her body with a speed of $80\,\mathrm{cm\,s^{-1}}$.

- **a** Outline a reason why the word 'stationary' has been put in quotation marks.
- **b** Predict what happened to the astronaut.
- **c** Suggest how she can stop moving.

63 Cannons have been used extensively in wars for hundreds of years. A large cannon from 200 years ago could fire a 25 kg cannon ball with a speed of over $150\,\mathrm{km\,h^{-1}}$ ($42\,\mathrm{m\,s^{-1}}$). The recoil momentum of these cannons could be dangerous, although the recoil speed was limited by the very large mass of the cannon. If the speed of recoil was $0.30\,\mathrm{m\,s^{-1}}$, calculate the mass of the cannon.

64 Figure A2.66 shows a heavy ball being thrown from one end of a canoe to the other. Describe what will happen to the canoe (and the passenger) when:

- **a** the ball is being thrown
- **b** the ball is in the air
- **c** the ball lands back in the canoe. Assume that the water does not resist any possible movement of the canoe.

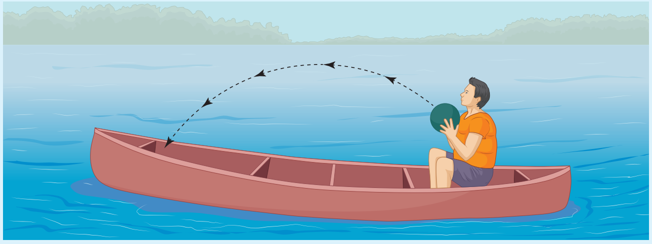


Figure A2.66 A heavy ball being thrown from one end of a canoe to the other

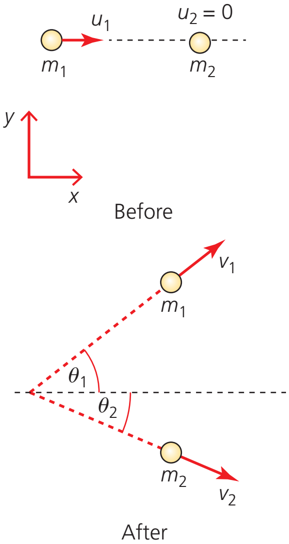


Figure A2.67 Two objects colliding in two dimensions

Collisions and explosions in two dimensions

So far, we have only considered interactions in one direction. This section extends the study to two dimensions. *It is aimed at Higher Level students only.* We will consider that the interacting objects behave as point particles.

Figure A2.67 shows two objects, m_1 and m_2, before and after a collision. In this example m_2 was stationary before the collision.

For collisions or explosions in two dimensions the law of conservation of momentum can be applied in two perpendicular directions.

For the example shown in Figure A2.67 we need to know the components of v_1 and v_2 in the x and y directions:

$$v_{1x} = v_1 \cos\theta_1 \qquad v_{1y} = v_1 \sin\theta_1$$

$$v_{2x} = v_2 \cos\theta_2 \qquad v_{2y} = v_2 \sin\theta_2$$

HL ONLY

WORKED EXAMPLE A2.13

Considering Figure A2.67, let $m_1 = 0.50\,\text{kg}$, $m_2 = 0.30\,\text{kg}$ and $u_1 = 4.0\,\text{m s}^{-1}$.

If the angles were $\theta_1 = 36.9°$ and $\theta_2 = 26.6°$, determine the value of v_2 if v_1 was $2.0\,\text{m s}^{-1}$.

Answer

Applying conservation of momentum in the y-direction:

$0 = m_1v_1 \sin\theta_1 + m_2v_2 \sin\theta_2$

$0 = 0.600 + 0.134v_2$

$v_2 = -4.5\,\text{m s}^{-1}$, the negative sign shows that it is moving in the negative y-direction.

Alternatively, and less simply, we can apply conservation of momentum in the x-direction:

$m_1u_1 = m_1v_1 \cos\theta_1 + m_2v_2 \cos\theta_2$

$(0.50 \times 4.0) = (0.50 \times 2.0 \times \cos 36.9°) + (0.30 \times v_2 \times \cos 26.6°)$

$2.0 = 0.800 + (0.268 \times v_2)$

$v_2 = 4.5\,\text{m s}^{-1}$, as before.

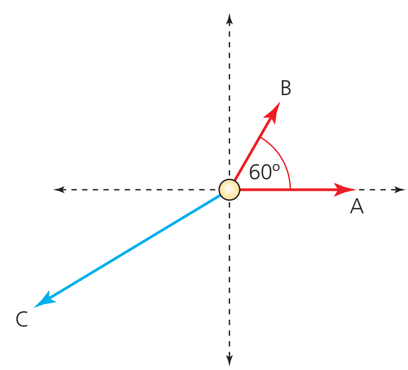
WORKED EXAMPLE A2.14

A stationary small mass of mass 5.0 g explodes into three particles.

One particle, A, of mass 1.5 g moves with a velocity of $23\,\text{m s}^{-1}$ in a direction that we will call the x-direction.

The second particle, B, of mass 2.0 g moves with a velocity of $18\,\text{m s}^{-1}$ in a direction of 60° to the first.

Determine what happened to the third particle, C.



■ **Figure A2.68** A stationary small mass explodes into three particles

Answer

Before the explosion there is zero momentum. After the explosion, in any chosen direction momentum must be conserved: $p_A + p_B + p_C = 0$.

Resolving the velocity of the B particle into two components:

In x-direction, $v_x = 18 \cos 60° = 9.0\,\text{m s}^{-1}$

In y direction, $v_y = 18 \sin 60° = 15.6\,\text{m s}^{-1}$

Consider momentum in the x-direction: $p_A + p_B + p_C = 0$.

$(1.5 \times 23) + (2.0 \times 9.0) = 52.5\,\text{g m s}^{-1} = -p_C$

Consider momentum in the y-direction: $0 + (2.0 \times 15.6) = 31.2\,\text{g m s}^{-1} = -p_C$

Dividing momentum by mass ($m = 5.0 - 1.5 - 2.0 = 1.5$) gives us the two components of the velocity of C: $-35\,\text{m s}^{-1}$ and $-21\,\text{m s}^{-1}$.

These two components can be added (using a scale drawing or trigonometry) to determine the actual velocity of C: $41\,\text{m s}^{-1}$ at an angle of 31° to the $-x$-direction.

65 Masses of 200 g and 500 g are travelling directly towards each other with speeds of $1.2\,\mathrm{m\,s^{-1}}$ and $0.30\,\mathrm{m\,s^{-1}}$, respectively. After they collide, the speed of the 200 g mass reduces to $0.10\,\mathrm{m\,s^{-1}}$ as it continues in a direction at 30° to its original motion.
Determine what happened to the other mass.

66 A mass of mass 1.0 kg moving at $2.0\,\mathrm{m\,s^{-1}}$ explodes into three parts. One part, which has a mass of 250 g, has a velocity of $8.5\,\mathrm{m\,s^{-1}}$ in the original direction of motion. The second part has a mass of 450 g and moves with a velocity of $5.6\,\mathrm{m\,s^{-1}}$ at an angle of 90° to the first part. Show that the third part has a speed of $8.4\,\mathrm{m\,s^{-1}}$.

HL ONLY

◆ **Propel** Provide a force for an intended motion.

◆ **Jet engine** An engine that achieves propulsion by emitting a fast-moving stream of gas or liquid in the opposite direction from the intended motion.

◆ **Rocket engine** Similar to a jet engine, but there is no air intake. Instead, an oxidant is carried on the vehicle, together with the fuel.

Propulsion

If the ball shown in Figure A2.66 had been thrown over the end of the canoe, the canoe would keep moving to the right (until resistive forces stopped it). An unusual example perhaps, but this shows us a very useful concept: to start, or maintain motion (**propel**), we can create momentum in the opposite direction. This can be restated using Newton's third law: if we want a force to move us to the right (for example), we exert a force on the surroundings to the left. The person in the boat pushes the ball to the left and the ball pushes the person (and the boat) to the right. Using friction for walking and car movement has already been discussed.

A boat can be pushed forward by pushing water backwards, using an oar, or a propeller. See Figure A2.69. The momentum of the boat forwards is equal and opposite to the momentum of the water backwards.

A propeller can also be used for a small airplane, but typically the propeller needs to be much larger and rotate faster, because the density of air is much less than water. Larger aircraft use the same conservation of momentum principle, but in a different way:

There are many designs of **jet engines**, but the basic principle is that they take in the surrounding air and use it to burn vaporized fuel. The resulting hot exhaust gases are ejected at the back of the engine with considerable momentum (much greater than the momentum of the air input), the difference is equal and opposite to the forward momentum given to the aircraft.

Rocket engines use the same principle, but they travel where there is little or no air, so they use oxygen that has been stored on the vehicle.

■ **Figure A2.69** Boat propeller

■ **Figure A2.70** A Chinese rocket launching a spacecraft to Mars

WORKED EXAMPLE A2.15

A rocket is ejecting exhaust gases at a rate of $1.5 \times 10^4\,kg\,s^{-1}$. If the speed of the exhaust gases (relative to the rocket) is $2.3 \times 10^3\,m\,s^{-1}$, what is the forward force acting on the rocket?

Answer

$$F = \frac{\Delta p}{\Delta t} \Rightarrow \left(\frac{\Delta m}{\Delta t}\right) \times v = (1.5 \times 10^4) \times (2.3 \times 10^3) = 3.5 \times 10^7\,N$$

67 A rocket's mass at lift-off was $2.7 \times 10^6\,kg$. If gases were ejected at a rate of $1.9 \times 10^4\,kg\,s^{-1}$ with a speed of $2.0 \times 10^3\,m\,s^{-1}$:

- **a** determine the initial acceleration of the rocket
- **b** explain why the acceleration will increase as the rocket rises, while the engines provide the same force.

Forces acting for short times: impulses

SYLLABUS CONTENT

- ▶ A resultant force applied to a system constitutes an impulse, J, as given by: $J = F\Delta t$, where F is the average resultant force and Δt is the time of contact.
- ▶ The applied external impulse equals the change in momentum of the system.

◆ **Impulse** The product of force and the time for which the force acts.

Many forces only act for a short time, Δt. Clearly the longer the time for which a force acts, the greater its possible effect, so the concept of **impulse**, J, becomes useful:

impulse, $\boldsymbol{J = F\Delta t}$ SI unit: N s

If a force varies during an interaction, we can use an average value to determine the impulse.

We have seen that $F = \frac{\Delta p}{\Delta t}$ which can be rearranged to give $F\Delta t = \Delta p$, showing us that

impulse, $J = \Delta p$ (change of momentum)

An impulse on an isolated object results in a change of momentum, which is numerically equal to the impulse. The units N s and $kg\,m\,s^{-1}$ are equivalent to each other.

WORKED EXAMPLE A2.16

A constant force of 12.0 N acts on a stationary mass of 0.620 kg for 0.580 s.

- **a** Calculate the impulse applied to the mass.
- **b** Calculate the change of momentum of the mass.
- **c** Calculate the final velocity of the mass.

Answer

a $J = F\Delta t = 12.0 \times 0.580 = 6.96\,Ns$

b $6.96\,kg\,m\,s^{-1}$ (or Ns)

c $\Delta p = m\Delta v$

$6.96 = 0.620 \times \Delta v$

$\Delta v = 11.2\,m\,s^{-1}$

The final velocity could also be determined by use of $F = ma$ and $v = u + at$.

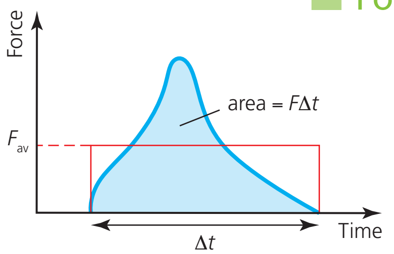


Figure A2.71 Graph showing how a force varies with time

Force–time graphs

In many simple impulse calculations, we may assume that the forces involved are constant, or that the average force is half of the maximum force. For more accurate work this is not good enough, and we need to know in detail how a force varies during an interaction. Such details are commonly represented by force–time graphs. The curved line in Figure A2.71 shows an example of a force varying over a time Δt.

The area under any force–time graph for an interaction equals force × time, which equals the impulse (change of momentum).

This is true whatever the shape of the graph. The area under the curve in Figure A2.71 can be estimated by drawing a rectangle of the same area (as judged by eye), as shown in red. F_{av} is then the average force during the interaction.

Inquiry 2: Collecting and processing data

Applying technology to collect data

Force sensors that can measure the magnitude of forces over short intervals of time can be used with data loggers to gather data and draw force–time graphs for a variety of interactions, both inside and outside a laboratory. Stop-motion replay of video recordings of collisions can also be very interesting and instructive.

Force–time graphs can be helpful when analysing any interaction, but especially impacts involved in road accidents and sports.

WORKED EXAMPLE A2.17

Figure A2.72 shows how the force on a 57 g tennis ball moving at 24 m s^{-1} to the right varied when it was struck by a racket moving in the opposite direction.

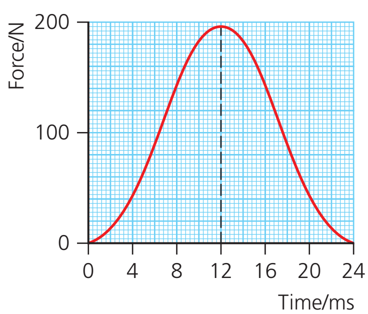


Figure A2.72 Force–time graph for striking a tennis ball

- **a** Estimate the impulse given to the ball.
- **b** Calculate the velocity of the ball after being struck by the racket.
- **c** The ball is struck with the same force with different rackets. Explain why a racket with looser strings could return the ball with greater speed.
- **d** Suggest a disadvantage of playing tennis with a racket with looser strings.

Answer

a Impulse = area under graph ≈ 200 × (12 × 10^{-3}) = 2.4 Ns to the left

b $m\Delta v = 2.4$

$$\Delta v = \frac{2.4}{0.057} = 42\ \text{m s}^{-1} \text{ to the left}$$

$v_{final} - v_{initial} = -42$ (velocity to the left chosen to be negative)

$v_{final} - (+24) = -42$

$v_{final} = -18\ \text{m s}^{-1}$ (to the left)

c The time of contact with the ball, Δt, will be longer with looser strings, so that the same force will produce a greater impulse (change of momentum).

d There is less control over the direction of the ball.

68 A ball of mass 260 g falls vertically downwards and hits the ground with a speed of $7.3\,\mathrm{m\,s^{-1}}$.

a What was its greatest momentum?

b If it rebounded with a speed of $5.5\,\mathrm{m\,s^{-1}}$, calculate the change of momentum.

c Determine the impulse on the ground.

d If the duration of the impact was 0.38 s, calculate the average force on the ball during the collision.

e Estimate the maximum force on the ball.

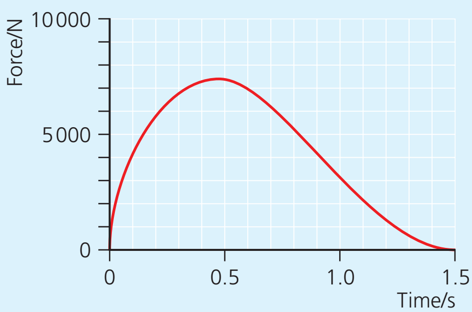
69 A baseball bat hits a ball with an average force of 970 N that acts for 0.0088 s.

a What impulse is given to the ball?

b What is the change of momentum of the ball?

c The ball was hit back in the same direction that it came from. If its speed before being hit was $32\,\mathrm{m\,s^{-1}}$, calculate its speed afterwards. (Mass of baseball is 145 g.)

70 Figure A2.73 shows how the force between two colliding cars changed with time. Both cars were driving in the same direction and after the collision they did not stick together.



■ **Figure A2.73** How the force between two colliding cars changed with time

a Show that the impulse was approximately 6500 Ns.

b Before the collision the faster car (mass 1200 kg) was travelling at $18\,\mathrm{m\,s^{-1}}$. Estimate its speed immediately after the collision.

71 Consider Figure A2.74.

a Discuss how the movement of the karate expert can maximize the force exerted on the boards.

b What features of the boards will help to make this an impressive demonstration?

■ **Figure A2.74** Karate expert

72 A soft ball, A, of mass 500 g is moving to the right with a speed of $3.0\,\mathrm{m\,s^{-1}}$ when it collides with another soft ball, B, moving to the left. The time of impact is 0.34 s, after which ball A rebounds with a speed of $2.0\,\mathrm{m\,s^{-1}}$.

a What was the change of velocity of ball A?

b What was the change of momentum of ball A?

c Calculate the average force exerted on ball A.

d Sketch a force–time graph for the impact.

e Add to your sketch a possible force–time graph for the collision of hard balls of similar masses and velocities.

f Suggest how a force–time graph for ball B would be different (or the same) as for ball A.

Circular motion and centripetal forces

SYLLABUS CONTENT

- Circular motion is caused by a centripetal force acting perpendicularly to the velocity.
- A centripetal force causes the body to change direction even if the magnitude of its velocity may remain constant.

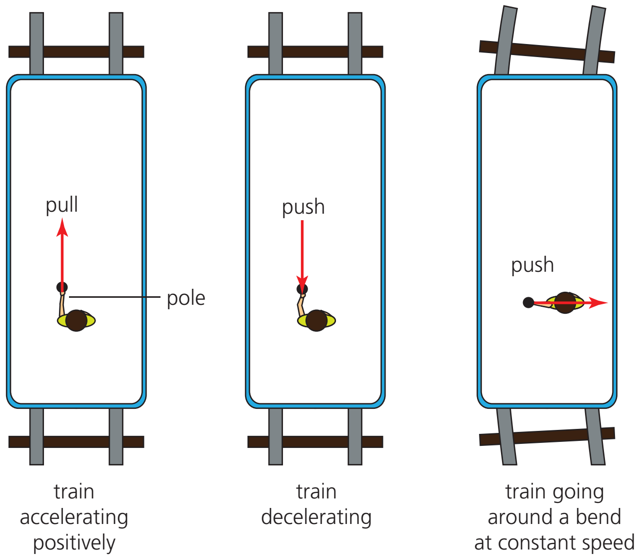


Figure A2.75 Forces which make a passenger accelerate in a train

An object moving along a circular path with a constant speed has a continuously changing velocity because its *direction* of motion is changing all the time. From Newton's first law, we know that any object that is not moving in a straight line must be accelerating and, therefore, it must have a resultant force acting on it, even if it is moving with a constant speed.

Perfectly uniform motion in complete circles may not be a common everyday observation, but the theory for circular motion can also be used with objects, such as people or vehicles, moving along arcs of circles and around curves and bends. Circular motion theory is also very useful when discussing the orbits of planets, moons and satellites. It is also needed to explain the motion of subatomic particles in magnetic fields, as discussed in Topic D.3.

Imagine yourself to be standing in a train on a slippery floor, holding on to a post (Figure A2.75). While you and the train are travelling in a straight line with a constant speed (constant velocity) there is no resultant force acting on you and you do not need to hold on to the post, but as soon as the train changes its motion (accelerates in some way) there needs to be a resultant force on you to keep you in the same place in the train. If there is little or no friction with the floor, the post is the only thing that can exert a force on you to change your motion. The directions of these forces (from the post) are shown in the diagram for different types of acceleration. If the post pushes or pulls on you, then by Newton's third law you must be pushing or pulling on the post, and that is the force you would be most aware of.

◆ **Centripetal force** The name given to any force which results in motion along a circular path.

In particular, note that the direction of the force needed to produce a curved, circular path is *perpendicular* to the motion.

The term **centripetal force** is used to describe any type of force which results in motion in a circle (or part of a circle). See Figure A2.76.

A centripetal force continuously changes direction so that it is always acting perpendicularly to the instantaneous velocity.

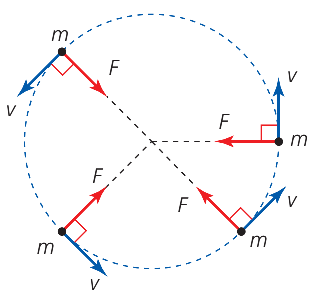


Figure A2.76 Velocity and centripetal force vectors during circular motion

Identifying different types of centripetal force

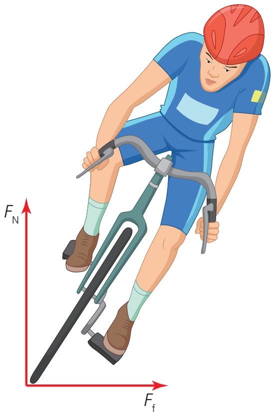
Gravity provides the centripetal force for planets moving around the Sun, and for satellites moving around the Earth (including the Moon).

Tension provides the centripetal force for an object being spun around on a string in an (almost) horizontal circle.

Friction provides the centripetal force for a vehicle, cyclist or a person, moving in a curved path on a horizontal surface. As an example, consider the cyclist shown in Figure A2.77. To move in a curved path there needs to be a centripetal force perpendicular to his motion. This is provided by friction: The cyclist leans 'into the bend' so that the tyre pushes outwards on the ground and the ground pushes inwards on the tyre (another example of Newton's third law).

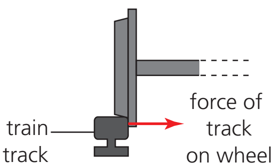
◆ **Banked track** A sloping surface to enable faster motion around curves.

If a greater speed is desired for movement around a curved track, friction may not be enough. By having a **banked track** greater speeds are possible (and safer). See Figure A2.78. A component of the contact force can then act in the necessary direction.



■ **Figure A2.77** Cycling around a bend

■ **Figure A2.78** Banked track at Daytona 500 race



■ **Figure A2.79** The contact force of the train track pushes inwards on the wheel of a train moving in a circular path

A train can travel around a curved track because of the contact force acting on the rim of the wheel. See Figure A2.79.

A driver or passenger in a car can move in a curved path because of the forces between them and the seat.

An aircraft can change direction by tilting, so that the air pushes the airplane perpendicular to its motion (Figure A2.80).

■ **Figure A2.80** Aircraft changing direction

Electrical forces provide the centripetal force for electrons moving around the nuclei of atoms.

Common mistake

Centripetal force is *not* a different type of force, like for example, tension or gravity. It is simply a way of describing the results of a force. Centripetal force should not be labelled as such in a free-body diagram.

It is common for people to refer to *centrifugal* forces, but this will only lead to confusion and the term is best avoided in this course. It is a matter of point of view (frame of reference): if a system is seen from 'outside', a centripetal force is needed for circular motion, but 'inside' the system an object seems to experience a force moving it outwards from its circular path.

LINKING QUESTION

- Why is no work done on a body moving along a circular trajectory?

This question links to understandings in Topic A.3.

73 Draw a free-body diagram for an aircraft changing direction ('banking') at constant altitude. Ignore air resistance for this question.

74 Consider a cyclist on a horizontal curved track. State three factors which will result in the need for a greater centripetal force.

75 If you were a passenger in a car going 'too fast' around a bend, outline what you would do to exert more centripetal force on yourself.

76 Draw a free-body diagram for a car on a banked curved surface.

Centripetal acceleration

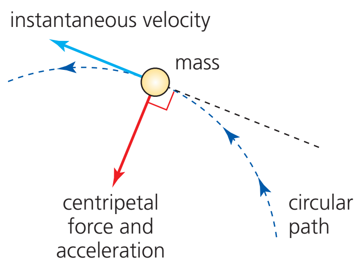


Figure A2.81 Centripetal force and acceleration

◆ **Centripetal acceleration** The constantly changing velocity of any object moving along a circular path is equivalent to an acceleration towards the centre of the circle.

We know that a resultant force causes an acceleration, a. Therefore, a centripetal force towards the centre of any circular motion must result in a **centripetal acceleration**, also towards the centre of the circle. This is shown more clearly in Figure A2.81. Although there is an acceleration directed towards the centre, there is no movement in that direction, or change in the magnitude of the velocity of the mass. Instead, the action of the force continually changes the direction of the motion of the mass.

Remember that acceleration means a change of velocity, and the velocity of a mass can change by going faster, going slower, or *changing direction*.

Any body moving in a circular path has a centripetal acceleration towards the centre of the circle.

The mathematics of uniform circular motion

SYLLABUS CONTENT

- Motion along a circular trajectory can be described in terms of the angular velocity, ω, which is related to the linear speed, v, by the equation as given by: $v = \frac{2\pi r}{T} = \omega r$.
- Bodies moving along a circular trajectory at a constant speed experience an acceleration that is directed radially towards the centre of the circle – known as a centripetal acceleration as given by: $a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$.

Tool 3: Mathematics

Use of units whenever appropriate: radians

◆ **Radians** Unit of measurement of angle. There are 2π radians in 360°.

In physics, it is usually much easier in calculations to use angles measured in **radians**, rather than degrees (which are based on the historical and arbitrary choice of 360 degrees for a complete circle). If you are studying Mathematics: Applications and Interpretations Standard Level, this may be a new concept for you.

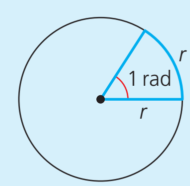


Figure A2.82 One radian

One radian (rad) is defined as the angle which has a length of arc equal to the radius of the circle. (See Figure A2.82.) Rotation through a complete circle passes through an angle of $\frac{2\pi r}{r} = 2\pi$ rad, so that:

$$1 \text{ rad} = \frac{180°}{\pi} (= 57.3°)$$

Consider an object of mass m moving with a constant linear speed, v, in a circle of radius r, as shown in Figure A2.83. The linear speed, v, of the mass can be calculated from:

$$v = \frac{2\pi r}{T}$$

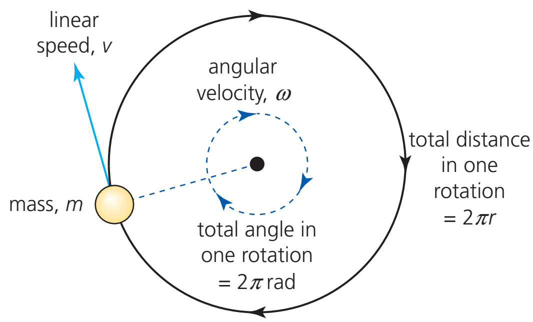
◆ **Time period, T** The duration of an event which occurs regularly. $T = \frac{1}{f}$.

◆ **Frequency, f** The number of repeating events per unit time.

◆ **Hertz, Hz** Derived SI unit of measurement of frequency. 1 Hz = one event per second.

◆ **Angular velocity, ω** Change of angle / change of time. Sometimes called angular speed.

where T is called the **time period** of the repeating motion, the time taken for one complete rotation. SI unit: s.



■ **Figure A2.83** Relating linear speed to angular velocity

The **frequency**, f, of the motion is the number of rotations in unit time (per second). SI unit: **hertz, Hz**. A frequency of 1 Hz means one rotation per second.

$$f = \frac{1}{T} \quad \text{SI unit: hertz, Hz}$$

We also commonly refer to **angular velocity**, ω, the rate at which an object rotates. In the context of uniform circular motion, the vector nature of a constant angular velocity is not important. It is also sometimes called angular speed.

Angular velocity = angle moved through / time taken. It can be measured in degrees per second, so that a constant angular velocity in degrees per second would be 360 / T. However, the use of radians per second is considered more convenient.

$$\omega = \frac{2\pi}{T} = 2\pi f \quad \text{SI unit: rad s}^{-1}$$

Comparing the last equation with $v = \frac{2\pi r}{T}$, it should be clear that:

$$v = \omega r$$

Top tip!

Period, frequency and angular velocity represent exactly the same information about a constant circular motion. (Given any one, we can calculate the other two.) In a question, we are most likely to be told the period, or the frequency of a rotation, but in calculations the angular velocity is often needed.

WORKED EXAMPLE A2.18

A car is travelling at a constant speed of 12 m s^{-1}. Its wheels each have a radius (including tyres) of 26 cm.

a What is the linear speed of a point on the surface of the tyre?

b Calculate the frequency and time period of the wheel's rotation.

c Determine the angular velocity of the wheels.

d Through what total angle does the wheel rotate in 10 seconds in:

  i radians

  ii degrees?

Answer

a 12 m s^{-1}

b $f = \frac{12}{2\pi r} = \frac{12}{(2 \times \pi \times 0.26)} = 7.3\text{ Hz}$ (7.3456... seen on calculator display)

$T = \frac{1}{f} = \frac{1}{7.3456} = 0.14\text{ s}$

c $\omega = 2\pi f = 2 \times \pi \times 7.3456 = 46\text{ rad s}^{-1}$ (46.1538... seen on calculator display)

d **i** $46.1538 \times 10 = 4.6 \times 10^2$ rad

  ii $461.538 \times 57.3 = 2.6 \times 10^{4\circ}$

77 a Convert an angle of 157° to radians.

  b i How many degrees does a rotating object pass through in five complete rotations?

   ii How many radians does a rotating object pass through in five complete rotations?

78 The diameter of the clock face seen in Figure A2.84 is 43 m.

Figure A2.84 The clock face on the Abraj Al-Bait Tower in Mecca is the largest in the world

  a Determine the linear speed of the tip of the minute hand.

  b What is the angular velocity of the minute hand?

79 If a rotating object completes 30.0 rotations in 47.4 s, calculate:

  a its time period

  b its frequency

  c its angular velocity.

80 a Calculate the angular velocity of the Earth's motion around the Sun.

  b What is your angular velocity as you rotate on the Earth's surface?

  c Determine the linear speed of someone on the equator spinning on the Earth's surface.
  (radius of Earth = 6.4×10^6 m)

81 A bicycle wheel which has a radius of 31 cm is rotating with an angular velocity of 41.9 rad s^{-1}.

  a Calculate the linear speed of a point on the circumference of the wheel.

  b What is the speed of the bicycle along the road?

Equations for centripetal acceleration and force

Even if an object moving in a circle of radius r has a constant linear speed, v, its centripetal acceleration, a, will have a numerical value, which represents how quickly the object's direction of motion is changing. The equation for calculating centripetal acceleration is shown below, and its derivation is included below.

$$a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$$

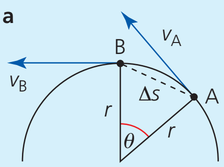
Tool 3: Mathematics

Derive relationships algebraically

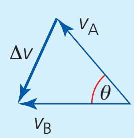
When basic principles of physics are used to explain the origin of an equation it is called **deriving** the equation.

Consider a mass moving in a circular path of radius, r, as shown in Figure A2.85a. It moves through an angle, θ, and a distance, Δs, along the circumference as it moves from A to B, while its velocity changes from v_A to v_B.

a



b



■ **Figure A2.85** Deriving an equation for centripetal acceleration

To calculate acceleration, we need to know the change of velocity, Δv. This is done using the vector diagram shown in Figure A2.85b. Note that the direction of the change of velocity (and therefore the acceleration) is towards the centre of motion. The two triangles are similar and, if the angle is small enough that Δs can be approximated to a straight line, we can write:

$$\theta = \frac{\Delta v}{v} = \frac{\Delta s}{r}$$

(The magnitudes of v_A and v_B are equal and represented by the speed, v.)

Dividing both sides of the equation by Δt we get:

$$\frac{\Delta v}{(\Delta t \times v)} = \frac{\Delta s}{(\Delta t \times r)}$$

Then, because $a = \frac{\Delta v}{\Delta t}$ and $\frac{\Delta s}{\Delta t} = v$:

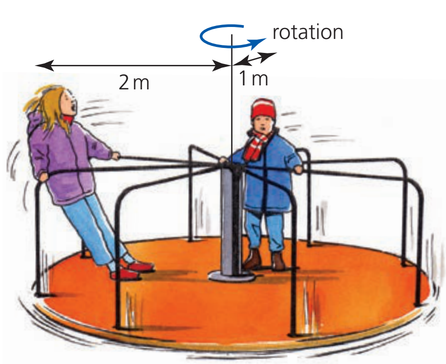
$$a = \frac{v^2}{r}$$

◆ **Derive** Explain in detail the origin of an equation.

We know $F = ma$ from Newton's second law of motion, so, the equation for the centripetal force acting on a mass m moving in a circle is:

$$F = \frac{mv^2}{r} = m\omega^2 r$$

In Figure A2.86, although both children have the same angular velocity, the bigger child needs a much greater centripetal force, so she should hold on tighter. This is because she has greater mass and is travelling with a greater linear speed.



■ **Figure A2.86** Children on a playground ride

WORKED EXAMPLE A2.19

Consider a ball of mass 72 g whirled with a constant speed of 3.4 m s^{-1} around in a (nearly) horizontal circle of radius 65 cm on the end of a thin piece of string, as shown in Figure A2.87.

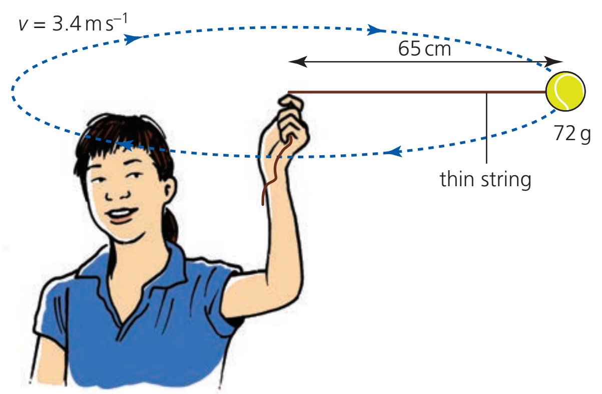


Figure A2.87 Ball whirled with a constant speed in a (nearly) horizontal circle

a Calculate the centripetal acceleration and force.

b Explain why the force provided by the string cannot act horizontally.

c Suggest a probable reason why the string breaks when the speed is increased to 5.0 m s^{-1}.

d Predict in which direction the ball moves immediately after the string breaks.

Answer

a $a = \frac{v^2}{r} = \frac{3.4^2}{0.65} = 18\,\text{m s}^{-2}$

$F = ma = 0.072 \times 18 = 1.3\,\text{N}$

b If the force is horizontal, it cannot have a vertical component with which to support the weight of the ball (see Figure A2.88).

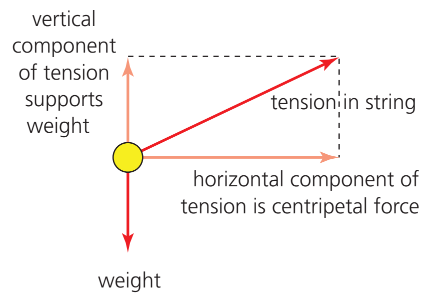


Figure A2.88 Free-body diagram for a ball whirled in a circle

c As the speed of the ball is increased, a greater centripetal force is needed for the same radius. If this force is greater than can be provided by the string, the string will break. This occurs when the speed reaches 5 m s^{-1}.

d The ball will continue its instantaneous velocity in a straight line after the string breaks. It will move at a tangent to the circle, but gravity will also affect its motion.

82 Estimate how much greater is the size of the centripetal force acting on the larger child (than the smaller child) in Figure A2.86. Explain your answer.

Figure A2.89 Throwing the hammer

83 The hammer being thrown in Figure A2.89 completed two full circles of radius 2.60 m at a constant speed in 1.38 s just before it was released. Assuming that the motion was horizontal:

a Calculate its centripetal acceleration.

b What force did the thrower need to exert on the hammer if its mass was 4.00 kg?

c The thrower will aim to release the hammer when it is moving at an angle of 45° to the horizontal. Explain why.

84 What is the centripetal acceleration of an object moving in a circular path of radius 84 cm if there are exactly two revolutions every second?

85 The Moon's distance from the Earth varies but averages about 380 000 km. The Moon orbits the Earth in an approximately circular path every 27.3 days.

a Show that the Moon's orbital speed is about 1 km s^{-1}.

b Calculate the centripetal acceleration of the Moon towards the Earth.

86 A car of mass 1240 kg moved around a bend of radius 37 m at a speed of $16\,\mathrm{m\,s^{-1}}$ (see Figure A2.90). If the car was to be driven any faster, there would not have been enough friction and it would have begun to skid off the road.

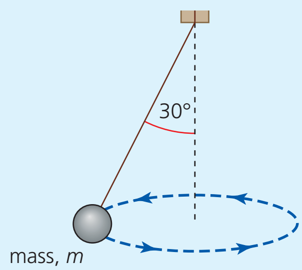
■ **Figure A2.90** Car driving around a tight mountain bend

a Calculate the magnitude of the centripetal force assuming that the road is horizontal.
b Determine a value for the coefficient of friction between the road and the tyre.
c State whether this is a coefficient of static friction or dynamic friction.
d Discuss whether a heavier car would be able to move faster around this bend.

87 A girl of mass 42 kg living in Sydney is moving (like everyone else) in a circle because of the rotation of the Earth. Sydney is 5.31×10^6 m from the Earth's axis of rotation.

a Calculate her linear speed of rotation.
b What is her centripetal acceleration?
c Determine the resultant force needed on her to maintain her circular motion.
d What provides this centripetal force?
e Your answer to part **c** should be much less than 1% of the girl's weight. It is so small that this force is usually considered to be insignificant. However, draw a free-body diagram of her standing on the Earth's surface that includes numerical values of the forces involved.

88 Figure A2.91 shows a pendulum of mass 120 g being swung in a horizontal circle.



■ **Figure A2.91** Pendulum

a Draw a free-body diagram of the mass, *m*.
b Calculate the centripetal force acting on the mass.
c If the radius of the circle is 28.5 cm, i what is the speed of the pendulum and ii how long does it take to complete one circle?

TOK

Knowledge and the knower

- How do our expectations and assumptions have an impact on how we perceive things?
- What constitutes a 'good reason' for us to accept a claim?

Most people accept that we live on a spherical rotating planet, but they have no 'direct' evidence of that. And as the Earth spins, we are told that the invisible force of gravity provides the necessary centripetal force that keeps us attracted to the Earth's surface. Our own observations are more likely to suggest that we live on a mostly flat Earth, and that the Sun and stars move around us.

Foucault's pendulum (Figure A2.92) provides evidence of the Earth's rotation, but not in an obvious way, and it needs to be explained to non-scientists.

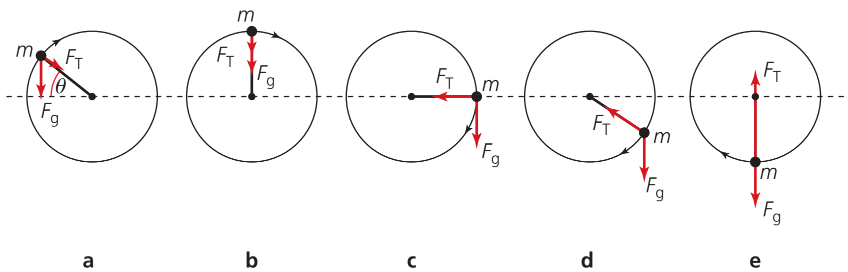
■ **Figure A2.92** Foucault's pendulum

Are we sensible to believe what we are told by 'experts' and teachers, rather than to trust our own senses? How can we decide whether or not to accept knowledge claims made by our predecessors?

Non-uniform circular motion

Vertical motion

As an example, consider a ball of mass, m, on the end of string which is being spun in a vertical circle of radius r, as shown in Figure A2.93. There are two forces which can act on the ball: the weight of the ball, $F_g = mg$, and the tension in the string, F_T.



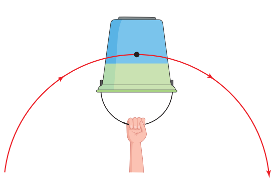
■ **Figure A2.93** Forces on a mass moving in a vertical circle

Vertical motion is more complicated than horizontal motion because the centripetal force is affected by the combination of the tension in the string and the component of the ball's weight, both of which vary continuously during the motion.

If the ball was moving at a constant speed in a circle of constant radius:

- In position **b**, centripetal force $(mv^2/r) = F_T + F_g$, so that the required tension will have its minimum value. If the weight is greater than the necessary centripetal force, the string will lose tension and the ball will move inwards from its circular path. See further example below.
- In position **e**, centripetal force $= F_T - F_g$, so that the required tension will have its maximum value. It is at this position that the string is most likely to break.
- In position **c**, centripetal force $= F_T$, because there is no component of weight acting to, or from, the centre.
- In position **a** there will be a component of weight acting towards the centre.
- In position **d** there will be a component of weight acting away the centre.

In practice, it is unlikely that the tension can be continuously adjusted to keep the centripetal force constant. This means that the speed of the ball will change during its rotation. It will not be uniform circular motion.



■ **Figure A2.94** Bucket of water spinning in a circle

The situation shown in **b** is the most widely discussed, and whirling a bucket of water in a vertical circle always makes for an interesting demonstration. See Figure A2.94.

The hand pulling on the handle provides a force on the bucket. The normal contact force from the bottom of the bucket provides a force on the water. In order for the water to stay in contact with the bucket, it must be spinning with a speed which requires a centripetal force which is equal to, or greater than, its weight (so that a force from the bucket is also needed):

$$F = mv^2 \geq mg$$

$$v_{min}{}^2 = gr$$

$$v_{min} = \sqrt{gr}$$

WORKED EXAMPLE A2.20

a What is the minimum linear speed needed to keep water in a bucket rotating in a circle of radius of 0.95 m?

b What is the minimum angular velocity needed?

c How long will each rotation take if the bucket could be kept moving at the same speed?

d What maximum pulling force would be needed to maintain that speed if the bucket and water had a combined mass of 2.1 kg?

Answer

a $v_{min} = \sqrt{gr} = \sqrt{(9.8 \times 0.95)} = 3.1\,\mathrm{m\,s^{-1}}$ (3.0512... seen on calculator display)

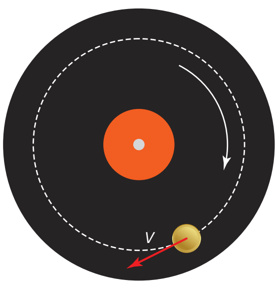
b $\omega = \dfrac{v}{r} = \dfrac{3.0512}{0.95} = 3.2\,\mathrm{rad\,s^{-1}}$

c $T = \dfrac{2\pi r}{v} = \dfrac{(2\pi \times 0.95)}{3.0512} = 2.0\,\mathrm{s}$

d Maximum pull (tension) is needed when the bucket is at its lowest point:

$$\frac{mv^2}{r} = F_T - F_g$$

$$F_T = F_g + \frac{mv^2}{r} = (2.1 \times 9.8) + \frac{(2.1 \times 3.0512^2)}{0.95} = 20.6 + 20.6 = 41\,\mathrm{N}$$



■ **Figure A2.95** Coin on rotating turntable

Horizontal motion

We have seen that any object moving along the arc of a circle requires a centripetal force of magnitude: $F = \dfrac{mv^2}{r}$.

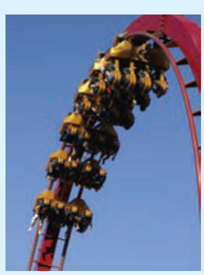
But what happens if the circumstances change? As an example, consider a coin on a rotating horizontal turntable, as shown from above in Figure A2.95. Friction provides the centripetal force on the coin. As the rotational speed increases, a greater frictional force is needed to keep the coin in the same place on the turntable. If the speed continues to increase, eventually there will not be enough friction and the coin will be thrown off the turntable (approximately along a tangent). A similar coin (to the first) placed closer to the centre will be able to stay on the turntable at greater speeds because less centripetal force (friction) is needed.

Consider again the car shown in Figure A2.90. If the radius, r, of a bend changes, the centripetal force needed changes. For example, if the radius reduces (the bend gets 'tighter'), a greater frictional force is needed to maintain the same speed. This may not be possible, so the driver should reduce speed. Similarly, a slower speed is advisable if water or ice on the road reduces frictional forces.

89 **a** Outline how the passengers seen in Figure A2.96 remain in their seats even though they are upside down (and even if they were not secured by safety harnesses!).

b What is the minimum speed needed if the carriage moves in a vertical arc of radius 15 m?

c In another part of the track the passenger carriage is upside down in a vertical arc of radius 20 m. Predict if the carriage needs to move faster, slower or the same speed. Explain your answer.



■ **Figure A2.96** Upside down on a fairground ride

90 Consider Figure A2.93a. If the mass was 240 g, the radius of the circle was 52 cm, $\theta = 40°$, and the mass was moving with a linear speed of $2.12\,\mathrm{m\,s^{-1}}$:

a Write down an expression for the component of weight acting towards the centre of motion.

b What was the necessary centripetal force in this position?

c Hence, determine the tension in the string.

91 A coin of mass 10 g was rotating on a turntable turning at 34 revolutions/minute. The coin was 17 cm from the centre.

a Calculate the magnitude of the centripetal force acting on the coin.

b Friction provides this force. The turntable's speed is increased so that more friction is required to keep the coin in place. If the coefficient of static friction is 0.43, what is the greatest possible value for the frictional force between the turntable and the coin?

c Determine the maximum angular velocity of the coin which will enable it to stay in the same place.

d How would your answers change if an identical coin had been fixed on top of the first coin?

A.3 Work, energy and power

Guiding questions

- How are concepts of work, energy and power used to predict changes within a system?
- How can a consideration of energetics be used as a method to solve problems in kinematics?
- How can transfer of energy be used to do work?

◆ **Work, *W*** The energy transfer that occurs when an object is moved with a force. More precisely, work done = force × displacement in the direction of the force.

◆ **Energy** Ability to do work.

In the last two topics, we have discussed movement (A.1), and how forces can change the motion of objects (A.2). In this topic (A.3) we will move on to introduce two very important, closely related, numerical concepts: **work** and **energy**. Together these provide the 'accounting system' for science, enabling explanations and useful predictions to be made.

Of course, 'work' and 'energy' are words in common use in everyday language, but in physics they have much more precise definitions.

Nature of science: Science as a shared endeavour

Same words, different meanings

The terms used in physics have very precise meanings – but the same words are often used differently in everyday life. There is a long list of such words, such as 'work', 'energy' and 'power', as well as the various meanings of 'conservation', 'law', 'momentum', 'pressure', 'stress', 'efficiency', 'heat', 'interference', 'temperature' and so on.

This ambiguity is a problem that all students must overcome when learning physics. For example, what is the connection, if any, between work = force × displacement and 'I have a lot of homework to do tonight'?

Work

SYLLABUS CONTENT

▶ Work, W, done on a body by a constant force depends on the component of the force along the line of displacement as given by: $W = Fs\cos\theta$.

Work done by constant forces

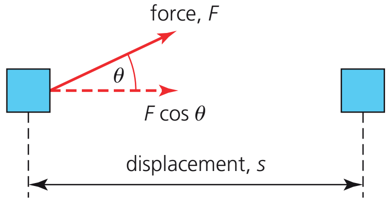
We say that work is done when any force moves an object: work is done *on* the object *by* the force. The work done, W, can be calculated by multiplying the displacement, s, by the component of the force acting in that direction, $F\cos\theta$.

See Figure A3.1 and note that θ is the angle between the force and the direction of motion.

work done, $W = Fs\cos\theta$ SI unit: Joule, J

One **joule** is defined to be the work done when a force of 1 N moves through a distance of 1 m.

◆ **Joule, J** Derived SI unit of work and energy. 1 J = 1 N m.



■ **Figure A3.1** Work done by a force

Commonly, a force acts in the same direction as the motion, in which case, the equation reduces to $W = Fs$.

WORKED EXAMPLE A3.1

Calculate how much work is done when a 1.5 kg mass is raised 80 cm vertically upwards.

Answer

The force needed to raise an object (at constant velocity) is equal to its weight (mg). The symbol h is widely used for vertical distances. (To avoid confusion, W will normally be used to represent work, and not weight.)

$W = Fs = mg \times h = 1.5 \times 9.8 \times 0.80 = 12\,\text{J}$

WORKED EXAMPLE A3.2

The 150 kg box in Figure A3.2 was pulled 2.27 m across horizontal ground by a force of 248 N, as shown.

a Determine how much work was done by the force.

b Suggest why it may make it easier to move the box if it is pulled in the direction shown by the dashed line.

c When the box was pulled at an angle of 20.0° to the horizontal, the force used to slide the box was 248 N. Calculate the work done by this force in moving the box horizontally the same distance.

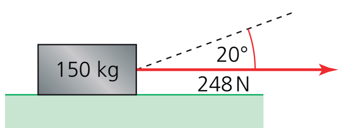


Figure A3.2 Box being pulled across the ground

Answer

a $W = Fs = 248 \times 2.27 = 563\,\text{J}$

b When the box is pulled in this direction, the force has a vertical component that helps reduce the normal contact force between the box and the ground. This will reduce the friction opposing horizontal movement.

c The force is not acting in the same direction as the movement. To calculate the work done we need to use the horizontal component of the 248 N force.

$W = F\cos 20° \times s = 248 \times 0.940 \times 2.27 = 529\,\text{J}$

It is important to realize that there are some surprising examples involving forces where *no* work is being done, as shown in Figures A3.3 and A3.4.

Figure A3.3 No work is being done on the weights at this moment

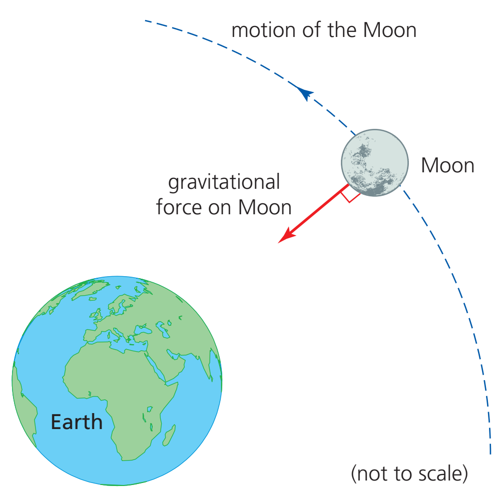


Figure A3.4 No work is done as the Moon orbits the Earth

In Figure A3.3 a large upwards force is being exerted on the stationary weights, but since there is no movement at that moment, no mechanical work is being done on the weights. However, work *is* being done in the weightlifter's muscles. In Figure A3.4, the Moon is moving *perpendicularly* to the force of gravity (cos 90° = 0), so there is no component of force in the direction of motion. A satellite in a similar circular path would not need to do any work, so that it would not need an engine to maintain the motion.

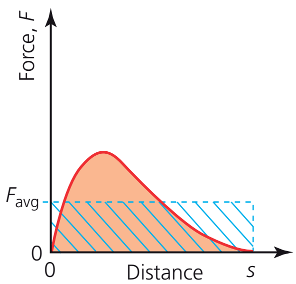


Figure A3.5 A force varying with distance

Work done by varying forces

When making calculations with the equation $W = Fs\cos\theta$ we are assuming a single, constant value for the force, but in reality, forces are rarely constant. In order to calculate the work done by a *varying* force, we have to make an estimate of the *average* force involved. This may be best done with the help of a force–time graph, or a force–distance graph, as shown in Figure A3.5, which could represent, for example, the resultant force used to decelerate a car.

The horizontal dotted blue line represents the average force, as judged by eye.

Then, work done = $F_{avg} \times s$, which is the same as the rectangular area, and it is also equal to the area under the original curved line.

Work done is equal to the area under a force–distance graph.

WORKED EXAMPLE A3.3

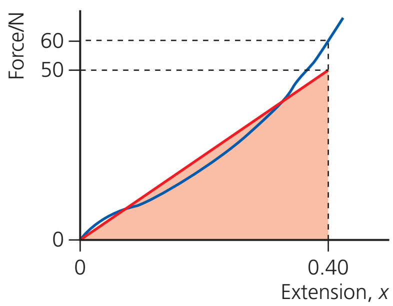


Figure A3.6 Force–extension (displacement) graph for rubber under tension

The blue line in Figure A3.6 represents the variations in force and extension as a rubber band was stretched. Estimate the work done as the force increased from zero to 60 N.

The work done is equal to the area under the graph. The red line has been drawn so that the area under it is the same as the area under the blue line (as judged by eye):

$$W = \frac{1}{2} \times 50 \times 0.40 = 10\,\text{J}$$

1 Calculate the work done to:
   a lift an 18 kg suitcase a vertical height of 1.05 m
   b push the same suitcase the same distance horizontally against an average frictional force of 37 N.

2 What was the magnitude of the average resistive force opposing the forward motion of a car if 2.3×10^6 J of work were done while maintaining a constant speed over a distance of 1.4 km?

3 In Figure A3.7 a gardener is pushing a lawnmower at a constant speed of 0.85 m s^{-1} with a force, F, of 70 N at an angle of 40° to the ground.
   a Calculate the component of F in the direction of movement.
   b What is the magnitude of the frictional force?
   c Determine the work done in moving the lawnmower for 3.0 s.

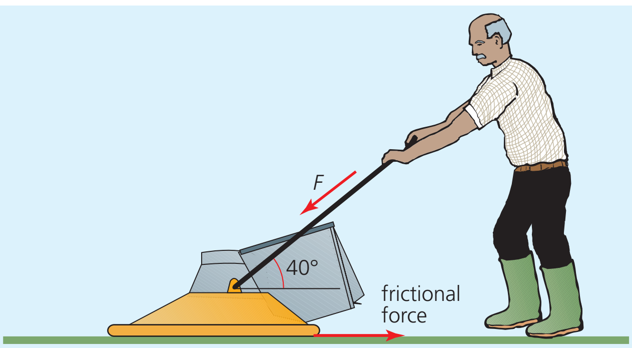


Figure A3.7 Gardener pushing a lawnmower

4 A spring which obeyed Hooke's law was stretched with a force which increased from zero to 24 N. The spring extended by 12 cm.
   a What was the average force used during the extension of the spring?
   b Calculate the work done on the spring.

Nature of science: Models

Macroscopic and microscopic work

The term ‘work’ is normally used to describe large-scale (macroscopic) movements and forces – such as kicking a ball – in which countless billions of particles move as a whole object. We do not concern ourselves with the individual particles.

But the concept of ‘doing work’ can also be applied to microscopic processes involving individual particles. For example, work is done when two particles collide with each other.

Work done during the random and unknown motions of individual particles can result in a different kind of energy transfer, called thermal energy. This important distinction is discussed in Topic B.4.

Energy

SYLLABUS CONTENT

- Work done by a force is equivalent to a transfer of energy.

Energy is probably the most widely used concept in the whole of science. However, the idea of energy is not easy to fully explain or truly understand.

When a battery is placed in a child’s toy dog (Figure A3.8), it moves, jumps up in the air and barks. After a certain length of time, the toy stops working. In order to try to explain these observations we will almost certainly need to use the concept of energy: chemical energy in the battery is transferred to electrical energy, which produces motion energy in a small electric motor. Some energy is also transferred from electricity to sound in a loudspeaker. Eventually, all the useful energy in the battery will be transferred to the surroundings and the toy will stop activity. Without the concept of *energy* all this is very difficult to explain.

Figure A3.8 The toy dogs get their energy from batteries

We can talk about the energy *in* the gasoline (petrol) we put in the tanks in our cars (for example) and go on to describe that energy being transferred to the movement of the car. But nothing has actually flowed out of the gasoline into the car, and this is just a convenient way of expressing the idea that the controlled combustion of gasoline with oxygen in the air can do something that we consider to be useful.

Perhaps the easiest way to understand the concept of energy is this: energy is needed to make things happen. Whenever anything changes, energy is transferred from one form to another. *Most importantly, energy transfers can be calculated*, and this provides the basic ‘accounting system’ for science. Any event will require a certain amount of energy for it to happen and, if there is not enough energy available, it cannot happen. For example, if you do not get enough energy (originally from your food), you will not be able to climb a 500 m hill; if your phone battery is not charged, you cannot call your friends; if you do not put enough gasoline in your car, you will not get to where you want to go; if energy is not transferred quickly enough from an electrical heater, your shower will not be hot enough.

A person, device or machine which provides a force to do work must be *able* to do the work. We say that they must have enough *energy* to do the work. Energy is often described as the capacity to do work. To do work, there must be a ‘source’ of energy.

When work is done *energy is transferred* from a source to the object.

In Figure A3.9, the archer uses her store of energy (originally from food) to do work on the bow, which then stores energy because it is stretched out of shape. The bow then does work when energy is transferred to the movement of the arrow. When the arrow hits the target, work is done on it by the arrow as energy is transferred causing a change of shape and a small rise in temperature.

Figure A3.9 Archer and target

TOK

Knowledge and the knower

- What criteria can we use to distinguish between knowledge, belief and opinion?

Abstract concepts

Energy is one of many **abstract concepts** in physics. Everyday abstract concepts include 'hope', 'justice' and 'freedom'. They are all very useful ideas that can be explained (often with difficulty!) and people can understand them at various levels, and in differing, often personal and **subjective** ways, but they have no actual physical form.

Should we believe that a non-abstract physics concept, like force, for example, is more 'real' than the abstract concept of energy?

◆ **Abstract concept** An idea which has no physical form.

◆ **Subjective** Describes an opinion based on personal experiences and emotions. Compare with **objective**, meaning free from emotion and bias.

◆ **Potential energy** Energy that arises because of forces between different parts of the system. Sometimes described as stored energy.

Different forms of energy

The capacity of something to do work can exist in many different *forms* (of energy). These forms can be difficult to classify, and no two sources of information ever seem to agree on a simple, definitive list or even on how to use the term 'forms'! The different forms of energy are a constant background to the study of physics and need to be well understood. The following is a broad initial summary.

Potential energy sources store energy because of their position or arrangement, and the forces between different parts of the system.

- **Gravitational potential energy**: The energy stored due to the position of a mass in a gravitational field. For example, a weight raised above the ground.
- **Electric potential energy**: The energy stored due to the position of a charge in an electric field (see Theme D).
- **Magnetic potential energy**: The energy stored due to position in a magnetic field (see Theme D).
- **Elastic potential energy**: The energy stored in a deformed elastic material, or a spring.
- **Chemical potential energy**: The energy stored in the bonding of chemical compounds, released in chemical reactions.
- **Nuclear potential energy**: The energy stored in the arrangement of particles in the nuclei of atoms.

Figure A3.10 Water behind the Three Gorges Dam (China) stores enormous amounts of gravitational potential energy

Figure A3.11 Kori nuclear power station (S. Korea) is the world's largest

A macroscopic object that is able to do work is said to possess *mechanical energy*.

Mechanical energy can come in one of three forms:

- **Kinetic energy**: The energy of all moving masses which could do work on anything they collide with. (Includes wind and mechanical waves, including sound).
- Elastic potential energy: As described above.
- Gravitational potential energy: As described above.

All matter contains large amounts of energy inside it.

Internal energy is the name we give to the enormous amount of energy which exists within all matter because of the motions and positions of the particles it contains.

The following types of energy transfer will be discussed in later topics:

- **Thermal energy**: Energy transferred because of a temperature difference.
- **Electrical energy**: Energy carried along metal wires because of a potential difference (voltage – see Topic B.5).
- **Radiation energy**: Light, for example: energy transferred as electromagnetic waves.

In Theme E we will discuss the equivalence of energy and mass.

◆ **Gravitational potential energy** Energy that masses have because of the gravitational forces between them.

◆ **Electric potential energy** Energy that charges have because of the electric forces between them.

◆ **Elastic potential energy** Energy that is stored in a material that has been deformed elastically.

◆ **Chemical potential energy** Energy related to the arrangement of electrons within the structure of atoms and molecules.

◆ **Nuclear potential energy** Energy related to the forces between particles in the centres (nuclei) of atoms.

◆ **Kinetic energy** Energy of moving masses.

◆ **Thermal energy (heat)** The (non-mechanical) transfer of energy between two or more bodies at different temperatures (from hotter to colder).

◆ **Electrical energy** Energy delivered in a circuit by an electrical current.

◆ **Radiation energy** Energy transferred by electromagnetic waves.

◆ **Internal energy** Total potential energies and random kinetic energies of the particles in a substance.

Energy transfers

Energy transfer (between the forms listed above) is the central, recurring theme of all of physics. Energy can be quantified and measured, and the total amount within a system remains the same, although some useful energy is always 'wasted' in every macroscopic energy transfer. Some examples of useful energy transfers:

- Our bodies transfer chemical energy from food to internal energy and kinetic energy.
- A rocket transfers chemical energy to gravitational potential energy.
- A 'clockwork' toy transfers elastic potential energy to kinetic energy.
- An electric light transfers electrical energy to radiation energy.
- A nuclear power station transfers nuclear potential energy to electrical energy.

These are just a few random examples. Any list like this can be very long!

Principle of conservation of energy

SYLLABUS CONTENT

- ▶ The principle of the conservation of energy.

The total energy of an isolated system remains constant.

An alternative way to state the same law is 'energy cannot be created or destroyed'. We can move energy around and transfer it from one form to another, but the total amount remains the same.

This is one of the most important principles in the whole of science, not only because it is one of very few principles of science which is always true, but also because it is highly relevant to every event that occurs, helping us to predict what can, and what cannot, happen.

A financial analogy may help: if you leave home with \$10 cash in your pocket, spend \$5 and arrive back home with \$2, then you will probably assume that you lost \$3 somewhere. And you would not expect to have, say, \$7 left in your pocket. You believe in the 'conservation' of cash, even if you are not sure of where it has gone.

Similarly, we know that if, for example, 20 000 J of energy is transferred into a system (when charging a mobile phone for example), we cannot take 25 000 J out, and if only 4000 J remain, then we know that 16 000 J has been transferred somewhere else.

Common mistake

'Energy conservation' has evolved to have a slightly different meaning in everyday use. We are often advised to 'conserve' energy, meaning that we should not use too much now, because there is a limited supply and we may not have enough for when we want it in the future.

LINKING QUESTION

- Where do the laws of conservation apply in other areas of physics? (NOS – see page 92.)

This question links to understandings in Topics A.2, B.4 and E.3.

◆ **Conservation laws** In isolated systems, some physical quantities remain constant under all circumstances: energy/mass, charge, momentum.

Nature of science: Theories

Conservation laws

A **conservation law** tells us that a measurable physical quantity does not change after any event, or with the passage of time. The following quantities, which occur in this course, are always conserved in any well-defined system.

- Energy
- Mass (the equivalence of mass and energy will be outlined in Topics E.4 and E.5)
- Linear momentum (see Topic A.2)
- Angular momentum (see Topic A.4)
- Electric charge (see Topic B.5)

Other quantities, such as force, velocity and temperature, are not conserved.

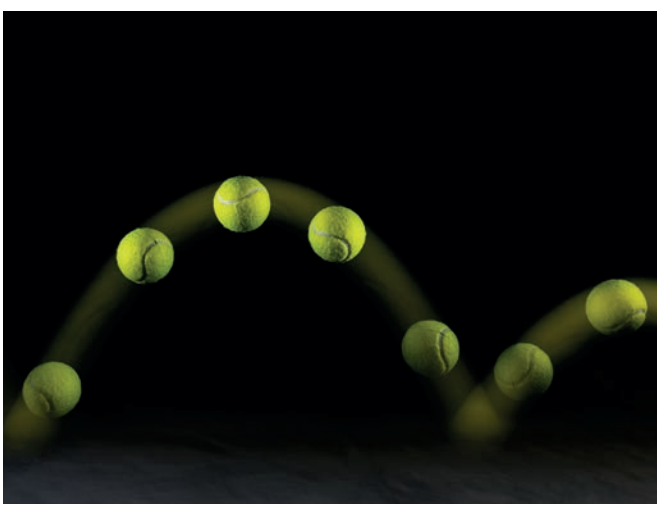
Conservation laws are important in predicting the outcome of events.

Dissipation of energy

So, we cannot create energy: the total energy after any event cannot be more than we started with, but in the macroscopic world in which we live, it seems that *every* event seems to 'lose' energy. Take a bouncing ball as an example, as shown in Figure A3.12. The height of each bounce is less than the one before, which shows us that the ball is losing (gravitational potential) energy. Also, just before each time the ball hits the ground it will also have less kinetic energy.

◆ **Dissipated** or **degraded energy** Energy that has spread into the surroundings and cannot be recovered to do useful work.

But if we think of the ball and the ground together as the 'system', the mechanical energy 'lost' by the ball has been 'gained' by the ground, keeping the total energy constant. The energy gained by the ground is in the form of *internal energy* and a very accurate temperature measurement would show that it had become a little warmer at the points of contact. (A little sound energy will also be present.) Some of the mechanical energy of the ball will also have been transferred to internal energy in the ball, which will also be slightly warmer.



■ **Figure A3.12** Bouncing ball

It is very important to appreciate that gravitational potential energy and kinetic energy can be considered as 'useful' mechanical energies, but the energy transferred to the ground spreads out into the surroundings and can never be recovered to do any useful work. It is sometimes called 'wasted' energy, but it may be better described as **dissipated energy**. Sometimes it is called *degraded energy*.

All macroscopic processes dissipate energy into the surroundings.

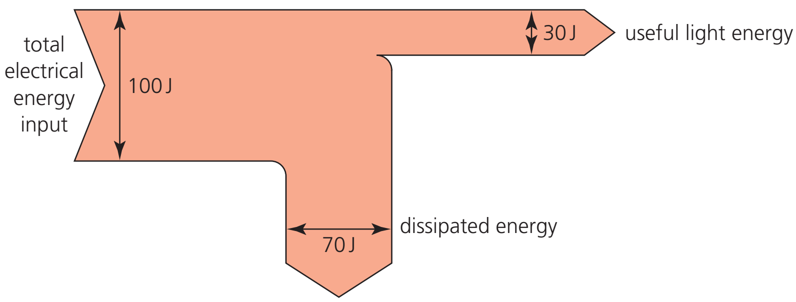
The law of conservation of energy is a constant theme throughout science, but in this chapter, we will concentrate on energy transfers in *mechanical systems*. Numerical examples will be given later.

Using Sankey diagrams to represent energy transfers

SYLLABUS CONTENT

- Energy transfers can be represented on a Sankey diagram.

Energy transfers can be usefully shown in flow diagrams, such as shown in Figure A3.13, which represents the energy transfers in a small LED lamp in a specified time.



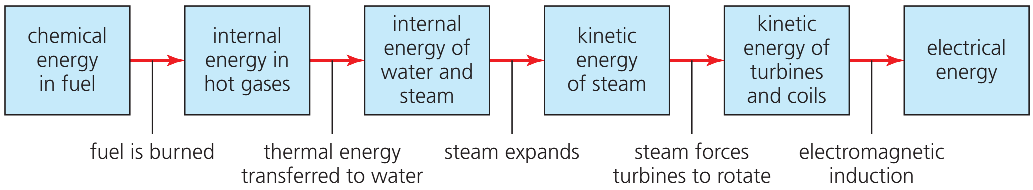
■ **Figure A3.13** A simple Sankey diagram

◆ **Sankey diagram**
Diagram representing the flow of energy in a system.

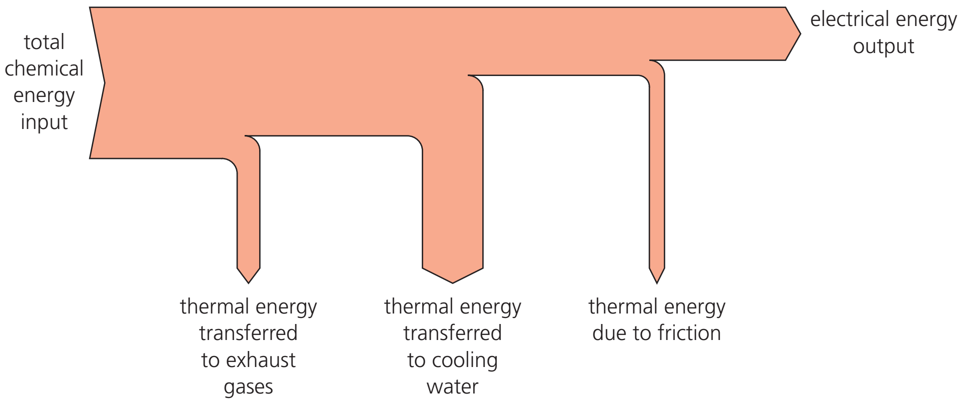
The width of each section is proportional to the amount of energy (or power), starting with the energy input shown at the left of the diagram. Dissipated energy is shown with downwards arrows and useful energy flows to the right. Diagrams like these are known as **Sankey diagrams** and they can be used to help represent many energy transformations.

Sankey energy flow diagrams can be used to visualize quantified transfers of energy.

As a more complicated example, Figure A3.14 represents the useful energy transformations in a fossil-fuel power station and Figure A3.15 shows a Sankey diagram representing the energy flow through the same system, including the dissipated energy.



■ **Figure A3.14** Energy transfers in a fossil-fuel power station



■ **Figure A3.15** Sankey diagram for a fossil-fuel power station

Nature of science: Theories

Law or principle?

Many physics books refer to the 'law' of conservation of energy, others refer to the 'principle' of conservation of energy. Is there any difference? And why is it not called the 'theory' of conservation of energy?

Research to determine if there are any differences between 'theories', 'laws' and 'principles.'

5 Outline the energy transfers that occur when a mass hanging vertically on the end of a metal spring is displaced and allowed to move up and down (oscillate) freely until it stops moving.

6 State the main energy transfers that occur in the use of a mobile phone.

7 State the names of devices whose main uses are to perform the following energy transfers:
- a electricity to sound
- b chemical to electricity
- c sound to electricity
- d chemical to radiation
- e chemical to kinetic
- f elastic to kinetic
- g kinetic to electricity
- h chemical to internal
- i electromagnetic radiation to electricity.

8 Sketch and annotate a Sankey diagram to represent the following process. A car transfers 1500 kJ from its fuel as it gains 100 kJ of kinetic energy and 200 kJ of gravitational potential energy. The rest of the energy is dissipated into the environment.

9 a What are the two main types of energy of an aircraft flying at a height of 12 km?
- b After the airplane has landed at an airport, what has happened to most of that energy?

10 An adult male body transfers about 10^7 J of energy every day.
- a Name the source of this energy.
- b i Outline the principle uses for this energy and
  - ii why does it have to be replaced?

11 A car slows down for traffic lights. State two causes of energy dissipation.

Common mistake

Many students believe (wrongly) that batteries store electrical energy. Batteries contain chemical compounds that react when a circuit containing the battery is connected. Chemical energy is then transferred to an electric current. Many batteries can be recharged if the direction of current is reversed. This reverses the chemical changes.

Calculating mechanical energies

SYLLABUS CONTENT

- Mechanical energy is the sum of kinetic energy, gravitational potential energy and elastic potential energy.
- Work done by the resultant force on a system is equal to the change in the energy of the system.
- Kinetic energy of translational motion as given by: $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$.
- Gravitational potential energy, when close to the surface of the Earth as given by: $\Delta E_p = mg\Delta h$.
- Elastic potential energy as given by: $E_H = \frac{1}{2}k\Delta x^2$.

◆ **Mechanical energy** Energy of a macroscopic object which can do useful work.

Work can be done on a macroscopic object / system to give it kinetic energy, gravitational potential energy, or elastic potential energy. The object then has the ability to transfer that energy to do useful work. We say that it has **mechanical energy**. We will now show how these three types of energy can be calculated.

The symbol E can be used to represent energy, usually with a subscript to represent the particular type of energy. (Note: E is also used for electric field.)

Kinetic energy

Tool 3: Mathematics

Derive relationships algebraically

Kinetic energy can be calculated from the equation $E_k = \frac{1}{2}mv^2$. This is explained / derived below. (Ideally, we would like to fully explain the origin of all of the equations used in this course, but that is not always possible.)

Consider a mass m accelerated horizontally from rest by a constant resultant force, F, acting in the direction of motion, as shown in Figure A3.16.

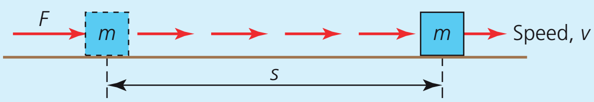


Figure A3.16 Doing work to increase movement

Using the equation of motion $v^2 = u^2 + 2as$ (from Topic A.1) and noting that, in this example, $u = 0$, we see that the distance travelled, s, can be determined from the equation: $s = \frac{v^2}{2a}$.

The work done W, in a distance s, can be calculated from: $W = Fs = ma \times \frac{v^2}{2a} = \frac{1}{2}mv^2$.

This amount of energy has been transferred from the origin of the force to the moving mass. We say that the mass has *gained* kinetic energy, E_k.

$\frac{1}{2}mv^2$ can also be written as: $\frac{[mv]^2}{2m}$.

Since momentum, mv, is given the symbol p, kinetic energy can also be determined from the equation: $E_k = \frac{p^2}{2m}$.

This version is most commonly used with the kinetic energy and momentum of atomic particles (See Topic E.5).

The kinetic energy of a moving mass can be calculated using: $\boldsymbol{E_k = \frac{1}{2}mv^2}$

Or from: $\boldsymbol{E_k = \frac{p^2}{2m}}$

◆ **Order of magnitude** An approximate value rounded to the nearest power of ten.

Some typical examples of kinetic energies (to an **order of magnitude**):

- a molecule in air at 20° C: 10^{-20} J
- a falling snowflake: 10^{-7} J
- a boy running in a 100 m race: 10^3 J
- a bullet from a rifle: 10^4 J
- a car moving along an open road: 10^5 J
- a large meteor moving towards Earth: 10^{17} J
- the Earth in its motion around the Sun: 10^{33} J.

Compare and quote values and approximations to the nearest order of magnitude

Physics is the fundamental science that tries to explain how and why everything in the Universe behaves in the way that it does. Physicists study everything from the smallest parts of atoms to distant objects in our galaxy and beyond (Figure A3.17).

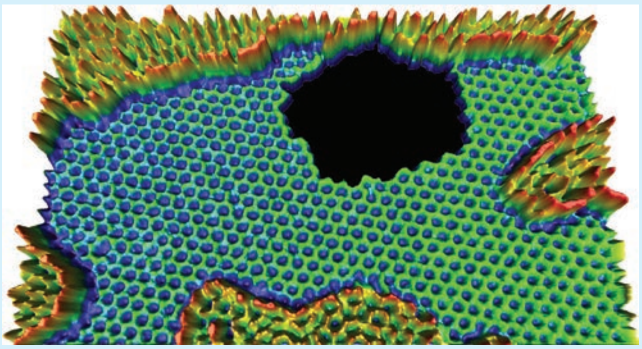


Figure A3.17 a The arrangement of individual atoms in graphene (a material made from a single layer of carbon atoms) can be seen using a special type of electron microscope; **b** complex gas and dust clouds in the Cat's Eye nebula, 3000 light-years away

Measurements and calculations commonly relate to the world that we can see around us (the macroscopic world), but our observations may require microscopic explanations, often including an understanding of molecules, atoms, ions and subatomic particles. Astronomy is a branch of physics that deals with the other extreme – quantities that are very much bigger than anything we experience in everyday life.

The study of physics therefore involves dealing with both very large and very small numbers. When numbers are so different from our everyday experiences, it can be difficult to appreciate their true size. For example, the age of the Universe is believed to be about 10^{18} s, but just how big is that number?

When comparing quantities of very different sizes (magnitudes), for simplicity we often make approximations to the nearest power of 10. When numbers are approximated and quoted to the nearest power of 10, it is called giving them an order of magnitude. For example, when comparing the lifetime of a human (the worldwide average is about 70 years) with the age of the Universe (1.4×10^{10} y), we can use the approximate ratio $10^{10} / 10^2$. That is, the age of the Universe is about 10^8 human lifetimes, or we could say that there are eight orders of magnitude between them.

Here are three further examples:

- The mass of a hydrogen atom is 1.67×10^{-27} kg. To an order of magnitude this is 10^{-27} kg.
- The distance to the nearest star (*Proxima Centauri*) is 4.01×10^{16} m. To an order of magnitude this is 10^{17} m. (Note: log of $4.01 \times 10^{16} = 16.60$, which is nearer to 17 than to 16.)
- There are 86 400 seconds in a day. To an order of magnitude this is 10^5 s.

Tables A3.1, A3.2 and A3.3 list the ranges of mass, distance and time that occur in the Universe.

Table A3.1 The range of masses in the Universe

Object mass / kg	Object mass / kg
the observable Universe	10^{53}
our galaxy (the Milky Way)	10^{42}
the Sun	10^{30}
the Earth	10^{24}
a large passenger airplane	10^5
a large adult human	10^2
a large book	1
a raindrop	10^{-6}
a virus	10^{-20}
a hydrogen atom	10^{-27}
an electron	10^{-30}

■ **Table A3.2** The range of distances in the Universe

Distance size / m	Distance size / m
distance to the edge of the visible Universe	10^{27}
diameter of our galaxy (the Milky Way)	10^{21}
distance to the nearest star	10^{16}
distance to the Sun	10^{11}
distance to the Moon	10^{8}
radius of the Earth	10^{7}
altitude of a cruising airplane	10^{4}
height of a child	1
how much human hair grows by in one day	10^{-4}
diameter of an atom	10^{-10}
diameter of a nucleus	10^{-15}

■ **Table A3.3** The range of times in the Universe

Time period time interval / s	Time period time interval / s
age of the Universe	10^{18}
time since dinosaurs became extinct	10^{15}
time since humans first appeared on Earth	10^{13}
time since the pyramids were built in Egypt	10^{11}
typical human lifetime	10^{9}
one day	10^{5}
time between human heartbeats	1
time period of high-frequency sound	10^{-4}
time for light to travel across a room	10^{-8}
time period of oscillation of a light wave	10^{-15}
time for light to travel across a nucleus	10^{-23}

Estimation

Sometimes we do not have the data needed for accurate calculations, or maybe calculations need to be made quickly. Sometimes a question is so vague that an accurate answer is simply not possible. The ability to make sensible estimates is a very useful skill that needs plenty of practice.

When making estimates, different people will produce different answers and it is usually sensible to use only 1 (maybe 2) significant figures. Sometimes only an order of magnitude is needed.

The numbers in the list of kinetic energies given above cannot be given with precision because the situations are vague and there are a wide range of possibilities (with the exception of the Earth's kinetic energy). For example, boys (of all ages) in 100 m races could have masses between 20 kg and 70 kg (or more), and they could run at speeds between $2\,\text{m s}^{-1}$ and $9\,\text{m s}^{-1}$. These figures correspond to a kinetic energy range of 90–2800 J. To give a value with 2, or 3, significant figures would be misleading. It is more sensible to give a typical value to the nearest power of 10 (or sometimes, 1 significant figure). In the case of boys running 100 m, values of 10^2 J or 10^3 J may be considered typical (a matter of opinion).

WORKED EXAMPLE A3.4

A constant resultant horizontal force of 40 N accelerated a box over a distance of 50 cm.

a How much work was done on the box?

b State the assumption that you made answering part **a**.

c Calculate the kinetic energy that was gained by the box.

d If the box was initially at rest and then reached a speed of $2.9\,\text{m s}^{-1}$, what was its mass?

Answer

a $W = Fs = 40 \times 0.50 = 20\,\text{J}$

b The force was in the same direction as the motion of the box.

c 20 J

d $E_k = \frac{1}{2}mv^2$

$$20 = \frac{1}{2} \times m \times 2.9^2$$

$$m = \frac{(2 \times 20)}{2.9^2}$$

$$= 4.8\,\text{kg}$$

Understand the significance of uncertainties in raw and processed data

Although scientists are perceived as working towards finding 'exact' answers, an unavoidable **uncertainty** exists in every measurement. The results of all scientific investigations have uncertainties and errors, although good experimenters will try to keep these as small as possible.

When we receive numerical data of any kind (scientific or otherwise) we need to know how much belief we should place in the information that we are reading or hearing. The presentation of the results of serious scientific research should always have an assessment of the uncertainties in the findings, because this is an integral part of the scientific process. Unfortunately, the same is not true of much of the information we receive through the media, where data is too often presented uncritically and unscientifically, without any reference to its source, uncertainties or reliability.

No matter how hard we try, even with the very best of measuring instruments, it is simply not possible to measure anything exactly. For one reason, the things that we can measure do not exist as perfectly exact quantities; there is no reason why they should.

This means that every measurement is an approximation. A measurement could be the most accurate ever made, for example the width of a ruler might be stated as 2.283 891 03 cm, but that is still not perfect, and even if it was, we would not know because we would always need a more accurate instrument to check it. In this example we also have the added complication of the fact that when measurements of length become very small, we have to deal with the atomic nature of the objects that we are measuring.

A measurement is **accurate** if is close to its true value. For example, if you weigh a mass of 90.1 g and the result is 90.1 g, then the measurement can be described as accurate. However, be aware that in scientific research, true values are usually not known.

If the same mass (90.1 g) was measured by a different method, or a different person and the result was 89.8 g, then there was a clear **error** in the measurement. An error has occurred if the measurement is different from its true value (using an appropriate number of significant figures for the comparison).

Significant errors are often due to faulty apparatus or poor experimental skills. It is often possible to correct the source of such errors.

A **systematic error** occurs because there is something consistently wrong with the measuring instrument or the method used. A reading with a systematic error is always either bigger or smaller than the correct value by the same amount. Common causes are instruments that have an incorrect scale (wrongly calibrated), or instruments that have an incorrect value to begin with, such as a meter that displays a reading when it should read zero. This is called a **zero-offset error** – an example is shown in Figure A3.18. A thermometer that incorrectly records room temperature will produce systematic errors when used to measure other temperatures.

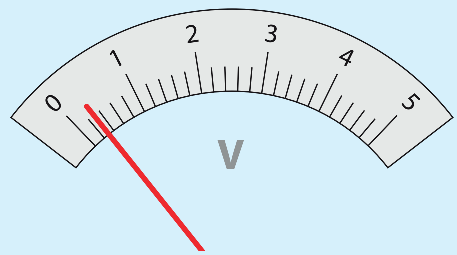


Figure A3.18 This voltmeter has a zero-offset error of 0.3 V, so that all readings will be too large by this amount.

Uncertainties in measurements are an indication of the amount of variation seen in the readings taken (without considering their accuracy). For example, if you weigh the same (unknown) mass five times you might get the following results: 53.2 g, 53.4 g, 52.9 g. 53.0 g, and 53.1 g. The results are not all the same, so there is clearly some random uncertainty in the results. The uncertainty in the use of the measuring instrument itself is usually assumed to be equal to the smallest division on its scale / display. In this example this is ± 0.1 g.

All experimental data has uncertainties. Sometimes uncertainties will arise because of difficulties in taking measurements. For example, human reaction times affect measurements made when using a stopwatch, or measuring the distance moved by something moving quickly can be difficult.

It is usually not possible to reduce uncertainties using the same apparatus and techniques.

In science the word **precise** means that measurements have low uncertainty. If, and when, measurements are repeated they will produce similar results. Consider Figure A3.19, which shows where arrows fired at a target (not shown) landed on eight separate occasions. Because we do not know where the target is, we cannot tell if the arrows were fired accurately, or not. But we can describe **b** and **d** as more precise than **a** and **c**.

◆ **Uncertainty (random)** The range, above and below a stated value, over which we would expect any repeated measurements to occur. Uncertainty can be expressed in absolute, fractional or percentage terms.

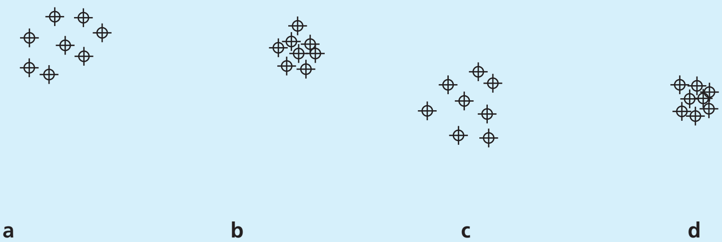
◆ **Accuracy** A single measurement is described as accurate if it is close to the correct result. A series of measurements of the same quantity can be described as accurate if their mean is close to the correct result.

◆ **Error** When a measurement is not the same as the correct value.

◆ **Systematic error** An error which is always either bigger or smaller than the correct value by the same amount, for example a zero-offset error.

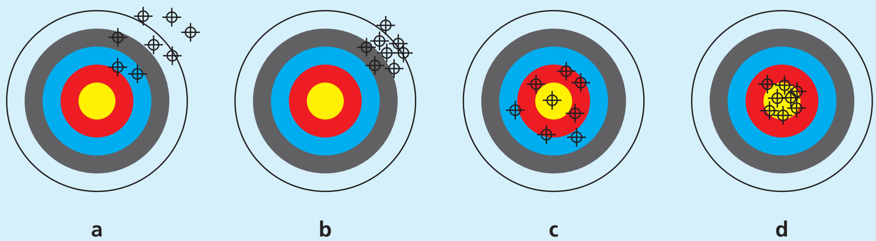
◆ **Zero offset error** A measuring instrument has a zero offset error if it records a non-zero reading when it should be zero.

◆ **Precision** A measurement is described as precise if a similar result would be obtained if the measurement was repeated.



■ **Figure A3.19** Precision of arrows hitting unseen target

In Figure A3.20 the positions of the targets have been included and we can see that the accuracy was good in **d**. But note that the accuracy seen in **c** is also good, because the average position of the arrows is close to the centre of the target.



■ **Figure A3.20** Accuracy of arrows hitting target

Common mistake

Uncertainties and errors are often confused, and different sources may define them slightly differently. **See Tool 3: Mathematics (Propagating uncertainties) on page 131.**

WORKED EXAMPLE A3.5

A student wanted to determine the increase of a trolley's kinetic energy as it accelerated down a slope. The trolley had a mass of 576 g ± 5 g and its length was 28.0 cm ± 0.5 cm. Using a stopwatch the student measured the time for the trolley to pass a point near the top of the slope to be 1.26 s. Near the bottom of the slope the trolley took 0.73 s to pass a particular point. Because it was difficult to start and stop the stopwatch at exactly the right time, it was estimated that the uncertainty in each time measurement was 0.10 s.

Calculate a value for the increase in kinetic energy of the trolley. Determine the absolute and percentage uncertainties in the answer. Comment on your answer

Answer

Near the top:

$$E_\text{k} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.576 \times \left(\frac{0.28}{1.26}\right)^2 = 1.42 \times 10^{-2}\,\text{J}$$

- Percentage uncertainty in mass = 100 × (5/576) = 0.87%
- Percentage uncertainty in length = 100 × (0.5/28) = 1.79%
- Percentage uncertainty in time = 100 × (0.10/1.26) = 7.94%
- Percentage uncertainty in kinetic energy = 0.87 + 1.79 + 1.79 + 7.94 + 7.94 = 20.33%
- Absolute uncertainty in kinetic energy = (20.33/100) × (1.42 × 10^{-2}) = 0.29 × 10^{-2} J

Near the bottom:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.576 \times \left(\frac{0.28}{0.73}\right)^2 = 4.24 \times 10^{-2}\,\text{J}$$

- Percentage uncertainty in mass = 100 × (5/576) = 0.87%
- Percentage uncertainty in length = 100 × (0.5/28) = 1.79%
- Percentage uncertainty in time = 100 × (0.10/0.73) = 13.70%
- Percentage uncertainty in kinetic energy = 0.87 + 1.79 + 1.79 + 13.70 + 13.70 = 31.85%
- Absolute uncertainty in kinetic energy = (31.85/100) × 4.24 × 10^{-2} = 1.35 × 10^{-2} J
- Difference in kinetic energies = (4.24 × 10^{-2}) − (1.42 × 10^{-2}) = 2.82 × 10^{-2} J
- Absolute uncertainty in difference = (0.29 × 10^{-2}) + (1.35 × 10^{-2}) = 1.64 × 10^{-2} J
- Percentage uncertainty in difference = $\left(\frac{1.64}{2.82}\right) \times 100 = 58\%$

This high percentage uncertainty in the results of this experiment may be surprising. The experiment needs redesigning if accurate results are needed.

Other types of kinetic energy

We can only use $E_k = \frac{1}{2}mv^2$ to determine the translational kinetic energy of objects travelling from place to place. Objects which are vibrating or rotating also have kinetic energy, but we need different equations to calculate their values. This will be covered in Topics A.4 and C.1 (for HL students).

12 Calculate the kinetic energy of a 57 g tennis ball served with a speed of 50 m s^{-1}.

13 a Determine the work needed to be done on a 1800 kg car to accelerate it from rest to 20 m s^{-1}.

b What average resultant force is needed to do this in a horizontal distance of 100 m?

c If the car decelerates from the same speed to rest in 70 m, calculate the average force exerted on the car

d What are the locations of this force?

14 a Calculate the kinetic energy of an electron (mass = 9.110 × 10^{-31} kg) moving with a speed which is 5% of the speed of light (3.0 × 10^8 m s^{-1}).

b What is the momentum of this electron?

15 Suppose that the force on the box in Worked example A3.4 continued to act for another 50 cm. Determine the final speed of the box after it had moved a total distance of 1.0 m.

16 a Calculate the kinetic energy of a high-speed train (see Figure A3.21) which has a mass of 7.5 × 10^5 kg and is moving with a speed of 300 km h^{-1}.

b Compare your answer to the kinetic energy of a typical family car on a motorway.

c What average force is needed to reduce the speed of the train uniformly to zero in a distance of 5.0 km?

d Suggest two reasons why high-speed railway systems do not have many stations.

Figure A3.21 A high-speed train

Gravitational potential energy

Consider raising a mass m a vertical height Δh, as shown in Figure A3.22. The minimum force required is equal to the weight of the mass, $F_g = mg$.

The work done by the force on the mass, $W = Fs = mg\Delta h$

This amount of energy has been transferred from the origin of the force to the raised mass. We say that the mass has gained gravitational potential energy, ΔE_p. If the mass is allowed to fall back down the same distance, the same amount of energy could be transferred to do useful work. Falling water in a **hydroelectric power** station uses this principle.

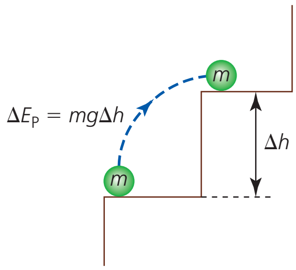


Figure A3.22 A mass gaining gravitational potential energy

The gravitational potential energy of a mass raised a height Δh, close to the Earth's surface can be calculated using $\boldsymbol{\Delta E_p = mg\Delta h}$

◆ **Hydroelectric power** The generation of electrical power from falling water.

It is important to realize that this equation can only be used accurately where a value of g can be considered as constant over the distance involved. For example, $g = 9.8\,\text{N kg}^{-1}$ cannot be used accurately when moving masses large distances up from the Earth's surface.

LINKING QUESTION

- Why is the equation for the change in gravitational potential energy only relevant close to the surface of the Earth, and what happens when moving further away from the surface?

This is discussed in Topic D.1 for HL students.

Top tip!

A mass resting on a table, or on the ground, does not have *zero* gravitational potential energy. When we use $\Delta E_p = mg\Delta h$ we are calculating how much more, or less, gravitational potential energy the object has in its new position compared to the place from where it was moved.

In other words, we are calculating a *change* in gravitational potential energy.

In the detailed study of gravitational fields (Topic D.1) HL students will need to consider if there is any place where an object really does have zero gravitational energy.

WORKED EXAMPLE A3.6

Estimate the gravitational potential energy gained by a teenage girl who moves from ground level to the viewing platform on the 124th floor of the Burj Khalifa in Dubai (see Figure A3.23).

Assume that the girl has a mass of 40 kg and the height of each floor is 3.5 m.

Figure A3.23 The Burj Khalifa in Dubai

Answer

$\Delta E_p = mg\Delta h = 40 \times 9.8 \times (124 \times 3.5) \approx 2 \times 10^5\,\text{J}$

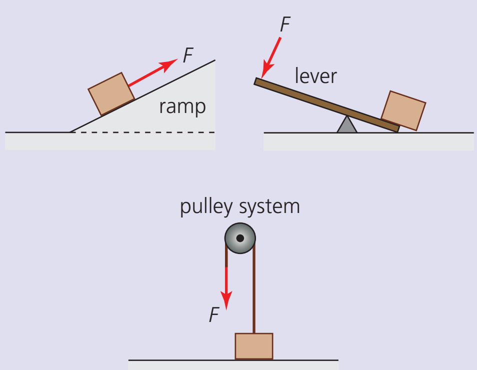
ATL A3A: Research skills

Use search engines and libraries effectively; evaluate information sources for accuracy, bias, credibility and relevance

When lifting a heavy object, the amount of gravitational potential energy that we need to transfer to it is decided only by its weight and the vertical height ($\Delta E_p = mg\Delta h$). For example, when a 50 kg mass is raised a height of 1 m it gains about 500 J of gravitational potential energy. Although 500 J may not be a lot of work to do, that does not mean that we can do this job easily.

There are two main reasons why this job could be difficult. Firstly, we may not be able to transfer that amount of energy in the time required to do the work. Another way of saying this is that we may not be powerful enough. (Power is discussed later in this topic.) Secondly, we may not be strong enough because we are not able to provide the required upwards force of 500 N. Power and strength are often confused with each other in everyday language.

Lifting (heavy) weights is a common human activity and many types of simple 'machine' were invented many years ago to make this type of work easier, by reducing the force needed. These include the ramp (inclined plane), the **lever** and the **pulley** (Figure A3.24).



■ **Figure A3.24** Simple machines which can be used to raise loads

In each of these simple machines the force needed to do the job is reduced, but the distance moved by the force is increased. If there was no energy dissipation (mainly due to friction), the work done by the force (Fs) would equal the useful energy transferred to the object being raised ($mg\Delta h$). In practice, because of energy dissipation, we will transfer more energy using a machine than if we lifted the load directly, without the machine. However, this is not a problem because we are usually much more concerned about how easy it is to do a job, rather than the total energy needed, or the efficiency of the process.

Figure A3.25 shows another example of a simple machine: a car jack being used to raise one side of a car.

■ **Figure A3.25** Changing a car tyre using a simple machine (car jack)

By changing the design of a car jack, it is possible in theory to raise the car with any sized force that we choose. For example, a force of 50 N may raise the wheel off the ground if the handle is rotated 10 times, whereas a force of 10 N would require about 50 rotations of the handle to transfer the same amount of energy. The heavy weight of the car will produce a lot of friction in the car jack.

The construction of many ancient structures was only possible because of the simple machines seen in Figure A3.24. Research online to learn some details of how the pyramids (or any other building of a similar age) were made. Compare the information you retrieve from different sources. Do the sources agree? Which sources do you consider most credible? Why?

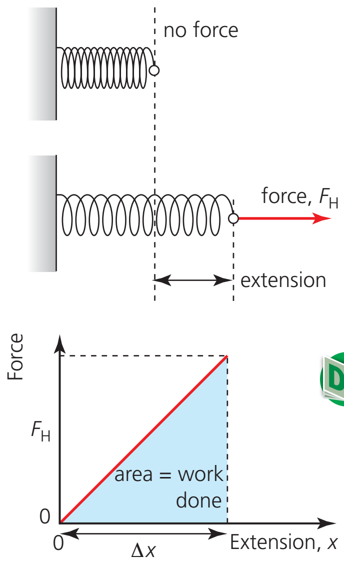
◆ **Lever** A simple machine consisting of a rigid bar and a pivot. Used to change the direction and magnitude of a force.

◆ **Pulley** A rotating wheel used to change the direction of a force. When two or more pulleys are combined, the system can reduce the force needed to do work.

Elastic potential energy

When a spring, or an elastic material, is deformed from the shape it had when there was no force acting on it, as in Figure A3.26, it will become a store of elastic potential energy that can later be used to transfer useful energy when the spring / material returns to its original shape.

Remember Hooke's law for elastic stretching from Topic A.2: $F_H = -kx$, where k is the 'spring constant'. Note also that, if a force on a spring / elastic material increases from 0 to F_H, the average force used during the deformation is $\frac{1}{2}F_H$, assuming that the deformation is proportional to the force (as shown in Figure A3.26).



■ **Figure A3.26** Force and extension when stretching a spring

For a spring / material which obeys Hooke's law, the work done, W, when it is deformed a distance Δx by a force F_H, is given by:

$$W = \text{average force} \times \text{distance} = \frac{1}{2}F_H \times \Delta x = \frac{1}{2}(k\Delta x)x = \frac{1}{2}k\Delta x^2$$

The work done is equal to the stored elastic potential energy, E_H. (There is no need to include a negative sign because work is not a vector quantity.)

The elastic potential energy of a deformed spring / material which obeys Hooke's law can be calculated by using:

$$\boldsymbol{E_H = \frac{1}{2}k\Delta x^2}.$$

As explained earlier in this topic, the work done is equal to the area under force–extension graph.

WORKED EXAMPLE A3.7

An open-coiled compression spring, like the one seen in Figure A3.27 has an overall length of 5.64 cm. A compressive force of 59.0 N reduced its length to 5.30 cm.

a Determine the spring constant, assuming that Hooke's law was obeyed.

b Calculate how much work was done to compress the spring.

c Use $E_H = \frac{1}{2}k\Delta x^2$ to confirm the amount of elastic potential energy stored in the spring.

■ **Figure A3.27** An open-coiled compression spring

Answer

a $k = \frac{F_H}{x} = \frac{59.0}{(5.64 - 5.30)} = 174\,\text{N cm}^{-1}$ (or $1.74 \times 10^4\,\text{N m}^{-1}$)

b $W = \frac{1}{2}F_H\Delta x = 29.5 \times (5.64 - 5.30) \times 10^{-2} = 0.100\,\text{J}$

c $E_H = \frac{1}{2}k\Delta x^2 = 0.5 \times (1.74 \times 10^4) \times (3.4 \times 10^{-3})^2 = 0.100\,\text{J}$

The answers to **b** and **c** are the same, as we would expect.

See Figure A3.6 for an example of how to determine elastic potential energy from a force–extension graph.

17 How much potential energy is transferred when:

a a 1.2 kg box is raised from the floor to a table top 0.85 m higher

b a 670 g book falls to the floor from the same table?

18 A cable car rises a vertical height of 700 m in a total distance travelled of 6.0 km.

a Show that approximately 15 MJ of gravitational potential energy must be transferred to a car of mass 1800 kg during the journey if it has six passengers with an average mass of 47 kg.

b Suggest why considerably more energy (than your answer to part **a** has to be transferred in making this journey.

■ **Figure A3.28** Ngong Ping cable car in Hong Kong

19 A rocket launches a 500 kg satellite to a height of 400 km above the Earth's surface.

a Outline why the equation $\Delta E_p = mg\Delta h$, with $g = 9.8\,\mathrm{N\,kg^{-1}}$ cannot be used to accurately determine the gravitational potential energy that has to be transferred to the satellite.

b However, the actual value of g at that height has not reduced as much as many students expect: during the launch it reduced from $9.8\,\mathrm{N\,kg^{-1}}$ to a value of $8.7\,\mathrm{N\,kg^{-1}}$ at a height of 400 km. Predict the gravitational potential energy gained by the satellite.

20 A spring has a spring constant of $384\,\mathrm{N\,m^{-1}}$ and is stretched by 2.0 cm.

a Calculate the elastic potential energy stored in the spring. Assume that it obeys Hooke's law.

b Predict a value for the extension that would be needed to store 1.0 J of energy.

c Explain why the answer to part **b** is uncertain.

21 a Calculate the work done in raising the centre of gravity of a trampolinist of mass 62 kg through a vertical height of 3.48 m (see Figure A3.29).

b When he lands on the trampoline he is brought to rest for a moment before being pushed up in the air again. If the maximum displacement of the trampoline is 0.90 m, sketch a possible force–displacement graph for the surface of the trampoline.

■ **Figure A3.29** The more the trampoline stretches, the higher the trampolinist can jump

Conservation of mechanical energy

SYLLABUS CONTENT

- In the absence of frictional resistive forces, the total mechanical energy of a system is conserved.
- If mechanical energy is conserved, work is the amount of energy transformed between different forms of mechanical energy in a system, such as: kinetic energy, gravitational potential energy and elastic potential energy.

Applying the law of conservation of energy to mechanical systems:

kinetic energy + gravitational potential energy + elastic potential energy = constant

◆ **Conservative force** A force, the action of which conserves mechanical energy. There is no energy dissipation.

But this is only true if there are no frictional (resistive) forces acting. Such forces can be described as 'non-conservative'. **Conservative forces** (such as gravitational forces, for example) conserve mechanical energy and do not involve the dissipation of energy. (More precisely: a *conservative force* is one for which the total work done in moving between two points is independent of the path taken.)

We have already explained that energy dissipation occurs in all mechanical systems to a greater or lesser extent, because of ever-present frictional / resistive forces (non-conservative forces). However, the equation above remains very useful for predicting 'ideal' outcomes, and for determining the amount of energy dissipated in other situations.

One of the most common examples of mechanical energy transfers is that between gravitational potential energy and kinetic energy. Assuming no resistive forces: change of gravitational potential energy = change of kinetic energy

$$mg\Delta h = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

If the mass starts or ends with zero velocity during the time being considered: v^2(or u^2) = $2g\Delta h$. leading to: v (or u) = $\sqrt{2g\Delta h}$.

This equation can be used to relate height with speed for any mass falling from rest, or any mass projected upwards to its highest point (assuming that gravity is the only force acting).

Note that the same equation can be obtained using the equation of motion (see Topic A.1): $v^2 = u^2 + 2as$, with u or v equal to zero, and using s instead of h.

WORKED EXAMPLE A3.8

A ball is thrown upwards with a speed of $23\,\text{m}\,\text{s}^{-1}$. Calculate the maximum height that it can reach.

Answer

$u^2 = 2g\Delta h$

$23^2 = 2 \times 9.8 \times \Delta h$

$\Delta h = 27\,\text{m}$

WORKED EXAMPLE A3.9

A ball of mass 7.8 g was fired horizontally by a spring, as shown in Figure A3.30. The spring has a spring constant of 620 N m^{-1} and was pulled back 3.7 cm from its uncompressed length.

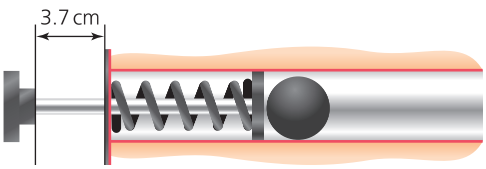


Figure A3.30 Ball being fired horizontally by a spring

a Calculate how much elastic potential energy was stored in the spring. Assume that it obeys Hooke's law.

b What average force was used to compress the spring?

c Determine the maximum possible speed of the ball after the spring was released.

d Explain why the actual speed will be less than your answer to part **c**.

Answer

a $E_H = \frac{1}{2}k\Delta x^2 = 0.5 \times 620 \times 0.037^2 = 0.42\,\text{J}$

b $E_H = W = Fs$

$0.42 = F \times 0.037$

$F = 11\,\text{N}$ (This was half of the maximum force.)

c $0.42 = \frac{1}{2}mv^2 = 0.5 \times 0.0078 \times v^2$

$v = 10\,\text{m s}^{-1}$

d All of the elastic potential energy in the spring was not transferred to the kinetic energy of the ball. For example, there was some friction with the sides of the tube, the spring did not stop moving after propelling the ball, there was some sound produced.

WORKED EXAMPLE A3.10

A box of mass 4.7 kg slid down a slope of vertical height 80 cm.

a Calculate the gravitational potential energy of the box at the top of the slope (compared to the bottom of the slope).

b Assuming the conservation of mechanical energy, what would the speed of the box be at the bottom of the slope?

c But the actual speed of the box was measured to be 2.2 m s^{-1}. Explain why the speed was less than the answer to part **b**.

d Determine how much energy was dissipated into the surroundings.

e In what form(s) was this dissipated energy?

Answer

a $\Delta E_p = mg\Delta h = 4.7 \times 9.8 \times 0.80 = 37\,\text{J}$

b $37 = \frac{1}{2}mv^2 = 0.5 \times 4.7 \times v^2$

$v = 4.0\,\text{m s}^{-1}$

Note that this answer (which assumes no energy dissipation) would be the same for any mass on any slope, or a vertical fall.

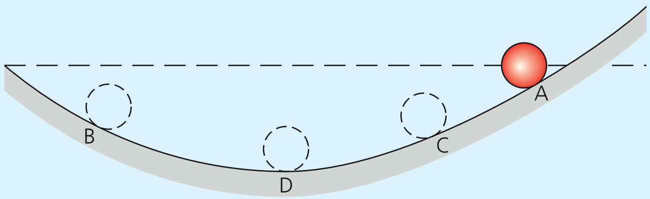
c Friction with the slope (and a little air resistance) acted in the opposite direction to motion.

d $37 - \frac{1}{2}mv^2 = 37 - (0.5 \times 4.7 \times 2.2^2) = 26\,\text{J}$

e Internal energy in the surfaces of the slope and box (which then spreads out as thermal energy).

22 What is the maximum speed with which a mass can hit the ground after being dropped from a height of 1.80 m?

23 The ball shown in Figure A3.31 was released from rest in position A. It accelerated down the slope and had its highest speed at the lowest point. It then moved up the slope on the other side, reaching its highest point at B.

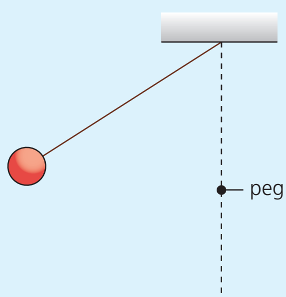


■ **Figure A3.31** Ball rolling down slope

a Explain why B is lower than A.

b Describe the motion of the ball after leaving position B, explaining the energy transfers until it finally comes to rest at D.

24 The large angle swing of a pendulum is interrupted by a peg as shown in Figure A3.32. Sketch a copy of the diagram and indicate the position of the pendulum after it has momentarily stopped moving on the right-hand side of the peg.



■ **Figure A3.32** Pendulum

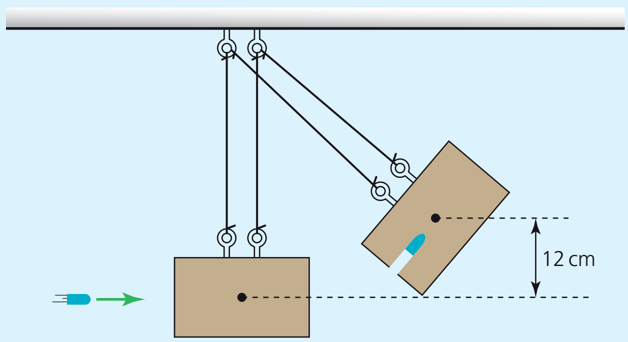
25 Figure A3.33 shows an experiment to measure the speed of a bullet which was fired into a block of wood. The bullet is embedded in the wood, so this was a totally inelastic collision. The block of wood and the bullet had kinetic energy and this was transferred to gravitational potential energy as they swung upwards.

a Outline what is meant by a totally inelastic collision.

b If the combined mass of the block and the bullet was 1.23 kg, determine their maximum gravitational potential energy.

c Use the law of conservation of energy to show that the initial velocity of the combined block and bullet was 1.5 m s^{-1}.

d If the bullet's mass was 15 g, use the law of conservation of momentum to determine its speed.



■ **Figure A3.33** Experiment to measure the speed of a bullet

26 a A 10 g steel sphere moving to the left at 2.0 m s^{-1} collided with a similar sphere of mass 2.0 g moving in the opposite direction at 4.0 m s^{-1}. If after the collision the 10 g sphere remained stationary, determine what happened to the other sphere.

b Calculate the total kinetic energy:

i before the collision

ii after the collision.

c Was mechanical energy conserved in this collision?

d State the term we use to describe collisions like this.

27 In a laboratory experiment an 8.6 g wooden sphere moving at 0.39 m s^{-1} collided with a 5.7 g wooden sphere moving in the opposite direction with a speed of 0.72 m s^{-1}. After the collision they were both observed to move with a speed of 0.25 m s^{-1}, but in opposite directions.

a Show that these results are in good agreement with the law of conservation of momentum.

b Calculate the total kinetic energy:

i before the collision

ii after the collision.

b Was kinetic energy conserved in this collision?

c State the term we use to describe collisions like this.

28 A rubber band of mass 1.2 g was extended by 8.4 cm. The extension was proportional to the force and the band had a spring constant of 280 N m^{-1}

a If the force was released and the band fired vertically upwards, predict the maximum theoretical height that it could reach.

b Explain why, in practice, the height will be a lot less than your answer to part **a**.

29 A long steel wire of mass 150 g was extended by 3.2 cm by a force which had increased slowly from 0 N to 240 N.

a Assuming that the extension was proportional to the force, how much elastic potential energy was stored in the wire?

b The wire then snapped and the stored energy was transferred to the kinetic energy of the wire. Calculate an average value for the speed of the wire.

c Discuss why breaking some metal wires can be dangerous.

30 'Crumple zones' are a design feature of most vehicles (Figure A3.34). They are designed to compress and deform permanently if they are in a collision. Use the equation $Fs = \frac{1}{2}mv^2$ to help explain why a vehicle should not be too stiff and rigid.

Figure A3.34 The front of the car is deformed but the passenger compartment is intact

31 A bungee jumper (Figure A3.35) of mass 61 kg is moving at 23 m s^{-1} when the rubber bungee cord begins to become stretched.

a Calculate her kinetic energy at that moment.

b Figure A3.36 shows how the extension of the cord varies with the applied force. State what quantity is represented by the area under this graph.

c Describe the relationship between force and extension shown by this graph.

d Use the graph to estimate how much the cord has extended by the time it has brought the jumper to a stop.

Figure A3.35 Bungee jumping in Taupo, New Zealand

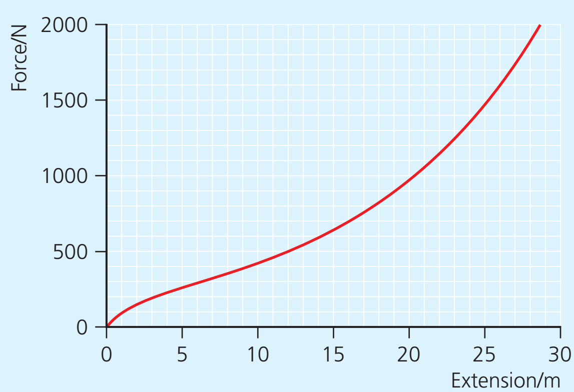


Figure A3.36 Force–extension graph for a bungee cord

32 A pole-vaulter of mass 59.7 kg falls from a height of 4.76 m onto foam.

a Calculate the maximum possible kinetic energy on impact.

b Will air resistance have had a significant effect in reducing the velocity of impact? Explain your answer.

c If the foam deforms by 81 cm, estimate the average force exerted on the pole-vaulter.

ATL A3B: Thinking skills

Apply key ideas and facts in new contexts

Regenerative braking

Most ways of stopping moving vehicles involve braking systems in which the kinetic energy of the vehicle is transferred to internal energy because of friction in the braking system and with the ground / track. The internal energy is dissipated into the surroundings as thermal energy and cannot be recovered.

The kinetic energy of a long, fast-moving train is considerable. Values of 10^8 J, or more, are not unusual. When the train stops, all of that energy has to be transferred to other forms and, unless the energy can be recovered, the same amount of energy then has to be transferred to accelerate the train again. This is very wasteful, so the train and its operation should be designed to keep the energy wasted to a minimum. One way of doing this is to make sure that large, fast trains stop at as few stations as possible, perhaps only at their origin and final destination.

A lot of research has gone into designing efficient **regenerative braking** systems in recent years, usually involving the generation of an electric current, which can be used to transfer energy to stored chemical changes in batteries, or to operate on-board electrical equipment.

The process of converting the kinetic energy of the train into electrical energy decelerates the train, so that there is much less need for frictional braking. This also reduces the wear on the brakes and the thermal energy transferred to the environment.

Small electric trains, which are often operated underground or on overhead tracks, are a feature of most large cities around the world (Figure A3.37). Such trains usually have stations every few kilometres or less, so regenerative braking systems and other energy-saving policies are very important.

■ **Figure A3.37** The Light Rail Transit trains on the SBS network in Singapore have regenerative braking

When designing a new urban train system, it has been suggested that energy could be saved by having a track shaped as shown in Figure A3.38.

What might be the advantages of such a track profile? Consider the energy transfers taking place as the train moves between stations.

■ **Figure A3.38** Possible track profile

◆ **Regenerative braking** Decelerating a vehicle by transferring kinetic energy into a form that can be of later use (rather than dissipating the energy into the surroundings). For example, by generating an electric current that charges a battery.

Power

SYLLABUS CONTENT

▶ Power, P, is the rate of work done, or the rate of energy transfer, as given by: $P = \frac{E}{t} = Fv$.

Power is the rate of transferring energy. When energy is transferred by people, animals or machines to do something useful, we are often concerned about how much time it takes for the change to take place. If the same amount of useful work is done by two people (or machines), the one that does it faster is said to be more *powerful*. (In everyday use the word power is used more vaguely, often related to strength and without any connection to time.)

◆ **Power, *P*** $\frac{\text{energy transferred}}{\text{time taken}}$ or, for mechanical energies, $\frac{\text{work done}}{\text{time taken}}$.

◆ **Watt, W** Derived SI unit of power. 1 W = 1 J s^{-1}.

$$\text{power} = \frac{\text{energy transferred}}{\text{time taken}} \qquad P = \frac{E}{t} \qquad \text{SI unit: } \textbf{watt, W}$$

$$\text{Alternatively: power} = \frac{\text{work done}}{\text{time taken}} \qquad \boldsymbol{P = \frac{\Delta W}{\Delta t}}$$

1 W = 1 J s^{-1}. The units mW, kW, MW and GW are all in common use. (Avoid confusion between work done, W and the unit the watt, W.)

The following are some examples of values of power in everyday life.

■ **Figure A3.39** Transferring energy at a rate of about 250 W

- A 0.0001 W calculator transfers energy at a rate of 0.0001 J every second.
- A 7 W light bulb transfers energy from electricity to light and thermal energy at a rate of 7 J every second.
- A girl walking upstairs may transfer chemical energy to gravitational energy at a rate of about 250 W.
- In many countries, homes use electrical energy at an average rate of about 1 kW.
- A 2 kW water heater transfers energy from electricity to internal energy at a rate of 2000 J every second.
- A typical family car might have a maximum output power of 100 kW.
- A 500 MW oil-fired power station transfers chemical energy to electrical energy at a rate of 500 000 000 J every second.

WORKED EXAMPLE A3.11

Calculate the average power of a 65 kg climber moving up a height of 40 m in 3 minutes.

Answer

$$P = \frac{\Delta W}{\Delta t} = \frac{(mg\Delta h)}{\Delta t} = \frac{(65 \times 9.8 \times 40)}{3 \times 60} = 1.4 \times 10^2\ \text{W}$$

Power needed to maintain a constant speed

◆ **Resistive force, F** Any force that opposes motion, for example friction, air resistance, drag.

It is common for a vehicle to maintain a constant velocity. Under those circumstances, the forward force, F, from the engines is equal in magnitude, but opposite in direction to the magnitude of the total **resistive forces**, F. Then:

Power, P, needed to maintain a constant velocity, v, against the resistive forces, can be determined from:

$$P = \frac{\Delta W}{\Delta t} = \frac{F\Delta s}{\Delta t} = Fv$$

DB Power needed to maintain a constant velocity, $\boldsymbol{P = Fv}$

WORKED EXAMPLE A3.12

a What average power is needed to accelerate a 1600 kg car from rest to 25 m s^{-1} in 12.0 s?

b What power is needed to maintain the same speed if the resultant resistive force is a constant 2300 N?

Answer

a $P = \dfrac{\Delta W}{\Delta t} = \dfrac{\text{kinetic energy gained}}{\text{time taken}} = \dfrac{\frac{1}{2}mv^2}{\Delta t} = \dfrac{(0.5 \times 1600 \times 25^2)}{12.0} = 4.2 \times 10^4\ \text{W}\ (= 42\ \text{kW})$

b $P = Fv = 2300 \times 25 = 5.8 \times 10^4\ \text{W}\ (= 58\ \text{kW})$

33 a How much useful energy must be transferred to lift twelve 1.7 kg bottles from the ground to a shelf that is 1.2 m higher?

b If this task takes 18 s, what was the average useful power involved?

34 Estimate the output power of an electric motor that can raise an elevator of mass 800 kg and six passengers 38 floors in 52 s. (Assume there is no counterweight.)

35 a Calculate the average power needed for a cyclist of mass 72.0 kg to accelerate from 8.00 m s^{-1} to 12.0 m s^{-1} in 22.0 s on a horizontal road. Assume that the resistive forces are negligible and the bicycle has a mass of 8.00 kg.

b Compare your answer to 6.5 W kg^{-1} (using body mass) for the best athletes.

36 A small boat is powered by an outboard motor with a maximum output power of 40 kW. The greatest speed of the boat is 27 knots (1 knot = 1.85 km h^{-1}). Determine the magnitude of the forward force provided by the motor at this speed.

37 a What is the constant speed of a car which has an output power of 22 kW when the resistive forces are 2.0 kN?

b What assumption did you make in answering part **a**?

38 What is the output power of a jet aircraft that has a forward thrust of 6.60×10^5 N when travelling at its top speed of 950 km h^{-1} (264 m s^{-1}) through still air?

39 a Find out which countries of the world have the highest average power consumption per person.

b Suggest reasons why they use so much energy.

Efficiency

SYLLABUS CONTENT

- Efficiency, η, in terms of energy transfer or power, as given by: $\frac{E_{\text{output}}}{E_{\text{input}}} = \frac{P_{\text{output}}}{P_{\text{input}}}$.

It is an ever-present theme of physics that, whatever we do, some of the energy transferred is 'wasted' (dissipated) because it is transferred to less 'useful' forms. In mechanics this is usually because friction, or air resistance, transfers kinetic energy to internal energy and thermal energy. The useful energy we get out of any energy transfer is *always* less than the total energy transferred.

When an electrical water heater is used, nearly all of the energy transferred makes the water hotter and it can therefore be described as 'useful', but when a mobile phone charger is used, for example, only some of the energy is transferred to the battery (most of the rest is transferred to thermal energy). Driving a car involves transferring chemical energy from the fuel and the useful energy is considered to be the kinetic energy of the vehicle, although at the end of the journey there is no kinetic energy remaining.

◆ **Efficiency (thermodynamic), η** Ratio of useful energy (or power) output to the total energy (or power) input.

A process that results in a greater useful energy output (for a given energy input) is described as being more efficient. In thermodynamics, **efficiency** is defined as follows:

$$\textbf{efficiency, } \eta = \frac{\textbf{useful energy output}}{\textbf{total energy input}} = \frac{E_{\text{output}}}{E_{\text{input}}}$$

Dividing energy by time to get power, we see that efficiency may also be defined as:

$$\textbf{efficiency, } \eta = \frac{\textbf{useful power output}}{\textbf{total power input}} = \frac{P_{\text{output}}}{P_{\text{input}}}$$

Because it is a ratio of two energies (or powers), efficiency has no units. It is often expressed as a percentage. It should be clear that, because of the principle of conservation of energy, efficiencies will always be less than 1 (or 100%).

It is possible to discuss the efficiency of any energy transfer, such as the efficiency with which our bodies transfer the chemical energy in our food to other forms. However, the concept of efficiency is most commonly used when referring to electrical devices and engines of various kinds, especially those in which the input energy or power is easily calculated. Sometimes we need to make it clear exactly what we are talking about. For example, when discussing the efficiency of a car, do we mean only the engine, or the whole car in motion along a road with all the energy dissipation due to resistive forces?

The efficiencies of machines and engines usually change with the operating conditions. For example, there will be a certain load at which an electric motor operates with maximum efficiency; if it is used to raise a very small or a very large mass it will be less efficient. Similarly, cars are designed to have their greatest efficiency at a certain speed, usually about $100\,\text{km}\,\text{h}^{-1}$. If a car is driven faster (or slower), then its efficiency decreases, which means that more fuel is used for every kilometre travelled.

Car engines, like all other engines that rely on burning fuels to transfer energy, are inefficient because of fundamental physics principles (see Topic B.4). There is nothing that we can do to change that, although better engine design and maintenance can make some improvements to efficiency.

In recent years we have all become very aware of the need to conserve the world's energy resources and limit the effects of burning fossil fuels in power stations and various modes of transport on global warming (see Topic B.2). Improving the efficiency of such 'heat engines' has an important role to play in this worldwide issue.

Figure A3.40 Power stations which use natural gas have the greatest overall efficiency (≈ 60%)

WORKED EXAMPLE A3.13

Figure A3.41 shows an experiment designed to measure the efficiency of a small electric motor. Electrical power was supplied to the motor at a rate of 0.72 W. The hanging mass (25 g) went up a distance of 1.12 m in 2.46 s.

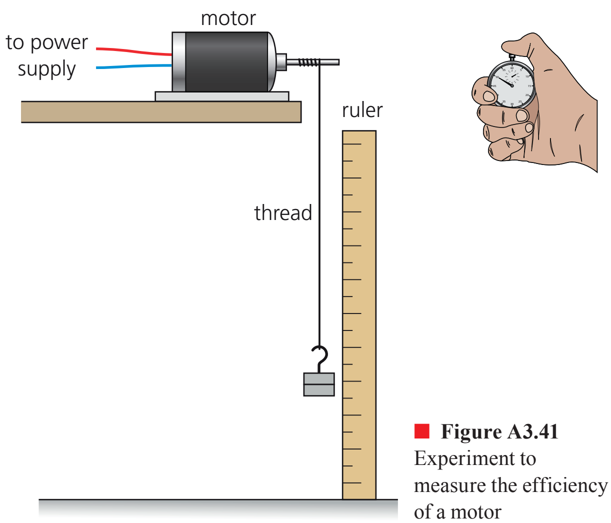
a Calculate how much gravitational potential energy was transferred to the mass.

b How much electrical energy was transferred in this time?

c Determine the efficiency of the motor.

d How much energy was dissipated into the surroundings?

e State the forms of this dissipated energy.



■ **Figure A3.41** Experiment to measure the efficiency of a motor

Answer

a $\Delta E_p = mg\Delta h = 0.025 \times 9.8 \times 1.12 = 0.27\,\text{J}$ (seen on calculator display as 0.2744...)

b power, $P = \dfrac{\text{energy transferred}}{\text{time taken}}$

energy transferred $= P \times \Delta t = 0.72 \times 2.46 = 1.8\,\text{J}$ (seen on calculator display as 1.7712...)

c efficiency, $\eta = \dfrac{\text{useful energy output}}{\text{total energy input}} = \dfrac{0.2744}{1.7712} = 0.15$ (15%)

d $1.7712 - 0.2744 = 1.5\,\text{J}$

e internal energy, thermal energy (+ sound)

We will see in Topic B.5 that the power input to any electric device can be determined from:
power = voltage × current.

Inquiry 3: Concluding and evaluating

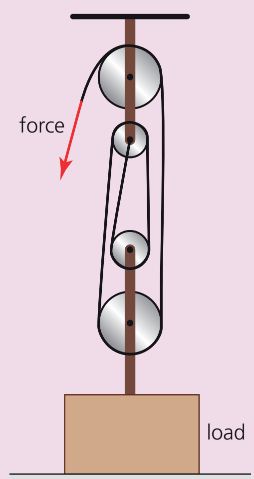
Concluding

Figure A3.42 represents the apparatus used by a student to investigate the efficiency of a pulley system.

The student measured the forces needed to lift loads of 0.5 kg, 1.0 kg, 4.0 kg and 5.0 kg: 0.6 N, 1.5 N, 7.5 N and 12.7 N. He assumed that the force always moves four times further than the load. His conclusion was that the efficiency of the system was 66%.

1 Is this a correct conclusion from the data collected?

2 Discuss the quantity and range of readings taken by the student.



■ **Figure A3.42** Pulley system

Energy density

SYLLABUS CONTENT

- Energy density of the fuel sources.

◆ **Energy density** The energy transferred from unit volume of fuel (SI unit: $J\,m^{-3}$).

◆ **Specific energy** Amount of energy that can be transferred from unit mass of an energy source (SI unit: $J\,kg^{-1}$).

One of the advantages of fossil fuels is the large amount of energy that can be transferred from each cubic metre of the fuels. Although, nuclear fuels are much more energy dense.

Energy density is the amount of energy that can be transferred from each cubic metre.

SI unit: $J\,m^{-3}$

Table A3.4 shows some typical energy densities of energy sources.

When discussing gaseous energy resources, it is more common to use **specific energy**, which is the amount of energy that can be transferred from each kilogram. For example, the specific energy of natural gas is $55\,MJ\,kg^{-1}$.

Table A3.4 Some approximate energy densities

Source	Energy density / $MJ\,m^{-3}$
Reactor-grade uranium-235	66 000 000 000
coal	43 000
gasoline (petrol)	36 000
crude oil	37 000
ethanol	24 000
wood	15 000
electrical batteries	1000
hydroelectric	1

Tool 2: Technology

Represent data in a graphical form

Use data from the internet to determine specific energies for a variety of different energy sources.

Use a spreadsheet to present the information in the form of a bar chart.

ATL A3C: Research skills

How did you check the reliability of your online sources?

Common mistake

Energy density and specific energy are often confused. Some sources define energy/mass as energy density.

Inquiry 1: Exploring and designing

Designing

Controlling heat losses

Figure A3.43 shows how a student intended to investigate the energy density of ethanol. She knows that for every 4184 J of energy that are transferred to 1.0 kg of water, the temperature will rise by 1.0 °C (see Topic B.2).

1 Identify the student's choice of dependent, independent and control variables. How would the results be used to calculate energy density?

2 Suggest how the experiment (as shown) could be improved. Hint: how much of the energy stored in the fuel is transferred to the water?

3 What other energy transfers are taking place?

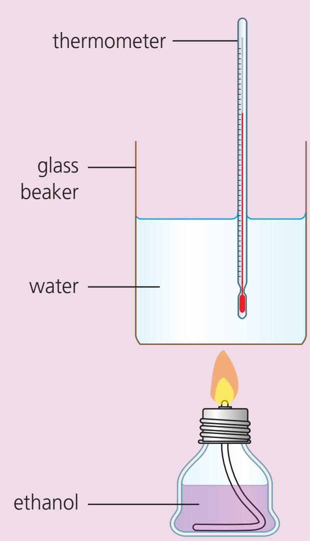


Figure A3.43 Estimating the energy density or specific energy of a fuel

40 The power output from a natural gas-fired electrical power station is 540 MW.

a If its efficiency is 48%, what is the input power?

b Calculate how much fuel the power station uses in one hour if 49 MJ can be obtained from each kilogramme.

41 An elevator (lift) which has a mass of 2400 kg when empty is connected as shown in Figure A3.44 to a counterweight of the same mass.

When the elevator goes up, the counterweight goes down. Five people of total mass 265 kg got in the elevator and went up four floors, each floor of height 3.2 m.

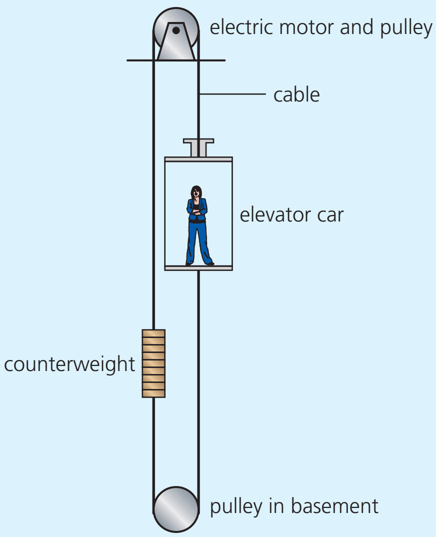


Figure A3.44 An elevator (lift) and its counterweight

a Outline the reason for using a counterweight.

b Calculate the useful work done by the motor.

c If, in fact, 1.3×10^5 J of energy was transferred from the electrical supply to the motor, determine the efficiency of the process.

d If the process took a total time of 18 s, what was the average input power to the motor?

e How much energy was transferred in overcoming friction?

f Discuss where friction would occur in this system.

42 Explain why it may be more useful to refer to the specific energy of natural gas, rather than its energy density.

43 A petrol-driven car was accelerated from rest in order to determine its overall efficiency.

a Calculate the efficiency if it gained kinetic energy of 4.0×10^5 J while using 55 ml (5.5×10^{-5} m^3) of fuel.

b Draw an annotated Sankey diagram to represent this process.

c Discuss whether it is reasonable to state that the car has zero efficiency while travelling with constant speed.

44 An airplane has a mass of 200 tonnes (2.0×10^5 kg) and take-off speed of 265 km h^{-1} (73 m s^{-1}) at the end of a distance of 2.24 km from where it began.

a Calculate the kinetic energy of the airplane when it takes off.

b Estimate the average power output from the airplane's engines while it is on the runway.

c What average resultant forward force was acting on the airplane during its movement along the runway?

d If 76 kg of fuel was used during take off, calculate the efficiency of the process if 1 kg of fuel can transfer 43 MJ.

45 An oil burning power station has an efficiency of 39% and an output of 770 MW.

a Calculate the mass of oil burned:

i every second **ii** every year.

b Estimate how much reactor grade uranium-235 would be needed every year to produce the same output power (assume the same efficiency).

46 Show that the energy density of wind blowing at a speed of 5 m s^{-1} (as could be used with a wind turbine to generate electricity) is about 15 J m^{-3}. The density of air is 1.3 kg m^{-3}.

ATL A3D: Thinking skills

Evaluating and defending ethical positions

After we have used any mode of transport, all of the energy used by the vehicle will have been dissipated into the environment. In a scientific sense, their efficiency is zero but, of course, they will have normally served a useful purpose.

For the sake of reducing pollution, conserving materials and limiting global warming, do individuals have any responsibility for limiting their travel? Or do people have the right to travel wherever and whenever they like, in whatever mode of transport they choose?

Should governments enact laws to limit our travel, and/or introduce or raise taxation on transportation and its fuels?

A.4 Rigid body mechanics

Guiding questions

- How can the understanding of linear motion be applied to rotational motion?
- How is the understanding of the torques acting on a system used to predict changes in rotational motion?
- How does the distribution of mass within a body affect its rotational motion?

Rotational dynamics

◆ **Dynamics** The science which explains the motion of objects.

◆ **Rotate** To move around a central point or axis (usually inside the rotating object).

◆ **Revolve** To move around a central point or axis (usually outside of the revolving object).

◆ **Rigid** Does not change shape.

◆ **Extended object** An object that has dimensions. Not a point.

◆ **Axis of rotation** Line about which an object can rotate.

Dynamics is the name we give to that branch of physics which is concerned with motion and its causes (forces). In Topics A.1, A.2 and A.3 we have mostly considered linear dynamics: bodies moving in straight lines. In this topic we will study the rotational motion of objects/bodies about a fixed axis. The term **rotation** usually refers to movement about an axis within the body, for example the rotation of the Earth every 24 hours. The rotation of a body does not affect its location, whereas **revolution** usually concerns movement of a body around an exterior point, for example the Earth revolves around the Sun every year. (Some situations may be described as a rotation or a revolution.)

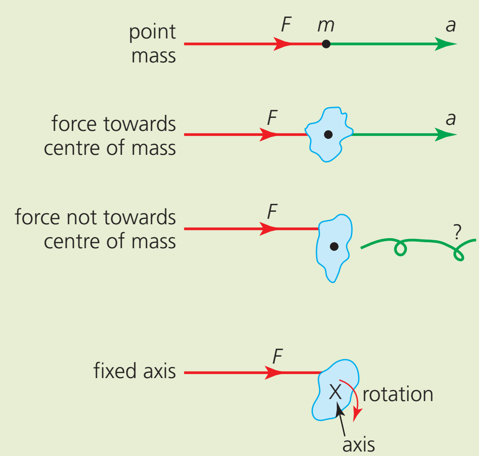
We will assume that the bodies are **rigid**, that is, their shape does not deform significantly under the action of forces. The simplest everyday examples of rotation include a wheel and a door handle.

Nature of science: Models

Point objects and extended objects

In Topics A.1–A.3 we have generally considered all bodies to be point objects. This was done in order to simplify the situations. This topic is different, as the focus of our attention will be on **extended objects** which are able to rotate.

We know that a resultant force acting on a point mass results in a linear translational acceleration ($F = ma$ from Topic A.2). This remains true if the force acts on an extended object, but only if the force is directed at the centre of mass of the object. However, when resultant forces are directed elsewhere on objects, the situations are complicated and the results will depend on how the magnitude and direction of the force vary after initial contact. See Figure A4.1.



■ **Figure A4.1** Resultant forces acting on point objects and extended objects

All of Topic A.4 is about rotations caused by forces on objects which are able to rotate in the same place because they have a fixed **axis of rotation** (wheels for example).

Comparing rotational motion to linear motion

We can greatly simplify our introduction to **rotational dynamics** by using our existing knowledge and understanding of linear dynamics. All the concepts of linear dynamics have rotational dynamics equivalents, with similar equations. They are summarized in Table A4.1.

Table A4.1 Comparing linear and rotational motion

Linear motion	Rotational motion
force	torque
mass (a measure of inertia)	moment of inertia
linear displacement	angular displacement
linear speed / velocity	angular speed / velocity
linear acceleration	angular acceleration
linear momentum	angular momentum
linear kinetic energy	rotational kinetic energy

◆ **Rotational dynamics** Branch of physics and engineering that deals with rotating objects.

◆ **Analogy** Applying knowledge of one subject to another because of some similarities.

TOK

Knowledge and the knower

- How do we acquire knowledge?

Analogies

An **analogy** is a useful comparison between two different things that have some features in common, with the intention that knowledge of one can be applied to the other. Making an analogy between linear and rotational motion is probably not surprising and, as we shall see, it is very useful. However, many other analogies may not be so obvious. For example, can the study of economics find useful analogies in the laws of physics?

Apart from assisting in the teaching and learning of a new situation, there may be two major purposes for using analogies.

1. An analogy may be used to make reliable predictions about the behaviour of the system to which it is applied and that, in itself, may be sufficient reason to justify the use of an analogy.
2. An analogy may help to provide a deeper understanding of the system to which it is applied. However, without further justification, analogies should not be assumed to be accurate descriptions of the systems to which they are applied.

Torque

SYLLABUS CONTENT

▶ The torque, τ, of a force about an axis, as given by: $\tau = Fr\sin\theta$

◆ **Pivot** A fixed point supporting something which turns or balances.

◆ **Hinge** Device which connects two solid objects allowing one (or both) to rotate in one direction.

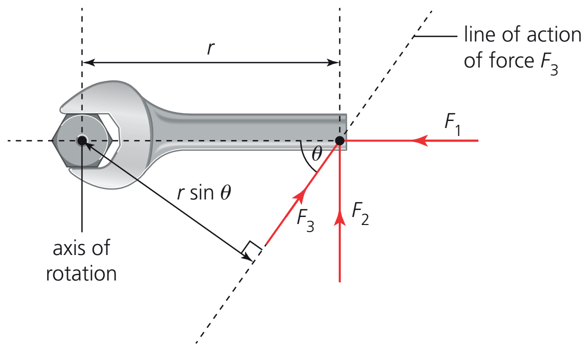
◆ **Fulcrum** See *pivot*.

◆ **Line of action (of a force)** A line through the point of action of a force, showing the direction in which the force is applied.

In situations where rotation may be possible, it is important to identify the place about which the rotation can occur. Most commonly this will involve an axis of rotation. (The terms **pivot**, **hinge** and **fulcrum** are widely used for various situations in which movement is not complete, nor continuous.)

A straight line showing the direction in which a force is applied is called its **line of action**. Any force applied to an object whose line of action is not through the axis of rotation will tend to start, or change, rotational motion, if that is possible.

Bigger forces will tend to produce larger rotational accelerations but the line of action (direction) of a force is also very important. See Figure A4.2.



■ **Figure A4.2** Forces producing rotation of a spanner (wrench) and bolt

In Figure A4.2, F_1 has no turning effect because its line of action is through the axis of rotation. F_2 has the biggest turning effect because its line of action is perpendicular to a line joining its point of application to the axis. F_3 has an effect between these two extremes. The turning effect also depends on the distance, r, from the axis of rotation to the line of action of the force.

The 'turning effect' of a force, F, is known as its **torque**, τ, and it depends on the magnitude of the force and the perpendicular distance from the axis of rotation to the line of action of the force. In Figure A4.2 this is shown for force F_3 as $r \sin\theta$, where θ is the angle between the line of action of the force and a line joining the point of application of the force to the axis of rotation; r is the distance from the axis of rotation to the point of application of the force.

DB **torque $\tau = Fr \sin\theta$**

◆ **Torque** Product of a force and the perpendicular distance from the axis of rotation to its line of action.

◆ **Moment (of a force)** Term sometimes used as an alternative to torque, especially if rotation is incomplete.

◆ **Principle of moments** If an object is in rotational equilibrium, the sum of the clockwise moments (torques) equals the sum of the anticlockwise moments (torques).

When there is no actual rotation, torque is sometimes called the **moment** of a force. (You may be familiar with the '**principle of moments**' for a body in equilibrium.)

Torque has the SI unit Nm (not Nm^{-1}) but note that it is not equivalent to the unit of energy, the joule, which is also Nm.

Inquiry 1: Exploring and designing

Exploring

The right tool for the job

A torque wrench is a device which limits the torque that can applied when tightening a bolt. This is to prevent over-tightening and damage to the bolt and, for example, an engine. There are various designs. Figure A4.3 shows a modern digital type. Demonstrate insight by reflecting on the image to suggest how this type of torque wrench might be used. Check your ideas through research.

■ **Figure A4.3** Torque wrench

WORKED EXAMPLE A4.1

Look again at Figure A4.2.

a If $r = 48$ cm, calculate the torque produced by a force of 35 N applied along the line of action of F_2.

b Determine the value of F_3 that would produce the same torque as in part **a**, if the angle $\theta = 55°$.

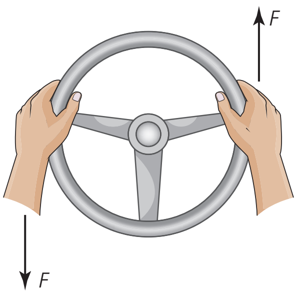
Answer

a $\tau = Fr \sin\theta = 35 \times 0.48 \times \sin 90° = 17\,\text{Nm}$

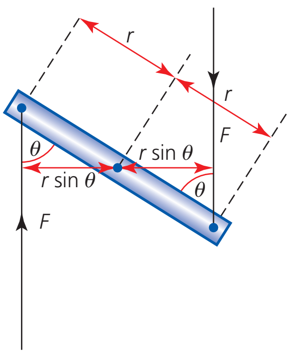
b $17 = F_3 \times 0.48 \times \sin 55°$

$$F_3 = \frac{17}{0.393} = 43\,\text{N}$$

◆ **Couple (forces)** Pair of equal-sized forces that have different lines of action, but which are parallel to each other and act in opposite directions, tending to cause rotation.



■ **Figure A4.4** A couple used to turn a steering wheel



■ **Figure A4.5** Calculating the torque provided by a couple

LINKING QUESTION

- How does a torque lead to simple harmonic motion?

This question links to understandings in Topic C.1.

Combining torques

Torque is a vector quantity, but generally we will only be concerned about its 'sense': whether it tends to produce clockwise or anticlockwise motion.

When more than one torque acts on a body the resultant (net) torque can be found by simple addition, but clockwise and anticlockwise torques will oppose each. For example, when an object is acted upon by a 12 Nm clockwise torque and a 15 Nm anticlockwise torque, the resultant torque is (15 − 12) = 3 Nm anticlockwise.

Couples

A **couple** is the name we give to a pair of equal-sized forces that have different lines of action but which are parallel to each other and act in opposite directions, either side of the axis of rotation.

A couple produces no resultant force on an object, so there is no translational acceleration; the object will remain in the same location. Figure A4.4 shows a typical example, a couple used to turn a steering wheel.

Other examples of using couples include the forces on a bar magnet placed in a uniform magnetic field, the forces on the handlebar of a bicycle and the forces on a spinning motor.

The magnitude of the torque provided by a couple is simply twice the magnitude of the torque provided by each of the two individual forces, $\tau = 2Fr \sin\theta$. See Figure A4.5.

Rotational equilibrium

SYLLABUS CONTENT

- Bodies in rotational equilibrium have a resultant torque of zero.
- An unbalanced torque applied to an extended, rigid body, will cause rotational acceleration.

If an object remains at rest, or continues to move in exactly the same way, it is described as being in *equilibrium*. *Translational equilibrium* occurs when there is no resultant force acting on an object (Newton's first law – Topic A.2), so that it remains stationary or continues to move with a constant velocity (that is, in a straight line at a constant speed). A similar definition applies to rotational motion:

Rotational equilibrium occurs when there is no resultant torque acting on an object, so that it remains stationary or continues to rotate with a constant angular speed (defined below).

If an object is in rotational equilibrium, there is no resultant torque, so that:
clockwise torque = anticlockwise torque.

If there is a resultant torque acting on a body, it will produce an angular acceleration. More details to follow.

WORKED EXAMPLE A4.2

Figure A4.6 shows a view of a door from above. A person is trying to push the door open with a force of 74 N in the direction and position shown.

Calculate the minimum force, F (magnitude and direction) needed at the handle to stop this happening.

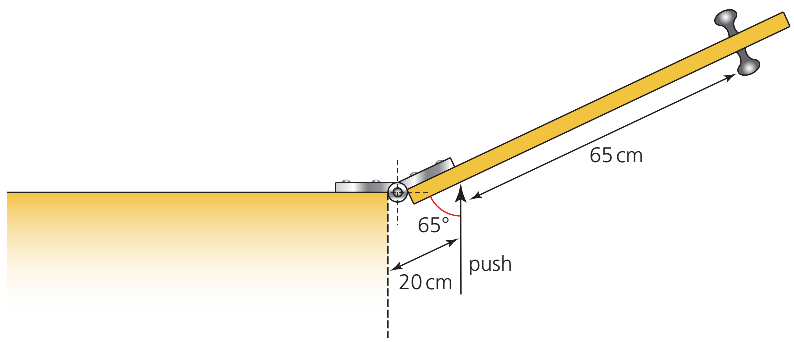


Figure A4.6 A view of a door from above

Answer

The maximum torque obtained with a given force at the door handle will be perpendicular to the door.

For the door to be in rotational equilibrium, the two torques must be equal in magnitude.

$Fr \sin\theta$ clockwise = $Fr \sin\theta$ anticlockwise

$F \times 0.65 \times \sin 90° = 74 \times 0.20 \times \sin 65°$

$0.65F = 13.4$

$F = 21$ N clockwise, perpendicular to the door.

1 A torque of 55 Nm is required to loosen a nut on an engine. Calculate the minimum force with which this can be achieved, if the length of the spanner (wrench) used is 25 cm.

2 Figure A4.7 shows a side-view of one pedal on a bicycle. The distance from the pedal to the axis is 21 cm.

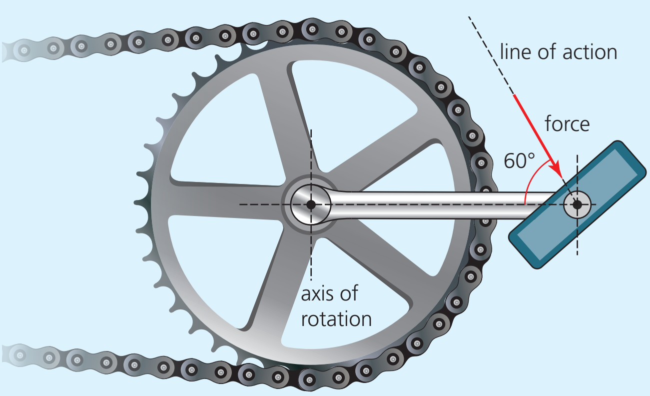


Figure A4.7 Bicycle pedal

a If a cyclist pushes down the line of action, as shown, with a force of 48 N, determine the torque that is being applied.

b Sketch a copy of the diagram but move the pedal to the position where the cyclist can probably apply the greatest torque.

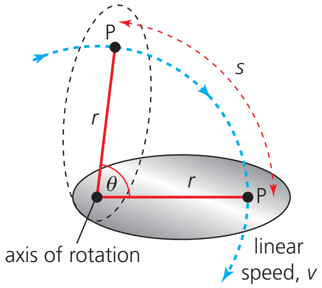
c In which part(s) of each rotation should no torque be applied by the cyclist?

3 a Determine the torque provided by the couple shown in Figure A4.5 if the force is 37 N, $r = 7.7$ cm and the angle θ is 49°.

b Sketch a graph to show how the torque would vary if the object moved from horizontal to vertical, as seen (assume that the directions of the forces do not change).

c Does the magnitude of the torque provided by the couple depend on the position of the axis of rotation? Explain your answer.

Angular displacement, velocity and acceleration



■ **Figure A4.8** Angular displacement, θ, of a point P on a rotating body

◆ **Angular displacement** The angle through which a rigid body has been rotated from a reference position.

SYLLABUS CONTENT

▶ The rotation of a body can be described in terms of angular displacement, angular velocity, and angular acceleration.

Angular displacement

Any point on a rigid rotating body will be moving along a circular path. See Figure A4.8 for an example.

Angular displacement is defined as the total angle, θ, through which a rigid body has rotated from a fixed reference position. It is measured in radians (or degrees).

In Figure A4.8, the point P is a distance r from the axis of rotation and it has travelled a distance s along the circumference of the circle (*arc length*). So that angular displacement in radians:

$$\theta = \frac{s}{r}$$

Tool 1: Experimental techniques

Understand how to accurately measure quantities to an appropriate level of precision: angle

Describe how you would measure the total angular displacement (in radians) through which the car jack handle seen in Figure A4.9 would need to be rotated in order to raise the side of the car exactly 5.0 cm. Estimate the percentage uncertainty in your measurement.

■ **Figure A4.9** A car jack

Angular velocity

Angular velocity is a vector quantity, but its direction will not be important here, so that angular speed and angular velocity can be considered to be equivalent.

Angular velocity has already been discussed in Topic A.2, where it was considered to be constant during uniform circular motion. More precisely:

Angular velocity, ω, is defined as the change of angular displacement divided by the time taken: $\omega = \frac{\Delta\theta}{\Delta t}$ SI unit: rad s^{-1}, radians (or degrees) per second

Angular velocities are often quoted in rotations per minute (rpm). 1 rpm is equal to 0.10 rad s^{-1} (to 2 significant figures).

■ **Figure A4.10** A car's tachometer displays rpm/1000

All points on a rigid rotating object will have the same angular velocity, but their linear speeds will be greater if they are further from the axis of rotation. We have seen in that for a body rotating with constant angular speed:

$$\omega = \frac{2\pi}{T}$$

and:

$$v = \frac{2\pi r}{T}$$

so that (as seen in Topic A.2):

linear speed, $v = \omega r$

WORKED EXAMPLE A4.3

In a fairground ride (Figure A4.11) which is moving with a constant linear speed, the passengers complete one rotation in 3.9 s.

■ **Figure A4.11** A fairground ride

a Calculate their angular velocity.

b What is their total angular displacement after one minute?

c Determine the angle between their current position and their starting position.

Answer

a $\omega = \frac{\Delta\theta}{\Delta t} = \frac{360}{3.9} = 92°\,s^{-1}$ or $\frac{2\pi}{3.9} = 1.6$ rad s^{-1}

b In 60 s they will have completed 60/3.9 = 15 rotations. (15.38... seen on calculator display)

Total angle moved through = 15.38 × 2π = 97 rad (5.5 × 10^{3}°)

c They are 0.38 of a complete rotation from their position at the beginning of the minute.

displacement = 0.38 × 2π = 2.4 radians (1.4 × 10^{2}°)

◆ **Angular acceleration, *α*** The rate of change of angular velocity with time, $\Delta\omega/\Delta t$ (SI unit: rad s^{-2}). It is related to the linear acceleration, a, of a point on the circumference by $\alpha = a/r$.

Common mistake

Do not confuse angular acceleration with centripetal acceleration.

Angular acceleration

Angular acceleration, *α*, is defined as the rate of change of angular velocity with time:

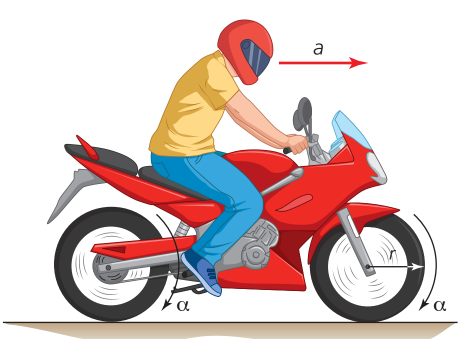
$$\alpha = \frac{\Delta\omega}{\Delta t}$$ SI unit: rad s^{-2} (or degrees per second squared)

There is a simple relationship between angular acceleration and linear acceleration of a point which is a distance r from the axis of rotation.

Since:

$$\Delta\omega = \frac{\Delta v}{r} \Rightarrow \alpha = \frac{\Delta v}{\Delta t r}$$

$$\alpha = \frac{a}{r}$$



■ **Figure A4.12** Comparing linear and angular acceleration $\alpha = \frac{a}{r}$

For example, Figure A4.12 shows a motor cyclist accelerating linearly at a rate a = 5.3 m s^{-2}. This is also the rate at which the edge of the tyre is accelerating. If the wheel and tyre have an outer radius of 34 cm, the angular acceleration of a wheel, $\alpha = \frac{a}{r}$ = 5.3 / 0.34 = 16 rad s^{-2}.

WORKED EXAMPLE A4.4

A motor spinning at 24 rotations per second accelerates uniformly to 33 rotations per second in 6.7 s.

Calculate its angular acceleration.

Answer

Initial angular velocity = $24 \times 2 \times \pi = 151\,\text{rad}\,\text{s}^{-1}$

Final angular velocity = $33 \times 2 \times \pi = 207\,\text{rad}\,\text{s}^{-1}$

Acceleration = $(207 - 151) / 6.7 = 8.4\,\text{rad}\,\text{s}^{-2}$

Equations of motion for angular acceleration

SYLLABUS CONTENT

- Equations of motion for uniform angular acceleration can be used to predict the body's angular position, θ, angular displacement $\Delta\theta$, angular speed, ω, and angular acceleration.

ATL A4A: Thinking skills

Write 'Linking questions' for the end of Topic A.1, A.2 and A.3 relating to the content of this topic (A.4) so far.

By direct analogy we can write down the equations for uniform angular acceleration. See Table A4.2. ω_i is the initial angular velocity (speed) at the start of time t. ω_f is the final angular velocity at the end of that time.

Table A4.2 Equations of motion

Equations for uniform linear acceleration	Equations for uniform angular acceleration
$s = \frac{(u+v)}{2}t$	$\Delta\theta = \frac{(\omega_f + \omega_i)}{2}t$
$v = u + at$	$\omega_f = \omega_i + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

WORKED EXAMPLE A4.5

An object which is rotating with an angular velocity of $54\,\text{rad}\,\text{s}^{-1}$ accelerates uniformly for 3.2 s and reaches an angular velocity of $97\,\text{rad}\,\text{s}^{-1}$.

a Calculate its angular acceleration.

b What was its angular displacement during the acceleration?

c It then decelerated to rest during an angular displacement of 156 rad. Determine the angular deceleration (negative acceleration).

Answer

a $\omega_f = \omega_i + \alpha t$

$97 = 54 + 3.2\alpha$

$\alpha = \frac{(97 - 54)}{3.2} = 13\,\text{rad}\,\text{s}^{-2}$

b $\Delta\theta = \frac{(\omega_f + \omega_i)}{2}t = \frac{(97 + 54) \times 3.2}{2} =$

$2.4 \times 10^3\,\text{rad}$

c $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

$0^2 = 97^2 + (2 \times \alpha \times 156)$

$\alpha = -\frac{9409}{312} = -30\,\text{rad}\,\text{s}^{-2}$

HL ONLY

4 A carriage on the London Eye (Figure A4.13) can rotate continuously at a speed of $26\,\text{cm}\,\text{s}^{-1}$. The wheel has a radius of 60 m.

a Calculate its angular velocity.

b Calculate how many minutes it takes the wheel to complete one revolution.

■ **Figure A4.13** The London Eye

5 A very large wind turbine (similar to those seen in Figure A4.14) has blades of length 80 m and has a maximum rotational speed of 15 rpm.
Determine the linear speed and the angular velocity of:

a the end of the blade

b a point 10 m from the axis of rotation

c Suggest why engineers limit the speed of rotation of the blades.

■ **Figure A4.14** Wind turbines

6 The outer rim of a bicycle wheel of radius 32 cm has a linear acceleration of $0.46\,\text{m}\,\text{s}^{-2}$.

a Calculate the angular acceleration of the wheel.

b If it starts from rest, determine the time needed for the wheel to accelerate to a rate of three rotations every second.

7 A wheel accelerates uniformly from rest at $5.2\,\text{rad}\,\text{s}^{-2}$.

a What is its angular velocity at the end of 5.0 s?

b Calculate its total angular displacement in this time.

c How many rotations does it complete in 5.0 s?

d After 5.0 s the accelerating torque is removed and the wheel decelerates at a constant rate to become stationary again after 18.2 s.
Calculate how many rotations are completed during this time.

8 A blade of a rotating fan has an angular velocity of $7.4\,\text{rad}\,\text{s}^{-1}$. It is then made to accelerate for 1.8 s, during which time it passes through a total angle of 26.1 rad. Calculate the angular acceleration of the fan blade.

9 A machine spinning at 3000 rpm is accelerated to 6000 rpm while the machine made 12 revolutions.

a Convert 3000 rpm to $\text{rad}\,\text{s}^{-1}$.

b Calculate the angular acceleration.

LINKING QUESTION

- How are the laws of conservation and equations of motion in the context of rotational motion analogous to those governing rectilinear motion?

This question links to understandings in Topics A.1, A.2 and A.3.