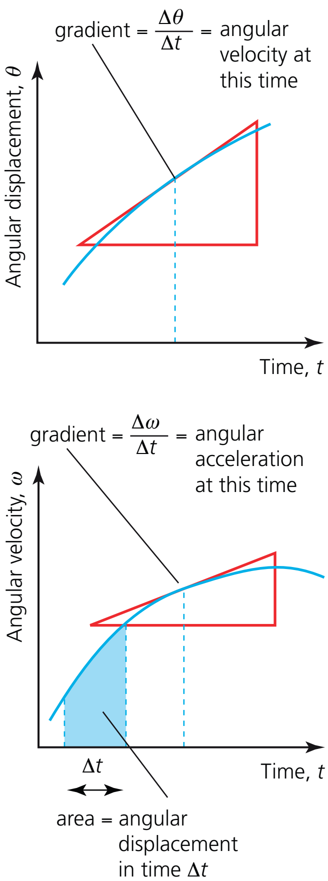


■ **Figure A4.15** Interpreting graphs of rotational motion

Graphs of rotational motion

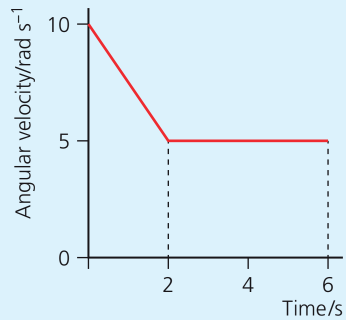
Graphs for rotational motion can be interpreted in the same way as graphs for linear motion. See Figure A4.15.

10 Sketch an angular displacement–time graph for the following rotational motion: an object rotates at 3 rad s^{-1} for 4 s, it then very rapidly decelerates and then remains stationary for a further 6 s. The rotation is then reversed so that it accelerates uniformly back to its original position after a total time of 15 s.

11 Figure A4.16 shows how the angular velocity of an object changed during 6.0 s.

a Determine the angular acceleration during the first 2.0 s.

b Through what total angle did the object rotate in 6.0 s?



■ **Figure A4.16** Change in angular velocity

Moment of inertia

SYLLABUS CONTENT

- ▶ The moment of inertia, I, depends on the distribution of mass of an extended body about an axis of rotation.
- ▶ The moment of inertia for a system of point masses: $I = \Sigma mr^2$.

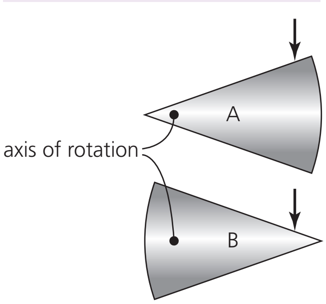
◆ **Moment of inertia, I** The resistance to a change of rotational motion of an object, which depends on the distribution of mass around the chosen axis of rotation. The moment of inertia of a point mass is given by $I = mr^2$ (SI unit: kg m^2). The moment of inertia of any real, extended mass can be determined by the addition of the individual moments of inertia of its particles. This is represented by $I = \Sigma mr^2$.

In Topic A.2 we saw that when a resultant *force* is applied to an object, the result is a linear *acceleration*, the magnitude of which depends on the *mass* of the object. Resistance to a change of motion (acceleration) is called *inertia*.

However, in the case of rotational motion, we also need to consider how the mass is distributed around the axis. Consider Figure A4.17.

Object A will require more force to produce a certain acceleration than object B, which has the same mass and shape but a different axis of rotation. We say that A has a larger **moment of inertia** than B.

Resistance to a change of rotational motion of an object is quantified by its moment of inertia, I, which depends on the distribution of mass around the chosen axis of rotation.



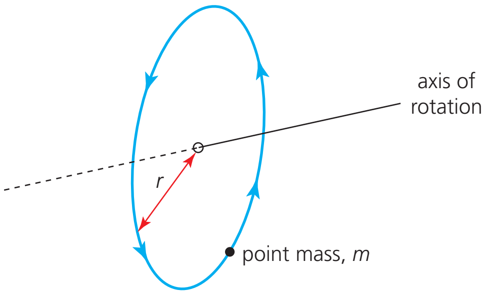
■ **Figure A4.17**

The simplest object to consider is a *point mass*, as seen in Figure A4.18.

The moment of inertia of a point mass, m, rotating at a distance r from its axis is given by:

$$I = mr^2$$

The SI unit of moment of inertia is kg m^2. Most spherical objects can be considered to behave like masses concentrated at their centre points. That is, their centre of mass is at the centre of the sphere.



■ **Figure A4.18** Rotation of a point mass

WORKED EXAMPLE A4.6

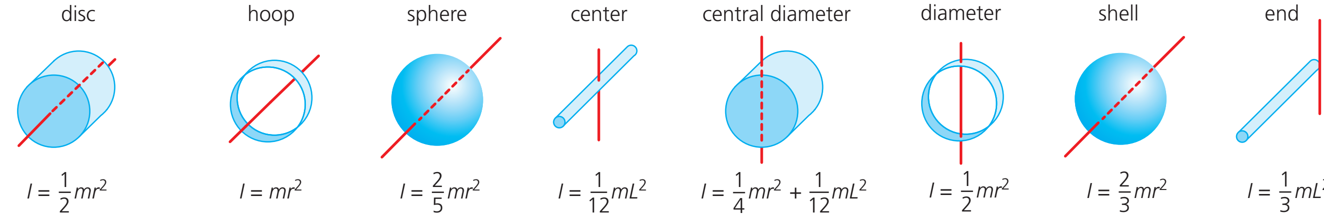
Determine the moment of inertia of a 24 g simple pendulum of length 90 cm.

Answer

$I = mr^2 = 0.024 \times 0.90^2 = 1.9 \times 10^{-2}\,\text{kg m}^2$ (Assuming that the pendulum can be considered to act as a point mass, and the effect of the string or connecting rod is negligible.)

In principle, the moment of inertia of any real, extended mass can be determined by the addition of the individual moments of inertia of its point masses: $\boldsymbol{I = \Sigma mr^2}$ (The symbol Σ means 'sum of'.)

In practice, the moments of inertia of most simple-shaped objects about specific axes are well known. Some examples are shown in Figure A4.19, but there is no need to remember them, or to know how they were derived, because equations will be provided in the examination paper if needed.



■ **Figure A4.19** Examples of moments of inertia (*r* represents radius and *L* represents length)

Tool 3: Mathematics

Calculate and interpret percentage change and percentage difference

Example: using Figure A4.19, determine the percentage difference between the moments of inertia (about a central axis) of a solid sphere of mass 1.0 kg and a thin spherical shell of mass 100 g. Assume they have the same radius of 12 cm. Use 3 significant figures for all answers.

Moment of inertia of solid sphere = $2/5 \times 1.0 \times 0.12^2 = 5.76 \times 10^{-3}\,\text{kg m}^2$

Moment of inertia of shell = $2/3 \times 0.10 \times 0.12^2 = 9.60 \times 10^{-4}\,\text{kg m}^2$

The difference between these two = $4.80 \times 10^{-3}\,\text{kg m}^2$

The mean of these two = $3.36 \times 10^{-3}\,\text{kg m}^2$

Percentage difference = 100 × (actual difference) / mean = $100 \times (4.80 \times 10^{-3}) / (3.36 \times 10^{-3}) = 143\%$

Perhaps more often, we are concerned with percentage changes. For example, consider a torque which increased from 12 Nm to 18 Nm: the change is 6 Nm (18 − 12)

Percentage change = 100 × (change / original value) = 100 × 6/12 = +50%

Alternatively, if the torque changed from 18 Nm to 12 Nm,

Percentage change = 100 × (change / original value) = 100 × −6/18 = −33%

WORKED EXAMPLE A4.7

Figure A4.20 shows a 'dumb-bell' arrangement in which two spherical masses, each of mass $m_1 = 2.0\,\text{kg}$, are rotating about an axis that is a distance $r_1 = 35\,\text{cm}$ from both of their centres.

a If the rod has a mass of $m_2 = 400\,\text{g}$, length $L = 56\,\text{cm}$ and radius $r_2 = 1.2\,\text{cm}$, determine the overall moment of inertia of this arrangement.

b Calculate the percentage that the rod contributes to the overall moment of inertia of the system.

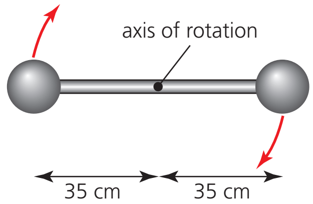


Figure A4.20 'Dumb-bell' arrangement

Answer

a $I = (2 \times m_1 r^2) + \left(\frac{1}{12} \times m_2 L^2\right) = (2 \times 2.0 \times 0.35^2) + \left(\frac{1}{12} \times 0.400 \times 0.56^2\right)$

$= 0.490 + (1.045 \times 10^{-2}) = 0.50\,\text{kg}\,\text{m}^2$ (0.50045... seen on calculator display)

b $100 \times \left(\frac{0.01045}{0.50045}\right) = 2.1\%$

◆ **Flywheel** Dense, cylindrical mass with a high moment of inertia – added to the axes of rotating machinery to resist changes of motion and/or to store rotational kinetic energy.

ATL A4B: Research skills

Comparing, contrasting and validating information

Flywheels

Flywheels are added to the axes of rotating machinery to resist changes of motion and/or to store rotational kinetic energy. They need to have large moments of inertia and are used in modern machinery, but Figure A4.21 shows an old-fashioned example, a potter's wheel. The large wheel at the bottom is kicked for a while until it is spinning quickly. After that, because it has a large moment of inertia, there will be no need to keep kicking the wheel continuously to maintain its motion.

Figure A4.21 A flywheel on a potter's wheel

Flywheels can be useful for maintaining rotations in machines that do not have continuous power supplies. To do this they will usually need to be able to store relatively large amounts of kinetic energy. If the potter's flywheel had a mass of 20 kg and radius 50 cm, it would store about 200 J of rotational kinetic energy if it was spinning with a frequency of 2 Hz. (This can be confirmed by using $E_k = \frac{1}{2}I\omega^2$, which is discussed later in this topic.)

A modern flywheel can be seen in Figure A4.22.

In Topic A.3 we outlined the technology of regenerative braking.

1 Use a search engine to find out how flywheels are used:
   a in vehicles which employ regenerative braking
   b in wind turbines for generating electricity.
2 Compare and contrast the use of the flywheel in each application. What is similar; what is different?

Figure A4.22 Flywheel on a two-wheeled tractor

Inquiry 1: Exploring and designing

Exploring

Formulating a research question

A student is planning to investigate the behaviour of a flywheel, using the apparatus shown in Figure A4.23. Suggest a possible research question for this investigation.

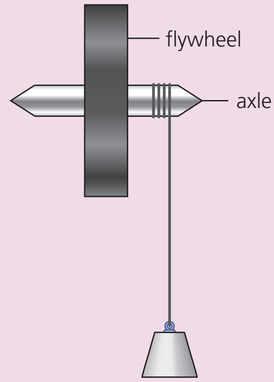


Figure A4.23 Flywheel investigation

12 Estimate the moment of inertia of the Earth in its orbit around the Sun (mass of Earth ≈ 6×10^{24} kg, distance to Sun ≈ 150 million km).

13 a Calculate the moment of inertia of the rotating dumb–bell arrangement seen in Figure A4.24. Assume that the connecting rod has no significant effect.

b By what factor would the moment of inertia change if 20 cm was increased to 30 cm?

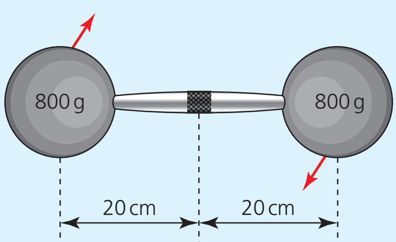


Figure A4.24 A dumb-bell

14 a Suggest why the equation $I = mr^2$ could be used to determine an approximate value for the moment of inertia of a bicycle wheel.

b Estimate a value for the moment of inertia of a typical bicycle wheel.

15 The flywheel shown in Figure A4.23 may be considered to be a steel cylinder (density 7800 kg m^{-3}) of outer radius 40 cm and thickness 12 cm. Use this simplification to estimate its moment of inertia. Ignore other features.

16 Figure A4.25 shows a thick-walled tube and its axis of rotation. Determine its moment of inertia about axis Y if its length, L, is 20.0 cm..

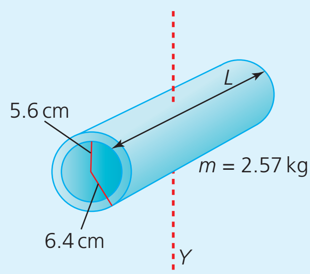


Figure A4.25 Thick-walled tube and its axis of rotation

17 Figure A4.26 shows a boy of mass 25 kg on a playground merry-go-round of mass 370 kg. Estimate a value for the moment of inertia of the system.

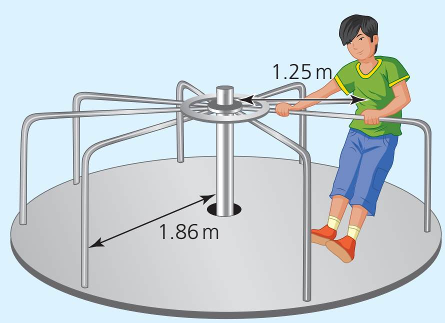


Figure A4.26 Boy on a playground merry-go-round

Newton's second law for rotational motion

◆ **Newton's second law for angular motion** $\tau = I\alpha$

◆ **Inverse proportionality** Two quantities are inversely proportional if, when one increases by a factor x, the other decreases by the same factor. For example: $x \propto \frac{1}{y}$ (xy = constant).

SYLLABUS CONTENT

▶ Newton's second law for rotation, as given by: $\tau = I\alpha$, where τ is the average torque.

For linear motion, a resultant force, F, acting on an object of mass, m, produces an acceleration, a, as given by: $F = ma$.

For rotational motion, a resultant torque, τ, acting on an object which has a moment of inertia I, produces an acceleration, α.

DB

torque, $\boldsymbol{\tau = I\alpha}$

Tool 3: Mathematics

Understand direct and inverse proportionality

The simplest possible relationship between two variables (like α and τ for a constant moment of inertia) is that they are directly proportional to each other (often just called proportional). This means that if one variable, x, doubles, then the other variable, y, also doubles; if y is divided by five, then x is divided by five; if x is multiplied by 17, then y is multiplied by 17, and so on. In other words, the ratio of the two variables ($\frac{x}{y}$ or $\frac{y}{x}$) is constant. Proportionality is shown using the following symbol:

Proportionality:

$$y \propto x \text{ and } \frac{x}{y} = \text{constant}$$

To check if two variables are proportional to each other, we can either

1 calculate their ratios for different values to confirm that they are constant (see Table A4.3), or
2 draw an x–y graph to determine if it is a straight line through the origin (as discussed later in this chapter).

■ **Table A4.3** The data in either of the last two columns confirms $y \propto x$ (allowing for experimental uncertainties)

x	y	$\frac{x}{y}$	$\frac{y}{x}$
0	0	-	-
0.32	1.6	0.20	5.0
0.81	4.2	0.19	5.2
1.4	6.9	0.20	4.9
2.5	12.8	0.20	5.1
6.4	30.0	0.21	4.7
10.9	55.2	0.20	5.1

If one variable increases while the other decreases (like α and I for a constant torque), we describe it as an *inverse relationship*. The simplest inverse relationship is when one variable, x, doubles, while the other variable, y, halves. if y is divided by five, then x is multiplied by five; if x is multiplied by 17, then y is divided by 17, and so on. In other words multiplying the two variables together always produces the same result: xy = constant. This is called **inverse proportionality**.

Inverse proportionality:

$$y \propto \frac{1}{x} \text{ and } xy = \text{constant}$$

To check if two variables are inversely proportional to each other, we can either

1 calculate values for when they are multiplied together, to confirm that they are constant (see Table A4.4), or
2 draw an $x = \frac{1}{y}$ graph to determine if it is a straight line through the origin (as discussed later in this chapter).

■ **Table A4.4** The data in the last column confirms $y \propto \frac{1}{x}$ (allowing for experimental uncertainties)

x	y	xy
0	0	-
2.0	17	34
11	3.1	34
22	1.6	35
37	0.94	35
43	0.79	34
64	0.55	35

This topic provides another example. Do a quick mathematical check to see if the following data represents an inversely proportional relationship and sketch a graph to show this relationship.

Moment of inertia / 10^{-2} kg m^{-2}	Angular acceleration / rad s^{-2}
0.058	5.3
0.051	5.8
0.039	7.8
0.029	9.9
0.021	14.6

WORKED EXAMPLE A4.8

a Calculate the acceleration produced when a system that has a moment of inertia of 1.23 kg m^2 is acted on by a resultant torque of 0.83 Nm.

b If the system was already rotating at 2.7 rad s^{-1}, determine its maximum angular velocity if the torque is applied for exactly 4 s.

c State the assumption that you made when answering these questions.

Answer

a $\tau = I\alpha$

$\alpha = \frac{\tau}{I} = \frac{0.83}{1.23} = 0.67\,\text{rad s}^{-2}$

b $\omega_f = \omega_i + \alpha t = 2.7 + (0.67 \times 4.0) = 5.4\,\text{rad s}^{-1}$

c There are no frictional forces acting on the system.

WORKED EXAMPLE A4.9

Figure A4.27 shows a falling mass, m, attached to a string which is wrapped around a cylinder of radius r and moment of inertia $I = \frac{1}{2}Mr^2$. Derive an equation for the downward linear acceleration, a, of the mass.

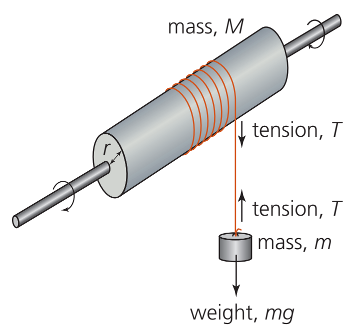


Figure A4.27 A falling mass attached to a string wrapped around a cylinder

Answer

Since $\alpha = \frac{a}{r}$, torque acting on cylinder, $\tau = Tr = I\alpha = \frac{1}{2}Mr^2 \times \frac{a}{r}$

So that, $T = \frac{1}{2}Ma$

Resultant downwards force acting on falling mass, $F = mg - T$

Linear acceleration of falling mass, $a = \frac{F}{m} = \frac{(mg - T)}{m} = \frac{\left(mg - \frac{1}{2}Ma\right)}{m}$

Rearranging gives: $a = \frac{mg}{\left(m + \frac{1}{2}M\right)}$

Common mistake

Note that, because the mass is accelerating downwards, the tension in the string is not equal to the weight of the mass on the end of the string.

HL ONLY

Tool 3: Mathematics

Propagate uncertainties in processed data

The term **processed data** is used to describe the results obtained after calculations have been made using **raw data**.

In this section we will consider how uncertainties in raw data affect the results of processed data.

Processed data should not have more significant figures than the raw data used to calculate it.

Consider a simple example: a trolley moving with constant speed was measured to have travelled a distance of 76 cm ± 2cm (±2.6%) in a time of 4.3 s ± 0.2 s (±4.7%).

The speed can be calculated from distance / time = 76 / 4.3 = 17.674…, which is 18 m s^{-1} when rounded to 2 significant figures, consistent with the experimental data.

To determine the uncertainty in this answer we consider the uncertainties in distance and time. Using the largest distance and shortest time, the largest possible answer for speed is 78 / 4.1= 19.024…. m s^{-1}. Using the smallest distance and the longest time, the smallest possible answer for speed is 74 / 4.5 = 16.444…. m s^{-1}. (The numbers will be rounded at the end of the calculations.)

The speed is therefore between 16.444 cm s^{-1} and 19.024 cm s^{-1}. The value 19.024 has the greater difference (1.350) from 17.674. So, the final result can be expressed as 17.674 ± 1.350 cm s^{-1}, which is a maximum uncertainty of 7.6%. Rounding to 2 significant figures, the more realistic result is 18 ± 1 cm s^{-1}.

Uncertainty calculations like these can be very time consuming and, for this course, approximate methods are acceptable. For example, in the calculation for speed shown above, the uncertainty in the data was ±2.6% for distance and ±4.7% for time. The percentage uncertainty in the final result is approximated by adding the percentage uncertainties in the data: 2.6 + 4.7 = 7.3%. This gives approximately the same value as calculated using the largest and smallest possible values for speed. Rules for finding uncertainties in calculated results are given below.

- For quantities that are added or subtracted: add the absolute uncertainties:

if $y = a \pm b$, then $\Delta y = \Delta a + \Delta b$

- For quantities that are multiplied or divided: add the individual fractional or percentage uncertainties:

if $y = abc$, then $\dfrac{\Delta y}{y} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b} + \dfrac{\Delta c}{c}$

- For quantities that are raised to a power, n:

if $y = a^n$, then $\dfrac{\Delta y}{y} = \left|n\left(\dfrac{\Delta a}{a}\right)\right|$

- For other functions (such as trigonometric functions or logarithms) calculate the highest and lowest absolute values possible and compare with the mean value, as shown in Worked example A4.10. But note that, although such calculations can occur in connection with laboratory work, they will not be required in examinations.

◆ **Processed data** Data produced by calculations made from raw experimental data.

◆ **Raw data** Measurements made during an investigation.

WORKED EXAMPLE A4.10

An angle, θ, was measured to be 34° ± 1°. Determine the uncertainty in the tangent of this angle.

Answer

tan34° = 0.6745, tan 33° = 0.6494, tan 35° = 0.7002

Larger absolute uncertainty = 0.7002 – 0.6745 = 0.0257

(0.6745 − 0.6494 = 0.0251, which is smaller than 0.0257)

So, tan θ = 0.67 ± 0.03 (using the same number of significant figures as in the original data).

HL ONLY

Nature of science: Falsification

Uncertainties

Most people believe that science deals with 'facts'. That is a reasonable comment – but it also gives an incomplete impression of the nature of science. The statement is misleading if it suggests that scientists believe they are always discovering 'truths' that will last forever. In reality, scientific knowledge is open to change, if and when we make new discoveries. More than that, it is the essential nature of science and good scientists to encourage the re-examination of existing 'knowledge' and 'truths' and to look for improvements and progress.

'All scientific knowledge is uncertain…' Richard P. Feynman (1998), *The Meaning of It All: Thoughts of a Citizen-Scientist.*

'One aim of the physical sciences has been to give an exact picture of the material world. One achievement of physics in the twentieth century has been to prove that this aim is unattainable.' Jacob Bronowski

18 A resultant torque of 2.4 Nm ± 0.2 N, accelerated a large metal hoop (with axis of rotation through a diameter) of radius 42 cm ± 1 cm from rest to 5.7 rad s^{-1} ± 0.1 rad s^{-1} in 3.2 ± 0.2 s. Determine a value for the mass of the hoop and the absolute uncertainty in your answer.

19 Calculate the torque needed to accelerate a rotating object which has a moment of inertia 3.2 kg m from 1.3 rad s^{-1} to 4.9 rad s^{-1} in 8.8 s.

20 An object was accelerated from 300 rpm (revolutions per minute) to 1100 rpm in 2.3 s when a resultant torque of 112 Nm was applied. Determine its moment of inertia.

21 Two parallel forces each of 26 N are separated by a distance of 8.7 cm. If this couple provides the resultant torque to a rotating system that has a moment of inertia of 17.3 kg m^2, determine the angular acceleration produced.

22 A torque of 14.0 Nm is applied to a stationary wheel, but resistive forces provide an opposing torque of 6.1 Nm. If the wheel has a moment of inertia of 1.2 kg m^2, show that the total angular displacement after 2.0 s is about two rotations.

23 Consider Figure A4.27. What mass, m, will produce a linear acceleration of 2.5 m s^{-2} when acting on an 8.3 kg cylinder?

Conservation of angular momentum

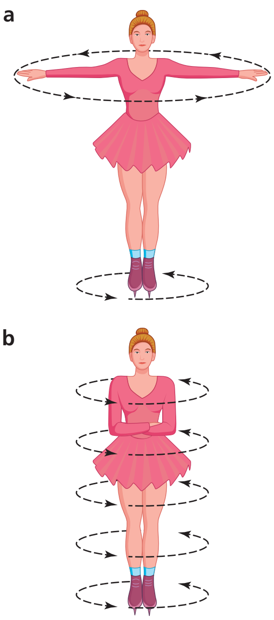
SYLLABUS CONTENT

- An extended body rotating with an angular speed has an angular momentum, L, as given by: $L = I\omega$.
- Angular momentum remains constant unless the body is acted upon by a resultant torque.

◆ **Angular momentum, L** Moment of inertia multiplied by angular velocity: $L = I\omega$ (SI unit: kg m^2 s^{-1}).

Angular momentum, L, of a rotating object is the rotational equivalent of linear momentum ($p = mv$). It depends on the moment of inertia, I, of the object and its angular velocity (speed), ω.

angular momentum, $\boldsymbol{L = I\omega}$ SI unit: kg m^2 s^{-1}



■ **Figure A4.28** Ice-skater

◆ **Conservation of angular momentum** The total resultant angular momentum of a system is constant provided that no resultant external torque is acting on it.

The law of conservation of linear momentum (Topic A.2), which has no exceptions, was shown to be very useful when predicting the outcome of interactions between masses exerting forces on each other. In a similar way, the law of **conservation of angular momentum** (as follows) has no exceptions and can be used to predict changes to rotating systems.

The total angular momentum of a system is constant provided that no resultant (net) external torque is acting on it.

Figure A4.28 shows a spinning ice-skater in two positions, **a** and **b**. In moving from position **a** to position **b**, the skater lowers her arms and brings them closer to her body, and so reduces her moment of inertia. Assuming there are no external torques acting, her rotational momentum will be constant so that her angular velocity must increase. Similar rotational behaviour can be seen in the motions of gymnasts, divers and ballet dancers.

WORKED EXAMPLE A4.11

A sphere of mass 2.1 kg and radius 38 cm is spinning around a diameter at a rate of 44 rpm. Calculate its angular momentum.

Answer

$$L = I\omega = \frac{2}{5}mr^2 \times \frac{2\pi}{T}$$

$$= \frac{2}{5} \times 2.1 \times 0.38^2 \times 2 \times \frac{\pi}{(60/44)}$$

$$= 0.56\,\text{kg}\,\text{m}^2\,\text{s}^{-1}$$

WORKED EXAMPLE A4.12

A solid metal disc of mass 960 g and radius 8.8 cm is rotating horizontally at 4.7 rad s^{-1}.

a Calculate the moment of inertia of the disc.

b Calculate the new angular velocity after a mass of 500 g is dropped quickly and carefully on to the disc at a distance of 6.0 cm from the centre.

Answer

a $I = \frac{1}{2}mr^2 = \frac{1}{2} \times 0.96 \times (8.8 \times 10^{-2})^2 = 3.7 \times 10^{-3}\,\text{kg}\,\text{m}^2$

b moment of inertia of added mass $= mr^2 = 0.5 \times (6.0 \times 10^{-2})^2 = 1.8 \times 10^{-3}\,\text{kg}\,\text{m}^2$

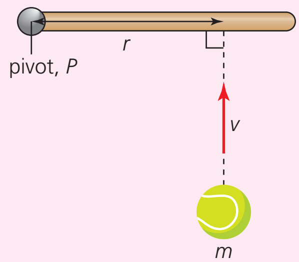
$L = I\omega = \text{constant}$

$(3.7 \times 10^{-3}) \times 4.7 = [(3.7 \times 10^{-3}) + (1.8 \times 10^{-3})] \times \omega$

$\omega = 3.2\,\text{rad}\,\text{s}^{-1}$

Common mistake

Students often believe that an object travelling in a straight line must have zero angular momentum, but consider Figure A4.29. A ball of mass m and speed v is just about to strike a stationary rod perpendicularly at a distance r from where it is pivoted.



■ **Figure A4.29** Ball striking a pivoted rod (seen from above)

The rod will be made to rotate anticlockwise, gaining angular momentum. From the law of conservation of angular momentum in the rod and ball 'system', the ball must have lost angular momentum. The ball initially had angular momentum, L, about point P, not around any point within the ball itself. Instantaneous value:

$$L = I\omega = (mr^2) \times \left(\frac{v}{r}\right) = mvr$$

LINKING QUESTION

- How does conservation of angular momentum lead to the determination of the Bohr radius?

This question links to understandings in Topic E.1 for HL students.

24 Calculate the angular momentum of a 1.34 kg disc of radius 56 cm spinning at an angular speed of 37 rad s^{-1} around an axis passing perpendicularly through its centre.

25 The three blades of a rotating fan each have a moment of inertia of 0.042 kg m^2. If they have a combined angular momentum of 0.74 kg m^2 s^{-1}, determine how many times the fan rotates every minute.

26 An unpowered merry-go-round of radius 4.0 m and moment of inertia 1200 kg m^2 is rotating with a constant angular velocity of 0.56 rad s^{-1}. A child of mass 36 kg is standing close to the merry-go-round and decides to jump onto its edge.

- **a** Predict the new angular velocity of the merry-go-round. State any assumptions you made.
- **b** Discuss whether the merry-go-round would return to its original speed if the child jumped off again.

27 Neutron stars are the very dense collapsed remnants of much larger spinning stars. Suggest why they have extremely high rotational velocities.

LINKING QUESTION

- How does rotation apply to the motion of charged particles or satellites in orbit?

This question links to understandings in Topics D.1 and D.3.

Angular impulse

SYLLABUS CONTENT

▶ The action of a resultant torque constitutes an angular impulse, ΔL, as given by: $\Delta L = \tau t = \Delta(I\omega)$.

In Topic A.2, we noted that the effect of an (average resultant) force is greater if it acts for a longer time. So, it was convenient to introduce the term linear impulse, $J = F\Delta t$. Using Newton's second law, a change of linear momentum, $\Delta p = \Delta(mv)$ occurs because of a linear impulse $F\Delta t$.

Similarly, for rotational motion: an average resultant torque, τ, acting for a time Δt, produces a change of angular momentum, ΔL, called angular impulse (no symbol).

DB

A change of angular momentum, $\boldsymbol{\Delta L = \Delta(I\omega)}$ occurs because of an angular impulse, $\tau\Delta t$:
$\boldsymbol{\Delta L = \tau\Delta t}$ SI unit: kg m^2 s^{-1}

N m s is an equivalent and alternative unit.

Nature of science: Science as a shared endeavour

Communicating inter-connected concepts

It could be argued that we do not need to give [force (or torque) × time] a name: impulse. Have we added to our understanding if we say that impulse = change of momentum, rather than force × time = change of momentum, or is it just easier to say one word instead of three? Would it be possible to understand this topic without ever referring to 'impulse'?

Perhaps the main reason why impulse is important is that forces are rarely constant. Forces usually change to, or from, zero over time and they may also vary depending on many other factors. It is much simpler to refer to the overall effect.

If the applied resultant torque changes, an average value should be used to determine an impulse. For a torque which varies in a regular way, this can be assumed to be midway between the starting and final values. In other examples, we may need to determine an average value from looking at a torque–time graph. We know from Topic A.2, that the area under a force–time graph is equal to the change of linear momentum of the system (impulse). Similarly, in rotational dynamics:

The area under a torque–time graph is equal to the change in angular momentum (angular impulse). This is true for any shape of graph.

WORKED EXAMPLE A4.13

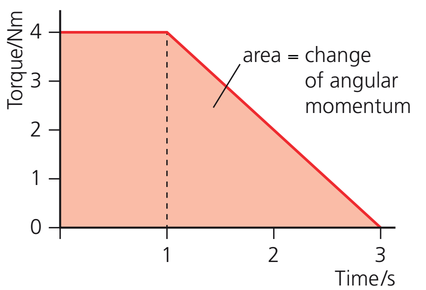


Figure A4.30 Example of a torque–time graph

Consider Figure A4.30.

a Determine the angular impulse represented.

b If this impulse accelerated an object already rotating at 25 rad s^{-1}, calculate the final angular velocity if the object had a moment of inertia of 0.51 kg m^2.

Answer

a Area under graph = angular impulse = change of angular momentum

$$\Delta L = (4.0 \times 1.0) + \left(\frac{1}{2} \times 4.0 \times 2.0\right) =$$

8.0 N m s (or kg m^2 s^{-1}).

b $\Delta L = \Delta(I\omega) = I(\omega_f - \omega_i) = 8.0$

$\omega_f - 25 = 8.0/0.51$

$\omega_f = 41\ \text{rad s}^{-1}$

28 An object which has a moment of inertia of 4.8 kg m^2 is initially at rest. A torque is then applied which increases uniformly from zero until the object is rotating with an angular velocity of 79 rad s^{-1} after 23 s.
Show that the maximum torque applied was approximately 30 Nm.

29 A solid sphere of mass 1.47 kg and radius 12 cm is rotating about a diameter with an angular speed of 57 rad s^{-1}. Determine what constant torque will bring it to rest in 10.0 s.

30 Figure A4.31 shows the variation of torque applied to a stationary system that has a moment of inertia of 0.68 kg m^2.

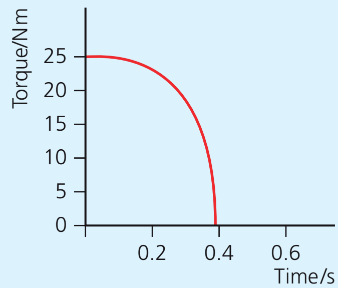


Figure A4.31 Variation of torque applied to a stationary system

a Estimate the change of angular momentum of the system.

b Predict a value for its final angular velocity.

Rotational kinetic energy

SYLLABUS CONTENT

- The kinetic energy of rotational motion, as given by: $E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

◆ **Rotational kinetic energy, E_k** Kinetic energy due to rotation, rather than translation. $E_k = \frac{1}{2}I\omega^2$

Knowing, from Topic A.3, that linear kinetic energy:

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

we can simply write down the equivalent equations for **rotational kinetic energy** by analogy.

DB

Rotational kinetic energy: $\boldsymbol{E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}}$

It is common for an object to have both linear and kinetic energy, the wheel on a bicycle, for example.

WORKED EXAMPLE A4.14

A car of mass 1340 kg is moving with a speed of 12 m s^{-1}.

a Calculate its linear kinetic energy.

b If each wheel and tyre (of four) has a radius of 29 cm and a moment of inertia of 0.59 kg m^2, determine its rotational kinetic energy.

c What is the total kinetic energy of the car?

Answer

a $E_k \text{ (linear)} = \frac{1}{2}mv^2 = \frac{1}{2} \times 1340 \times 12^2 = 9.6 \times 10^4\text{ J}$

b $\omega = \frac{v}{r} = \frac{12}{0.29} = 41.4\text{ rad s}^{-1}$ (41.379... seen on calculator display)

$E_k \text{ (rotational)} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.59 \times 41.379^2 = 5.1 \times 10^2\text{ J}$

c $(9.6 \times 10^4) + (4 \times 5.1 \times 10^2) = 9.8 \times 10^4\text{ J}$

HL ONLY

◆ **Slipping (wheel)** Occurs when there is not enough friction between a wheel and the surface to maintain a rolling motion.

◆ **Sliding** Surfaces moving over each other without any rotation involved.

Common mistake

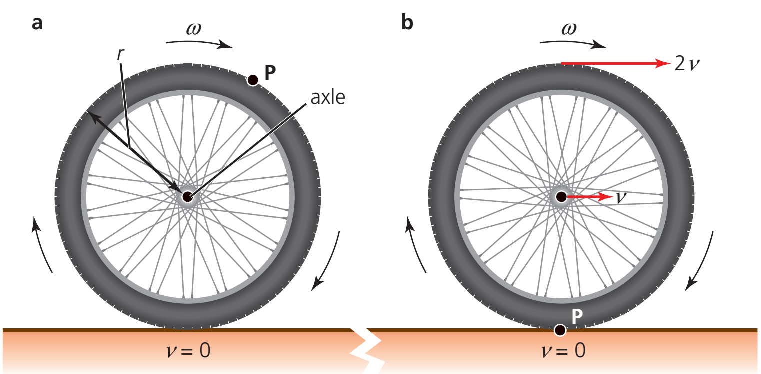
When a vehicle accelerates, many students think that there is just one frictional force, acting against the direction of motion. But *all* forces occur in pairs: because of friction, there is a force backwards on the road *and* an equal and opposite force acting forwards on the vehicle. This force accelerates the vehicle or opposes air resistance if it is moving with constant speed. When there is no forward propulsion, the frictional force on the vehicle from the road will be in the opposite direction.

◆ **Roll** Rotation of an object along a surface in which the lowest point of the object is instantaneously stationary. Requires friction. Compare with slipping.

Rolling (without slipping)

We will assume that there is sufficient surface friction to prevent any **slipping** or **sliding**. This means that there is no relative motion between the lower surface of the rotating object and the surface on which it is moving.

Consider a wheel of radius r rotating with an angular speed ω. All points on its circumference will be moving with linear speed $v = \omega r$. Figure A4.32a shows the wheel of a motor bike (for example), on a road surface, and Figure A4.32b shows the same wheel a short time later. If point P is moving with linear speed v, this must also be the overall speed of the motor bike, as shown on the central axle.



■ **Figure A4.32** Moving wheel showing instantaneous speeds relative to the road surface

The wheel is rotating clockwise due to the action of the engine, and it is pushing backwards (to the left) on the road surface because of friction. Friction with the road surface pushes the car forward. (Newton's third law.)

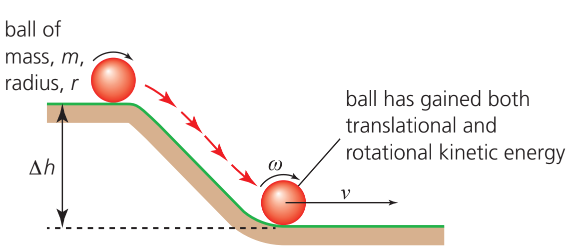
Under normal circumstances, because of friction, there will be no slipping of the wheel on the road surface, which would be dangerous. This means that point P on the wheel in Figure A4.32b must be *momentarily* stationary.

A point on the top of the wheel will be moving with speed $2v$ relative to the road surface.

Because there is no movement, the frictional force does not do any work, so no energy is dissipated at that point. (This is a simplified interpretation.)

Rolling down a slope

An object, such as a ball or a wheel, which can **roll** down a hill will transfer its gravitational potential energy to both translational kinetic energy and rotational kinetic energy.



■ **Figure A4.33** Rolling down a slope

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

At the bottom of a slope, a sliding object will reach a higher speed than a rolling object. Also, rotating objects that have bigger moments of inertia will travel slower at the bottom of the same slope. If the angle of the slope is too steep, rolling will not be possible.

Consider the example of a solid sphere, for which:

$$I = \frac{2}{5}mr^2$$

Remembering that $v = \omega r$, the equation above becomes:

$$\cancel{m}g\Delta h = \frac{1}{2}\cancel{m}\omega^2 r^2 + \left(\frac{1}{2}\right)\frac{2}{5}\cancel{m}r^2\omega^2 = \frac{7}{10}\omega^2 r^2$$

Note that, with the assumptions made, the angular velocity at the bottom of the slope does not depend on the slope angle or the mass of the ball.

WORKED EXAMPLE A4.15

Determine the:

a angular speed

b linear speed of the centre of mass of a 500 g ball,

which has a radius of 10 cm, after it has rolled down a slope of vertical height 1.0 m.

Answer

a $$g\Delta h = \frac{7}{10}\omega^2 r^2$$

$$9.8 \times 1.0 = 0.7 \times \omega^2 \times 0.1^2$$

$$\omega = 37\,\text{rad s}^{-1}$$

b $v = \omega r = 37 \times 0.1 = 3.7\,\text{m s}^{-1}$

31 Calculate the rotational kinetic energy of a tossed coin if it has a mass of 8.7 g, radius 7.1 mm and completes one rotation in 0.52 s.

32 Calculate the rotational kinetic energy of the Earth spinning on its axis. (Research relevant information.)

33 A flywheel is spinning with rotational kinetic energy of 4.6×10^4 J. Calculate its moment of inertia if it has an angular momentum of $98\,\text{kg m}^2\,\text{s}^{-1}$.

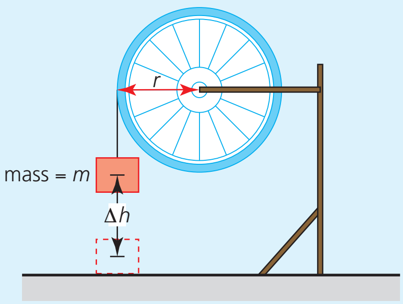
34 The moment of inertia of the windmill seen in Figure A4.34 is $96\,\text{kg m}^2$. Estimate how many rotations every minute are needed for it to have 250 J of kinetic energy.

Figure A4.34 A windmill

35 Figure A4.35 shows an experimental arrangement that could be used to determine a value for the moment of inertia of a wheel. A string is wrapped around the outside of a wheel and provides a torque as the attached mass accelerates downwards and starts the wheel rotating.

a Write down an equation to represent the transfer of energy when the mass has fallen a distance h.

b Calculate a value for the moment of inertia of the wheel, of radius 24 cm, if a mass of 500 g is moving down with a speed of $1.14\,\mathrm{m\,s^{-1}}$ after falling a distance of 50 cm.



■ **Figure A4.35** Determining the moment of inertia of a wheel

36 The cyclist seen in Figure A4.36 is moving to the right with a constant linear speed of $4.0\,\mathrm{m\,s^{-1}}$.

a State the linear speed of all points on the circumference of the wheel (with respect to the bicycle).

b State the speed of the lowest point of the wheel with respect to the ground.

c Use the picture to show that the angular speed of the wheel is approximately $15\,\mathrm{rad\,s^{-1}}$.

d What is the instantaneous velocity of the top of the wheel, with respect to the ground?

■ **Figure A4.36** Cyclist

37 A solid ball and a hollow ball of the same mass and radius roll down a hill. At the bottom, discuss which ball

a will be rotating faster

b has the greater linear speed.

38 a Calculate the greatest

i angular speed

ii linear speed of a solid ball of radius 1.2 cm rolling down a slope from a vertical height of 6.0 cm.

b What assumption did you make?

c Compare your answer to the greatest speed of the same ball dropped the same vertical distance.

39 Use the analogies between linear and rotational mechanics to write down equations for rotational work and power.

Inquiry 1: Exploring and designing

Designing

Identify variables

A student wishes to investigate balls rolling down slopes. Identify all the possible variables involved, and select one independent and one dependent variable that could be investigated. State how the other variable would be controlled.

HL ONLY

A.5 Relativity

Guiding questions

- How do observers in different reference frames describe events in terms of space and time?
- How does special relativity change our understanding of motion compared to Galilean relativity?
- How are space–time diagrams used to represent relativistic motion?

◆ **Relativistic motion** Travelling at a significant fraction of the speed of light.

◆ **Reference frame** A coordinate system from which events in space and time are measured.

◆ **Coordinate system** An agreed numerical way of identifying the location and time of an event.

◆ **Event** Single incident that occurs exactly at a precise time and location.

Einstein's theory of special relativity (1905) was a complete revolution in scientific thinking. Before then, Newtonian Mechanics (as explained in Topics A.1 to A.3) had accurately described and predicted motion in the Universe as it was understood at that time. However, the theories of Newton, Galileo and others cannot be applied accurately to objects which are moving very fast (with speeds close to the speed of light called **relativistic motion**). The (unexpected) discovery that the speed of light is always the same for all observers was one reason that Einstein proposed his theory of relativity, one aspect of which is that all motions are observed relative to each other, there is no 'correct point' of view, there is nowhere at absolute rest.

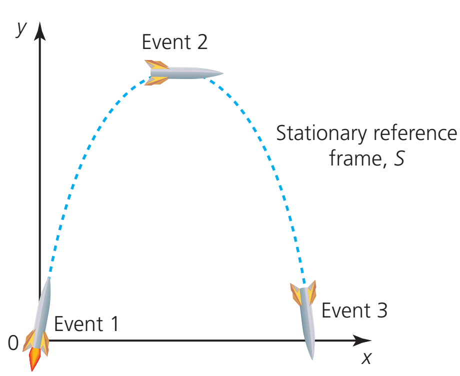
Reference frames

When making calculations on motion (in Topics A.1 to A.3) you will usually have assumed that the objects concerned were moving over the stationary surface of the Earth. The Earth's surface was the *reference frame* and values of displacement, velocity and acceleration were relative to a stationary point on that surface.

A **reference frame** is a **coordinate system** that allows a single value of time and position to be assigned to an event.

Precise **events** are used to simplify our understanding:

An **event** is considered to be a single, instantaneous incident that occurs at a specific time and point in space.



■ **Figure A5.1** Graphical representation of the Earth's reference frame for a rocket in flight

Examples of events could be a flash of light, the moment when two objects collide, or the high point of an object in parabolic flight. Lightning strikes and balloon bursts are commonly used visualizations.

Reference frames are often represented by a set of axes, usually given the label S or S', as shown for two dimensions (only) in Figure A5.1.

To fully define a four-dimensional reference frame, we must specify the origin, the directions of the x-, y- and z-axes, and the event from which the measurement of time, t, is started. The example shown in Figure A5.1 is limited to the x- and y-axes (for simplicity) and it uses the obvious reference frame that is the Earth's surface. However, if we wanted, we could alternatively consider the rocket's reference frame, in which the rocket is stationary and it is the Earth that is seen to move (at the same speed in the opposite direction).

The success of Newtonian mechanics is that it allows the accurate calculation of properties such as displacement, velocity, acceleration and time using the equations of motion (Topic A.1), as Worked example A5.1 illustrates.

WORKED EXAMPLE A5.1

In reference frame, S, shown in Figure A5.1, calculate for the three events shown the x-, y- and t-coordinates of an unpowered rocket with an initial vertical velocity of $400\,\mathrm{m\,s^{-1}}$ and a horizontal velocity of $100\,\mathrm{m\,s^{-1}}$. Ignore the effects of air resistance.

Answer

Event 1: This is the event that defines the origin and also the start of the timing, so, $x = 0\,\mathrm{m}$, $y = 0\,\mathrm{m}$ and $t = 0\,\mathrm{s}$. The coordinates are (0.0 m, 0.0 m, 0.0 s).

Event 2: This is the event defined by the rocket reaching its maximum height. We can use an equation of motion ($v^2 = u^2 + 2as$) to calculate x, y and t: this gives us the height, $y = 8200\,\mathrm{m}$.

The equation $s = \frac{(u + v)}{2}t$ can be used to show that the time to reach the top of the flight, $t = 41\,\mathrm{s}$.

Since there is no horizontal acceleration, it is straightforward to calculate the horizontal position, x, using $s = ut = 4100\,\mathrm{m}$.

Hence the (x, y, t) coordinates of Event 2 are (4100 m, 8200 m, 41 s).

Event 3: This event occurs when the rocket is the same height as it was originally. The symmetry of parabolic motion means that it occurs at (8200 m, 0 m, 82 s).

In the rest of Topic A.5, to explain principles without involving extra complications, we will restrict the discussion of reference frames to just the x-direction and time.

1 A car is caught by a speed camera travelling at $35.0\,\mathrm{m\,s^{-1}}$. If the speed camera photograph is taken at point (0.00 m, 0.0 s) determine the coordinates of the car 23.0 s later.

2 A naughty child throws a tomato out of a car at a stationary pedestrian the car has just passed. The car is travelling at $16\,\mathrm{m\,s^{-1}}$ and the child throws the tomato directly towards the pedestrian so that it leaves the car with a speed of $4\,\mathrm{m\,s^{-1}}$.
Explain why the tomato will not hit the pedestrian.

Different reference frames

◆ **Observer:** Often a hypothetical person able to use their senses, or instrumentation, to record information about events.

Have you ever walked along a moving train and wondered what your speed was? If you happened to bang your head twice on bags that stuck out too far from the luggage rack as you walked, what were the coordinates of these two events and the distance between them? The answer to this depends on the reference frame from which an **observer** is taking the measurements.

In the study of relativity, an *observer* is a hypothetical person who takes measurements from only one specific reference frame. An observer is always stationary relative to their own reference frame.

In the example of the train, there are three possible reference frames that could be occupied by three different observers:

1 an observer taking measurements sitting on the platform as the train moves past
2 an observer taking measurements sitting on a seat in the train
3 an observer taking measurements walking up the train at the same velocity as you.

According to Newton, each of these three observers will record different values for how fast you are moving and your position when you bang your head. However, they will all agree on the time between the two events occurring and the distance you have moved up the carriage between the two events.

Inertial reference frames

The problem that most students have when trying to understand relativity is that, when they observe movement while they are standing on the ground, they instinctively think that the Earth's surface is not moving, which makes it the 'correct' frame of reference (point of view). We think that the points of view of others – in planes and boats and trains – are temporary and misleading. It needs to be repeatedly stressed that, in physics:

all reference frames are equally valid; there is no 'correct' reference frame.

■ **Figure A5.2** Observer in a rocket

It may help to think about a thought experiment, far away from the Earth: consider an observer in a space vehicle (without windows) in deep space, where any effects of gravity are negligible. See Figure A5.2.

Can the observer determine if the vehicle is 'stationary', moving with constant velocity, or accelerating?

If the observer slowly and carefully releases an object (which has no weight in deep space) in mid-air, the object will appear to stay in exactly the same place if the vehicle and all its contents are moving with constant velocity. If the rocket was considered to be 'stationary' the same observations would be made, but most importantly, there is no such thing as being absolutely 'stationary' – it can never be distinguished from motion at constant velocity.

A resultant force is needed for acceleration. That force originates with the rocket engines and the observer is accelerated by contact forces with the vehicle. An object released in mid-air would not have any resultant force acting on it, so that it will maintain its original motion while the vehicle accelerates around it. To the observer, the object moves 'backwards' compared to them and the accelerating rocket.

This last point is very important: For an observer in the frame of reference of an accelerating vehicle, Newton's laws of motion appear to be broken.

◆ **Inertial reference frame** A frame of reference that is neither accelerating nor experiencing a gravitational field, in which masses obey Newton's laws of motion.

An **inertial reference frame** is one which is not accelerating and in which Newton's laws of motion can be applied.

If there were places which were truly stationary, they would be perfect inertial reference frames, the ideal background for observations of motions. (The adjective 'inertial' suggests lack of movement.) However, a frame of reference moving with constant velocity (zero acceleration) fulfils the same purpose.

The reference frames discussed in the rest of Topic A.5 will all be inertial (non-accelerating) reference frames.

For non-relativistic applications we usually use the Earth's surface as our inertial reference frame (despite its movement).

HL ONLY

◆ **Global positioning system (GPS)** A navigation system that provides accurate information on the location of the GPS receiver, by continually communicating with several orbiting satellites.

3 State which of the the following can be thought of as truly inertial reference frames, almost inertial reference frames (objects measured over a small distance appear to be travelling at constant velocity) or clearly not inertial reference frames (unbalanced forces or gravity are clearly present):

a a rocket stationary in deep space so that it is a long way from any gravitational fields.

b a rocket travelling through deep space in a straight line with constant speed

c a **GPS** communication satellite in orbit around the Earth

d a space probe hovering above the surface of the Sun

e a proton travelling close to the speed of light through a straight section of tubing in the CERN particle accelerator in Geneva.

Newton's postulates concerning time and space

◆ **Postulate** See *axiom*.

◆ **Axiom** An unproven assumption that is accepted to be true, which is then used as starting point for further discussion. Similar in meaning to a postulate.

◆ **Simultaneous events** Events that occur at the same time in a specific reference frame, so that in this reference frame they have the same time coordinates. Events that are simultaneous in one frame may not be simultaneous in another frame.

TOK

The natural sciences

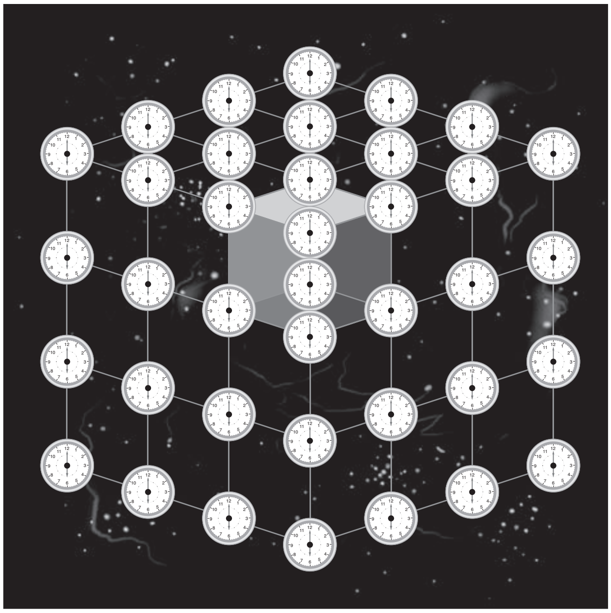
- Do the natural sciences rely on any assumptions that are themselves unprovable by science? How is an axiomatic system of knowledge different from, or similar to, other systems of knowledge?

Mathematics and the arts

A **postulate** is a starting point for the development of more advanced reasoning and discussions.

An historical example, from Euclid more than two thousand years ago, concerning geometry: a straight line can be drawn from any one point to any other point. Such fundamental postulates in mathematics are referred to as **axioms**.

Postulates and axioms may not be directly provable, but they are usually simple and unambiguous statements which are agreed by everybody (at that time). They are considered to be necessarily true, in the sense that they are logically necessary.



■ **Figure A5.3** A cubic matrix of clocks spreading out regularly throughout space and all reading exactly the same time

Before Einstein's theory of relativity, Newton's description of the Universe had made important assumptions. These assumptions are still used by everybody in their everyday lives.

1. The universality of time: All observers agree on the time interval between two events. In particular, they will agree on whether two events are **simultaneous**, or not.
2. The universality of distance: All observers agree on the distance between two simultaneous events.

Simultaneous means that two events are observed to occur at exactly the same time. That is, the time interval between the two events is zero.

To understand these postulates better, imagine a universe with a tiny clock placed in the centre of every cubic metre as shown in Figure A5.3.

The first postulate implies that every clock would always be reading the same time and ticking at exactly the same rate. Any observer moving through the Universe carrying a clock would find that their clock also read the same time as the background clocks and would tick at the same rate. If an observer also carried a metre rule with them as they moved around, they would find that it always exactly matched the shortest distance between any two adjacent clocks.

The central theme of this topic is that Newton's postulates are not totally accurate: they do not apply if relativistic effects are significant.

Galilean relativity

SYLLABUS CONTENT

- Newton's laws of motion are the same in all inertial reference frames and this is known as Galilean relativity.
- In Galilean relativity the position x' and time t' of an event are given by: $x' = x - vt$ and $t' = t$.
- Galilean transformation equations lead to the velocity addition equation as given by: $u' = u - v$.

◆ **Galilean relativity** How relative motions were described before the discovery of special relativity.

Galilean relativity refers to relative motion as described and explained (using the principles we have already used in Topics A.1 to A.3) by Galileo, Newton and others. That is, relativity as understood before special relativity, which was first introduced by Einstein in 1905 (see below). Galilean relativity can be assumed to be a very good approximation for special relativity for speeds which are low compared to the speed of light.

In Galilean relativity, Newton's laws of motion are the same in all (inertial) frames of reference.

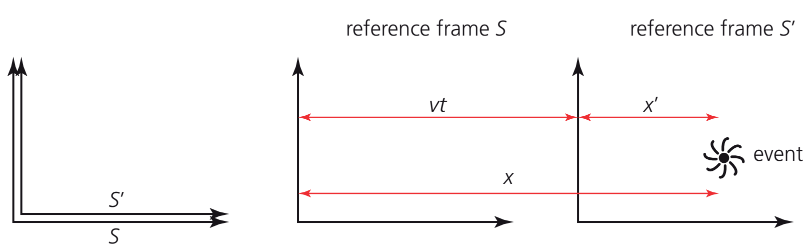
Whenever we move our point of view from one reference frame to another, we need to do what is called a *transformation* by applying standard equations. This becomes very important when we study relativity, so it is worth ensuring that Galileo and Newton's simpler vision of the Universe (as follows) is expressed in a similar way and fully understood.

Observations of position, distance and speed made within our own reference frame are straightforward, but if we make measurements from our reference frame of similar quantities in another reference frame which is moving compared to us, we need to know in what way our measurements are different from measurements made in the other reference frame.

Consider first, the widely used example of an event that occurs on a moving train and how it is seen by observers on the train and on the ground. Consider that both observers start their clocks when the observer sitting on the train travelling with a constant velocity of $20\,\text{m s}^{-1}$ passes the observer sitting on the ground. After 5 s, the observer on the ground sees a flash of light (an event) at a point on the train 120 m away. The coordinates of this event in his frame of reference are $x = 120\,\text{m}$, $t = 5\,\text{s}$.

The train has moved forward $5 \times 20 = 100\,\text{m}$ in the 5 s since the timings began, so that, in the frame of reference of the observer on the train, the flash of light occurs a distance of 20 m in front of her. The coordinates of this event in her frame of reference are $x = 20\,\text{m}$, $t = 5\,\text{s}$.

More generally, Figure A5.4 compares where an event is observed to occur in two different reference frames (S and S') which were coincident (in the same place) at time $t = t' = 0$, and before they separated.



a time $t = t' = 0$
Two reference frames coincident

b time $t = t'$
Reference frames have separated and an event occurs

Figure A5.4 Comparing an event in two frames of reference

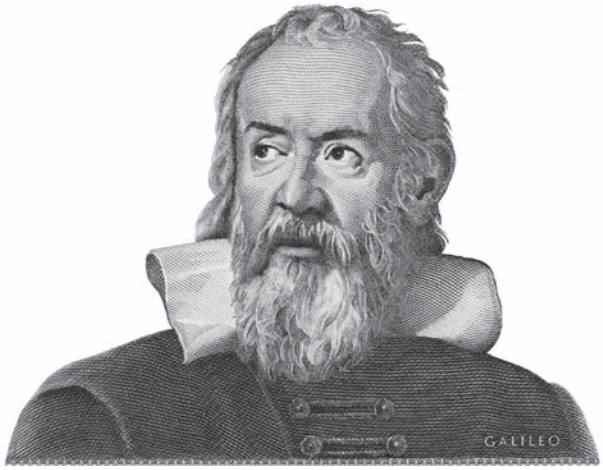


Figure A5.5 Galileo Galilei (1564–1642)

◆ **Galilean transformation** The non-relativistic method of mathematically relating observations between reference frames.

If in reference frame S the coordinates of an event are (x, t), in reference frame S', which has relative velocity v compared to S, the coordinates (x', t') of the same event are

$\boldsymbol{x' = x - vt}$ and $\boldsymbol{t' = t}$

These are known as the **Galilean transformation** equations.

Common mistake

It is easy to think that an event primarily occurs in one reference frame and then it is observed in another, but we need to remember that any given event occurs in all reference frames.

Remember that this assumes that the two reference frames are coincident when $t = t' = 0$.

Velocity addition equation

In this topic we will often need to calculate the velocity of an object as observed from different reference frames. This called **velocity addition**. In Galilean relativity this is straightforward. Returning to our previous example of the train: suppose that an object on a train (moving with velocity v) is moving forward with a constant speed and an observer on the ground (reference frame S) records this as a velocity u. This will be greater than the velocity of the object, u', recorded on the train (reference frame S'). $u' = u - v$

If in reference frame S the velocity of an object is u, in a reference frame S', which has relative velocity of v compared to S, the same movement will be recorded as having a velocity, $\boldsymbol{u' = u - v}$

◆ **Velocity addition** Equation which connects the velocities of the same object as observed in two different reference frames.

WORKED EXAMPLE A5.2

A train is moving with a constant velocity of 16.0 m s^{-1}. A ball is rolling along the floor of the train in the direction of travel with a constant velocity of 3.0 m s^{-1}.

a Calculate the velocity of the ball as recorded by an observer on the ground outside.

b Determine how your answer would change if

- **i** the ball was rolling towards the back of the train with the same speed
- **ii** the train was moving in the opposite direction (with the ball moving towards the front of the train). .

Answer

a $u' = u - v$

$3.0 = u - 16.0$

$u = +19\ \mathrm{m\,s^{-1}}$

b i $u' = u - v$

$-3.0 = u - 16.0$

$u = +13\ \mathrm{m\,s^{-1}}$

ii $u' = u - v$

$-3.0 = u - (-16.0)$

$u = -19\ \mathrm{m\,s^{-1}}$

If we choose to reverse which reference frames are S and S', the same answers will be obtained (as we should expect)

WORKED EXAMPLE A5.3

In deep space, rocket A leaves a space-station with a constant velocity of $300\,\text{m s}^{-1}$. At the same time rocket B travels in the same direction with a constant velocity of $200\,\text{m s}^{-1}$.

a Calculate the distance between rocket A and the space-station after one hour.

b According to an observer in rocket B, what is the distance to rocket A after one hour?

c In rocket B's reference frame, determine how fast an observer would measure the speed of rocket A.

Answer

a $x = ut$ where $t = 1 \times 60 \times 60 = 3600\,\text{s}$

$= 300 \times 3600 = 1.08 \times 10^6\,\text{m}$

b $x' = x - vt$

$= 1.08 \times 10^6\,\text{m} - (200 \times 3600)$

$= 3.6 \times 10^5\,\text{m}$

c $u' = u - v$

$= 300 - 200 = 100\,\text{m s}^{-1}$

Assume that the Newtonian model of the Universe is correct and use Galilean transformations to answer the following questions. *(Note that the answers to some of these questions will contradict the rules of relativity that are introduced later.)* The speed of light, $c = 3.00 \times 10^8\,\text{m s}^{-1}$.

4 In Worked example A5.3 the rockets travel in the same direction. Use the Galilean transformation equations to calculate the answers to Worked example A5.3 questions **b** and **c**, if the rockets travel in opposite directions.

5 A rocket travelling at one-tenth of the speed of light away from Earth shines a laser beam forwards into space.

- **a** Determine how fast an observer inside the rocket measures the light beam photons to be travelling.
- **b** Calculate how fast an observer floating stationary, relative to the Earth, measures the light beam photons to be travelling.

6 Two rockets travelling towards each other are measured by an observer on Earth to each be moving with a speed of $0.6c$. Calculate how fast an observer in one rocket thinks that the other rocket is travelling.

7 If you were in an incredibly fast spaceship that was travelling past a space-station at $0.35c$ and you accelerated a proton inside the ship so that it was travelling forwards through the ship at $0.95c$, what speed would an observer in the space-station measure the proton to be travelling?

Limitations of Galilean relativity

The discovery of the constancy of the speed of light (in a vacuum) – see opposite – was evidence that Galilean relativity could not be applied under all circumstances.

Consider the Galilean velocity addition equation applied to light: if a beam of light was sent forward by a passenger on the train (instead of a ball as in Worked example A5.2), they would *correctly* record the speed of light leaving them to be $c = 3.00 \times 10^8\,\text{m s}^{-1}$, but using the Galilean velocity equation *incorrectly* predicts that the observer on the ground would record a higher speed.

◆ **Michelson–Morley experiment** An experiment designed to measure the Earth's speed through the ether. The famous null result was the prime reason for the abandonment of the ether idea, which then contributed to the development of special relativity.

◆ **Ether (or aether)** A hypothetical substance, proposed (falsely) to be the medium through which electromagnetic waves travel.

◆ **Special relativity** Theory connecting space and time developed by Albert Einstein based on two postulates concerning relativistic motion in inertial reference frames. The consequences lead to time dilation, length contraction and the equivalence of mass and energy.

Nature of science: Hypotheses, and falsification

The Michelson–Morley experiment

In the nineteenth century it was assumed that light needed a medium through which to travel as with other types of waves, such as sound. It was thought that there was a not-understood or detected, 'luminiferous aether', that was present in all space, including vacuum.

Many experiments were designed to discover and investigate this **aether**. The most famous were the experiments carried out in 1887 by Albert Michelson and Edward Morley. The experiments were technically difficult because of the high speed of light, but it was expected that it would be possible to detect small differences in the speed of light beams sent in different directions through the aether (because of the motion of the Earth).

The **Michelson-Morley experiment** failed to demonstrate (verify) the hypothesis that the speed of light would be affected by its direction of travel through the aether. Repeated tests then, and subsequently, have not detected any difference in the speed of light. It has been called 'the most famous failed experiment in history', although perhaps it would be fairer to refer to Michelson and Morley's important result as a 'null finding.'

The results of experiments similar to this confirm that the speed of light is *always* observed to have the same value, regardless of the motions of the source or observer.

Introducing special relativity

Towards the end of the nineteenth century, the classical physics of Newton and Galileo faced two very big problems:

1 The work of James Clerk Maxwell on electromagnetism in the middle of the nineteenth century had combined the phenomena of electricity, magnetism and light. However, Maxwell's (correct) theories of electromagnetism contradicted classical physics in some important respects. James Clerk Maxwell was undoubtedly one of the greatest physicists/mathematicians of all time but his work is *not* included in this course.

2 Experiments were unable to show that light travelled at different speeds depending on its direction of travel with respect to the rotating Earth.

■ **Figure A5.6** Albert Einstein in 1905

Einstein proposed the theory of **special relativity**, connecting space and time, in 1905 in order to resolve these problems. His theory adjusts the Galilean / Newtonian model for speeds close to the speed of light, but classical physics is still valid for slower speeds.

It is called 'special' relativity because it is restricted to inertial frames of reference.

The effects of special relativity only become significant at speeds close to the speed of light, but none of us have any direct experiences of such phenomena in our everyday lives. Throughout the rest of Topic A.5, we will be using examples of events involving such speeds: the motion of imaginary rockets and atomic particles, with imaginary observers travelling with them.

LINKING QUESTION

- Why is the equation for the Doppler effect for light so different from that for sound?

This question links to understandings in Topic C.5.

HL ONLY

HL ONLY

◆ **Postulates of special relativity** The speed of light in a vacuum is the same for all inertial observers. The laws of physics are the same for all inertial observers.

◆ **Invariant quantity** A quantity that has a value that is the same in all reference frames. In relativity, examples are the speed of light in a vacuum, space–time interval, proper time interval, proper length, rest mass and electrical charge.

The two postulates of special relativity

SYLLABUS CONTENT

- The two postulates of special relativity.

First postulate: the laws of physics are identical (**invariant**) in all inertial reference frames.

The first postulate does not initially appear to be profound. However, it can be interpreted as:

- Observations in different inertial reference frames all have equal worth; there is no single 'correct' frame of reference.
- The Universe has no unique stationary reference frame.
- No experiment is possible that can show an observer's absolute velocity through the Universe.

Second postulate: the speed of light in a vacuum is a constant, $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$, in all inertial reference frames.

This simple statement has enormous implications and needs to be carefully considered (as explained in the rest of this topic). It does not appear to make any sense as judged by our experiences from everyday life. However, many experiments have confirmed it to be true.

It implies that if a rocket in deep space passes a space-station at a tenth of the speed of light, and fires a laser beam forwards as it does so, then both the observer in the rocket and on the space-station must measure the speed of light to be $3.00 \times 10^8\,\mathrm{m\,s^{-1}}$, even though they are moving relative to each other. For this to be the case, space and time must behave in profoundly different ways to how we have learnt to expect.

LINKING QUESTION

- Special relativity places a limit on the speed of light. What other limits exist in physics? (NOS)

TOK

Knowledge and the knower

- How do our expectations and assumptions have an impact on how we perceive things? Is the truth what the majority of people accept?

The constancy of the speed of light (and its implications) conflicts with our expectations and experiences but is undoubtedly true. This counterintuitive knowledge cannot be denied and is a cornerstone of modern physics. Scientists are faced with similar issues when dealing with quantum physics.

Implications of the two postulates

Since the constant speed of light equals distance travelled / time taken, and we know from Galilean relativity that observers in different frames of reference can measure different distances between the same events, then the observers must also measure different times for the travel of a light beam. This means that observers moving relative to each other will disagree about the measurement of time.

Time cannot be an invariant quantity in a relativistic universe.

Our earlier model of the matrix of clocks (Figure A5.2) was wrong. Not only do clocks read different times and tick at different rates but for any pair of events different clocks can record different time *intervals*. In other words, the time interval between two events is not the same for different observers taking measurements from different (inertial) reference frames.

◆ **Space–time** The combination of space and time into a single entity that is used to describe the fabric of the Universe. Fundamentally, in relativity, time and space are not independent of each other. They are observed differently depending on the relative motion of an observer.

◆ **Paradigm** The complete set of concepts and practices etc. that characterize a particular area of knowledge at a particular time. When these change significantly, it is described as a **paradigm shift**.

HL ONLY

Space and time are linked as **space–time**. Space and time are not independent of each other, as they were assumed to be in Newtonian mechanics.

The rest of this topic will provide more details about 'space–time' and the implications of special relativity, including

- time dilation
- length contraction
- relativistic velocity additions
- invariant space–time intervals
- space–time diagrams
- relativity of simultaneous events.

TOK

The natural sciences

- What role do paradigm shifts play in the progression of scientific knowledge?

When referring to a particular area of knowledge, a **paradigm** is the name given to the collected, relevant and widely accepted theories, understandings and practices (and so on) that characterize that topic at that time. In the late nineteenth century, the paradigm of knowledge about motion had been centred on the work of Galileo and Newton (and others).

In his 1962 work 'The Structure of Scientific Revolutions' the American philosopher of science Thomas Kuhn suggested that a **paradigm shift** occurs when, for whatever reason, an existing paradigm is replaced with a new paradigm. In science, this might occur after a significant new discovery, or after a completely new, and probably widely unexpected, theory is developed.

A totally new way of thinking about existing knowledge requires great imagination and individuality, which many would describe as genius. Einstein's theory of relativity was possibly the greatest paradigm shift in the history of physics.

Human nature is such that it is usually difficult for people, who may have spent many years living with a particular paradigm, to accept something totally new, which contradicts what they had previously believed.

ATL A5A: Thinking skills

Providing a reasoned argument to support a conclusion

Apart from relativity, another famous paradigm shift in physics knowledge was the confirmation that the Earth was not at the centre of the Universe.

Describe another paradigm shift (of any kind, perhaps from your studies in other IB Diploma subjects) and explain why you think it had significant and far-reaching effects.

Lorentz transformations

SYLLABUS CONTENT

- The postulates of special relativity lead to the Lorentz transformation equations for the coordinates of an event in two inertial reference frames as given by:

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

where

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

◆ **Lorentz transformation** The equations, involving the Lorentz factor, used to calculate the new position and time coordinates, or spatial and temporal intervals, when transferring from one relativistic reference frame to another.

◆ **Lorentz factor, γ** Scaling factor that describes the distortion of non-invariant quantities when moving between different relativistic reference frames: $\gamma = \frac{1}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)}$

In Newton's model of the Universe, we used Galilean transformation equations to move from one reference frame to another, allowing us to change one coordinate into another (from x, y, z and t to x', y', z' and t'). We can do the same in Einstein's relativistic Universe, but we must instead use the **Lorentz transformation** equations, as follows (for the x-direction only). The equations can be used to transform x-coordinates and t-coordinates of a single event, but only if the origins of the two reference frames coincided at $t = 0$ s.

If in reference frame S the coordinates of an event are x and t, in reference frame S', which has relative velocity of v compared to S, the coordinates (x', t') of the same event are given by:

$$x' = \gamma(x - vt)$$

and

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

γ is known as the **Lorentz factor**. It can be calculated using the following equation.

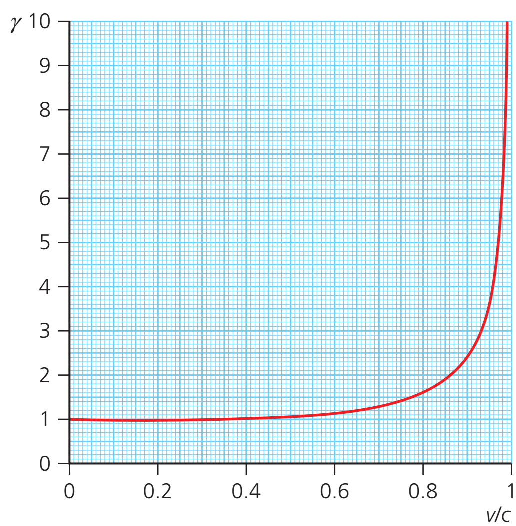
Lorentz factor:

$$\gamma = \frac{1}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)}$$

The factor involves a ratio, so it does not have any units.

For the purposes of your IB Diploma Physics course, you do not need to know the origin of these three equations.

γ is always greater than one. For everyday macroscopic speeds, $v << c$, so that γ has a value very close to one, which shows us that relativistic effects are not significant in our daily lives. For speeds close to the speed of light, γ becomes significantly greater than one, so that relativistic effects dominate. See Figure A5.7.



■ **Figure A5.7** Graph showing how the Lorentz factor, γ, varies with speed, v (shown as v/c).

WORKED EXAMPLE A5.4

a Calculate the Lorentz factor for a relative speed of $1.50 \times 10^8\,\mathrm{m\,s^{-1}}$ ($0.50c$).

b An event in reference frame S occurs at $x = 5000\,\mathrm{m}$ and $t = 2.0\,\mathrm{s}$. Calculate when and where the same event occurs as observed from a rocket (reference frame S') which has a relative velocity of $0.50c$.

c State what assumptions you have made to answer part **b**.

Answer

a $$\gamma = \frac{1}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)} = \frac{1}{\left(\sqrt{1 - \frac{(1.50 \times 10^8)^2}{(3.00 \times 10^8)^2}}\right)} = 1.15$$

b $x' = \gamma(x - vt) = 1.15 \times (5000 - [0.50 \times 3.00 \times 10^8 \times 2.0]) = -3.5 \times 10^8\,\mathrm{m}$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = 1.15 \times \left(2.0 - \frac{0.50 \times 3.00 \times 10^8 \times 500}{(3.00 \times 10^8)^2}\right) = 2.3\,\mathrm{s}$$

c It was assumed that the two frames of reference were coincident at $t = 0\,\mathrm{s}$.

Tools 3: Mathematics

Use units where appropriate: light year

Astronomical distances are huge. The nearest star to Earth, other than our Sun, (*Alpha Proxima*), is $4.02 \times 10^{16}\,\mathrm{m}$ away. It becomes convenient to use larger units than metres and kilometres in astronomy. The following are non-SI units.

DB

The **light-year (ly)** is the *distance* travelled by light in one year: **1 ly** = $(3.00 \times 10^8) \times 365 \times 24 \times 360$ = $\mathbf{9.46 \times 10^{15}\,m}$

(A light-year is defined to be exactly a distance of 9 460 730 472 580 800 m.)

In light-years, the distance to *Alpha Proxima* is 4.25 ly.

Using the light-year as the unit of distance makes many relativity questions easier to answer.

The *parsec* is another widely used unit for distance in astronomy (see Topic E.5).

◆ **Light-year, ly** Unit of distance used by astronomers equal to the distance travelled by light in a vacuum in 1 year.

WORKED EXAMPLE A5.5

According to a rest observer in reference frame S, a rocket reaches a point 20 light-years away after 30 years. This gives (x, t) coordinates for the rocket as (20 ly, 30 y). Another reference frame S' is moving at $0.50c$ relative to S. Determine the coordinates of the rocket according to an observer in S'. (The two reference frames were coincident at $t = 0$.)

Answer

We have already calculated $\gamma = 1.15$ for $v = 0.50c$.

$x' = \gamma(x - vt) = 1.15 \times [20\,\mathrm{ly} - (0.50c \times 30\,\mathrm{y})] = 1.15 \times (20\,\mathrm{ly} - 15\,\mathrm{ly}) = 5.8\,\mathrm{ly}$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = 1.15 \times \left(30\,\mathrm{y} - \frac{0.50c \times 20\,\mathrm{ly}}{c^2}\right) = 1.15 \times (30\,\mathrm{y} - 10\,\mathrm{y}) = 23\,\mathrm{y}$$

Therefore, according to an observer in reference frame S', the rocket has only travelled 5.8 ly in 23 years, which means that it is travelling at only $0.25c$. This example is straightforward because the unit being used (ly) allows c to be cancelled easily.

WORKED EXAMPLE A5.6

One observer records an event at $x = 250\,\text{m}$ and $t = 1.7 \times 10^{-6}\,\text{s}$. Determine the coordinates of this event as recorded by a second observer travelling at $0.75c$ to the right according to the first observer. Assume the frames of reference were coincident at $t = 0$.

Answer

$$\gamma = \frac{1}{\left(\sqrt{1-\frac{v^2}{c^2}}\right)} = \frac{1}{\left(\sqrt{1-0.75^2}\right)} = 1.51$$

$$x' = \gamma(x - vt)$$

$$= 1.51 \times (250 - [0.75 \times (3.00 \times 10^8) \times 1.7 \times 10^{-6}] = -200\,\text{m}$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$= 1.51 \times \left(1.7 \times 10^{-6} - \frac{0.75 \times (3.00 \times 10^8) \times 250}{\left(3.00 \times 10^8\right)^2}\right) = 1.6 \times 10^{-6}\,\text{s}$$

Equations for transforming distances and time intervals between two events

Often, we are interested in the differences in x-coordinates and t-coordinates between events. That is, distances and time *intervals*. The transformation equations (Δx to $\Delta x'$) and Δt to $\Delta t'$) are similar to those shown above:

Distance between two events:

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

Time interval between two events:

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

For each of these problems assume one-dimensional motion and assume that, in each case, the observers start timing when the origins of the two reference frames coincide.

8 Imagine a situation where a rocket passes the Earth at $0.5c$. There are two observers – one in the Earth's frame of reference and the other in the rocket's frame of reference.

a Calculate the value of γ.

b A star explosion event occurs at a point 20 light-years from the Earth. The rocket passes the Earth heading towards the star. According to the Earth-based observer the rocket passes the Earth 20 years before the light arrives.
Determine x' and t' coordinates of the explosion event for the observer in the rocket's reference frame.

9 From Earth, the Milky Way galaxy is measured to be 100 000 light-years in diameter, so the time taken for light to travel from one side of the Milky Way to the other is 100 000 years. Calculate the diameter of the Milky Way for an observer in a distant galaxy moving at a speed of $0.2c$ away from Earth.
Assume that they are travelling in the same plane as the measured diameter of the Milky Way.

◆ **Inertial observer** An observer who is neither accelerating nor experiencing a gravitational field.

◆ **Spatial** To do with the dimensions of space. A spatial interval is a length in space.

◆ **Temporal** To do with time. A temporal interval is an interval of time.

10 According to Earth-based astronomers a star near the centre of the Milky Way exploded 800 years before a star 2000 ly beyond it.
Determine how much later the second explosion is according to a rocket travelling towards the explosions at 0.2*c*.

11 In a laboratory an electron is measured to be travelling at 0.9*c*. According to an observer in the laboratory at $t = 9.6 \times 10^{-9}$ s it is at a position of $x = 2.6$ m down the length of a vacuum tube.
Calculate the value of the Lorentz factor and use it to work out the time and position of the electron according to an observer in the electron's reference frame.

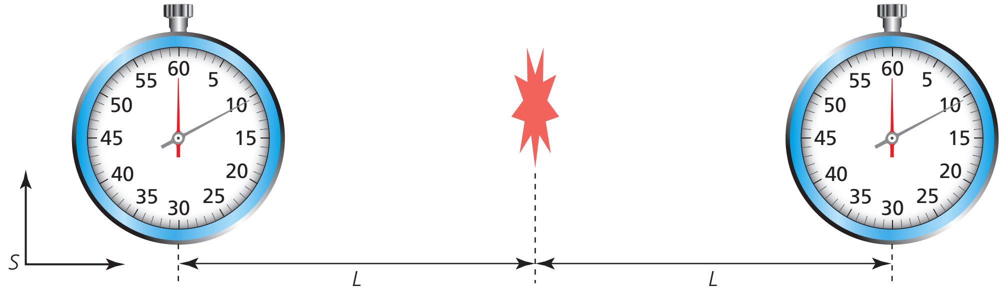
12 Two **inertial observers** are travelling with a relative velocity of 0.8*c* and both see two events occur. According to one observer the events occur 4.2 m apart and with a time interval of 2.4×10^{-8} s between them.
According to the other observer, determine the **spatial** ($\Delta x'$) and **temporal** ($\Delta t'$) intervals between the two events.

Clock synchronization

◆ **Synchronized** Two clocks are said to be synchronized if according to an observer they are reading the same time.

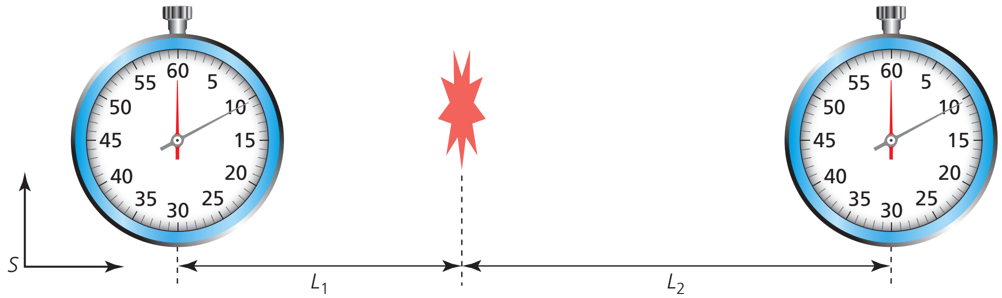
If we wish to compare times of events at different locations in the same reference frame, we need to **synchronize** the clocks. That is, we need to make sure that both / all clocks in the same reference frame always show exactly the same time.

The simplest method is for a flash of light (or a sound) to originate exactly midway between the two clocks and the clocks are both started when the flash of light is detected. The flashes of light are received simultaneously.



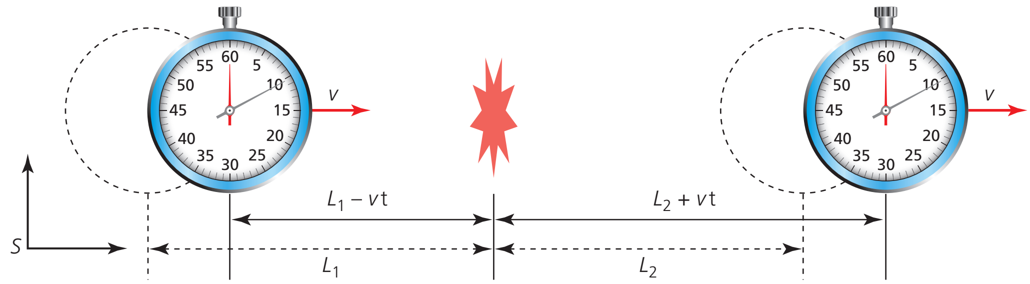
■ **Figure A5.8** Synchronizing two equidistant clocks

The arrangement seen in Figure A5.8 may not be practicable and it is more likely that the clocks are placed at different distances from the flash of light. See Figure A5.9. If the distances are known (L_1, L_2), then the clocks can be synchronized by setting them initially to $\frac{L_1}{c}$ and $\frac{L_2}{c}$, and then starting the clocks from those values when the flash is received.



■ **Figure A5.9** Synchronizing two clocks at different distances

Now consider how an observer in a different reference frame, S', would detect the process seen in Figure A5.8. See Figure A5.10. An observer who sees the clocks moving will see one clock moving towards the flash and the other clock moving away from the flash. Remember that the speed of light, c, is the same in all reference frames.



■ **Figure A5.10** Clocks cannot be synchronized by an observer with a velocity relative to them

As seen in reference frame S', the light takes time to travel outwards from the flash and, in this time, one clock is further from the source of the flash than the other clock, so that the light cannot reach the two clocks at the same time in S' (simultaneously). In reference frame S' the two clocks will not be synchronized, and the clock on the left will read an earlier time than the one on the right.

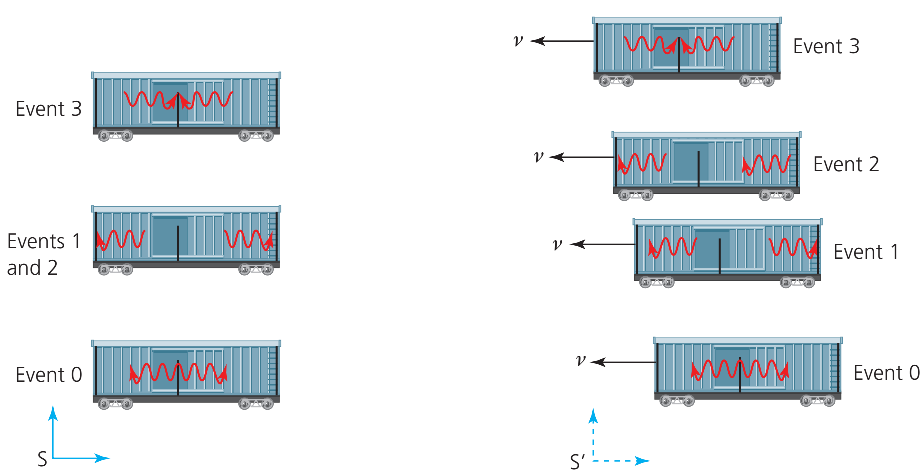
In other words, the observer in reference frame S thinks that two events (light arriving at the two clocks) are simultaneous, but the observer in reference frame S' records that the event at the left-hand clock occurred first.

Simultaneity

SYLLABUS CONTENT

▶ The relativity of simultaneity.

The previous section has shown that we need to reconsider our ideas about whether two or more events occur at the same time (are simultaneous) and the order of events. Consider Figure A5.11, which shows a similar situation to Figure A 5.10, but this time the figure shows a flash of light sent from the centre of a train carriage (Event 0) in both directions to mirrors at the end of the carriage. The light beams are then reflected back (Events 1 and 2) to their origin (Event 3).



■ **Figure A5.11** Simultaneous and non-simultaneous events

HL ONLY

The reference frame shown on the left-hand side (S) is that of an observer on the train. Both beams of light travel equal distances in equal times. The observer will record that:

- The pulses are sent out simultaneously.
- The pulses reach each end of the carriage simultaneously.
- The pulses return to observer S and are recorded simultaneously.

The reference frame shown on the right-hand side is that of an observer on the ground outside the train (S'). The observer in S' will record that:

- The pulses are sent out simultaneously.
- The pulse that travels down the carriage against the motion of the carriage must arrive at the end of the carriage (Event 1) before the pulse that travels up the carriage (Event 2) because it must have travelled a shorter distance at the same speed.
- However, the observer in S' still sees the pulses return to observer S simultaneously because the effect is reversed for the reflected rays.

If two (or more) events occur at the same place at the same time, then they are simultaneous for all observers in all reference frames.

If two (or more) events occur at different places, then it is possible that they could be simultaneous for one observer in one reference frame, but not be simultaneous for other observers in other reference frames.

We will return to the concept of simultaneity when discussing space–time diagrams.

WORKED EXAMPLE A5.7

A spacecraft travelling at 0.6c passes a space-station and at that moment clocks in both locations are set to zero. A short time later the spacecraft passes a point which is 3000 m away from the origin of the space-station's reference frame. Determine what time a clock on the spacecraft will record for this event.

Answer

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

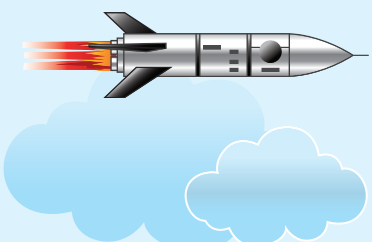
with

$$\gamma = \frac{1}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)} = 1.25$$

and time in reference frame of space-station,

$$t' = 1.25 \times \left(1.67 \times 10^{-5} - \frac{0.6 \times 3.00 \times 10^8 \times 3000}{\left(3.00 \times 10^8\right)^2}\right) = 1.34 \times 10^{-5}\,\text{s}$$

13 A *light clock* is a concept sometimes used as a way of comparing observations that are made by observers in two different inertial reference frames. A light clock is a very simple device that reflects a light beam between two parallel mirrors separated by a fixed distance, d. The speed of light in a vacuum is constant for all observers, but the path length taken by the light varies. See Figure A5.12; one physicist (Rachel) is in the rocket, while another (Mateo) is hiding in the cloud.



■ **Figure A5.12**

One of the diagrams in Figure A5.13 shows the path of the light beam as seen by Rachel, while the other is seen by Mateo, who sees the rocket moving to the right with speed v. Which is which?

14 According to Mateo the time that the light pulse takes to travel from M_1 to M_2 is Δt. Therefore, state how far the rocket moves sideways in this time.

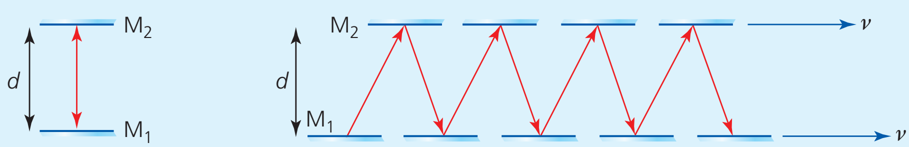
15 Use a Galilean transformation to determine the speed of the light beam according to Mateo.

16 Using Newtonian physics, calculate how far the light beam has to travel when reflecting between M_1 and M_2, according to Mateo.

17 Mateo sees the rocket moving sideways with speed v. In terms of c and Δt, how far has the light beam travelled from M_1 and M_2 according to Mateo?

18 According to Rachel in the rocket, the time taken to travel from M_1 to M_2 is $\Delta t'$. Utilizing Pythagoras's theorem, use your understanding of the postulates of Newtonian physics to derive an expression for Δt in terms of $\Delta t'$, v and c.

19 Explain in terms of the constancy of the speed of light, why the two observers must disagree about the time it takes for the light beam to travel between M_1 and M_2.



■ **Figure A5.13**

Velocity addition transformations

SYLLABUS CONTENT

▶ Lorentz transformation equations lead to the relativistic velocity addition equation as given by:

$$u' = \frac{u - v}{\left(1 - \frac{uv}{c^2}\right)}$$

Apart from times and distances, we need to know how to make velocity transformations between different frames of reference. Earlier in this topic we used the Galilean velocity transformation: $u' = u - v$, but for situations in which relativistic effects are significant, we need to use the following equation:

$$\boldsymbol{u' = \frac{u - v}{\left(1 - \frac{uv}{c^2}\right)}}$$

u is a velocity of an object in one reference frame (S), u' is the velocity of the same object as seen by an observer who is in a reference frame (S') moving with a velocity v with respect to the first reference frame. Remember that u' must always be less than c.

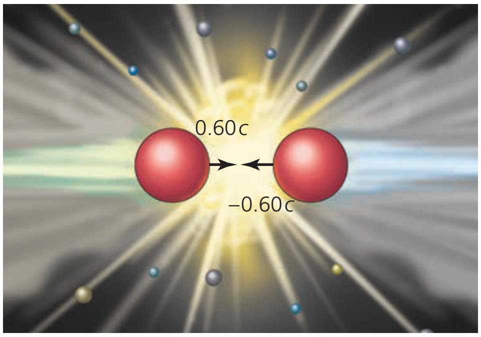
Note that if u and/or v are small compared to c, the equation reduces to the Galilean form.

There is no need to understand the origin of this equation, although it is linked to the Lorentz transformations.

HL ONLY

WORKED EXAMPLE A5.8

Two particles are seen from an external reference frame to be travelling towards each other, each with a velocity of $0.60c$ (Figure A5.14).



■ **Figure A5.14** Two particles

An observer with one particle measures the velocity of the other particle; determine what speed they record.

Answer

$$u' = \frac{u - v}{\left(1 - \frac{uv}{c^2}\right)}$$

$$= \frac{0.60c - -0.60c}{1 - \frac{(0.60c \times -0.60c)}{c^2}}$$

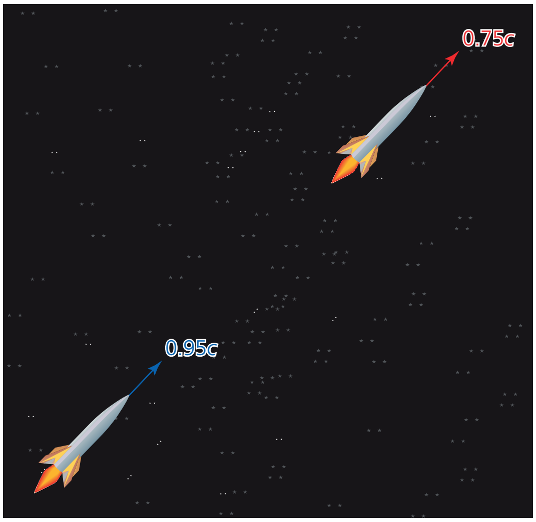
$$= \frac{1.2c}{1 - \frac{-0.36c^2}{c^2}} = \frac{1.2}{1.36}c = 0.88c$$

The situation is symmetrical. Observers with each particle travelling at $0.60c$ will measure the speed of the other to be $0.88c$.

It is very easy to miss out the negative signs when doing this calculation. Remember that u and v are both vectors and so can be positive or negative depending on their direction.

WORKED EXAMPLE A5.9

Two rockets are observed from an external reference frame (S) to be travelling in the same direction – the first is measured to be travelling through empty space at $0.75c$, and a second rocket, which is sent after it, is measured to be travelling at $0.95c$ (Figure A5.15).



■ **Figure A5.15** Two rockets

Calculate what an inertial observer travelling with the first rocket would measure the approach of the second rocket to be.

Answer

$$u' = \frac{(u - v)}{\left(1 - \frac{uv}{c^2}\right)}$$

$$= \frac{(0.95c - 0.75c)}{1 - \frac{(0.95c \times 0.75c)}{c^2}} = \frac{0.20c}{1 - \left(\frac{0.71c^2}{c^2}\right)}$$

$$= \frac{0.20}{0.29}c = 0.69c$$

Some of these questions refer to photons (light 'particles') and atomic particles, which have not yet been discussed in this course. However, no detailed knowledge is needed to answer the questions.

20 A rocket travelling at one-tenth of the speed of light away from Earth shines a laser beam forwards into space.

a An observer inside the rocket accurately measures the speed of the light beam photons. What value would you expect them to obtain?

b An observer floating stationary, relative to the Earth, also accurately measures the light beam photons. State what value they will obtain.

21 Two rockets are flying towards each other; each are measured by an observer on Earth to be moving with a speed of $0.7c$.
How fast does an observer in one rocket think that the other rocket is travelling?

22 If you were in an incredibly fast spaceship that was travelling past a space-station at $0.35c$ and you accelerated a proton inside the ship so that it was travelling forwards through the ship at $0.95c$, relative to the ship. Determine the speed an observer in the space-station would measure the proton to have.

23 In an alpha-decay experiment the parent nucleus may be considered to be stationary in the laboratory. When it decays, the alpha particle travels in one direction with a velocity of $0.7c$ while the daughter nucleus travels in exactly the opposite direction at $0.2c$.
According to an observer travelling with the daughter nucleus, calculate how fast the alpha particle is travelling.

24 In a beta particle decay experiment an electron and an anti-neutrino that are produced happen to travel away in exactly the same direction. In the laboratory reference frame, the anti-neutrino has a velocity of $0.95c$ and the electron has a velocity of $0.75c$.
What is the anti-neutrino's velocity according to an observer travelling in the electron's reference frame?

25 Protons in CERN's Large Hadron Collider travel in opposite directions around the ring at over $0.999\,000\,0c$.
According to an observer travelling with one group of protons, how fast are the approaching protons travelling?

26 Two light beams are travelling towards each other in exactly opposite directions. According to a laboratory observer their relative velocity is $2c$.
Calculate how fast an observer travelling in the reference frame of one of the light beam's photons measures the speed of the approaching light beam's photons to be.

27 In a space race two spaceships pass a mark and are measured by the race officials at the mark to be travelling in the same direction and travelling at $0.6c$ and $0.7c$ respectively.
According to the faster spaceship, calculate how fast the other ship is moving.

Time dilation

SYLLABUS CONTENT

- Time dilation as given by: $\Delta t = \gamma \Delta t_0$.
- Proper time interval.

We have already seen that observers in relative motion will not agree about measurements of time. We will now consider the measurement of time intervals in more detail.

An observer in reference frame S measures a time interval, Δt, between successive ticks of a clock that is stationary in their reference frame. Another observer in reference frame S' moving with relative velocity, v, will measure a greater time interval for the same ticks, as given by the Lorentz transformation:

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

(As shown previously.)

However, since the ticks of the clock occur at the same place, $\Delta x = 0$, so that the equation can be simplified:

If in reference frame S the time interval between two events that occur at the same place, which is stationary in that reference frame, is Δt, then in reference frame S', which has relative velocity of v compared to S, the time interval between the same two events will be observed to be $\Delta t' = \gamma \Delta t$.

◆ **Time dilation** Relative to an observer who sees the two events occurring in the same place. All other observers measure an increase in the time interval between two events.

◆ **Twin paradox** A paradox that appears to challenge special relativity, based on the impossibility that two twins should each find that they are older than the other. One twin remains on Earth while the other travels at high speed to a distant star and returns.

Since the Lorentz factor, γ, is always greater than one, the time intervals between the ticks of a clock in a reference frame (S') that is moving relative to where the clock is at rest (reference frame S) are greater. This is known as **time dilation**.

WORKED EXAMPLE A5.10

Imagine that a rocket is travelling at $2.0 \times 10^8\,\mathrm{m\,s^{-1}}$ away from Earth.

a The rocket carries a clock which ticks once every second. Calculate the time interval between ticks of the clock as observed on Earth.

b How would your answer change if the clock was on Earth and observed from the rocket?

Answer

a $\Delta t' = \gamma \Delta t$

$$\gamma = \frac{1}{\left(\sqrt{1-\frac{v^2}{c^2}}\right)} = 1.7$$

$\Delta t' = \gamma \Delta t = 1.7 \times 1.0 = 1.7\,\mathrm{s}$

1.0 s on the rocket corresponds to 1.7 s on Earth.

b The situation is symmetrical: 1.0 s on Earth corresponds to 1.7 s on the rocket.

It is important to realize that time dilation refers to all processes, not just the ticking of a clock. Time proceeds at different rates for frames of reference that are moving relative to each other, but this only becomes relevant at very high speeds. In this course we will *not* be considering what happens to accelerating frames of reference, or frames of reference that are moving closer together.

ATL A5B: Research skills

The '**twin paradox**' is a widely used situation used when discussing time dilation. Research into this thought experiment, explain why it is described as a paradox, and outline how it is resolved.

TOK

Knowledge and the knower

- Does knowledge always require some kind of rational basis?

A paradox is a situation, question or statement that contains an embedded contradiction. For example: Perhaps the most famous example of a simple paradox is 'this statement is false'. What might paradoxes suggest to us about the nature or reason and its value in determining what is true?

Proper time interval

◆ **Proper time interval** The time interval between two events as measured by an observer who records the two events occurring at the same point in space. It is the shortest time interval between events measured by any observer.

It should be clear that the magnitude of a time interval between two events depends on whether the observer is moving with respect to the events being observed. If the observer is in the same frame of reference as the events, this corresponds to the shortest possible time interval, which is called the **proper time**. If the observer is moving with respect to the events, the time interval will be greater.

A proper time interval, Δt_0, is the time interval between two events that take place at the same location as the observer.

We can then re-write the **time-dilation formula**:

time dilation:

$$\Delta t = \gamma \Delta t_0$$

◆ **Time dilation formula** $\Delta t = \gamma \Delta t_0$, where Δt_0 represents the proper time interval as measured by an observer who sees the first and second events occur in the same place.

You need to try to imagine yourself in the reference frame of each observer – are the x, y and z coordinates of the two events the same? If they are, then this observer measures the proper time between the two events.

28 In a laboratory, an electron is accelerated by a potential difference of 100 kV. Its speed is then measured by timing how long it takes to pass between two different points measured in the laboratory as being 5.00 m apart. Is the observer in the electron's reference frame or the observer in the laboratory reference frame recording proper time?

29 A rod measured in its rest frame to be one metre in length is accelerated to $0.33c$. The rod is then timed as it passes a fixed point. Is the observer at the fixed point or the observer travelling with the rod measuring proper time?

30 The same rod is timed by both observers as it travels between two fixed points in a laboratory. If the observers are recording when the front of the rod passes each fixed point, is either observer measuring the proper time?

31 In a third experiment the two observers start timing when the front of the rod passes the first point and stop timing when the end of the rod passes the second point. Is either observer measuring the proper time?

Length contraction

SYLLABUS CONTENT

- Length contraction as given by:
  $L = \dfrac{L_0}{\gamma}$
- Proper length.

Suppose that we observe an object in another reference frame, S', which is moving relative to our reference frame, S. The length of the object, a rod for example, $\Delta x'$, can be determined in reference frame S' from a measurement of the position of its ends.

We know from the Lorentz transformation equation that $\Delta x' = \gamma(\Delta x - v\Delta t)$ (as seen above), where Δx is the length of the rod as observed from reference frame S. Assuming that the measurement of the positions of both ends of the rod are done at the same time, $\Delta t = 0$, so that the equation simplifies:

If in reference frame S', the length of an object, which is stationary in that reference frame, is recorded to be $\Delta x'$, then in reference frame S, which has relative velocity of v compared to S, the length will be recorded as:

$$\Delta x = \frac{\Delta x'}{\gamma}$$

◆ **Length contraction** The contraction of a measured length of an object relative to the proper length of the object due to the relative motion of an observer.

Since γ is always greater than one, any length in another reference frame, which is moving relative to us, will always be less than the length we would measure if the object was in our own reference frame. This is called **length contraction**.

Top tip!

It may seem strange that relativistic effects *increase* time intervals but *decrease* lengths. For example, from observations of a spacecraft travelling at very high speed away from Earth, time expands but lengths contract (both as seen from Earth). This is because we are considering a clock at rest in the Earth's reference frame, but a length at rest in the spacecraft's reference frame.

WORKED EXAMPLE A5.11

The length of an object measured in a spacecraft travelling at 20% of the speed of light away from Earth is 1.00 m. Determine what length would be detected by an observer on Earth.

Answer

$$\gamma = \frac{1}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)} = 1.02$$

$$\Delta x = \frac{\Delta x'}{\gamma} = \frac{1.00}{1.02} = 0.980\,\text{m}$$

Because the situation is symmetrical, a 1.00 m length on Earth would be recorded as 0.980 m from the spacecraft.

Proper length

The magnitude of a length between two points depends on whether the observer is moving with respect to the observations. If the observer is in the same frame of reference as the object, this will correspond to the longest possible length, which is called the **proper length**, L_0. If the observer is moving with respect to the object, the length will decrease.

The proper length of an object, L_0, is its length when stationary in the same reference frame as the observer.

We can then re-write the **length contraction formula** as:

DB

length contraction: $\boldsymbol{L} = \dfrac{\boldsymbol{L_0}}{\gamma}$

◆ **Proper length** The proper length of an object is the length measured by an observer who is at rest relative to the length being measured. The proper length is always the longest length measurable by any observer.

◆ **Length contraction formula** $L = L_0/\gamma$, where L represents the length and L_0 represents the proper length.

32 In a laboratory, an electron is accelerated by a potential difference of 100 kV. Its speed is then measured by timing how long it takes to pass between two different points measured in the laboratory as being 5.00 m apart.
Is an observer in the electron's reference frame, or the observer in the laboratory reference frame recording the proper length between the two points?

33 A rod measured in its rest frame to be one metre in length is accelerated to $0.33c$. The rod is then timed as it passes a fixed point.
Is the observer at the fixed point or the observer travelling with the rod measuring proper length between the start and finish events?

34 The same rod is timed by both observers as it travels between two fixed points in the laboratory.
If the observers are recording when the front of the rod passes each fixed point, is either observer measuring the proper length for the distance between the start and finish events?

35 In a third experiment the two observers start timing when the front of the rod passes the first point and stop timing when the end of the rod passes the second point.
Is either observer measuring the proper length for the distance between the start and finish events?

36 A rod is measured to have a proper length of exactly 1.00 m.
Calculate how long you would measure it to be if it was to fly past you at $0.80c$.

37 The time taken for the rod in question 36 to pass a fixed point in the laboratory is 2.5×10^{-9} s.
Determine what time interval an observer travelling with the rod would measure between the same two events.

38 Fatima flies through space and, according to Fatima, her height is 1.60 m. Fatima flies headfirst past an alien spaceship and the aliens measure her speed to be 0.80*c*.

a How tall will the aliens on their spaceship measure Fatima to be?

b Oliver takes 6.1×10^{-9} s to fly past the same aliens at 0.90*c*. According to the aliens what time interval does it take Oliver to fly past them?

39 In a space race, a spaceship, measured to be 150 m long when stationary, is travelling at relativistic speeds when it crosses the finish line.

a According to the spaceship it takes 7.7×10^{-7} s to cross the finishing line. Calculate how fast it is travelling in terms of *c*.

b Determine what time interval the spaceship takes to cross the finishing line according to an observer at the finishing line.

c According to an observer at the finishing line, how long is the spaceship?

d According to the observer at the finishing line, how fast is the spaceship travelling?

40 In the same race as question 39 a sleek new space cruiser takes only 2.0×10^{-6} s to cross the finish line according to the race officials at the line. They measure the space cruiser to be 450 m long. How long is the space cruiser according to its sales brochure?

◆ **Muon decay experiment** An important experiment supporting both time dilation and length contraction. The experiment compares the levels of high-energy muons found in the atmosphere at around 10 km with those found at the Earth's surface, using the muon half-life as a means of measuring time.

◆ **Muon** Unstable elementary subatomic particle.

◆ **Half-life (radioactive)** The time taken for the number of unstable particles to be reduced to half.

◆ **Radioactive decay (radioactivity)** Spontaneous and random change of an unstable particle.

The muon-decay experiment: a test of special relativity

SYLLABUS CONTENT

▶ Muon-decay experiments provide experimental evidence for time dilation and length contraction.

The muon particle effectively provides us with a tiny clock that travels at a speed very close to the speed of light.

A **muon** (μ) is an unstable subatomic particle. Muons are produced naturally in the Earth's atmosphere as a result of collisions between atmospheric particles and very high-energy cosmic radiation that is continually bombarding the planet. This occurs at about 10 km above the Earth's surface; the muons produced have an average speed of around 0.995*c* in the Earth's frame of reference.

Top tip!

You do not need to understand or remember the nature of muon particles. However, an understanding of this section requires some knowledge of the concept of **half-life**, which is not described fully until Topic E.3. A brief outline follows:

Because they are unstable, muons exhibit random **radioactive decay** into other particles. Mathematically, decay is expressed by the concept of *half-life* which is the time taken for half of any given number of muons to decay. The half-life of muons as observed in the frame of reference of the Earth's surface is 1.56×10^{-6} s. This means that:

- After one half-life (1.56×10^{-6} s), half of the muons are still undecayed and half of the muons have decayed.
- After two half-lives ($2 \times 1.56 \times 10^{-6}$ s), $\frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ of the muons are still undecayed.
- After three half-lives ($3 \times 1.56 \times 10^{-6}$ s), $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ of the muons are still undecayed.
- After *n* half-lives ($n \times 1.56 \times 10^{-6}$ s), $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \ldots \left(= \frac{1}{2}\right)^n$ of the muons are still undecayed.

Muon decay and the predictions of classical physics

The time taken for the muons to travel the 10 km from the upper atmosphere down to the Earth's surface is calculated simply as:

$$t = \frac{x}{v} = \frac{(10 \times 10^3)}{(0.995 \times 3.00 \times 10^8)} = 3.35 \times 10^{-5}\ \text{s}$$

During this time, most of the muons will have decayed, so that only a very small fraction arrives at the Earth's surface. We can determine that fraction by calculating the number of half-lives involved as shown after the next Tools box.

Tool 3: Mathematics

Carry out calculations involving logarithmic and exponential functions

If we wish to determine the value of y in the exponential equation $y = x^a$, where a is not a whole number, we need to take logarithms:

$$\log y = a \times \log x$$

For example, if $y = 3.4^{7.4}$,

$$\log y = 7.4 \times \log 3.4 = 7.4 \times 0.5315 = 3.933$$

$$y = \text{antilog of } 3.933 = 8.6 \times 10^3$$

Alternatively, natural logarithms, ln, could be used.

Number of half-lives elapsed as muons travel down to the Earth's surface, n

$$\frac{\text{total time}}{\text{half-life}} = \frac{(3.35 \times 10^{-5})}{(1.56 \times 10^{-6})} = 21.5$$

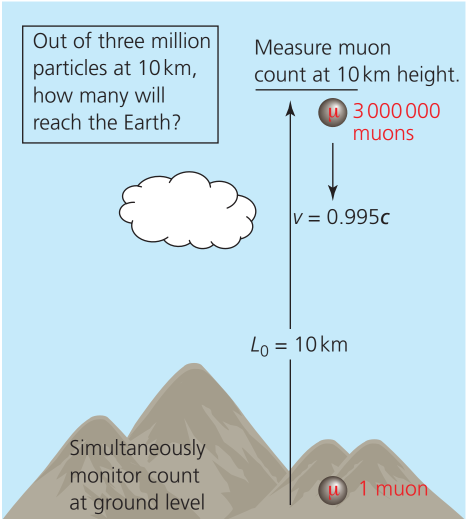
The fraction reaching surface,

$$f = \left(\frac{1}{2}\right)^{21.5}$$

$$\log f = 21.5 \log \left(\frac{1}{2}\right) = -6.47$$

Fraction (f) = 3.37×10^{-7}.

◆ **Classical physics** Physics theories that predated the paradigm shifts introduced by relativity and quantum physics.



■ **Figure A5.16** Using classical physics to predict muon count on Earth's surface

Classical physics predicts that only about 1 muon reaches the surface for every 3 million that are created in the upper atmosphere.

But, in reality, a far greater proportion of muons are detected than this classical physics calculation predicts. In fact, the fraction of the muons from the upper atmosphere reaching the Earth's surface is approximately 0.2 (two in every 10). In order to understand this, we need to replace classical physics with a relativistic interpretation.

Muon decay using relativity and time dilation

In the Earth reference frame, the proper length is 10.0 km and the speed of the muons is 0.995c. An observer in this frame of reference would also measure the time interval for the muons' travel, Δt, to be 3.35×10^{-5} s. However, an observer in the muons' frame of reference would be measuring the proper time interval (Δt_0).

$$\Delta t = \gamma \Delta t_0$$

with:

$$\gamma = \frac{1}{\left(\sqrt{1-\frac{v^2}{c^2}}\right)} = \frac{1}{\left(\sqrt{1-\frac{0.995^2c^2}{c^2}}\right)} = 10$$

so that:

$$3.35 \times 10^{-5} = 10\Delta t_0$$

$$\Delta t_0 = 3.35 \times 10^{-6}\,\text{s}$$

Alternatively, this can be calculated from the time dilation of the half-life.

$$\text{number of half-lives} = \frac{\text{total time}}{\text{half-life}} = \frac{(3.35 \times 10^{-6})}{(1.56 \times 10^{-6})} = 2.15$$

$$\text{The fraction reaching surface} = \left(\frac{1}{2}\right)^{2.15} = 0.23.$$

This is consistent with actual measurements, thus providing experimental evidence for time dilation.

Muon decay using relativity and length contraction

In the muon's frame of reference, the 10.0 km thickness of the lower atmosphere is contracted.

$$L = \frac{L_0}{\gamma} = \frac{10 \times 10^3}{10} = 1000\,\text{m}$$

Then:

$$t = \frac{x}{v} = \frac{1000}{(0.995 \times 3.00 \times 10^8)} = 3.35 \times 10^{-6}\,\text{s}$$

So, the fraction remaining to reach the Earth's surface is once again 0.23. In this analysis, from the muon's reference frame, the rest observer perceives what we measure to be 10 km of atmosphere to be only 1 km.

Experimental confirmation of this result has provided evidence in support of the concept of length contraction.

41 Some muons are generated in the Earth's atmosphere 8.00 km above the Earth's surface as a result of collisions between atmospheric molecules and cosmic rays. The muons that are created have an average speed of 0.99*c*.

- **a** Calculate the time it would take the muons to travel the 8.00 km through the Earth's atmosphere to detectors on the ground according to Newtonian physics.
- **b** Calculate the time it would take the muons to travel through the atmosphere according to a relativistic observer travelling with the muons.
- **c** Muons have a very short half-life. Explain how measurements of muon counts at an altitude of 8.0 km and at the Earth's surface can support the theory of special relativity.

42 In a high-energy physics laboratory electrons were accelerated to a speed of 0.950*c*.

- **a** How long would scientists in the laboratory record for these electrons to travel a distance of 2.00 km at this speed?
- **b** Calculate the time this takes in the reference frame of the electrons.

Space–time

In Newton's Universe, time and space are both invariant – they have fixed intervals that do not vary throughout either space or time. This means that in classical physics we can measure space and time independently. As we have seen, this is not true in a relativistic universe. If time and distance are not fixed, unvarying quantities in a relativistic universe, what quantities can we rely on? The answer is: space–time intervals.

In relativity, space and time are joined to form a single four-dimensional (x, y, z and t) concept called space–time.

◆ **Space–time diagrams** Graphs showing variations of objects' positions with time, adapted to compare different frames of reference.

This is very difficult to visualize, but **space–time diagrams** (discussed later) are very helpful.

TOK

The natural sciences

- What is the role of imagination and intuition in the creation of hypotheses in the natural sciences?

We have seen throughout this topic and elsewhere in Theme A that sometimes physics produces results that seem to contradict our commonsense expectations but must logically be true. What are the different roles of imagination and intuition in making scientific knowledge? How do they relate to reason as a way of knowing?

In essence: things cannot move through space without also moving through time, simply because it takes time to move anywhere. Conversely, we cannot measure time without referring to things moving through space. If everything stayed in exactly the same place and nothing moved, we would have no indication that time was passing.

Space–time was a concept first introduced by Minkowski in 1908. He was Einstein's former mathematics teacher. Einstein initially rejected the idea of space–time but then realized its importance and used it as a major stepping stone in the discovery of general relativity.

◆ **Space–time interval, Δs** A space–time interval combines both the spatial and temporal elements of space–time into a single value.

Space–time interval

SYLLABUS CONTENT

- The space–time interval Δs between two events is an invariant quantity as given by: $(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$.

In classical physics, a distance, Δs, between two points, the length of a rod for example, in three-dimensional space (x, y, z) can be calculated using Pythagoras's theorem: $(\Delta s)^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$. Although the coordinates of its ends may change when seen in different frames of reference, the length of the rod is always the same (invariant).

Similarly, a 'distance' (Δs) between two points in four-dimensional space–time (x, y, z, t) can be calculated from $(\Delta s)^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$. (Note: do not worry about the signs used here; different sources of information commonly use different sign conventions.) You are not expected to explain the origin of this equation.

The 'distance' Δs is called a **space–time interval**.

Since we are restricting direction to the x-direction only, space–time interval squared:

DB

$$(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$$

Different (inertial) observers may measure different time intervals and different distances between events, but they will agree on the space–time interval between events.

HL ONLY

We know that an event that is measured to have coordinates (x, t) in the S reference frame can have different coordinates (x', t') in the S' frame of reference. But observers in both reference frames will agree that the quantities $[c^2(\Delta t)^2 - (\Delta x)^2]$ and $[c^2(\Delta t')^2 - (\Delta x')^2]$ are equal.

$$c^2(\Delta t')^2 - (\Delta x')^2 = c^2(\Delta t)^2 - (\Delta x)^2$$

The importance of this will become clearer when we draw space–time diagrams in the next section.

WORKED EXAMPLE A5.12

A single laser pulse is made to trigger two explosion events as it travels through a long vacuum tube. The two events are 99 m apart and the time for the light to travel this distance is 3.3×10^{-7} s. Determine the space–time interval squared between the two events.

Answer

$$(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2 = \left[(3.00 \times 10^8)^2 \times (3.3 \times 10^{-7})^2\right] - 99^2 = 0.0\,\text{m}^2$$

The space–time interval for any two events linked by a photon travelling in a vacuum is always zero. Two events linked by an object travelling slower than c will have a positive space–time interval, while two events that are too far apart for a photon to travel between the two events in the time interval between them have a negative space–time interval.

WORKED EXAMPLE A5.13

Determine the space–time interval squared for an electron that is fired with a speed of $5.93 \times 10^{-7}\,\text{m s}^{-1}$ across a gap of 5.00 m.

Answer

$$\Delta t = \frac{\Delta x}{v} = \frac{5.00}{5.93 \times 10^7}$$

$$= 8.43 \times 10^{-8}\,\text{s}$$

$$(\Delta s)^2 = (c\Delta t)^2 - \Delta x^2$$

$$= (3.00 \times 10^8)^2 \times (8.43 \times 10^{-8})^2 - 5.00^2$$

$$= 6.14 \times 10^2\,\text{m}^2$$

The fact that the space–time interval between any two events is constant for all observers allows us to calculate how long a time an observer travelling in the electron's reference frame will record between the two events. In this reference frame the electron is stationary and the start and finish lines move towards it with the start and finish events occurring at the electron.

This means that this observer is recording proper time and $\Delta x' = 0$:

$$(\Delta s)^2 = (c\Delta t')^2 - (\Delta x')^2$$

$$(\Delta t')^2 = \frac{(\Delta s)^2}{c^2}$$

$$\Delta t' = \sqrt{\frac{6.14 \times 10^2}{(3.00 \times 10^8)^2}} = 8.26 \times 10^{-8}\,\text{s}$$

We have seen that proper time interval and proper length can be considered as invariant quantities.

- a proper time interval, Δt_0, is the time between two events that take place at the same location as the observer, so that $\Delta x = 0$. The space–time interval can then be written as:

  $$\Delta s^2 = c^2\Delta t_0^{\,2} - \Delta x^2 = c^2\Delta t_0^{\,2} - 0^2$$

  $$\Delta s^2 = c^2\Delta t_0^{\,2}$$

- the proper length of an object, L_0, is its length when stationary in the same reference frame as the observer, so that $\Delta t = 0$. The space–time interval can then be written as:

  $$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 = c^2 0^2 - \Delta x^2 = -\Delta x^2$$

  $$\Delta s^2 = -\Delta x^2$$

The fact that Δs^2 is negative is to do with how Δs^2 is defined.

Space–time diagrams

SYLLABUS CONTENT

- Space–time diagrams.
- The angle between the worldline of a moving particle and the time axis on a space–time diagram is related to the particle's speed as given by: $\tan\theta = \frac{v}{c}$.

The graphical representation of events seen in Figure A5.17 is called a space–time diagram. There are some similarities (but not many) with distance–time graphs. The axes represent the reference frame (coordinate system) of a specific inertial observer.

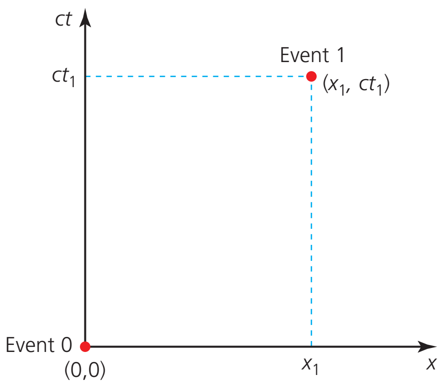


Figure A5.17 Space–time diagram for an inertial reference frame, *S*, showing two events and their coordinates

Space–time diagrams can be a very powerful method of explaining relativistic physics. They contain a lot of information, so we will try to build up the parts before putting a complete diagram together.

Space–time diagrams are normally drawn with the distance, x, on the horizontal axis and time, t, on the vertical axis. Although the vertical axis could just show time, more commonly it shows the speed of light multiplied by time, ct, because, as we will explain, this simplifies interpretation of the diagram with respect to space–time intervals. The units of the vertical axis, ct, are m, which is the same as on the horizontal axis. The scales on both axes are the same.

Events are represented as points in space–time. Just like an ordinary graph, the coordinates of the event are read off from the axes. In Figure A5.17 it is clear that, for an inertial observer in reference frame *S*, Event 0 occurs before Event 1. Event 0 occurs at $x = 0$, $ct = 0$. Event 1 occurs at $x = x_1$, $t = t_1$ (ct_1).

Events that occur on the same horizontal line are simultaneous. Events that occur on the same vertical line occur at the same location.

◆ **World line** The path that an object traces on a space–time diagram.

World lines

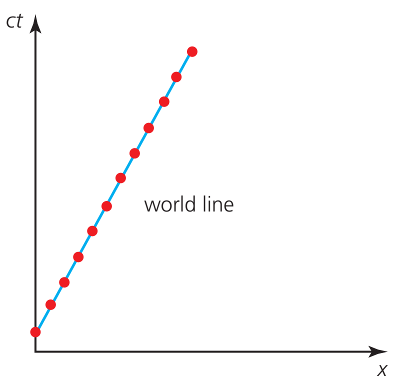


Figure A5.18 Space–time diagram showing how a series of events joined together produces an object's world line

An object travelling through space–time can be imagined as a series of consecutive events. If we join up these events with a line, then we are plotting an object's path through space–time. We call this path the object's **world line**. In Figure A5.18 a straight world line is drawn showing that the object is moving through space with constant velocity relative to the observer. This particular world line does not pass through the origin because the object is observed a short time after the observer started their clock.

Consider Figure A5.19, which shows world lines with different gradients.

Since:

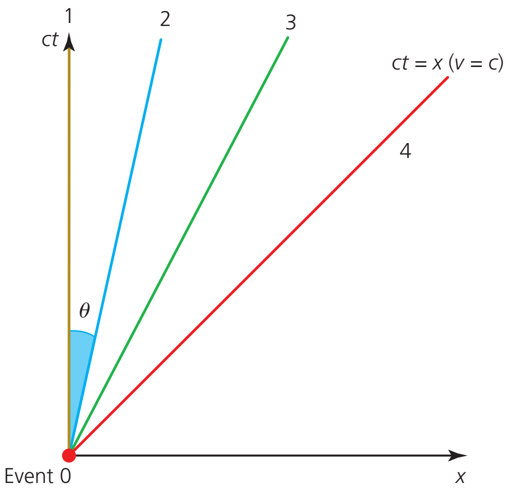
$$v = \frac{x}{t}$$

the gradient of a world line is given by:

$$\frac{ct}{x} = \frac{c}{v}$$

So, the steeper the gradient, the slower the object is travelling. The units are the same on each axis so that the gradient has no units.

An object that is stationary as seen by an observer in the same reference frame will appear as a vertical line because its x-coordinate does not change (Line 1). Line 2 represents a moving object. Line 3 represent faster movement.



■ **Figure A5.19** Straight world lines

Line 4 is a central feature of space–time diagrams. It has a gradient of 1. (Assuming equal scales, the line will be at angle of 45° to both axes.) It represents $v = c$, that is, motion at the speed of light, which can only be light itself.

All inertial observers agree on the value of c, so all observers must agree on the world line for light.

Note that it is not possible for a world line to have an angle θ of greater than 45°, because that would represent an object moving with a speed greater than the speed of light.

Angles between world lines and *ct*-axis

Looking at Figure A5.19, we can see that:

$$\tan\theta = \frac{x}{ct} = \frac{v}{c}$$

The angle between the world line of a moving object and the ct-axis on a space–time diagram is related to the object's speed as given by:

$$\tan\theta = \frac{v}{c}, \quad \theta = \tan^{-1}\left(\frac{v}{c}\right)$$

Adding another frame of reference to a space–time diagram

This is what makes space–time diagrams so useful. Representing a second inertial reference frame on the same diagram is straightforward because the background space–time does not change and events remain in the same places, making it possible to compare how different observers perceive the same events.

The world lines seen in Figure A5.19 represent observations made of four objects in a particular frame of reference, S. Now let us consider how to represent the motion of any of those objects, number 2 for example, in its own frame of reference, S', which is moving to the right with a speed v $(= c\tan\theta)$.

If we were to draw a separate space–time diagram for S', the world line of object 2 would be a vertical line, showing that it was stationary in its own frame of reference. However, the intention here is to draw and compare two (or more) frames of reference on the same diagram.

When we add the S' frame of reference to the original S frame of reference, its space–time axis (ct'), which represents being stationary in its own frame of reference, must coincide with its world line in S. Its other space–time axis (x') must be placed so that the pair of axes are symmetrical about the $ct = x$ and $ct' = x'$ line. See Figure A5.20.

The axes of the S' frame of reference are tilted and not perpendicular to each other. To determine the coordinates of an event we draw lines parallel to the S' axes, just as we do in the S frame of reference, as seen in Figure A5.17.

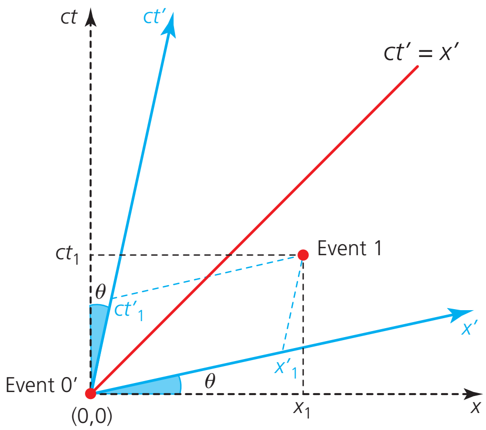
The coordinates of event 1 in S, x_1 and ct_1, transform to x'_1 and ct'_1 in S'.

Space–time diagrams like these give an immediate visualization that $x_1 \neq x'_1$ and $t_1 \neq t'_1$.

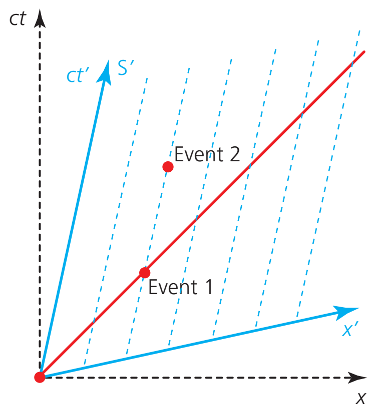
Space–time diagrams can be understood in a similar way to the graphs that you are used to, except that the grid lines are skewed rather than vertical and horizontal.

See Figure A5.21 in which, in reference frame S', Events 1 and 2 occur at the same place, but at different times. The same two events in reference frame S occur at different places and different times.

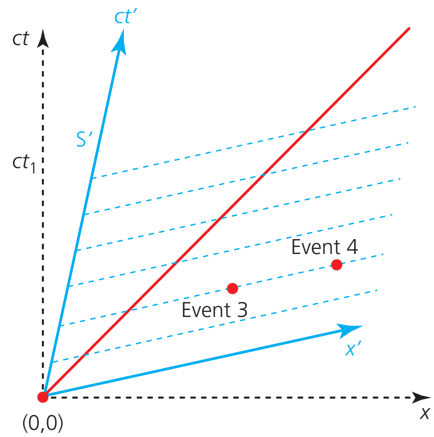
Now consider Figure A5.22, in which Events 3 and 4 are simultaneous for an observer in reference frame S', but at different locations. The same two events in reference frame S occur at different locations and different times.



■ **Figure A5.20** Space–time diagram showing the additional axes for reference frame S' in blue



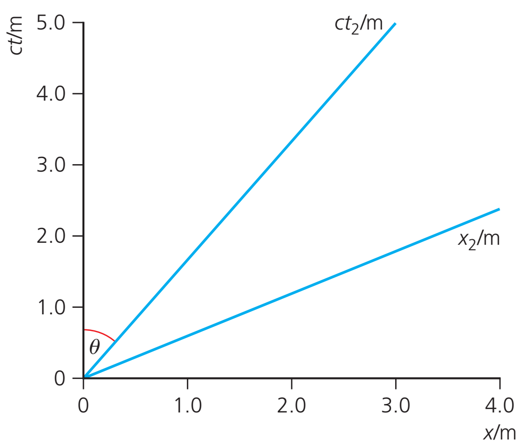
■ **Figure A5.21** Space–time diagram with dashed blue lines representing separate world lines for different points that are each stationary in reference frame S'



■ **Figure A5.22** Space–time diagram with dashed blue lines representing separate world lines for different points that occur at the same time in reference frame S'

WORKED EXAMPLE A5.14

Use Figure A5.23 to determine the speed of the reference frame shown in blue relative to the other reference frame.



■ **Figure A5.23**

Answer

$$\tan\theta = \frac{v}{c}$$

$$\frac{3.0}{5.0} = \frac{v}{c}$$

$$v = 0.60c$$

WORKED EXAMPLE A5.15

Use the Lorentz transformation equations to show that the x'-line has a gradient of v/c, and hence confirm that the angle between the x- and x'-axes is given by: $\tan\theta^{-1} = \left(\frac{v}{c}\right)$, as shown in Figure A5.24.

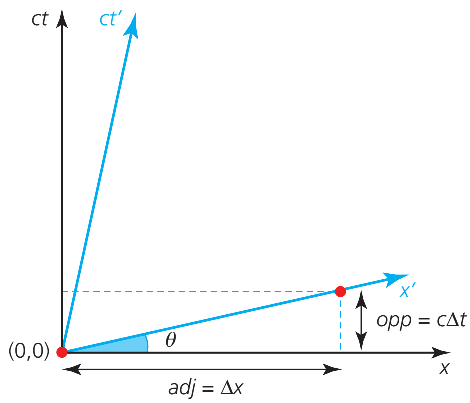


Figure A5.24 Space–time diagram showing the calculation of the angle formula

Answer

The equation for the x'-axis in terms of x and t can be found by setting $t' = 0$ and using the Lorentz transformation for t':

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = 0$$

So, the bracket = 0

$$t = \frac{vx}{c^2}$$

$$ct = \left(\frac{v}{c}\right)x$$

which is of the form $y = mx$

$$\text{gradient} = \left(\frac{v}{c}\right) = \frac{c\Delta t}{\Delta x} = \frac{opp}{adj}$$

$$\tan\theta = \frac{opp}{adj} = \left(\frac{v}{c}\right)$$

$$\theta = \tan^{-1}\left(\frac{v}{c}\right) \text{ as required}$$

WORKED EXAMPLE A5.16

Remember how an observer could demonstrate that two events were simultaneous (see Figure A5.11). Draw a space–time diagram with reference frame S representing the observer on the train and S' representing the reference frame on the platform.

The light rays are sent out in opposite directions, so we need to draw a positive and a negative x-axis to allow us to position both the events (Figure A5.25).

Observer S sees:

- The pulses are sent out simultaneously (Event 0).
- The pulses reach each end of the carriage simultaneously (Events 1 and 2).
- The pulses return to observer S simultaneously (Event 3).

Observer S' sees:

- The pulses are sent out simultaneously (Event 0).
- The pulse that is fired down the carriage against the motion of the carriage must arrive at the end of the carriage (Event 1) before the pulse that is fired up the carriage arrives at the other end of the carriage (Event 2).
- However, S' still sees the pulses return to observer S simultaneously (Event 3); the geometry of the space–time diagram gives us exactly the same outcome, demonstrating that events with no space–time interval are simultaneous for all observers, but events that occur in two separate places can be simultaneous for some observers but not for others.

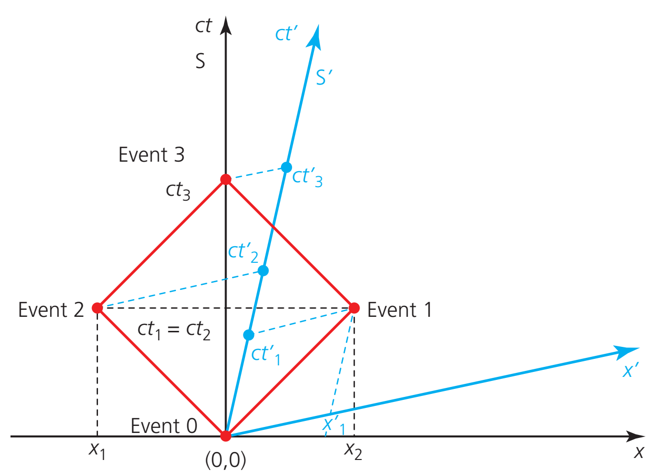


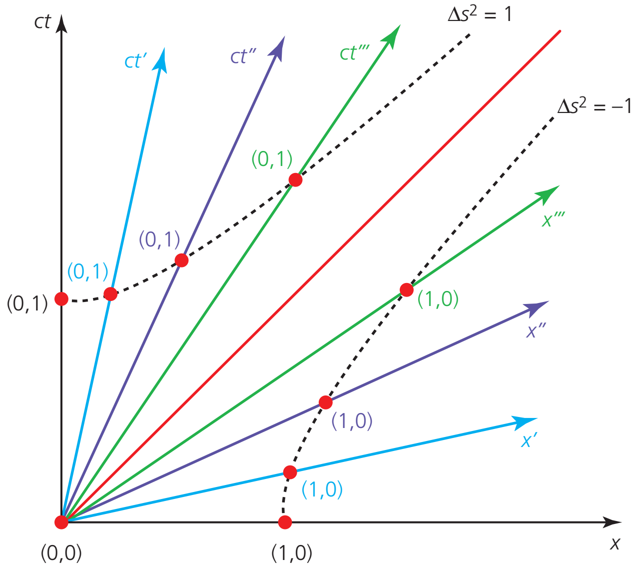
Figure A5.25 Space–time diagram for the thought experiment considering simultaneity, carried out in Figure A5.11. The red lines represent the world lines of the two reflecting light rays. The grey axes represent the inertial reference frame of the train carriage, S, while the blue axes represent the inertial frame of the platform, S', with the train moving to the left. The dashed intersections with the timeline of each observer give their version of the order of events.

Lines of constant space–time interval

We have seen that space–time interval is in invariant quantity:

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 = c^2\Delta t'^2 - \Delta x'^2$$

This means that we can draw lines of constant space–time interval (sometimes called invariant hyperbole) on space–time diagrams.



■ **Figure A5.26** Two lines of constant space–time interval

Figure A5.26 shows four different frames of reference (travelling at different velocities) in different colours. The dotted lines are two examples of lines of constant space–time: $\Delta s^2 = 1$ and $\Delta s^2 = -1$.

Calculations using the coordinates (in any frame of reference) of any point on a line of constant space–time interval will give the same numerical result.

First consider the $\Delta s^2 = -1$ line. The points at which the dotted line crosses all the x-axes corresponds to $t = 0$ and so on, so that all these points, in their different frames of reference, must have coordinates of (1, 0). $c^2\Delta t^2 - \Delta x^2 = -1$ and so on, with $t = 0$.

If observers in each of the reference frames were measuring the length of a stationary rod, with one end at the origin of their reference frame (0, 0), they would all record the same proper length of the rod, 1 m.

We can see immediately that:

The scales of the axes on space–time diagrams are not equal for different frames of reference.

The scales expand with greater velocity, as can be seen by comparing the lengths between the red dots in Figure A5.26.

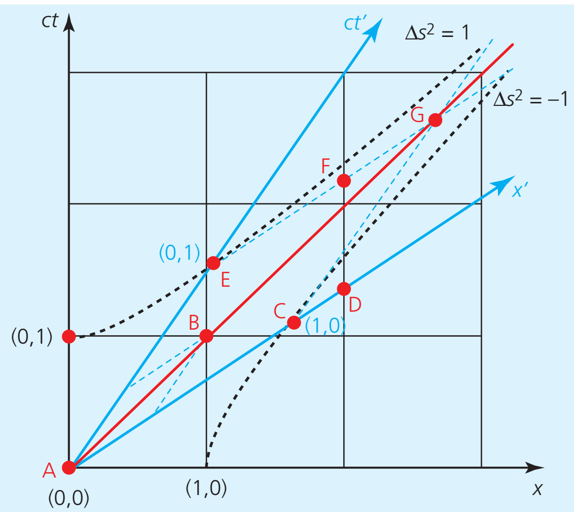
Now consider the $\Delta s^2 = +1$ line. The points at which the dotted line crosses all the ct-axes corresponds to $x = 0$ and so on, so that all these points, in their different frames of reference must have coordinates of (0, 1). $c^2\Delta t^2 - \Delta x^2 = 1$ and so on, with $x = 0$.

43 a Use a ruler, calculator and Figure A5.27 to complete Table A5.1.

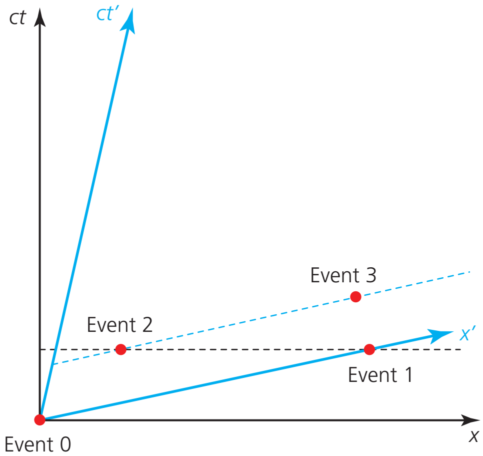
■ **Table A5.1**

Event	Coordinates in S (x, ct)	Coordinates in S' (x', ct')
A	(0, 0)	(0, 0)
B		(0.4, 0.4)
C	(1.6, 1.1)	(1, 0)
D		
E		(0, 1)
F		
G		

b List the order in which the events occur according to observers in both reference frame S and reference frame S'.



■ **Figure A5.27** Space–time diagram showing seven events, labelled A to G, from two different reference frames



■ **Figure A5.28** Space–time diagram comparing simultaneity in different reference frames

Simultaneity in space–time diagrams

Remember that all inertial observers will agree that two events are simultaneous if they occur in the same place, but they may disagree as to the order of two events that occur at two different points in space. Figure A5.28 shows a space–time diagram with four different events. According to one observer, Event 0 occurs first followed by Events 1 and 2 occurring simultaneously, with Event 3 happening last. However, for the other observer Events 0 and 1 both occur simultaneously followed by Events 2 and 3 occurring simultaneously.

Nature of science: Models

Visualization of models

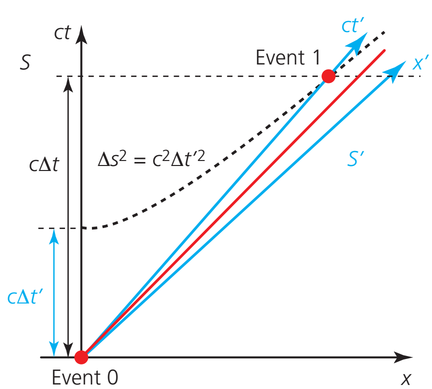
The visualization of events in terms of space–time diagrams is an enormous advance in understanding the concept of space–time.

The Lorentz transformations we use to describe how we transfer from one reference frame to another are highly mathematical and make the topic very difficult to interpret in a non-mathematical way. The geometry of space–time diagrams appears initially quite confusing, but with practice provides an entirely different way of approaching relativity. This new dimension means that aspects of relativity become significantly more accessible – in particular, space–time diagrams readily explain whether or not events are simultaneous in different reference frames and explain the order of events seen by different observers.

With more practice, space–time diagrams also explain concepts such as time dilation and length contraction but they can also be used to understand relativistic velocity additions and to visualize why it is impossible to exceed the speed of light in a vacuum.

Time dilation in space–time diagrams

Look at a space–time diagram for the muon experiment (Figure A5.29). Using the angle formula actually gives an angle of 44.9° for $v = 0.995c$, but ct' is drawn at less than this for clarity.



Event 0 is the formation of a muon by the incoming cosmic radiation, while Event 1 is the arrival of the muon at the Earth's surface. The black reference frame S is the Earth reference frame, while the blue reference frame S' is that of the muon. An observer travelling with the muon measures the proper time between Events 0 and 1. To measure this on the scale of the vertical ct-axis we follow the dashed line of constant space–time interval from Event 1 to where it crosses the ct-axis, where it can be easily calculated by measuring the interval labelled $c\Delta t'$. The interval according to the observer in reference frame S can be calculated using $c\Delta t$.

■ **Figure A5.29** Space–time diagram of the muon-decay experiment.

HL ONLY

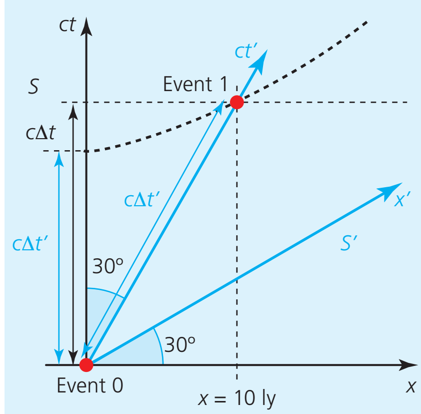
One of the problems with space–time diagrams is that the scales on the axes are not the same. We could use Lorentz transformations to carefully mark the scales on each axis, but there is a neat little trick that allows us to avoid this.

In the muon reference frame, Event 1 occurs a time $\Delta t'$ after Event 0, and both events occur at $x' = 0$. This is because in the muon reference frame the rest observer will see the stationary muon formed in the atmosphere (Event 0) and then the Earth's surface colliding with the stationary muon (Event 1) – the muon is stationary throughout. Thus, the observer measures the spatial separation between the two events to be zero, so the time, $\Delta t'$, is the proper time between the two events, shown correctly to scale on the *ct*-axis.

This occurs at a specific space–time interval, where $\Delta s^2 = c^2\Delta t'^2$, and we can follow the dashed line that joins all the points with this same space–time interval. Where this crosses the vertical *ct*-axis it marks the equivalent interval as measured on the scale of the *ct*-axis. On the space–time diagram this is labelled $c\Delta t'$.

In the Earth reference frame, *S*, the time interval between Events 0 and 1 is significantly longer and can be found from the vertical coordinate of Event 1. This is marked as $c\Delta t$ on the *ct*-axis. Since both measurements have been correctly scaled onto the *ct*-axis their lengths can now be directly compared, and it is clear that the proper time interval $\Delta t'$ is shorter than the stretched (or dilated) Δt time interval. Careful measurement from the *ct*-axis would show that $c\Delta t = \gamma c\Delta t'$. Note that this equation may appear to be the wrong way round because S' is measuring proper time.

44 Use the space–time diagram in Figure A5.30 and the Lorentz transformation equations to do the following calculations.

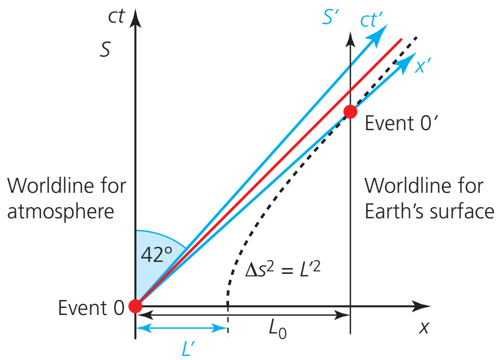


■ **Figure A5.30**

- **a** the velocity of an object that has a world line at 30°
- **b** the value of γ for this speed
- **c** the time, *t*, at which an observer in reference frame *S* will record the object to have travelled 10 ly (Event 1) at this speed
- **d** the value of $c\Delta t$ between Event 0 and Event 1.
- **e** The graph is drawn correctly to scale. Measure the length of $c\Delta t$ and $c\Delta t'$ on the *ct*-axis and show that the ratio of the measured lengths $\dfrac{c\Delta t}{c\Delta t'} \approx \gamma$.
- **f** State which reference frame is measuring proper time.
- **g** Use time dilation to calculate the value of $c\Delta t'$.
- **h** Mark the position of 14 ly on both the vertical black axes (*S* reference frame) and blue axes (S' reference frame) to show that the scales on the axes are different.

Length contraction in space–time diagrams

Once again let us turn to the muon experiment as shown in Figure A5.31. The length being measured is the distance between the formation of the muons in the Earth's atmosphere and the surface of the Earth.



■ **Figure A5.31** Space–time diagram for the muon-decay experiment used to demonstrate length contraction. The instantaneous separation between the atmosphere and the Earth's surface in the muon reference frame must be measured in each reference frame.

In the previous section, the length contraction equation was harder to derive than the time-dilation equation because it required one more key piece of information – in order to measure a length correctly we must measure the position of each end of the length at the same time. In other words, the two space–time events used to determine the two ends of the length in a given reference frame must occur simultaneously in that reference frame.

This length is straightforward to measure in the Earth reference frame because it is simply the horizontal separation between the vertical worldline of the Earth's atmosphere and the world line of the Earth's surface. These are shown on the space–time diagram as the two vertical black axes. Because each is stationary in the Earth reference frame, the separation between them is a proper length and is labelled L_0 on the diagram, where it could easily be measured on the x-axis.

In the muon reference frame, S', the distance between the Earth's atmosphere and the Earth's surface can be measured using simultaneous events 0 and 0′, where the world line of the atmosphere and the world line of the Earth's surface each cross the x'-axis. In the muon reference frame both events occur when $t' = 0$, so they can be used to correctly measure the separation, L' – it could be measured off the scale on the x'-axis but we would need to calculate the scale to do this.

Instead, we can sketch on the curve that links all the points with space–time interval $\Delta s^2 = -L'^2$. Extending this to the x-axis gives the separation between Events 0 and 0′ on the x-axis scale, where it can easily be measured. It can clearly be seen that the proper length is much larger than the contracted length, L', confirming that length contraction occurs. Careful measurement would also show that:

$$L' = \frac{L_0}{\gamma}$$

Once again, the space–time diagram's geometry has been able to represent the dynamics of relativity.

45 In Figure A5.31 the angle between the x and x'-axes is also 42°.

a Calculate the relative velocity of the two frames of reference.

b Calculate the value of γ.

c Calculate the length of L', if $L_0 = 1.0\,\text{m}$.

d Using a ruler, measure the ratio of L_0 / L' to confirm that this gives the value of γ.

46 Einstein's first postulate stated that the laws of physics are the same in all inertial reference frames. This means that we should be able to show on a space–time diagram that an object that is stationary in reference frame S' will also be measured as having a contracted length by an observer in S.

Use Figure A5.32 to show that this is indeed the case by marking on the length as measured by S' on the x' axis and using the space–time interval curve to mark on the x'-axis the equivalent length as measured by S. Hence, use the measured lengths along the x'-axis to estimate the value for γ.

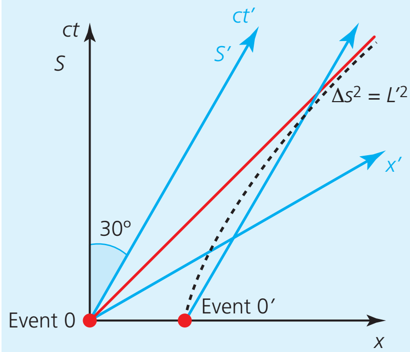


Figure A5.32

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B.1 Thermal energy transfers

Guiding questions

- How do macroscopic observations provide a model of the microscopic properties of a substance?
- How is energy transferred within and between systems?
- How can observations of one physical quantity be used to determine the other properties of a system?

Thermal energy, internal energy and heat

These three terms are often used interchangeably, which can be confusing! Unfortunately, different teachers and different books have varying interpretations, so it is important to clarify, from the beginning of this topic, exactly how these terms will be used in this course.

All substances contain particles / molecules which can have individual kinetic energies and potential energies. There are more details about this later in this topic. We will describe the total of all these particle energies as the *internal energy* of a substance. We will not describe the energy inside substances as thermal energy, or heat.

Energy is always transferred from hotter objects to cooler objects. We will describe this transfer as *thermal energy*, although the word 'heat' is also widely used to describe this type of energy transfer.

TOK

The natural sciences

- Does the precision of the language used in the natural sciences successfully eliminate all **ambiguity**?

Ambiguities

There is, perhaps, nowhere else in the study of physics where such important terms have such ambiguous uses. It is interesting to consider how this has arisen, why it is not corrected, and whether it is truly important.

If you understand the theory of particle energies inside matter, does it really matter if your teacher calls it 'internal energy', while a book refers to is as 'thermal energy' and your friend calls it 'heat'? To what extent is precise language important to our understanding of underlying physics?

◆ **Ambiguity** Open to different interpretations.

Kinetic theory of matter

SYLLABUS CONTENT

- Molecular theory in solids, liquids and gases.
- Density, ρ, is given by: $\rho = \frac{m}{V}$.

◆ **Kinetic theory of matter** All matter is composed of a very large number of small particles that are in constant motion.

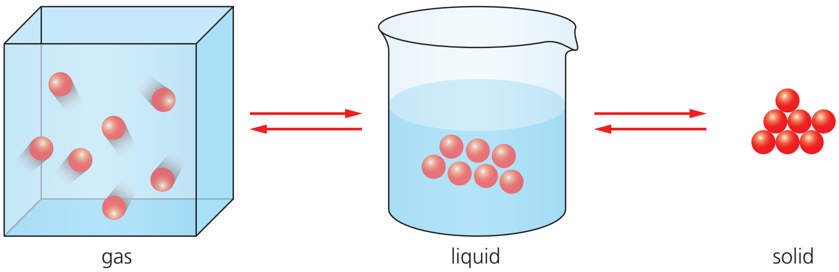
◆ **Ions** An atom or molecule that has gained or lost one or more electrons.

◆ **Atoms** The particles from which chemical elements are composed. They contain subatomic particles.

The essential idea that all matter contains countless billions of particles constantly moving (in various ways), with forces between them when they get close enough, is possibly the most important theory in the whole of science. The microscopic **kinetic theory of matter** is the starting point that can be used to help explain so much of what we observe in our macroscopic everyday life. To begin with, it can explain the different properties of solids, liquids and gases.

Solids, liquids and gases

The 'particles' we are referring to in the kinetic theory are usually molecules, but they could also be **ions**, or **atoms**. Figure B1.1 shows a simplified visual model of particle arrangements. Table B1.1 offers generalized comments on the major differences.



■ **Figure B1.1** Particle arrangements

■ **Table B1.1** Differences between particles in solids, liquids and gases

	Solid	**Liquid**	**Gas**
Arrangement of particles	regular patterns	no pattern	no pattern
Forces between particles	attractive and large enough to keep particles in their positions	some particles have enough energy to overcome attractive forces	negligible (except in collisions) under most conditions
Separation of particles	close together	still close together	much further apart
Motion of particles	vibrate in fixed positions	some limited random movement is possible	all move in random directions and usually with high speeds

Density

Clearly, the more massive the individual particles are, and/or the closer they are together, the greater will be the total mass of a given volume of a substance.

$$\text{density} = \frac{\text{mass}}{\text{volume}} \qquad \rho = \frac{m}{V} \qquad \text{SI unit: kg m}^{-3}$$

◆ **Expand** Increasing in size. An expansion of a gas is an increase in volume.

When solids and liquids are heated to a higher temperature, they will usually **expand** slightly in size because there will be a very small increase in the separation of particles. This means that there will be a small decrease in their densities.

When gases are heated, they will only expand if they are in a container that will allow that to happen.

Table B1.2 Typical values for densities (gases at 0 °C and normal atmospheric pressure)

Substance	Density / $kg\,m^{-3}$
helium	0.18
air	1.23
carbon dioxide	1.98
wood (pine, approx.)	500
ethanol	810
ice	910
water (at 20 °C)	998
water (at 4 °C)	1000
sea water (approx.)	1030
aluminium	2710
average density of Earth	5520
iron	7870
gold	19 300
black hole	1×10^{15}

The particles in solids and liquids can be considered to be as close together as possible, that is, they are effectively 'touching' each other. Since gases are about 1000 × less dense than solids and liquids, their molecules are typically 10 molecular diameters apart from each other.

1 Calculate the density of olive oil in SI units, if a mass of 125 g has a volume of 137 cm^3.

2 An iron bar has the dimensions 5.0 × 5.0 × 25.0 cm. What is its mass?

3 Explain how we can conclude from Table B1.2 that the molecules in air are approximately 10 times further apart than the molecules in water.

4 Outline why ice floats on water. (Refer to buoyancy forces from Topic A.2.)

5 Water has its maximum density at 4 °C. How does this affect the formation of ice on, for example, a lake in very cold weather?

6 In everyday life the volume of liquids is more often measured in litres, l, (and cl and ml) rather than in m^3 or cm^3. 1 litre is a volume of 1000 cm^3.
Determine the volume, in litres, of some ethanol which has a mass of 1.0 kg.

7 It is common practice in many Asian countries to see people making merit by placing gold leaf on Buddhist images, as shown in Figure B1.2.

Figure B1.2 Placing gold foil on a statue of Buddha

Gold is extremely *malleable* – meaning that it can be hammered relatively easily into different shapes, including *very* thin foil (approximately 2×10^{-7} m).

Predict what area of gold foil of thickness 1.80×10^{-5} cm can be made from each 1.0 g of gold.

Temperature

SYLLABUS CONTENT

- Temperature difference determines the direction of the resultant thermal energy transfer between bodies.
- Kelvin and Celsius scales are used to express temperature.
- A change in temperature of a system is the same when expressed with the Kelvin or Celsius scales.
- Kelvin temperature is a measure of the average kinetic energy of particles: $\bar{E}_k = \frac{3}{2}k_B T$.

Macroscopic understanding of temperature

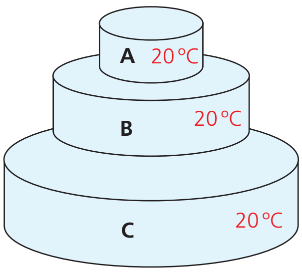
◆ **Thermal equilibrium** All temperatures within a system are constant.

◆ **Thermal contact** Objects can be considered to be in thermal contact if thermal energy (of any kind) can be transferred between them.

In everyday life, temperature is a number obtained from a thermometer, which informs us how hot, or cold, something is. But this is a long way from an acceptable scientific definition. Thinking a little more deeply, we realize that temperature values tell us something about energy transfer: Energy always flows spontaneously from a hotter place to a colder place. As we have already noted, we will call this a flow of *thermal* energy.

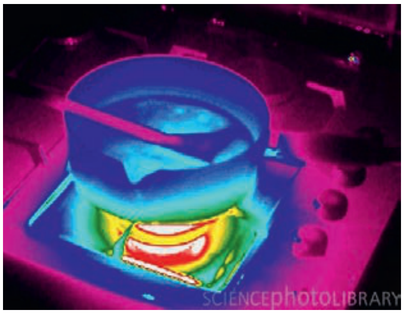
The resultant flow of thermal energy is always from higher temperature to lower temperature.

Consider an isolated system such as shown in Figure B1.3. If there is no net transfer of thermal energy between A and B, then they must be at the same temperature, and we describe them as being in **thermal equilibrium**. Similarly for B and C. If B is in thermal equilibrium with A and C, then A and C are also in thermal equilibrium with each other, and at the same temperature.

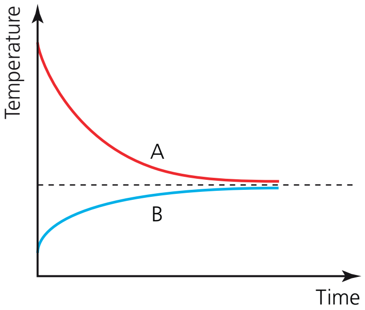


■ **Figure B1.3** Thermal equilibrium: same temperature, no flow of thermal energy

Consider the simple example of a system of two objects (or substances) at different temperatures able to transfer thermal energy between themselves, but isolated from everything around them (their surroundings). The hotter object will transfer energy to the colder object and cool down, while at the same time the colder object warms up. As the temperature difference between the two objects gets smaller, so too does the rate of thermal energy transfer (as represented by the gradients of the graphs). This is represented in Figure B1.5, which shows how the temperature of two objects (A and B) might change when they are placed in good **thermal contact** with each other. Being in 'thermal contact' means that thermal energy can be transferred between them, by any means. But the phrase is most often used when referring to significant physical contact.



■ **Figure B1.4** This thermogram, taken using infrared radiation, uses colour to show different temperatures in a saucepan on a cooker. The scale runs from white (hottest) through red, yellow, green and blue to pink (coldest)



■ **Figure B1.5** Two objects (A and B) at different temperatures, insulated from their surroundings but not from each other, will reach thermal equilibrium

Eventually A and B will reach the same temperature. If the temperatures have stopped changing and both objects are at the same temperature, the objects are in thermal equilibrium and there will be no net flow of thermal energy between them. In any realistic situation, it is not possible to completely isolate / insulate two objects from their surroundings, so the concept of thermal equilibrium may seem to be idealized.

Common mistake

0 °C is simply the freezing point of pure water. It has no other meaning. It is not a 'true' zero. For example, it would be wrong to think that 20 °C was double the temperature of 10 °C.

◆ **Celsius (scale of temperature)** Temperature scale based on the melting point (0 °C) and boiling point (100 °C) of pure water.

◆ **Thermometer** An instrument for measuring temperature.

◆ **Kelvin scale of temperature** Also known as the **absolute temperature scale**. Temperature scale based on absolute zero (0 K) and the melting point of water (273 K). The kelvin, K, is the fundamental SI unit of temperature. T (in K) = 0 °C + 273.

◆ **Absolute zero** Temperature at which (almost) all molecular motion has stopped (0 K or −273 °C).

The concept of hotter objects always getting colder, and colder objects always getting hotter, suggests an important concept: eventually everything will end up at the same temperature.

Temperature scales

Celsius scale of temperature

A temperature scale needs two fixed points. For the **Celsius scale**, these are the freezing point and the boiling point of pure water (under specified conditions). An instrument for measuring temperature, a **thermometer**, can then calibrated by marking these two points as 0 °C and 100 °C, and then dividing the interval between them into one hundred equal divisions. Higher and lower temperatures can then be determined by *extrapolation*.

Kelvin scale of temperature

The choice of the two fixed points on the Celsius scale is arbitrary and mainly for convenience. However, there is a more logical scale – the **Kelvin scale** – that is widely used in science, but in everyday life, people around the world have become used to the Celsius scale (and Fahrenheit scale).

There is a temperature at which the kinetic energy of all particles reduces to (almost) zero. This is known as **absolute zero** and it is discussed in more detail in Topic B.3. On the Celsius scale absolute zero has the value of –273.15 °C. This temperature is the lower fixed point, zero, of the Kelvin temperature scale, which is sometimes described as the **absolute temperature scale**.

Zero kelvin (0 K) is the lowest possible temperature (= –273.15 °C)

The upper fixed point of the Kelvin scale also effectively uses the melting point of pure water which, for convenience, is given the value of +273.15 K. This scale defines the SI unit of temperature, the kelvin, K. Defining the Kelvin temperature scale in this way means that a *change* of temperature has the same numerical value in both the Celsius and the Kelvin scales. Table B1.3 compares some Celsius and kelvin temperatures (to the nearest whole number).

The symbol T is used for temperature in kelvin. Θ is often used for a temperature in degrees Celsius.

Table B1.3 A comparison of temperatures in degrees Celsius and Kelvin

Temperature	°C	K
absolute zero	−273	0
melting point of water	0	273
body temperature	37	310
boiling point of water	100	373

DB

Temperature in kelvin, T/K = temperature in Celsius, Θ/°C + 273

WORKED EXAMPLE B1.1

a The freezing point of ethanol is –114 °C. Convert this temperature into kelvin.

b The melting point of aluminium is 933 K. Convert this temperature to degrees Celsius.

c On a cold night the temperature dropped from +10 °C to −10 °C. Calculate this change of temperature in kelvin.

Answer

a $T = -114 + 273 = 159\,\text{K}$

b $\Theta = 933 - 273 = 660\,°\text{C}$

c $(-10) - (+10) = -20\,\text{K}$

Tool 2: Technology

Use sensors

Electronic sensors respond to a particular physical quantity by producing a corresponding voltage. That voltage must then be converted to digital form before it can be understood, processed and displayed by appropriate software. There are several possibilities, including where a separate data logger/interface, is connected between the sensor and a computer. See Figure B1.6.

Alternatively and more conveniently, the sensor may be connected to a corresponding all-in-one unit which processes and displays results. In some cases, a mobile phone app can be used. Bluetooth connections are also available.

The advantage of using sensors in this way are obvious:

- Data can be collected over very short times (less than a second), or times which are otherwise inconveniently long.
- A large amount of data can be gathered.
- The data can be stored.
- The data can be very quickly processed and graphs drawn.

The sensors which are in common use in physics experiments at this level include:

- position and motion
- pressure
- temperature
- sound level
- light level
- current and voltage
- magnetic field.

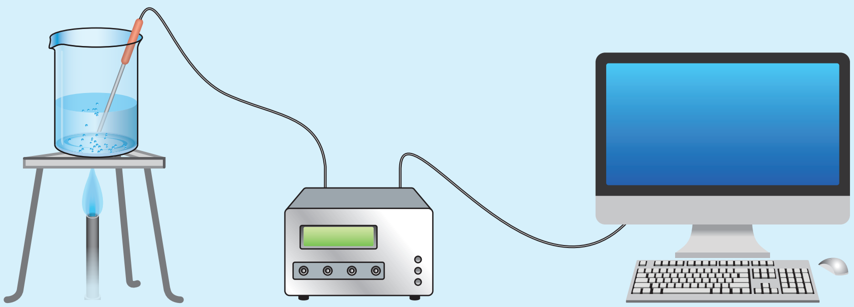


Figure B1.6 Separate sensor, interface and computer

Measuring temperature, using sensors

In principle, any physical quantity which varies with temperature could be used to construct a thermometer. However, it is better to use a physical quantity that varies significantly and regularly over a wide range of temperatures. The most common types of thermometer involve:

- variation in length of a liquid along a thin (capillary) tube
- variation in pressure of a fixed volume of gas
- variation of electrical resistance
- variation in the voltage generated by wires of different metals joined together
- variation in infrared radiation from a surface.

The part of the thermometer which is used to measure the temperature is placed in good thermal contact with the material whose temperature is to be measured (except for infrared thermometers). After sufficient time for the thermometer to reach thermal equilibrium with the material, a reading can be taken. Other sources of thermal energy should be avoided. For example, when measuring air temperature, the thermometer should not be receiving thermal energy directly from the Sun.

Electronic resistance thermometers (sensors) have an obvious advantage in that they can provide immediate digital data when they are connected to a computer through an interface.

8 Convert the following kelvin temperatures into degrees Celsius: **a** 175 **b** 275 **c** 10000.

9 The world's highest and lowest recorded weather temperatures are reported to be 56.7 °C (California – see Figure B1.7) and −89.2 °C (Antarctica).

a State values for these temperatures in kelvin.

b Some hot water at 68 °C cools down to 22 °C. What is this change of temperature in kelvin?

Figure B1.7 The world's highest temperature to date was recorded in California, USA.

10 The volume of a gas was 67 cm^3 when the temperature was 22 °C.

a If the volume is proportional to the absolute temperature, calculate the volume if the temperature increases to 92 °C.

b Predict the volume *if* the gas temperature could be reduced to 0 K.

11 A student calibrated an unmarked liquid-in-glass thermometer. The liquid expands up a thin capillary tube as it gets hotter. She has correctly marked the upper and lower fixed points as 0 °C and 100 °C. The two marks were 10.7 cm apart.

a She then wanted to use her thermometer to measure room temperature and she left it in the laboratory for 10 minutes. The level of the liquid was then 2.6 cm above the lower fixed point.
Explain why the thermometer was left undisturbed for 10 minutes.

b Calculate a value for room temperature.

c State an assumption that you have to make to answer part **b**.

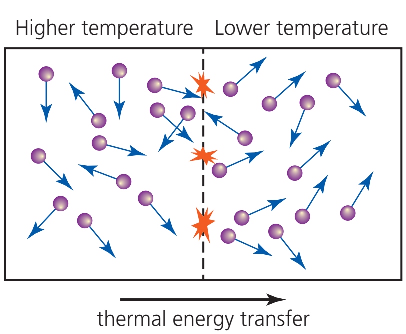


Figure B1.8 Energy transfers between molecules

Microscopic understanding of temperature

A true understanding of temperature is to be found in the kinetic theory of matter. Consider the example of two samples of the same gas, as shown in Figure B1.8. Suppose that the molecules on the right have a lower average kinetic energy (and speed) than the molecules on the left. After they have collisions, the faster moving molecules will slow down and the slower molecules will speed up (conservation of momentum in elastic collisions). In this way energy is transferred from the left to the right. This is equivalent to a transfer of thermal energy, so we must conclude that the left-hand side of the figure represents a higher temperature. Eventually, the average energies and speeds on both sides will become equal and a macroscopic interpretation would be that they were in thermal equilibrium at the same temperature.

Average particle speed indicates different temperatures when we compare samples of the *same* gas. Increasing temperature corresponds to greater average speed. More generally, when we compare *different* gases, we need to consider the average kinetic energy of the particles, rather than their speeds.

Temperature (K) is proportional to the average random translational kinetic energy, $\bar{E}_k$, of particles in a gas.

◆ **Vibrational kinetic energy** Kinetic energy due to vibration/oscillation.

◆ **Boltzmann constant, k_B** Important constant that links microscopic particle energies to macroscopic temperature measurements.

All gases, at the same temperature, contain particles with the same average translational kinetic energy.

The particles in most gases are molecules, which means that they also have other forms of kinetic energy (not just translational), for example, rotational kinetic energy and **vibrational kinetic energy**.

The all-important mathematical connection between macroscopic measurements of kelvin temperature, T, and the microscopic concept of individual molecular kinetic energies is provided by the Boltzmann constant in the following equation:

DB

Average random translational kinetic energy of a gas particle: $\bar{E} = \frac{3}{2}k_B T$

DB

k_B is known as the **Boltzmann constant**. It has the value $\mathbf{1.38 \times 10^{-23}\,J\,K^{-1}}$

LINKING QUESTION

- How is the understanding of systems applied to other areas of physics?

Top tip!

In Topics A.2 and A.3 we discussed collisions between *macroscopic* objects, describing the collisions as either elastic (total kinetic energy of the objects is conserved) or inelastic. During inelastic collisions, energy is transferred to the surroundings, dissipated, mostly in the form of internal energy and thermal energy. That is, energy is transferred from the ordered kinetic energy of countless billions of particles moving together in the same direction in the objects as a whole, to the disordered random kinetic energies of individual particles.

Energy dissipation is a macroscopic concept and cannot be applied to microscopic particle collisions. Total kinetic energy can only decrease in a collision between two particles if it is used to cause ionization (see Topic B.5).

WORKED EXAMPLE B1.2

Calculate the average kinetic energy of translation of gas molecules at room temperature.

Answer

Using 20 °C (293 K) as room temperature,

$$\bar{E}_k = \frac{3}{2}k_B T = 1.5 \times (1.38 \times 10^{-23}) \times 293 = 6.1 \times 10^{-21}\,J$$

The energy of particles in liquids and solids is more complicated because of the significant forces between the particles. In general, however, the following is always true:

A temperature rise is equivalent to the particles gaining kinetic energy.

12 Consider the gas in the previous worked example. If the particles have a mass of 5.3×10^{-26} kg, use the equation for linear kinetic energy to estimate their average speed.

13 The surface temperature of the Sun is 5800 K. Calculate the average kinetic energy of the particles it contains, assuming that the equation in Worked example B1.2 can be applied.

14 A cylinder of gas contains gas molecules moving with an average speed of $400\,\text{m}\,\text{s}^{-1}$, which is characteristic of their temperature of 20 °C. If the cylinder is then put on an aircraft which is moving at $200\,\text{m}\,\text{s}^{-1}$, discuss what will happen to the average speed of the gas molecules and the temperature of the gas.

15 Nitrogen and oxygen are the two principal gases in air. Oxygen molecules are slightly more massive than nitrogen molecules. Explain how the average speeds of the molecules will compare in the air you are breathing.

◆ **Sense perception** How we receive information, using the five human senses.

TOK

Knowledge and the knower

- How do we acquire knowledge?
- To what extent are technologies merely extensions to the human senses, or do they introduce radically new ways of seeing the world?

Sense perception (of temperature)

'Information' received directly by receptors in the human body and then processed by our brains, is described as **sense perception**.

It is often said that we have five senses (hearing, sight, smell, touch and taste), but we also have a limited ability to detect changes in temperature and the flow of thermal energy into, or out of, our bodies. Most people are able to estimate the approximate temperature of the air around them.

However, this 'way of knowing' using sense perception can be unreliable. Whether we are hot or cold is a very common topic of conversation, but people in the same environment can sometimes disagree about the temperature that they sense. Being able to consult an instrument capable of measuring the temperature (a thermometer) has obvious advantages, in everyday life as well as in scientific experiments. However, such reliable measurements were not possible until about 300 years ago.

■ **Figure B1.9** Fahrenheit with his thermometer

A German physicist, Daniel Fahrenheit (Figure B1.9) invented the first accurate thermometer in 1709. He used the expansion of mercury along a thin tube.

Internal energy

All substances contain moving particles. Moving particles have kinetic energy. The particles might be moving in different ways, which gives rise to three different forms of kinetic energy:

- Particles might be vibrating about fixed positions (as in a solid) – this gives the particles *vibrational* kinetic energy.
- Particles might be moving from place to place (translational motion) – this gives the particles *translational* kinetic energy in liquids and gases.
- Molecules might also be rotating – this gives the molecules *rotational* kinetic energy.

Particles can have potential energy as well as kinetic energy. In solids and liquids, it is the electrical forces (between charged particles) that keep particles from moving apart or moving closer together. Wherever there are electrical forces there will be electrical potential energy in a system, in much the same way as gravitational potential energy is associated with gravitational force.

If the average separation of the particles in a solid or liquid increases, so too does their potential energy (and the total internal energy of the substance.)

In gases, however, the forces between molecules (or atoms) are usually negligible because of the larger separation between molecules. This is why gas molecules can move freely and randomly. The molecules in a gas, therefore, usually have negligible electrical potential energy – all the energy is in the form of kinetic energy.

So, to describe the total energy of the particles in a substance, we need to take account of both the kinetic energies and the potential energies. This is called the internal energy of the substance and is defined as follows:

The internal energy of a substance is the sum of the total random kinetic energies and total potential energies of all the particles inside it.

In the definition of internal energy given above, the word 'random' means that the particle movements are disordered and unpredictable. That is, they are not linked in any way to each other, or ordered – as their motions would be if they were all moving together, such as the particles in a macroscopic motion of a moving object. The particles in a moving object have both the ordered kinetic energy of macroscopic movement together and the random kinetic energy of internal energy.

Nature of science: Theories

Caloric fluid

An understanding of thermal energy and internal energy depends on the kinetic theory of matter, but that theory is less than 200 years old. Before that, 'heat' was often explained in terms of a vague invisible 'caloric fluid' that flowed out of a hot object, where it was concentrated, to a colder place where it was less concentrated.

This is an example of one of many serious scientific theories that were developed to explain observed phenomena, but which were never totally satisfactory because they could not explain all observations. The earlier 'phlogiston' theory of combustion is another such theory related to heat and combustion.

Looking back from the twenty-first century, these theories may seem unsophisticated and inaccurate (but imaginative!). However, they should be judged in the context of their times, and at the time of these theories (seventeenth and eighteenth centuries) the kinetic theory of matter had not been developed, so the current understanding of the flow of thermal energy was not possible.

Thermal energy

SYLLABUS CONTENT

- Conduction, convection and thermal radiation are the primary mechanisms for thermal energy transfer.

Thermal energy is the name we give to the transfer of energy because of a temperature difference: a net flow from hotter to colder.

There are three principal ways in which thermal energy can be transferred:

- **Thermal conduction**. In which kinetic energy is transferred between particles.
- **Convection**. In which differences in the densities of liquids and gases result in their movement.
- **Thermal radiation**. In which electromagnetic radiation is emitted by surfaces.

We will discuss each of these in detail in the next three sections.

Thermal conduction

SYLLABUS CONTENT

- Conduction in terms of the difference in kinetic energy of particles.
- Quantitative analysis of rate of thermal energy transferred by conduction in terms of the type of material and cross-sectional area of the material and the temperature gradient as given by: $\frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x}$

As mentioned earlier in this topic, when gas particles (usually molecules) have elastic collisions, the slower moving particles gain kinetic energy and the faster moving particles lose kinetic energy. In this way, over time, there will be a net transfer of energy from a place where particles, on average, are moving faster to a place where they are moving slower on average. That is, from hotter to colder. Such a transfer will continue until the whole of the gas has particles with the same average kinetic energy, when thermal equilibrium has been reached and a constant temperature reached.

◆ **Conduction (thermal)** Passage of thermal energy through a substance as energy is transferred from particle to particle.

Similar ideas can be applied to the transfer of energy between particles in liquids and solids. This type of thermal energy transfer, from particle to particle, is called **thermal conduction**. Figure B1.10 gives an impression of thermal conduction through a solid, although vibrations and increasing kinetic energy are not easily represented in a single picture!

Thermal conduction occurs because of the transfer of kinetic energy between particles.

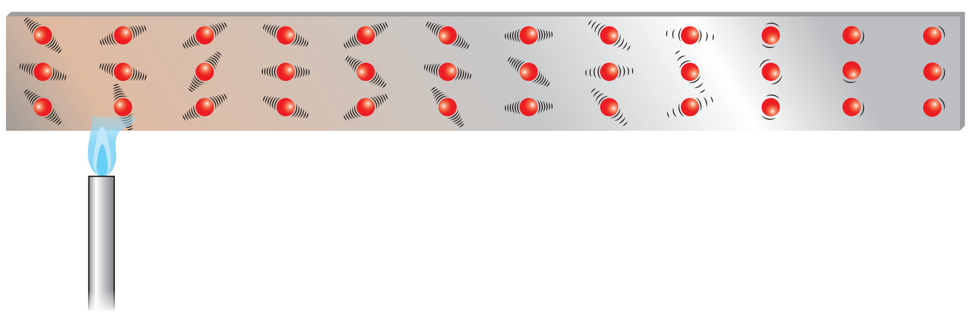


Figure B1.10 Thermal conduction through a solid

Common mistake

Many students think that thermal conduction only occurs in solids. This is not true, although some solids, especially metals, are by far the best thermal conductors. See Table B1.4.

Consider again Figure B1.10. The solid bar has gained its thermal energy by conduction from the hot gas in the flame.

In solids, the particles vibrate in fixed positions, with forces between them. In Figure B1.10, the particles on the left-hand side are vibrating faster and have greater vibrational kinetic energy (on average) because the solid is at a higher temperature. Energy is transferred through the solid, to the right, because of the forces / interactions between particles.

Solids are generally better thermal conductors than liquids, and liquids conduct better than gases. This can be explained by considering the closeness of particles and the strength of forces between them.

Table B1.4 lists various substances and their *thermal conductivities*, which are explained later in this topic. A larger number means that the substance is better at conducting thermal energy: more energy is transferred under similar conditions. (Metals are good conductors because they contain many *free / delocalized electrons*.)

■ **Table B1.4** Typical thermal conductivities at room temperature (some approximate)

Substance	**Thermal conductivity / W m^{-1} K^{-1}**
vacuum	0.00
carbon dioxide	0.15
air	0.025
polyurethane foam	0.03
paper	0.05
rubber	0.13
wood	0.15
common plastics	0.2
water	0.59
concrete and brick	0.72
glass	0.86
carbon	1.7
ice	2.1
iron	84
aluminium	237
copper	385

◆ **Insulator (thermal)** A material that significantly reduces the flow of thermal energy.

We often describe a substance as being either a good thermal conductor or a good **thermal insulator** (poor conductor). However, these are not precisely defined terms, although it should be clear from looking at Table B1.4 that the last three are much better at conducting thermal energy than any of the rest. These three would be described as good thermal conductors; the rest are usually described as insulators.

16 How can you explain that a vacuum has a thermal conductivity of zero?

17 The last three substances in Table B1.4 are all good conductors of thermal energy. State what they have in common.

18 Compare the ability of air, water, glass and copper to conduct thermal energy. (Determine ratios.)

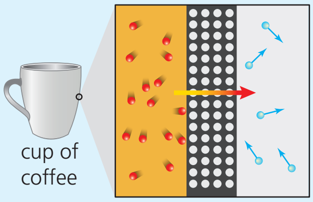
19 Explain why a metal door handle will often feel cooler than a plastic handle at the same temperature.

20 Discuss whether you would describe carbon as a conductor, or an insulator.

21 Explain the choice of materials in the manufacture of the frying pan shown in Figure B1.11.

■ **Figure B1.11** Frying pan

22 Outline the transfers of thermal energy represented in Figure B1.12.



■ **Figure B1.12** Transfer of thermal energy

23 Wetsuits are made from neoprene foam rubber (see Figure B1.13). Suggest how this can keep the surfer from getting too cold.

Figure B1.13 Surfer wearing a wetsuit

Quantitative treatment of thermal conductivity

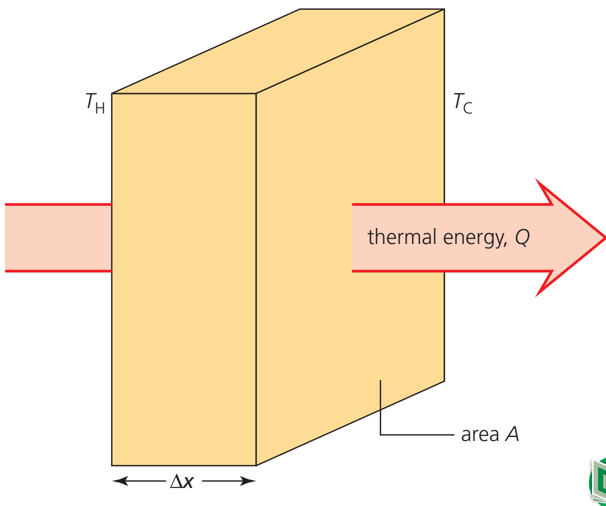


Figure B1.14 Thermal energy flowing through a block

Consider Figure B1.14, which represents the flow of thermal energy by conduction through an isolated system of a specimen of a single substance, which has an area A, and a thickness Δx. The symbol Q will be used for thermal energy.

A flow of thermal energy occurs because the left-hand side is at a higher temperature than the right-hand side: $T_H > T_C$.

The rate of thermal energy flow, $\Delta Q / \Delta t$, will be proportional to the temperature difference, ΔT, and the area, A, but inversely proportional to the thickness, Δx. It also obviously depends on the thermal properties of the substance involved. In summary:

Rate of transfer of thermal energy by conduction:

$$\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$$

◆ **Thermal conductivity, k**
Constant that represents the ability of a substance to conduct thermal energy.

k is a constant, different for each substance. It is called the **thermal conductivity** of the substance (as shown in Table B1.4). Unit: $W\,m^{-1}\,K^{-1}$.

$\frac{\Delta Q}{\Delta t}$ is a flow of energy per second (a power) so it is measured in watts.

WORKED EXAMPLE B1.3

The outside brick wall (single layer) of a house is 4.85 m × 2.88 m. It contains a closed glass window of dimensions 1.67 m × 1.23 m. On a hot afternoon the outside air temperature is 34.0 °C, while it is 27.0 °C inside the room.

Use the equation above to calculate the flow of thermal energy through:

a the wall of thickness 25 cm

b the window of thickness 4.5 mm.

Use data from Table B1.4.

Answer

a $$\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$$

$$= 0.72 \times \left((4.85 \times 2.88) - (1.67 \times 1.23)\right) \times \frac{7.0}{0.25}$$

$= 2.4 \times 10^2$ W into the room.

b $$\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$$

$$= 0.86 \times (1.67 \times 1.23) \times \frac{7.0}{0.0045}$$

$= 2.7 \times 10^3$ W into the room.

The thermal conductivities of brick and glass are similar. Much more thermal energy flows through each cm^2 of the glass because it is significantly thinner.

It should be noted that these calculations *considerably overestimate* the magnitude of thermal energy flows. This is because the surface temperatures of the glass and brick cannot be assumed to be the same as the surrounding air temperatures (as was done in answering the question).

The best insulator for limiting thermal energy flowing out of, or into, homes is air. See Table B1.4. However, if the air can move, thermal energy can also be transferred by *convection currents* (see next section). Various kinds of foam consist mainly of air, but the foam limits the movement of that air. Figure B1.15 shows polyurethane foam between the outer and inner walls of the outside of a house. Similar insulation can be used under the roof and below the ground floor.

Parallel sheets of glass (known as *double glazing*), as seen in Figure B1.16, can be used to trap air and limit thermal energy flow through a window. Obviously, no foam can be put between the sheets of glass, but convection is limited by keeping the separation small. Double glazing has the added benefit of reducing the transfer of sound.

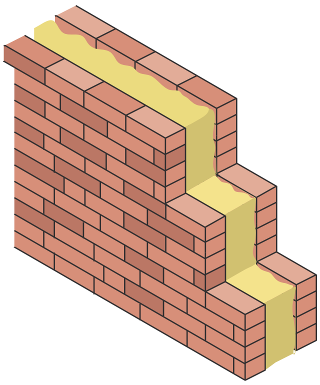


Figure B1.15 Foam insulation in a 'cavity wall'.

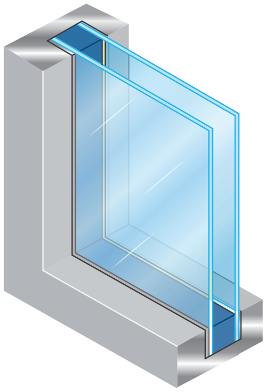


Figure B1.16 Double glazing

24 In an experiment to measure the thermal conductivity of a disc of wood, a sample of area $65\,cm^2$ was used, with a thickness of 5.2 mm. When the surfaces of the wooden disc were kept at 0 °C and 100 °C, the flow of thermal energy through the wood was determined to be 16 W.
Use this data to calculate a value for the thermal conductivity of the wood.

25 In 10 minutes, a total of 275 J of thermal energy was conducted through a block of material of area $12.5\,cm^2$ when it had a temperature gradient of $4.2\,°C\,cm^{-1}$ across it.
- **a** Determine a value for the thermal conductivity of the material.
- **b** Would you describe this material as a conductor or an insulator?
- **c** Suggest a material it might have been.

26 **a** If 5.6 W of thermal energy was flowing through each square metre of the insulating polyurethane foam seen in Figure B1.15, calculate the temperature difference between its surfaces if the foam had a thickness of 7.8 cm.
- **b** Determine a value for the outside temperature if the thickness of the brick walls was 10.9 cm and the inside temperature of the outer brick wall was 5.4 °C.
- **c** Determine the inside temperature of the interior wall.
- **d** Sketch the arrangement and annotate your drawing with all the known data.

Thermal convection

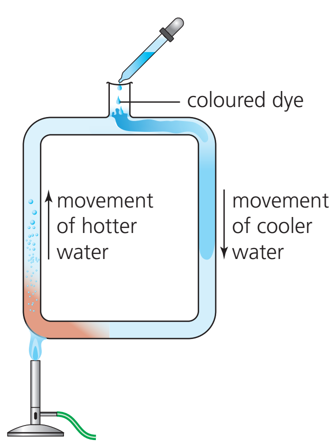
SYLLABUS CONTENT

- Qualitative description of thermal energy transferred by convection due to a fluid density difference.

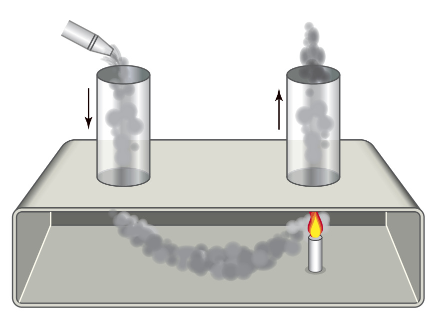
◆ **Convection** Passage of thermal energy through liquids and gases due to the movement of the substance because of differences in density.

When part of a fluid (gas or liquid) is heated, there will be a localized decrease in density. Because of increased buoyancy (see Topic A.2), the warmer part of the fluid will then rise and flow above the cooler part of the fluid, which has a slightly greater density. This movement of thermal energy in a fluid is called thermal **convection**. It is common for convection to produce currents and a circulation of a gas or liquid. Figures B1.17 and B1.18 show two common laboratory demonstrations of convection.

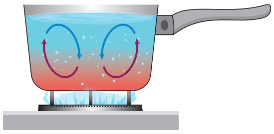
Thermal convection is the transfer of thermal energy because of the movement of a fluid due to changes in density.



■ **Figure B1.17** Demonstrating convection in water



■ **Figure B1.18** Demonstrating convection in air



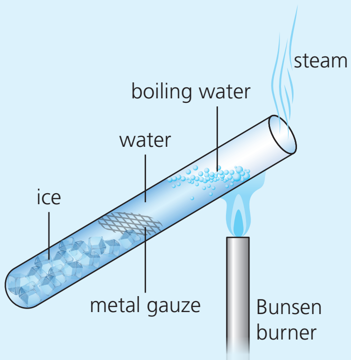
■ **Figure B1.19** Convection of water in a saucepan

There are large number of examples of convection currents, including:

- The water in heaters, saucepans, kettles, and so on, is supplied with energy (by thermal conduction) near the bottom of the container. The heated water rises, to be replaced by cooler water, which in turn will be heated. Convection currents ensure that the thermal energy spreads evenly throughout the water. See Figure B1.19.
- Room heaters are placed near to the floor, but air-conditioners are near the ceiling.
- The coolest part of a refrigerator is near the bottom.
- Water is mixed in the oceans and lakes by convection currents.
- Molten material in the Earth's core circulates because of convection.
- Convection currents occur in the very hot cores of stars, including the Sun.
- Formation and movement of clouds and storms depend on convection.
- The Earth's climate and weather patterns are controlled by convection.
- The direction of winds near coasts depends on convection.
- Smoke usually rises because of convection, but a lack of convection can make air pollution problems worse.

27 Outline how the experiment shown in Figure B1.18 is demonstrating convection in air.

28 Figure B1.20 shows a tube of water being heated near to the water surface. A metal gauze is keeping some ice at the bottom of the tube.



■ **Figure B1.20** Tube of ice being heated

a Explain what this demonstration shows us about the transfer of thermal energy in water.

b Predict and explain how the observations will change if the gauze is removed, allowing the ice to rise, and the water is heated at the bottom of the tube.

29 Convection currents in the air often flow from the sea towards the land. This is because, in the daytime, the land changes temperature quicker, and gets warmer, than the sea. Sketch an annotated diagram to help to explain this phenomenon.

30 Discuss what features of the clothing of the Antarctic explorer seen in Figure B1.21 keep the explorer warm.

■ **Figure B1.21** Antarctic explorer

31 Outline the cooking process (in terms of thermal energy transfers) for the pizza seen in Figure B1.22.

■ **Figure B1.22** Pizza oven

ATL B1A: Thinking skills

Being curious about the natural world

Under certain weather conditions, the normal convection currents which rise from the Earth's surface can be greatly reduced. This can result in trapping air pollutants that would usually disperse. Use the search term 'temperature inversion' to research into this phenomenon and write a 200–300 word summary.

Figure B1.23 The polluting effects of a temperature inversion over Almaty in Kazakhstan

Top tip!

This section on thermal radiation requires some understanding of waves, radiation and spectra, all of which are covered in Theme C. If you have not been introduced to these topics before, it may be better to delay the study of this section (thermal radiation) until after Topics C.2 and C.3 have been studied.

◆ **Emit** To send out from a source.

◆ **Thermal radiation** Electromagnetic radiation emitted because of the movement of charged particles in the atoms of all matter at all temperatures. Most commonly, infrared.

◆ **Infrared** Electromagnetic radiation emitted by all objects (depending on temperature) with wavelengths longer than visible light.

Thermal radiation

SYLLABUS CONTENT

- Quantitative description of energy transferred by radiation as a result of the emission of electromagnetic waves from the surface of a body, which in the case of a black body can be modelled by the Stefan–Boltzmann law as given by: $L = \sigma A T^4$, where L is the luminosity, A is the surface area and T is the absolute temperature of the body.
- The emission spectrum of a black body and the determination of the temperature of the body using Wien's law: $\lambda_{max} T = 2.9 \times 10^{-3}$ mK, where λ_{max} is the peak wavelength.

All matter / objects **emit** electromagnetic waves because of the movement of charged particles within their atoms. (There is no need to understand this process.) This is called **thermal radiation**. Electromagnetic waves are explained in Topic C.2. Most commonly this radiation is called **infrared**, but if the temperature is hot enough, visible light is also emitted.

A flame (Figure B1.24) is an obvious example, producing significant amounts of electromagnetic radiation: we can detect the infrared by holding a hand near the flame, and detect the light with our eyes. Figure B1.4 showed the infrared emitted by a saucepan.

Although it is true to say that thermal radiation is emitted continuously by all matter at all temperatures, we tend to only notice it coming from hot objects. The power of the emitted radiation from *any* surface depends on:

Figure B1.24 Thermal radiation from a flame

1. **Surface temperature** The radiated power is proportional to the *fourth* power of the surface temperature (in kelvin), T^4. This means, for example, a metal bar at 600 K (323 °C) will emit $2^4 = 16$ times as much radiation as the same bar at 300 K (23 °C).
2. **Surface area** The radiated power is proportional to the area, A.
3. **Nature of the surface** See next section.

Note that the emitted power is *not* dependent on the chemical nature of the material.

■ **Figure B1.25** Sydney Opera House.

Good absorbers and good emitters of thermal radiation

Dark surfaces, especially black, are the best absorbers of thermal radiation. White and shiny surfaces reflect and scatter radiation well, so that they are poor absorbers. (Scattering is explained later in this topic.) The Sydney Opera House (Figure B1.25) is a poor absorber of thermal radiation. (The word **absorption** describes something taking in something else, a sponge absorbing water, for example.)

Any surface which is a good absorber of radiation will also be a good emitter. Black surfaces emit and absorb radiation well; white surfaces are poor at absorbing and emitting.

◆ **Absorption** When the energy of incident particles or radiation is transferred to other forms within a material.

◆ **Black body** An idealized object that absorbs all the electromagnetic radiation that falls upon it. A perfect black body also emits the maximum possible radiation.

◆ **Black-body radiation (spectrum)** Radiation emitted from a 'perfect' emitter. The characteristic ranges of different radiations emitted at different temperatures are commonly shown in graphs of intensity against wavelength.

Black bodies

A perfect **black body** is the term we use to describe an object which has a surface which absorbs *all* of the infrared and light (and other electromagnetic radiation) that falls on it.

No light is reflected, so we are unable to see a black body, except in outline (unless it is also hot enough to emit visible radiation: light). This is easily defined and understood; however, a perfect black body is also a perfect *emitter* of thermal radiation, but what exactly does that mean? Obviously, it cannot mean that all of the energy in the surface is emitted instantaneously!

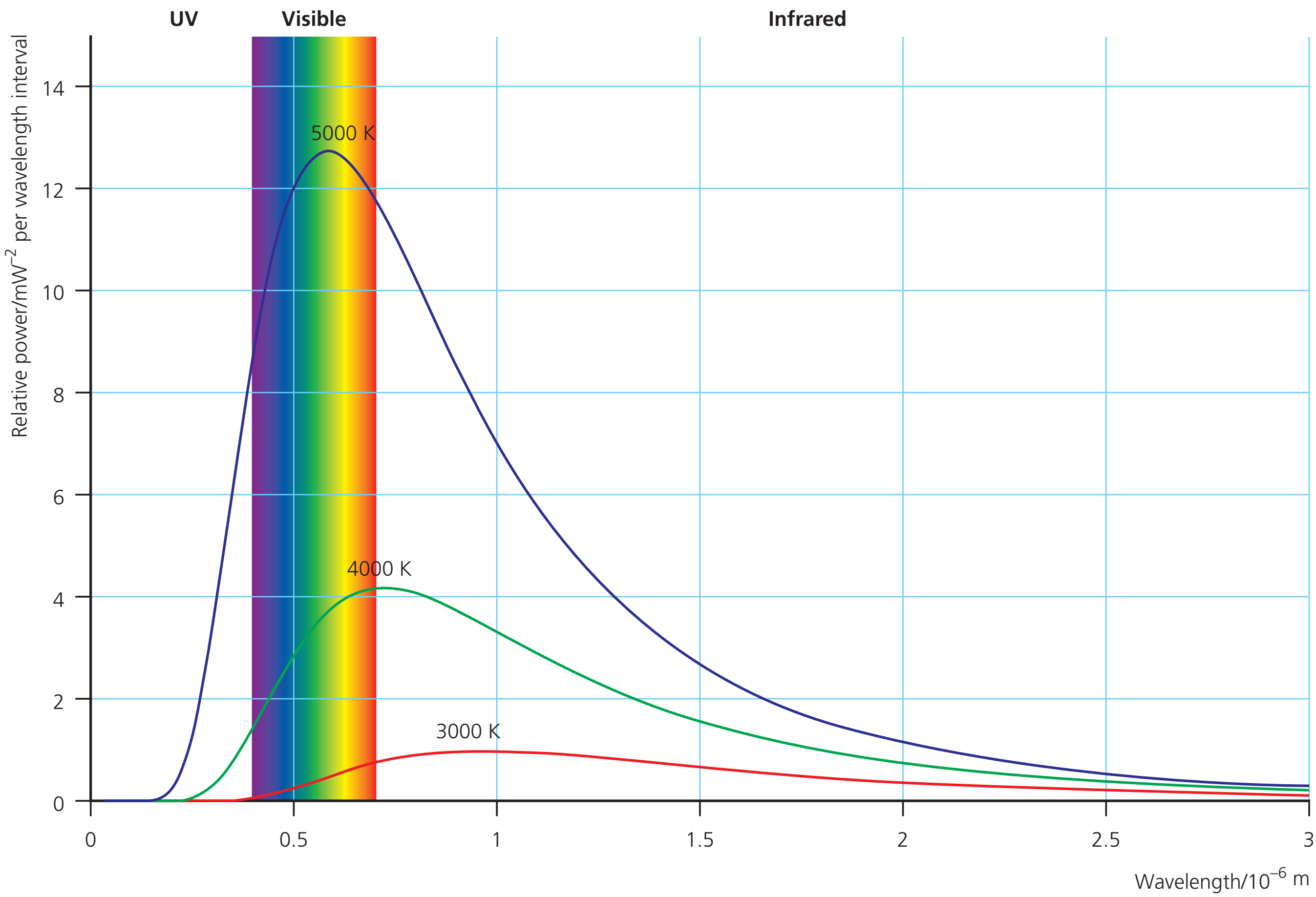
All surfaces emit a range of different wavelengths with different powers, and this varies with temperature. A perfect black body emits the maximum possible thermal radiation, and this is best described graphically by a **black-body emission spectrum**, as shown in Figure B1.26, for three different high temperatures. A curve for 300 K (27 °C) would be too small to show on the scale of this graph and it would have its maximum value at a wavelength of about 10×10^{-6} m, which is well off the horizontal scale to the right.

We can see from the graph that, as temperatures increase, more power is emitted and the wavelength at which the maximum power is emitted, λ_{max}, becomes smaller.

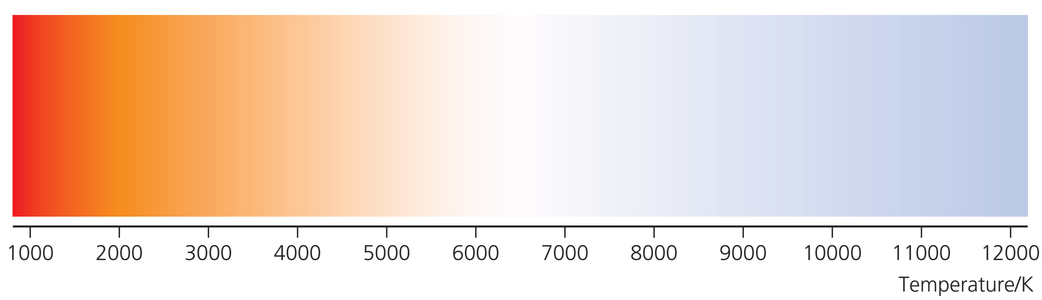
At 3000 K, only a small proportion of the emitted radiation is visible light. This proportion increases with temperature and, if the surface it hot enough, some ultraviolet radiation will also be emitted.

If an object is heated (without chemical reactions occurring), a metal bar for example, it will begin to emit visible light (the red end of the spectrum) at about 850 K. If the temperature rises, other colours will be emitted, combining to give the overall effects seen in Figure B1.27.

Note that 'perfect' emitters are called black bodies, but that does not mean that they will always appear black. The Sun has a surface temperature of about 5800 K and it is a good example of a black body, so we may assume that it absorbs all the radiation falling on it; however, it is so hot that it emits enormous quantities of visible light. We are familiar with the visible spectrum, but its full black-body spectrum extends into the infrared and ultraviolet.



■ **Figure B1.26** Black-body emission spectra at three different temperatures



■ **Figure B1.27** Colours of hot surfaces

◆ **Stefan–Boltzmann law** An equation that can be used to calculate the total power radiated from the surface of a black body, $P = \sigma AT^4$. σ is known as the **Stefan–Boltzmann constant**.

Since everything emits thermal radiation, all bodies are continuously emitting *and* receiving radiation. In practice, we may assume that one of these is insignificant compared to the other. For example, the radiation absorbed by the Sun is insignificant compared to the energy it emits. However, this is not true when we consider the Earth. See next Topic B.2: Greenhouse effect.

The total power, P, emitted (across all wavelengths) from a perfect black body of surface area A can be calculated from the **Stefan–Boltzmann law**:

power emitted from a black body, $P = \sigma AT^4$

σ is known as the **Stefan–Boltzmann constant**. It has the value of $\mathbf{5.67 \times 10^{-8}\,W\,m^{-2}\,K^{-4}}$

◆ **Celestial objects** Any naturally occurring objects that can be observed in space.

When referring to **celestial objects** (stars, for example), the emitted power is usually called **luminosity**, as discussed below.

WORKED EXAMPLE B1.4

A metal wire is heated (by an electric current) to a uniform 632 °C. If its length, l, is 80 cm and its radius, r, is 1.6 mm, calculate the total power it radiates into its surroundings.

Assume that it acts as a perfect black body (which is almost true).

Answer

Surface area, $A = 2\pi rl$

$= 2\pi \times 0.0016 \times 0.80 = 8.04 \times 10^{-3}\ \text{m}^2$

$P = A\sigma T^4 = (8.04 \times 10^{-3}) \times (5.67 \times 10^{-8}) \times (632 + 273)^4 = 3.1 \times 10^2\ \text{W}$

Top tip!

We can also use the same equation for the energy *absorbed* by an object from its surroundings:

$$\frac{P}{A} = \sigma T^4$$

If the surrounding temperature is T_s, then the overall (net) radiant energy flow per second from, or to, a black body of area A is: $P = \sigma AT^4 - \sigma AT_s^4$.

For example, using this equation, we can calculate that a black-body surface at 100 °C radiates thermal energy at a rate of 1.1 kW m^{-2}.

At the same time, if the surrounding temperature is 20 °C, it will be receiving energy at a rate of 0.42 kW m^{-2}.

Tool 2: Technology

Use sensors

Infrared scanners and hand-held thermometers (Figure B1.28) have become commonplace in recent times. They detect the thermal radiation emitted by our skins and other surfaces. Their advantages are obvious: they are quick and easy to use, and they do not involve any physical contact. But they have their limitations.

Infrared scanners assume that all skin behaves as a perfect black body. That is, the results from skins of different colours or textures are approximately the same. The radiation coming from the skin is focused onto a detector which effectively determines the power and calculates the corresponding temperature of the emitting surface.

If the distance between the skin and the detector increases, the detector may receive less radiation from each square millimetre but may receive from a greater overall area: it depends on the geometry of the situation.

■ **Figure B1.28** Infrared thermometer

Wien's displacement law

There is a straightforward inverse relationship between surface temperature, T, and the wavelength at which the maximum power is received, λ_{max}.

$$T \propto \frac{1}{\lambda_{max}}$$

This is known as **Wien's displacement law**:

$$\lambda_{max} T = 2.9 \times 10^{-3}\,\text{mK}$$

◆ **Wien's displacement law** Relationship between absolute temperature and the wavelength emitted with maximum power by a black body at that temperature.

WORKED EXAMPLE B1.5

Determine the temperature corresponding to a surface which emits its maximum power with a wavelength of 5.8×10^{-7} m.

Answer

$\lambda_{max} T = 2.9 \times 10^{-3}$

$5.8 \times 10^{-7} \times T = 2.9 \times 10^{-3}$

$T = 5.0 \times 10^{3}$ K This is consistent with Figure B1.26.

■ **Figure B1.29** The Great Lakes in North America appearing dark / black from Space

Although a 'perfect' black body is an idealized concept, the following may approximate to the ideal:

- very hot objects
- dark and dull surfaces
- water
- human skin
- ice
- soil
- vegetation

In Topic B.2, we will introduce the numerical concept of *emissivity*: the ratio of the power radiated per unit area by a surface compared to that of an ideal black surface at the same temperature.

32 Give an everyday example of:
- **a** a dark surface being good at absorbing thermal energy
- **b** a dark surface being good at emitting thermal energy
- **c** a white or shiny surface being good at reflecting thermal energy
- **d** a white or shiny surface being poor at emitting thermal energy.

33 a Calculate the maximum thermal power radiated away from each square centimetre of a coffee cup which has a surface temperature of 40 °C.
- **b** Explain why your answer will be an overestimate of the actual power emitted.

34 An object's surface is at 25 °C. Determine the temperature (°C) to which it would have to be heated in order to double the thermal radiation that it radiates.

35 A water storage tank is in sunlight most of the day and its surface reaches a constant temperature of 36 °C. At night the surroundings cool to 23 °C. Estimate the net flow of radiant thermal energy from each square metre of the tank's surface assuming that the surface temperature remains constant.

36 An (unclothed) human body has an average skin temperature of about 35 °C.

a Estimate a value for the area of the skin of a typical adult.

b **i** Show that the power emitted from this area is about 1 kW.

ii What assumption did you need to make?

c 1 kW is a large radiated power, outline why the body does not cool down quickly. What assumption do you need to make?

37 At what wavelength is the maximum infrared power emitted from your skin?

38 Explain how emergency 'survival blankets', as seen in Figure B1.30, can protect people against dangerous loss of thermal energy. They are made of thin plastic sheets with reflective coatings.

Figure B1.30 Survival blankets for long distance runners

Thermal radiation and stars

SYLLABUS CONTENT

- The concept of apparent brightness, b.
- The luminosity of a body as given by: $b = \frac{L}{4\pi d^2}$.

◆ **Luminosity (stellar)** Total power of electromagnetic radiation emitted by a star (SI unit: W).

Luminosity

As explained previously, the power emitted by a celestial body (across all wavelengths) is called its luminosity. The Stefan–Boltzman law can be restated with respect to stars as:

luminosity of a star (or other body), $L = \sigma A T^4$

WORKED EXAMPLE B1.6

The Pole Star (north), Polaris, has a surface area of 8.5×10^{21} m^2 and a surface temperature of 6.0×10^3 K.

a Determine an approximate value for its luminosity.

b Compare its luminosity to that of the Sun (3.8×10^{26} W).

Answer

a $L = \sigma A T^4 \approx (5.67 \times 10^{-8}) \times (8.5 \times 10^{21}) \times (6.0 \times 10^3)^4 \approx 6.2 \times 10^{29}$ W

b $\frac{6.2 \times 10^{29}}{3.8 \times 10^{26}} \approx 1600$

Polaris has a luminosity about 1600 times greater than the Sun.

We can write Polaris's luminosity as $L = 1600\ L_{\odot}$

Luminosity of stars is often given in terms of the luminosity of the Sun, $L_{\odot}$

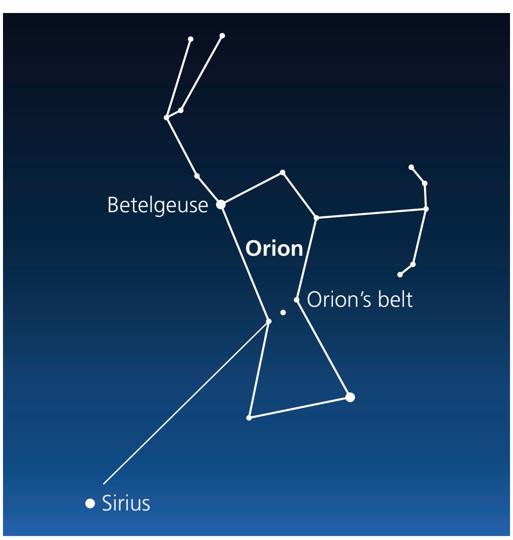
$L_{\odot} = 3.8 \times 10^{26}$ W

There is no conduction or convection across the near-vacuum of space and the stars can be considered to be perfect black bodies. There is also no significant absorption of thermal energy in space (except where the distances are enormous). All this means that developing a basic understanding of thermal energy transfers across space is fairly straightforward.

All stars (except the Sun) appear as *point sources* of light. The only differences we can see with telescopes are their brightness and slight differences in colour. More than 2000 stars can be seen in Figure B1.31. Some may *appear* larger than others in the picture, but this effect is only because they are brighter.

■ **Figure B1.31** Sagittarius Star Cloud taken from the Hubble telescope

◆ **Star map** Two-dimensional representation of the relative positions of stars as seen from Earth.



■ **Figure B1.32** Locating *Sirius* in the night sky

There are two possible reasons why one star may appear brighter than another. The brighter star may be emitting more power (more luminous), and/or it may be closer to Earth.

Differences in the colours of stars seen in Figure B1.31 may be attributed to differences in surface temperatures, as explained above.

Sirius is the brightest star in the night sky. Observations of its spectrum show that $\lambda_{max} = 2.92 \times 10^{-7}$ m. Using Wien's law, astronomers can determine that it has a surface temperature of 9930 K. Figure B1.27 confirms that Sirius will appear slightly blue in colour. To locate Sirius in the night sky we can use a **star map**, which highlights groups of stars (constellations). Sirius can be seen close to the constellation of Orion, with its well-known three stars apparently in a line – Orion's belt. See Figure B1.32.

◆ Apparent brightness, b Intensity (power / area) of radiation received on Earth from a star (SI unit: $W\,m^{-2}$).

◆ Intensity, I Wave power / area: $I = P/A$ (SI unit: $W\,m^{-2}$).

Apparent brightness and intensity

We could describe the luminosity of a star as its *actual brightness*, but that is different from what we detect here on Earth, which is called a star's **apparent brightness**. Stars of equal luminosity will have different apparent brightnesses on Earth if they are different distances away. Consider Figure B1.33, which represents three stars of *equal* luminosity at different distances from Earth. If the apparent brightness of star A is b, then apparent brightness of star B is $b/2^2$, while star C has an apparent brightness of $b/3^2$.

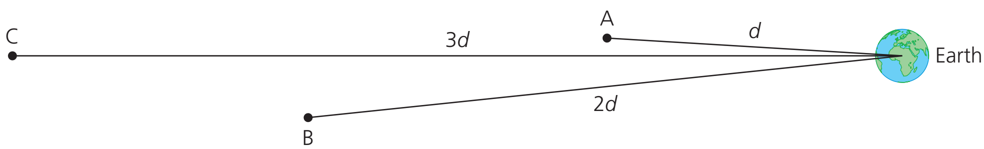


Figure B1.33 Comparing apparent brightnesses

Apparent brightness is a measure of the **intensity** of the radiation from the star which reaches Earth. SI unit: $W\,m^{-2}$

The intensity, I, of radiation (or waves) is the power, P, transferred through unit area (perpendicular to the direction of energy transfer).

$$\text{intensity} = \frac{\text{power}}{\text{area}} \qquad I = \frac{P}{A} \qquad \text{SI unit: } W\,m^{-2}$$

WORKED EXAMPLE B1.7

A solar panel close to the Earth's surface has dimensions 3.2 m × 1.6 m. How much total power is received when the intensity of radiation falling perpendicularly on the panel is 739 $W\,m^{-2}$?

Answer

$$I = \frac{P}{A}$$

$$739 = \frac{P}{(3.2 \times 1.6)} \Rightarrow P = 3.8 \times 10^3\,W$$

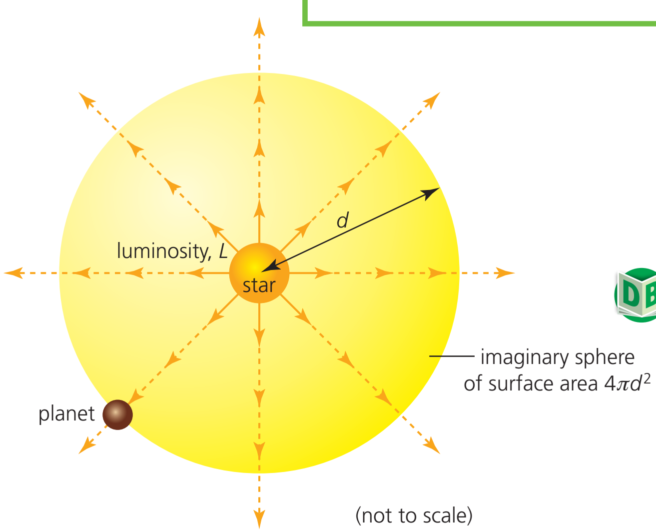


Figure B1.34 A star's radiation spreading out

If we assume that thermal radiation from a star like the Sun spreads out equally in all directions without absorption, then at a distance d from the star, the same total power, L, is passing through an area $4\pi d^2$ (the surface area of an imaginary sphere), as shown in Figure B1.34.

$$\text{apparent brightness: } \boldsymbol{b = \frac{L}{4\pi d^2}}$$

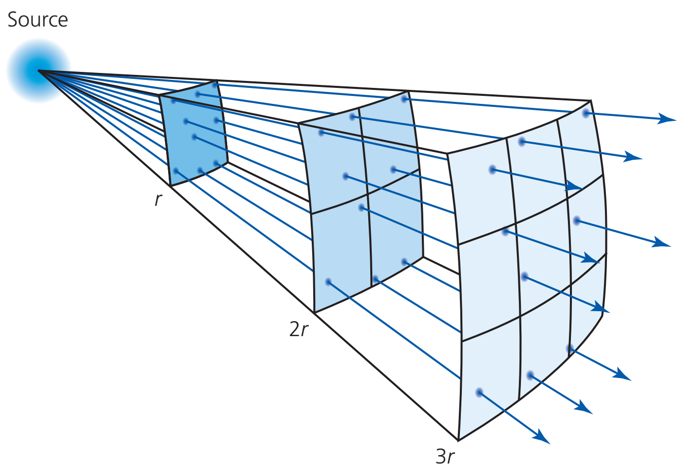
The apparent brightness of the Sun is important information in Topic B.2.

This equation is an example of an **inverse square law** (the apparent brightness is inversely proportional to the distance squared), of which there are several in this course.

◆ Inverse square law For waves / energy / particles / fields spreading equally in all directions from a point source without absorption or scattering, the intensity is inversely proportional to the distance squared, $I \propto 1/x^2$ (Ix^2 = constant).

Figure B1.35 shows an alternative visual representation: radiation is spreading out from a point source equally in all directions, *but without absorption.* The same power passes through greater areas as it travels away from the source.

An *inverse* square law represents the fact that a physical quantity is *divided* by 2^2 if the distance from a source is doubled and divided by 3^2 if the distance from a source is trebled, and so on.



■ **Figure B1.35** Radiation spreads to cover four times the area at twice the distance ($2r$) and nine times the area at three times the distance ($3r$)

More generally, intensity: $I \propto \frac{1}{r^2}$ or Ir^2 = constant

Tool 3: Mathematics

Linearize graphs

Straight lines are much easier to understand and analyse than curved lines, but when raw experimental data are plotted against each other (x and y, for example), the lines are often curved rather than linear.

Data that give an x–y curve can be used to draw other graphs to check different possible relationships. For example:

- A graph of y against x^2 could be drawn to see if a straight line through the origin is obtained, which would confirm that y was proportional to x^2.
- A graph of y against $\frac{1}{x}$ that passed through the origin would confirm that y was proportional to $\frac{1}{x}$. In which case x and y are said to be inversely proportional to each other.
- A graph of y against $\frac{1}{x^2}$ passing through the origin would represent an inverse square relationship.

Figure B1.36 shows graphs of the most common relationships.

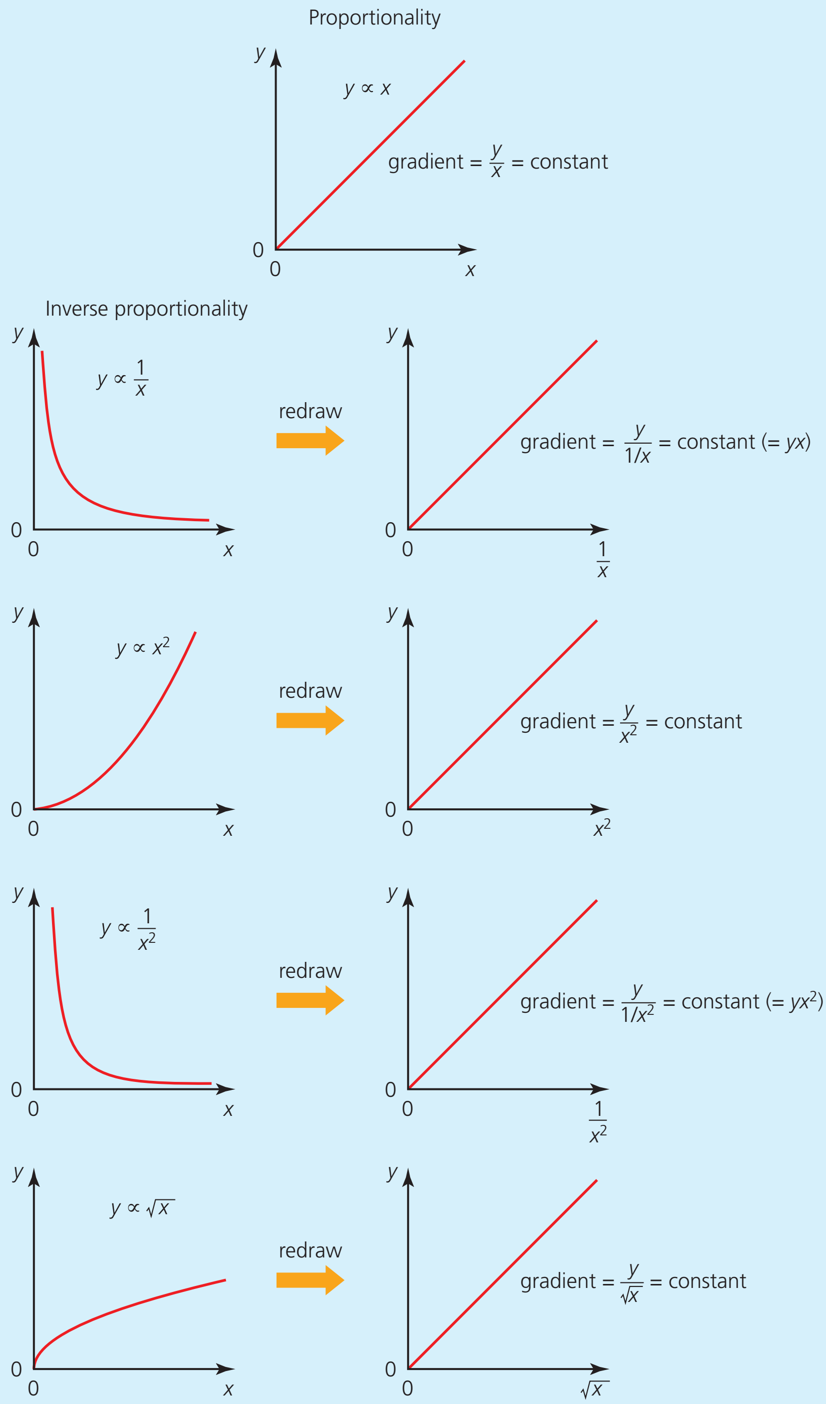


Figure B1.36 Some common graphical relationships, showing how curves can be replotted to produce straight lines

LINKING QUESTION

- Where do inverse square relationships appear in other areas of physics?

Forces around point sources in gravitational and electric fields follow inverse square laws (Topics D.1 and D.2).

WORKED EXAMPLE B1.8

The star Betelgeuse has a luminosity of $126\,000\,L_\odot$ and an apparent brightness of $1.4 \times 10^{-7}\,\text{W}\,\text{m}^{-2}$. Determine its distance from Earth.

Answer

$$b = \frac{L}{4\pi d^2}$$

$$1.4 \times 10^{-7} = \frac{(126\,000 \times 3.8 \times 10^{26})}{4\pi d^2} \Rightarrow d = 5.2 \times 10^{18}\,\text{m}$$

Determining astronomical distances

The distance to 'nearby' stars can be determined from geometrical calculations made with measurements of the apparent locations of the stars at different times of the year (explained in Topic E.5), But this method is not possible with most stars because they are so far away that there is no detectable movement in their apparent locations: most stars remain in *exactly* the same positions as seen on a map of the stars. See Figure B1.32 for an example of part of a star map.

In principle, the equation $b = \frac{L}{4\pi d^2}$ can be used to determine the distance, d, to any star if we measure its apparent brightness, b, but *only if* we know its luminosity, L. However, for most stars we have no direct way of knowing their luminosities.

Fortunately, astronomers have identified a few '**standard candles**'. These are stars which have known luminosities, including a type of supernova and Cepheid variables, as explained below.

◆ **Standard candles**
Term used by astronomers to describe the fact that the distance to a galaxy can be estimated from a knowledge of the luminosity of a certain kind of star within it.

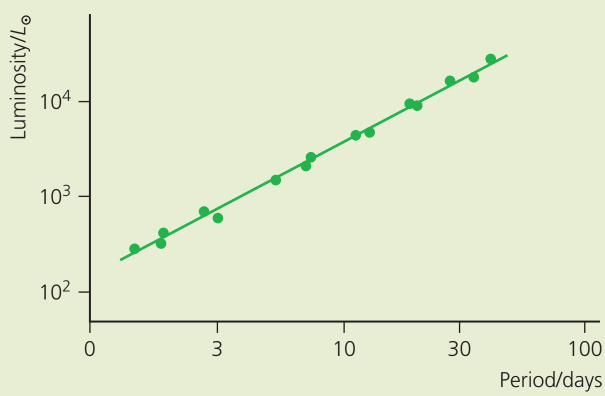
Nature of science: Patterns and trends

Cepheid variable stars, a type of 'standard candle'

In 1908, Henrietta Swan Leavitt (see Figure B1.37) discovered that a certain type of star, called a *Cepheid variable*, had a variable luminosity, the maximum value of which could be determined from the time period of its variation (typically a few days).

■ **Figure B1.37** Henrietta Swan Leavitt

Figure B1.38 shows the graphical relationship between the periods of Cepheid variable stars and their luminosity as multiples of the Sun's luminosity, $L_\odot$.



■ **Figure B1.38** Cepheids' variable luminosities

From the graph, for a period of 30 days, the luminosity, $L \approx (1.0 \times 10^4) \times (3.83 \times 10^{26}) \approx 3.8 \times 10^{30}\,\text{W}$. If a Cepheid variable star has an apparent brightness of 8.4×10^{-8},

$$b = \frac{L}{4\pi d^2}$$

$$8.4 \times 10^{-8} = \frac{3.8 \times 10^{30}}{4\pi d^2} \Rightarrow d \approx 1.9 \times 10^{18}\,\text{m}$$

This distance is approximately 200 light years. A *light year* is the distance travelled by light in one year.

Tool 3: Mathematics

Use units where appropriate: Light year

Astronomical distances are huge. The nearest star to Earth, other than our Sun, (*Alpha Proxima*), is 4.02×10^{16} m away. It becomes convenient to use larger units than metres and kilometres in astronomy. The following are non-SI units.

The light year (ly) is the *distance* travelled by light in one year:

$$1\,\text{ly} = (3.00 \times 10^8) \times 365 \times 24 \times 360 = \mathbf{9.46 \times 10^{15}\,m}$$

(A light year is defined to be exactly a distance of 9 460 730 472 580 800 m.)

In light years, the distance to *Alpha Proxima* is 4.25 ly.

The *parsec* is another widely used unit for distance in astronomy (see Topic E.5).

39 The surface area of the Sun is 6.1×10^{18} m^2 and it has a surface temperature of 5780 K.

- **a** Determine the total thermal power that it emits (luminosity).
- **b** Describe the colour of the visible light emitted by the Sun.

40 The star Betelgeuse, seen in Figure B1.32, has a surface temperature of 3500 K.

- **a** Describe its colour.
- **b** At what wavelength does it emit radiation at the greatest rate?

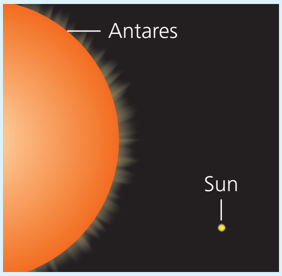
41 If a star has a luminosity which is 1000 times greater than the Sun and a surface temperature of 15 000 K, predict its surface area compared to the Sun.

42 An LED lamp emits light energy equally in all directions with a total power of 4.1 W.

- **a** Calculate the intensity falling on a book which is 2.34 m away.
- **b** Determine the distance from the lamp where the intensity is 0.40 W m^{-2}.

43 The star Antares is 550 light years from Earth and it has a luminosity of 2.9×10^{31} W.

- **a** Calculate its apparent brightness as viewed from Earth. (Antares is the 15th brightest star in the sky.)
- **b** Its radius is approximately 700× the radius of the Sun (7.0×10^8 m). Calculate its surface temperature.
- **c** Suggest why this star is described as a 'red (super) giant'. Figure B1.39 compares the size of Antares to the Sun.



■ **Figure B1.39** The size of Antares compared to the Sun

44 Outline the concept of a 'standard candle', as used to determine the distance to galaxies.

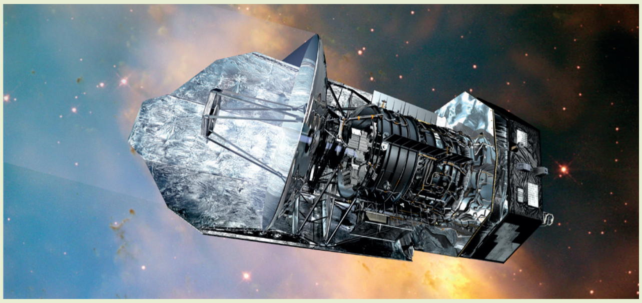
45 A certain type of supernova had a luminosity of 1.4×10^{36} W. If its apparent brightness was 1.9×10^{-6} W m^{-2}, determine its distance from Earth in light-years.

46 Calculating a distance in answers to questions similar to question 43 assumes that there is no absorption of thermal energy as it travels through space. However, there will certainly be *some* absorption over distances as large as these. Discuss how this could affect the calculated answer to question 43.

Nature of science: Observations

Understanding the Universe

Astronomers have developed an impressive understanding of the Universe, especially with recent technological advances in the detection of remote sources of radiation. Amazingly, all this knowledge has been deduced from thermal and electromagnetic radiation received on Earth, or satellites in orbit.



■ **Figure B1.40** Herschel Space Telescope (ESA: 2009–2013)

Heating and cooling

SYLLABUS CONTENT

- Quantitative analysis of thermal energy transfers, Q, with the use of specific heat capacity, c, using: $Q = mc\Delta T$.

The dotted blue straight line in Figure B1.41 shows how the temperature of an object, heated at a constant rate, would change with time under the idealized circumstances of no thermal energy losses to the surroundings. The best way of supplying a constant power is with an electrical heater. The temperature rises by equal amounts in equal times. However, thermal energy losses to the surroundings are unavoidable, so the curved red line represents a more realistic situation. The curve shows that the rate of temperature rise decreases as the object gets hotter. This is because thermal energy losses are higher with larger temperature differences. If energy continues to be supplied, the object will eventually reach a constant temperature when the input power and rate of thermal energy loss to the surroundings are equal (assuming that there are no chemical or physical changes).

When something is left to cool naturally, the rate at which thermal energy is transferred away decreases with time because it also depends on the temperature difference between the object and its surroundings. See Figure B1.42.

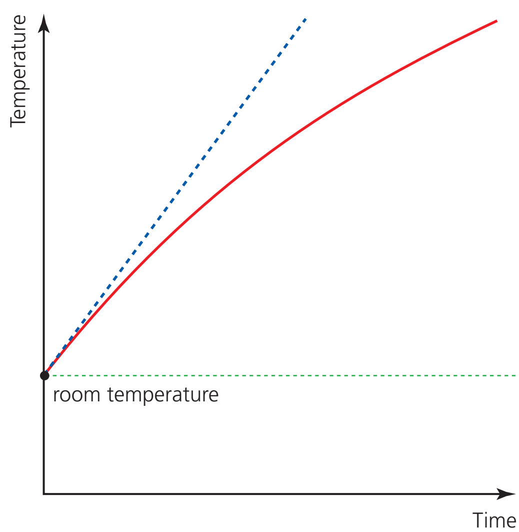


Figure B1.41 A typical graph of temperature against time for heating at a constant rate

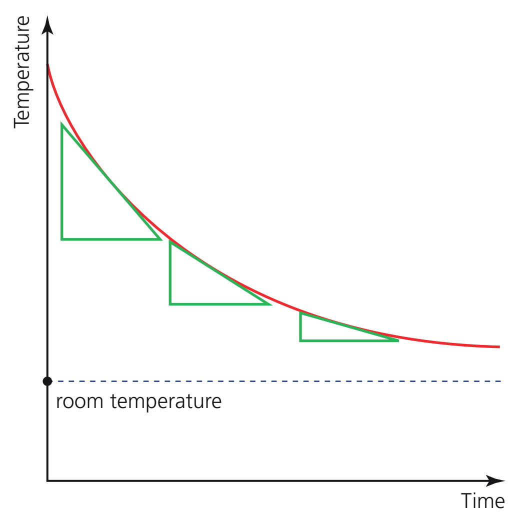


Figure B1.42 A typical graph of temperature against time for an object cooling down naturally to room temperature. Note how the gradient decreases with time

Tool 3: Mathematics

Interpret features of graphs: gradient

Figure B1.42 shows another example of determining rates of change from gradients of a graph. In this case, as can be seen on the figure, the gradients and rates of change are negative and decrease in magnitude with time. Three examples are shown in Figure B1.42, but ideally, larger triangles should be used.

Specific heat capacity

In order to compare how different substances respond to heating, we need to know how much thermal energy will increase the temperature of the same mass (1 kg) of each substance by the same amount (1 K, or 1 °C). This is called the *specific heat capacity*, *c*, of the substance. (The word 'specific' is used here simply to mean that the heat capacity is related to a specified amount of the material, namely 1 kg.)

◆ **Specific heat capacity, *c*** The amount of energy needed to raise the temperature of 1 kg of a substance by 1 K.

The **specific heat capacity** of a substance is the amount of energy needed to raise the temperature of 1 kg of the substance by 1 K. (SI Unit: $J\,kg^{-1}\,K^{-1}$, but $°C^{-1}$ can be used instead of K^{-1}.)

The values of specific heat capacity for some common materials are given in Table B1.5.

Table B1.5 Specific heat capacities of some common materials

Material	Specific heat capacity / $J\,kg^{-1}\,K^{-1}$
copper	390
aluminium	910
water	4180
air	1000
dry earth	1250
glass (typical)	800
concrete (typical)	800
steel	420

Common mistake

The unit for specific heat capacity is very often written incorrectly by students.

Substances with high specific heat capacities heat up slowly compared with equal masses of substances with lower specific heat capacities (given the same power input). Similarly, substances with high specific heat capacities will cool down more slowly. It should be noted that water has an unusually large specific heat capacity. This is why it takes the transfer of a large amount of energy to change the temperature of water and the reason why water is used widely to transfer energy in heating and cooling systems.

If a quantity of thermal energy, Q, was supplied to a mass, m, and produced a temperature rise of ΔT, we could calculate the specific heat capacity from the equation:

$$c = \frac{Q}{m\Delta T}$$

This equation is more usually written as follows:

DB

thermal energy transferred, $\boldsymbol{Q = mc\Delta T}$

When a substance cools, the thermal energy transferred away can be calculated using the same equation.

◆ **Immersion heater** Heater placed inside a liquid or object.

Figure B1.43 shows two laboratory experiments to determine specific heat capacities of **a** a metal, and **b** water, or another liquid. The energy is supplied by **immersion heaters** at a constant rate electrically and can be measured directly by a 'joulemeter'. (Alternatively, the energy can be calculated from voltage × current × time. See Topic B.5.)

To use the equation shown above to determine specific heat capacity it is necessary to determine the amount of thermal energy that was transferred to raise the temperature of a known mass by a known amount.

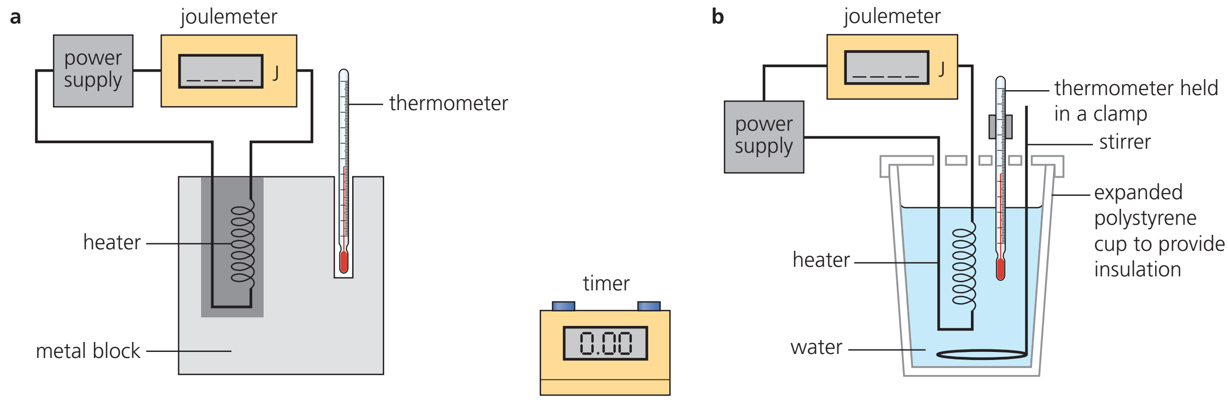


Figure B1.43 Determining the specific heat capacity of **a** a metal, **b** water

WORKED EXAMPLE B1.9

Suppose that in an experiment similar to that shown in Figure B1.43a, a metal block of mass 1500 g was heated for exactly 5 minutes with an 18 W heater. If the temperature of the block rose from 18.0 °C to 27.5 °C, calculate its specific heat capacity, assuming that no energy was transferred to the surroundings.

Answer

$c = \dfrac{Q}{m\Delta t}$ and $Q = Pt$

$c = \dfrac{18 \times (5 \times 60)}{[1.5 \times (27.5 - 18.0)]} = 3.8 \times 10^2\,\text{J kg}^{-1}\,°\text{C}^{-1}$

In making such calculations, it has to be assumed that all of the substance was at same temperature and that the thermometer recorded that temperature accurately at the relevant times. (The thermometer is measuring a temperature that is changing.) In practice, both of these assumptions might lead to significant inaccuracies in the calculated value. Furthermore, in any experiment involving thermal energy transfers and changes in temperature, there will be unavoidable losses (or gains) from the surroundings. If accurate results are required, it will be necessary to use insulation to limit these energy transfers, which in this example would have led to an overestimate of the substance's specific heat capacity (because some of the energy input went to the surroundings rather than into the substance). The process of insulating something usually involves surrounding it with a material that traps air (a poor conductor) and is often called **lagging**.

◆ **Lagging** Thermal insulation.

Inquiry 2: Collecting and processing data

Interpreting results

In an experiment similar to that seen in Figure B1.43b, the temperature of 210 g of water rose from 22.4 °C to 30.7 °C when 7880 J were supplied.

1. Calculate a value for the specific heat of water.
2. Is this an accurate experiment? (Calculate percentage difference from the accepted value.)
3. Estimate the uncertainty in the raw data and then calculate the absolute uncertainty in the processed result.
4. Compare your answers to 2 and 3 and suggest why they are different.

Thermal capacity

◆ **Thermal capacity** The amount of energy needed to raise the temperature of a particular object by 1 kelvin.

Many everyday objects are not made of only one substance, so that referring to a specific amount (a kilogram) of such objects is not useful. In such cases we refer to the **thermal capacity** of the whole object. For example, we might want to know the thermal capacity of a room and its contents when choosing a suitable heater or air conditioner. The thermal capacity of an object is the amount of energy needed to raise its temperature by 1 K. (Unit: $\mathrm{J\,K^{-1}}$ or $\mathrm{J\,°C^{-1}}$)

$$\text{thermal capacity} = \frac{Q}{\Delta T}$$

WORKED EXAMPLE B1.10

How much thermal energy is needed to increase the temperature of a kettle and the water inside it from 23 °C to 77 °C if its thermal capacity is $4800\,\mathrm{J\,K^{-1}}$?

Answer

$Q = \text{thermal capacity} \times \Delta T = 4800 \times (77 - 23) = 2.6 \times 10^5\,\mathrm{J}$

When answering these questions, assume that no energy was transferred to, or from, the surroundings.

47 Calculate how much energy is needed to raise the temperature of a block of metal of mass 3.87 kg by 54 °C if the metal has a specific heat capacity of $456\,\mathrm{J\,kg^{-1}\,K^{-1}}$.

48 Determine the specific heat capacity of a liquid that requires 3840 J to raise the temperature of a mass of 156 g by 18.0 K.

49 A drink of mass 500 g has been poured into a glass of mass 250 g (of specific heat capacity $850\,\mathrm{J\,kg^{-1}\,°C^{-1}}$) in a refrigerator. Calculate how much energy must be removed to cool the drink and the glass from 25 °C to 4 °C. (Assume the drink has the same specific heat capacity as water.)

50 A 20 W immersion heater is placed in a 2.0 kg iron block at 24 °C for 12 minutes. Calculate the final temperature. (Specific heat capacity of iron = $444\,\mathrm{J\,kg^{-1}\,°C^{-1}}$.)

51 An air conditioner has a cooling power of 1200 W and is located in a room containing 100 kg of air (specific heat capacity $1000\,\mathrm{J\,kg^{-1}\,°C^{-1}}$) at 30 °C. Determine the minimum possible temperature after the air conditioner has been switched on for 10 minutes.

52 A water heater for a shower is rated at 9.0 kW. Water at 15 °C flows through it at a rate of 15 kg every 3 minutes. Predict the temperature of the water in the shower.

53 A burner on a gas cooker raises the temperature of 500 g of water from 24 °C to 80 °C in exactly 2 minutes. What is the effective average power of the burner?

54 If the thermal capacity of a room and its contents were $3.5 \times 10^5\,\mathrm{J\,K^{-1}}$, estimate how long it would take a 2.5 kW heater to raise the temperature from 9 °C to 22 °C.

Exchanges of thermal energy

Figure B1.5 showed the temperature–time graphs of two objects, originally at different temperatures, placed in good thermal contact so that thermal energy can be transferred relatively quickly, assuming that the system is insulated from its surroundings. Under these circumstances the thermal energy given out by one object is equal to the thermal energy absorbed by the other object. Exchanges of thermal energy can be used as an alternative means of determining a specific heat capacity, or in the determination of the energy that can be transferred from a food or a fuel.

◆ **Calorimeter** Apparatus designed for (**calorimetry**) experiments investigating thermal energy transfers.

Calorimetry is the name used to describe experiments that try to accurately measure the temperature changes produced by various physical or chemical processes. Energy transfers can then be calculated if the masses and specific heat capacities are known. Calorimetric techniques may involve specially designed pieces of apparatus, called **calorimeters**, which are designed to limit thermal energy transfer to, or from, the surroundings.

WORKED EXAMPLE B1.11

A large metal bolt of mass 26.5 g was heated in an oven until it reached a constant temperature of 312 °C. (See Figure B1.44.)

It was then quickly transferred into 294 g of water initially at 22.1 °C.

The water was stirred and it reached a maximum temperature of 24.7 °C.

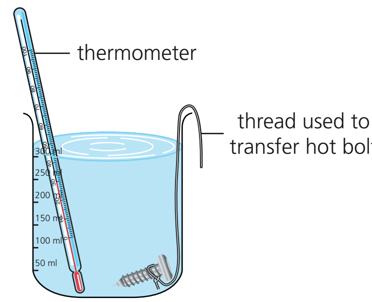


Figure B1.44 A hot metal bolt placed in cold water

a Explain why it was necessary to stir the water.

b Calculate how much thermal energy was transferred from the bolt to the water. (Assume no energy went to the surroundings.)

c Determine a value for the specific heat capacity of the metal from which the bolt was made.

d Suggest which metal the bolt was made from.

Answer

a To make sure that all the water was at the same temperature.

b $Q = mc\Delta T = 0.294 \times 4180 \times (24.7 - 22.1) = 3.20 \times 10^3$ J

c Thermal energy transferred to the water = thermal energy transferred from the bolt

$3.20 \times 10^3 = (mc\Delta T)_{\text{bolt}} = 0.0265 \times c \times (312 - 24.7)$

$c = 4.20 \times 10^2\,\text{J kg}^{-1}\,°\text{C}^{-1}$

d Steel (see Table B1.5)

When answering these questions, assume that no energy was transferred to, or from, the surroundings.

55 What mass of water with a temperature of 18 °C has to be mixed with 1.5 kg of water at 83 °C to produce a combined temperature of 55 °C?

56 5 coins, each of mass 8.8 g, were left in some boiling water for a few minutes. They were then very quickly transferred to 98 g of water at 20.7 °C. The water was stirred and its temperature rose to a maximum of 23.6 °C.

a Calculate the specific heat capacity of the metal alloy used in the coins.

b Explain why the coins were transferred quickly.

57 When 5.6 g of wood was completely burned in a calorimeter the temperature of 480 g of water rose from 22.7 °C to 64.6 °C.

a How much thermal energy was transferred to the water from the burning wood?

b Calculate a value for how much energy can be obtained from the combustion of a kilogramme of this wood.

Changes of phase

SYLLABUS CONTENT

- A phase change represents a change in particle behaviour arising from a change in energy at constant temperature.
- Quantitative analysis of thermal energy transfers, Q, with the use of specific latent heat of fusion and vaporization of substances, L, using: $Q = mL$.

◆ **Phase (of matter)** A substance in which all the physical and chemical properties are uniform. In physics, the term **phase change** is used to describe changes between solids, liquids and gases of the same substance.

◆ **States of matter** Solid, liquid or gas (or plasma).

◆ **Melting** Change from a solid to a liquid. Usually at a specific temperature (**melting point**).

◆ **Fusion (thermal)** Melting.

◆ **Freeze** Change from a liquid to a solid. Also called solidify.

◆ **Evaporation** The change from a liquid to a gas (vapour) at any temperature below the boiling point of the liquid. Occurs only at the liquid surface.

◆ **Vaporization** Change from a liquid to a vapour (gas) by boiling or evaporation. A **vapour** is a gas which can be condensed by pressure.

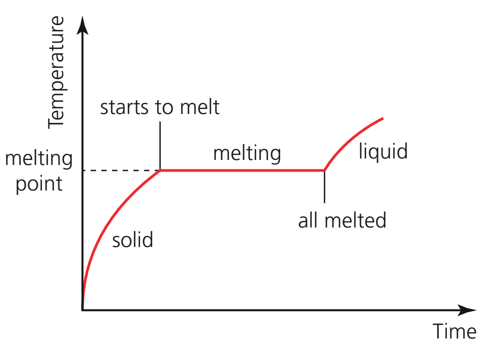
◆ **Boiling** Change from a liquid to a gas / vapour throughout the liquid at a precise temperature.

A **phase of matter** is a definite region of space in which all the physical and chemical properties of the substance contained in that space are the same. For example, a bottle containing water and oil has two different (chemical) phases of matter. A bottle containing water and ice has two different physical phases of matter.

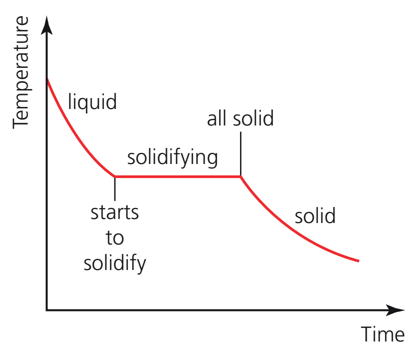
(The word phase has another, totally different, meaning in physics: as explained in Topic C.1, the phase of an oscillation (or a wave) describes the fraction of an oscillation that has occurred since an agreed reference point.)

Water, ice and steam are commonly described as examples of the three **states of matter**. Generally, in physics, the terms 'phase' and 'state' tend to be used interchangeably.

When thermal energy is transferred to a solid it will usually get hotter. However, for many solid substances, once they reach a certain temperature they will begin to **melt** (change from a solid to a liquid), and while they are melting the temperature does not change (Figure B1.45), even as energy continues to be supplied. This temperature is called the **melting point** of the substance, and it has a fixed value at a particular air pressure (Table B1.6). Melting is an example of a **phase change**. Another word for melting is **fusion**.



■ **Figure B1.45** Temperature changes as a solid is heated and melted (note that the lines are curved only because of energy transferred to the surroundings)



■ **Figure B1.46** Temperature changes when a liquid cools and freezes (solidifies)

Similarly, when a liquid cools, its temperature will be constant at its melting point while it changes phase from a liquid to a solid (Figure B1.46). This process is known as solidifying or **freezing**. But be careful – the word 'freezing' suggests that this happens at a low temperature, but this is not necessarily true (unless we are referring to turning water into ice, for example). The phase changes of water are such common events in everyday life that we all tend to think of them as the obvious examples but, of course, many other substances can melt and freeze. For example, the freezing (melting) point of chocolate is variable but is approximately 30 °C to 40 °C, so many chocolates, but not all, will melt when in contact with skin, see Figure B1.47.

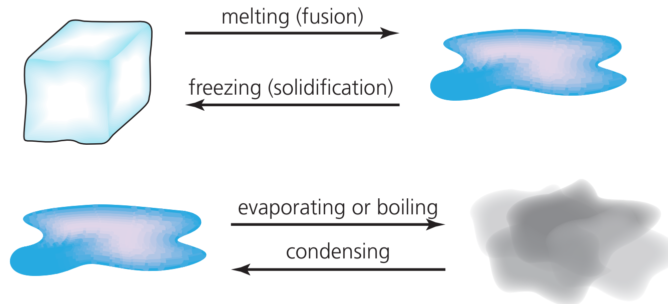
A change of phase also occurs when a liquid becomes a gas (or vapour), or when a gas (or vapour) becomes a liquid (Figure B1.48). A **vapour** is any gas at a temperature such that it can be condensed by pressure alone. The change of phase from a gas (or vapour) to a liquid can be by **boiling** or **evaporation** (it may also be called **vaporization**). Changing from a gas (or vapour) to a

◆ **Condense** Change from a gas or vapour to a liquid.

liquid is called **condensation**. The temperature at which boiling occurs is called the boiling point of the substance, and it has a fixed value for a particular air pressure (see Table B1.6).

The shape of graph showing the temperature change of a liquid being heated to boiling will look very similar to that for a solid melting (Figure B1.45), while the graph for a gas being cooled will look very similar to that for a liquid freezing (Figure B1.46).

■ **Figure B1.47** Melting chocolate



■ **Figure B1.48** Changes of phase

■ **Table B1.6** Melting points and boiling points of some substances (at normal atmospheric pressure)

	Melting point		Boiling point	
Substance	**°C**	**K**	**°C**	**K**
water	0	273	100	373
mercury	−39	234	357	630
alcohol (ethanol)	−117	156	78	351
oxygen	−219	54	−183	90
copper	1083	1356	2580	2853
iron	1538	1811	2750	3023

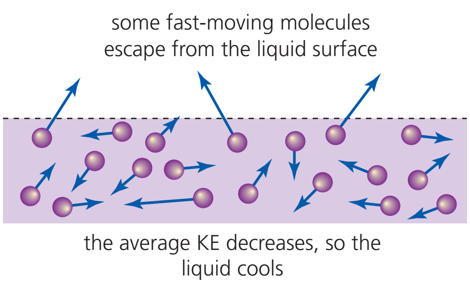
LINKING QUESTION

- How can the phase change of water be used in the process of electricity generation?

This question links to understandings in Topic D.4.

Melting, freezing, boiling and condensing are known as **phase changes**.

Boiling and evaporation



■ **Figure B1.49** Molecules leaving a surface during evaporation

In a liquid the molecules will always have a range of different random kinetic energies that are continuously transferred in interactions / collisions between them. This means there will always be some molecules near the surface that have enough energy to overcome the attractive forces that hold the molecules together in the liquid. Such molecules can escape from the surface and this effect is called *evaporation*. See Figure B1.49.

Evaporation occurs only from the surface of a liquid and can occur at any temperature, although the rate of evaporation increases significantly with rising temperature (between the melting and boiling points).

Boiling occurs at a precise temperature – the temperature at which the molecules have enough kinetic energy to form bubbles *inside* the liquid. Boiling points can vary considerably with different surrounding air pressures.

Evaporation occurs from the surface of a liquid over a range of temperatures. Boiling occurs throughout a liquid at a precise temperature.

LINKING QUESTION

- What role does the molecular model play in understanding other areas of physics?

This question links to understandings in Topics B.3 and B.4.

The loss of the most energetic molecules during evaporation means that the average kinetic energy of the molecules remaining in the liquid must decrease (until thermal energy flows in from the surroundings). This microscopic effect explains the macroscopic fall in temperature (cooling) that always accompanies evaporation from a liquid.

Using the cooling effect of evaporation

Sweating – Our bodies produce tiny droplets of water on our skin. See Figure B1.52. When the water evaporates it removes thermal energy from our bodies and helps to keep us cool.

Cooling buildings – The cooling effect produced by water evaporating has been used for thousands of years to keep people and buildings cool. For example, in central Asia open towers in buildings encouraged air flow over open pools of water, increasing the rate of evaporation and the transfer of thermal energy up the tower by convection currents. The flow of air past people in buildings also encourages the human body's natural process of cooling by sweating.

◆ **Refrigerant** Fluid used in the refrigeration cycle of refrigerators, air conditioners and heat pumps.

Refrigerators – Modern refrigerators rely on the cooling produced when a liquid evaporates. The liquid/gas used is called the **refrigerant**. Ideally it should take a large amount of thermal energy to turn the refrigerant from a liquid into a dense gas at a little below the desired temperature. In a refrigerator, for example, after the refrigerant has removed thermal energy from the food compartment, it will have become a gas and be hotter. In order to re-use it and turn it back into a cooler liquid again, it must be compressed and its temperature reduced. To help achieve this thermal energy is transferred from the hot, gaseous refrigerant to the outside of the refrigerator (Figure B1.50).

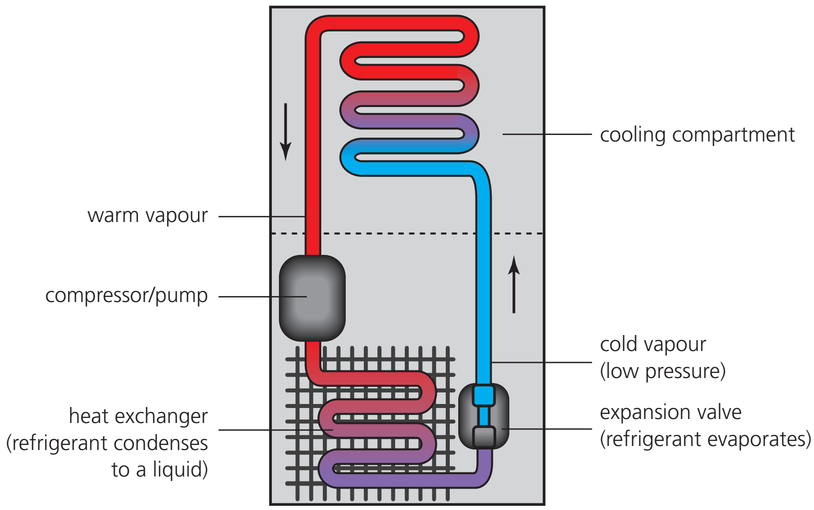


Figure B1.50 Schematic diagram of a refrigerator

Air-conditioners use the same principle as refrigerators.

Outdoor misting systems – Outdoor misting systems (see Figure B1.51) are becoming increasingly popular. Tiny water droplets are sprayed out of nozzles and quickly evaporate, cooling the air, or people on whom the droplets fall.

■ **Figure B1.51** Cooling mists in a restaurant

◆ **Humidity** A measure of the amount of water vapour present in air.

ATL B1B: Communication skills

Clearly communicate complex ideas in response to open-ended questions

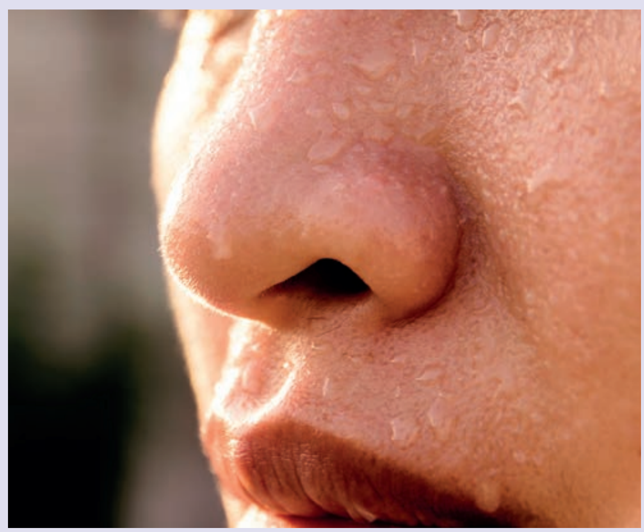
Humidity and fans

The cooling effect of sweating on the human body is greatly affected by the **humidity** of the surrounding air.

Air contains unseen water **vapour** that has evaporated from plants and various water surfaces. At 20 °C each cubic metre of air can contain up to a maximum of 17 g of invisible water vapour. As the temperature increases, each cubic meter of air can contain more water. For example, at 30 °C the maximum is 30 g.

Humidity is a measure of the amount of water vapour in air compared to the maximum possible. For example, at 20 °C if there is 17 $g\,m^{-3}$, the humidity is said to be 100%, but, if there is 8.5 $g\,m^{-3}$, the humidity is 50%, which is often reported to be about the most comfortable humidity for people.

With greater humidity in the surrounding air, it is more difficult for water to be evaporated from the skin in the process of sweating, and so the cooling effects are reduced.



■ **Figure B1.52** Sweat cools us by evaporation

Fans can be very useful in helping to keep people cool, but they do not directly reduce temperatures.

Research and explain in your own words how a fan might help to keep someone cool.

58 Suggest why the boiling point of a liquid depends on the surrounding air pressure.

59 Some spaghetti is being cooked in boiling water in an open pan on a gas cooker. Discuss what happens if the gas flow is increased so that more thermal energy is transferred to the water.

60 Explain why wet clothes will dry more quickly outside on a windy day.

61 Why will water spilt on the floor dry more quickly if it is spread out?

Latent heat

◆ **Latent heat** Thermal energy that is transferred at constant temperature during any change of physical phase.

To melt a solid, or boil a liquid, it is necessary to transfer thermal energy to the substance. However, as we have seen, melting and boiling occur at constant temperatures, so that the energy supplied is not being used to increase molecular kinetic energies (which change with temperature). Because there is no change of temperature, the thermal energy transferred during a phase change is called **latent heat** (*latent* means hidden).

The latent heat supplied is used to produce the molecular re-arrangements that characterize the differences between solids and liquids, and liquids and gases. Latent heat is used to overcome intermolecular forces and to increase molecular separations. This will increase molecular potential energies. In the case of melting, some forces are overcome and there is a slight increase in average separation, but in the case of boiling all the remaining forces are overcome as the molecules move much further apart.

When a liquid freezes (solidifies), the same amount of energy per kilogram is emitted as was needed to melt it (without a change in temperature). Similarly, boiling and condensing involve equal energy transfers.

The concept of *specific* latent heat brings a mathematical treatment to this subject:

The specific latent heat of a substance, L, is the amount of energy transferred when 1 kilogram of the substance changes phase at a constant temperature. $Q = mL$ SI units: J kg^{-1}

◆ **Specific latent heat, L_f or L_v** The amount of energy needed to melt (fusion) or vaporize 1 kg of a substance at constant temperature.

The latent heat associated with melting or freezing is called **specific latent heat of fusion**, L_f. The latent heat associated with boiling or condensing is known as **specific latent heat of vaporization**, L_v.

As an example, the specific latent heat of fusion of lead is $2.45 \times 10^4\,\text{J kg}^{-1}$ and its melting point is 327 °C. This means that 2.45×10^4 J is needed to melt 1 kg of lead at a constant temperature of 327 °C.

Experiments to determine specific latent heats (water is often used as a convenient example) have many similarities with specific heat capacity experiments. Usually, an electric heater of known power is used to melt or boil a substance – Question 65 overleaf describes such an experiment.

WORKED EXAMPLE B1.13

The latent heats of vaporization of water and ethanol are $2.27 \times 10^6\,\text{J kg}^{-1}$ and $8.55 \times 10^5\,\text{J kg}^{-1}$.

a State and explain which one is ‘easier’ to boil (at the same pressure).

b Calculate how much thermal energy is needed to turn 50 g of ethanol into a gas at its boiling point of 78 °C.

Answer

a It is ‘easier’ to boil ethanol because less energy is needed to turn each kilogram into a gas.

b $Q = mL = 0.050 \times (8.55 \times 10^5) = 4.3 \times 10^4\,\text{J}$

62 Some water was heated in an open 2250 W electric kettle. When it reached 100 °C the water boiled and in the next 180 s the mass of water reduced from 987 g to 829 g.

a Use these figures to estimate the latent heat of vaporization of water.

b Explain why your answer is only an estimate. Is it an underestimate, or an overestimate?

63 The latent heat of fusion of a certain kind of chocolate is $1.6 \times 10^5\,\mathrm{J\,kg^{-1}}$. Predict how much thermal energy is removed from you when a 10 g bar of chocolate melts in your mouth.

64 Outline why you would expect that the latent heats of vaporization of substances are larger than their latent heats of fusion.

65 The apparatus shown in Figure B1.53 was used to determine the specific latent heat of fusion of ice. Two identical 50 W immersion heaters were placed in some ice in two separate funnels. The heater above beaker A was switched on, but the heater above B was left off. After 5 minutes it was noted that the mass of melted ice in beaker A was 54.7 g, while the mass in beaker B was 16.8 g.

a Explain the reason for having ice in two funnels.

b Use these figures to estimate the latent heat of fusion of ice.

c Suggest a reason why this experiment does not provide an accurate result.

d Describe one change to the experiment that would improve its accuracy.

66 0.53 g of steam at 100 °C condensed and then the water rapidly cooled to 35 °C.

a How much thermal energy was transferred from the steam:

i when it condensed

ii when the water cooled down?

b Suggest why a burn received from steam is much worse than from water at the same temperature (100 °C).

67 120 g of water at 23.5 °C was poured into a plastic tray for making ice cubes. If the tray was already at 0 °C, calculate the thermal energy that has to be removed from the water to turn it to ice at 0 °C.
(The latent heat of fusion of water is $3.35 \times 10^5\,\mathrm{J\,kg^{-1}}$.)

68 Clouds are condensed droplets of water and sometimes they freeze to become ice particles. Suppose a typical cloud had a mass of 24 000 kg:

a Determine how much thermal energy would be released if it all turned to ice at 0 °C.

b Discuss how your answer compares to a typical value of 5×10^9 J of energy released in a single lightning strike.

69 Some water and a glass container are both at a temperature of 23 °C and they have a combined thermal capacity of $1500\,\mathrm{J\,K^{-1}}$. If a 48 g lump of ice at −8.5 °C is placed in the water and the mixture is stirred until all the ice has melted, determine the final temperature.
(The specific heat capacity of ice is $2.1 \times 10^3\,\mathrm{J\,kg^{-1}\,K^{-1}}$. The latent heat of fusion of water is $3.35 \times 10^5\,\mathrm{J\,kg^{-1}}$.)

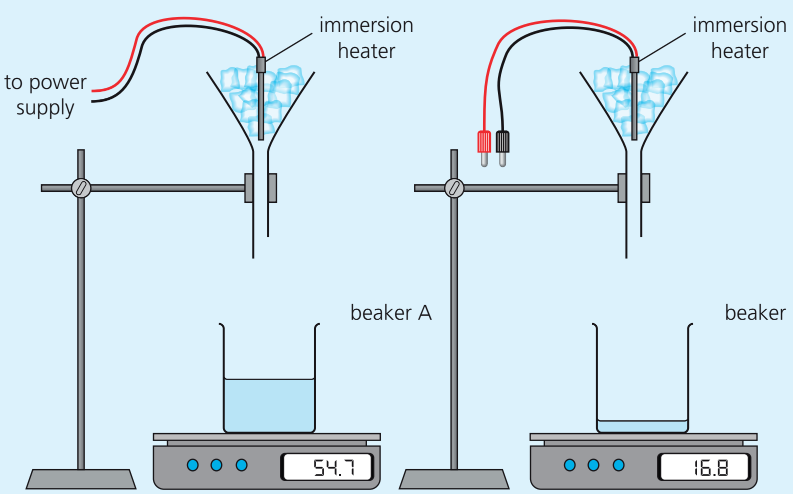


Figure B1.53 An experiment to determine the latent heat of fusion of ice

B.2 Greenhouse effect

Guiding questions

- How does the greenhouse effect help to maintain life on Earth and how does human activity enhance this effect?
- How is the atmosphere as a system modelled to quantify the Earth–atmosphere energy balance?

◆ **Greenhouse effect** The natural effect that a planet's atmosphere has on reducing the amount of radiation emitted into space, resulting in a planet warmer than it would be without an atmosphere.

◆ **Greenhouse effect (enhanced)** The reduction in radiation emitted into space from Earth due to an increasing concentration of *greenhouse gases* in the atmosphere (especially carbon dioxide) caused by human activities; believed by most scientists to be the cause of global warming.

◆ **Anthropogenic climate change** Changes in the climate due to human activities. Also called global warming.

◆ **Solar System** The Sun and all the objects that orbit around it.

We use the term **greenhouse effect** to describe the fact that the Earth's atmosphere keeps the planet warmer than it would be without the atmosphere. The effect of the Earth's atmosphere is similar in some ways to how the glass in the walls and roof of a greenhouse keep the plants warmer than if they were left in the open air.

From the beginning, it is important to understand that the basic greenhouse effect is essential for life on Earth. However, human activity has changed, and continues to change, the atmosphere in ways that are making the Earth warmer. This is known as the ***enhanced* greenhouse effect**, or **anthropogenic climate change**.

A planet's energy balance: an introduction

SYLLABUS CONTENT

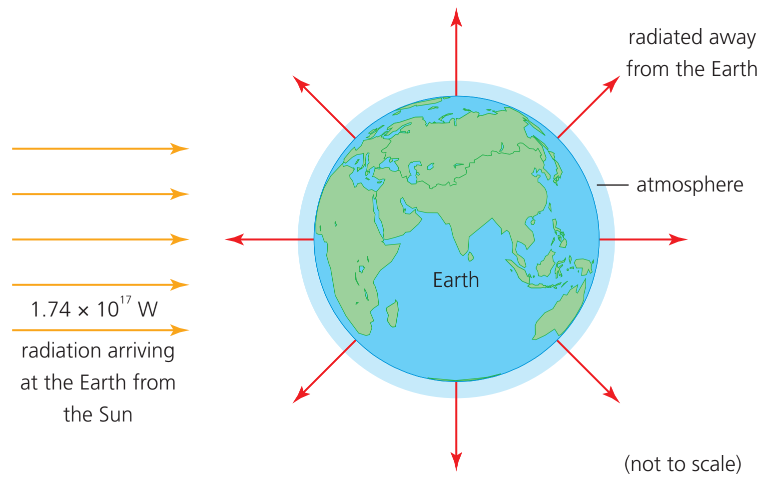
- Conservation of energy.

The planets of the **Solar System**, including the Earth, have existed for billions of years, so it is reasonable to assume that they should each have reached a steady (average) temperature, over human timescales at least. This means that each planet should be receiving and emitting thermal energy at the same rate, assuming that there are no significant internal energy sources of its own.

Since the only thermal energy which can travel across Space is radiation, for a planet at constant average temperature,

radiant thermal energy received by a planet (or moon) = radiant thermal energy emitted by a planet (or moon) in the same time interval.

Figure B2.1 represents this energy balance for the Earth.



■ **Figure B2.1** Earth receiving and emitting radiation

◆ **Simulation** Simplified visualization (imitation) of a real physical system and how it changes with time. Usually part of a computer modelling process.

In the rest of this topic, we will discuss the various factors which affect this energy balance, and how human activity is upsetting this balance on planet Earth.

We will begin by developing an understanding of the thermal energy (including light) arriving at the Earth and other planets from the Sun.

Tool 2: Technology

Identify and extract data from databases

A database is information (often extensive) which is stored electronically in a structured and organized way. A database will usually be continually updated with new data. There are many different types and sizes of database, which may be accessed in different ways, which might be only available to certain people (information within a school, for example).

Google's 'Bigtable' is an example of an enormous database which helps to run internet searches, Google Maps and so on. At the other extreme, you may wish to set up and monitor your own physical fitness database on a *Microsoft Excel* spreadsheet.

There are a few areas of study in this course for which enormous quantities of data are readily available, including climate change, energy resources and astronomy.

Generate data from models and simulations

Many situations in physics can be reduced to simplified mathematical models. These provide the essential basis for understanding, but they can later be expanded to include more details.

Many computer **simulations** are available to visually represent these models. For example, the movement of a mass bouncing up and down on the end of a spring (Topic C.1). These simulations can be very useful in the learning process, especially when you can investigate the effects of changing the variables (for example, mass on spring, stiffness of spring, air resistance and so on).

While virtual experiments like these should not replace actual experimental work, they are guaranteed to quickly produce results which are consistent with the physics theory and they enable a wider range of tests to be carried out than would usually be done in a laboratory.

As we shall see, the word equation highlighted on page 215 is the starting assumption for a mathematical analysis of the Earth's surface temperature. Without too much difficulty, we will be able to predict the average surface temperatures of planets and moons with reasonable accuracy. But, as we are all now aware, relatively small changes in the Earth's temperature can have disastrous effects. A simple model is inadequate for making predictions about how the Earth's temperature and climate may change.

The factors affecting climate change are numerous, complicated and interconnected. Computer models are needed in order to cope with this complexity and the vast amount of data available.

Luminosity and apparent brightness of the Sun: the solar constant

SYLLABUS CONTENT

- The solar constant, S.
- The incoming radiative power is dependent on the projected surface of a planet along the direction of the path of the rays, resulting in a mean value of the incoming intensity being $S/4$.

Using Wien's law (Topic B.1), we can determine that the surface temperature of the Sun is 5780 K. Geometrical measurements made from Earth inform us that the Sun is an average distance of 1.50×10^{11} m away, and it has a surface area of $6.05 \times 10^{18}\,\text{m}^2$. With this information we can determine its luminosity, $L_\odot$ (as defined in Topic B.1), assuming that its acts as a black body:

$$L_\odot = \sigma A T^4 = (5.67 \times 10^{-8}) \times (6.05 \times 10^{18}) \times 5780^4 = 3.83 \times 10^{26}\,\text{W}$$

Knowing its luminosity, we can then calculate the apparent brightness, b, of the Sun just above the Earth's atmosphere. See Figure B2.2.

$$b = \frac{L_\odot}{4\pi d^2} = \frac{3.83 \times 10^{26}}{4\pi \times (1.50 \times 10^{11})^2} = 1.35 \times 10^3\,\text{W m}^{-2}$$

◆ **Solar constant** Intensity of the Sun's radiation arriving perpendicularly on an area above the Earth's atmosphere.

The value of the intensity of thermal radiation (including light) from the Sun which passes perpendicularly through an area just above the Earth's atmosphere, is called the **solar constant**, S.

DB The accepted average value for the solar constant of the Earth is $\mathbf{1.36 \times 10^3\,W\,m^{-2}}$

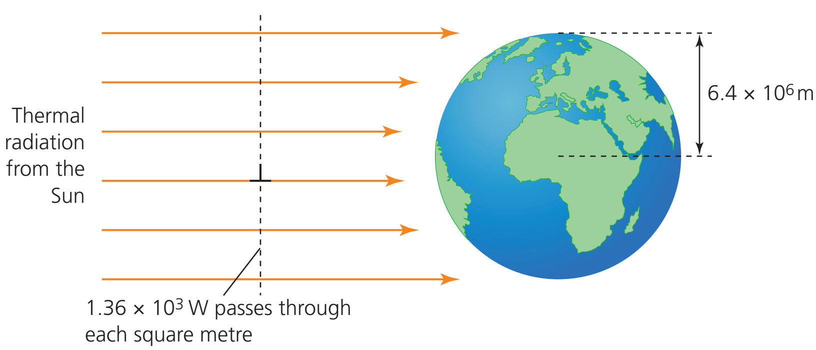


Figure B2.2 Solar constant

Although S is called the solar *constant*, it is well-known to vary very slightly, by about 0.1% every 11 years. However, it is not believed that this has a significant effect on the Earth's climate.

Different planets will have different solar constants. For example, Venus has a higher solar constant, $2.6 \times 10^3\,\text{W m}^{-2}$, because it is closer to the Sun. Solar constant values change slightly with periodic changes in the behaviour of the Sun and variations in the distances of planets from the Sun.

This constant will play an important part in calculations concerning the greenhouse effect, later in this topic.

If a planet has a radius r, the incoming radiative intensity extends over a cross-sectional area of πr^2, but the whole planet has a surface area of $4\pi r^2$ (the surface area of a sphere). This means that:

the mean value of the radiative intensity directed at the surface of the planet is: $S \times \left(\frac{\pi r^2}{4\pi r^2}\right) = \frac{S}{4}$

In the case of the Earth:

$$\frac{S}{4} = \frac{1360}{4} = 340\,\text{W m}^{-2}$$

This would be the average intensity of thermal radiation reaching the Earth's surface if it did not have an atmosphere.

Non-black bodies: albedo and emissivity

SYLLABUS CONTENT

- Albedo as a measure of the average energy reflected off a macroscopic system as given by: $\text{albedo} = \dfrac{\text{total scattered power}}{\text{total incident power}}$
- Earth's albedo varies daily and is dependent on cloud formations and latitude.
- Emissivity as the ratio of the power radiated per unit area by a surface compared to that of an ideal black surface at the same temperature as given by: $\text{emissivity} = \dfrac{\text{power radiated per unit area}}{\sigma T^4}$

Before we can return to a discussion of the energy balances of planets (and moons), we have to consider how the surfaces of these bodies compare to the idealized concept of the surfaces of black bodies.

◆ **Albedo** The total scattered or reflected power/total incident power (on part of a planet's surface). Albedo depends on the nature of the surface and inclination of the radiation to the surface.

◆ **Incident wave, or ray** Wave (or ray) arriving at an object or a boundary.

Although we can assume that the Sun, with its high surface temperature, behaves as a perfect black body, we cannot make the same assumptions for the Earth, or other planets and moons. We need to quantify the ability of non-black-body surfaces to:

- absorb, reflect and scatter thermal radiation; and
- emit thermal radiation.

To do this, we will introduce the two concepts of **albedo** and **emissivity**.

Albedo

The surface of any planet or moon will reflect / scatter some of the thermal radiation that is **incident** upon it (arrives at the surface). We use the term albedo to quantify this:

$$\textbf{albedo} = \frac{\textbf{total scattered power}}{\textbf{total incident power}} \quad \text{(A ratio, so no units.)}$$

◆ **Scattering** Irregular reflections of waves or particles from their original path by interactions with matter.

Scattering can be considered to be unpredictable and small-scale random reflections. For example, a plane mirror reflects light so that images may be seen in it, but a mirror broken into many small pieces scatters light. A very smooth surface may reflect light, but a rough surface scatters light.

A perfect black body would have an albedo of 0: all incident thermal energy would be absorbed. An albedo of 1 would represent a surface which scatters all of the incident thermal radiation.

Table B2.1 lists approximate values for the albedos of the surfaces of some common materials, but there is a lot of variation from the surfaces of each material. The albedo of a surface also varies with the angle at which the incident radiation strikes the surface: albedo increases as the radiation is incident at greater angles to the perpendicular. This means that, at any particular location, there are variations during each day and at different times of the year. For the same reason, albedo will vary with different latitudes around the Earth.

■ **Table B2.1** Approximate albedo values

Material	Albedo
(ocean) water	0.1
forest	0.1
road surface	0.1
soil	0.2
grass	0.3
desert sand	0.4
clouds (very variable)	0.5
ocean ice	0.6
snow	0.8

■ **Figure B2.3** Snow has high albedo, but water has low albedo

The Earth (including its atmosphere) has an average albedo of 0.315. That is, about 68.5% of the incident thermal radiation is absorbed by the Earth and its atmosphere.

WORKED EXAMPLE B2.1

Radiation of intensity 610 W m^{-2} was incident perpendicularly on the surface of a lake at midday.

a If the water had an albedo of 0.18 at that time, calculate how much energy was absorbed every second by each square metre.

b Describe how your answer would change much later in the day

Answer

a $610 \times (1.0 - 0.18) = 500$ W (110 W was reflected)

b Later in the day the radiation will be incident at a greater angle to the perpendicular. The albedo will increase and more radiation is reflected. The incident intensity will also decrease because the radiation passes through a greater length of the atmosphere. The answer to **a** will decrease.

◆ **Emissivity** The power radiated by an object divided by the power radiated from a black body of the same surface area and temperature.

Emissivity

The concept of **emissivity** compares the power of the thermal radiation emitted by a surface (from unit area) to that of a perfect black body at the same temperature (σT^4). A surface with a greater emissivity emits thermal energy with more power, under the same conditions.

$$\text{emissivity} = \frac{\text{power radiated per unit area}}{\sigma T^4}$$

Emissivity is a ratio, so it has no unit. The symbol e (or ε) is sometimes used for emissivity.

Table B2.2 lists approximate values for the emissivity of the surfaces of the same materials as seen in Table B2.1.

The average emissivity of the Earth and its atmosphere is estimated to be 0.61.

A black body has an emissivity of one.

■ **Table B2.2** Typical values for the emissivity of materials also seen in Table B2.1

Material	Emissivity
(ocean) water	0.99
forest	0.97
road surface	0.96
soil	0.95
grass	0.91
desert sand	0.90
clouds (variable)	0.55
ocean ice	0.97
snow	0.94

WORKED EXAMPLE B2.2

Determine the average emissivity of a planet if it has an average surface temperature of −63 °C, a surface area of $1.4 \times 10^{14}\,m^2$ and it emits thermal energy at a rate of 1.3×10^{16} W.

Answer

$$\text{emissivity} = \frac{\text{power radiated per unit area}}{\sigma T^4}$$

$$= \frac{(1.3 \times 10^{16}/1.4 \times 10^{14})}{5.67 \times 10^{-8} \times (273 - 63)^4}$$

$$= 0.84$$

1 580 W of thermal radiation is incident perpendicularly on photovoltaic solar panels of dimensions 1.86 × 2.12 m, similar to those seen in Figure B2.4
   - a Calculate the intensity of the radiation.
   - b If the surface of the panels has an albedo of 0.21, determine the total rate at which the panels absorb thermal radiation.
   - c Describe how the albedo of the panels is minimized.
   - d State two reasons why it is best if the panels are perpendicular to the incident radiation.

Figure B2.4 Photovoltaic panels

2
   - a State why it would be reasonable to expect that, if the temperature of the Earth's surface were to rise, snow and ice would melt quicker.
   - b Conversely, explain why it would also be reasonable to expect that, if large amounts of snow and ice were to melt, the temperature of the Earth's surface would rise.

3 Explain why the average albedo of ocean water will tend to increase
   - a in winter
   - b closer to the poles.

4 Discuss why the emissivity of clouds make them an important factor affecting the average emissivity of Earth.

5 A brick wall is 2.34 m high and 3.80 m long. The bricks have an emissivity of 0.72. At what rate is thermal energy radiated away from the wall if its temperature is 18 °C?

6 Show that the total power radiated away from the Earth's surface is approximately 1×10^{17} W. (Assume surface temperature is 288 K. Radius of the Earth is 6.4×10^6 m.)

Modelling a planet's energy balance

Having established an understanding of emissivity and albedo, we can now return to the subject of the energy balances of planets and moons.

We will take planet Earth as our first and most obvious example, but similar calculations are possible for other planets and moons.

Accurately modelling the Earth's temperature is a very complex process, which, because of its enormous consequences, has preoccupied some of the best scientific minds and fastest computers, using enormous quantities of data, for decades. However, we can use the physics already discussed to make a broad prediction:

We have the following relevant data:

- Solar constant = $1.36 \times 10^3\,\mathrm{W\,m^{-2}}$
- Radius of the Earth = $6.4 \times 10^6\,\mathrm{m}$
- Average emissivity of the Earth (including its atmosphere) = 0.61
- Average albedo of the Earth (including its atmosphere) = 0.315

As explained near the start of this topic:

Radiant thermal energy received by a planet = radiant thermal energy emitted by a planet (in the same time interval).

We can now be more detailed:

solar constant × cross-sectional area of planet × (1 – albedo)
= emissivity × σ × surface area of planet × T^4

$$1.36 \times 10^3 \times \cancel{\pi r^2} \times (1 - 0.315) = 0.61 \times (5.67 \times 10^{-8}) \times 4 \times \cancel{\pi r^2} \times T^4$$

$$T = 286\ \mathrm{K}\ \text{or}\ 13\,^\circ\mathrm{C}$$

The current mean temperature of the Earth's surface is 288 K (15 °C), so it would appear that our basic model is reasonably accurate.

Three things should be very clear:

- The values of emissivity and albedo have a fundamental effect on a planet's temperature. They will have different values for a planet without an atmosphere.
- The values of emissivity and albedo on Earth are being changed by human activity.
- Small changes in the Earth's temperature may produce large changes in the Earth's climate, and such changes are mostly harmful to our lives.

WORKED EXAMPLE B2.3

Calculate a value for the average emissivity of the Moon. Assume that it has an average albedo of 0.12, and an average surface temperature of 274 K (but it should be noted that the Moon's surface temperature is *very* variable and an average is not really accurately defined).

Figure B2.5 The Moon

Answer

solar constant × cross-sectional area of Moon × (1 – albedo)
= emissivity × σ × surface area of Moon × T^4

$1.36 \times 10^3 \times \cancel{\pi r^2} \times (1 - 0.12) = \text{emissivity} \times (5.67 \times 10^{-8}) \times 4 \times \cancel{\pi r^2} \times 274^4$

Mean emissivity = 0.94

7 To determine a value for the Earth's surface temperature if it never had an atmosphere, we need to estimate values for emissivity and albedo under those conditions. Use estimated values of 0.9 (emissivity) and 0.3 (albedo) to make the calculation.

8 Determine a value for the surface temperature of the Earth if it never had an atmosphere and behaved as a perfect black body.

9 Calculate the average surface temperature of Mars, assuming that it has an average emissivity of 0.95 and an average albedo of 0.21. Its solar constant is $590\,\mathrm{W\,m^{-2}}$.

10 Suppose that climate change resulted in a 5% change in both the emissivity and the albedo of the Earth. Predict a new value for the maximum average surface temperature.

11 State the factors which will affect the surface temperatures of planets orbiting a distant star.

Nature of science: Models

Using computers to expand human knowledge

Trying to predict the future, or to answer the question 'what would happen if …' has always been a common and enjoyable human activity. But it seems that our predictions are usually much more likely to be wrong than right. This is partly because, in all but the simplest of examples, there are just too many variables and unknown factors. Of course, the inconsistencies of human nature play an important part when dealing with people's behaviour, but accurately predicting events governed mostly by the laws of physics – such as next week's weather – can also be difficult.

Mathematical modelling is a powerful tool to understand a situation that can be represented by equations and numbers. But even in the simplest situations, there are nearly always simplifications and assumptions that result in uncertainty in predictions. When dealing with complex situations – such as predicting next month's weather, the value of a financial stock next year, or the climate in 50 years' time – even the most able people in the world will struggle with the complexity and amount of data. The rapid increase in computing power in recent years has changed this.

Modern computers have computing power and memory far in advance of human beings. They are able to handle masses of data and make enormous numbers of calculations that would never be possible without them. They are ideal for making predictions about the future climate, but that does not necessarily mean that the predictions will turn out to be correct. Computer predictions are limited by the input data provided to them and, more particularly, by the specific tasks that human beings have asked them to perform. To check the accuracy of predictions, computer models can be used to model known complex situations from the past to see if they are able to predict what actually happened next. But predicting the past is always much easier than predicting the future.

Effect of the Earth's atmosphere: greenhouse effect

So far, we have modelled the Earth's energy balance by treating the planet and its atmosphere as one system. In order to fully understand what is happening, we now need to consider transfers of energy within that system: between the Earth and its atmosphere. This is where a knowledge of the greenhouse effect becomes important.

Nature of science: Models

Greenhouses and the Earth's atmosphere

A **greenhouse** is a building for controlling the temperature of growing plants. The walls and roof are made of glass, or transparent plastic. A significant number of large, impressive, ornate structures have been constructed over the last two centuries in colder countries, where there was much enthusiasm for growing plants, especially fruit and vegetables, native to hotter climates. See Figure B2.6.

Figure B2.6 The Palm House at Schönbrunn Palace Park in Vienna

The major advantage of a greenhouse is that the temperature inside is hotter than outside. To understand the main reason why, we need to compare the radiation arriving at the greenhouse from the Sun to the radiation emitted from the contents of the greenhouse. See Figure B2.7.

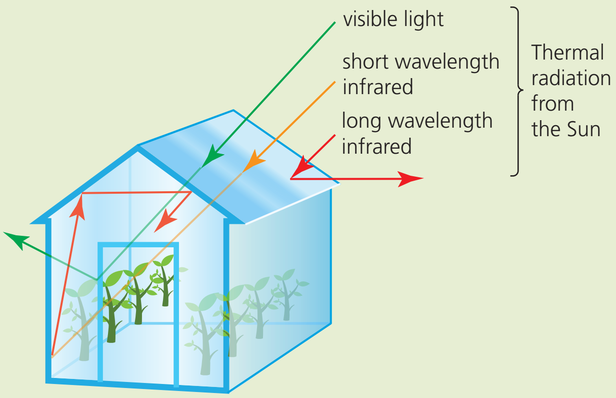


Figure B2.7 Thermal radiation in a greenhouse

We have already discussed the black-body spectrum from the Sun's surface (at 5780 K) and most of this thermal radiation falling on the glass of the greenhouse (light and infrared) will be transmitted through into the inside, where it will raise the temperature of the contents. Some longer wavelengths will be reflected back by the glass. Much of the light radiation entering the greenhouse will pass back out of the glass after reflection from surfaces in the interior.

The ground, other surfaces, pots and plants inside the greenhouse will get warmer because of the thermal radiation (infrared) that they absorb. Then they transfer thermal energy to the air by conduction. Convection currents move the warmer air upwards, but it mostly remains within the greenhouse.

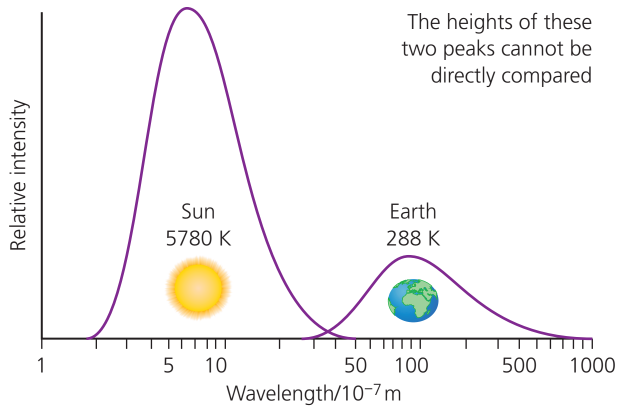
The thermal radiation emitted from the warmed contents of the greenhouse comes from much cooler surfaces ($\approx$ 300 K) than the Sun (infrared, but no light). The spectrum of this radiation is very different from that of the Sun. Most importantly, the infrared wavelengths are much longer and they are not able to pass back out through the glass. In this way we may describe thermal energy as being 'trapped' inside the greenhouse.

When you have completed this topic, discuss in pairs: how valid is the comparison between the physics of greenhouses described here and the physics of the Earth's atmosphere? Is 'greenhouse effect' a useful term for these effects? To what extent might it mislead?

◆ **Greenhouse** Structure made mostly from a transparent material (usually glass) used for controlling plant growth.

Comparing the radiation emitted by the Sun to the radiation emitted by the Earth

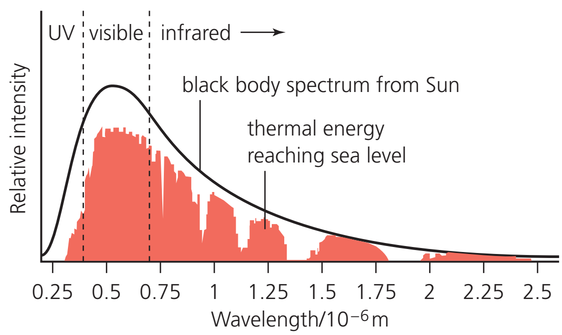
The spectrum of the black-body thermal radiation received from the Sun is very different from the thermal radiation spectrum radiated from the Earth's much cooler surface, as can be seen in Figure B2.8. Note that the horizontal scale is not linear (it is logarithmic) and the relative height of the peak for the Earth's spectrum has been *greatly* exaggerated for clarity.



■ **Figure B2.8** Comparing the radiation emitted by the Sun to the radiation emitted by the Earth

Compared to radiation from the Sun, the thermal radiation from the Earth is at a much lower power, with much longer wavelengths.

Most of the radiation from the Sun passes through the Earth's atmosphere, as can be interpreted from Figure B2.9.



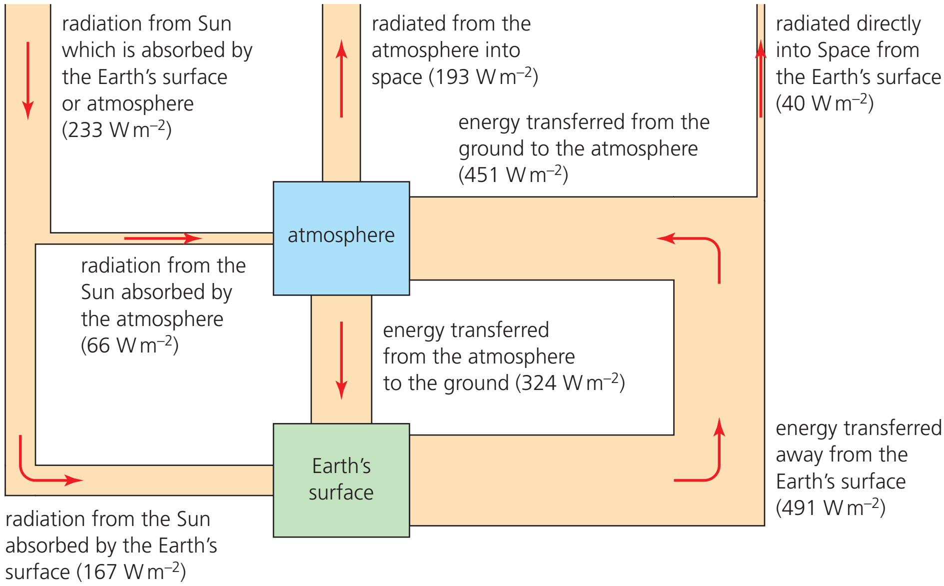
■ **Figure B2.9** The effect of the atmosphere on radiation arriving at the Earth's surface from the Sun

Because of its different wavelength range, a smaller proportion of the infrared from the Earth's surface is able to pass back out through the same atmosphere. However, over time, the temperature of the Earth's surface adjusts so that the *total* thermal power received by the Earth (including its atmosphere) = total thermal power radiated by the Earth (including its atmosphere).

Energy flow through the Earth's atmosphere

We have already seen that the mean intensity of thermal radiation arriving at the Earth from the Sun, averaged over the whole planet, over an extended period of time, is 340 W m^{-2}. We have also noted that the mean albedo of the Earth and its atmosphere is 0.315, meaning that only 68.5% of that 340 (= 233 W m^{-2}) is absorbed in the atmosphere or the Earth's surface.

Figure B2.10 summarizes what happens to that 233 W m^{-2}. You will find that numerical data varies slightly, depending on your source of information and its date. Note that data shows that the whole Earth, its surface and its atmosphere, are each receiving and emitting thermal energy at the same rates. Note that thermal energy transfer between the Earth's surface and its atmosphere is not just by radiation. Convection is also very important.



■ **Figure B2.10** Energy flow through the Earth's atmosphere

The importance of the atmosphere should be clear from studying Figure B2.9. We will now consider *how* the atmosphere absorbs and radiates energy.

ATL B2A: Research skills

Comparing, contrasting and validating information

Use the internet to research into data which either supports or conflicts with that shown in Figure B2.10. Is there much disagreement? Which sources do you think are the most reliable and/or up-to-date? Do you think that different websites get their information from the same original sources?

Greenhouse gases

SYLLABUS CONTENT

- Methane, CH_4, water vapour, H_2O, carbon dioxide, CO_2, and nitrous oxide, N_2O are the main greenhouse gases and each of these has origins that are both natural and created by human activity.
- The absorption of infrared radiation by the main greenhouse gases in terms of the molecular energy levels and the subsequent emission of radiation in all directions.
- The greenhouse effect can be explained in terms of either a resonance model or molecular energy levels.

◆ **Greenhouse gases** Gases in the Earth's atmosphere that absorb and re-emit infrared radiation, thereby affecting the temperature of the Earth. The principal greenhouse gases are water vapour, carbon dioxide, methane and nitrous oxide. Atmospheric concentrations of the last three of these have been increasing significantly in recent years.

The Earth's atmosphere has been formed over millions of years by naturally occurring volcanic and biological processes and from collisions with comets and asteroids. The air in the atmosphere contains approximately (by volume) 78% nitrogen, 21% oxygen and 0.9% argon. There are also naturally occurring traces of many other gases, including carbon dioxide and water vapour. Some of these trace gases are called **greenhouse gases** because they play a very important part in controlling the temperature of the Earth in the greenhouse effect. Greenhouse gases absorb (and then re-emit) infrared radiation.

There are many greenhouse gases but the four most important, in decreasing order of their contribution to the greenhouse effect, are:

- water vapour, H_2O (but see Top tip!)
- carbon dioxide, CO_2
- methane, CH_4
- nitrous oxide (dinitrogen monoxide), N_2O.

Nitrogen, oxygen and argon have no greenhouse effect (because they have non-polar molecules or atoms).

The relative importance of these gases in causing the greenhouse effect depends on their abundance in the atmosphere as well as their ability to absorb infrared radiation. Each of the gases has natural as well as human origins.

Carbon dioxide contributes most to the overall greenhouse effect. Methane and nitrous oxide absorb infrared radiation more strongly than carbon dioxide, but their concentrations in the atmosphere are much lower.

Top tip!

Water vapour is by far the most abundant of the greenhouse gases. However, for a variety of reasons, scientists do not believe that any future *changes* in water vapour concentrations in the atmosphere will significantly affect the Earth's climate.

Molecules of the greenhouse gases absorb some of the thermal radiation (infrared) emitted by the Earth's surface. Without these gases, the radiation would continue to travel away from the planet. The molecules will re-radiate the same energy a short time later, but in random directions, which means that about half is directed back towards the Earth's surface, keeping it warmer than it would be without the greenhouse gases.

LINKING QUESTIONS

- What relevance do simple harmonic motion and resonance have to climate change?
- What limitations are there in using a resonance model to explain the greenhouse effect?

These questions link to understandings in Topics C.1 and C.4.

We will explain the absorption of infrared radiation by molecules of greenhouse gases by using carbon dioxide, CO_2, as an example. See Figure B2.11.

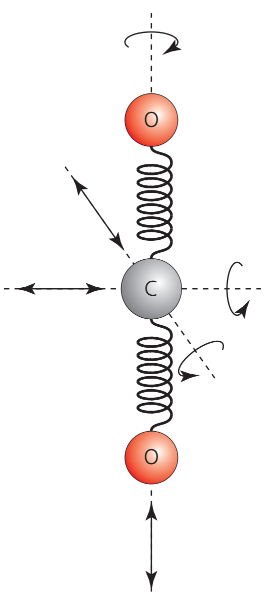


Figure B2.11 A few possible molecular vibrations in carbon dioxide

Molecules of greenhouse gases absorb infrared radiation because the atoms within their molecule are not at rest – they vibrate at high frequencies. (See Topic C.1: they oscillate with *simple harmonic motion*, like masses connected by springs.) Figure B2.11 shows a simplified example of possible ways in which a carbon dioxide molecule can vibrate.

If the atoms in a molecule vibrate at the same frequency as the infrared radiation passing through the greenhouse gas, then energy can be absorbed (an example of an effect known as *resonance* – see Topic C.4), raising the molecule to a higher energy level. The energy is quickly released again as the molecule returns to its lower energy level, but the released energy is radiated in random directions.

Since most of the radiation from the Sun is at higher frequencies it is much less likely to be absorbed than the mostly lower frequencies of radiation emitted from the cooler surface of the Earth.

As we have said before, the greenhouse effect, as described above, is essential for life on Earth. Next, we will discuss how the situation is changing.

12 Write a word equation representing the energy balance of a greenhouse at constant temperature. Assume that the inside is hotter than the outside and that there are no open windows.

13 Consider Figure B2.9. Outline the reasons for the decreases of intensity at some wavelengths (because of water vapour in the atmosphere).

14 One natural frequency of vibration of a carbon dioxide molecule is 2.0×10^{13} Hz.
- a Determine the wavelength of thermal radiation which this molecule will absorb.
- b Explain why carbon dioxide can absorb radiation from the Earth but not from the Sun.

15 Use data from Figure B2.10 to show that it is representing an atmosphere in thermal equilibrium.

16 Many people are concerned about the increasing levels of methane in the atmosphere. Use the internet to research the reasons for these concerns.

◆ **Global warming** Increasing average temperatures of the Earth's surface, atmosphere and oceans.

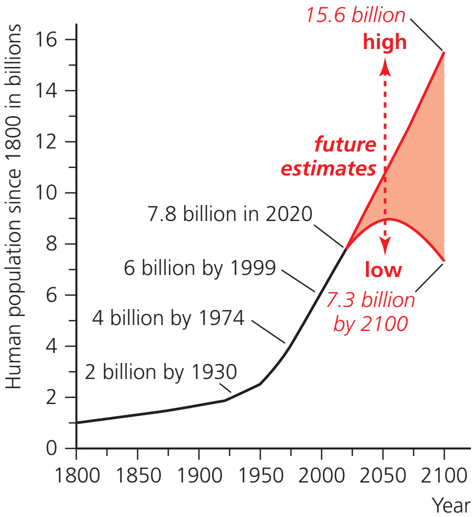
◆ **Combustion (of fuels)** Burning. Release of thermal energy from a chemical reaction between the fuel and oxygen in the air.

◆ **Natural gas** Naturally occurring fossil fuel: mixture of gases (mainly methane).

Enhanced greenhouse effect: global warming

SYLLABUS CONTENT

- The augmentation of the greenhouse effect due to human activities is known as the enhanced greenhouse effect.



■ **Figure B2.12** Prediction of World population growth (*source of data: UN World Population Prospects 2019*)

The population of the Earth passed eight billion towards the end of 2022, having quadrupled in less than a century. See Figure B2.12.

Understandably, everyone wants easy, enjoyable lifestyles, to be well fed, to be protected from heat and cold, to travel ... and so much more. All of these human activities involve transfers of energy. That energy has to be provided from somewhere and, after our activities, the energy is mostly dissipated into the surroundings and cannot be recovered.

For the last 300 years, we have been using the vast store of chemical potential energy available from the **combustion** (burning) of coal, oil and **natural gas** to power various types of machines and engines. These energy sources are called **fossil fuels** because they are made over the course of millions of years under the surface of the ground by the decomposition of once living material in the absence of oxygen. The enormous benefits to society from the use of fossil fuels (for example in the generation of electricity and in transport) are undeniable.

◆ **Fossil fuels** (A **fuel** is a store of chemical or nuclear energy that can be used to do useful work.) Naturally occurring fuels that have been produced by the effects of high pressure and temperature on dead organisms (in the absence of oxygen) over a period of millions of years. Coal, oil and natural gas are all fossil fuels.

However, we have become increasingly aware of the considerable disadvantages of the continued use of fossil fuels. Most notably, the release of increased amounts of greenhouse gases (principally carbon dioxide) into the atmosphere is responsible for **global warming**.

Increased concentrations of greenhouse gases in the atmosphere results in more of the thermal energy that is radiated away from the Earth's surface being absorbed in the atmosphere, some of which is re-radiated randomly back to the surface, increasing its average temperature.

This is known as the enhanced greenhouse effect. (Enhanced means increased.) It is currently believed that the enhanced greenhouse effect has resulted in increasing the average temperature of the Earth's surface by just over 1 °C during the last 60 years. Further rises are considered to be inevitable.

◆ Non-renewable energy sources Energy sources that take a very long time to form and which are being rapidly used up (*depleted*). Oil, natural gas and coal.

The disadvantages of using fossil fuels have been well understood and discussed among scientists for well over 50 years. To begin with, attention was mainly on the polluting effects on their immediate environments and when accidents occurred, and the fact that they were **non-renewable sources**: there was a limited supply (some predictions were made that supplies would be running low by now). In more recent times, the world's attention has shifted to their effect on the global climate.

The burning of fossil fuels is almost certainly the greatest cause of the enhanced greenhouse effect.

With enormous quantities of relevant data available and the use of super-computers, nearly all scientists agree with the last two highlighted statements and their consequence: global warming. Curiously, some of the general public and some politicians have been less easy to convince. It is difficult to understand why.

Nature of science: Models

Simple and complex

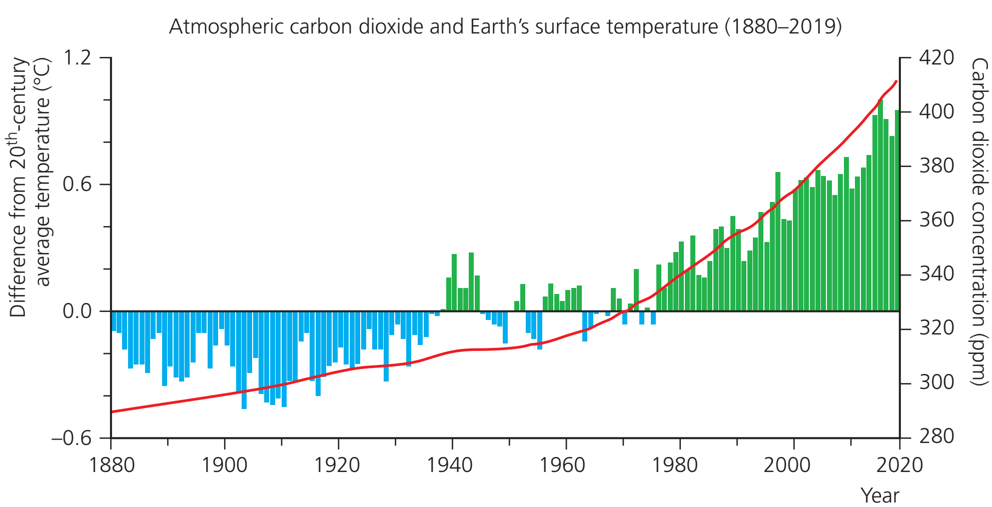
The kinetic theory of gases is a model that can be applied successfully to relatively small amounts of gases in closed containers. The Earth's atmosphere is also a gaseous system, but one which is vastly larger and more complex. Meteorologists use computer modelling to predict the weather in a particular location with reasonable accuracy up to about 10 days in advance. But predicting the climate of an entire planet with any certainty for many years in the future is a near impossible task. However, climate modelling is a problem which has understandably attracted an enormous amount of scientific attention in recent years and, with the availability of better data and faster processing, together with international **collaboration**, long-term **climate models** are believed to have become more consistent and reliable. We will have to wait to see how accurate they are.

◆ Collaboration (scientific) Two or more scientists sharing information or working together on the same project.

◆ Climate model A complex computerized model that attempts to predict the future climate of the planet, especially how it will be affected by global warming.

◆ Correlation There is a correlation between two sets of varying data if they show similarities that would not be expected to occur because of chance alone.

Figure B2.13 compares the levels of carbon dioxide in the atmosphere (in parts per million, ppm) with the Earth's average surface temperature, which is represented by the difference of the annual average with the average over the hundred years of the twentieth century. The **correlation** is easy to see, and it is believed by scientists that increased carbon dioxide levels caused the increasing temperatures, although the data in this chart is not conclusive by itself. Variations in carbon dioxide levels correlate well with variations in temperature over tens of thousands of years, long before humans began burning fossil fuels in vast quantities. See Figure B2.14. However, one important aspect of Figure B2.13 is the unusually short timescales involved.



■ **Figure B2.13** Correlating the concentration of carbon dioxide in the atmosphere with the Earth's temperature (*source of data: NOAA Climate.gov; ESRL / ETHZ / NCEI*)

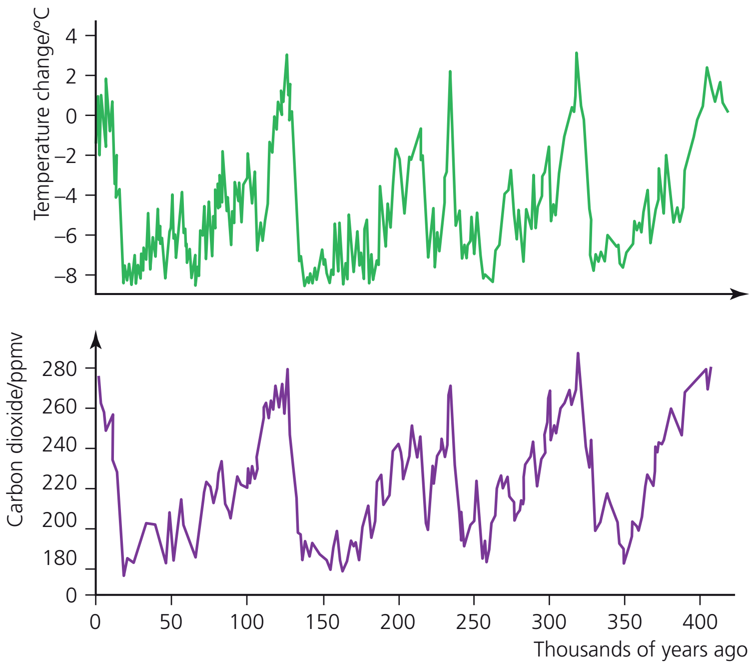


Figure B2.14 There is a strong correlation between average global temperatures and the amount of carbon dioxide in the atmosphere (ppmv = parts per million by volume)

The correlation between increasing concentrations of greenhouse gases and rising global temperatures is well established and accepted by (almost) everybody. However, that does not mean that we can be 100% sure that global warming is caused by the release of more greenhouse gases. Obviously, controlled experiments to test such a theory cannot be carried out and we must rely on statistical evidence, computer modelling and scientific reasoning. In such cases, 100% certainty is never possible and individuals and societies must make informed judgements based on the best possible scientific evidence. Of course, some people will always choose to disagree with, or ignore, the opinions of the majority.

Tool 3: Mathematics

Determine the effect of changes to variables on other variables in a relationship

When the similarities between two sets of data are as clear as that seen in Figures B2.13 and B2.14, we describe it as a *correlation*. Without any further evidence, it is easy to believe that one effect (A) caused the other (B), which we would describe as *cause and effect*. However, it may be possible that effect A was caused by effect B, or maybe they are inter-dependent in some way. Two other possibilities need to be considered: both effects are a consequence of another cause (C), or maybe the correlation is just an unlikely random phenomenon, which has no known explanation.

It is also possible that A did cause B, but only because there was some other effect involved, that has not been identified. Greater certainty can only be gained by gathering further data.

Consequences of the enhanced greenhouse effect

These are well documented elsewhere and there is no intention to go into detail here. These interconnected consequences, which are expected to get worse, are mostly detrimental to the lives of people and animals on the Earth, although some places will be more affected than others, and richer countries will be better able to cope with the changes.

- Increasing temperatures of the oceans, land and atmosphere
- Climate change, melting snow and ice
- More frequent extreme weather conditions (storms, floods, drought, fires and so on)
- Rising sea levels with increased acid levels.

Figure B2.15 Wildfires in Greece 2021

Figure B2.16 Flooding in Germany 2021

Common mistake

Many people wrongly believe that increasing global temperatures is a consequence of the basic greenhouse effect. More correctly, we should say that human activities are changing global temperatures because of an *enhanced* greenhouse effect.

TOK

Knowledge and the knower

- What criteria can we use to distinguish between knowledge, belief and opinion? Is the truth what the majority of people accept?
- Can probability become certainty?

The four bullet points listed above are empirical facts, based on very extensive measurements. But do we know *for certain* that they are a consequence of burning fossil fuels? Clearly, the vast majority of scientists and the general public now believe so. Ten or twenty years ago more people were doubtful, but the increasing weight of evidence is convincing.

At what point does something like this become accepted knowledge, or will there always be some uncertainty? It seems likely that, under any circumstances, some people will continue to believe that climate change is unconnected to burning fossil fuels.

What are the possible solutions to climate change?

There is a wide range of actions that individuals and governments can take. These include:

- Support governments who will take appropriate and prompt action. (Including an *increasing use of renewable energy sources* – see below, discouraging energy-intense activities and investing in scientific research and development, for example, into 'carbon capture').
- Accept higher taxation as a method of discouraging energy intense activities and the use of fossil fuels.
- Invest in renewable energy sources for individual homes, such as solar panels on the roof.
- Use energy-efficient devices and use them less frequently.
- Improve home insulation against both hot and cold weather. See Figure B2.17.
- In colder countries: reduce the temperatures of homes and the hot water used in showers and washing machines.
- In hotter countries: increase the temperature settings on air-conditioners.

■ **Figure B2.17** Improving insulation under the roof

- Waste less (of everything, but especially food). Re-use items and resist the temptation to keep buying new things.
- Reduce methane emissions.
- Many populations could eat less, especially meat.
- Make public transport cheaper and more plentiful.
- Support businesses which are taking action against climate change.
- Use electric vehicles. Use smaller, fewer and less powerful vehicles.
- Limit travel (especially for pleasure).
- Recycle.
- Change lifestyles and expectations.
- Reforestation.

LINKING QUESTION

- How do different methods of electricity production affect the energy balance of the atmosphere?

This question links to understandings in Topics B.2 and D.4.

ATL B2B: Thinking skills

Evaluating and defending ethical positions

Do people in rich and developed countries have any ethical authority to complain about the way in which fossils fuels are used in poorer countries?

Do we have individual responsibilities to take action against climate change? Is it acceptable to do nothing, or assume that the government will act on our behalf?

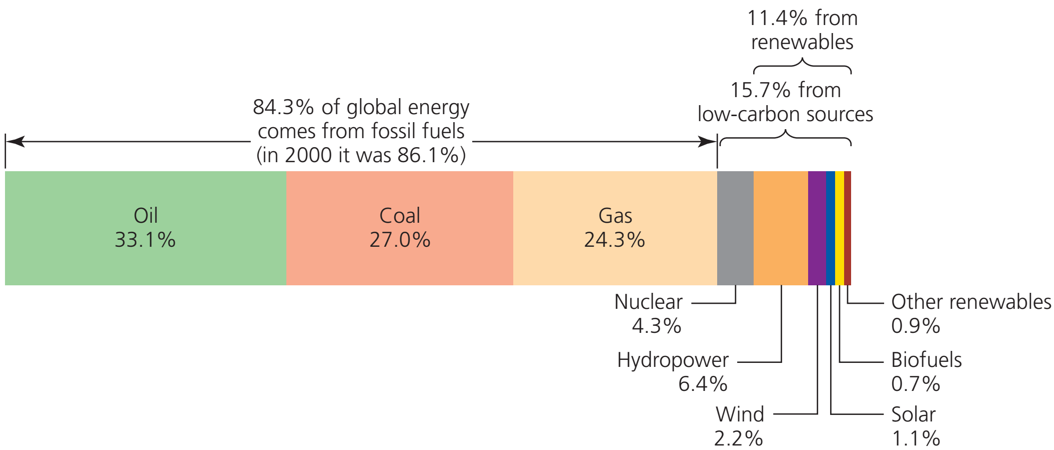
Would you be happy if the country where you lived took strong measures to try to limit climate change, while other countries did much less, or nothing?

◆ **Renewable energy** Energy from sources that will continue to be available for our use for a very long time. They cannot be used up (depleted), except in billions of years, when the Sun reaches the end of its lifetime.

World use of energy resources

Using more **renewable energy** sources should reduce our dependence on fossil fuels. As their name suggests, renewable energy sources will continue to be available to us in the future, because they are continuously being renewed by the energy arriving from the Sun. Their other major advantage is that they do not contribute to climate change.

Figure B2.18 shows recent information (2020) about the world's energy resources. Other sources of information will show some variations from these figures, but the trends are generally agreed.



■ **Figure B2.18** BP's 2020 Statistical Review of World Energy (*source of data: BP Statistical Review of World Energy*)

Figure B2.19 shows how the uses of various energy sources has changed over the last 120 years. (1 TWh = 3.6×10^{15} J). There are justified high hopes for the continued rapid increases in the use of wind power and solar power, but currently they still amount to less than 4% of our global needs.

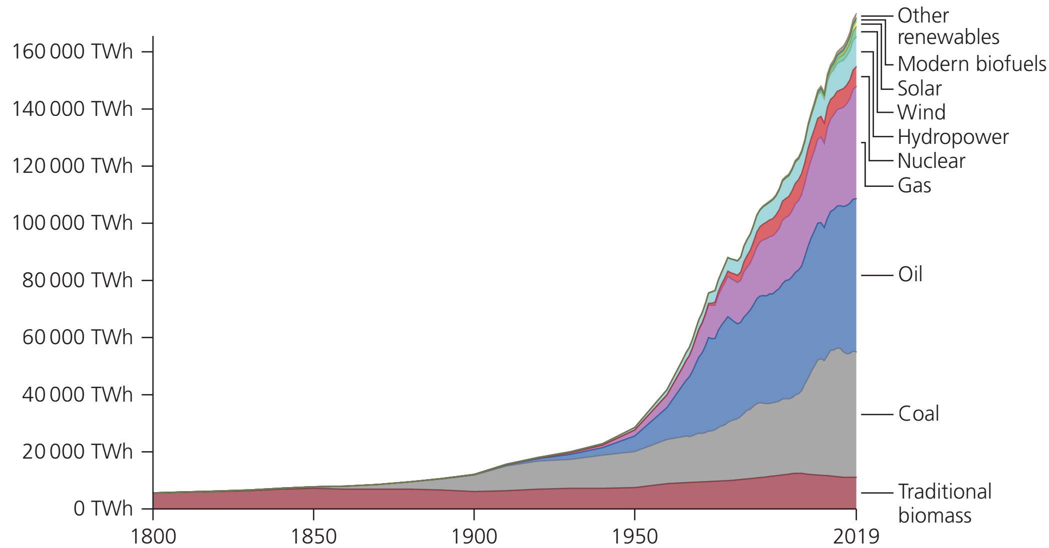


Figure B2.19 Changes in energy consumption over 120 years (*source of data: Vaclav Smil (2017) and BP Statistical Review of World Energy*)

Perhaps the most important information that can be seen in Figure B2.19 is the increase in the planet's *overall* demand for energy. Although the Covid pandemic has had a temporary effect in reducing demand, the overall trend is likely to continue upwards because the populations of poorer countries will understandably hope to match the living standards of those in richer countries.

Most of the energy for this increased demand has come from fossil fuels. Over the last 20 years, fossil fuels have continued to supply about 85% of the world's increasing energy needs. This means that we are burning about 50% more fossil fuels now than in the year 2000. All this is despite the demand for a greater use of renewable energy sources. Fossil fuels remain plentiful and relatively cheap, millions of people around the world are employed in the fossil-fuel businesses, and we already have the systems and infrastructures in place to continue their use.

ATL B2C: Communication skills

Reflecting on the needs of the audience

Prepare a 5 minute presentation on the enhanced greenhouse effect which could be understood by 10–12 year-old students. What simplifications will you need to make? How can you present the scientific information in an engaging and accessible way?

17 Use Figure B2.13 to estimate the percentage by which the concentration of carbon dioxide in the atmosphere increased in the 50 years between 1970 and 2020.

18 Sketch Sankey diagrams which show all the principal energy transfers that have resulted in:

- **a** a wind generator producing an electric current
- **b** an oil-fired power station producing an electric current.

19 Explain why hydroelectric power is considered to be a renewable energy source.

Figure B2.20 What a 2 °C rise in ocean temperature could do to the Hard Rock 2020 Super Bowl Stadium at Miami Beach, Florida

20 Use the internet to determine
- a two reasons why increasing temperatures result in increasing sea levels
- b the latest predictions for future sea levels.

21 a Discuss whether nuclear energy is renewable or non-renewable.
- b Figure B2.21 shows how use of nuclear power has changed since the 1960s. Suggest some reasons why it is not used more widely.
- c State what major incidents happened in 1986 and 2011 which affected people's opinions about nuclear power.

22 Use the internet to find the primary sources of energy used to generate electricity in the country where you live.

23 Would you like the government of the country where you live to take more action in an attempt to limit climate change? If not, why not? If yes, what changes would you recommend?

24 Use the internet to gain information and data about the risks associated with the generation of electricity from non-nuclear sources.

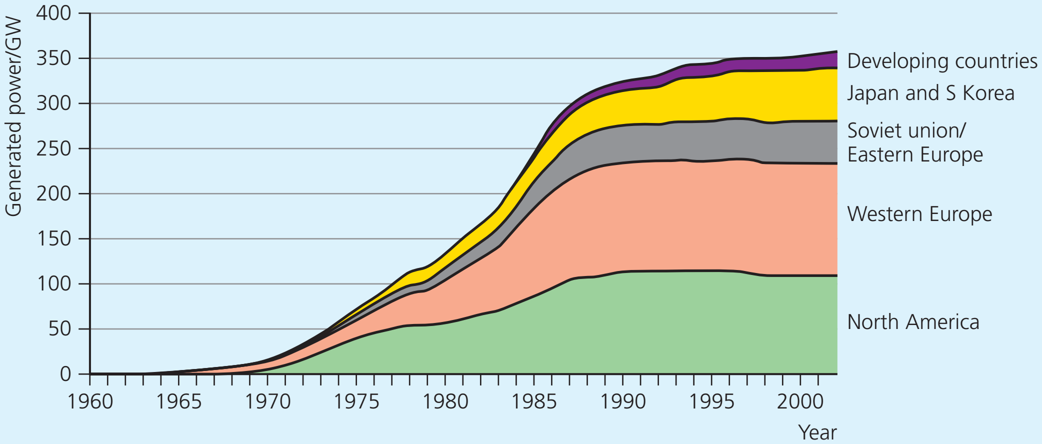


Figure B2.21 Use of nuclear power

Nature of science: Global impact of science

United Nations Climate Change Conferences

After years of relative inaction, at the time of writing, the urgent need for significant and widespread action on climate change finally seems to have become widely accepted, especially among younger people.

These important annual COP meetings involve detailed discussions about how the effects of climate change could be reduced. They involve scientists and representatives from the governments of most of the countries of the world.

Undoubtedly, important agreements are reached. Research online to find out what these were at the latest COP meeting.

However, there are many voices raised in opposition: Did the proposed changes go far enough? Was it all just talk? Is keeping temperature rises to 1.5 °C already impossible? Why were some important countries apparently not fully involved? Will countries do in the future what they have promised to do? We will see.

Figure B2.22 Activists at COP 26

B.3 Gas laws

Guiding questions

- How are macroscopic characteristics of a gas related to the behaviour of individual molecules?
- What assumptions and observations lead to universal gas laws?
- How can models be used to help explain observed phenomena?

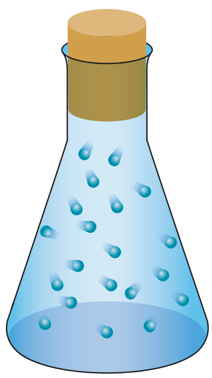


Figure B3.1 Model of gas molecules in a container

◆ **Gas laws** Laws of physics relating the temperature, volume and pressure of a fixed amount of a gas: Boyle's law, Charles' law and the pressure law.

◆ **Pressure, *P*** Force acting normally per unit area: pressure = force / area (SI unit: pascal, Pa). $1\,\text{Pa} = 1\,\text{N}\,\text{m}^{-2}$.

◆ **Amount of gas** The quantity of gas in a container, expressed in term of the number of particles it contains.

At the beginning of Topic B.1 we introduced the kinetic theory of matter and explained that all gases contain particles (usually molecules) moving in random directions, usually at high speeds, as represented in Figure B3.1.

Because we can usually assume that there are no forces acting between the molecules (except in collisions), the physical properties of gases are much easier to study and understand than solids and liquids. As we will see, using an idealized model of the motion of molecules in a gas, we can use knowledge of dynamics (from Topic A.2) to predict the physical behaviour of gases. It will be important to understand this link between microscopic motions of molecules and the macroscopic properties of gases.

Under most conditions, all gases show similar physical behaviour. The (universal) **gas laws** is the name that scientists give to the straightforward relationships that describe the physical behaviour of all gases.

We can determine the following four physical properties of any gas in any container:

- volume, V
- temperature, T
- **pressure**, P (explained below)
- **amount of gas**, n, in terms of the number of gas molecules it contains (explained below).

In the kinetic theory of gases, the number of molecules in a gas is usually more useful information than their overall mass. The *density* of a gas may also be of interest but can be determined from mass / volume.

Before looking at the physical properties of gases, we need to explain more about two of the four properties in the above list: pressure and amount of a substance.

Pressure

SYLLABUS CONTENT

▶ Pressure, $P = \dfrac{F}{A}$ where F is the force exerted perpendicular to the surface.

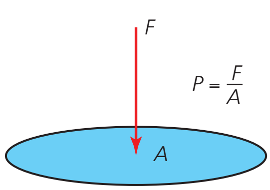


Figure B3.2 Calculation of pressure from force and area

Generally, the effect of a force, F, often depends on the area, A, on which it acts. For example, when the weight of a solid pushes down on a surface, the consequences usually depend on the area underneath it, as well as the magnitude of the weight (see Figure B3.2).

Pressure is defined as perpendicular (normal) force per unit area: $\boldsymbol{P = \dfrac{F}{A}}$

SI unit: **pascal**, Pa. $1\,\text{Pa} = 1\,\text{N}\,\text{m}^{-2}$

◆ **Pascal** Derived SI unit for pressure. $1\,\text{Pa} = 1\,\text{N}\,\text{m}^{-2}$.

The concept of pressure is needed to explain, for example, why one finger in Figure B3.3 will be less comfortable than the other. From Newton's third law, we know that the forces on both fingers are the same.

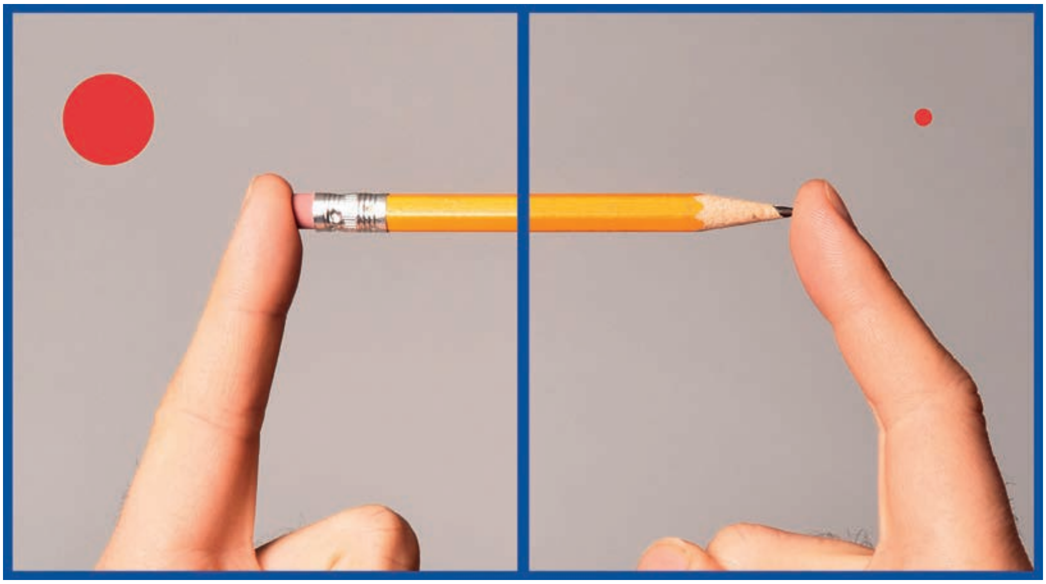


Figure B3.3 Same force, different pressure

WORKED EXAMPLE B3.1

A girl of mass 51 kg stands on one leg. If the effective area of her foot in contact with the ground is 62 cm², calculate the pressure she is exerting on the ground.

Answer

$$P = \frac{F}{A} = \frac{(51 \times 9.8)}{62 \times 10^{-4}} = 8.1 \times 10^4\,\text{Pa}$$

WORKED EXAMPLE B3.2

Figure B3.4 shows some water coming out of a container with three holes. This is often shown to students as a demonstration about pressure in liquids.

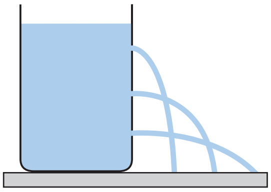


Figure B3.4 Pressure with depth apparatus

a What conclusion should students reach after watching this demonstration?

b If the depth of the water is 26 cm and the area of the bottom of the container is 84 cm², calculate:

i the volume of water in the container

ii the mass of water (density = 1000 kg m^{-3})

iii the weight of the water.

c Determine the pressure of the water at the bottom of the container. (Ignore the pressure due to the air above the water.)

Answer

a The pressure in the water increases with depth.

b **i** volume = depth × area = $0.26 \times (84 \times 10^{-4}) = 2.2 \times 10^{-3}\,\text{m}^3$ (2.184×10^{-3} seen on calculator display)

ii mass = volume × density = $2.184 \times 10^{-3} \times 1000 = 2.2$ kg (2.184 seen on calculator display)

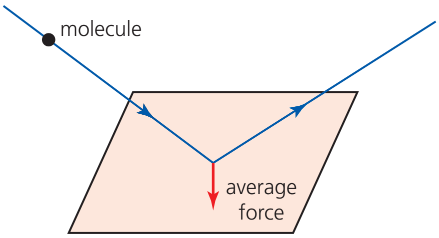
iii weight = $mg = 2.184 \times 9.8 = 21$ N (21.403... seen on calculator display)

c $P = \frac{F}{A} = \frac{21.403...}{84 \times 10^{-4}} = 2.5 \times 10^3\,\text{Pa}$

To determine the *total* pressure at the bottom of the container we would need to add the pressure due to the water to the gas pressure of the air above the water surface (1.0×10^5 Pa).

Pressure in gases

Our main concern in this topic is the pressure created by a gas in a container, but we explain gas pressure in a different way to the pressure under solids and liquids. Consider Figure B3.5.



■ **Figure B3.5** Every molecular collision with a wall creates a tiny force on the wall

Each collision of a molecule with the containing walls creates a tiny outwards force. The average forces are perpendicular (normal) to the surfaces. Because the molecular motions are random, each collision can result in a different sized force, but the enormous number of molecules colliding every second with a small area means that the average force / area will stay the same. That is, the pressure is constant. Note that molecules also collide with other molecules, but this simply re-directs their motion and has no other overall effect.

Note that gas pressure acts in *all* directions, not just downwards. This is also true for the pressure in liquids.

◆ **Atmospheric pressure** Pressure due to the motions of the gas molecules in the air. Can be considered as being due to the weight of the air above an area of 1 m^2. Acts equally in all directions.

The pressure of the air around us (**atmospheric pressure**) is 1.0×10^5 Pa at sea level. It decreases with height above the ground.

Amount of a substance

SYLLABUS CONTENT

▶ The amount of substance, n, as given by: $\boldsymbol{n} = \frac{\boldsymbol{N}}{\boldsymbol{N_A}}$, where N is the number of molecules and N_A is the Avogadro constant.

The **amount of a substance** (symbol: n) is a measure of the number of atomic-scale particles it contains. The nature of the 'particles' depends on the substance being considered. For example, the gas helium, He, consists of separate atoms, but most other gases are molecular. Examples include H_2, O_2, CO_2, CH_4 and N_2.

Even very small amounts of gas can contain an enormously large number of particles ($\approx 10^{19}$ or more). The SI unit of amount of a substance is more manageable. It is called the **mole** (mol):

One mole is the amount of a substance that contains exactly $6.022\,140\,76 \times 10^{23}$ of its particles. This number is known as the **Avogadro constant**. It is given the symbol $\boldsymbol{N_A}$. For most calculations we can use a value of $\mathbf{6.02 \times 10^{23}}$.

◆ **Amount of substance, *n*** Measure of the number of atomic-scale particles (atoms or molecules) it contains (SI unit: mole).

◆ **Mole, mol** SI unit of amount of substance (fundamental).

◆ **Avogadro constant, N_A** The number of particles in 1 mole of a substance: 6.02×10^{23}.

Top tip!

Since 2018, the mole and the Avogadro constant have been defined only by the number shown above. Previously, the Avogadro constant was defined with reference to a standard substance: carbon. N_A was the same number of particles as there are atoms in exactly 12 g of the isotope carbon-12. (Isotopes are explained in Topic E.3.) This link has now been removed from the definition, but the number is still the same.

Figure B3.6 shows one mole of various substances. Figure B3.7 shows a ball which contains about 0.25 moles of air at a pressure of about 9×10^4 Pa.

Figure B3.6 One mole of water (in the form of ice), sugar, copper and aluminium

Figure B3.7 A football

If we know the number of particles in a substance, N, the number of moles, n, is determined by dividing by the Avogadro constant:

$$\text{amount of substance in moles} = \frac{\text{number of particles}}{\text{Avogadro constant}}$$

$$n = \frac{N}{N_A}$$

◆ **Molar mass** The mass of a substance that contains 1 mole of its defining particles.

The **molar mass** of a substance is the mass which contains one mole. Usual unit: g mol^{-1}. Table B3.1 shows some common molar masses.

Table B3.1 Molar masses

Substance	Molar mass / g mol^{-1}
hydrogen molecules	2.02
helium atoms	4.00
carbon-12 atoms	12.00
carbon atoms	12.01
water molecules	18.01
aluminium atoms	26.98
nitrogen molecules	28.02
oxygen molecules	32.00
carbon dioxide molecules	44.01
gold atoms	197.00

Top tip!

The molar mass of a substance depends on the number of particles (protons and neutrons) in the nucleus of each atom or molecule. (Details are provided in Topic E.3.) More massive atoms have more protons and neutrons, so that a mole of their atoms will have a greater mass. For example, carbon atoms usually have 12 particles, carbon dioxide molecules usually have 44 particles and hydrogen molecules usually have two particles.

A numerical example: an oxygen molecule has two atoms, each has 16 particles, each with a mass of 1.67×10^{-27} kg.

Total mass of one mole (6.02×10^{23} particles) $= 6.02 \times 10^{23} \times 2 \times 16 \times 1.67 \times 10^{-27} = 3.2 \times 10^{-2}$ kg (32 g)

WORKED EXAMPLE B3.3

a How many moles are there in 0.50 kg of molecular oxygen gas?

b How many molecules are there in 0.50 kg of the same gas?

Answer

a $\frac{\text{total mass}}{\text{molar mass}} = \frac{0.50}{32 \times 10^{-3}} = 16\,\text{mol}$ (15.625 seen on calculator display)

b $N = nN_A = 15.625 \times (6.02 \times 10^{23}) = 9.4 \times 10^{24}$ molecules

1 A car of weight 1500 kg has four tyres, each of which has an area of 180 cm^2 in contact with the road.

- **a** Calculate the pressure under each tyre (in Pa).
- **b** State the pressure of the air inside the tyre.
- **c** If the driver puts more air into the tyre, what will happen to:
  - **i** the area in contact with the ground
  - **ii** the pressure on the road?

2 The air pressure at a height of 10 km above sea level is 2.6×10^4 Pa. Inside a passenger aircraft at this height, the air pressure is maintained at 80% of the pressure at sea level (1.0×10^5 Pa).

- **a** What is the difference in pressure between inside and outside the aircraft?
- **b** What resultant force due to the air is acting on a window which is 27 × 47 cm?

Figure B3.8 Aircraft window

3 **a** Using the concept of pressure, explain how it is possible for the water to remain in the upside-down glass shown in Figure B3.9. (Assume that the glass is full of water.)

- **b** If the glass is only half full of water to begin with, how will the demonstration change?

Figure B3.9 Water in an upside-down glass

4 Consider the ball shown in Figure B3.7.

- **a** If the molar mass of the air in the ball is approximately 29 g mol^{-1}, what is the mass of the air in the ball?
- **b** Compare the pressure of the air inside the ball to the pressure of the air outside the ball.

5 **a** Estimate the volume of the water seen in the glass in Figure B3.9.

- **b** Water has a density of 1000 kg m^{-3}. Using your answer to part **a**, estimate:
  - **i** the mass of water in the glass
  - **ii** the number of moles of water in the glass
  - **iii** the number of water molecules in the glass.

6 The thickness of a car tyre decreased by 5.0 mm over a distance of 30 000 km. The circumference of the tyre was 1.9 m and its surface area was 3000 cm^2.

■ **Figure B3.10** Car tyre

a Calculate the volume of the rubber in the tyre which was worn away in this distance.

b Assuming the density of the rubber was 950 kg m^{-3}, what mass was worn away?

c If about 90% of the mass in rubber is due to carbon atoms, estimate:

i the number of moles of carbon spread into the environment in travelling each 1000 km

ii the number of carbon atoms lost from the tyre in each rotation of the wheel.

7 Nitrogen is often put inside potato crisps / chips packets. This is to keep the crisps fresh and limit damage to them. Estimate the amount of nitrogen in a typical packet (see Figure B3.11). Assume the density of the nitrogen is 1.4 kg m^{-3}.

■ **Figure B3.11** Nitrogen was used inside the pack to keep these crisps / chips fresh.

8 Most iron atoms contain a total of 54 protons and neutrons, each with a mass of 1.67 × 10^{-27} kg. Determine a value for the molar mass of iron.

Investigating the physical properties of gases

SYLLABUS CONTENT

- The empirical gas laws for constant pressure, constant volume and constant temperature.

As we have already noted, given a closed container with a gas sealed inside, there are four physical properties of the gas which can be easily measured: pressure, volume, temperature and mass (P, V, T and m). The amount of gas in a mole, n, can be calculated from its mass, as explained above.

All gases exhibit the same physical properties (under most conditions), so that any gas, or mixture of gases, can be used in the following experiments, and the same conclusions should be reached. Air is the obvious and easy choice.

The following are three classic investigations of the ‘gas laws’. Each keeps the amount of gas and one other variable constant:

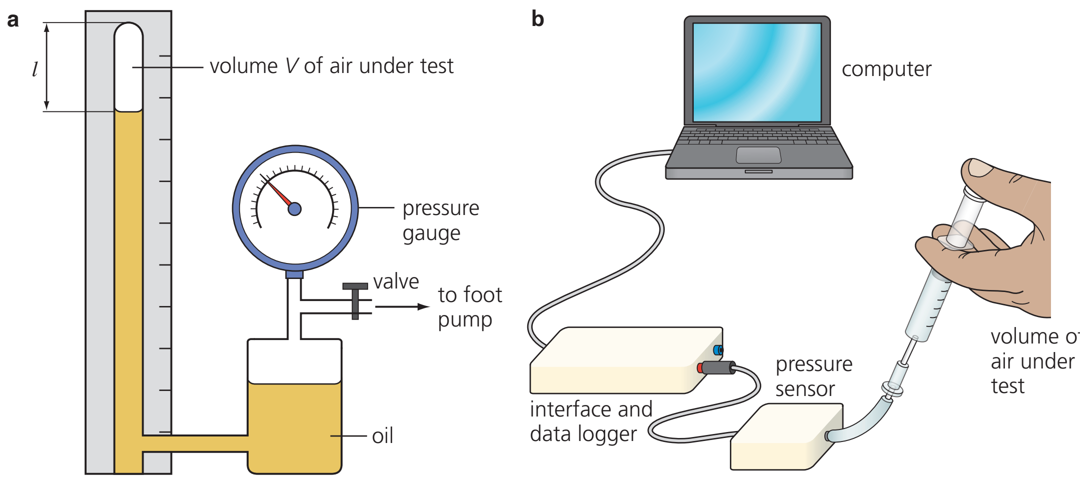
- How does the volume of a gas change with pressure (at constant temperature)?
- How does the volume of a gas change with temperature (at constant pressure)?
- How does the pressure of a gas change with temperature (at constant volume)?

We will look at each of these in more detail.

Variation of gas volume with pressure at constant temperature (Boyle’s law)

Figure B3.12 shows two sets of apparatus that could be used to investigate how changing the pressure on a fixed mass of gas affects its volume, at a constant temperature. However, note that using a force to do work and change the volume of a gas will tend to change its temperature, which could complicate the results. To minimize this unwanted effect, the changes should be made slowly.

■ **Figure B3.12** Investigating Boyle's law



Tool 1: Experimental techniques

Recognize and address relevant safety, ethical or environmental issues in an investigation

The safety of you (and your teachers) in a science laboratory is an important concern. However, it is not a major issue if your behaviour is appropriate. Most schools have a list of behaviours expected of students in a laboratory. The few accidents which occur are usually because of inattention or carelessness.

Experimental situations requiring particular attention include the following:

- **Using electrical equipment which is connected to the mains supply (110–230 V).** Such equipment should have regular safety checks and never be used close to water. Bare wires connected to the mains should not be used for experimentation. The laboratory should have appropriate **circuit breakers** and an RCCB which can cut off the electrical supply very quickly and protect lives.
- **High voltage supplies** (for example, 5000 V) may be needed for a few teacher demonstrations, but they are provided with protection due to the very high internal resistance included deliberately in their design.
- **Electrostatic high voltage generators** always make interesting and dramatic demonstrations. They are very safe to use, but people with serious medical conditions are usually advised not to get involved.
- **Hazardous chemicals** are not usually used in physics experiments.
- **Equipment made from glass** always needs to be treated carefully to avoid breakage. Goggles should be worn.
- **Containers with a gas at high or low pressure** should have a shield around them for protection in the unlikely event of an explosion or implosion.
- **Radioactive sources** will usually be used only by a teacher, although some schools allow older students to use the sources under close supervision. They should be labelled and stored securely and used for as short a time as possible. Students should not be too close to the experiments and the sources should be shielded and never directed towards students.
- **The light from a laser** (or other very intense light source) should not be allowed to fall on anyone's eyes.
- **High temperatures** are required for a few experiments. Appropriate care is needed, especially if very hot water is involved.

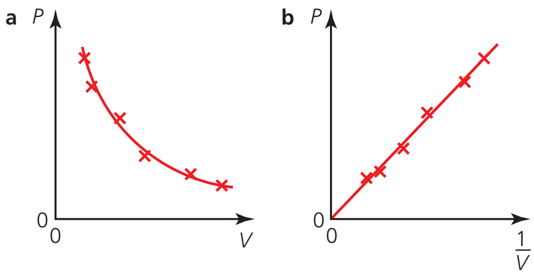
The gas pressure in the apparatus seen in Figure B3.12 can be high enough that there is small possibility that a glass tube could explode. It should be surrounded by a clear plastic protective screen.

◆ **Circuit breaker** Electromagnetic device used to disconnect an electrical circuit in the event of a fault.

◆ **Isothermal** Occurring at constant temperature.

Figure B3.13 shows graphs of typical results. The lines are called **isotherms**, which means that all points are at the same temperature.

The graph shown in Figure B3.13b represents the same data as in Figure B3.13a, but the graph has been re-drawn to produce a straight line to show that the pressure and volume are inversely proportional to each other.



■ **Figure B3.13** Two graphs showing that gas pressure is inversely proportional to volume

For a fixed amount of gas at constant temperature: $P \propto \dfrac{I}{V}$

This is known as **Boyle's law**.

Boyle's law can be stated as PV = constant. If the pressure and/or volume of a fixed amount of gas are changed from initial values of P_1 and V_1 to final values of P_2 and V_2, then, provided that the temperature has not changed:

$$P_1V_1 = P_2V_2$$

◆ **Boyle's law** Pressure of a fixed amount of gas is inversely proportional to volume (at constant temperature).

We can explain this relationship in terms of molecular behaviour. If the volume of a container is reduced, the molecules (travelling with the same average speed) will collide with a given area of the walls more frequently. In other words, there will be more molecular collisions with each square centimetre every second, which will increase the gas pressure.

WORKED EXAMPLE B3.4

A sample of gas has a volume of 43 cm^3 when at normal atmospheric pressure (1.0×10^5 Pa).

a If the pressure on the same amount of gas is increased to 3.7×10^5 Pa, calculate its new volume.

b State the assumption you made in answering part **a**.

Answer

a $P_1V_1 = P_2V_2$

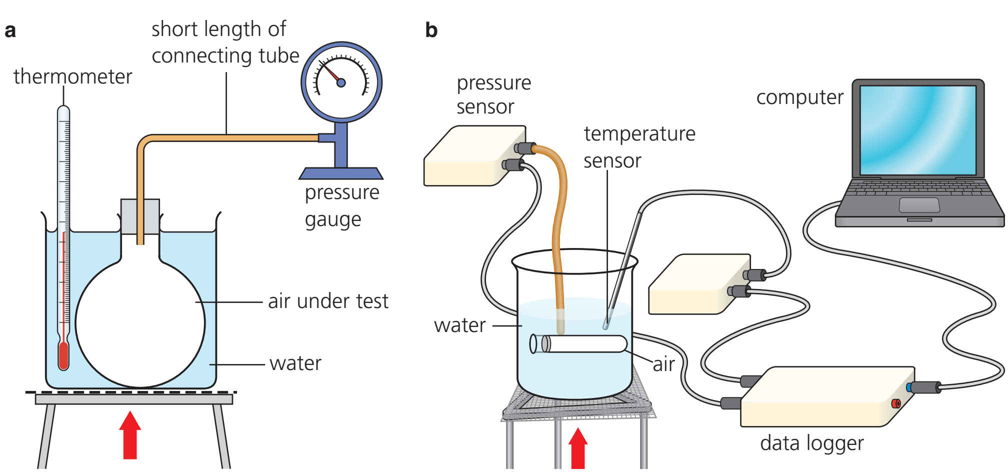
$(1.0 \times 10^5\ \text{Pa}) \times 43 = (3.7 \times 10^5) \times V_2$

$V_2 = 12\ \text{cm}^3$

b The temperature of the gas did not change.

Variation of gas pressure with temperature at constant volume (pressure law)

This relationship can be investigated with either of the two sets of apparatus shown in Figure B3.14. Measurements are usually taken for temperatures between 0 °C and 100 °C.



■ **Figure B3.14** Two sets of apparatus that can be used to investigate the pressure law

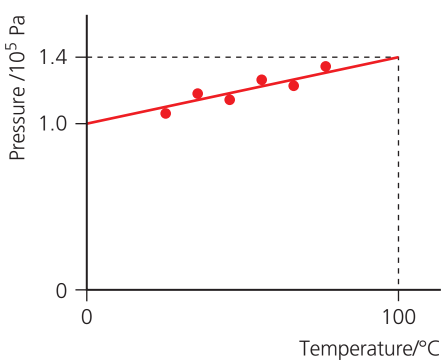
Tool 1: Experimental techniques

Recognize and address relevant safety, ethical or environmental issues in an investigation

There are several possible hazards when using the apparatus seen in Figure B3.14. There is the possibility that a larger beaker of very hot water could be knocked over, so the apparatus should not be near the edge of the table and nobody should be sitting nearby. There needs to be a good seal where the bung is pushed into the flask, but gloves should be worn when pushing them firmly together, in case the glass breaks.

Figure B3.15 shows a graph representing typical raw data.

The results represent a linear relationship between pressure and temperature in degrees Celsius.

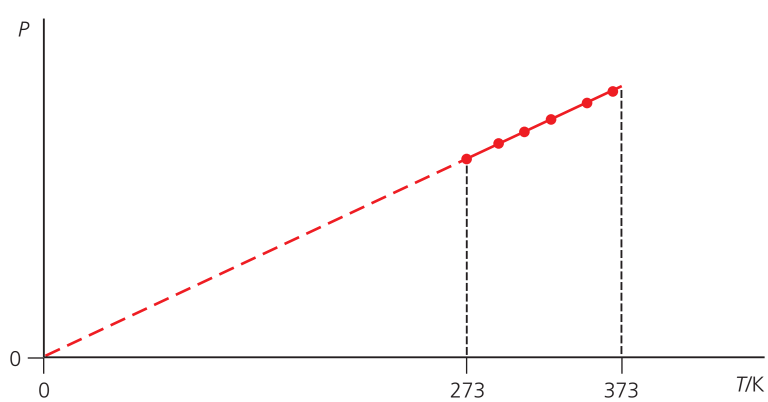


■ **Figure B3.15** Variation of gas pressure with temperature

We can explain this relationship in terms of molecular behaviour. If the temperature is reduced, the molecules move more slowly, so that they will collide with a given area of the walls less frequently, so that the pressure is reduced.

If the straight-line graph seen in Figure B3.15 is extrapolated to lower and lower temperatures and pressures, the temperature at which the pressure will be predicted to be zero is −273 °C. The same result is obtained with any gas. We can assume that this is the temperature at which (almost) all molecular motion has stopped, molecules are no longer colliding with the walls. −273 °C is called absolute zero, and it is the lower fixed point for the Kelvin temperature scale, as already explained in Topic B.1.

In practice, most gases will condense and then freeze at various low temperatures, but that does not change the concept of an absolute zero at −273 °C (0 K).



■ **Figure B3.16** Gas pressure is proportional to temperature in kelvin

Figure B3.16 shows the results of Figure B3.15 re-drawn. It represents a proportional relationship but remember that temperature must be measured in kelvin.

◆ **Pressure law** For a fixed mass of gas with a constant volume, the pressure is proportional to the kelvin temperature.

For a fixed amount of gas at constant volume: $P \propto T$

This is known as the **pressure law**.

If the pressure and/or temperature of a fixed amount of gas are changed from initial values of P_1 and T_1 to final values of P_2 and T_2, then, provided that the volume has not changed:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Common mistake

Remember that when making calculations involving temperatures (rather than temperature *changes*), you should always use kelvin.

Common mistake

It is widely stated that the volume of a gas increases when the temperature rises. But the volume of a gas will *only* increase if we allow it to, as in the next investigation.

Tool 3: Mathematics

Extrapolate and interpolate graphs

The considerable extrapolation seen in Figure B3.16 (and Figure B3.18) would not normally be recommended. However, in this case, we are not predicting the unknown behaviour of a gas all the way down to zero pressure. Rather, we are specifically asking the question: '*if* the gas continued to behave in this way, at what temperature would its pressure reduce to zero?'

WORKED EXAMPLE B3.5

Some air in a sealed container was heated from 60 °C to 92 °C. If its final pressure was 1.82×10^5 Pa, determine its pressure at 60 °C.

Answer

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

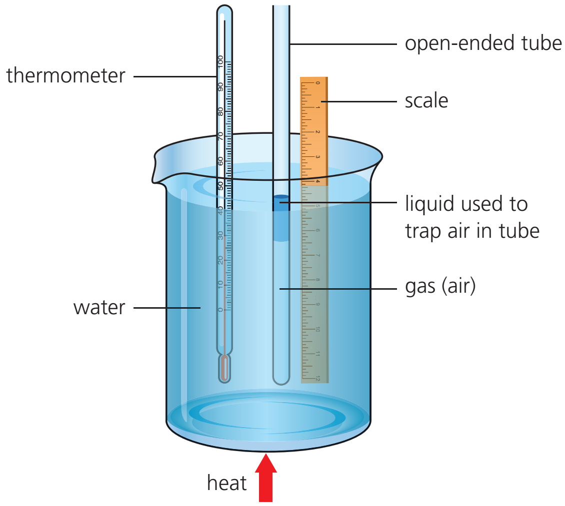
$$\frac{P_1}{(273 + 60)} = \frac{1.82 \times 10^5}{(273 + 92)}$$

$$P_1 = 1.7 \times 10^5 \text{ Pa}$$

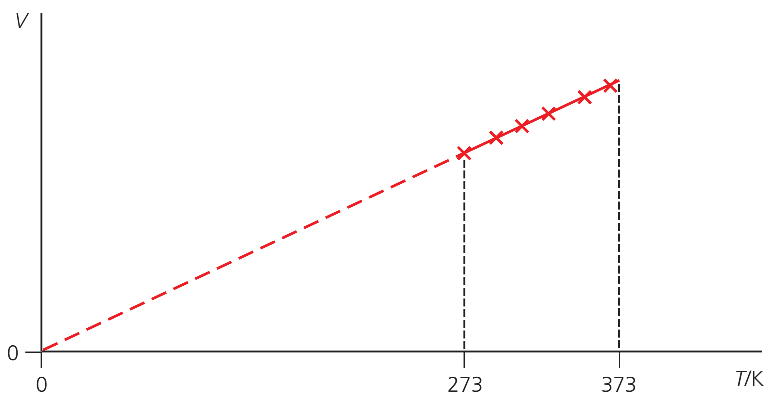
Variation of gas volume with temperature at constant pressure (Charles' law)

The apparatus seen in Figure B3.17 can be used for this investigation. When the gas is warmed by thermal energy conducted from the surrounding water, it expands along the tube, keeping the pressure in the gas equal to the pressure outside from the air in the atmosphere.

Figure B3.18 shows the concluding graph from this investigation. It is similar to the pressure–temperature graph.



■ **Figure B3.17** Simple apparatus for investigating Charles' law



■ **Figure B3.18** Gas volume is proportional to absolute temperature (K)

We can explain this relationship in terms of molecular behaviour. If the temperature of a gas is reduced, the molecules move more slowly, so that they would collide less frequently with the same area. But if the volume is also reduced, the rate of collision can be kept constant.

◆ **Charles' law** Volume of a fixed amount of gas is proportional to absolute temperature (at constant pressure).

For a fixed amount of gas at constant pressure: $V \propto T$ (K)

This is known as **Charles' law**.

If the pressure and/or temperature of a fixed amount of gas are changed from initial values of V_1 and T_1 to final values of V_2 and T_2, then, provided that the pressure has not changed:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

WORKED EXAMPLE B3.6

In an experiment such as shown in Figure B3.17, the temperature of the gas was increased from 5.0 °C to 95.0 °C. If the initial volume was 3.1 cm³, calculate the final volume of the gas.

Answer

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{3.1}{(273 + 5.0)} = \frac{V_2}{(273 + 95)}$$

$$V_2 = 4.1\,\text{cm}^3$$

9 After the pressure on a gas was increased from 1.0×10^5 Pa to 4.5×10^5 Pa its volume had become 280 cm³. What was its original volume, assuming that its temperature was constant?

10 The temperature of a gas was reduced from 80 °C to 20 °C, while keeping it at the same pressure. If the starting volume was 110 cm³, what was the final volume?

11 A fixed volume of gas at 310 K and a pressure of 1.2×10^5 Pa was heated in an oven. At what temperature (°C) will the pressure have risen to 1.9×10^5 Pa?

ATL B3A: Thinking skills

Applying key ideas and facts in new contexts

Using knowledge from this topic and from Topic A.2, explain how a hot air balloon can be made to rise and fall.

Hint: Consider the motion of air particles inside and outside the balloon, and the forces the particles apply to the balloon skin.

Figure B3.19 Hot air balloons over Cappadocia, Turkey

Combined gas laws

SYLLABUS CONTENT

- The ideal gas law equation can be derived from the empirical gas laws for constant pressure, constant volume and constant temperature as given by: $\frac{PV}{T}$ = constant.
- The equation governing the behaviour of ideal gases as given by: $PV = nRT$.

◆ **Empirical** Based on observation or experiment.

The three separate **empirical** gas laws described above can be combined into one equation:

For a fixed amount of gas, $PV \propto T$, or:

$$\frac{PV}{T} = \text{constant} \quad \text{or} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Nature of science: Observations

Empirical science and theories

By describing the gas laws as *empirical*, we mean that they are based only on direct observation and experiment, using the human senses. The gas laws, as such, are not *theoretical*. Based *only* on the empirical results of these experiments, we can make reasonably accurate predictions about the way gases will behave under most circumstances. More detailed observations result in more accurate and improved predictions of real gas behaviour.

Clearly, empirical research is the basis of the *scientific method* and most scientific and technological advances. *Theoretical thinking* is then needed to explain the experimental results. The *ideal gas theory* later in this topic is a good example of a theory used to explain experimental results and make predictions. Theories can only be accepted if they have been widely tested by further experimentation.

In everyday language, the word *theory* is often used much more loosely, as a casual explanation, or even a guess. However, in science, a theory describes an explanation that has been extensively tested and is widely accepted.

More generally, *empiricism* is the view that all knowledge originates from our experiences.

WORKED EXAMPLE B3.7

Gas in a container at a temperature of 289 K was heated in an oven to a temperature of 423 K. If the volume expanded from 0.27 m^3 to 0.35 m^3, and the initial pressure was 1.1×10^5 Pa, determine the final pressure.

Answer

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{(1.1 \times 10^5) \times 0.27}{289} = P_2 \times \frac{0.35}{423}$$

$$P_2 = 1.2 \times 10^5 \text{ Pa}$$

So far, we have only discussed fixed amounts of gas. We need to expand the discussion to include *any* amount of gas.

Investigations can show that the pressure of a fixed volume of gas at constant temperature is proportional to the amount of gas. This is what we would expect from our molecular understanding: If we doubled the number of molecules in a fixed volume, then the frequency of collisions with the walls would double if they still had the same average speed.

This leads us to $PV \propto nT$, or:

DB

$$PV = nRT$$

The constant R is known as the **universal (molar) gas constant.** It has the value **8.31 J K^{-1} mol^{-1}.**

◆ **Universal (molar) gas constant** The constant, R, that appears in the equation of state for an ideal gas ($pV = nRT$). $R = 8.31$ J K^{-1} mol^{-1}.

R is the macroscopic equivalent to the *Boltzmann constant, k,* which was introduced in the microscopic interpretation of temperature in Topic B.1.

This equation is being presented here as a summary of empirical results. It can be used to accurately predict the physical behaviour of most gases under most conditions. For example, for a fixed amount of gas at constant temperature, the equation predicts an inverse proportionality between volume and pressure.

◆ **Equation of state for an ideal gas, $pV = nRT$:** Describes the macroscopic physical behaviour of ideal gases. Also called the **ideal gas law**.

◆ **Piston** A solid cylinder that fits tightly inside a hollow cylinder, trapping a fluid. Designed to move as a result of pressure differences.

Ideal gases (see below) obey this relationship perfectly, so it is called the **ideal gas law** (also called the **equation of state**).

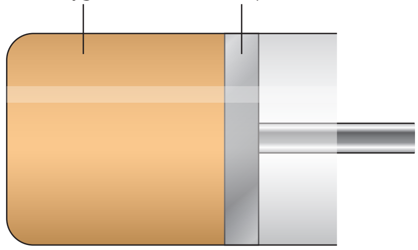
WORKED EXAMPLE B3.8

Some oxygen, O_2, is contained in the cylinder seen in Figure B3.20. No gas can move past the movable **piston**. The temperature of the gas is 298 K, it has a volume of 27 cm^3 and its pressure is equal to normal atmospheric pressure (1.0×10^5 Pa).

a Calculate the amount of gas in the cylinder.

b What was the mass of oxygen in the cylinder?

c The piston was suddenly pulled out increasing the volume to 32 cm^3. The pressure fell to 8.2×10^4 Pa. Determine the new temperature of the gas.



■ **Figure B3.20** Cylinder with movable piston

Answer

a $PV = nRT$

$(1.0 \times 10^5) \times (27 \times 10^{-6}) = n \times 8.31 \times 298$

$n = 0.0011$ mol

b $0.11 \times 32 = 3.5$ g

c $PV = nRT$

$(8.2 \times 10^4) \times (32 \times 10^{-6}) = 0.0011 \times 8.31 \times T$

$T = 287$ K

The expansion of the gas has resulted in a temperature fall of 11 K.

Alternatively, $\frac{PV}{T}$ = constant can be used to answer part **c**.

Inquiry process: Processing data

Processing data

A student carried out an experiment similar to that shown in Figure B3.12 and obtained the following results (Table B3.2). She used 6.1×10^{-4} mol of gas at a temperature of 21 °C.

Use these results to draw a straight line graph. One reading was an outlier and should not be considered when drawing the line of best fit. Which one? Use the gradient of the graph to determine a value for the molar gas constant.

■ **Table B3.2**

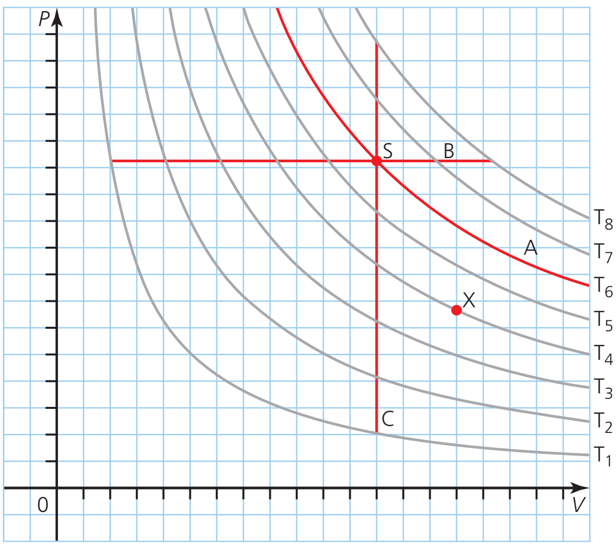
P / 10^5 Pa	V / cm^3
1.0	15.0
1.2	12.6
1.5	9.8
1.8	8.5
2.0	6.6
2.3	6.4
2.5	5.9
2.7	5.6

◆ ***PV* diagram** A graphical way of representing changes to the state of a gas during a thermodynamic process.

Pressure–volume diagrams

The state of a known amount of gas, as defined by values of its pressure, volume and temperature, can be identified as a point on a pressure–volume ***(PV)* diagram**.

For example, the point S in Figure B3.21 could represent one mole of a gas at a temperature of 300 K in a container of volume $1.0 \times 10^{-2}\,\text{m}^3$ and with a pressure of $2.5 \times 10^5\,\text{Pa}$ (data needed for later question).



■ **Figure B3.21** *PV* diagram

We know that there are three interconnected variables for a fixed amount of gas: pressure, volume and temperature. In order to represent these three variables on a graph with two axes, different temperatures are shown by the curved isothermal lines, labelled on Figure B3.21 by T_1 (lowest temperature) to T_8 (highest temperature).

Changes in the state of the gas can be represented by paths on *PV* diagrams. For example:

- Line A represents variations in pressure and volume at constant temperature.
- Line B represents variations in temperature and volume at constant pressure.
- Line C represents variations in pressure and temperature at constant volume.
- Any other path through point S (to point X, for example) will involve changing all three variables.

12 The volume of a scuba diving tank similar to that seen in Figure B3.22 is $11 \times 10^{-3}\,\text{m}^3$. The air inside is compressed to a pressure 210× atmospheric pressure.

■ **Figure B3.22** Scuba diving

a What is that air pressure in pascals?

b What volume of air at atmospheric pressure was pumped into the tank?

c The water temperature decreases with depth below the surface. Would you expect that to significantly affect the pressure in the tank?

d As she breathes out, the diver releases air bubbles into the water. Explain why the bubbles get bigger as they rise towards the surface.

13 The volume of a gas in a cylinder such as seen in Figure B3.20 was $38\,\text{cm}^3$ when the temperature was 20 °C.

a The apparatus was heated. Explain why the piston moved to the right as thermal energy flowed into the gas through the cylinder.

b What was the temperature (°C) of the gas when the volume had increased to $50\,\text{cm}^3$?

14 Helium gas is widely used to fill balloons (Figure B3.23). If the density of the helium in a balloon is $2.0 \times 10^{-4}\,g\,cm^3$, *estimate* its:

a volume

b mass

c amount in moles

d number of atoms.

■ **Figure B3.23** Helium-filled balloons

15 A container of gas has a volume of $120\,cm^3$. At a temperature of 300 K the gas has a pressure of $1.5 \times 10^6\,Pa$. Determine the amount of gas in the container.

16 How can you be sure that the isotherm T_5 on Figure B3.21 is representing a higher temperature than T_2?

17 Consider Figure B3.21. What changes must be made to a fixed amount of gas at point S on the diagram to move it to point X?

18 Represent the following changes to a fixed mass of gas on a *PV* diagram:

a a gas expanding at constant pressure, as its temperature increases

b a compression at constant temperature

c a gas cooled so that its volume and pressure decreased.

Modelling gas behaviour: ideal gases

SYLLABUS CONTENT

- Ideal gases are described in terms of the kinetic theory and constitute a modelled system used to approximate the behaviour of real gases.
- The change in momentum of particles due to collisions with a given surface gives rise to pressure in gases, and, from that analysis, pressure is related to the average translational speed of molecules: $P = \frac{1}{3}\rho v^2$

LINKING QUESTION

- How do the concepts of force and momentum link mechanics and thermodynamics?

This question links with understandings in Topics A.2, A.3.

◆ **Ideal gas** Gas which obeys the ideal gas law equation perfectly. The microscopic particle model of an ideal gas makes several important assumptions about the particles and their motions.

We now want to extend our microscopic kinetic theory of gases to include a mathematical treatment, which will predict macroscopic behaviour, including the equation $PV = nRT$.

First, we will provide a more detailed definition of an **ideal gas**. Although most gases are molecular, we will often use the term *particles* in order to be more general.

Top tip!

For a gas all at the same temperature, if collisions between gas particles resulted in a loss of kinetic energy, that would mean that the gas would keep getting colder. Collisions of gas particles with particles in the containing walls will result in a transfer of kinetic energy only if there is a temperature difference (thermal conduction).

◆ **Average value** Any single number used to represent a quantity which is varying.

◆ **Range (data)** Spread of data from smallest to largest values.

◆ **Anomalous** Different from the pattern of other similar observations.

◆ **Outlier** A value which is significantly different from the others in the same data set.

◆ **Mean** A certain type of average: the sum of all of the numbers divided by the number of values involved.

Assumptions about the particles in an ideal gas:

- The gas contains a very large number of identical particles.
- The volume of the particles is negligible compared with the total volume occupied by the gas.
- The particles are moving in random directions, with a wide variety of speeds.
- There are no forces between the particles, except when they collide. Because there are no forces, the particles have no (electrical) potential energy. This means that any changes of internal energy of an ideal gas are assumed to be only in the form of changes of random kinetic energy.
- The motion of the particles obeys Newton's laws of motion.
- All collisions between particles are elastic. (This means that the total kinetic energy of the particles remains constant at the same temperature.)

Tool 3: Mathematics

Calculate mean and range

We have often referred to the **average values** of particle speeds but have not really explained what that means.

Consider a more straightforward example: the following times (s) recorded for a ball to roll down a particular slope: 15, 12, 11, 21, 14, 11, 14, 13.

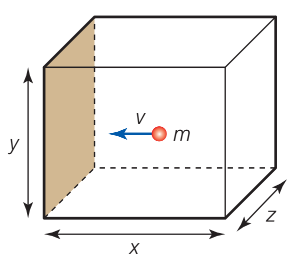
The **range** of this data is from 11 s to 21 s.

The value of 21 s is **anomalous** and is inconsistent with the other values. It may be described as an **outlier**. It may have been a mistake and should be excluded from calculations, unless checked and confirmed.

An *average value* is any single number that has been chosen to represent a range of values. There are several possibilities, including the central (*median*) value (13), or the most common value (11 or 14). However, in physics, average values are usually **mean** values. A *mean value* is obtained by adding all the values and dividing by the number of values. In this example, the mean is 12.9, which may be better limited to 2 significant figures (13).

In this topic, average particle speeds cannot be determined as straightforward means. As we shall see, (average speed)^2 is calculated from average kinetic energies.

Mathematical model for gas behaviour



■ **Figure B3.24** One particle in a box

We will start by considering just one particle in a rectangular box. See Figure B3.24. The particle has a mass m and is moving with a velocity v perpendicularly towards the end wall.

After the particle has an elastic collision with the wall it will return along the same path, with velocity $-v$.

Its change of velocity is: $v - (-v) = 2v$

average force on end wall, F = change of momentum $= m \times 2v = 2mv$

time interval between collisions of the particle with the same wall, $t = \dfrac{2x}{v}$

$$\text{average force on end wall} = \frac{\text{change of momentum}}{\text{time between collisions}} = \frac{2mv}{\left(\dfrac{2x}{v}\right)} = \frac{mv^2}{x}$$

$$\text{average pressure on end wall, } P = \frac{\text{average force}}{\text{area}} = \frac{\left(\dfrac{mv^2}{x}\right)}{yz} = \frac{mv^2}{xyz} = \frac{mv^2}{V}$$

where V is the total volume of the box $(= xyz)$.

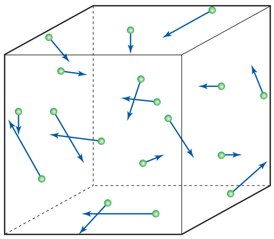


Figure B3.25 Gas molecules moving around at random in a container

However, of course, the container may not be rectagular, and all gases have a very large number of particles moving in random directions with different speeds. See Figure B3.25.

- If there are N particles in the box (all moving perpendicular to the end wall):
  average pressure on end wall becomes: $\dfrac{Nmv^2}{V}$
- If the particles are moving in random directions:
  average pressure on end wall becomes:
  $$\frac{\left(\frac{1}{3}Nmv^2\right)}{V}$$
  because there are three perpendicular directions, so that on average $\frac{1}{3}$ of the velocities are directed in any one of these directions.
- If the particles have different speeds:
  v^2 is the average value of their speeds-squared.
- If the container is a different shape:
  it can be shown that the shape of the container does not affect the pressure.

Common mistake

This equation contains two Vs. One upper case (volume), and one lower case (velocity). It is easy to get them confused.

So, finally we have an equation which links the macroscopic properties of a gas (pressure and volume) to its microscopic properties (the number of particles, their mass and their speed):

Pressure of an ideal gas: $P = \dfrac{1}{3}Nm\dfrac{v^2}{V}$

This long derivation need not be remembered, but it should be understood. It provides an all-important link between the theories of particle behaviour and measurements that can easily be made in a laboratory.

Since Nm is the total mass of the gas:

$$\frac{Nm}{V}$$

is the density of the gas, ρ, leading to an alternative expression:

DB

Pressure of an ideal gas: $\boldsymbol{P = \dfrac{1}{3}\rho v^2}$

Tool 3: Mathematics

Check an expression using dimensional analysis of units

The units of an expression on the left-hand side of an equation must be the same as the units of an expression on the right-hand side of the equation.

This can be used to check if a suggested equation could be correct. It is a simplified version of what is known as **dimensional analysis**.

◆ **Dimensional analysis** Method of checking if an equation may be correct. The units (dimensions) of all terms should be the same.

To check that there is no obvious mistake in the equation $P = \frac{1}{3}\rho v^2$, we can see if the units on both sides of the equation are the same.

The units of pressure are pascals, which are the same as $\mathrm{N\,m^{-2}}$. We also know that newtons are equivalent to $\mathrm{kg\,m\,s^{-2}}$ ($F = ma$). So that, the units of pressure can be reduced to the SI base units: $\mathrm{kg\,m\,s^{-2}/m^{-2} = kg\,m^{-1}\,s^{-2}}$.

On the right-hand side of the equation: $\frac{1}{3}$ has no units, ρ has the units $\mathrm{kg\,m^{-3}}$ and v has the units $\mathrm{m\,s^{-1}}$. Combining these, we get: $\mathrm{kg\,m^{-1}\,s^{-2}}$ which is the same as on the left-hand side of the equation. This check has found no obvious mistake in the equation.

WORKED EXAMPLE B3.9

a Determine a value for the average speed of nitrogen molecules at normal air pressure (1.0×10^5 Pa) if a mass of 1.4 g of the gas was in a container of volume 1200 cm^3.

b How many moles of nitrogen were in the container (see Table B3.1)?

c Calculate the temperature of the gas.

d Determine a value for the average molecular speed if the temperature rose to 350 K in the same container.

Answer

a $\rho = \dfrac{1.4 \times 10^{-3}}{1200 \times 10^{-6}} = 1.17\,\text{kg m}^{-3}$

$P = \dfrac{1}{3}\rho v^2$

$1.0 \times 10^5 = \dfrac{1}{3} \times 1.17 \times v^2$

$v = 5.1 \times 10^2\,\text{m s}^{-1}$

(This value is obtained from the square root of the average value of speeds^2. This is not exactly the same as the value of the average speed.)

b $\dfrac{1.4}{28} = 5.6 \times 10^2\,\text{mol}$

c $PV = nRT$

$(1.0 \times 10^5) \times (1200 \times 10^{-6}) = 0.050 \times 8.31 \times T$

$T = 289\,\text{K}$

d We can use $\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$ to calculate the new pressure:

$\dfrac{1.0 \times 10^5}{289} = \dfrac{P_2}{350}$

$P_2 = 1.21 \times 10^5\,\text{Pa}$

Then, $P = \dfrac{1}{3}\rho v^2$

$1.21 \times 10^5 = \dfrac{1}{3} \times 1.17 \times v^2$ (density is unchanged)

$v = 560\,\text{m s}^{-1}$

19 Make a list of all the basic physics principles used to derive the equation $P = \frac{1}{3}\rho v^2$, as shown above.

20 What pressure would be created by 5.0×10^{23} ideal gas molecules, each of mass 5.3×10^{-26} kg in a volume of 0.010 m^3 if the temperature of the gas was such that the average speed of the molecules was 500 m s^{-1}?

21 An ideal gas has a density of 2.4 kg m^{-3}. If it creates a pressure of 1.5×10^5 Pa on its container, determine a value for the average speed of its molecules.

22 At room temperature the average speed of oxygen molecules is 500 m s^{-1}. What pressure will these molecules create if the density of oxygen was the same as in air (1.3 kg m^{-3})? Assume that oxygen behaves as an ideal gas.

23 Three particles have speeds 440 m s^{-1}, 480 m s^{-1} and 520 m s^{-1}.

a What is their average speed?

b What is the square of their average speed?

c What is the average of their speeds-squared?

d What is the square root of the average of their speed squared?

Nature of science: Models

Randomness

A key feature of the motions of the particles in an ideal gas is that they are 'random'. But what exactly does 'random' mean? The word has various uses throughout science and more generally, often with slight differences in meaning. For example, we might say that the result of throwing a six-sided die is random because we cannot predict what will happen, although we probably appreciate that there is a one-in-six chance of any particular number ending up on top. In this case, all outcomes should be equally likely. Another similar example could be if we were asked to 'pick a card at random' from a pack of 52.

■ **Figure B3.26** Six-sided dice

Sometimes we use the word random to suggest that something is unplanned. For example a tourist might walk randomly around the streets of a town.

Unpredictability is a key feature of random events and that certainly is a large part of what we mean when we say a gas molecule moves randomly. All possible directions of motion may be equally likely, but the same cannot be said for speeds. Some speeds are definitely more likely than others. For example, at room temperature a molecular speed of $500\,\mathrm{m\,s^{-1}}$ is much more likely than one of $50\,\mathrm{m\,s^{-1}}$. Similarly, when we refer to random kinetic energies of molecules, we mean that we cannot know or predict the energy of individual molecules, although some values are more likely than others. But there is a further meaning: we are suggesting that individual molecules behave independently and that their energies are disordered.

Perhaps surprisingly, in the kinetic theory, the random behaviour of a very large number of individual molecules on the microscopic scale leads to complete predictability in our everyday macroscopic world. Similar ideas occur in other areas of physics, notably in radioactive decay (Topic E.1), where the behaviour of an individual atom is unknowable, but the total activity of a radioactive source is predictable. Of course, insurance companies, betting companies and casinos can make good profits by understanding the statistics of probability, without being too concerned about individual events.

LINKING QUESTIONS

- How can gas particles of high kinetic energy be used to perform work?
- How does a consideration of the kinetic energy of molecules relate to the development of the gas laws?

These questions link to understandings in Topic B.4 (HL).

Internal energy of an ideal gas

SYLLABUS CONTENT

- The relation between the internal energy, U, of an ideal monatomic gas and the number of molecules, or amount of substance as given by: $U = \frac{3}{2}Nk_BT$ or $U = \frac{3}{2}RnT$.
- The equations governing the behaviour of ideal gases are given by: $PV = Nk_BT$ and $PV = nRT$.

◆ **Internal energy of an ideal monatomic gas, U** The sum of the random translational kinetic energies of all the molecules.

The **internal energy of an ideal (monatomic) gas** is the total random *translational* kinetic energy of its particles. Ideal monatomic gas particles do not have any potential energies, or vibrational or rotational kinetic energies.

Internal energy is given the symbol U. It can be calculated by multiplying the number of particles by their average random translational kinetic energy:

$$U = N \times \frac{1}{2}mv^2$$

Comparing $PV = nRT$ with $PV = \frac{1}{3}Nmv^2$ we see that: $nRT = \frac{1}{3}Nmv^2$.

But $\frac{1}{3}Nmv^2$ can be rewritten as $\frac{2}{3} \times \frac{1}{2}Nmv^2$ or $\frac{2}{3}U$ so: $nRT = \frac{2}{3}U$.

Or:

internal energy of an ideal (monatomic) gas: $\boldsymbol{U = \frac{3}{2}nRT}$ (using macroscopic quantities R and T.)

Most gases consist of molecules, rather than atoms. They still approximate well to the macroscopic behaviour of an ideal gas, but their internal energy is more complicated, because of the vibrational and rotational kinetic energies of their molecules.

The forces between particles in solids and liquids makes calculating their internal energies much more complicated.

WORKED EXAMPLE B3.10

Calculate the total internal energy of one mole of a monatomic ideal gas at 300 K.

Answer

$U = \frac{3}{2}nRT = 1.5 \times 1.0 \times 8.31 \times 300 = 3.7 \times 10^3\,\text{J}$

If we divide both sides of the highlighted equation for U by the number of particles, N, we get the average random translational kinetic energy of a single atom in an ideal gas, $\overline{E}_k$ (=U/N):

$\overline{E}_k = \frac{3}{2}\frac{nRT}{N}$, but we know that $\frac{n}{N}$ = the Avogadro constant, N_A, so that: $\overline{E}_k = \frac{3}{2}\frac{RT}{N_A}$.

$\frac{R}{N_A}$ is the molar gas constant divided by the number of particles in a mole.

It is called the Boltzmann constant, k_B (unit: J K^{-1}), which was intoduced in Topic B.1:

$$k_B = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23}\,\text{J K}^{-1}$$

Boltzmann constant: $k_B = \frac{R}{N_A}$

This leads us to the important relationship between temperature (K) and the average random translational kinetic energy of particles, that we first met in Topic B.1, and which is repeated here:

$$\boldsymbol{E_k = \frac{3}{2}k_B T} \left(= \frac{1}{2}mv^2\right)$$

This equation is *not* restricted to the particles in ideal gases. When particles collide / interact, translational kinetic energy is exchanged, so that the particles in all gases in good thermal contact with each other will eventually all have the same average *translational* kinetic energy. In other words:

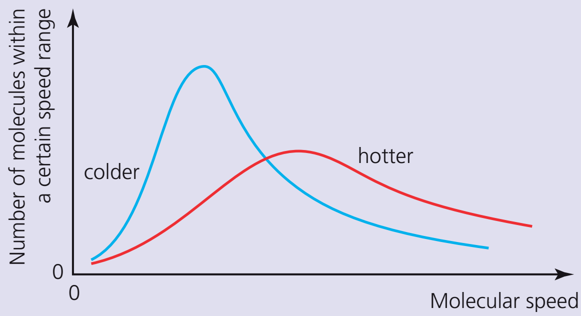
At the same temperature, all gases contain molecules with the same average random translational kinetic energy.

ATL B3B: Research skills

Providing a reasoned argument to support conclusions

Kinetic theory and the distribution of molecular speeds

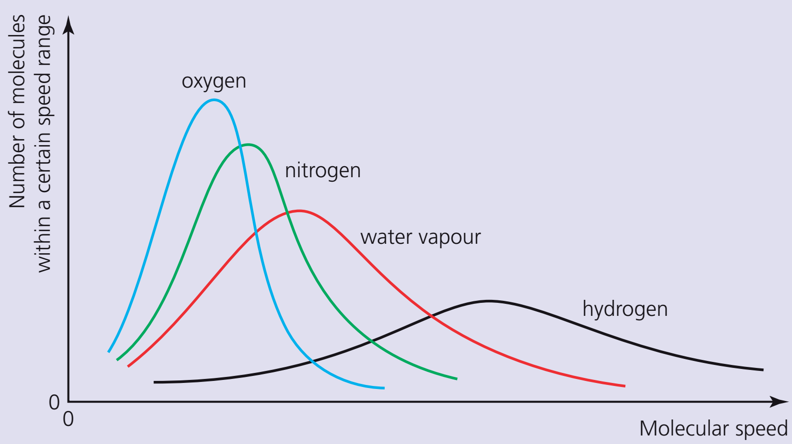
Figure B3.27 shows the range and distribution of molecular speeds in a typical gas, and how it changes as the temperature increases. This is known as the Maxwell–Boltzmann distribution.



■ **Figure B3.27** Typical distributions of molecular speeds in a gas

Note that there are no molecules with zero speed and few with very high speeds. Molecular speeds and directions (that is, molecular velocities) are continually changing as the result of intermolecular collisions. As we have seen, higher temperature means higher kinetic energies and therefore higher molecular speeds. But the range of speeds also broadens and so the peak becomes lower, keeping the area under the graph, constant.

Figure B3.28 illustrates the ranges of molecular speeds for different gases at the same temperature. It shows that, as we have noted before, at the same temperature less massive molecules have higher speeds than more massive molecules.



■ **Figure B3.28** Distribution of molecular speeds in different gases at the same temperature

For a given sample of a gas, the area under the curves in Figure B3.27 is the same. Explain why this must be true.

Understanding that $k_B = \frac{R}{N_A} = \frac{Rn}{N}$, internal energy, $U = \frac{3}{2}nRT$ can now be expressed in terms of k (rather than R) if we prefer:

Internal energy of an ideal monatomic gas: $\boldsymbol{U = \frac{3}{2}Nk_BT}$ (using the microscopic quantities N and k_B.)

We can also re-write the ideal gas equation ($PV = nRT$) in terms of k (rather than R):

$$\boldsymbol{PV = Nk_BT}$$

WORKED EXAMPLE B3.11

a A sample of an ideal monatomic gas in a closed container at 18 °C has a total internal energy of 380 J. Determine how many particles (atoms) it contains.

b If the container has a volume of 435 cm^3, calculate the pressure of the gas.

Answer

a $U = \frac{3}{2}Nk_BT$

$380 = 1.5 \times N \times (1.38 \times 10^{-23}) \times (273 + 18)$

$N = 6.3 \times 10^{22}$

b $PV = Nk_BT$

$P \times (435 \times 10^{-6}) = (6.3 \times 10^{22}) \times (1.38 \times 10^{-23}) \times (273 + 18)$

$P = 5.8 \times 10^5$ Pa

Real gases compared to ideal gases

SYLLABUS CONTENT

- Temperature, pressure and density conditions under which an ideal gas is a good approximation to a real gas.

◆ **Real gases** Modelling of gas behaviour is idealized. Real gases do not behave exactly the same as the model of an ideal gas.

An ideal gas is impossible to achieve, but we have seen that **real gases** obey the ideal gas equation ($PV = nRT$) under most circumstances. However, this will not be true if there are significant differences from the stated assumptions about the particles in an ideal gas, such as:

- At high densities and pressures the particles will be closer together than has been assumed. The forces between the particles may no longer be negligible.
- At low temperatures the forces between particles will have a greater effect because the particles are moving slower. Most real gases will also turn into liquids and solids if the temperature is low enough.

Figure B3.29 Liquid nitrogen has a boiling point of 77 K

Most real gases behave like ideal gases unless their pressure or density is very high, or the temperature is very low.

24 a Calculate the average random translational kinetic energy of oxygen (O_2) molecules at 300 K.

b Oxygen molecules each have a mass of 5.31×10^{-26} kg. Determine their average speed at 300 K.

c Explain why an average carbon dioxide (CO_2) molecule travels slower than oxygen molecules (O_2) at the same temperature.

25 a Determine how much energy is needed to raise the temperature of two moles of a monatomic gas from 20 °C to 50 °C.

b Why will more energy be needed to raise two moles of a *molecular* gas through the same temperature rise?

26 At what temperature will 1.0×10^{23} atoms of an ideal gas have a total internal energy of 1000 J?

27 Some nitrogen gas was cooled from a temperature of 300 K to 100 K.

a Estimate by what percentage its pressure was reduced.

b State two assumptions you had to make in order to answer part **a**.

c Explain why the actual final pressure was less than that predicted by theory.

28 What is the pressure of an ideal gas which has a temperature of 47 °C and contains 4.2×10^{25} particles in a volume of 0.037 m^3?

29 a Explain why the internal energy of an ideal gas can be determined from: 1.5 × pressure × volume.

b Discuss why temperature does not appear in this equation.

c What volume (cm^3) of an ideal gas has an internal enegy of 10 J and exerts a pressure of twice normal atmospheric pressure?

30 The *molar heat capacity* of a gas is a similar concept to specific heat capacity. It equals the thermal energy needed to raise the temperature of one mole of the gas by one kelvin.

a Calculate the molar heat capacity of argon. (Assume that the gas is kept in a constant volume.)

b Explain why the molar heat capacity of a *molecular* gas, oxygen for example, will be greater.

c Why will the molar heat capacity of argon be greater than your answer to part **a** if it is allowed to exand?

TOK

Knowledge and the knower

- How do our interactions with the material world shape our knowledge?

Models always have limitations

We cannot directly use our human perception to help understand the behaviour of molecules in a gas. We need a *model* to help to understand the situation. In this topic we have presented both a visual and a mathematical model of an 'ideal' gas. These models are extremely useful and accurate predictors of the behaviour of real gases, but they are not perfect.

Can modelling any system which is too big or too small to be seen ever be 'good enough' to count as true knowledge? Can we ever be sure that a model is a true representation of reality, if we can never observe events directly? Added to which, one purpose of a model is to simplify reality, to make it easier to understand.

Or will there always be some doubt? And, if the model is useful, does it really matter if the model is a 'true' representation of something that we cannot observe directly anyway? Can a model of the solar system, for example, be considered to be knowledge of a higher level (than the model of an ideal gas), because we can directly observe and record the motion of individual planets?

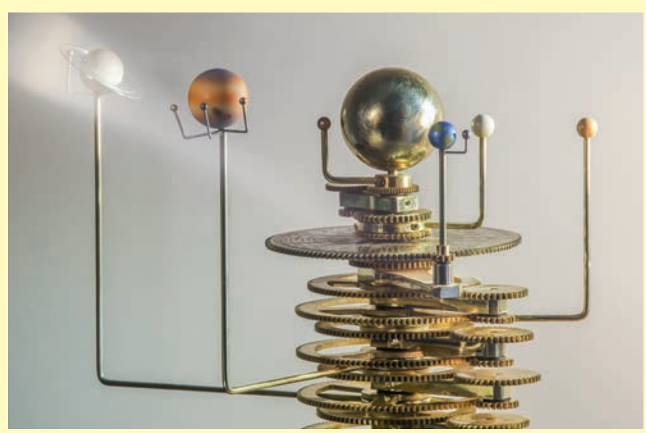


Figure B3.30 Simple model of the Solar System

B.4 Thermodynamics

HL ONLY

◆ **Heat engine** Device that uses the flow of thermal energy to do useful work.

◆ **Thermodynamics** Branch of physics involving transfers of thermal energy to do useful work.

Guiding questions

- How can energy transfers and storage within a system be analysed?
- How can the future evolution of a system be determined?
- In what way is entropy fundamental to the evolution of the Universe?

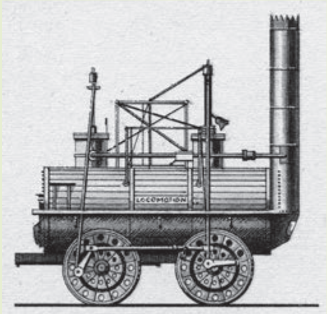
Nature of science: Global impact of science

Heat engines

The invention of devices that could continuously use the thermal energy transferred from a burning fuel to do useful mechanical work changed the world completely. No longer did people, or animals, have to do such hard work – they could get engines to do it for them, and much quicker than they could do the same work themselves.

The idea that burning a fuel to heat water to make steam, which could then be used to make something move, had been understood for a very long time. But, using this in a practical way was much more difficult, and it was not until the early eighteenth century that the first commercial steam engines were produced. It was about 100 years afterwards when George Stephenson built the engine 'Locomotion' (See Figure B4.1) for the first public steam railway, opened in the UK in 1825.

About 200 years later, things are very different. We live in a world that is dominated by **heat engines** (devices that get useful mechanical work from a flow of thermal energy). We are surrounded by all sorts of different engines – in cars, boats, trains, planes, factories and power stations producing electricity (see Figure B4.2).



■ **Figure B4.1** The 'Locomotion'

All these engines need a transfer of thermal energy from fuels in order to work. The fuels are usually fossil fuels. It is difficult to overstate the importance that these devices have had in modern life, because without them our lives would be very different. Of course, we are now also very much aware of the problems associated with the use of heat engines, as discussed in Topic B.2: limited fossil-fuel resources, inefficient devices, pollution and global warming.

This topic describes the process of using thermal energy to do useful mechanical work in heat engines. This branch of physics is known as **thermodynamics**. Although thermodynamics grew out of a need to understand heat engines, it has much wider applications. The study of thermodynamics leads to a better understanding of key scientific concepts such as internal energy, heat, temperature, work and pressure, and how they are all connected to each other and to the microscopic behaviour of particles.

■ **Figure B4.2** Using hot gases in heat engines

LINKING QUESTION

- What paradigm shifts enabling changes to human society, such as harnessing the power of steam, can be attributed to advancements in physics understanding? (NOS)

◆ **Closed system** Allows the free flow of thermal energy, but not matter.

◆ **Reservoir (thermal)** Part of the surroundings of a thermodynamic system that is kept at approximately constant temperature and is used to encourage the flow of thermal energy.

◆ **Isobaric** Occurring at constant pressure. $\Delta P = 0$.

In this topic we will concentrate our attention on understanding the basic principles of the processes that involve the volume increase (expansion) of fixed masses of gases.

In the rest of this chapter, and throughout physics, there are many references to thermodynamic 'systems' and 'surroundings'. Before going any further, we should make sure that these simple and widely used terms are clearly understood.

A *system* is simply the thing that we are studying or talking about. In this topic it will be a gas.

In this topic we will be discussing **closed systems**, in which energy can be transferred into or out of the system as heat, or work, but no mass can be transferred in or out. Compare this to an *isolated system*: one in which neither mass nor energy can be transferred in or out. For example, when discussing the conservation of momentum in Topic A.2, we were referring to an *isolated* system.

The *surroundings* are everything else – the gas container and the rest of the Universe. Sometimes the surroundings are called the *environment*. If we wish to suggest that a part of the surroundings was deliberately designed for thermal energy to flow into it or out of it, we may use the term **(thermal) reservoir**.

A thermodynamic system can be as complex as a rocket engine, planet Earth or a human body, but in this topic, we will develop understanding by considering the behaviour of gases in heat engines.

Work done when a gas expands (or is compressed)

SYLLABUS CONTENT

- ▶ The work done by or on a closed system, as given by $W = P\Delta V$, when its boundaries are changed, can be described in terms of pressure and changes of volume of the system.

For simplicity, the thermodynamic system that we are considering is often shown as a gas in a regularly shaped cylindrical container, constrained by a gas-tight piston that can move without friction (Figure B4.3).

First consider the example of a gas expanding so that there is no change in pressure. (This is called an **isobaric** change, as described later.) If the gas trapped in the cylinder in Figure B4.4 is given thermal energy it will exert a resultant force on the piston (because the pressure in the cylinder is momentarily higher than the pressure from the surroundings) and the piston will move outwards as the gas expands, keeping the pressures equal.

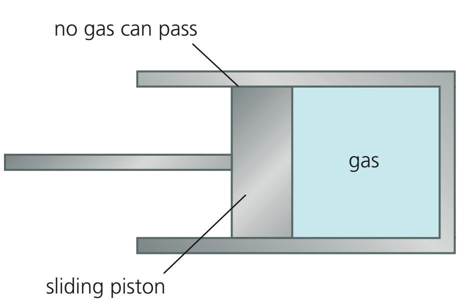


Figure B4.3 Gas in a cylinder with a movable piston

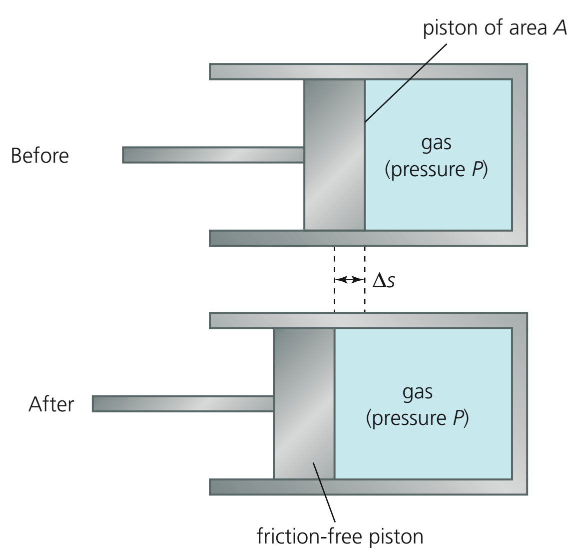


Figure B4.4 Gas expanding in a cylinder

We say that work has been done *by* the gas in pushing back the surrounding air (remember that we are assuming that there is no friction).

work done by gas = force × distance moved in direction of force, or

work done by gas = $(PA)\Delta s$ (because force = pressure × area)

Because change of volume, $\Delta V = A\Delta s$:

work done when a gas changes volume at constant pressure, $W = P\Delta V$

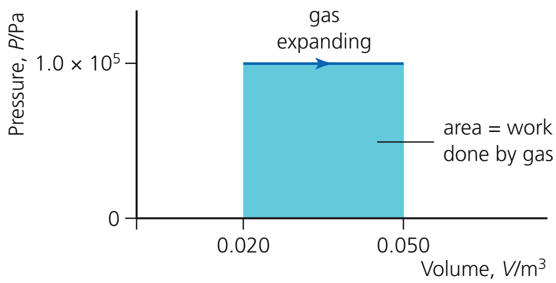


Figure B4.5 Work done during expansion of an ideal gas

If work is done *on* the gas to reduce its volume, ΔV and the work done, W, will have negative values.

We introduced PV diagrams in Topic B.3 and they are very useful in representing changes in the **state of a gas** during thermodynamic processes. An example is shown in Figure B4.5, which shows the expansion of a gas from $0.020\,m^3$ to $0.050\,m^3$ at a constant pressure of 1.0×10^5 Pa.

The work done *by* the gas in expansion in this example:

$W = P\Delta V = (1.0 \times 10^5) \times (0.050 - 0.020) = 3.0 \times 10^3$ J

◆ **Work done when a gas changes volume, W** Work is done by a gas when it expands (W is positive). Work is done on a gas when it is compressed (W is negative). At constant pressure $W = P\Delta V$. If the pressure changes, the work done can be determined from the area under a PV diagram.

◆ **State of a gas** Specified by quoting the pressure, P, temperature, T, and volume, V, of a known amount, n, of gas.

Note that this calculation to determine the work done, $P\Delta V$, is numerically equal to calculating the area under the PV diagram. This is true for all thermodynamic processes, regardless of the shape of the graph, and this is one reason why PV diagrams are so widely used in thermodynamics to represent various processes. Figure B4.6 shows two further examples.

The work done when a gas changes pressure and/or volume can be determined from the area under a PV diagram.

WORKED EXAMPLE B4.1

Determine values for the work done in the two changes of state represented in Figure B4.6.

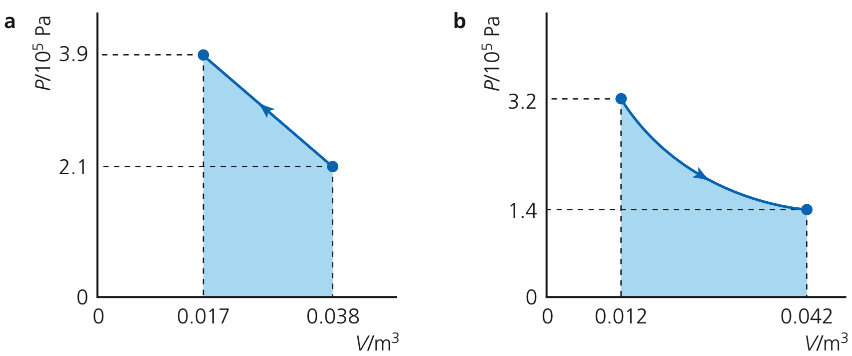


Figure B4.6 Determining areas under pressure–volume graphs

Answer

a $W = P\Delta V$ = area under graph

$$= \left[\frac{1}{2} \times (3.9 - 2.1) \times 10^5 \times (0.038 - 0.017)\right] + \left[(0.038 - 0.017) \times 2.1 \times 10^5\right] = 6.3 \times 10^3 \text{ J}$$

The work is done *on* the gas as it is compressed into a smaller volume.

b In this example, work is done *by* the gas as the volume increases. Because the graph is curved, the area underneath it must be estimated, as explained below.

HL ONLY

Tool 3: Mathematics

Interpret features of graphs: areas under the graph

The areas under curved graphs can be estimated by drawing a rectangle that is judged by eye to have the same area, as shown by the example in Figure B4.7.

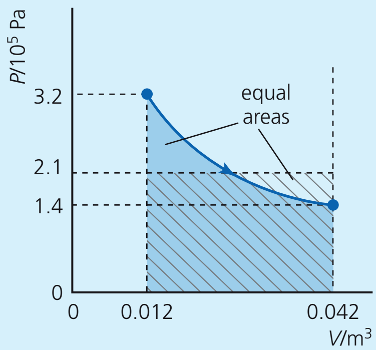


Figure B4.7 Estimating the area under a curved graph

W = area under graph $\approx (2.1 \times 10^5) \times (0.042 - 0.012) = 6.3 \times 10^3$ J

1 The volume of a gas expanded from 43 cm³ to 51 cm³ while its pressure remained constant at 1.1×10^5 Pa.
   a Calculate the work done.
   b State whether work was done on the gas, or by the gas.
2 0.84 J of work was done on a gas of volume 7.6×10^{-5} m³. If the pressure was constant at 1.4×10^5 Pa, calculate the new volume.
3 Figure B4.8 shows how the volume of a gas changed as the pressure on it was increased. Show that the work done during the expansion was approximately 230 J.

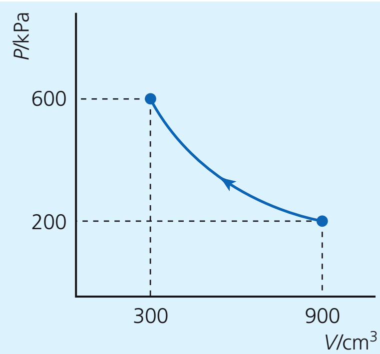


Figure B4.8 PV graph for a gas

First law of thermodynamics

SYLLABUS CONTENT

- The first law of thermodynamics, as given by $Q = \Delta U + W$, results from the application of conservation of energy to a closed system and relates the internal energy of a system to the transfer of energy as heat and as work.
- The change in internal energy, as given by $\Delta U = \frac{3}{2}Nk_B\Delta T$ is related to the change of its temperature.

If an amount of thermal energy, $+Q$, is transferred *into* a system, such as that seen in Figure B4.3, then, depending on the particular circumstances, the gas may gain internal energy, $+\Delta U$, and/or the gas will expand and do work *on* the surroundings, $+W$. We can use the *principle of conservation of energy* (from Topic A.2) to describe how these quantities are connected:

DB

Thermal energy supplied to a gas:

$$Q = \Delta U + W$$

◆ **First law of thermodynamics** If an amount of thermal energy, $+Q$, is transferred into a system, then the system will gain internal energy, $+\Delta U$, and/or the system will expand and do work on the surroundings, $+W$: $Q = \Delta U + W$.

HL ONLY

This important equation, known as the **first law of thermodynamics**, covers all the possibilities of expanding or compressing gases, and/or supplying or removing thermal energy from a system.

Common mistake

Students often get confused over the signs used in this equation. They are restated here:

- Thermal energy transferred *into* the gas will be given positive values: $+Q$; thermal energy *removed* from the gas will be considered to be negative: $-Q$.
- An *increase* in the internal energy of a gas will be given positive values: $+\Delta U$; a *decrease* in internal energy will be considered to be negative: $-\Delta U$.
- Work done *by* the gas during *expansion* will be given positive values: $+W$; work done *on* the gas during *compression* will be given negative values: $-W$.

WORKED EXAMPLE B4.2

80 J of work was done by a gas when 120 J of thermal energy was transferred to it. Determine the change in internal energy of the gas.

Answer

$Q = \Delta U + W$

$(+120) = \Delta U + (+80)$

$\Delta U = (+40)$ J. The positive sign shows that the internal energy increased.

WORKED EXAMPLE B4.3

150 J of work was done when a gas was compressed. At the same time, its internal energy increased by 50 J. Calculate how much thermal energy flowed into, or out of, the system during this process.

Answer

$Q = \Delta U + W$

$Q = (+50) + (-150) = (-100)$ J of thermal energy was transferred. The negative sign shows that the transfer was *out of* the gas.

Top tip!

When applying the first law of thermodynamics, show the energies as shown in the above worked example: in brackets with either a + or – sign.

Changes in internal energy of an ideal monatomic gas

We have seen in Topic B.3 that the internal energy of an ideal monatomic gas can be calculated from:

$$U = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$

This means that:

changes in internal energy, ΔU, of an ideal monatomic gas can be calculated from:

DB

$$\boldsymbol{\Delta U = \frac{3}{2}Nk_B\Delta T = \frac{3}{2}nR\Delta T}$$

WORKED EXAMPLE B4.4

Consider the previous worked example. If the gas contained 3.4×10^{23} particles, determine the temperature rise. Assume the gas was an ideal monatomic gas.

Answer

$\Delta U = \frac{3}{2}Nk_B\Delta T$

$50 = 1.5 \times (3.4 \times 10^{23}) \times (1.38 \times 10^{-23}) \times \Delta T$

$\Delta T = 7.1$ K

4 200 J of work was done when a gas expanded at constant temperature. How much thermal energy was transferred?

5 70 J of work was done while a gas was compressed. At the same time 30 J of thermal energy was transferred out of the gas. Calculate the change in internal energy.

6 50 J of thermal energy was removed from a gas and its internal energy decreased by 10 J.
  a How much work was done?
  b Was the work done on the gas, or by the gas?

7 An ideal monatomic gas containing 3.4×10^{24} particles was heated from 297 K to 348 K.
  a What was the change in internal energy of the gas?
  b If, in this time, 5600 J of thermal energy was supplied to the gas, what was the amount of work done by, or on, the gas?

8 The internal energy of 3.2 moles of some helium gas, at an initial temperature of 302 K, rose by 540 J.
  a What was the final temperature of the gas?
  b If, at the same time, the gas expanded from a volume of 220 cm^3 to 380 cm^3 at constant pressure of 2.3×10^5 Pa, how much work was done?
  c How much thermal energy was transferred?

9 During an investigation of how the volume of a fixed mass of air changed with temperature at constant atmospheric pressure (1.0×10^5 Pa), the volume increased from 2.3 cm^3 to 3.1 cm^3.
  a Calculate how much work was done.
  b Was the work done on the gas, or by the gas?
  c The expansion of the air happened when 0.150 J of thermal energy was transferred into the system. Determine the change in internal energy of the gas.

10 1.82 J of thermal energy was supplied to a gas, increasing its pressure steadily from 1.0×10^5 Pa to 1.2×10^5 Pa. During this time the volume was increased steadily from 35 cm^3 to 45 cm^3.
  a Represent this process on a *PV* diagram.
  b Indicate the work done during this process on your drawing.
  c Calculate the work done.
  d Determine the change in internal energy of the gas.

11 Consider Figure B4.9. Work has been done to change the state of a gas from point A to point B on the *PV* diagram.

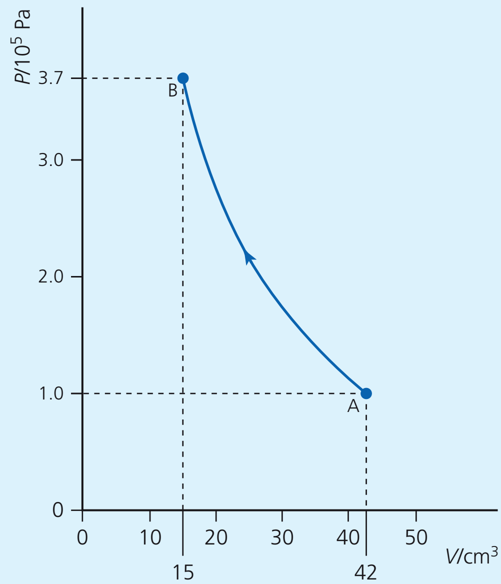


Figure B4.9 *PV* graph

  a Describe the process.
  b Did the process obey Boyle's law?
  c What has happened to the temperature of the gas? Explain your answer.
  d Estimate the work done during the process.
  e During the process, 1.9 J of thermal energy flowed out of the gas. Determine the change in its internal energy.

12 1.74 mol of an ideal monatomic gas expanded and during the process its temperature changed. The container was well insulated and no thermal energy was able to flow into, or out of, the gas.
  a If the internal energy of the gas decreased by 29.7 J, calculate the temperature change of the gas.
  b Determine how much work was done by the gas during this expansion.

13 Explain why a gas gets hotter when it is compressed rapidly.

Four thermodynamic processes

SYLLABUS CONTENT

- Isovolumetric, isobaric, isothermal and adiabatic processes are obtained by keeping one variable fixed.
- Adiabatic processes in monatomic ideal gases can be modelled by the equation as given by:
  $PV^{\frac{5}{3}} = \text{constant}$

Among all the various possible changes of state that a gas might experience, it is convenient to consider the first law of thermodynamics under four extremes:

- $\Delta U = 0$
- $W = 0$
- $Q = 0$
- $\Delta P = 0$

These are represented in Figure B4.10, in which the dotted lines are all isothermals.

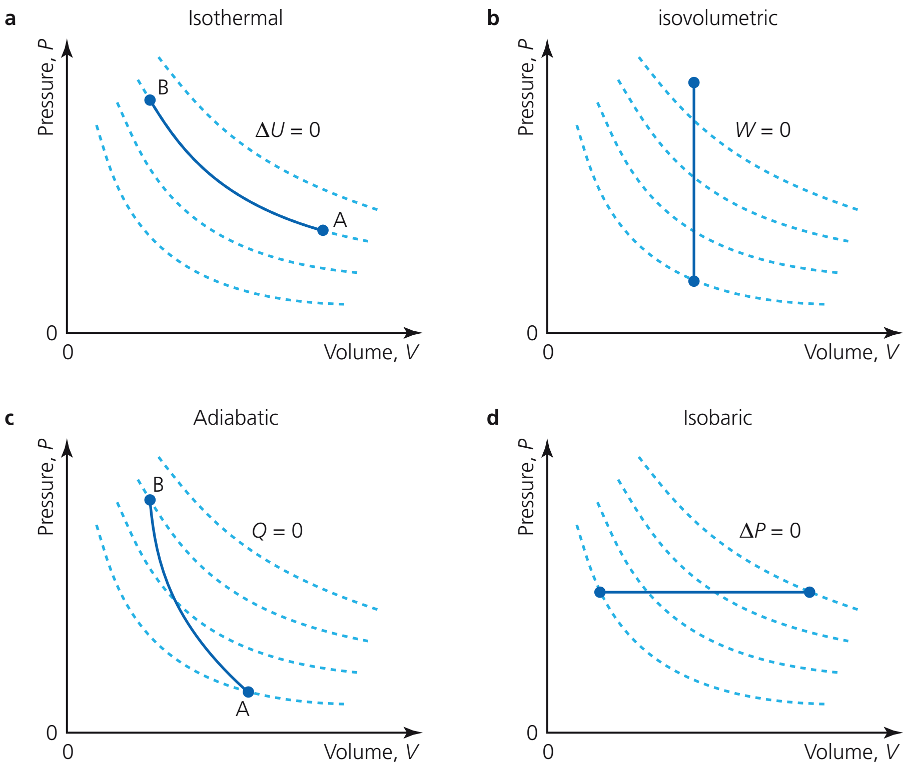


Figure B4.10 Four thermodynamic processes

ΔU = 0 (isothermal process)

(The prefix iso- means equal.)

There is no change in the internal energy of the gas because its temperature is constant. Therefore,

in an isothermal change:

$$Q = 0 + W \quad \text{or } Q = W$$

In an isothermal expansion (B → A) all the work done by the gas on the surroundings is supplied by thermal energy transferred into the gas. In an isothermal compression (A → B), the work done on the gas is all transferred away from the gas as thermal energy. For a process to approximate to the ideal of being isothermal, the change must be as slow as possible. Isothermal changes obey Boyle's law (as described in Topic B.3): PV = constant.

$W = 0$ (isovolumetric process)

There is no work done by or on the gas because there is no change in volume. Therefore,

◆ **Isovolumetric** Occurring at constant volume.

in an **isovolumetric change**:

$$Q = \Delta U + 0 \quad \text{or } Q = \Delta U$$

In this straightforward process, if thermal energy is transferred into a gas, it simply gains internal energy and its temperature rises. If thermal energy is transferred away from a gas, its internal energy and temperature decrease.

$Q = 0$ (adiabatic process)

No thermal energy is transferred between the gas and its surroundings. Therefore

◆ **Adiabatic** Occurring without thermal energy being transferred into or out of a thermodynamic closed system.

in an **adiabatic change**:

$$0 = \Delta U + W$$

$\Delta U = -W$ for a compression and $-\Delta U = W$ for an expansion.

In an adiabatic expansion (B → A) all the work done by the gas is transferred from the internal energy within the gas, ΔU is negative and the temperature decreases. In an adiabatic compression (A → B) all the work done on the gas ($-\Delta W$) is transferred to the internal energy of the gas, which gets hotter.

When gas molecules hit the inwardly moving piston during a compression, they gain kinetic energy and the temperature rises. When gas molecules hit the outwardly moving piston during an expansion, they lose kinetic energy and the temperature falls.

For a process to approximate to the ideal of being adiabatic, the change must be as rapid as possible in a well-insulated container.

Note that adiabatic lines on PV diagrams must be steeper than isothermal lines, because in equal expansions, the temperature decreases during an adiabatic change, but is constant (by definition) during an isothermal change. This difference can be quantified by considering PV relationships:

We know that PV = constant during an isothermal change (that is, PV^1 = constant) but,

DB

in an adiabatic change of an ideal monatomic gas:

$$PV^{\frac{5}{3}} = \text{constant}$$

For gases other than monatomic ideal gases, V is raised to a different power, but this is not included in this course.

Comparing this equation to that for an isothermal change, we see that similar changes in volume are associated with greater changes in pressure during adiabatic changes. This is because there are also accompanying temperature changes.

$\Delta P = 0$ (isobaric process)

Any expansion or compression that occurs at constant pressure. Therefore,

in an isobaric change:

$$Q = \Delta U + W \text{ (and } W = P\Delta V)$$

Most isobaric changes occur when gases are allowed to expand or contract freely when their temperature changes, keeping their pressure the same as the surrounding pressure.