The $N-k$ Problem using AC Power Flows

Sean Harnett

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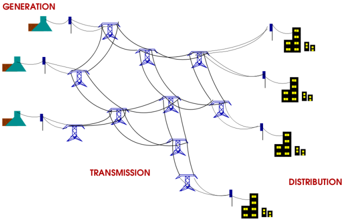
Outline

Goal: find a small set of lines whose removal will cause the power grid to fail.



There are a number of ways the grid can fail; one is due to voltage instability (voltage collapse)

- Normal voltages: $|V| \approx [1\ 1\ \dots\ 1]^T$.
- Voltage disturbance: $\sum_{i=1}^{\#buses}(1-|V_i|)^2$

Why the AC model

DC model is widely used, can compute power flows by simply solving a linear system. Why use the more complicated non-linear AC model?

- The DC model ignores reactive power, very important when considering voltage collapse
- The DC model assumes all voltage magnitudes are equal

AC allows for study of wider range of phenomena, particularly useful for voltage instability

The power flow problem

A. Bergen and V. Vittal, *Power Systems Analysis*

Goal: find $V_i = |V_i|e^{j\theta_i}$ at each bus i given the known quantities:

- complex power $S_i = P_i + jQ_i$ demanded at each demand bus
- real power P_i and voltage magnitude $|V_i|$ at each generator
- parameters for transmission lines (impedance, capacitance, transformer parameters, etc.)

After finding the V_i, can calculate other possible quantities of interest, such as Q_i at generators, power flow on lines, etc

Current injected into the grid at bus i (Kirchoff):

$$I_i = I_{G_i} - I_{D_i} = \sum_{k=1}^{n} I_{ik} = \sum_{k=1}^{n} \frac{V_i - V_k}{Z_{ik}} = \sum_{k=1}^{n} Y_{ik} V_k$$

Y is the *admittance matrix*. $Y_{ii} = \sum_{k \neq i} \frac{1}{Z_{ik}}$, $Y_{ik} = -\frac{1}{Z_{ik}}$. Captures the physical parameters of the transmission lines

admittance is the inverse of impedance: $Y = Z^{-1} = (R + jX)^{-1}$

Example with two buses:

$$I_1 = Y_{11} V_1 + Y_{12} V_2 = \frac{V_1}{Z} + \frac{V_2}{-Z} = \frac{V_1 - V_2}{Z}$$

The power injected into the grid at bus i is:

$$S_i = S_{G_i} - S_{D_i} = V_i I_i^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

$$S = \mathrm{diag}[V](YV)^*$$

Recall that $S_i = P_i + jQ_i$, that P is known at the generators, and that both P and Q are known at the demand nodes

$\longrightarrow$ system of complex quadratic equations. Want to find vector V to satisfy the above

Solving the power flow problem

Use Newton's method and look for high-voltage solution with $|V_i| \approx 1$ (after scaling)

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Possible problems with Newton:

- there is no solution
- there is a solution, but it's too far from initial guess for Newton to find it
- Newton converges to one of many possible "bad" solutions

Other methods: enhanced Newton, general polynomial equation solvers, homotopy methods

"If Newton's method doesn't work, chances are nothing else will"

Steven Low - convex problem, semi-definite programming

Back to the $N - k$ problem

Recall:

- looking for small set of lines that will cause maximum damage
- this is known to be very hard, even in the linear (DC) case

Combinatorial approach is too hard → try something continuous

Imagine an adversary who modifies impedances of the power lines (within budgets) to maximize voltage disturbance

Why? We "expect" one of two interesting possibilities:

- The optimal adversarial action focuses on a handful of lines, or
- The optimal adversarial action focuses on a handful of lines, but spreads the remaining budget in a nontrivial way among many lines

In other words, we expect the solution to be combinatorial, but we are (superficially) bypassing the combinatorial complexity

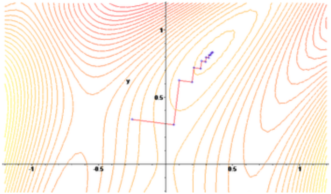
Find set of impedances which leads to maximum voltage disturbance

The optimization problem:

- variable is the vector of impedance multipliers, call it x
- objective is the voltage disturbance, call it $f(x) = \sum_{i=1}^{\#buses}(1 - |V_i|)^2$ ("black box")
- constraints are $1 \leq x \leq a$ and $\sum_{i=1}^{\#lines}(x_i - 1) \leq B$

Gradient search: Frank-Wolfe

Try a simple first-order method, fast and easy



- General idea: climb up the voltage disturbance hill while staying within the budget

Find the gradient by finite differences. Requires solving the AC power flow thousands of times $\rightarrow$ slow. (save LU factors of Jacobian, parallel)

More about Frank-Wolfe

Near the point x_k approximate f by a linear function

$$f(x_k + y) \approx f(x_k) + \nabla f(x_k)^T (y - x_k)$$

Finding the maximizer y_k (subject to the constraints) is an LP (gurobi). This gives the search direction $p_k = y_k - x_k$.

Find a satisfactory step length $\alpha \in [0, 1]$ and define $x_{k+1} = x_k + \alpha p_k$

Experiments and Observations

case2383wp: power flow data for Polish system - winter 1999-2000 peak.

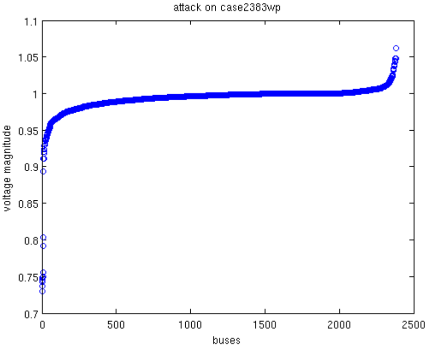
- 2896 lines
- a=3, B=6
- took about 40 seconds to run
- allocated entire budget to five lines (typical)

Results for case2383wp:

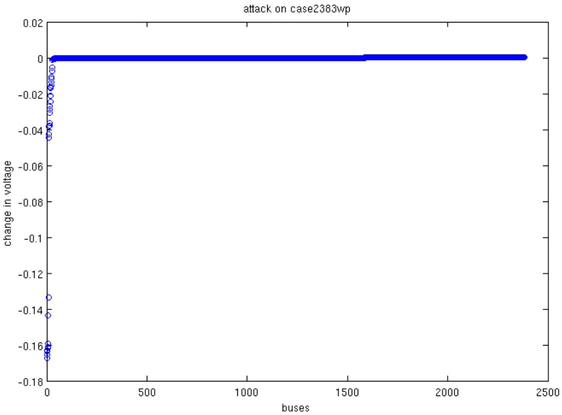
iteration	objective	step size	405	467	005	501	404
1	0.28032	.99993	3.000	3.000	3.000	1.000	1.000
2	0.29184	.39062	3.000	3.000	2.219	1.781	1.000
3	0.29295	.11902	3.000	3.000	2.074	1.688	1.238
4	0.29296	.0073841	3.000	3.000	2.066	1.698	1.236

(Recall the objective: voltage disturbance: $\sum_{i=1}^{\#buses}(1-|V_i|)^2$. Here I subtract off the voltage disturbance of the unperturbed initial case)

The voltage disturbance is concentrated on a small percentage of buses, and is mostly a decrease in voltage:



Looking at only the change from the base case:



Better methods?

Why use such a simple algorithm? Also tried IPOPT, modern library for large scale nonlinear optimization

- indeed finds an allocation with greater objective
- but chooses the same lines as Frank-Wolfe (slightly changing the allocation between those lines)
- slower

Comparison of Frank-Wolfe and IPOPT

Example - solution for the 2383-bus Polish case:

method	objective	405	467	005	501	404	time
FW	0.29296	3	3	2.066	1.698	1.236	40s
IPOPT	0.29299	3	3	2.051	1.709	1.240	766s

Solutions for the IEEE test cases (14, 30, 57, 118, and 300 bus cases) were identical for the two methods, with IPOPT being 5-10 times slower.

A second example

case15029: power flow data for Eastern interconnect, 2003 summer peak estimate

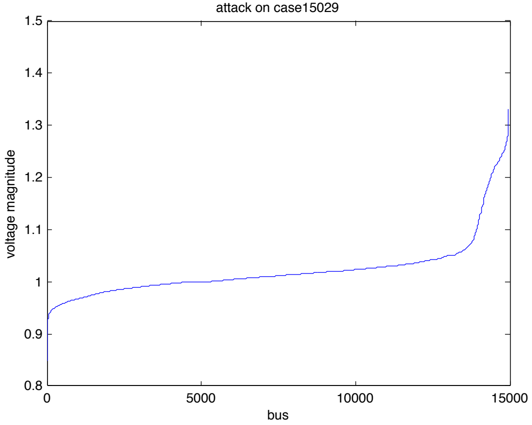
- 23772 lines
- a=5, B=12
- took Frank-Wolfe about two hours to run, IPOPT over ten hours
- allocated budget over $\sim$30 lines

Results for case15029, 23772 lines

IPOPT objective .217. Frank-Wolfe objective .2166, 12 iterations.

IPOPT		Frank-Wolfe	
line	multiplier	line	multiplier
22039	2.2134	22039	2.1502
22037	2.1495	22037	2.115
22046	1.8873	22046	1.8651
22047	1.8564	21667	1.8314
21667	1.7675	22047	1.7979
22348	1.6733	22348	1.6648
23091	1.6014	23091	1.6638
23090	1.597	22922	1.575
22922	1.4845	23090	1.575
21607	1.4153	23451	1.4254
23452	1.4091	23452	1.4254
23451	1.4091	21607	1.401
21836	1.4064	21833	1.401
21833	1.4034	21836	1.401
21606	1.3222	21606	1.3432
23371	1.2661	23371	1.2627
23368	1.2652	23368	1.2425
22350	1.2622	21919	1.2352
21919	1.2483	21920	1.2352
21920	1.2436	22350	1.2352
21893	1.18	21888	1.1728
21888	1.1728	21893	1.1728
22349	1.1481	22349	1.1526
23396	1.1402	22719	1.1526
23397	1.1395	23396	1.1342
22719	1.1317	23397	1.1342
21887	1.0911	21887	1.1174
21892	1.0909	21892	1.1174
22757	1.0165		
22756	1.0084		

Voltage magnitudes after attack



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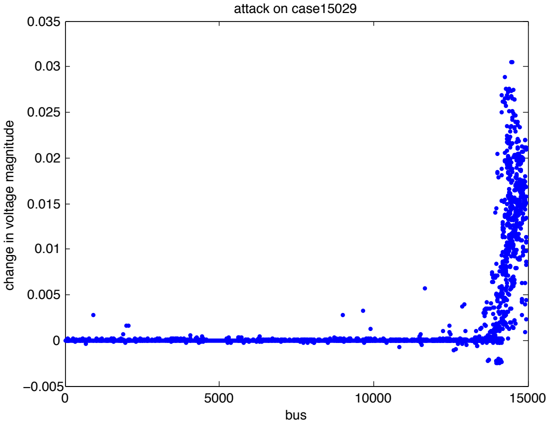

Thank you

ons are shown in Table 1.

Table 1

	|V1|	Angle 1	|V2|	Angle 2	|V3|	Angle 3
1)	1.0	0.0°	0.944	-5.88°	0.946	-5.66°
2)	1.0	0.0°	0.082	-62.33°	0.462	-14.76°
3)	1.0	0.0°	0.450	-16.03°	0.074	-63.14°
4)	1.0	0.0°	0.120	-56.54°	0.105	-58.75°

These four solutions have associated four regions of attraction shown in figure 7. The regions appear to be contiguous, but detailed examination shows their boundaries are not smooth. Note that the white areas represent initial conditions for which a solution was not found.

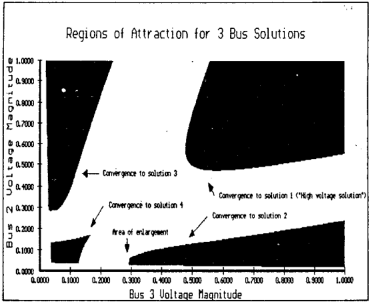

Figure 7

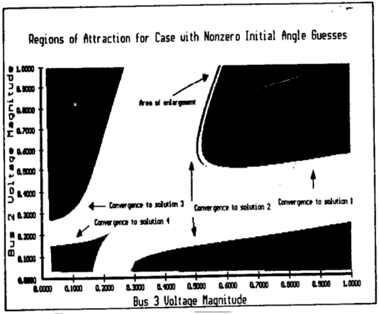

Figure 8

Low Voltage Power Flow Solutions and Their Role in Exit Time Based Security Measures for Voltage Collapse, C. DeMarco and T. Overbye