Robust Optimal Power Flow with Uncertain Renewables

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6-26-2012

THE ENERGY CHALLENGE

Wind Energy Bumps Into Power Grid's Limits

Mike Groll/Associated Press

The Maple Ridge Wind farm near Lowville, N.Y. It has been forced to shut down when regional electric lines become congested.

By MATTHEW L. WALD
Published: August 26, 2008

When the builders of the Maple Ridge Wind farm spent $320 million to put nearly 200 wind turbines in upstate New York, the idea was to get paid for producing electricity. But at times, regional electric lines have been so congested that Maple Ridge has been forced to shut down even with a brisk wind blowing.

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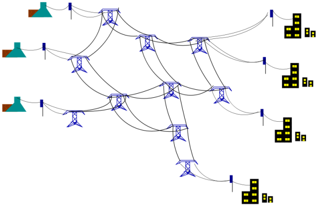
Optimal power flow
aka economic dispatch, tertiary control



- Choose generator outputs
- Minimize cost
- Meet loads, satisfy generator and network constraints

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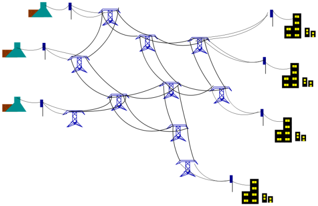
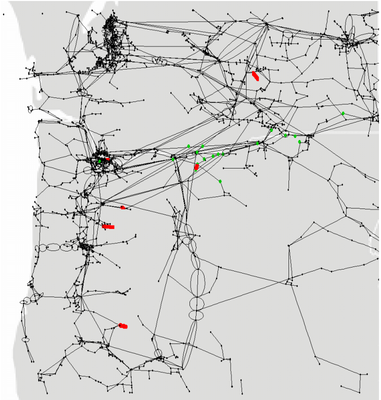


- Choose generator outputs
- Minimize cost
- Meet loads, satisfy generator and network constraints
- **Assume everything fixed**

Motivation

Bonneville Power Administration data, northwest US

- proportional control
- with standard solution, 7 lines exceed limit $\geq$ 8% of the time



Goals

- simple control
- aware of limits
- not too conservative
- computationally practicable

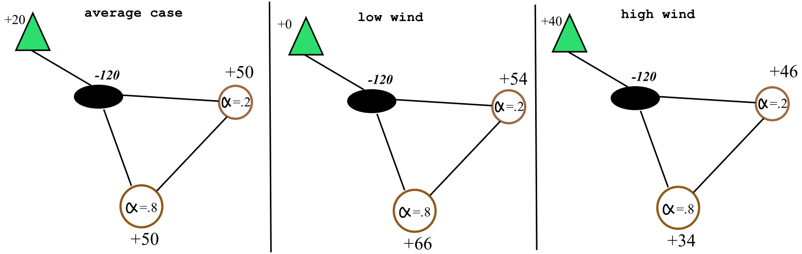
two parameters $\overline{p}$ and α per generator, affine control of form

$$p_i = \overline{p}_i - \alpha_i \sum_j \Delta\omega_j$$

$$\sum_i \alpha_i = 1$$

$\sim$ primary + secondary control

Set up
control



Anghel, Werley, Motter (2007): model of thermal dynamics of a power line

heat equation: $\frac{\partial T(x,t)}{\partial t} = \kappa \frac{\partial^2 T(x,t)}{\partial x^2} + \alpha I^2 - \nu(T(x,t) - T_0)$

solution: $\mathbf{T(t) = e^{-\nu t}(T(0) - T_e(P)) + T_e(P)}$, where

$$T_e(P) = \lim_{t \to \infty} T(t) = \frac{\alpha I^2}{\nu} + T_0 = \frac{\alpha}{\nu} \frac{P^2}{V^2} + T_0,$$

P = power flow, T_0 = ambient temperature

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if $P > P^{\max}$, line fails when its temperature reaches $T_e(P^{max})$:

$$t^* = \frac{1}{\nu} \ln \frac{T_e(P) - T(0)}{T_e(P) - T_e(P^{max})}$$

But...

- In 2003 event, many critical lines tripped due to thermal reasons, but well short of their line limit
- Thermal limit may be in terms of terminal equipment, not line itself
- Wind strength and wind direction is important
- Resistivity is a function of line temperature
- In medium-length lines ($\sim$ 100 miles) the line limit is due to voltage drop, not thermal reasons
- In long lines, it is due to phase angle change (stability), not thermal reasons

summary: exceeding limit for too long is bad, but complicated

want: "fraction time a line exceeds its limit is small"

proxy: prob(violation on line i) $< \epsilon$ for each line i

Need to model variation in wind power between dispatches

Wind at bus i of the form $\mu_i + \mathbf{w_i}$

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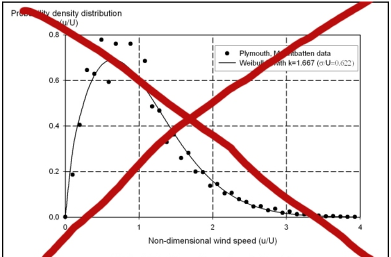

Weibull fit to Plymouth weather station data.

Computing line flows

wind power at bus i: $\mu_i + \mathbf{w}_i$

DC approximation

- $B\boldsymbol{\theta} = \overline{p} - d$

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DC approximation

- $B\boldsymbol{\theta} = \overline{p} - d + (\mu + \mathbf{w} - \alpha \sum_{i \in G} \mathbf{w}_i)$
- $\boldsymbol{\theta} = B^{+}(\bar{p} - d + \mu) + B^{+}(I - \alpha e^{T})\mathbf{w}$

Computing line flows

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DC approximation

- $B\boldsymbol{\theta} = \overline{p} - d + (\mu + \mathbf{w} - \alpha \sum_{i \in G} \mathbf{w}_i)$
- $\boldsymbol{\theta} = B^{+}(\bar{p} - d + \mu) + B^{+}(I - \alpha e^{T})\mathbf{w}$
- flow is a linear combination of bus power injections:

$$\mathbf{f_{ij}} = \beta_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)$$

Computing line flows

$$\mathbf{f_{ij}} = \beta_{ij}\left((B_i^+ - B_j^+)^T(\bar{p} - d + \mu) + (A_i - A_j)^T\mathbf{w}\right),$$
$$A = B^+(I - \alpha e^T)$$

Given distribution of wind can calculate moments of line flows:

Computing line flows

$$\mathbf{f_{ij}} = \beta_{ij}\left((B_i^+ - B_j^+)^T(\bar{p} - d + \mu) + (A_i - A_j)^T\mathbf{w}\right),$$

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Given distribution of wind can calculate moments of line flows:

- $\bar{f}_{ij} = \beta_{ij}(B_i^+ - B_j^+)^T(\bar{p} - d + \mu)$
- $var(\mathbf{f_{ij}}) = \beta_{ij}^2 \sum_k (A_{ik} - A_{jk})^2 \sigma_k^2 \leq s_{ij}^2$ (assuming ind.)
- and higher moments if necessary

Chance constraints to deterministic constraints

- recall chance constraints: $P(|\mathbf{f_{ij}}| > f_{ij}^{max}) < \epsilon_{ij}$
- from moments of $\mathbf{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality

Chance constraints to deterministic constraints

- recall chance constraints: $P(|\mathbf{f_{ij}}| > f_{ij}^{max}) < \epsilon_{ij}$
- from moments of $\mathbf{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality
- for Gaussian wind, can do better, since $\mathbf{f_{ij}}$ is Gaussian :

$$f_{ij}^{max} \pm \bar{f}_{ij} \geq s_{ij}\phi^{-1}(1 - \frac{\epsilon_{ij}}{2})$$

Formulation

Choose generator outputs and response parameters to minimize the expected cost, and so that the chance (fraction of the time) that each line overflows is small.

$$\min_{\overline{p},\alpha}\{\mathbb{E}[c(\overline{p})] : B\delta = \alpha, \delta_n = 0$$

$$s_{ij}^2 \geq \beta_{ij}^2 \sum_{k \in W} \sigma_k^2 (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2$$

$$B\overline{\theta} = \overline{p} + \mu - d, \overline{\theta}_n = 0$$

$$\overline{f}_{ij} = \beta_{ij}(\overline{\theta}_i - \overline{\theta}_j)$$

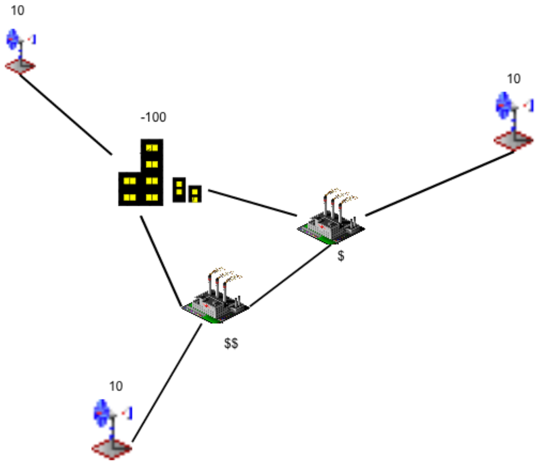
$$f_{ij}^{max} \pm \overline{f}_{ij} \geq s_{ij}\phi^{-1}(1 - \frac{\epsilon_{ij}}{2})$$

$$\sum_{i \in G} \overline{p}_i + \sum_{i \in W} \mu_i = \sum_{i \in D} d_i$$

$$\sum_{i \in G} \alpha_i = 1, \alpha \geq 0\}$$

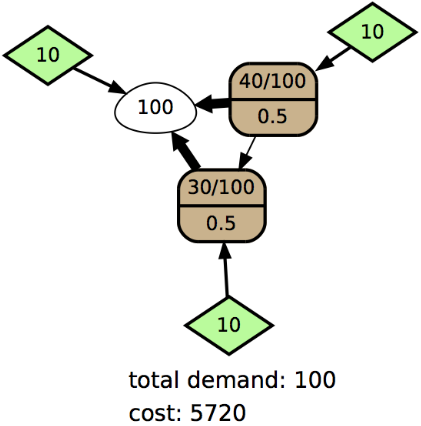
Toy example

1. What if no line limits?
2. What if tight limit on line connecting generators?



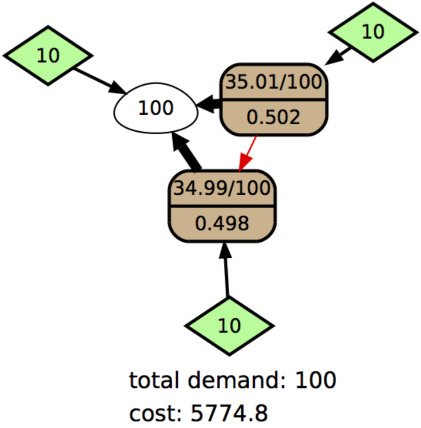
Answer 1

What if no line limits?



Answer 2

What if small limit on line connecting generators?



Experiment: NYTimes

How much more wind power can the CC-OPF method use?

And how much money does this save?

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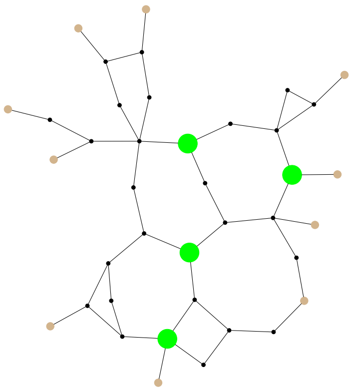
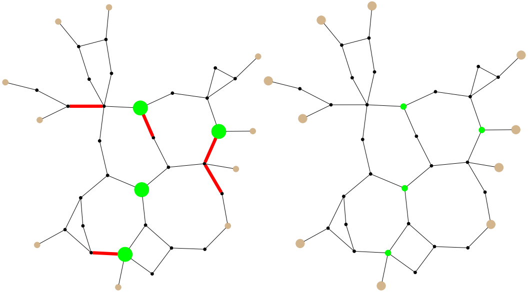
39-bus New England system from MATPOWER



30% penetration, CC-OPF cost 264,000

Experiment: NYTimes

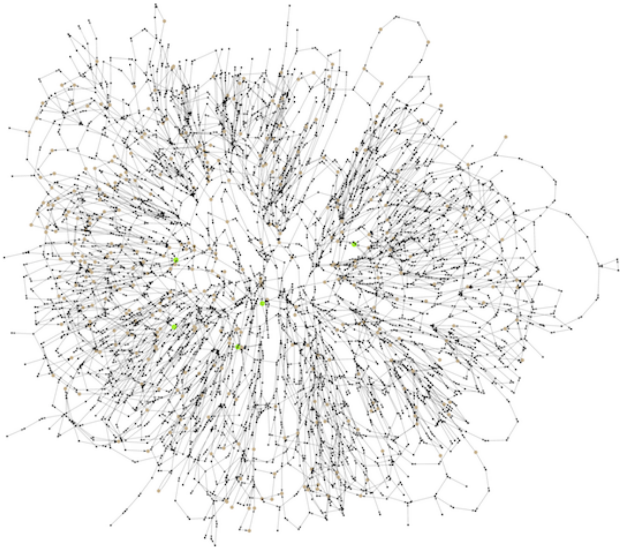
'standard' solution with 10% buffer
feasible only up to 5% penetration (right)



Cost 1,275,000 – almost 5(!) times greater than CC-OPF

Big cases

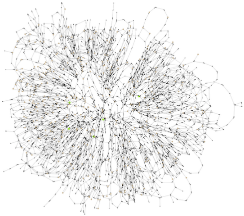
Polish system - winter 2003-04 evening peak



Big cases

Polish 2003-2004 winter peak case

- 2746 buses, 3514 branches, 8 wind sources
- 5% penetration and $\sigma = .3\mu$ each source



According to CPLEX, the optimization problem has

- 36625 variables
- 38507 constraints, 6242 conic constraints
- 128538 nonzeros, 87 dense columns

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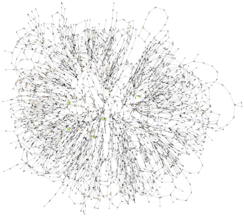

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Big cases

CPLEX:

- total time on 16 threads = 3393 seconds
- "optimization status 6"
- given solution is wildly infeasible

Gurobi:

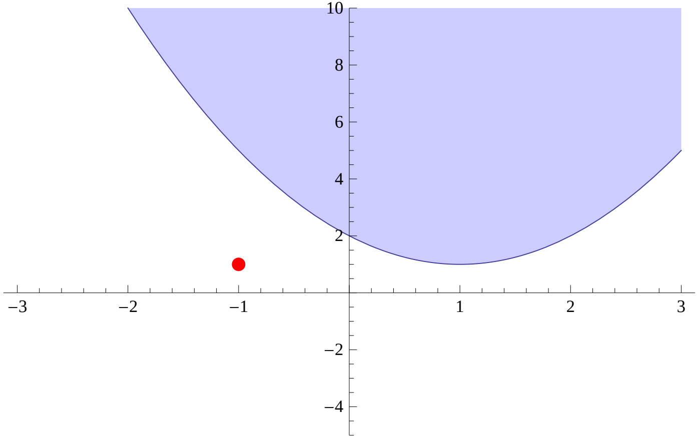
- time: 31.1 seconds
- "Numerical trouble encountered"

Cutting-plane algorithm:

```
remove all conic constraints
repeat until convergence:
    solve linearly constrained problem
    if no conic constraints violated: return
    find separating hyperplane for maximum violation
    add linear constraint to problem
```

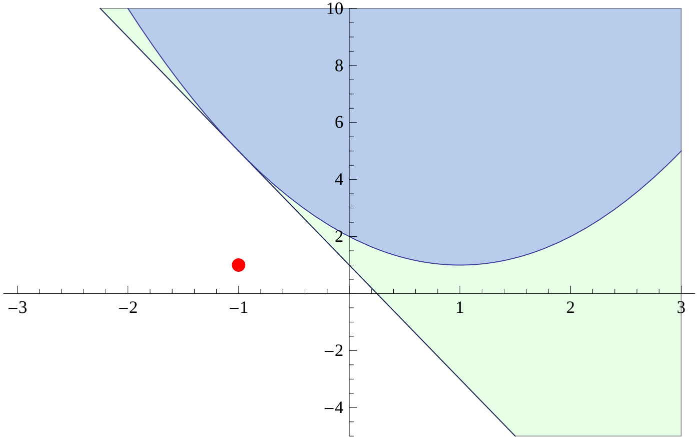
Cutting-plane method

Candidate solution violates conic constraint



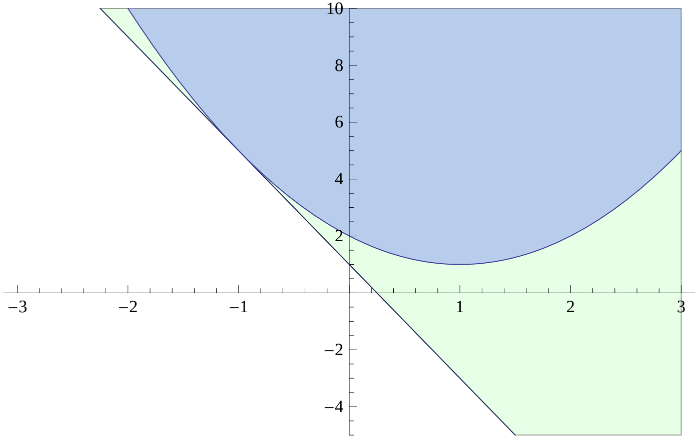
Cutting-plane method

Separate: find a linear constraint also violated



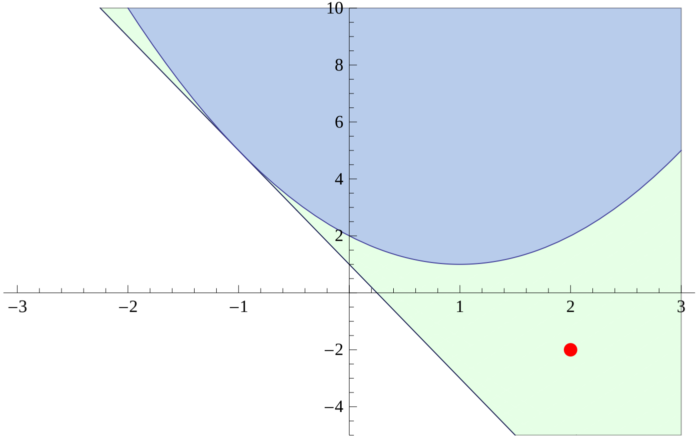
Cutting-plane method

Solve again with linear constraint



Cutting-plane method

New solution still violates conic constraint



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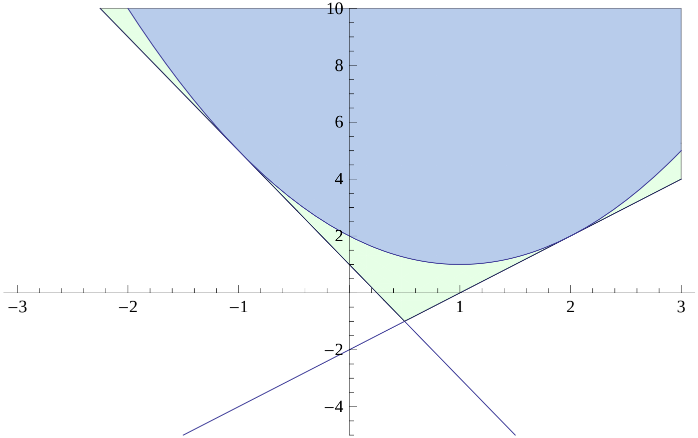
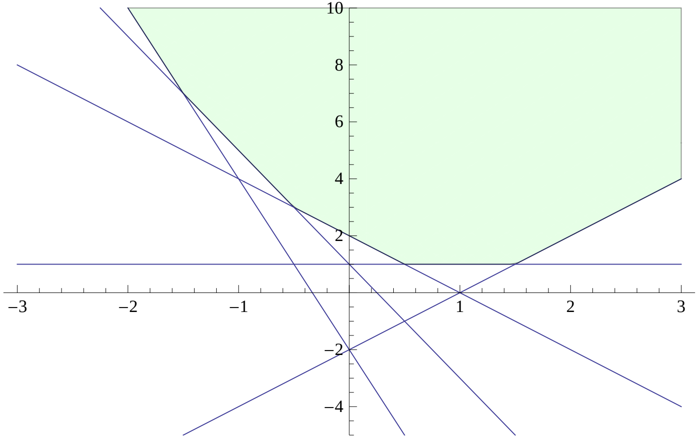


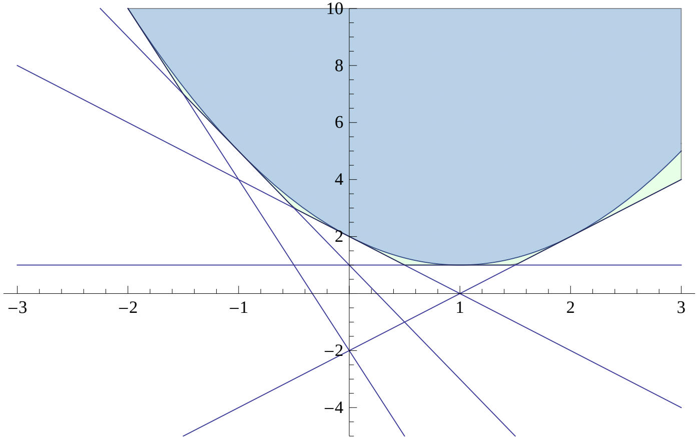
Cutting-plane method

We might end up with many linear constraints



Cutting-plane method

... which approximate the conic constraint



conic constraint:

$$\sqrt{x_1^2 + x_2^2 + ... + x_k^2} = \|x\|_2 \leq y$$

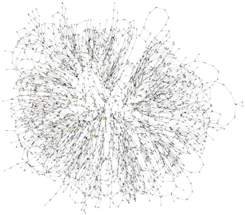
candidate solution:

$$(x^*, y^*)$$

cutting-plane (linear constraint):

$$\|x^*\|_2 + \frac{{x^*}^T}{\|x^*\|_2}(x - x^*) = \frac{{x^*}^T x}{\|x^*\|_2} \leq y$$

Polish 2003-2004 case
CPLEX: “opt status 6”
Gurobi: “numerical trouble”



Example run of cutting-plane algorithm:

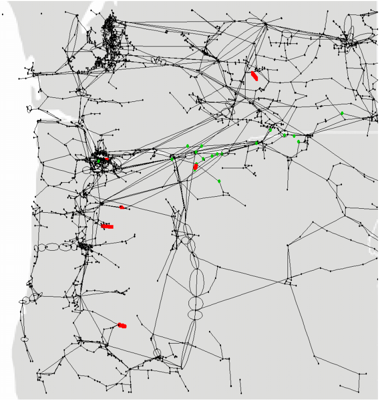
Iteration	Max rel. error	Objective
1	1.2e-1	7.0933e6
4	1.3e-3	7.0934e6
7	1.9e-3	7.0934e6
10	1.0e-4	7.0964e6
12	8.9e-7	7.0965e6

Total running time: 32.9 seconds

Back to motivating example

BPA case

- standard OPF: cost 235603, 7 lines exceed $\geq$8% of the time
- CC-OPF: cost 237297, all lines exceed $\leq$2% of the time
- run time 9.5 seconds, only one cutting plane



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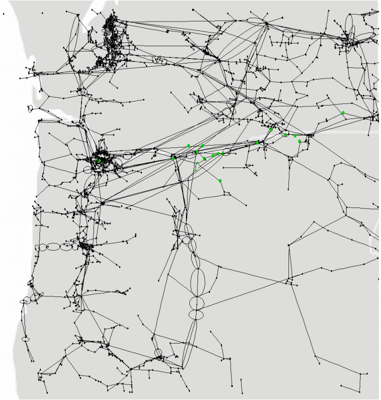

Conclusion

Our chance-constrained optimal power flow:

- safely accounts for variability in wind power between dispatches
- uses a simple control which is easily integrable into existing system
- is fast enough to be useful at the appropriate time scale