Title

Subtitle

Max Sharnoff
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candidate number
MCompSci Computer Science

Abstract

lorem ipsum or something

Contents

1 Introduction

General ideas: Full fluid dynamics simulations of the lungs is difficult. Thankfully, we don't need that. We can get pretty accurate results by building on Brody's work, simulating airflow as 1D and incompressible.

Using this model allows us to model how the airflow changes under a number of conditions, e.g. damage to the alveoli in a region, constricting bronchioles, etc.

2 Background

3 Methods

This section is mostly some temporary filler at the moment, just serving as a place to jot down the formulae that we're using inside the simulation *right now*.

3.1 Simultaneous Equations

There are essentially four simultaneous equations that we use to evaluate the state of the system at a given point in time. This section provides a summary and brief description of each, the first of which is the following:

$$P_{\text{parent}(i)} - P_i = R(i)Q_i \tag{1}$$

which specifies that the pressure differential between the distal end of branch i and its parent must equal the pressure from the resistance from the flow through this branch i. For the "root" branch, $P_{\text{parent}(i)}$ is the pressure at the trachea – typically atmospheric pressure.

The resistance term $R(i)$ is defined as following function, as given by Pedley et al. (1970):

$$R(i) = \frac{2\mu L_i c}{\pi r_i^4}\left(\frac{4\rho|Q_i|}{\mu\pi L_i}\right)^{\frac{1}{2}}$$

The parenthesized term corresponds to the Reynold's number of the flow, scaled by the ratio of the diameter of the branch to its length L_i. r_i is the radius of branch i, μ is the viscosity of the air, and $c = 1.85$ is a correction constant.

The second equation ensures incompressibility; the flow through a bifurcation must equal the sum of the flow through its children:

$$Q_i = \sum Q_{\text{child}} \tag{2}$$

where each *child* refers to any branch c with $\text{parent}(c) = i$.

The third equation maintains that the volume of an acinar region changes with the flow into or out of it for the given timestep:

$$V_i^t = V_i^{t-1} + dtQ_i^t \tag{3}$$

where dt is the timestep size, t refers to the current timestep, and V_i is the volume of the acinar region of branch i.

The final equation defines the elastic force of each acinar region, relating the pressure it exerts on its branch to the volume of the region itself and the pressure outside it:

$$P_i = \frac{1}{C_i}V_i + P_{pl}(t) \tag{4}$$

where $P_{pl}(t)$ is the pleural pressure (i.e. the "pressure" from the diaphragm, outside the acinar region) at the current time and C_i is the *compliance* of the acinar region of branch i. The pleural pressure changes over time to mimic human breathing patterns – hence why it is parameterised by t.

3.2 Units

It is worth making explict the units used for each value. After careful consideration, these were considered to provide the best spread of values with magnitude close to one, at which the accuracy of floating-point accuracy is maximized. These units are:

Type of thing ???	Units
Distance	m
Volume	m^3
Flow velocity	$\frac{\mathrm{m}^3}{\mathrm{s}}$
Density	$\frac{\mathrm{kg}}{\mathrm{m}^3}$
Pressure	Pascals $\left(\frac{\mathrm{kg}}{\mathrm{m}\cdot\mathrm{s}^2}\right)$
Compliance	$\frac{\mathrm{m}^3}{\mathrm{Pascal}} \left(\frac{\mathrm{m}^4\cdot\mathrm{s}^2}{\mathrm{kg}}\right)$
Resistance	$\frac{\mathrm{kg}}{\mathrm{m}^4\cdot\mathrm{s}}$
Viscosity	$\frac{\mathrm{kg}}{\mathrm{m}\cdot\mathrm{s}}$