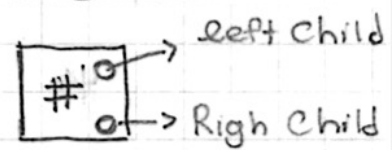
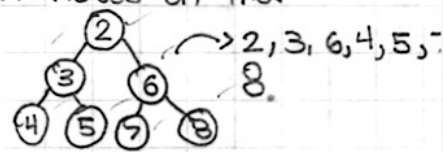


## Problem 1)

Breadth First will start at the tree root, return all nodes on that level before moving on to the next level.

Queue is a FiFo structure that will use the push(add) and pop(remove) operations



Node

Algorithm levelOrder(root):

if root is null  
return

Queue "queue"

tmp.push root

while "queue" is not empty

Node tmp = queue.front

Output the value of tmp

Pop the queue (removes tmp)

if left node of tmp exists

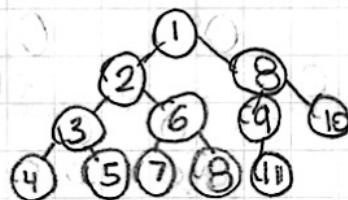
Push left node onto "queue"

1)

if the right node of tmp exists

Push right node onto "queue"

Example:



Queue: 1 2 8 3 6 9 10 4 5 7 8 11

Output: 1 2 8 3 6 9

\*we will stop here notice the nodes are queuing in level order, and the queue get longer compared to output

N is the number of nodes, with this algorithm for a level traversal we never have to visit a node more than once because we are working our way down the tree. Therefore at most we visit N nodes meaning this algorithm is  $O(N)$ , to add onto this we are always moving from left to right and down, we never move up.

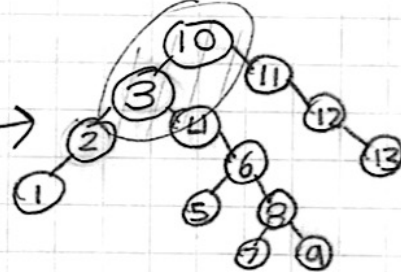
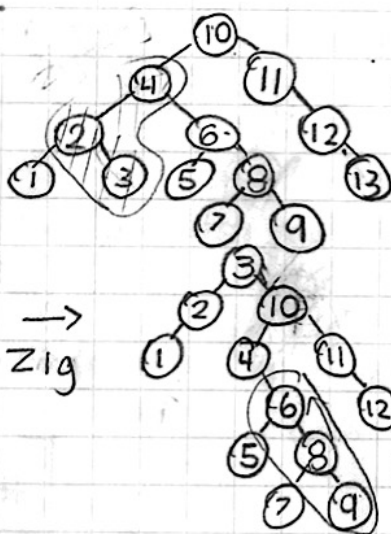
# Problem 2)

a)

Access  
Key  
3

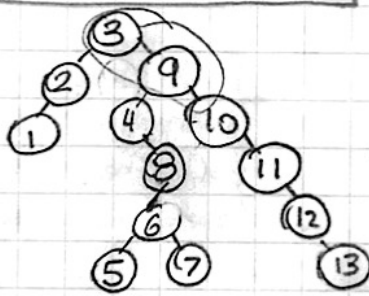
→  
Zig

Zig-Zag



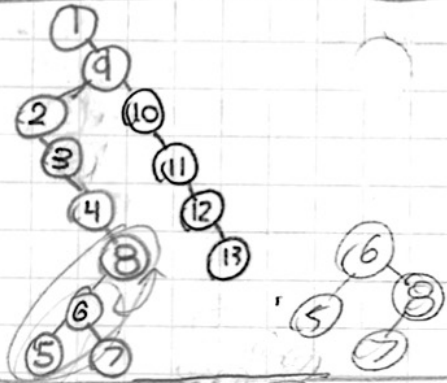
→  
Zig-Zag

→  
Zig



→  
Zig-Zig

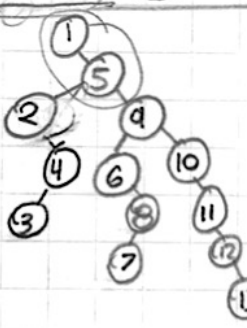
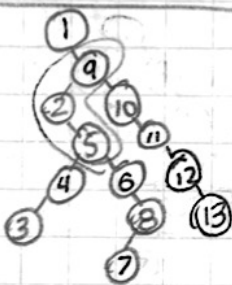
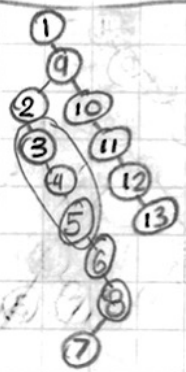
→  
Zig



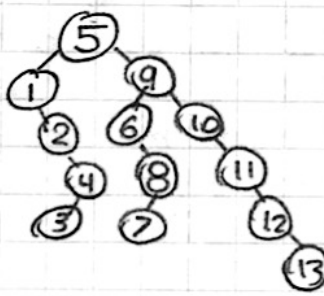
→  
ZigZig

→  
ZigZig

→  
ZigZag



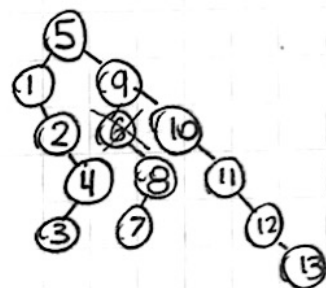
→  
Zig



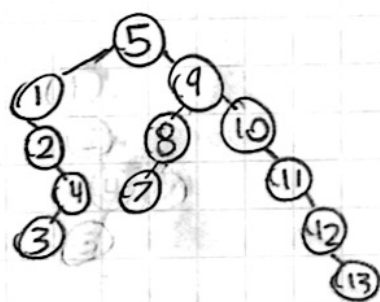
Access  
Key  
5

Problem 2)

b)



Delete 6



Problem 3)

(a)

$$T_{\text{worst}} = \Theta(N)$$

$$T_{\text{avg}} = \Theta(N)$$

$$T_{\text{best}} = \Theta(N)$$

regardless of the state of the array we will have to move all the current entries of the array to the right the average, best, and worst scenario are equal.

push 3  $\left\{ \begin{array}{l} S[] = \frac{4}{0} \quad \frac{5}{1} \quad \frac{6}{2} \quad \frac{4}{3} \quad n=4 \\ S[] = \frac{3}{0} \quad \frac{4}{1} \quad \frac{5}{2} \quad \frac{6}{3} \quad \frac{4}{4} \quad n=5 \end{array} \right\}$  as you can see it is directly proportional to length.

(b) 0. insert 1  $\frac{1}{0}$  1 move  
1. insert 2  $\frac{2}{0} \rightarrow \frac{1}{1}$  2 move  
2. insert 3  $\frac{3}{0} \rightarrow \frac{2}{1} \rightarrow \frac{1}{2}$  3 move  
3. insert 4  $\frac{4}{0} \rightarrow \frac{3}{1} \rightarrow \frac{2}{2} \rightarrow \frac{1}{3}$  4 move

as we observe more of one operation we see it increases with size for that operation.

$\therefore 1 + 2 + 3 + 4 + 5 \dots + N$  for each insertion there is less movements.

$$\sum_{i=0}^N i = \frac{(N)(N+1)}{2} = \left[ \frac{N^2 + N}{2} \right] \therefore \boxed{O(N^2)}$$

The total cost for  $N$  operations is  $O(N^2)$  therefore  $N^2/N = N$

$\therefore$  Amortized running time  $\Rightarrow O(N)$

# Problem 4)

(a) The worst case running time would be  $\Theta(N)$  this will happen when the array is all 1's because the while loop will check all entries.

$\therefore \Theta(N)$

(b)

$a[5]$	$a[4]$	$a[3]$	$a[2]$	$a[1]$	$a[0]$	
0	0	0	0	0	0	0
0	0	0	0	0	1	1
0	0	0	0	1	0	2
0	0	0	0	1	1	3
0	0	0	1	0	0	4
0	0	0	1	0	1	5
0	0	0	1	1	0	6
0	0	0	1	1	1	7
0	0	1	0	0	0	8
0	0	1	0	0	1	9
0	0	1	0	1	0	10
0	0	1	0	1	1	11
0	0	1	1	0	0	12
0	0	1	1	0	1	13
0	0	1	1	1	0	14
0	0	1	1	1	1	15
0	1	0	0	0	0	16

\* this example  $k=5$

(c) every step  $a[0]$  is flipped, every second step  $a[1]$  is flipped every fourth step  $a[2]$  is flipped.

Therefore if we have  $N$  operation  $a[0]$  will flip  $N$  times,  $a[1]$  will flip  $N/2$  times,  $a[2]$  will flip  $N/4$  times.

$$\hookrightarrow N + N/2 + N/4 + N/8 \dots N/2^i$$

let  $k$  be # of bits used.

$$\sum_{i=0}^{k-1} N/2^i = N \sum_{i=0}^{k-1} 1/2^i$$

\* Pull out to front

\* we will over estimate  $\sum_{i=0}^{\infty} 1/2^i = 2$

$$\therefore 2N$$

Now we divide by cost of  $N$  operation  $2N/N = 2$

Amortized running time  $\therefore \Theta(1)$