MA2C03: ASSIGNMENT 3
DUE BY FRIDAY, MARCH 2
IN THE MATHS OFFICE ROOM 0.6

Please write down clearly both your name and your student ID number on everything you hand in.

1) (10 points) Let (V, E) be the graph with vertices a, b, c, d, e, f, g, and h, and edges ab, bc, cd, de, ef, af, bg, and eh.

(a) Is this graph connected? Justify your answer.
(b) Does this graph have an Eulerian trail? Justify your answer.
(c) Does this graph have an Eulerian circuit? Justify your answer.
(d) Does this graph have a Hamiltonian circuit? Justify your answer.
(e) Is this graph a tree? Justify your answer.

2) Consider the connected graph with vertices A, B, C, D, E, F, G, H, I, J, K, L, M and N and with edges, listed with associated costs, in the following table:

HI	JL	MN	EH	AB	EF	FI	LM	JK	AD
2	2	2	3	4	5	6	6	6	7
GH	GJ	HK	DG	KM	BE	DE	BC	CF	IN
8	8	9	9	10	10	11	11	12	13

(a) (2 points) Draw the graph and label each edge with its cost.
(b) (9 points) Determine the minimum spanning tree generated by Kruskal's Algorithm, where that algorithm is applied with the queue specified in the table above. For each step of the algorithm, list the edge that is added and draw the graph.
(c) (9 points) Determine the minimum spanning tree generated by Prim's Algorithm, starting from the vertex F, where that algorithm is applied with the queue specified in the table above. For each step of the algorithm, list the edge that is added and draw the graph.

3) (10 points) Let $(\mathcal{V}, \mathcal{E})$ be the directed graph with vertices A, B, C, D, E, and F and edges (A, A), (A, B) (A, F), (C, B), (C, C), (C, D), (E, D), (E, E), and (E, F).

(a) Draw this graph.
(b) Write down this graph's adjacency matrix.

(c) Give an example of an isomorphism φ from the graph $(\mathcal{V}, \mathcal{E})$ to itself such that $\varphi(C) = C$. Note that an isomorphism of directed graphs should also respect the direction of the edges.

4) (20 points) Let R be a relation on a set $V = \{a, b, c, d\}$ given by

$$R = \{(a,a), (a,b), (b,d), (b,b), (a,c), (a,d), (b,c), (c,c), (d,c), (d,d)\}.$$

(a) Using the one-to-one correspondence between relations on finite sets and directed graphs, draw the directed graph corresponding to the relation R.
(b) Is R an equivalence relation? Justify your answer.
(c) If R is not an equivalence relation, which ordered pairs would have to be added to R to make it into an equivalence relation?

Let R' be a relation on a set $V = \{a, b, c\}$ with three elements.

(d) What is the minimum number of ordered pairs in R' for the relation R' to be reflexive? What should those pairs be? Justify your answer.
(e) What is the minimum number of ordered pairs in R' for the relation R' to be symmetric? Justify your answer.
(f) What is the minimum number of ordered pairs in R' for the relation R' to be transitive? Justify your answer.