Quiz No.3

1- Use three-digit rounding arithmetic to perform $\frac{\frac{13}{14}-\frac{6}{7}}{2e-5.4}$. Compute the relative error with the exact value determined to four digits.

Answer: $\frac{13}{14} = 0.929$ and $\frac{6}{7} = 0.857$ and $e = 2.72$. So, $\frac{\frac{13}{14}-\frac{6}{7}}{2e-5.4} = 1.80$ The exact value is 1.956 which gives the R.E as 0.0788.

2- How many multiplications and additions are required to determine a sum of the form $\sum_{i=1}^{n}\sum_{j=1}^{i} a_i b_j$

Answer: For each i the inner sum $\sum_{j=1}^{i} a_i b_j$ requires i multiplications and $i-1$ additions. So, totally there are $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ multiplications. Once the inner sum $\sum_{i=1}^{n} i-1 = \frac{n(n+1)}{2} - n$ are computed,$n-1$ additions are required to complete the sum. Therefore, total addition is $\frac{n(n+1)}{2} - n + (n-1)$.

3- Which one of the following methods is convergent to compute $\sqrt[3]{21}$, assuming $p_0 = 1$.(Hint: use the fixed-point iteration and convergence condition)

$$\textbf{(a).}\ \ p_n = \frac{20p_{n-1} + \frac{21}{p_{n-1}^2}}{21}$$

$$\textbf{(b).}\ \ p_n = p_{n-1} - \frac{p_{n-1}^4 - 21p_{n-1}}{p_{n-1}^2 - 21}$$

Answer: (a). $g(x) = \frac{20x+\frac{21}{x^2}}{21} \Rightarrow g'(x) = \frac{20}{21} - \frac{2}{x^3}$. So, $g'(\sqrt[3]{21}) = \frac{6}{7} \approx 0.857 < 1$

(b). $g(x) = x - \frac{x^4-21x}{x^2-21} \Rightarrow g'(x) = \frac{-2x^5+x^4+84x^3-63x^2}{(x221)^2}$. So, $g'(\sqrt[3]{21}) = 5.706 > 1$

By the corollary of **contraction mapping theorem**, algorithm **(a)** is convergent.

4- Construct an approximating polynomial for the following given data:

Use the obtained polynomial to find f(1).

Answer: As $\acute{f}$ is given, we can construct hermite polynomial which is of

x	f	$\acute{f}$
0	1	2
0.5	3	5

degree 3 in this question.
$H_3(x) = a + b(x - x_0) + c(x - x_0)^2 + d(x - x_0)^2(x - x_1)$. $x_0 = 0$ and $x_1 = 0.5$ so $H_3(x) = a + bx + cx^2 + dx^2(x - 0.5)$.

Interpolating condition should be satisfied: $H_3(0) = 1$, $H_3(0.5) = 3$, $\acute{H}_3(0) = 2$ and $\acute{H}_3(0.5) = 5$. These give $a = 1$, $a + 0.5b + 0.25c = 3$, $b = 2$ and $b + c + 0.25d = 5$. All these together yield $H_3(x) = 1 + 2x + 4x^2 - 4x^2(x - 0.5)$ and $f(1) \approx H_3(1) = 5$.

5- A natural cubic spline S on [0,2] is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3 & x \in [0, 1] \\ S_1(x) = 2 + b(x - 1) + c(x - 1)^2 + d(x - 1)^3 & x \in [1, 2] \end{cases}$$

(i). smoothness condition at $x = 1$: $S_0'(1) = S_1'(1)$ and $S"_0(1) = S"_1(1)$. These give: $2 - 3x^2|_{x=1} = b + 2c(x - 1) + 3d(x - 1)^2|_{x=1}$ and $-6x|_{x=1} = 2c + 6d(x - 1)|_{x=1}$. So, $b = -1$ and $c = -3$.
(ii). Free boundary condition: $S"_0(0) = 0$ and $S"_1(2) = 0$ which gives $d = 1$.