MA2C03: ASSIGNMENT 4
DUE BY FRIDAY, APRIL 13
IN THE MATHS OFFICE ROOM 0.6

Please write down clearly both your name and your student ID number on everything you hand in. Please attach a cover sheet with a declaration confirming that you know and understand College rules on plagiarism. Details can be found on `http://tcd-ie.libguides.com/plagiarism/declaration`.

1) (20 points)

(a) Is $\{(x, \sin(\pi x)) \mid x \in \mathbb{R}\} \cap \{\mathbb{Q} \times \mathbb{Z}\}$ finite, countably infinite, or uncountably infinite? Justify your answer.

(b) Is $\{(x, y) \in \mathbb{R}^2 \mid xy = 1\}$ finite, countably infinite, or uncountably infinite? Justify your answer.

(c) Is $\left\{(x, y) \in \mathbb{R}^2 \mid \frac{x^2}{4} + \frac{y^2}{9} = 1\right\} \cap \left\{(x, y) \in \mathbb{R}^2 \mid \frac{x^2}{9} + \frac{y^2}{4} = 1\right\}$ finite, countably infinite, or uncountably infinite? Justify your answer.

(d) Let $A = \{0, 1\}$. Is $(0^* \circ 1^*) \cap \{A^* \circ 11 \circ A^*\}$ finite, countably infinite, or uncountably infinite? Justify your answer.

2) (20 points)

(a) Let L be the language consisting of the binary representations of all odd natural numbers. Write down the algorithm of a Turing machine that recognizes L. Process the following strings according to your algorithm: ϵ, 0, 11, and 100.

(b) Write down the transition diagram of the Turing machine from part (a) carefully labelling the initial state, the accept state, the reject state, and all the transitions specified in your algorithm.

3) (10 points) Write down the algorithm of an enumerator that prints out EXACTLY ONCE every odd natural number divisible by 5. (Hint: Order the words on the input tape.)