Quiz No.4

1- **True or false:** There are arbitrarily many different mathematical functions that interpolate a given set of data points.

Answer: True

2- **True or false:** If the polynomial interpolating a given set of data points is unique, then so is the representation of that polynomial.

Answer: False

3- Let $y_2(x) = \sum_{i=0}^{2} L_i(x) f_i$ be a quadratic Lagrange polynomial that passes through the points (1,0), (3,1.1), and (5,1.6). Find $L_0(2)$.

Answer: $L_0(x) = \frac{1}{8}\left((x-3)(x-5)\right)$. So, $L_0(2) = \frac{3}{8}$

4- A clamped cubic spline s for a function f is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + Bx + 2x^2 - 2x^3, & x \in [0,1] \\ S_1(x) = 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3, & x \in [1,2] \end{cases}$$

find $f'(0)$.

Answer: Smoothness condition $S_0'(1) = S_1'(1)$ gives $B - 2 = b$, and continuity at x=1 gives $B = 0$. $f'(0) = S_0'(0) = B = 0$

5- Evaluate the definite integral $\int_1^3 \frac{x}{1+x^2}\, dx$ using MID-POINT and TRAPEZOIDAL rule.

Answer:$M(f) = (b-a) f(\frac{a+b}{2}) = 2 \times f(2) = 0.8$, $T(f) = \frac{b-a}{2}\left(f(a) + f(b)\right) = f(1) + f(3) = 0.8$

6- Take the integral of f(x) which is represented by the following data using the Simpson's rule.

x	1	3	5
f(x)	5	12	9

Answer: $h = \frac{b-a}{2} = 2$, $x_0 = a = 1$, $x_1 = a + h = 3$, and $x_2 = b = 5$. $S(f) = \frac{b-a}{6}\left(f(a) + 4f(x_1) + f(b)\right) = 41.3$