MA2C03: ASSIGNMENT 2
DUE BY FRIDAY, JANUARY 19
AT LECTURE OR IN THE MATHS OFFICE ROOM 0.6

Please write down clearly both your name and your student ID number on everything you hand in.

1) (20 points)

(a) Describe the formal language over the alphabet $\{a, b, c\}$ generated by the context-free grammar whose non-terminals are $\langle S \rangle$ and $\langle A \rangle$, whose start symbol is $\langle S \rangle$, and whose production rules are the following:
(1) $\langle S \rangle \to a\langle A \rangle$
(2) $\langle A \rangle \to b\langle S \rangle$
(3) $\langle S \rangle \to c$
In other words, describe the structure of the strings generated by this grammar.

(b) Is this grammar regular? Justify your answer.

(c) Is this grammar in normal form? If it is not in normal form, then modify it to make it be in normal form. Explain why it generates the same language after your modifications.

(d) Write down a regular expression that gives the language from part (a) and justify your answer.

2) (10 points) Let L be the language consisting of all the strings of the form $10^{2m+1}1$ for m a natural number, $m \geq 0$, i.e.

$$\{101,\ 10001,\ 1000001,\ 100000001,\ \ldots\}$$

(a) Draw a deterministic finite state acceptor that accepts the language L. Carefully label all the states including the starting state and the finishing states as well as all the transitions. Make sure you justify it accepts all strings in the language L and no others.

(b) Devise a regular grammar in normal form that generates the language L. Be sure to specify the start symbol, the non-terminals, and all the production rules. Make sure you justify it generates all strings in the language L and no others.

3) (10 points) Let M be the language

$$\{0101,\ 001001,\ 00010001,\ 0000100001,\ \ldots\}$$

whose words consist of some positive number n of occurrences of the digit 0, followed by the digit 1, followed by n further occurrences of the digit 0, and followed by the digit 1. (In particular, the number of occurrences of 0 preceding the first 1 is equal to the number of occurrences of 0 preceding the second 1.)

(a) Use the Pumping Lemma to show this language is not regular.
(b) Write down the production rules of a context-free grammar that generates exactly M.

4) (20 points) Let (V, E) be the graph with vertices a, b, c, d, e, f, g, and h, and edges ab, bc, cd, de, ef, af, bg, and eh.

(a) Draw this graph.
(b) Write down this graph's incidence table and its incidence matrix.
(c) Write down this graph's adjacency table and its adjacency matrix.
(d) Is this graph complete? Justify your answer.
(e) Is this graph bipartite? Justify your answer.
(f) Is this graph regular? Justify your answer.
(g) Does this graph have any regular subgraph? Justify your answer.
(h) Give an example of an isomorphism φ from the graph (V, E) to itself satisfying that $\varphi(b) = e$.
(i) Is the isomorphism from part (h) unique or can you find another isomorphism ψ that is distinct from φ but also satisfies that $\psi(b) = e$? Justify your answer.