Our PnP problem is (constrained on all valid rotations matrices SO(3)):

$$R, t = \underset{R,t}{\operatorname{argmin}} \sum_{i=1}^{n} \|(I - V_i)(RX_i + t - c_i)\|^2$$

When finding t_{opt} and substituting it back, our problem becomes:

$$R = \underset{R}{\operatorname{argmin}}\, r^\top M_r r + M_c r + M_{cc}$$

Where

$$M = \sum_i ([C(X_i)|c_i] + T_{3\times10})^\top Q_i ([C(X_i)|c_i] + T_{3\times10}) = \begin{bmatrix} M_r & \frac{1}{2} M_c \\ \frac{1}{2} M_c & M_{cc} \end{bmatrix} \in \begin{bmatrix} R_{9\times9} & 1 \\ 1 & 1 \end{bmatrix}$$

Under the assumption of $c_i = 0$, any M containing c gets zero and we left with:

$$R = \underset{r}{\operatorname{argmin}}\, r^\top M_r r$$

And

$$M_r = \sum_i (C(X_i) + T_{3\times9})^\top Q_i (C(X_i) + T_{3\times9})$$

Since $Q_i = (I - V_i)^\top (I - V_i)$, we can re-write M_r as $M_r = \sum_i B^\top B$ where:

$$B = (I - V_i)(C(X_i) + T_{3\times9})$$

(maybe this can be also derived from the first objective). according to this expression, M is semi positive definite. There are other matric A, such that $M_r = A^\top A$, we get:

$$R = \underset{r}{\operatorname{argmin}}\, r^\top M_r r = \underset{r}{\operatorname{argmin}}\, r^\top A^\top A r = \underset{r}{\operatorname{argmin}}\, r^\top A^\top A r = \|A \cdot r\|^2$$

Now, we will parameterize rotation vector r using quaternions:

$$R = \underset{r}{\operatorname{argmin}} \|A \cdot r\|^2 = \underset{q \in \mathbb{R}^4}{\operatorname{argmin}} \|A \cdot f(q)\|^2$$

Where, using [this article](#):

$$f: \mathbb{R}^4 \to \mathbb{R}^9, f(q) = \begin{bmatrix} q_1^2 + q_2^2 - q_3^2 - q_4^2, & 2q_2q_3 - 2q_1q_4 & 2q_2q_4 + 2q_1q_3 \\ 2q_2q_3 + 2q_1q_4 & q_1^2 + q_3^2 - q_2^2 - q_4^2 & 2q_3q_4 - 2q_1q_2 \\ 2q_2q_4 - 2q_1q_3 & 2q_3q_4 + 2q_1q_2 & q_1^2 + q_4^2 - q_2^2 - q_3^2 \end{bmatrix}$$

And the constraint of $R \in SO(3)$ is replaced in $\|q\| = 1$.

Therefore,

$$R = \underset{\|q\|=1}{\operatorname{argmin}} \|A \cdot f(q)\|^2 = \underset{\|q\|=1}{\operatorname{argmin}} \sum_{1}^{9} \left(R_i(A) f(q)\right)^2 = \underset{q \in \mathbb{R}^4}{\operatorname{argmin}}\, p\left(\frac{q}{\|q\|}\right)$$

Where $\forall x \in \mathbb{R}^4$: $p(x)$ as denoted above. Since f was squared, f^2 is quadratic (in 4 variables of x) and so does p. Moreover $p: \mathbb{R}^4 \to \mathbb{R}^9$.