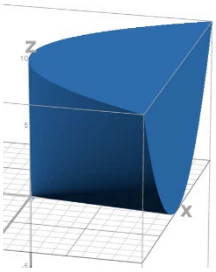
Let $S(3) \subset \mathbb{R}^{2\times 2}$ be the set of all symmetric matrices (of dimension 3).

Hence, we can describe them as $\begin{pmatrix} x & y \\ y & z \end{pmatrix}$ and represent them as $\{(x, y, z) \in \mathbb{R}^3\}$. That means, that each point $(x, y, z) \in \mathbb{R}^3$ corresponds to symmetric matrix.

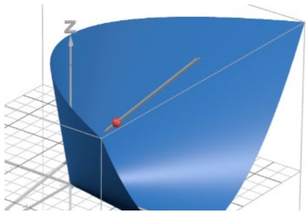
Let $S_+(3) \subset S(3)$ be the set of all psd matrices (2x2). This is represented by the group $\{(x, y, z) \in \mathbb{R}^3 | x \geq 0, xz \geq y^2\}$ (Sylvester's theorem).



Let $A \coloneqq B^\top B \in S_+(3)$ and let $A = \begin{pmatrix} x_0 & y_0 \\ y_0 & z_0 \end{pmatrix}$.

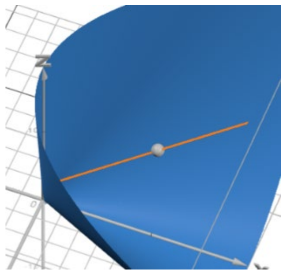
Now, define $\ell \coloneqq \{A + t \cdot I | t \in \mathbb{R}\} \subseteq S(3)$. Say $C \in \ell$,

We can describe C as $(x_0, y_0, z_0) + t \cdot (1,0,1) \in \mathbb{R}^3$ which is a parametric equation of a line.



Now we want the intersection of ℓ with $S_+(3)$. In other words,
$\ell' \subseteq S_+(3)$ and $\ell' \subseteq \ell$.

$$\ell' = \{(x_0 + t, y_0, z_0 + t) | t \in \mathbb{R}, x \geq 0, xz \geq y^2\}$$



So, the intersection of this line with $S_+(3)$ is the same line but restricted to the cone (in yellow).

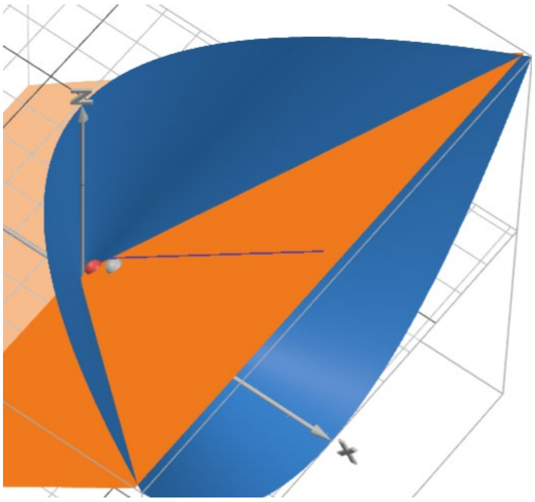
To finish with, we take a plane that contains ℓ. I chose plane π:

$$\pi: \{(x_0 + t, y, z_0 + t) | t, y \in \mathbb{R}\}$$

And the intersection $S_3^+ \cap \pi$ is:

$$\pi' = \{(x_0 + t, y, z_0 + t) | t, y \in \mathbb{R}, x \geq 0, xz \geq y^2\}$$

Which is infinite 2d cone (the line bolded in purple).



The full guided simulation is here: https://www.desmos.com/3d/8f904f3eaa

Appendix for myself: let $B = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$, I will prove that $A = B^{\top}B$ is PSD:

$$A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2 + c^2 & ab + cd \\ ab + cd & b^2 + d^2 \end{pmatrix}$$

Two things should hold. $a^2 + c^2$ is trivial.

For the second minor, recall from the averaging theorem inequality that:

$$abcd = \sqrt[2]{(ab)^2(cd)^2} \leq \frac{(ab)^2 + (cd)^2}{2}$$

$$(ab)^2 + (cd)^2 \geq 2abcd$$
$$(ab)^2 + (ad)^2 + (bc)^2 + (cd)^2 \geq (ad)^2 + 2abcd + (bc)^2$$
$$(a^2 + c^2)(b^2 + d^2) \geq (ad + bc)^2 \Longrightarrow \det A \geq 0$$