lab3

Q1

a. One problem in Monte Carlo methods is in obtaining sample points from complex distributions p(x). Many Monte Carlo methods can help us sample points from p(x).

b. Metropolis Hastings Algorithm generalizes Metropolis Algorithm. Metropolis Hastings Algorithm does not require q(.|.) to be symmetric (q(a1|a2) does not have to equal q(a2|a1)).

c. Both methods can reduce multicollinearity, reduce overfitting. Ridge can shrink coefficient to zero. Lasso has the effect of forcing some of the coefficient estimates to be exactly zero. To some extent, Lasso can do variable selection.

d. Independence of irrelevant alternatives (IIA): the ratio of the probabilities of choosing two alternatives is independent of the presence or attributes of any other alternative.

Q2 a

```
gas <- read.csv('gas_mileage.csv')
library(quantreg)
```

```
## Loading required package: SparseM
```

```
##
## Attaching package: 'SparseM'
```

```
## The following object is masked from 'package:base':
##
##     backsolve
```

```
fit1 <- rq(Mpg~.,tau=seq(0.05, 0.95, by=0.05), data=gas)
```

```
## Warning in rq.fit.br(x, y, tau = tau, ...): Solution may be nonunique
```

```
summary(fit1)
```

```
## Warning in rq.fit.br(x, y, tau = tau, ci = TRUE, ...): Solution may be
## nonunique
```

```
## Warning in rq.fit.br(x, y, tau = tau, ci = TRUE, ...): Solution may be
## nonunique

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## nonunique
```

```
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.05
##
## Coefficients:
##                 coefficients   lower bd       upper bd
## (Intercept)       7.505845e+01 -1.797693e+308  1.797693e+308
## Displacement     -3.701000e-02 -1.797693e+308  1.797693e+308
## Hpower           -1.893800e-01 -1.797693e+308  1.797693e+308
## Torque            1.094900e-01 -1.797693e+308  1.797693e+308
## Comp_ratio       -3.509360e+00 -1.797693e+308  1.797693e+308
## Rear_axle_ratio   3.866260e+00 -1.797693e+308  1.797693e+308
## Carb_barrels      2.145330e+00 -1.797693e+308  1.797693e+308
## No._speeds       -2.299040e+00 -1.797693e+308  1.797693e+308
## Length            1.753600e-01 -1.797693e+308  1.797693e+308
## Width            -6.623400e-01 -1.797693e+308  1.797693e+308
## Weight           -3.030000e-03 -1.797693e+308  1.797693e+308
## Trans._type      -9.004500e-01  -1.792682e+01  1.797693e+308
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.1
##
## Coefficients:
##                 coefficients   lower bd       upper bd
## (Intercept)       7.505845e+01  -2.640074e+02   1.965771e+02
## Displacement     -3.701000e-02  -3.574400e-01   6.540000e-02
## Hpower           -1.893800e-01  -7.592400e-01   1.053380e+00
## Torque            1.094900e-01  -3.856000e-01   8.116000e-01
## Comp_ratio       -3.509360e+00  -1.141334e+01   7.802265e+01
## Rear_axle_ratio   3.866260e+00  -1.949856e+01   3.144942e+01
## Carb_barrels      2.145330e+00  -1.083878e+01   1.214711e+01
## No._speeds       -2.299040e+00  -9.998130e+00   1.812914e+01
## Length            1.753600e-01  -2.232600e-01  1.797693e+308
## Width            -6.623400e-01 -1.797693e+308   1.918620e+00
## Weight           -3.030000e-03  -1.060100e-01   1.284000e-02
## Trans._type      -9.004500e-01  -1.561480e+00  1.797693e+308
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.15
##
## Coefficients:
##                 coefficients   lower bd       upper bd
## (Intercept)       7.505845e+01  -9.002075e+01   1.453873e+02
## Displacement     -3.701000e-02  -2.327100e-01   2.910000e-02
## Hpower           -1.893800e-01  -6.259600e-01   6.757800e-01
## Torque            1.094900e-01  -2.939300e-01   5.021700e-01
## Comp_ratio       -3.509360e+00  -6.623030e+00   2.989379e+01
## Rear_axle_ratio   3.866260e+00  -1.374687e+01   1.842395e+01
## Carb_barrels      2.145330e+00  -3.081880e+00   6.189830e+00
## No._speeds       -2.299040e+00  -9.698530e+00   1.010556e+01
## Length            1.753600e-01  -8.571000e-02   2.162340e+00
## Width            -6.623400e-01  -3.833210e+00   4.010500e-01
## Weight           -3.030000e-03  -1.328000e-02   1.131000e-02
## Trans._type      -9.004500e-01  -1.446450e+00  1.797693e+308
```

```
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.2
##
## Coefficients:
##                 coefficients   lower bd       upper bd
## (Intercept)       6.259344e+01  -8.228754e+01   1.409044e+02
## Displacement     -1.956000e-02  -2.040000e-01   3.166000e-02
## Hpower           -1.639200e-01  -6.078400e-01   4.992700e-01
## Torque            8.250000e-02  -3.315400e-01   4.444400e-01
## Comp_ratio       -2.796880e+00  -6.437820e+00   1.030132e+01
## Rear_axle_ratio   2.859870e+00  -4.345210e+00   1.796188e+01
## Carb_barrels      1.786780e+00  -1.398360e+00   3.303940e+00
## No._speeds       -1.428330e+00  -9.994610e+00   1.355025e+01
## Length            1.922900e-01  -1.138700e-01   1.237590e+00
## Width            -5.698600e-01  -3.078290e+00   5.256000e-02
## Weight           -4.420000e-03  -1.309000e-02   1.036000e-02
## Trans._type      -4.470000e-01  -7.606060e+00  1.797693e+308
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.25
##
## Coefficients:
##                 coefficients   lower bd       upper bd
## (Intercept)       5.939339e+01  -8.167520e+01   1.244924e+02
## Displacement     -1.917000e-02  -2.322600e-01   2.464000e-02
## Hpower           -1.745200e-01  -5.456900e-01   3.766700e-01
## Torque            8.982000e-02  -3.224100e-01   4.848900e-01
## Comp_ratio       -2.721790e+00  -6.584030e+00   1.024147e+01
## Rear_axle_ratio   2.507430e+00  -6.154160e+00   1.816992e+01
## Carb_barrels      1.825000e+00  -1.590480e+00   3.191410e+00
## No._speeds       -9.305200e-01  -1.021943e+01   1.580215e+01
## Length            1.858100e-01  -1.563300e-01   4.075000e-01
## Width            -5.308900e-01  -2.755050e+00   2.577000e-02
## Weight           -4.380000e-03  -1.345000e-02   9.000000e-03
## Trans._type      -4.767800e-01  -7.956070e+00  1.797693e+308
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.3
##
## Coefficients:
##                 coefficients lower bd  upper bd
## (Intercept)      54.06294    -68.83438 103.95882
## Displacement     -0.03751     -0.22369   0.02329
## Hpower           -0.14300     -0.49277   0.31943
## Torque            0.09195     -0.33155   0.43812
## Comp_ratio       -2.15210     -6.28234   9.89148
## Rear_axle_ratio   2.66851     -6.44198  18.14440
## Carb_barrels      1.70373     -3.17755   3.36442
## No._speeds       -1.60050    -10.35158  14.36612
## Length            0.19950     -0.16919   0.42062
## Width            -0.52344     -1.20202   0.04226
## Weight           -0.00444     -0.00998   0.00998
## Trans._type       0.00138     -9.84964  18.44084
##
```

```
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.35
##
## Coefficients:
##                 coefficients lower bd  upper bd
## (Intercept)      33.61471    -64.66366 114.81804
## Displacement     -0.03139     -0.21008   0.03422
## Hpower           -0.20400     -0.44658   0.30928
## Torque            0.13156     -0.27674   0.31270
## Comp_ratio       -0.25080     -5.45183   9.81983
## Rear_axle_ratio   3.65908     -7.03406  14.90364
## Carb_barrels      1.23102     -3.39051   3.63315
## No._speeds        1.41816    -10.18349  11.84650
## Length            0.23047     -0.16893   0.42550
## Width            -0.72708     -1.12616   0.06438
## Weight           -0.00460     -0.00969   0.01709
## Trans._type       1.21189    -13.55527  19.91186
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.4
##
## Coefficients:
##                 coefficients lower bd  upper bd
## (Intercept)      39.79782    -38.02844 113.89174
## Displacement     -0.13338     -0.20434   0.03074
## Hpower           -0.18288     -0.42267   0.26439
## Torque            0.24622     -0.04369   0.30530
## Comp_ratio       -0.46214     -5.25613   8.45928
## Rear_axle_ratio   9.72169     -7.02632  13.60216
## Carb_barrels      1.13543     -2.96256   3.81884
## No._speeds       -4.67178    -10.06583  11.59511
## Length            0.22521     -0.17691   0.45815
## Width            -0.71592     -0.96215   0.04934
## Weight           -0.00493     -0.00970   0.01547
## Trans._type       2.03764    -13.21112  13.78413
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.45
##
## Coefficients:
##                 coefficients lower bd  upper bd
## (Intercept)      39.79782    -56.56228 106.18042
## Displacement     -0.13338     -0.20343   0.02052
## Hpower           -0.18288     -0.41773   0.25501
## Torque            0.24622     -0.01230   0.30261
## Comp_ratio       -0.46214     -6.14907   8.28425
## Rear_axle_ratio   9.72169     -6.94519  13.35862
## Carb_barrels      1.13543     -2.98675   4.21629
## No._speeds       -4.67178    -10.00668  11.72722
## Length            0.22521     -0.18485   0.43406
## Width            -0.71592     -1.16886   0.17787
## Weight           -0.00493     -0.00847   0.01610
## Trans._type       2.03764    -15.49451   7.66150
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
```

```
##
## tau: [1] 0.5
##
## Coefficients:
##                 coefficients lower bd  upper bd
## (Intercept)      41.98707    -50.15249  99.41846
## Displacement     -0.13873     -0.19219   0.01530
## Hpower           -0.17596     -0.39591   0.25625
## Torque            0.24692     -0.02048   0.29231
## Comp_ratio       -1.14223     -6.05074   8.13403
## Rear_axle_ratio   9.03682     -6.58867  12.87569
## Carb_barrels      1.14349     -2.74990   4.52378
## No._speeds       -3.91968     -9.28143   7.94056
## Length            0.17526     -0.17574   0.40710
## Width            -0.54095     -1.21406   0.19273
## Weight           -0.00472     -0.01453   0.01580
## Trans._type       1.99845    -16.08817  12.71580
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.55
##
## Coefficients:
##                 coefficients lower bd  upper bd
## (Intercept)      37.45543    -44.82510  83.71515
## Displacement     -0.15632     -0.18890   0.00376
## Hpower           -0.16826     -0.39300   0.25379
## Torque            0.26247     -0.01384   0.30666
## Comp_ratio       -0.66081     -6.06884   6.68266
## Rear_axle_ratio   9.51487     -6.24103  12.86802
## Carb_barrels      1.04178     -3.13414   4.18934
## No._speeds       -4.62124     -9.61926   8.96272
## Length            0.13267     -0.10225   0.52539
## Width            -0.40408     -1.49854   0.22254
## Weight           -0.00460     -0.01807   0.01441
## Trans._type       2.58728    -17.09597  11.63718
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.6
##
## Coefficients:
##                 coefficients lower bd  upper bd
## (Intercept)     -12.38280    -43.03643  95.08684
## Displacement     -0.12421     -0.41794  -0.00553
## Hpower           -0.03070     -0.35527   0.24415
## Torque            0.16519     -0.02707   0.42386
## Comp_ratio        2.08188     -5.70257   6.47639
## Rear_axle_ratio  10.01460     -6.14963  12.04353
## Carb_barrels      1.43890     -2.71410   4.09294
## No._speeds       -7.01770     -9.16567   8.71186
## Length            0.37290     -0.10354   0.51369
## Width            -0.29559     -1.54439   0.35325
## Weight           -0.01231     -0.02441   0.00933
## Trans._type       3.20547    -17.37450  10.84163
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
```

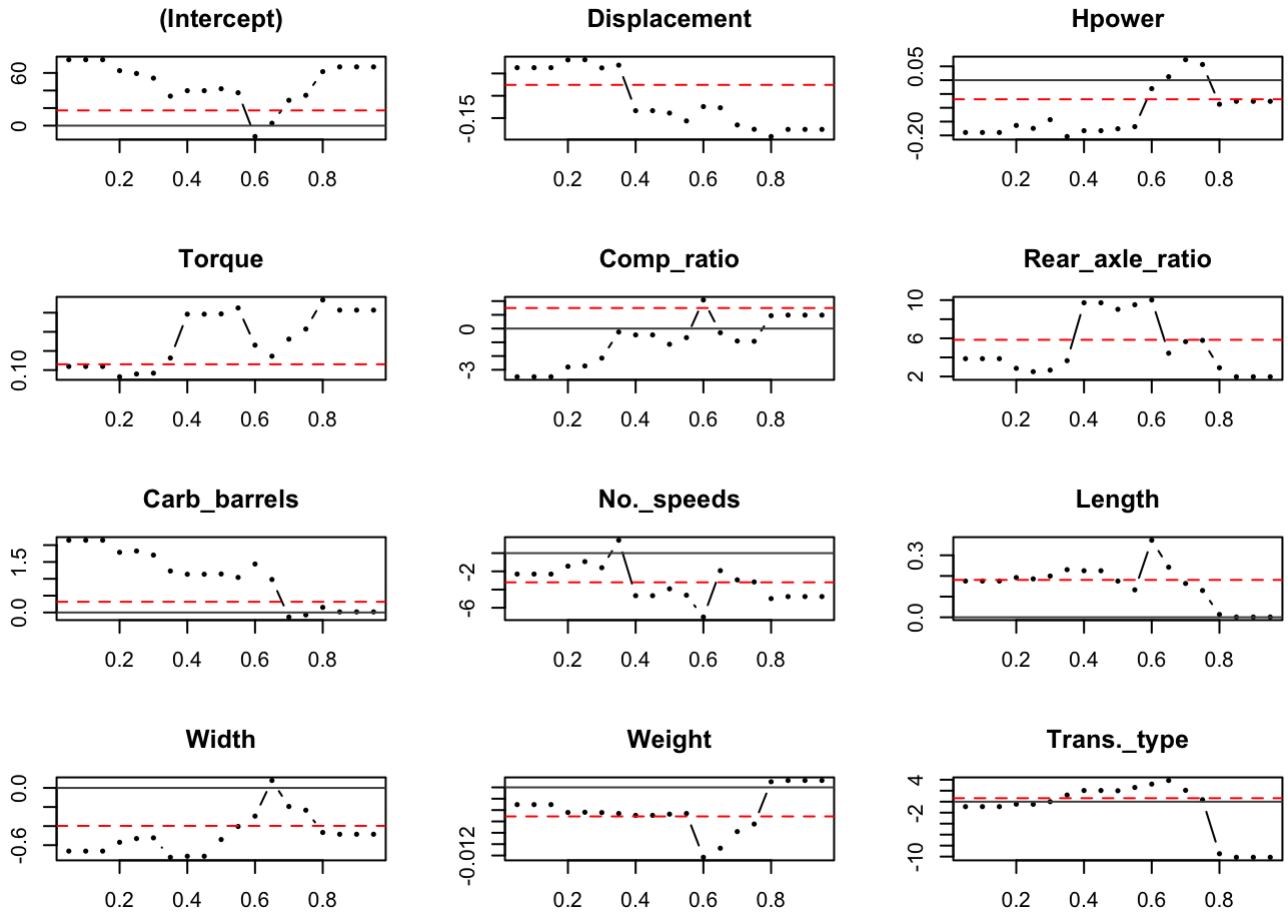
```
## tau: [1] 0.65
##
## Coefficients:
##                coefficients lower bd  upper bd
## (Intercept)       2.72420    -62.53270  90.21213
## Displacement     -0.12688     -0.45468   0.03413
## Hpower            0.01245     -0.33805   0.20142
## Torque            0.13632     -0.01474   0.71181
## Comp_ratio       -0.30299     -6.43194   7.23641
## Rear_axle_ratio   4.44313     -6.87306  12.41785
## Carb_barrels      0.97970     -3.14994   4.08618
## No._speeds       -1.92379     -9.72640  11.20294
## Length            0.24256     -0.02695   0.54294
## Width             0.07790     -1.54193   0.34287
## Weight           -0.01072     -0.02450   0.00551
## Trans._type       3.86325    -17.61289   6.83024
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.7
##
## Coefficients:
##                coefficients lower bd  upper bd
## (Intercept)      28.85096    -75.12977 102.50991
## Displacement     -0.16541     -0.47664   0.05931
## Hpower            0.07405     -0.33272   0.20573
## Torque            0.18091      0.03334   0.66419
## Comp_ratio       -0.90495     -6.34058   7.71359
## Rear_axle_ratio   5.65233     -7.01015  14.03433
## Carb_barrels     -0.13504     -2.96208   4.04653
## No._speeds       -2.93528    -10.54811  11.40447
## Length            0.16370     -0.07872   0.53613
## Width            -0.19469     -1.21537   0.36292
## Weight           -0.00779     -0.02598   0.00638
## Trans._type       2.07428    -23.65402   5.03042
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.75
##
## Coefficients:
##                coefficients   lower bd       upper bd
## (Intercept)       3.455691e+01  -8.684394e+01   1.032997e+02
## Displacement     -1.751100e-01  -4.660100e-01   6.019000e-02
## Hpower            5.674000e-02  -3.025600e-01   8.576000e-02
## Torque            2.073900e-01  -1.951000e-01   5.179700e-01
## Comp_ratio       -9.275300e-01  -7.579510e+00   9.662210e+00
## Rear_axle_ratio   5.785450e+00  -6.660930e+00   1.305027e+01
## Carb_barrels     -7.231000e-02  -3.181530e+00   4.833050e+00
## No._speeds       -3.165050e+00  -1.308105e+01   1.568430e+01
## Length            1.295500e-01  -1.320200e-01   6.347100e-01
## Width            -2.334800e-01  -1.300490e+00   3.444300e-01
## Weight           -6.460000e-03  -2.710000e-02   9.380000e-03
## Trans._type       3.597200e-01 -1.797693e+308   5.314290e+00
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.8
```

```
##
## Coefficients:
##                 coefficients   lower bd       upper bd
## (Intercept)       6.148552e+01  -1.049836e+02   8.566354e+01
## Displacement     -1.913300e-01  -4.137200e-01   6.737000e-02
## Hpower           -8.712000e-02  -2.164400e-01   7.954000e-02
## Torque            2.833300e-01  -2.153400e-01   4.907800e-01
## Comp_ratio        9.368600e-01  -7.735370e+00   9.631920e+00
## Rear_axle_ratio   2.917710e+00  -4.611710e+00   1.369960e+01
## Carb_barrels      1.512300e-01  -4.358200e+00   4.657640e+00
## No._speeds       -4.994060e+00  -1.314589e+01   1.682156e+01
## Length            1.373000e-02  -1.543800e-01   7.594600e-01
## Width            -4.669700e-01  -1.331300e+00   1.108440e+00
## Weight            9.900000e-04  -3.790000e-02   3.420000e-03
## Trans._type      -9.478690e+00 -1.797693e+308   7.201720e+00
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.85
##
## Coefficients:
##                 coefficients   lower bd       upper bd
## (Intercept)       6.690518e+01  -1.017219e+02   8.340677e+01
## Displacement     -1.753400e-01  -4.133800e-01   8.903000e-02
## Hpower           -7.653000e-02  -2.252300e-01   2.891000e-02
## Torque            2.567900e-01  -2.193400e-01   5.192900e-01
## Comp_ratio        9.785700e-01  -1.052048e+01   1.013836e+01
## Rear_axle_ratio   1.973560e+00  -4.461560e+00   1.404317e+01
## Carb_barrels      1.741000e-02  -5.369720e+00   4.663750e+00
## No._speeds       -4.769530e+00  -1.477001e+01   1.962953e+01
## Length            1.180000e-03  -2.910870e+00   7.777500e-01
## Width            -4.858100e-01  -1.369200e+00   4.014110e+00
## Weight            1.210000e-03  -4.319000e-02   3.710000e-03
## Trans._type      -1.012671e+01 -1.797693e+308   7.245470e+00
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.9
##
## Coefficients:
##                 coefficients   lower bd       upper bd
## (Intercept)       6.690518e+01  -9.810508e+01   8.661455e+01
## Displacement     -1.753400e-01  -4.236900e-01   1.289300e-01
## Hpower           -7.653000e-02  -2.656700e-01   4.391000e-02
## Torque            2.567900e-01  -3.484200e-01   5.416000e-01
## Comp_ratio        9.785700e-01  -3.524620e+01   2.352705e+01
## Rear_axle_ratio   1.973560e+00  -6.904900e+00   1.521520e+01
## Carb_barrels      1.741000e-02  -9.354370e+00   4.553580e+00
## No._speeds       -4.769530e+00  -2.477762e+01   2.793282e+01
## Length            1.180000e-03 -1.797693e+308   9.343800e-01
## Width            -4.858100e-01  -5.684390e+00  1.797693e+308
## Weight            1.210000e-03  -4.721000e-02   5.040000e-03
## Trans._type      -1.012671e+01 -1.797693e+308   7.331570e+00
##
## Call: rq(formula = Mpg ~ ., tau = seq(0.05, 0.95, by = 0.05), data = gas)
##
## tau: [1] 0.95
##
```

```
## Coefficients:
##                 coefficients   lower bd       upper bd
## (Intercept)       6.690518e+01 -1.797693e+308  1.797693e+308
## Displacement     -1.753400e-01 -1.797693e+308  1.797693e+308
## Hpower           -7.653000e-02 -1.797693e+308  1.797693e+308
## Torque            2.567900e-01 -1.797693e+308  1.797693e+308
## Comp_ratio        9.785700e-01 -1.797693e+308  1.797693e+308
## Rear_axle_ratio   1.973560e+00 -1.797693e+308  1.797693e+308
## Carb_barrels      1.741000e-02 -1.797693e+308  1.797693e+308
## No._speeds       -4.769530e+00 -1.797693e+308  1.797693e+308
## Length            1.180000e-03 -1.797693e+308  1.797693e+308
## Width            -4.858100e-01 -1.797693e+308  1.797693e+308
## Weight            1.210000e-03 -1.797693e+308  1.797693e+308
## Trans._type      -1.012671e+01 -1.797693e+308   7.544440e+00
```

b

```
plot(fit1)
```



c

Width: At 5th quantile, a unit increase in width will reduce Mpg by 0.7 unit. But at 80th quantile, a unit increase in width will only reduce Mpg by 0.45 unit.

Carb barrels: At 5th quantile, a unit increase in carb barrels will increase Mpg by 2.2 unit. But at 85th-95th quantile, the effect of carb barrels on Mpg is almost 0.

Torque: At 5th quantile, a unit increase in torque will increase Mpg by 0.11 unit. At 85th-95th quantile, a unit increase in torque will increase Mpg by 0.25 unit.

d

```
fit2 <- rq(Mpg ~ ., tau = .5, data = gas)
summary(fit2, se = "boot")
```

```
##
## Call: rq(formula = Mpg ~ ., tau = 0.5, data = gas)
##
## tau: [1] 0.5
##
## Coefficients:
##                 Value    Std. Error t value  Pr(>|t|)
## (Intercept)     41.98707 55.51216    0.75636  0.45922
## Displacement    -0.13873  0.10724   -1.29364  0.21214
## Hpower          -0.17596  0.22509   -0.78170  0.44455
## Torque           0.24692  0.17817    1.38590  0.18271
## Comp_ratio      -1.14223  4.76204   -0.23986  0.81315
## Rear_axle_ratio  9.03682  7.73542    1.16824  0.25795
## Carb_barrels     1.14349  2.59308    0.44098  0.66448
## No._speeds      -3.91968  8.12158   -0.48262  0.63518
## Length           0.17526  0.29860    0.58695  0.56453
## Width           -0.54095  0.73941   -0.73159  0.47384
## Weight          -0.00472  0.01023   -0.46108  0.65026
## Trans._type      1.99845  8.99377    0.22220  0.82666
```

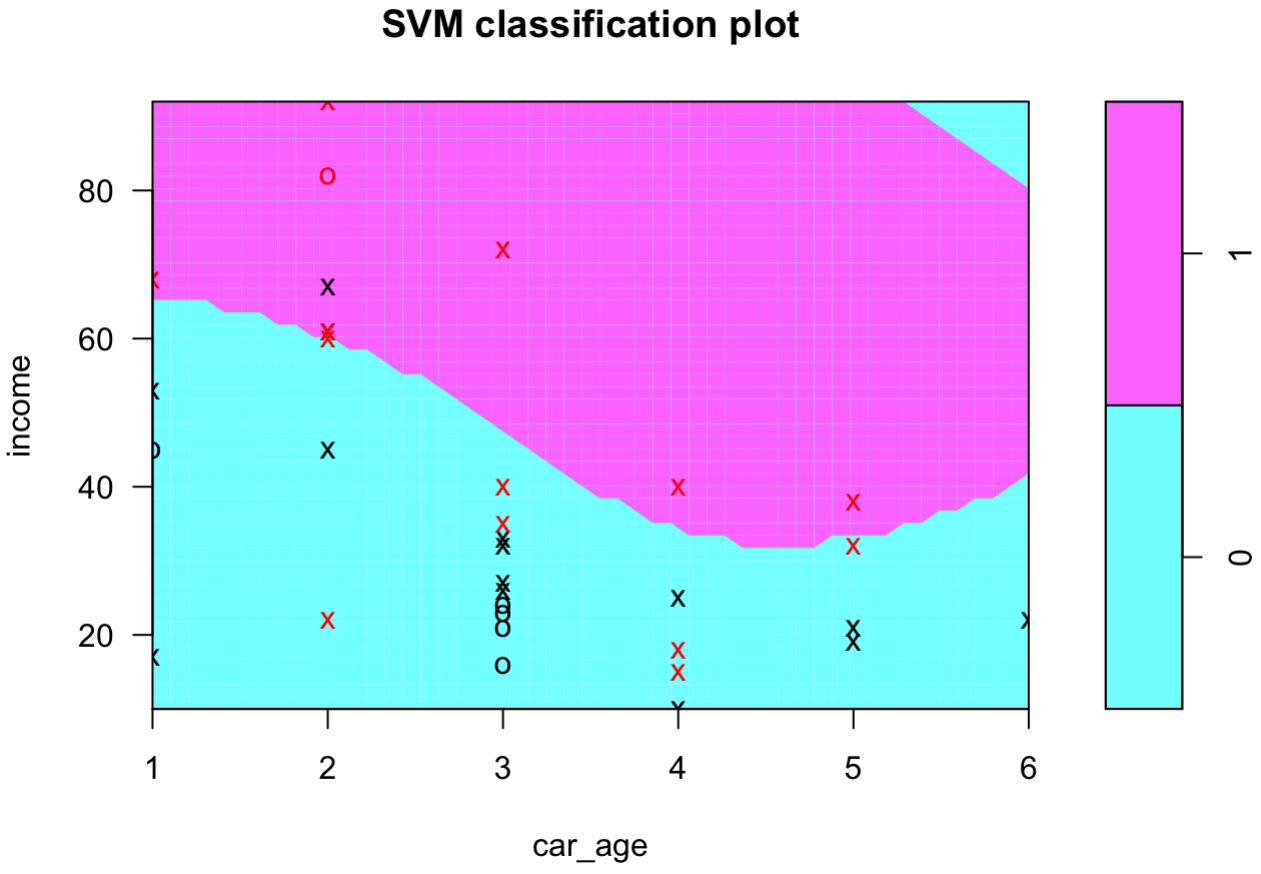
Q3 a

```
car <- read.csv('car.csv')
library(e1071)
svm <- svm(factor(y) ~ income+car_age, data = car)
summary(svm)
```

```
##
## Call:
## svm(formula = factor(y) ~ income + car_age, data = car)
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  1
##       gamma:  0.5
##
## Number of Support Vectors:  27
##
##  ( 14 13 )
##
##
## Number of Classes:  2
##
## Levels:
##  0 1
```

b

```
plot(svm, car, income~car_age)
```



c

```
newdata <- with(car, data.frame(income = 50, car_age = 5))
predicted <- predict(svm, newdata = newdata, type = "response")
print(paste0('predicted category: ', predicted))
```

```
## [1] "predicted category: 1"
```

The family purchased a new car.