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Vellore Institute of Technology

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CHENNAI

Final Assessment Test(FAT) - NOV/DEC 2025

Programme	**B.Tech.**	Semester	**Fall Semester 2025-26**
Course Code	**BMAT202L**	Faculty Name	**Prof. Revathi G K**
Course Title	**Probability and Statistics**	Slot	**E1+TE1**
		Class Nbr	**CH2025260101141**
Time	**3 hours**	Max. Marks	**100**

Instructions To Candidates

- Write only your registration number in the designated box on the question paper. Writing anything elsewhere on the question paper will be considered a violation.
- Only Non programmable calculator is allowed.
- Statistical table is provided in the exam.

Course Outcomes

CO1: Compute and interpret descriptive statistics using numerical and graphical techniques.
CO2: Understand the basic concepts of random variables and find an appropriate distribution for analyzing data specific to an experiment.
CO3: Apply statistical methods like correlation, regression analysis in analyzing, interpreting experimental data.
CO4: Make appropriate decisions using statistical inference that is the central to experimental research.
CO5: Use statistical methodology and tools in reliability engineering problems.

Answer any 10 Questions (10 × 10 Marks)

01. Find the median and mode of the following data and hence find the arithmetic mean using the empirical relation.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	10	18	30	20	12	5

[10] (CO1/K3)

02. (a) If the first four moments about the point x = 4 are $\mu_1' = 0, \mu_2' = 2, \mu_3' = 0$ and $\mu_4' = 11$, show that the distribution is symmetric. (5 marks)

(b) (i) A computer has calculated the correlation coefficients for a set of variables X_1, X_2, and X_3 as $r_{12} = 0.91$, $r_{13} = 0.33$ and $r_{23} = 0.81$. Determine whether these computed values are consistent and free from error. (2 marks)
(ii) In a three variate multiple correlation analysis, the following results were obtained: $r_{12} = 0.59$, $r_{13} = 0.46$, and $r_{23} = 0.77$. Find $R_{1.23}$ and $R_{2.13}$. What does that tell you about the relationships among these variables based on $R_{1.23}$ and $R_{2.13}$? (3 marks)

[10] (CO1,3/K3)

03. Let X denote the bandwidth allocation and Y denote the throughput performance in a network system. The joint probability density function is given by

$$f(x,y) = \begin{cases} \frac{3x^2}{20}(2+y), & 0 < x < 2, 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine whether bandwidth allocation affects throughput performance. (3 marks)
(b) Calculate the variance of $Z = 4X - 3Y + 2$. (7 marks)

[10] (CO2/K3)

04. The cumulative distribution function (CDF) of a continuous random variable X is given by

$$F(x) = \begin{cases} 0, & x \leq 3 \\ 1 - e^{-2(x-3)}, & x > 3 \end{cases}$$

(a) Determine the moment generating function (MGF) of X. (5 marks)
(b) Using the MGF, find the mean and standard deviation of X. (3 marks)
(c) Compute $E(3X - 2)$ and find the probability $P(X > 5)$. (2 marks)

[10] (CO2/K3)

05. The following table gives the data of the marks obtained by 8 students in Commerce and Mathematics. Compute the rank correlation coefficient.

Marks in Commerce	15	20	28	12	40	60	20	80
Marks in Mathematics	40	30	50	30	20	10	30	60

[10] (CO3/K3)

06. A binomial distribution consists of 5 independent trials such that $P(X = 1) = 0.4096$ and $P(X = 2) = 0.2048$.
Find:
(a) the value of p (2 marks)
(b) the moment generating function for Random Variable X (2 marks)
(c)$E(X)$, $E(2X + 1)$, $Var(X)$ and $Var(-3X)$ using the moment generating function (3 marks)
(d) $P\left(\frac{X+1}{2} > 1\right)$. (3 marks)

[10] (CO2/K2)

07. (a) The height of men is normally distributed with mean $\mu = 167$ cm and standard deviation $\sigma = 3$ cm. What is the probability of the population of men that have height
(i) greater than 167 cm, (2 marks)
(ii) between 161 cm and 173 cm. (3 marks)
(b) Suppose the survival time X (in weeks) of a randomly selected male mouse exposed to 240 rads of gamma radiation follows a gamma distribution with a specified shape parameter λ =1 and shape parameter θ =2.
(i) What is the expected survival time. (1 mark) (ii) Compute the probability that a mouse lives less than 3 weeks. (2 marks) (iii) Obtain the probability that a mouse survives between 1 and 3 weeks (2 marks)

[10] (CO2/K3)

08. (a) A manufacturer of ball point pen claims that a certain pen manufactured by him has a mean writing-life is 460 A-4 size pages. A purchasing agent selects a sample of 100 pens and puts them on the test. The mean writing life of the sample found 453 A-4 size pages with standard deviation 25 A-4 size pages. Should the purchasing agent reject the manufacturer's claim at 5% level of significance? (5 marks)
(b) A university conducts both face to face and distance mode classes for a particular course indented both to be identical. A sample of 50 students of face to face mode yields examination results mean and SD respectively as:

$$\overline{X} = 80.4, \quad S_1 = 12.8$$

and other sample of 100 distance-mode students yields mean and SD of their examination results in the same course respectively as:

$$\overline{Y} = 74.3, \quad S_2 = 20.5$$

Are both educational methods statistically equal at 5% level? (5 marks)

[10] (CO4/K4)

09. (a) A machine produces a large number of items out of which 25% are found to be defective. To check this, company manager takes a random sample of 100 items and found 35 items defective. Is there an evidence of more deterioration of quality at 5% level of significance? (5 marks)
(b) A power supply unit has a reliability of 0.88. To improve the reliability, a redundant power supply is added in parallel. However, to ensure proper load sharing, a power distributor must be installed before the power supplies and a load balancer must be connected after them. Each of these additional components (distributor and balancer) has a reliability of 0.96.
(i) Compute the system reliability with and without redundancy. (1 mark)
(ii) Does the addition of the redundant power supply improve the overall system reliability? (2 marks)
(iii) Comment on whether the improvement is significant enough to justify the additional hardware. (2 marks)

[10] (CO4,5/K2)

10. An experiment was designed to study the performance 4 different detergents for cleaning fuel injectors. The following "cleanness" reading were obtained with specially designed equipment for 12 tanks of gas distributed over 3 different models of engines

	E1	E2	E3
DA	45	43	51
DB	47	46	52
DC	48	50	55
DD	42	37	49

Looking at the detergents (D) as treatments and the engines (E) as blocks, obtain the appropriate ANOVA table and test the 0.01 level of significance whether there are differences in the detergents or in the engines.

[10] (CO4/K4)

11. (a) Two random samples were drawn from two normal populations and their values are

A	66	67	75	76	82	84	88	90	92	-	-
B	64	66	74	78	82	85	87	92	93	95	97

Test whether the two populations have the same variance at $\alpha = 0.05$ level of significance. (6 marks)

(b) A dietician believes a new supplement increases average weight by more than 2 kg. Sample of 12 people shows: mean gain = 2.8 kg, SD = 0.9 kg. Test at 5% significance (one-tailed). (4 marks)

[10] (CO4/K4)

12. The density function of the time to failure (in years) of a certain device is

$$f(t) = \frac{81}{(t+3)^4}, \quad t \geq 0.$$

(a) Derive the reliability (survival) function $R(t)$ and determine the reliability for the first year of operation, $R(1)$. (2 marks)

(b) Compute the mean time to failure (MTTF). (3 marks)

(c) Find the design life t_d such that the reliability is 0.95. (2 marks)

(d) If a two-year burn-in period is applied (units surviving the first 2 years are accepted), will the one-year reliability after burn-in improve compared with part (a)? If so, compute the new one-year reliability after burn-in (i.e. $P(T > 3 \mid T > 2) = \frac{R(3)}{R(2)}$). (3 marks)

[10] (CO5/K3)

BL-Bloom's Taxonomy Levels - (K1-Remembering,K2-Understanding,K3-Applying,K4-Analysing,K5-Evaluating,K6-Creating)

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