VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of the UGC Act, 1956)

Reg. No. :

Final Assessment Test (FAT) - November/December 2023

Programme	**B.Tech.**	Semester	**FALL SEMESTER 2023 - 24**
Course Title	**DISCRETE MATHEMATICS AND GRAPH THEORY**	Course Code	**BMAT205L**
Faculty Name	**Prof. Sakthidevi K**	Slot	**D2+TD2+TDD2**
		Class Nbr	**CH2023240101207**
Time	**3 Hours**	Max. Marks	**100**

PART-A (10 X 10 Marks)
Answer any 10 questions

01. a) Using algebra of logics, find the PCNF and PDNF of the following proposition: [10]
$P \wedge (P \vee Q) \wedge [P \vee (\neg Q \rightarrow R)]$. **[7 marks]**
b) Write the converse, contrapositive and inverse of the following proposition symbolically and in words: "I go to the beach whenever it is a sunny summer day". **[3 marks]**

02. a) Test the validity of the following argument: "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution". **[7 marks]** [10]
b) Translate the given statement into propositional logic: "You can see the movie only if you are over 18 years old or you have the permission of a parent". **[3 marks]**

03. a) Check whether $(Z_{12}, \times_{12})$ and $(Z_7, +_7)$ are groups or not. **[5 marks]** [10]
b) Does there exist subgroups of orders 6 and 4, respectively for the group $(Z_{13}^* = \{1, 2, 3, \ldots 11, 12\}, \times_{13})$? If so, find subgroups of these orders. **[5 marks]**

04. [10]
a) Given the generator matrix $G \equiv \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ corresponding the encoding function $e : B^3 \rightarrow B^6$, find the parity check matrix, use it to decode the following received words and hence find the original message: i) 1 0 1 1 1 1 ii) 0 1 1 0 1 0 iii) 1 0 1 1 1 0 iv) 1 1 1 1 1 1. Are all the words decoded uniquely? **[5 marks]**
b) There are 15 candidates for an examination. 7 candidates are appearing for Mathematics examination while the remaining 8 are appearing for different subjects. In how many ways can they be seated in a row so that no two Mathematics candidates are together? **[3 marks]**
c) A box of one dozen apple contains a rotten apple. If we choose 3 apples simultaneously, in how many ways can one get only good apples? **[2 marks]**

05. Solve the recurrence relation: $a_n = 4a_{n-1} - 3a_{n-2} + 2^n + n + 3$ with $a_0 = 1$ and $a_1 = 4$. [10]

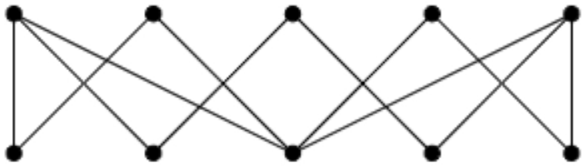
06. Consider D_{42} , the set of all divisors of 42 and | represents divisibility relation [10]
a) Prove this relation is a partial order relation
b) Draw the Hasse diagram for $(D_{42}, |)$
c) List all upper bounds, lower bounds and find LUB and GLB for the subset $A = \{3, 6, 7\}$
d) Prove that $(D_{42}, |)$ is a lattice. Also, check if it is a bounded lattice or not.

07. a) Minimize the function [10]

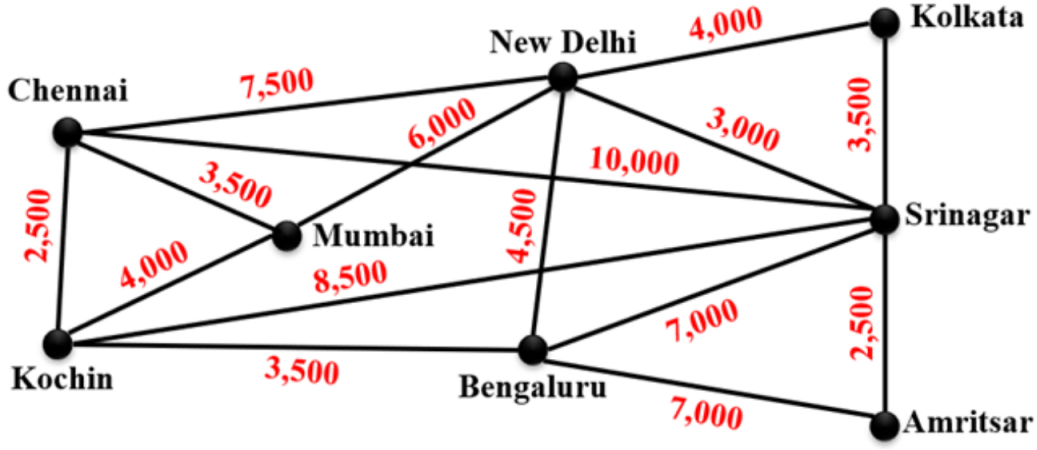
$$f(w,x,y,z) = (\bar{w}+\bar{x}+\bar{y}+\bar{z}).(\bar{w}+\bar{x}+\bar{y}+z).(\bar{w}+x+\bar{y}+\bar{z}).(\bar{w}+x+\bar{y}+z)$$

using K-map and draw the circuit for the minimised function. **[5 marks]**

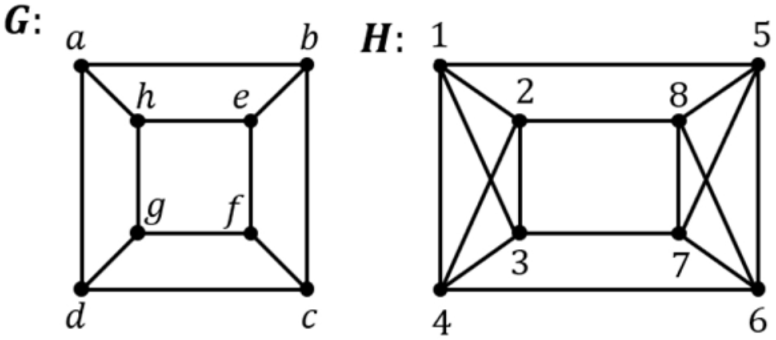
b) Find a maximum matching and a minimum vertex cover of the following graph. **[5 marks]**



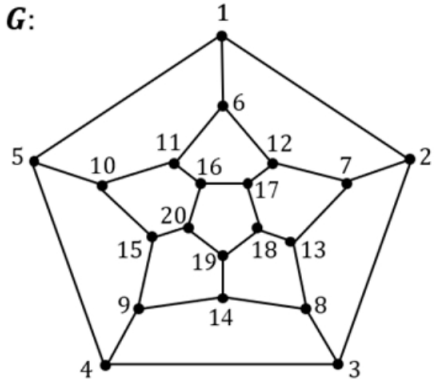
08. From the city of Amritsar, identify the route that has the lowest total airfare that visits each of the remaining cities shown in the following graph. Moreover, determine their minimum fare from Amritsar to all other cities. In this example, the weight at the edge represents the lowest available fare between the two cities in rupees. [10]



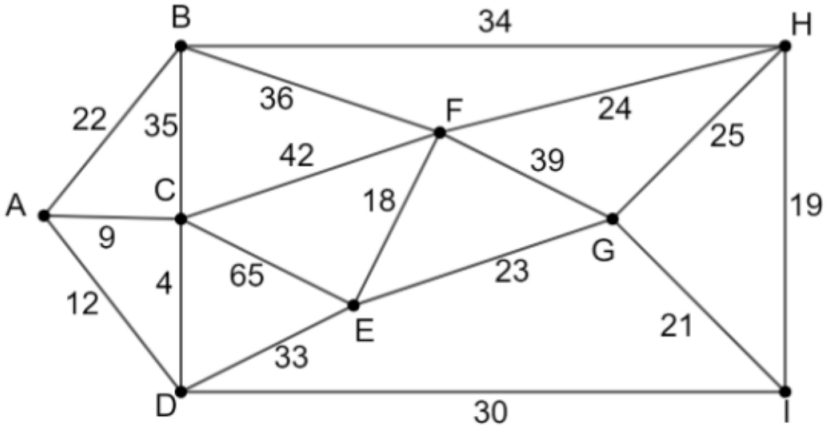
09. a) Verify whether the complement of graph G is isomorphic to graph H. **[7 marks]** [10]



b) Check whether the following graph is Hamiltonian or not. If so, find a Hamiltonian cycle. **[3 marks]**

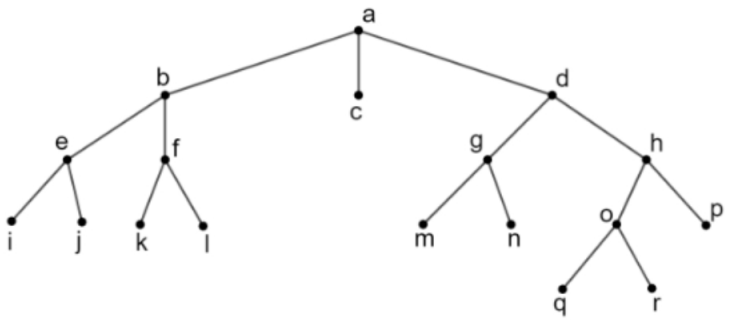


10. a) Tender is called for constructing roads to connect 9 villages. The possible roads that can be constructed and their costs are listed below as weights of the road. Which roads do you suggest to be constructed so that the total cost is minimum?. **[7 marks]** [**10**]



b) Is it possible to draw a tree on 6 vertices with degrees $1, 1, 3, 3, 3, 3$. Else, explain why it is not possible? **[3 marks]**

11. For the following tree [**10**]



a) Find the preorder, postorder and inorder traversal. (5 Marks)

b) Find the diameter and centre. (2.5 Marks)

c) Find the eccentricity of the vertices m, r, a. (2.5 Marks)

12. Find the chromatic number and chromatic polynomial of the following graph: [**10**]