Reg. Number:

VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Continuous Assessment Test (CAT) – I August 2024

Programme	:	B.Tech.	Semester	:	FALL 2024-2025
Course Code & Course Title	:	BMAT205L Discrete Mathematics and Graph Theory	Slot	:	C2+TC2+TCC2
Faculty	:	Prof. Aarthy B Dr. Amit Kumar Rahul Prof. Anitha G Dr. Ankit Kumar Dr. Padmaja N Dr. Poulomi De Dr. Surath Ghosh	Class Number	:	CH2024250102066 CH2024250102265 CH2024250102267 CH2024250102069 CH2024250102266 CH2024250102068 CH2024250102268
Duration	:	90 Minutes	Max. Mark	:	50

General Instructions:

- Write only your registration number on the question paper in the box provided and do not write other information.
- Use statistical tables supplied from the exam cell as necessary
- Use graph sheets supplied from the exam cell as necessary
- Only non-programmable calculator without storage is permitted

Answer all questions
(5×10=50)

Q. No	Sub Sec.	Description	Marks
1.	(a)	Without using truth table, find PDNF of $\neg(p \vee (\neg p \wedge \neg q \wedge r))$.	(5)
	(b)	Identify the bound variable, free variable and the scope of the following expression: $\forall x(P(x) \wedge Q(x)) \vee \forall y R(y)$. Also, write the converse, contrapositive and inverse of the following proposition symbolically and in words "If the weather is nice, then I'll wash the car".	(2+3)
2.	(a)	Prove that $\neg p \leftrightarrow q, q \rightarrow r, \neg r \Rightarrow p$ is valid.	(5)
	(b)	Show that the premises "An employee in my office has not completed his daily work" and "Everyone in my office completed his monthly files" imply the conclusion "Someone who completed his monthly files has not completed his daily work".	(5)
3.	(a)	Prove the following equivalences by proving the equivalences of the dual without using truth table: $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \equiv p \wedge q$	(5)

	(b)	Let $P(x)$ be the statement "x has visited universal studios" where the universe consists of the students at UCF. Express each of the following statements using quantifiers (i) Some students at UCF have not visited universal studios. (ii) Not all students at UCF have visited universal studios. (iii) No student at UCF has not visited universal studios.	(2+2+1)
4.	(a)	If $*$ is the binary operation on $S = Q \times Q$, the set of ordered pairs of rational numbers and given by $(a, b) * (c, d) = (ac, ad + b)$, i) Prove $(S, *)$ is a semi group. Is it commutative? ii) Find the identity element of S iii) Which elements, if any, have inverses, and what are they?	(6)
	(b)	Let $\mathbb{R} - \{0\}$ represents set of all nonzero real numbers and M denotes the set of all 2×2 invertible matrices over $\mathbb{R}$. Determine whether the following map is a homomorphism. If so, what is its kernel? Given the map $f: \mathbb{R} - \{0\} \to M$ defined by $f(a) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$.	(4)
5	(a)	Given the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ corresponding the encoding function $e: B^3 \longrightarrow B^6$ find the parity check matrix and use it to decode the following received words and hence find the original message. Are all the words decoded uniquely? (i) 111101 (ii) 100100 (iii) 111100 (iv) 010100	(5)
	(b)	In the group S_6, a permutation group over $\{1,2,3,4,5,6\}$ $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 5 & 3 & 6 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 6 & 1 & 2 & 4 \end{pmatrix}$ determine α^{-2} and $x \in S_6$ such that $\alpha x = \beta$.	(1+2)
	(c)	Provide a justification for why $U(8) = \{1,3,5,7\}$, under multiplication modulo 8, is not a cyclic group.	(2)
		**********All the best *************	