



Continuous Assessment Test II (CAT-2) – December 2022

Programme	: B.Tech	Semester	: Fall 2022-23
Course Code	: BMAT101L/IMAT101L	Class Nbr(s)	: CH2022231700262, CH2022231700260, CH2022231700270, CH2022231700272, CH2022231700268, CH2022231700481, CH2022231700612, CH2022231700751, CH2022231700482, CH2022231700484, CH2022231700479, CH2022231700362
Course Title	: Calculus		
Faculty(s)	: Dr. Karan Kumar Pradhan, Dr. Manoj Kumar Singh, Dr. Pankaj Shukla, Dr. Abhishek Kumar Singh, Dr. Dhasekhar, Dr. Berin Greeni A, Dr. Kirti Aarya, Dr. Kalyan Manna, Dr. Vijay Kumar Poshala, Dr. Sandeep Saha, Dr. David Raj Michel, Dr. Ankit Kumar	Slot	: B1+TB1
Time	: One and half Hours	Max. Marks	: 50

Answer all the Questions

1.	(i) Find all critical points of the function $f(x, y) = x^4 + y^4 - 2x^2 - 2y^2 + 4xy$ and check whether the function attains maximum or minimum at each of these points. (ii) Show that point (0,0) is neither a point of local minimum nor a point of local maximum for the function given by $f(x, y) = 3x^4 - 4x^2y + y^2$ for $(x, y) \in \mathbb{R}^2$.	10
2.	(i) If x, y and z are positive real numbers, then find the minimum value of function $x^2 + 8y^2 + 27z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$. (ii) Find the Taylor series expansion of $f(x, y) = \sin xy + x^2y + e^x$ in the power of $(x - 1)$ and $(y - \pi)$ up to second degree terms.	10

3.	<p>(i) Find the value of integral by using the polar coordinates.</p> $I = \iint_D \sqrt{x^2 + y^2} \, dydx \quad \text{where } D = \{(x, y) \in \mathbb{R}^2 : x \leq x^2 + y^2 \leq 2x\}$ <p>(ii) Find the value of integral by changing the order of integration</p> $I = \int_0^4 \int_{(4-x)^2}^2 e^{y^3} \, dydx$	10
4.	Using multiple integrals, find the volume of the solid region bounded above by hemisphere $z = 1 + \sqrt{1 - x^2 - y^2}$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$.	10
5.	<p>Solve the following integrals by using Beta and Gamma Function:</p> <p>(i) $I = \int_0^\infty \frac{e^{-\frac{k}{x^2}}}{x^6} dx \quad \text{where } k \neq 0$</p> <p>(ii) $I = \int_0^1 x^4 \sqrt{1 - x^2} \, dx$</p>	10