VIT®

Vellore Institute of Technology

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Reg. No. :

Final Assessment Test (FAT) - May 2024

Programme	B.Tech.	Semester	WINTER SEMESTER 2023 - 24
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Sankarsan Tarai	Slot	X11+X12+X21+Z21
		Class Nbr	CH2023240503686
Time	3 Hours	Max. Marks	100

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Answer any 10 questions (10 X 10 Marks = 100 Marks)

01. (a) Solve $(D^2+1)y = cosecx$. (5 M) [10]

(b) A current of 2amperes flows in an RLC circuit with resistance 40ohms, Inductance 0.2henrys and capacitance 10^{-5} farads. Find the current flowing in the circuit at $t > 0$ if the initial charge on the capacitor is 1 coulomb. Assume that $E(t) = 0$ for $t > 0$. (5 M)

02. Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2sin(log(1+x))$. [10]

03. (a) Find the complete integral of $p^2x^2 = z(z-qy)$. (6M) [10]

(b) Form the partial differential equation eliminating the function $z = xy + f(x^2+y^2)$. (4M)

04. Find the general integral of $(y-zx)p + (x+yz)q = x^2+y^2$. [10]

05. (a) Find the inverse Laplace transform of $\frac{1}{(s^2+1)(s^2+9)}$. (5M) [10]

(b) Evaluate $L\left(\frac{cosat-cosbt}{t} + tsinat\right)$. (5M)

06. Solve by method of transforms of the equation $ty'' + 2y' + ty = sint$, $y(0) = 1$ [10]

07. (a) Evaluate L $\left\{t\int_0^t \frac{e^{-t}sint}{t}dt\right\}$. (5 marks) [10]

(b) Obtain $L^{-1}\left\{\frac{s^2+2s-3}{s(s-3)(s+2)}\right\}$. (5 marks)

08. Solve $y'' + 2y' - 15y = 6\delta(t-1)$, $y(0) = 0$, $y'(0) = 2$. [10]

09. Verify the function [10]

$$f(x) = \begin{cases} -x+1, & \text{for } -\pi \le x \le 0 \\ x+1, & \text{for } 0 \le x \le \pi \end{cases}$$

is even or odd. Also, find the Fourier series of the function and deduce the value of $\Sigma_{n=0}^{\infty}\frac{1}{(2n+1)^2}$.

10. Find the half range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$ and hence, find the value of $\Sigma_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. [10]

11. Using Fourier transform, solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given $u(0,t) = 0$, and $u(x,0) = 2x$, where $0 < x < 4, t > 0$. [10]

12. (a) Find the Z-transform of $cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. (5 marks) [10]

(b) Find the inverse Z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. (5 marks)