VIT

Reg. No. :

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	**B.Tech.**	Semester	**Winter Semester 2022-23**
Course Title	**DIFFERENTIAL EQUATIONS AND TRANSFORMS**	Course Code	**BMAT102L**
Faculty Name	**Prof. Soumendu Roy**	Slot	**A2+TA2+TAA2**
		Class Nbr	**CH2022232300449**
Time	**3 Hours**	Max. Marks	**100**

PART-A (10 X 10 Marks)

Answer any 10 questions

01. Solve the ODE $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=(\ln x)\sin(\ln x)$. [10]

02. (a) Solve the ODE $\dfrac{d^2y}{dx^2}-y=e^x$ by using undetermined coefficient method. [5-marks] [10]
(b) Solve $y^2p-xyq=x(z-2y)$. [5-marks]

03. (a) Solve $p^2z^2+q^2=p^2q$. [5-marks] [10]
(b) Solve $p-q=\ln(x+y)$. [5-marks]

04. Evaluate $L^{-1}\left[\dfrac{s^2-5s+7}{(s+2)^2}\right]$. [10]

05. (a) Find $L[tH(t-1)+e^{2t}\delta(t-2)]$. [5-marks] [10]
(Note: $H(t-a)$ is the unit step function at the point a and $\delta(t-b)$ is the impulse function at the point b.)

(b) Find the Fourier sine series of $f(x)=\begin{cases}\dfrac{\pi x}{4}, & 0\le x<\pi/2\\ \dfrac{\pi(\pi-x)}{4}, & \pi/2\le x<\pi\end{cases}$. [5-marks]

06. Use Laplace transform to solve $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial t}=x,\ u(0,t)=0$ and $u(x,0)=0$ for $x>0,\ t>0$. [10]

07. Use Laplace transform to solve $\dfrac{d^2x}{dt^2}-2\dfrac{dx}{dt}+x=e^t$ with $x=2,\ \dfrac{dx}{dt}=-1$ at $t=0$. [10]

08. Obtain the Fourier cosine series of $x\sin(x)$ in $(0,\pi)$. Hence show that [10]
$\dfrac{1}{(1)(3)}-\dfrac{1}{(3)(5)}+\dfrac{1}{(5)(7)}-\cdots=\dfrac{\pi-2}{4}$.

09. Find the Fourier cosine transform of $f(x)=e^{-x}$. Hence evaluate $\displaystyle\int_0^\infty \frac{\cos mx}{1+x^2}dx$ for $m>0$. [10]

10. Solve $\dfrac{\partial u}{\partial t}=3\dfrac{\partial^2 u}{\partial x^2}$, by using Fourier sine transform, where the initial and boundary conditions are $u(x,0)=e^{-2x},\ u(0,t)=0$ for $x>0$ and $t>0$. [10]

11. Use Z-transform to solve $u_{n+2}-6u_{n+1}+8u_n=4^n$ such that $u_0=0$ and $u_1=1$. [10]

12. (a) Find $Z[\sinh 3n]$. [5-marks] [10]
(b) Find $Z^{-1}\left[\dfrac{z^3}{(z-1)^2(z-2)}\right]$. [5-marks]

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