



VIT

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test II (CAT-2) – December 2022

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| Programme | : B.Tech | Semester | : Fall 2022-23 |
| Course Code | : BMAT101L/IMAT101L | Class Nbr(s) | : CH2022231700262, CH2022231700260, CH2022231700270, CH2022231700272, CH2022231700268, CH2022231700481, CH2022231700612, CH2022231700751, CH2022231700482; CH2022231700484 , CH2022231700479. CH2022231700362 |
| Course Title | : Calculus | | |
| Faculty(s) | : Dr. Karan Kumar Pradhan, Dr. Manoj Kumar Singh, Dr. Pankaj Shukla, Dr. Abhishek Kumar Singh, Dr Dhansekhar, Dr Berin Greeni A , Dr. Kirti Aarya, Dr Kalyan Manna, Dr Vijay Kumar Poshala, Dr. Sandeep Saha,Dr David Raj Michel,Dr Ankit Kumar | Slot | : B1+TB1 |
| Time | : One and half Hours | Max. Marks | : 50 |

Answer all the Questions

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| 1. | (i) Find all critical points of the function $f(x, y) = x^4 + y^4 - 2x^2 - 2y^2 + 4xy$ and check whether the function attains maximum or minimum at each of these points. (ii) Show that point $(0,0)$ is neither a point of local minimum nor a point of local maximum for the function given by $f(x, y) = 3x^4 - 4x^2y + y^2$ for $(x, y) \in \mathbb{R}^2$. | 10 |
| 2. | (i) If x, y and z are positive real numbers, then find the minimum value of function $x^2 + 8y^2 + 27z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$. (ii) Find the Taylor series expansion of $f(x, y) = \sin xy + x^2y + e^x$ in the power of $(x - 1)$ and $(y - \pi)$ up to second degree terms. | 10 |

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| 3. | <p>(i) Find the value of integral by using the polar coordinates.</p> $I = \iint_D \sqrt{x^2 + y^2} dy dx \quad \text{where } D = \{(x, y) \in \mathbb{R}^2 : x \leq x^2 + y^2 \leq 2x\}$ | 10 |
| | <p>(ii) Find the value of integral by changing the order of integration</p> $I = \int_0^4 \int_{(4-x)^{\frac{1}{2}}}^{2} e^{y^2} dy dx$ | |
| 4. | <p>Using multiple integrals, find the volume of the solid region bounded above by hemisphere $z = 1 + \sqrt{1 - x^2 - y^2}$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$.</p> | 10 |
| 5. | <p>Solve the following integrals by using Beta and Gamma Function:</p> <p>(i) $I = \int_0^{\infty} \frac{e^{-x^2}}{x^6} dx \quad \text{where } k \neq 0$</p> <p>(ii) $I = \int_0^1 x^4 \sqrt{1 - x^2} dx$</p> | 10 |