Reg. No.:

Name :

VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test (CAT)- I– October 2022

Programme	:	**B.Tech.**	Semester	:	**Fall Semester Year I 2022-2023**
Course Title	:	**Calculus**	Code	:	**BMAT101L**
			Slot	:	**E1+TE1**
Faculty	:	**Dr. Saroj Kumar Dash, Dr. Manivannan A, Dr. C. Rajivganthi, Dr. Harshavarthini, Dr. Prosenjit, Dr. Ashis Bera, Dr. Ankit Kumar, Dr. Sandip Saha, Dr. Kriti Arya**	Class Nbr	:	**CH2022231700190, 189, 191, 192, 196, 194, 257, 323, 883**
Duration	:	**1 ½ Hours**	Max. Marks	:	**50**

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	a) Using Mean Value Theorem (MVT) prove that $0 < \frac{1}{x}\log\left(\frac{e^x-1}{x}\right) < 1$ for $x > 0$.	[5]
	b) Find the intervals on which the function $f(x) = 3x^2 - 4x^3, x \in \mathbb{R}$ is increasing or decreasing?	[5]
2.	Examine the extreme values of the function $f(x) = x^5 - 5x^4 + 5x^3 + 12, x \in \mathbb{R}$. Also find the intervals on which the function $f(x)$ is concave up and concave down.	[10]
3.	Find the volume of the solid formed by revolving the region enclosed by the parabola $y^2 = 4ax$ and the straight line $y = x$, (i) about x-axis, (ii) about y-axis.	[10]
4.	. Let $f(x, y) = (x^2 + y^2)^{2/3}$. Find f_x, f_y, f_{xy} and f_{yx} at each point in $\mathbb{R}^2$.	[10]
5.	a) The inductance L (in microhenrys) of a straight nonmagnetic wire in free space is: $L = 0.00021\left[ln\left(\frac{2h}{r}\right) - 0.75\right]$, where '$h$' is the length of the wire in the millimetre and 'r' is the radius of the circular cross section. Find the maximum possible error of L, when $r = 2 \pm \frac{1}{16}$ millimetres and $h = 100 \pm \frac{1}{100}$ millimetres.	[5]
	b) Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$, where $u = \cos x \cosh y$, $v = \sin x \cosh y$ and $w = \sinh z$.	[5]

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