VIT
Vellore Institute of Technology

Reg. No. :

Final Assessment Test (FAT) – January/February 2023

Programme	**B.Tech.**	Semester	**Fall Semester 2022-23**
Course Title	**CALCULUS**	Course Code	**BMAT101L**
Faculty Name	**Prof. SANDIP**	Slot	**E1+TE1**
		Class Nbr	**CH2022231700323**
Time	**3 Hours**	Max. Marks	**100**

Section-A (10 X 10 Marks)
Answer <u>any 10</u> questions

1. For the given function $f(x) = (x^2 - 5x + 7)e^x$, find the [10]
   (a) critical points and classify them,
   (b) intervals where the function is increasing and decreasing,
   (c) intervals where the function is concave up and down.
2. (a) Find the volume of the solid generated by revolving the region between the parabola $x = y^2$ and the line $x = 2$ about the axis $x = 2$. [5 marks] [10]
   (b) If $x + y + z = u$, $y + z = uv$, $z = uvw$, then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. [5 marks]
3. Let $y = F(x,t)$, where F is a differentiable function of two independent variables x and t which are related to two variables u, v by $u = x + ct$, $v = x - ct$. Prove that $\frac{\partial^2 y}{\partial u \partial v} = \frac{1}{4}\left[\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}\right]$. [10]
4. Find the absolute minimum and maximum of $f(x,y) = x^2 - y^2 + xy - 5x$ on the region bounded by $y = 5 - x^2$ and the x-axis. [10]
5. (a) Find a quadratic approximation to $f(x,y) = \cos x \cos y$ near the origin. How accurate is the approximation if $|x| \leq 0.1$ and $|y| \leq 0.1$? [5 marks] [10]
   (b) Evaluate $\int_0^1 \frac{dx}{\sqrt[3]{x^2 - x^3}}$. [5 marks]
6. Change the order of integral and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$. [10]
7. Using double integration, find the area enclosed by the curves $y = 2x^2$ and $y^2 = 4x$. [10]
8. Prove that $\iint_R x^{l-1} y^{m-1} \, dx \, dy = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m+1)} a^{l+m}$, $a > 0$. R is the region bounded by x-axis, y-axis and the line $x + y = a$. [10]
9. For the function given by $f(x,y,z) = x^2 + y^2 + z^2$, find ∇f and its value at $P(2,2,-1)$. Also, find the directional derivative of $f(x,y,z)$ at P in the direction of $\vec{a} = \hat{i} + \hat{j} + 3\hat{k}$. [10]
10. A fluid motion is given by $\vec{v} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$. Show that the motion is irrotational and hence find the velocity potential. [10]
11. Verify Gauss divergence theorem for $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ taken over the region bounded by the planes $x = -1$, $x = 1$, $y = -1$, $y = 1$, $z = -1$ and $z = 1$. [10]
12. Verify Green's theorem for $\int_C (x^2 - y^2)\, dx + 2xy \, dy$, where C is the boundary of the area between $y = x^2$, $y = x$. [10]