



Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. KRITI ARYA	Slot	C1+TC1+TCC1
		Class Nbr	CH2022232300682
Time	3 Hours	Max. Marks	100

Section A (10 X 10 Marks)

Answer any 10 questions

01. Solve the ODE $2x^2y'' + 5xy' + y = \frac{x-1}{2x}$. [10]
02. (a) Find the solution of the differential equation $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 13x = 0$. Hence find the Wronskian of the solutions and conclude that the solutions are linearly independent or not. [5]
(b) Form a partial differential equation by eliminating the arbitrary function from the family of curves $z = f(x + ct) + g(x - ct)$. [5]
03. Solve the PDE: $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$. [10]
04. Find the Laplace transforms of the following functions. [10]
 - (a) $f(t) = \begin{cases} \sin t, & \text{if } 0 < t < \pi \\ \sin 2t, & \text{if } \pi < t < 2\pi \\ \sin 3t, & \text{if } t > 2\pi \end{cases}$ [5]
 - (b) $f(t) = e^{-5t} \int_0^t \frac{\sin t}{t} dt$. [5]
05. Solve the following partial differential equation by the method of Laplace transform [10]

$$\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial t} = -8t; x, t > 0$$
 with the initial and boundary conditions $u(x, 0) = 0$ and $u(0, t) = 2t^2$.
06. Solve the following ODE using Laplace transform. [10]

$$y'' - 4y' - 5y = 30u(t - 1),$$
 given that $y = 0, y' = 6$, at $t = 0$ and $u(t)$ denotes the unit step function.
07. (a) Find the Laplace inverse of the function $\frac{1}{(s^2 + a^2)^2}$, where a is a constant. [5]
(b) Let $f(x) = (\pi - x)^2, x \in (0, \pi)$. Find the half range cosine series of $f(x)$ and hence find the value of $\sum_{n=1}^{\infty} \frac{1}{n^4}$. [5]
08. Let $f(x) = \begin{cases} x & 0 < x < \pi \\ 2\pi - x & \pi < x < 2\pi \end{cases}$ and $f(x + 2\pi) = f(x)$, for all real number x . Find the Fourier series of $f(x)$ and hence find the value $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$. [10]
09. Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence evaluate the Fourier sine transform of $xe^{-a^2x^2}$. [10]
10. Find the Fourier sine transform of $e^{-2x}, x > 0$ Hence evaluate $\int_0^{\infty} \frac{x^2}{(x^2 + 4)^2} dx$. [10]
11. (a) Find the Z-transform of the function $f(n) = ke^{-an} - 2\sin(bn) + 2^n(n^2 - n)$. [5]

(b) Find the inverse Z -transform of the function $\frac{4z^2}{4z^2 - 2z\sqrt{3} + 1}$. [5]

12. Solve the following difference equation.

[10]

$8y(n+2) + 6y(n+1) + y(n) = 5$, where $y(0) = 0$, $y(1) = -1$.

