VIT
Vellore Institute of Technology

Reg. No. :

Final Assessment Test (FAT) - November/December 2023

Programme	**B.Tech.**	Semester	**FALL SEMESTER 2023 - 24**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. Sudip Debnath**	Slot	**A1+TA1+TAA1**
		Class Nbr	**CH2023240101008**
Time	**3 Hours**	Max. Marks	**100**

PART A (10 X 10 Marks)

Answer <u>any 10</u> questions

01. Find the analytic function $w = u + iv$. Given that $v = e^{-2xy}\sin(x^2 - y^2)$. [10]

02. (a) Show that $\log|f(z)|$ is harmonic where $f(z) = u(x,y) + iv(x,y)$ [5 marks] [10]
(b) Find the image of the circle $|z| = 2$ under the transformation $w = \left(\sqrt{2}e^{\frac{i\pi}{4}}\right)z$. [5 marks]

03. Determine the bilinear transformation which maps the points $z = 0, -i, 2i$ into the points $w = 5i, \infty, -\frac{i}{3}$ respectively. Find all the invariant points of this transformation. Sketch the image of $|z - i| < 1$ under the obtained transformation. [10]

04. Evaluate $\int_0^\infty \frac{dx}{x^4+1}$ using contour integration. [10]

05. (a) Find the Laurent's series for the function $\frac{z^2-1}{z^2+5z+6}$ in the region $2 < |z| < 3$. [5 Marks] [10]
(b) Evaluate $\int_C \frac{tan\frac{z}{2}}{(z-1-i)^2}dz$, where C is the boundary of the square whose sides are the lines $x = \pm 2$ and the y-lines as $y = \pm 2$. [5 Marks]

06. (a) Do the matrices $\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, and $\begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$ span $M_{2\times 2}$? Justify your answer. [5 Marks] [10]
(b) Does $W = \{(x,y,z) \mid \text{either } y = 0 \text{ or } z = 0\}$ subspace of $\mathbb{R}^3$? Justify your answer [5 Marks]

07. Find bases for the row space, column space, null space, rank and nullity for the given matrix [10]

$$A = \begin{bmatrix} 1 & 2 & 1 & 5 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & -1 & 1 \end{bmatrix}$$

08. Let $F : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear mapping given by [10]
$F(x,y,z,t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$. Find the basis, dimension of kernel of F, Image of F and the dimension of Image of F.

09. Find the matrix of change of basis P from the basis $\alpha = \{(1,2,1), (-1,2,1), (1,1,3)\}$ to $\beta = \{(-3,2,-3), (1,-1,-1), (5,4,9)\}$ for $\mathbb{R}^3$. Also find the matrix of change of basis from β to α. [10]

10. Consider $P_2(R)$ with basis $\{1, t, t^2\}$ and inner product $< f(t), g(t) >= \int_0^1 f(t)g(t)dt$. Find an orthogonal basis of $P_2(R)$ using Gram-Schmidt process. [10]

11. (a) For $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$ in $\mathbb{R}^2$ defined as [10]

$\langle \alpha, \beta \rangle = (x_1 + x_2)(y_1 + y_2) + (2x_1 + x_2)(2y_1 + y_2)$.
Show that $\langle , \rangle$ is an inner product in $\mathbb{R}^2$. [5 Marks]

(b) Verify the Cayley-Hamilton theorem for $A = \begin{bmatrix} 1/5 & 4 \\ 3/5 & -2 \end{bmatrix}$ and hence, find the inverse of A.

[5 Marks]

12. Find for what real value(s) of c, the following system of equations has non-trivial solution and hence solve: **[10]**

$x + 2y + 3z = cx$; $3x + y + 2z = cy$; $2x + 3y + z = cz$ using Gauss elimination