VIT
Vellore Institute of Technology

Reg. No. :

Final Assessment Test (FAT) - APRIL/MAY 2023

Programme	**B.Tech**	Semester	**Winter Semester 2022-23**
Course Title	**DIFFERENTIAL EQUATIONS AND TRANSFORMS**	Course Code	**BMAT102L**
Faculty Name	**Prof. SURATH**	Slot	**D1+TD1+TDD1**
		Class Nbr	**CH2022235002761**
Time	**3 Hours**	Max. Marks	**100**

Part-A (10 X 10 Marks)
Answer any 10 questions

01. Solve $(D^2+2D+1)y = e^{-x}\ln x$ by using method of variation of parameters, where $D = \frac{d}{dx}$. [10]

02. Solve the PDE $(y^2+z^2)p - xyq = -xz$ [10]

03. a) Find the Laplace transform of $t^2e^{-2t}\cos t$ [10]
b) Find the inverse Laplace transform of $\frac{s}{s^4+4a^4}$ **[5+5 marks]**

04. Solve $y'' - 8y' + 15y = 9te^{2t}$, with $y(0) = 5, y'(0) = 10$ by using Laplace transform. [10]

05. Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2 & \text{if } |x| < a \\ 0 & \text{if } |x| > a \end{cases}$ [10]
where $a > 0$. Also, deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3}dt$

06. Evaluate $\int_0^\infty \frac{x^2dx}{(a^2+x^2)(b^2+x^2)}$ using Fourier transform techniques and hence find $\int_0^\infty \left(\frac{x}{x^2+1}\right)^2 dx$. [10]

07. Find the Fourier series for $f(x) = \begin{cases} x & \text{if } -1 < x \le 0 \\ x+2 & \text{if } 0 < x \le 1 \end{cases}$. [10]
Also find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$

08. Find the Fourier series of $f(x) = x^2, x \in (-\pi, \pi)$. Use Parseval's identity to deduce the sum of [10]
$\sum_{n=1}^{\infty} \frac{1}{n^4}$

09. a) Find $Z\left(\frac{n-2}{n(n+1)}\right)$ [10]
b) Find the inverse Z-transform of $\frac{z}{z^2+7z+10}$ **[5+5 marks]**

10. Solve the difference equation $y(n+2) - 5y(n+1) + 6y(n) = 4^n$ by using Z-transform with [10]
$y(0) = 0, y(1) = 1$.

11. a) Solve the PDE by using Laplace transform $\frac{\partial y}{\partial t} + \alpha\frac{\partial y}{\partial x} = 0$ for $x > 0, t > 0$ and [10]
$y(0,t) = c, y(x,0) = 0$.
b) Use convolution theorem to find $f(t)$ if $L(f(t)) = \frac{16}{(s-2)(s+2)}$ **[5+5 marks]**

12. a) Find the complete solution of the PDE $z^2(p^2z^2 + q^2) = 1$ [10]
b) Solve $x^2y'' - 3xy' + 3y = 0$ with $y(1) = 0, y'(1) = -2$. **[5+5 marks]**