VIT®

Vellore Institute of Technology

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Reg. No. :

Final Assessment Test(FAT) - Nov/Dec 2024

Programme	**B.Tech.**	Semester	**Fall Semester 2024-25**
Course Code	**BMAT205L**	Faculty Name	**Prof. Pavithra R**
Course Title	**Discrete Mathematics and Graph Theory**	Slot	**C1+TC1+TCC1**
		Class Nbr	**CH2024250102263**
Time	**3 hours**	Max. Marks	**100**

General Instructions

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

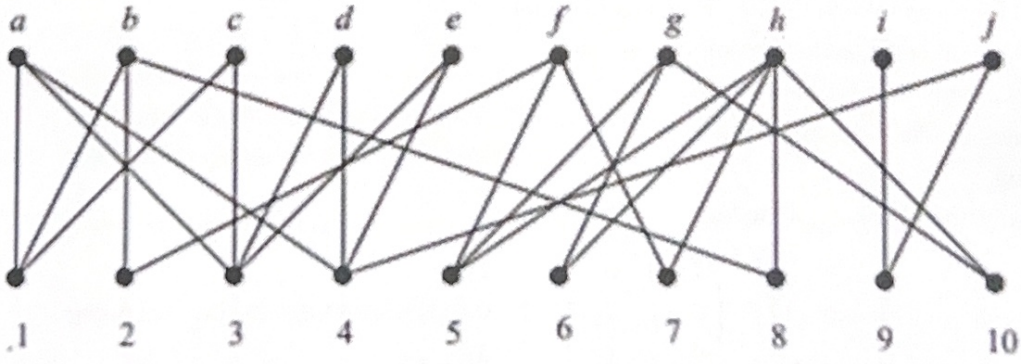
Course Outcomes

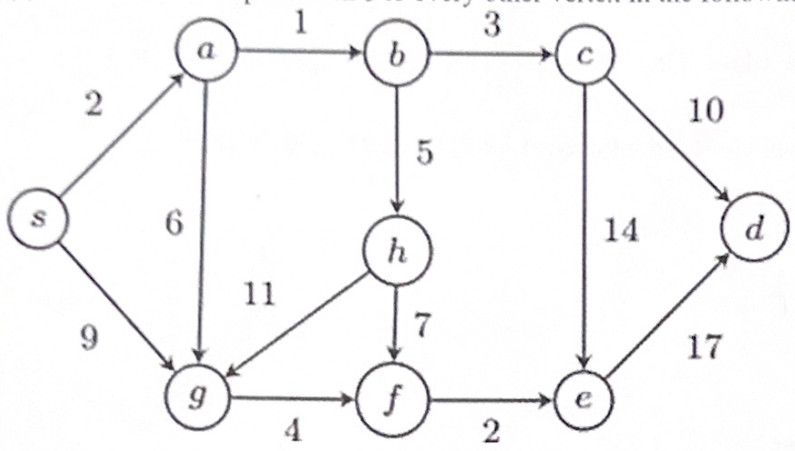
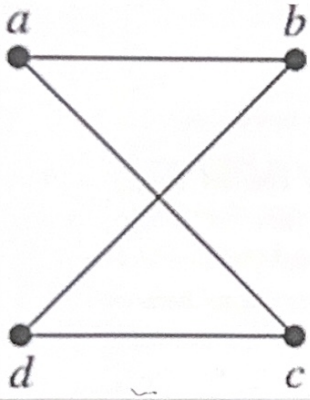
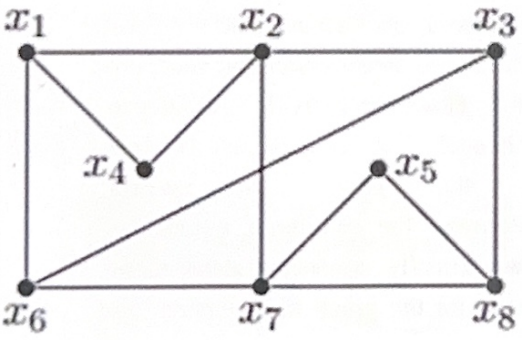
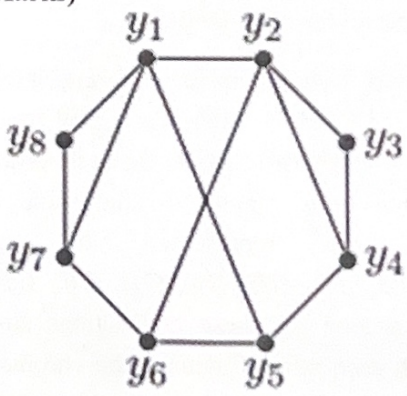
1. Learn proof techniques and concepts of inference theory
2. Use algebraic structures in applications
3. Counting techniques in engineering problems.
4. Use lattice and Boolean algebra properties in Digital circuits.
5. Solve Science and Engineering problems using Graph theory.

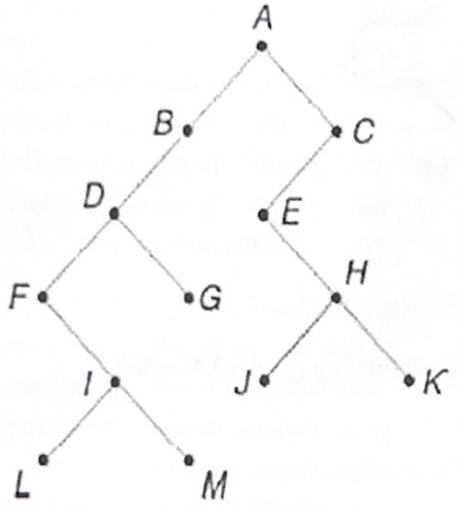
Section - I

Answer any 10 Questions (10 × 10 Marks) *M - Marks

Q.No	Question	*M	CO	BL
01.	(a) Verify if the following statement $[(p \wedge \neg q) \rightarrow r] \rightarrow [p \rightarrow (q \vee r)]$ is a Tautology. **(5 Marks)** (b) Use rules of inference to verify the validity of the following: If $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true. **(5 Marks)**	10	1	2
02.	Determine whether the following inference is valid or not: If it is not cloudy, then if I eat ice-cream, I do not study. Either it is not cloudy or it is raining. If I do not watch movie, I eat ice-cream. It is not raining. Hence, if I study, then I watch movie too.	10	1	3
03.	Given the generating matrix $G = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$ corresponding to the encoding function $e: B^3 \rightarrow B^6$, find the corresponding Parity check matrix and use it to decode the following received words and hence to find the original message. Are all the words decoded uniquely? (i) 101011 (ii) 110101 (iii) 011001	10	2	3

04.	(a) Let $\mathrm{GL}(2,\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a,b,c,d \in \mathbb{R},\ ad-bc \neq 0 \right\}$ be the non-commutative group with respect to usual matrix multiplication. Let $\mathbb{R}^*$ be group of all non-zero real numbers under multiplication. Define $f: GL(2,R) \to \mathbb{R}^*$ by $f\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = ad-bc$, for all $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{GL}(2,\mathbb{R})$, check if f is a homomorphism. **(5 Marks)** (b) (i) How many positive integers less than 1000 are divisible by 8? (ii) How many are divisible by 8 but not by 12? (iii) How many are divisible by both 8 and 12? (iv) How many are divisible by either 8 or 12? (v) How many are divisible by exactly one of 8 and 12? **(5 Marks)**	10	2,3	4
05.	(a) In how many ways can 5 software engineers be selected from 15 software engineers when: (i) There is no restriction on selection (ii) A particular engineer is included every time (iii) A particular engineer is never included. **(3 Marks)** (b) Solve the following recurrence relation using the generating function that satisfies the initial conditions given below: **(7 Marks)** $a_n - 4a_{n-1} + 3a_{n-2} = 2^n, a_1 = 1, a_2 = 11$	10	3	3
06.	Let R be a relation defined by$\{(x,y) \mid x \textit{ is a factor of } y\}$ on a set $A = \{1,2,3,4,6,10,12,20\}$. Write all elements of R. (i) Check whether the set A is a poset or not (ii) Draw a Hasse diagram that represents the relation (iii) What are the maximal and minimal elements of A? (iv) Write the GLB and LUB of {1, 2, 3} (v) Write the upper bound and lower bound of {2, 6} (vi) Whether the set A is a lattice or not?	10	4	4
07.	(a) Suppose that R is a relation on the set of strings of English letters such that aRb if and only if $l(a) = l(b)$, where $l(x)$ is the length of the string x. Is R an equivalence relation? **(5 Marks)** (b) Find a maximum matching and minimum vertex cover for the following graph: **(5 Marks)** 	10	4,5	4

08.	(a) Find the shortest path from s to every other vertex in the following graph **(7 Marks)**  (b) List the number of walks of length three from a to c in the following graph **(3 Marks)**  	10	5	5
09.	(a) Check whether the graphs are isomorphic or not. **(4 Marks)**   (b) Suppose that there are four employees in the computer support group of the School of Engineering of a large university. Each employee will be assigned to support one of four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking, and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software. **(6 Marks)** (i) Draw a graph to model the four employees and their qualifications. (ii) Check whether the graph is planar. (iii) Check whether the graph is Eulerian.	10	5	6

10.	(a) A tree has 10 vertices, and their degrees are 1, 1, 1, 2, 2, 2, 3, 3, 4, 5. Find the number of edges in the tree. **(2 Marks)** (b) A graph has 8 vertices and 10 edges. How many edges should be removed to form a spanning tree? **(2 Marks)** (c) Determine the Preorder, in order and post order traversal for the below tree. **(6 Marks)** 	10	5	3
11.	A renewable energy company is planning to construct power lines to connect several communities for a new solar and wind energy network. The communities are labeled Community A, Community B, Community C, Community D, Community E, and Community F. The cost of constructing the power lines between each pair of communities (in millions of dollars) varies due to geographical factors and construction challenges, with the following construction costs: A to B: 20; A to C: 25; A to D: 22; B to C: 18; B to D: 24; B to E: 19; C to D: 17; C to F: 28; D to E: 21; D to F: 23; and E to F: 16. Determine the minimum total construction cost to connect all the communities.	10	5	6
12.	A company is setting up a communication network across six departments, labeled A, B, C, D, E, and F. Each department needs to have a secure, direct communication line with certain other departments to facilitate efficient workflow. However, to avoid interference, adjacent connections cannot use the same frequency/channel. The connections between departments are represented by edges in the graph as follows: AB, AD, BC, BD, BE, BF, CE, CF, DE, EF. Determine the minimum number of frequencies needed for these connections so that no two directly connected departments interfere with each other. Calculate the chromatic polynomial for the graph to understand the number of ways we can assign different frequencies.	10	5	5

BL-Bloom's Taxonomy Levels - (1.Remembering, 2.Understanding, 3.Applying, 4.Analysing, 5.Evaluating, 6.Creating)