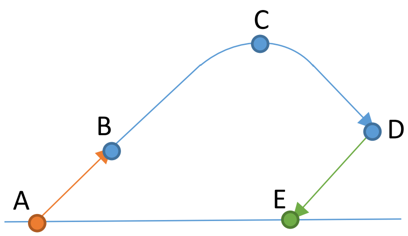
Description:

One breezy afternoon Algebra Alex decides to launch Hamster Huey into the air using a model rocket. The rocket is launched over level ground, from rest, at a specified angle above the East horizontal. The rocket engine is designed to burn for specified time while producing a constant net acceleration for the rocket. Assume the rocket travels in a straight-line path while the engine burns. After the engine stops the rocket continues in projectile motion. A parachute opens after the rocket falls a specified distance from its maximum height. When the parachute opens the rocket instantly changes speed and descends at a constant vertical speed. A horizontal wind blows the rocket, with parachute, from the East to West at the constant speed of the wind. Assume the wind affects the rocket only during the parachute stage.

Diagram:



Stage AB: Engine works
Stage BD: Engine fails, projectile motion
Stage DE: Parachute

Strategy:

Stage AB:

Givens:

a = 4.6 m/s^2
x_A = 0 m
V_A = 0 m/s
t = 8 s
Θ = 37° above horizontal

Step 1: Find the accelerations in the x- and y-directions using trigonometry.

$$a_x = a \times cos\theta \qquad a_y = a \times sin\theta$$
$$a_x = 4.6 \times cos37 \qquad a_y = 4.6 \times sin37$$
$$\underline{a_x = 3.674 m/s^2} \qquad \underline{a_y = 2.768 m/s^2}$$

Step 2: Find the velocity in the x-direction at B. Then, find the displacement for stage AB.

x-dir:

$$V_{Bx} = a\Delta t + V_A \qquad {V_{Bx}}^2 = {V_A}^2 + 2(a_x)(\Delta x)$$
$$V_{Bx} = (3.674)(8) + 0 \qquad (29.39)^2 = 0 + 2(3.674)(\Delta x)$$
$$\underline{V_{Bx} = 29.39 m/s} \qquad 863.76 = 7.347\Delta x$$
$$\underline{\Delta x_{AB} = 117.56 m}$$

Step 3: Find the height at B and the velocity in the y-direction at B using trigonometry.

y-dir:

$$tan\theta = \frac{y_B}{\Delta x_{AB}} \qquad tan\theta = \frac{V_{By}}{V_{Ay}}$$
$$tan37 = \frac{y_B}{117.56} \qquad tan37 = \frac{V_{By}}{29.39}$$
$$(tan37)(117.56) = y_B \qquad (tan37)(29.39) = V_{By}$$
$$\underline{y_B = 88.58 m} \qquad \underline{V_{By} = 22.15 m/s^2}$$

Stage BD:

Givens:

a = -9.8 m/s^2
x_B = 117.56 m
Θ = 37° above horiz.
$y_D = y_C$ – 70 m
V_{Bx} = 29.39 m/s
V_{By} = 22.15 m/s
V_{Cy} = 0 m/s

Step 1: Find the time at which the rocket is at C and the height at C.

y-dir:

$$V_{Cy} = a\Delta t + V_{By}$$
$$0 = (-9.8)t + 22.15$$
$$-22.15 = -9.8t$$
$$\underline{t = 2.260s}$$

$$y[t] = \frac{1}{2}at^2 + V_{By}t + y_B$$
$$y_C = \frac{1}{2}(-9.8)(2.260)^2 + (22.15)(2.260) + 88.59$$
$$y_C = (-4.9)(5.107) + 50.049 + 88.59$$
$$y_C = -25.025 + 138.64$$
$$\underline{y_C = 113.62m}$$

Step 2: Find the time the rocket takes to fall from C to D. Then, find the position at D.

y-dir:

$$y_D = \frac{1}{2}at^2 + V_{By}t + y_C$$
$$43.61 = \frac{1}{2}(-9.8)t^2 + 22.15t + 88.59$$
$$-44.98 = -4.9t^2 + 22.1t$$
$$0 = -4.9t^2 + 22.15t + 44.98, solver$$
$$\cancel{t = -1.52s} \qquad \underline{t = 6.04s}$$

x-dir:

$$x_D = \frac{1}{2}at^2 + V_{Bx}t + x_B$$
$$x_D = (29.39)(6.04) + 117.56$$
$$x_D = 177.5 + 117.56$$
$$\underline{x_D = 295.06m}$$

Stage DE:

Givens:

$x_D = 295.06$ m $V_{Dx} = -20$ m/s

$y_D = 43.61$ m $V_{Dy} = -8$ m/s

$y_E = 0$m

Step 1: Find the time the rocket takes to fall from D to E.

y-dir:

$$y_E = \frac{1}{2}at^2 + V_{Dy}t + y_D$$
$$0 = -8t + 43.61$$
$$\underline{t = 5.451s}$$

Step 2: Find the displacement from D to E.

x-dir:

$$\Delta x_{DE} = \frac{1}{2}(V_{Ex} - V_{Dx})t$$
$$\Delta x_{DE} = \frac{1}{2}(-20 + (-20))(5.451)$$
$$\Delta x_{DE} = (-20)(5.451)$$
$$\underline{\Delta x_{DE} = -109.03\ m}$$

Stage AE:

Givens:

$x_D = 295.06$ m $\Delta x_{DE} = -109.03$ m

Step 1: Find the total displacement.

$$\Delta x = x_D + \Delta x_{DE}$$
$$\Delta x = 295.06 + (-109.03)$$
$$\boxed{\Delta x = 186.0\ m\ East}$$