HiMCM 2016

Problem A: Swim, Bike, and Run

Team #6287

Dear Mayor,

A schedule for start times has been developed for your town's upcoming triathlon. This new schedule will fit for a range of attendants, if fewer attend or more than expected. There will be minimal backlog in portions of the race where it might be squishy in portions of the road.

To show which the effectiveness of the mathematical model, several graphs were analyzed, one showed minimal intersections of waves at certain point throughout the race. The more intersection there were the weaker the mathematical model was and thus could not be the optimal solution for scheduling. Continuing to improve upon the time between when different waves were released, fewer and fewer lines intersected while all allowed the roads to be open in under than five and a half hours. To find when the roads started to be closed and then until when they opened, the sum of the swimming time and the first transition time was calculated, and the minimum value among the different waves was obtained. Therefore, the roadways had to be blocked starting from 39 minutes since the beginning of the race, until the last wave had finished the triathlon. A table of values was created by subtracting 39 from the sum obtained in the last step and also from each wave's total times for the whole race.

In order to apply the analysis of this data to the triathlon in your town, follow the formatting that the chart illustrates. To find the start times pick the hour the race will start and add the minutes to that hour for each wave and that is when the wave will start. This is the most efficient solution because there might not be known times of people entering the race. If all is known is age then this will provide the best results for an efficient race.

Sincerely,

HiMCM Team #6287

Wave Description	Percentage of People	Heat Number	Start Time	Finish swimming	Finish T1	Finish Biking	Finish T2	Finish
Slowest Slowest	4.82	**Wave 1**	0	29	43	226	232	320
Faster Slowest	8.18	**Wave 2**	3	28	40	147	152	229
Pros & Premiers	2.10	**Wave 3**	20	35	39	104	105	142
18-25 & 60+	5.12	**Wave 4**	30	51	60	148	151	209
26-35	7.03	**Wave 5**	41	62	71	164	167	224
26-35	7.03	**Wave 6**	15	38	45	129	132	189
26-35	7.03	**Wave 7**	53	77	84	166	169	226
26-35	6.99	**Wave 8**	62	85	92	175	178	233
36-45	7.06	**Wave 9**	71	91	100	184	187	242
36-45	7.06	**Wave 10**	105	126	135	224	227	284
36-45	7.06	**Wave 11**	25	48	55	141	144	204
36-45	7.03	**Wave 12**	92	117	124	206	209	266
46-60	7.62	**Wave 13**	98	119	128	216	219	278
46-60	7.58	**Wave 14**	104	127	134	219	222	282
46-60	7.55	**Wave 15**	120	145	152	233	236	296

Summary

To organize and execute a successful triathlon, many parameters must be taken into consideration - the schedule of runners must be optimized in order to prevent congestion, and the amount of time that the roads are blocked off for the runners and the bikers must also be minimized. In previous triathlons, numerous start methods were used: Iron Man, where everyone starts together, Staggered, where one person or a group begin at a certain increment, or Waves where a large group of people begin in large division. (Complete Tri, 2016). Analyzing these previous methods and studying the data provided, a schedule for the triathlon was established that prevented congestion and had roadways blocked for less than five and a half hours. The schedule created can applied a triathlon of approximately 2,000 runners.

The novel method established has athletes with the overall slowest total times beginning in the first and second wave, Pros and Premiers beginning in the third and then the athletes in the Open beginning in the 12 other waves based on age and within age groups, by total time in their best performance. We created a graph that illustrated the congestion of each different schedule created.

The outline developed can be applied to any triathlon of relative size and relative constraints. The order must be as mentioned, which is 13% of all runners, then the athletes which establish themselves as a Pro or Premier, which account for about 2.4 % of all athletes, and the rest are all Opens. The Opens start from 18 and there is no older limit. The age group are 18-25, 26-35, 36-45, 46-60. 60+. The age group are broken into wave of 275 or less depending on their size and the wave go in order from oldest to youngest.

The method developed for creating a schedule is simple yet efficient because it can be applied to any town having a triathalon.

Introduction

A local town is holding a triathlon, and the mayor has hired an organization to develop a schedule that will prevent congestion, while minimizing the time the road is blocked for the race. Reducing the time the road is closed will allow the town to spend less money on police officers, who are needed to block the roads. All the athletes will be able to achieve their best time because of minimal congestion. The town is expecting approximately 2000 athletes to participate. Female and male athletes, ages 18 and older, will be competing at varying performance levels.

Assumptions

- ➔ The minimum age requirement to enter the race is 18 years old. There is no maximum age limit.
- ➔ The percentages of different performance levels of athletes competing in the triathlon are proportional to other triathlons.
- ➔ Athletes move at a constant speed for each portion of the race (swimming, biking, and running).
- ➔ The maximum amount of time that the road can be closed (five and a half hours) does not include the time taken to block and unblock the roads.
- ➔ Each athlete has a chip that measure his or her time accurately, so his or her time will be correct regardless of when he or she starts. The athlete's time starts when he or she passes

the start line into the water. This prevents an inaccurate time because it may take several minutes for he or she to enter the water.

- ➔ The roads are wide enough for people run next to several other athletes without causing congestion.
- ➔ Waves can not exceed 275 athletes for safety reasons.
- ➔ Congestion is defined as 750 people or more athletes within kilometer of each other.

Model Description

The first step in creating the mathematical model was analyzing the data and determining guidelines to sort the athletes into categories other than Pro and Premier. The categories created are “Fast,” “Standard,” “Slow,” and “Exceedingly Slow.” Athletes who are considered “Slow” are those who take four hours or more to complete the race in total, and “Exceedingly Slow” are those who take three hours or more to complete the road portion. Athletes who fall into the Clydesdale or Athena categories were included with the Open Athletes because there was no significant difference between their times and the times of the “Open” athletes.

The next step that was taken was to reorganize the data by removing Pros and Premiers from the list of athletes. This division left the team with two smaller groups: Pros and Premiers, and the Open athletes. The Slow and Exceedingly Slow athletes were then removed and put into their own group. The team now had four groups to split into waves. The Exceeding Slow athletes all ran together in the first wave. The Slow athletes ran together in the second wave.

Pros and Premiers ran in the third wave because they would take the shortest amount of time to finish and needed the minimal amount of people in front of them to prevent congestion. Because of the variance on times between the Slow athletes it is easier for the Pros and Premiers to pass them because they are the group that will spread out the most in the 20 minutes. The fact that they still go before many of the other athletes means that they will be more interested in racing in our race because they are not slowed down by many The Standard athletes were divided into age groups, 18-25, 26-35, 36-45, 46-60, 60+. The number of people in each group was divided by the maximum number of athletes a wave could hold. From there, the numbers of wave from each group was calculated. Athletes who were categorized as Standard ran in several waves based upon the age group, and the age groups ran in order from fastest to slowest total time. The time is calculated from each age group. The fourth wave was 18-25 and 60+ because both were too small to separately send out as different waves and because 18-25 had the fastest average total times. The 26-35 age group was divided into four even waves. The 36-45 age group was also divided into four even waves. The 46-60 age group was divided into three even age groups due to the lower number of participants in the age group.

The first mathematical model created was a piecewise function graph (Figure 2). In order to determine the wave size best performance time the runner, and status as an athlete was taken into consideration method that used to organize. The slowest people start running first. Then ten minutes after the slowest people start running, the Pros and Premiers start running. By organizing it this way, the slow people will still be in the water and the Pros will start biking, they will not be inhibited by the slowest people. Each line on Figure 2** is the average time of each wave that it takes them to complete the five distances of the triathlon. The model had to show minimal congestion. The organization defines congestion as more than two lines intersecting at any given point. If more than two lines intersected that area was deemed too congested, and then the lines would need to be reorganized. Allowing only two lines to intersect at any given point optimized use of the road space because no more than 750 will beat any given place.   The starting times at for each of waves was the same increments, 3 minutes in between each wave.

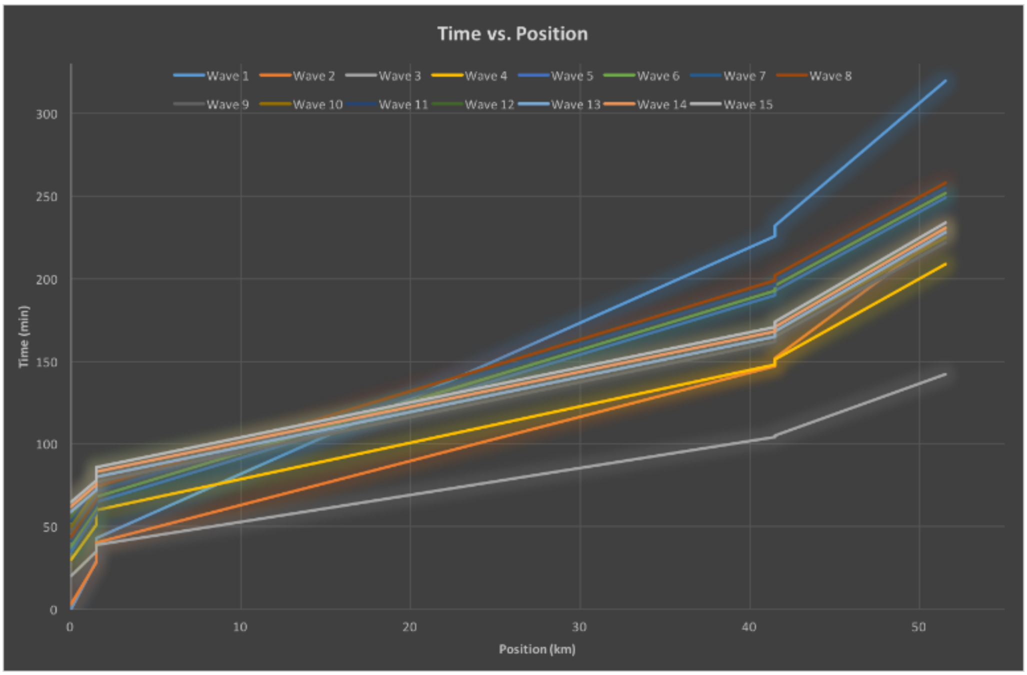

Figure 1. First version of "Time vs. Position" graph. Waves start at different times.

The second mathematical model created was an improved piecewise function graph (Figure 3), using Figure 1 as the starting point. Figure 2 also used the average times for each wave to finish the same amount of distance. Time was the dependent variable in this situation so we made a Time vs. Position graph rather than the usual vice versa. The y-value of each point on the line of Figure 2 is the average time it takes to complete each of the five distances of the triathlon. Changing the start time of the waves allowed the organization to find the optimal schedule. A graph with the least congestion would form a graph where no lines intersected, but that would elongate the time that the road is blocked off by an extremely unreasonable amount. Creating a schedule that showed no lines intersected would inefficient because the faster athletes would need to go first, then the Standard athletes, and at the very end the slowest athletes in order to avoid crossing paths. Then the average line for each wave was graphed, time vs. distance. The x-axis of Figure 1 was distance because that variable was the same for each time, even though presenting Figure 2 as distance as a function of time represents the model in a better fashion, Excel could not handle using different x values, but producing the same y value. Figure 1 could not show piecewise functions either. However, even though Figure 2 is not able to show piecewise functions, the data is still accurate because the slopes show average velocity if it is $\frac{x_2 - x_1}{y_2 - y_1} = vavg$, distance,/time. The line that has the highest maximum is the group that takes the longest time to complete the triathlon. The group that has the lowest maximum is the group that finishes in the shortest amount of time. The distance time from when the first athlete enters the graph

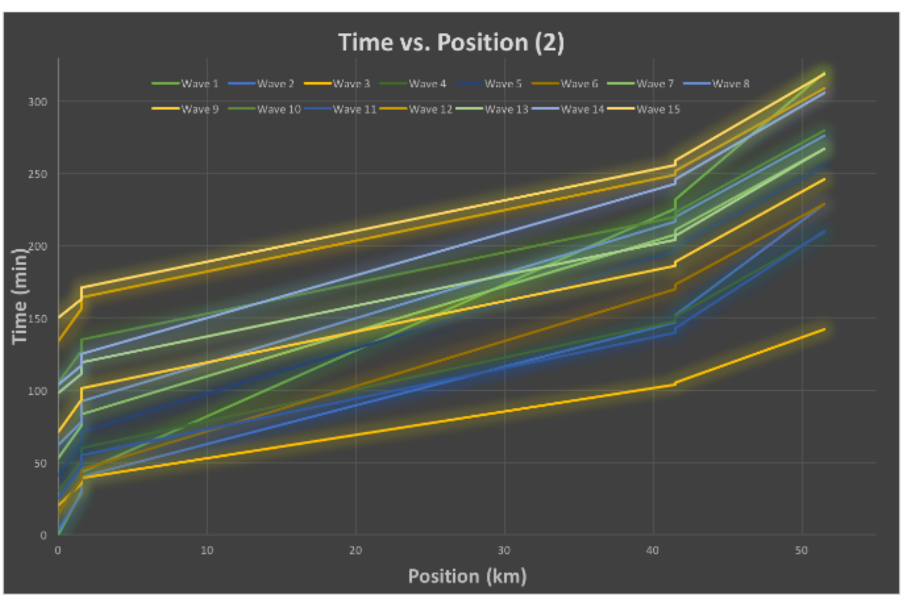

Figure 2. Second version of "Time vs. Position". Wave start times were changed.

Figure 1 and Figure 2 did not show the most accurate model of what time each wave would finish because within the section for different ages, they were broken up into 15 waves. So the average of the total group would be slightly different than the total averages. The Optimal Solution depicts the averages for each wave within the certain age group.

To further analyze the method developed, of first the Slowest, then the Pros and Premiers, then the divided age groups of the normal people (18-25, 26-35, 36-45, 46-60, and 60+), a program was developed to visually simulate the race. The program draws a box, divided into 3 parts, which are proportional to the race distances. Each division is a different color, blue representing the swimming portion, green representing the biking portion, and red representing the running portion. There are white pixels representing each runner and move through the race. The program tracks the minimum, maximum, and average, time for each wave throughout the race. The program allows the organization to see times where there is too much congestion. The program utilizes graphics class and JFrames.

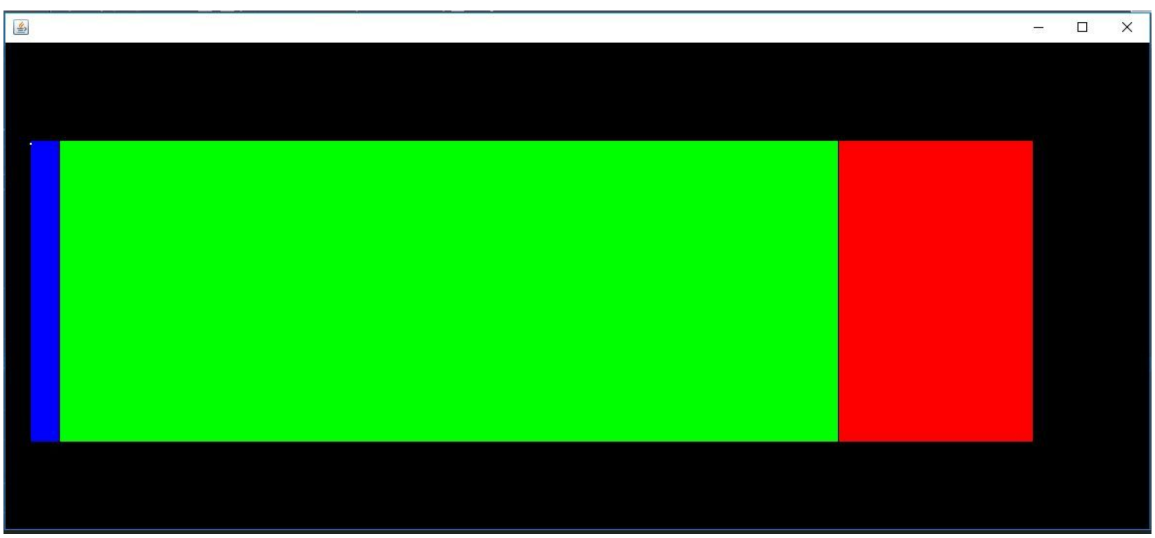

Figure 3. The output of the program.

<u>Part II:</u>

The program was also utilized in exploring the advantages to congestion when adjusting the distances. When adjusting the distance variables in the program, it became clear that the longer any of the distances were, the less congested everything became; however, this is unreasonable because we would have to totally disregard the 5.5 hour time limit that was given in the problem.

<u>Analysis</u>

The waves did not hold more than 275 runners, in order to minimize congestion.

To determine whether the model accurately represented the solution, the team analyzed the graphs that were created.

In order to prove the effectivity of the mathematical model, and in order to display how the goal of the problem was achieved, the average road time for each of the waves was calculated. In order to so, the sum of the swimming time and the first transition time was calculated, and the minimum value among the different waves was obtained. Therefore, the roadways had to be blocked starting from 39 minutes since the beginning of the race, until the last wave had finished the triathlon. So, a table of values was created by subtracting 39 from the sum obtained in the last step and also from each wave's total times for the whole race, as seen in *Table 1* The difference between these two values represents the average amount of time that all of the participants in each wave spent on the roads, also known as their road times. Next, a stacked bar graph was created using the data from *Table 1* Because the values in *Figure 5* as well as the values in *Table 1* did not exceed 330 minutes, the road does not need to be blocked for more than 5 and a half hours, indicating that this mathematical model achieved its goal.

Table 1: Table of Values for The Average Road Time per Wave

Wave Number	Amount of Time Before Reaching the Road (min)	Amount of Time On the Road (min)
1	4	277
2	1	189
3	0	103
4	26	149
5	32	185
6	6	144
7	45	142
8	53	141
9	61	142
10	111	149
11	71	149
12	85	142
13	89	150
14	100	148
15	128	144

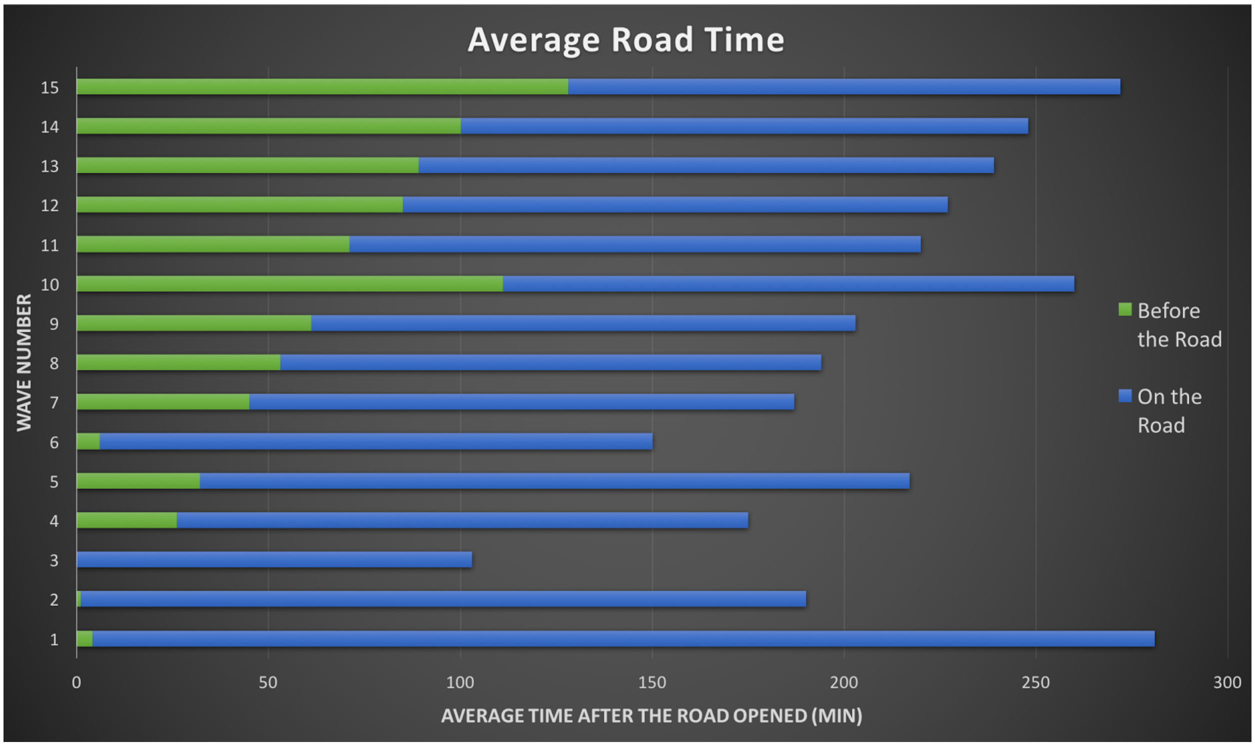

Figure 5. A stacked bar graph displaying the average road times for each wave.

The main goal of this mathematical model was to minimize congestion, as defined in the assumptions as over 750 people in a one kilometer long length of the race at any given time. To display the success of the model in this field, as shown in *Figure 5* no more than 2 lines, each one representing one division consisting of less than 275 people intersected at a certain time. By ensuring that on average, only one to two waves were present at a certain time and place, congestion of over 750 people was prevented.

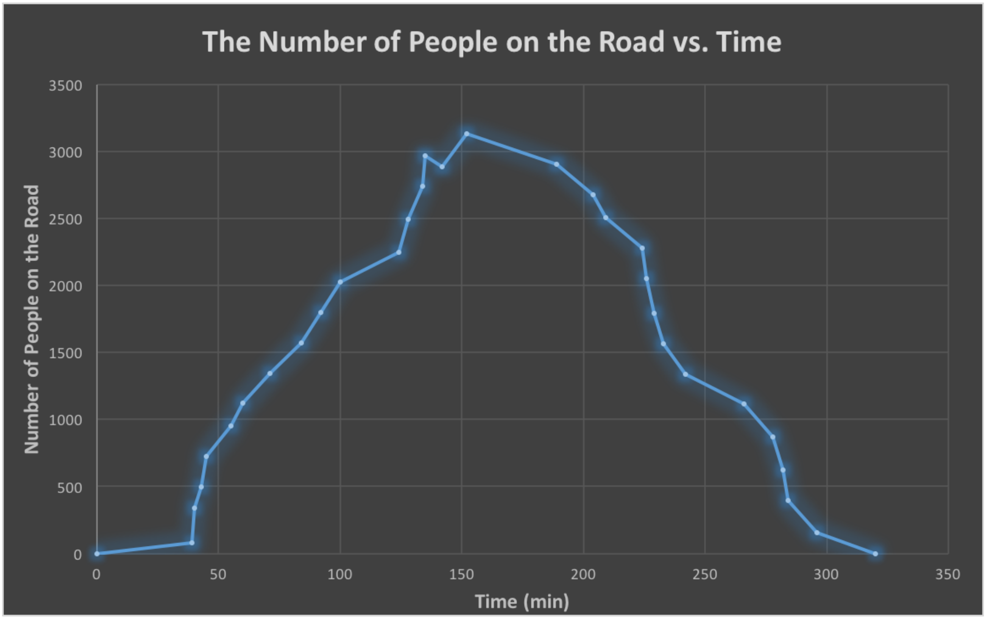

Figure 6. The number of people on the road at any given moment during the race (includes biking, T2 time, and running).

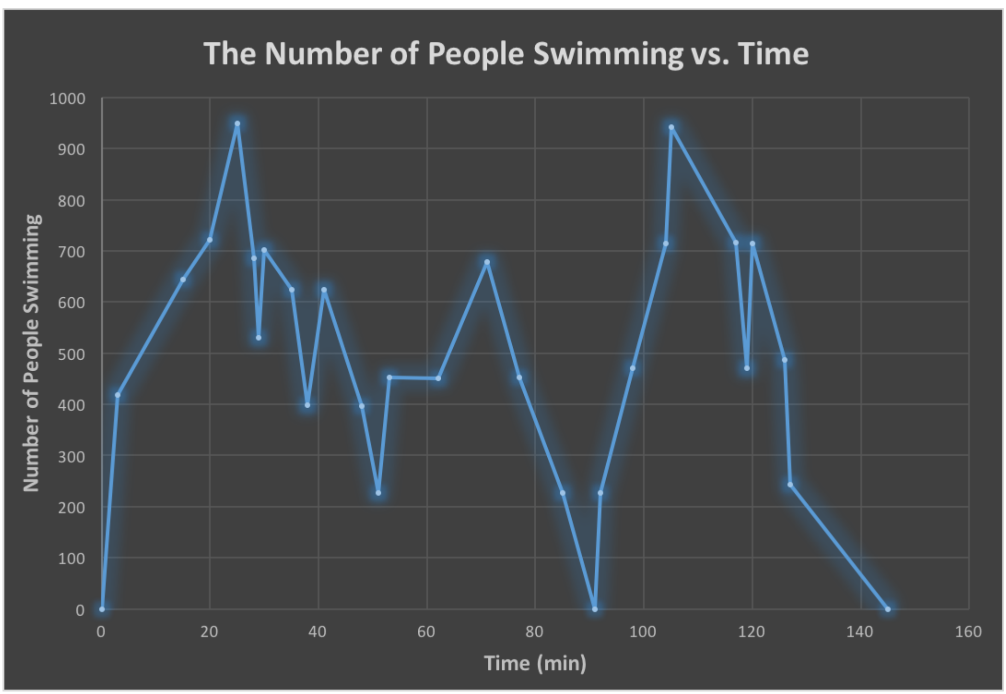

Figure 7. The number of people swimming at any given point in time during the race.

Results and Conclusion

Another method that was used to ensure that the roads were not closed for more than the given time was to find the first person who reached the road and the last person who finished to race, and calculate the overall time that the road needed to be closed for the participants by finding their difference. Based on the optimal solution, shown in *Figure* ***, the first person started running 30 minutes after the beginning of the triathlon, and the last person finished the race 328 minutes later; thus, the road must be blocked for 5 hours and 28 minutes, staying within the 5 and a half hour time limit that was provided in the question.

Table 2.

Wave Description	**Number of people**	**Heat Number**	**Start Time**	**Finish swimming**	**Finish T1**	**Finish Biking**	**Finish T2**	**Finish**
Slowest Slowest	155	**Wave 1**	0	29	43	226	232	320
Faster Slowest	263	**Wave 2**	3	28	40	147	152	229
Pros & Premiers	78	**Wave 3**	20	35	39	104	105	142
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46-60	244	**Wave 14**	104	127	134	219	222	282
46-60	243	**Wave 15**	120	145	152	233	236	296

References

Complete Tri. (September 3, 2016). TRIATHLON SWIM STARTS: WAVE VS. TIME TRIAL (AND OTHERS).

http://completetri.com/triathlon-swim-starts-wave-vs-time-trail-and-others/