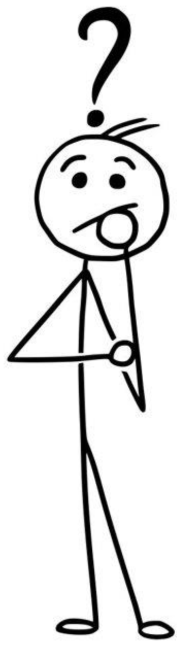
Ice-Breaker Question (while we wait to start):

Which superhero do you think would be a great econometrician?

Get ready to enter your answer at **menti.com** when we start.



Introduction to Causal Inference and the Potential Outcome Model

Topics in Data Analytics & Econometrics

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Lesson Plan

- **Coding Exercise**

  A Marvel Universe Example: Being Dr. Strange's Assistant!

- **The Potential Outcome Model**

  Causality can be illusive

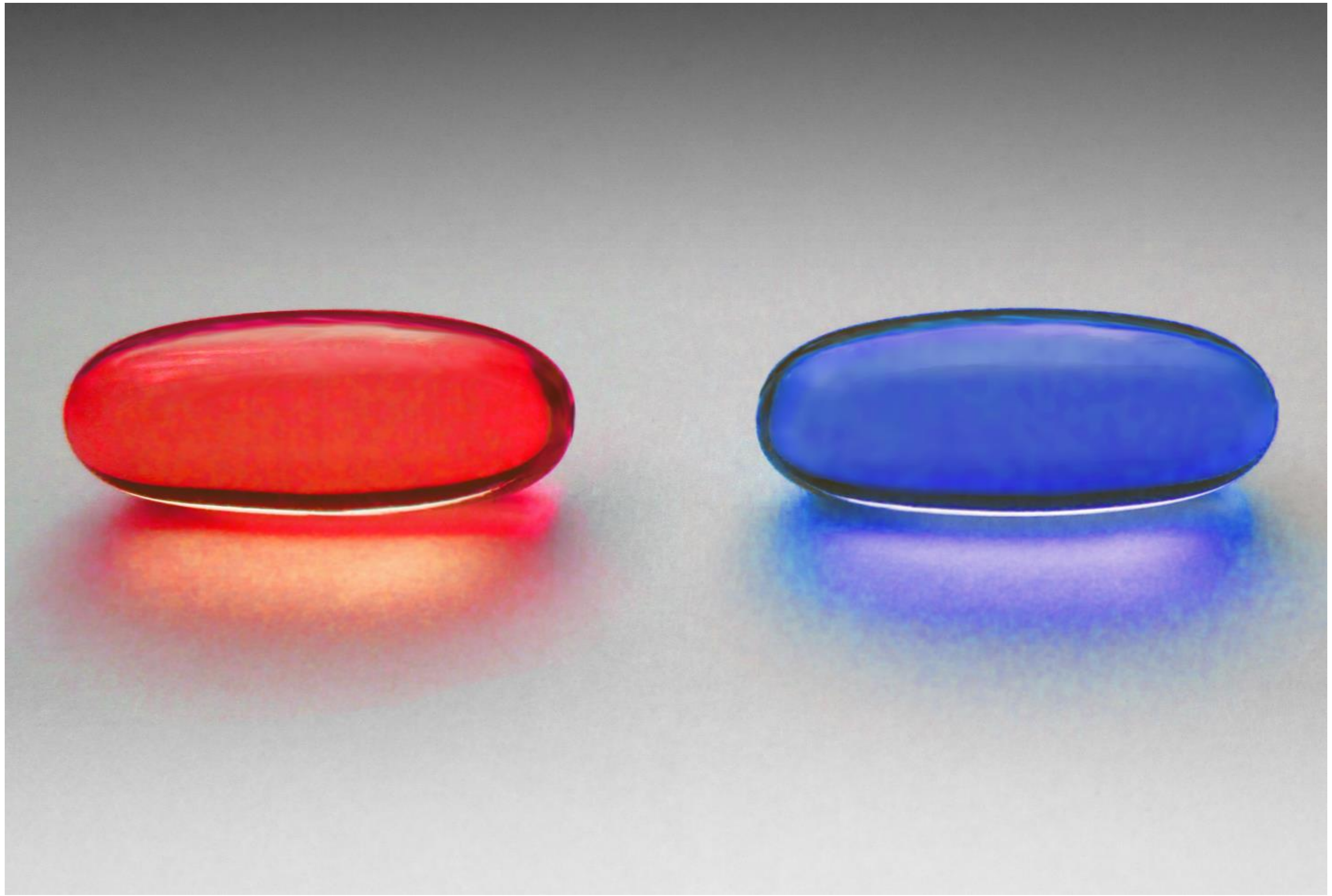
Coding Exercise:

Saving Cancer Patients! (As Dr. Strange)

Dr. Strange is a very well-known neurosurgeon.

Who also happens to be a superhero that can travel through different dimensions!

Red Pill or Blue Pill?



A rare brain cancer has infected 10 patients. The medical board has recently developed a red pill and blue pill as a possible cure.

Dr. Strange was given the task to assign the pills to the patient.

	Dim. 1	Dim 0
Patient ID	$Y_i^{Blue\ (1)}$	$Y_i^{Red\ (0)}$
1	7	1
2	5	6
3	5	1
4	7	8
5	4	2
6	10	1
7	1	10
8	5	6
9	3	7
10	9	8

Being Dr. Strange, he traveled to two different dimensions.

1) Blue pill is administered to all patients
2) Red pill is administered to all patients

In each dimension, Dr. Strange gets to observe how many years the patient lives after taking the pill.

Y_i^j = # years living well after taking the pill in dimension j

j = 1 in the dimension where everyone takes the blue pill

j = 0 in the dimension where everyone takes the red pill

	Dim. 1	Dim 0	Treatment Effect	What would Dr. Strange do?
Patient ID	Y_i^1	Y_i^0	α_i	D_i
1	7	1		
2	5	6		
3	5	1		
4	7	8		
5	4	2		
6	10	1		
7	1	10		
8	5	6		
9	3	7		
10	9	8		

Coding exercise 1 (10 minutes):

Treatment here is given blue pill instead of red.

1) Calculate what is the **individual treatment effect** (α_i) of a blue pill relative to a red pill?

2) Dr. Strange goes back to his own dimension and assign the patients into groups. $D_i = 1$ if the person recovers well with a blue pill, while $D_i = 0$ if a red pill is better. Create this variable D_i.

	Dim. 1	Dim 0	Treatment Effect	What would Dr. Strange do?
Patient ID	Y_i^1	Y_i^0	α_i	D_i
1	7	1		
2	5	6		
3	5	1		
4	7	8		
5	4	2		
6	10	1		
7	1	10		
8	5	6		
9	3	7		
10	9	8		

Coding exercise 1 (continue):

- What is the average treatment effect of a blue pill?
- What is the average treatment effect of the blue pill on the group that Dr. Strange assign as $D_i = 1$?
- What is the average treatment effect of the blue pill on the group that Dr. Strange assign as $D_i = 0$?

Did Dr. Strange strategy make sense?
What information does he has?

The Assistant's Dilemma

Patient ID	Y_i^1	Y_i^0	α_i	D_i	Y_i
1	7	1			?
2	5	6			?
3	5	1			?
4	7	8			?
5	4	2			?
6	10	1			?
7	1	10			?
8	5	6			?
9	3	7			?
10	9	8			?

- Darn, Dr. Strange left! (to some other dimension saving the world.)
- His assistant is left behind, and the medical board needs a report on the effectiveness of the blue pill.
- The assistant does not know Y_i^1 or Y_i^2. She only observes Y_i, the number of years lived after the pill that Dr. Strange gave each patient.
- Create variable Y_i in your data frame.

The Assistant's Report

The assistant decided to report SDO (simple differences of outcome).

$$\text{SDO} = E\left[Y_i^1 \mid D_i = 1\right] - E\left[Y_i^0 \middle| D_i = 0\right]$$

1) Calculate SDO.

2) Compare this number with the average treatment effect.

    - Is the assistant's calculation correct?
    - Is the red pill more effective than the blue pill as the SDO suggested, or something else is going on?

Potential Outcome Model

If only we knew what happens in the other dimension…

The Potential Outcome Model

- To understand how to investigate causal questions, the potential outcome model is a good starting point.
- We commonly wonder whether a variable or event **D** causes changes in outcome **Y**.

D (Treatment)

- Attend a data analytics course during undergrad
- Receiving tutoring from the department
- Take aspirin during a headache

Y (Outcome)

- Competitiveness in the market
- Job Earnings
- Condition of head 5 hours later

Problem with finding causal effect? Missing a data point!!!

You
with
a headache

There are two states of the world to compare.

Headache Level

Treatment (D = 1) Take aspirin → Y^1 5 hours after taking aspirin

Control (D = 0) Leave alone → Y^0 5 hours after leaving it alone

In the data, you only observe 1 outcome.

Back to the Potential Outcome Model

$$Y_i = D_i Y_i^1 + (1 - D_i) Y_i^0$$

- Y_i^1 Potential outcome when receiving treatment
- Y_i^0 Potential outcome when not receiving treatment
- Y_{i} Observed outcome

- Define treatment effect for individual *i* :

$$\delta_i = Y_i^1 - Y_i^0$$

Different Types of Treatment Effects

- Average Treatment Effect (ATE)

$$E[\delta_i] = E\left[Y_i^1 - Y_i^0\right] = E\left[Y_i^1\right] - E[Y_i^0]$$

- Average Treatment Effect on the Treatment Group (ATT)

$$E[\delta_i | D_i = 1] = E\left[Y_i^1 | D_i = 1\right] - E[Y_i^0 | D_i = 1]$$

- Average Treatment Effect on the Untreated Group (ATU)

$$E[\delta_i | D_i = 0] = E\left[Y_i^1 | D_i = 0\right] - E[Y_i^0 | D_i = 0]$$

Why is SDO not equal ATE?

- First, let us rewrite ATE.

$$
\begin{aligned}
\text{ATE} &= E[Y^1] - E[Y^0] \\
&= \{\pi E[Y^1|D=1] + (1-\pi)E[Y^1|D=0]\} \\
&\quad -\{\pi E[Y^0|D=1] + (1-\pi)E[Y^0|D=0]\}
\end{aligned}
$$

- Second, rearrange.

$$
\begin{aligned}
E[Y^1|D=1] - E[Y^0|D=0] &= ATE \\
&\quad +E[Y^0|D=1] - E[Y^0|D=0] \\
&\quad +(1-\pi)(ATT - ATU) \qquad (77)
\end{aligned}
$$

Intuition of the Equation

$$\begin{aligned} E[Y^1|D=1] - E[Y^0|D=0] &= ATE \\ &+ E[Y^0|D=1] - E[Y^0|D=0] \\ &+ (1-\pi)(ATT - ATU) \qquad (77) \end{aligned}$$

- **Second Term (selection bias)**

  Recall that the doctor "select" those who benefited from surgery to get the treatment.

  People can select themselves into treatment. Think about who select themselves into higher education, for example.

  The second term says that the treatment and control group differ when both groups did not get treatment Y^0.

- **Third Term (heterogeneous treatment effect bias)**

  The control and treatment group differ in terms of how they react to the treatment.

Checking SDO and ATE Relationship

Coding exercise 2:

$$\begin{aligned} E[Y^1|D=1] - E[Y^0|D=0] &= ATE \\ &+E[Y^0|D=1] - E[Y^0|D=0] \\ &+(1-\pi)(ATT - ATU) \qquad (77) \end{aligned}$$

1) Calculate each term, if you have not previously.

2) Confirm that the equation is correct.

How to find ATE with simple differences

1) Assume there is no heterogenous effect $\delta_i = \delta$

   The last term will then disappear.

2) Assume or make sure you have **conditional independence.**

$$(Y^1, Y^0) \perp\!\!\!\perp D$$

Treatment is assigned to individuals **INDEPENDENT** of potential outcome.

Goal is to get rid of the third term.

$$E[Y^1|D=1] - E[Y^1|D=0] = 0$$
$$E[Y^0|D=1] - E[Y^0|D=0] = 0$$

Randomization helps achieve independence.

$$E[Y^1|D=1]-E[Y^1|D=0]=0$$

$$E[Y^0|D=1]-E[Y^0|D=0]=0$$

In the surgery example, if there was randomization, patients who will benefit more from the surgery will be equally likely to be in the treatment group or the control group.

We will not have the problem of selection anymore.

The difference in potential outcomes of treatment and control group are thus 0.

The potential outcome model in a regression form.

$$Y_i^1 = \beta + \delta_i + u_i$$
$$Y_i^0 = \beta + u_i$$

Recall, δ_i is the effect received from treatment D_i

Therefore:

$$Y_i = \beta + \delta_i D_i + u_i$$

Does Randomization Help?

Coding exercise 3 (NEXT CLASS):

1) Can you randomly assign treatment to the patients?
2) Now calculate the SDO. This SDO under randomization, is it closer to the ATE now?
3) Calculate ATE/SDO with a regression. Are they the same?

Reference

Material taken from chapter Potential Outcomes Causal Model.

“Causal Inference: The Mixtape (V.1.7)” by Scott Cunningham.