Detailed Simulations to Address the Unsolved Problems of Lunar Formation

James Guillochon

ABSTRACT

The Giant Impact hypothesis has gradually become accepted as the correct theory for describing the formation of the Moon. But recent results have shown that the Moon's composition is almost identical to that of the Earth's mantle, in direct contradiction to the results of numerical simulations which predict that the Moon is primarily formed from material deposited by the impactor. Additionally, no simulations have successfully simulated both the impact and the coalescence of the Moon from the debris disk in a single simulation, ignoring a crucial phase in the lunar formation process. We propose to run detailed simulations of the Giant Impact hypothesis using the best available hydrodynamics and physics methods to address these outstanding issues and many of the unanswered questions relating to how the Moon formed.

1. Why Lunar Formation is Still an Unsolved Problem

In the past three decades, the planetary science community has reached a general consensus on a theory for the formation of the Earth's moon known as the Giant Impact (GI) hypothesis. GI postulates that the Earth suffered a cataclysmic collision early in the solar system's history with a protoplanet with a mass approximately equal to that of the planet Mars. This formation channel provides a satisfactory explanation for many of the Moon's atypical characteristics, such as its anomalously low bulk density and the large angular momentum of the Earth-Moon system (Canup 2005). Impacts between planetary bodies of similar mass are thought to have been a crucial determinant in the evolution of our solar system, and have left their mark on many other solar system bodies in addition to the Earth (Asphaug 2010). We are fortunate that our own planet has a companion that might originate from one of these spectacular collisions, which gives us a unique insight into how giant impacts operate.

However, despite our privileged position to study the Earth-Moon system, there are a number of outstanding issues with this proposed mechanism that have yet to be properly addressed. Recent work on comparing isotopic ratios present in the Moon and the Earth's mantle seems to indicate that they are nearly identical (O'Neill 1991; Humayun & Clayton 1995; Clayton & Mayeda 1996; Shukolyukov & Lugmair 2000). This is troubling because hydrodynamical simulations predict that as much as 80% of lunar mass originates from the impactor (Canup 2004), which almost certainly formed in a different location in the protostellar disk than the Earth and is unlikely to have an identical composition.

Another outstanding issue is how the debris disk created by the impact eventually coalesces into a moon, which requires detailed knowledge of disk physics, gravitational instability, radiative cooling, and the tracking of volatile evolution and mixed-phase fluids (Stevenson 1989). No hydrodynamics treatment of GI to date has been able to simulate the impact and subsequently produce a self-gravitating object with the correct final characteristics, with the best simulations only being able to place a consistent quantity of matter into orbit about Earth.

2. The State of Lunar Formation Simulations

GI has been tested numerous times using the method of smoothed-particle hydrodynamics (SPH), a Lagrangian method that follows the trajectories of particles with a fixed mass (Benz 1986). Unfortunately, SPH falls short in a number of ways. As the particles in stable SPH approaches have equal masses, spatial resolution is greatest in regions of high-density, and thus SPH is incapable of resolving high- and low-density regions within the same simulation. In the context of simulations of GI, SPH simulations cannot resolve densities smaller than $\sim 10^{-2}$ g cm^{-3}, a density which is significantly larger than those typically found in the post-impact debris disk. Even the most recent simulations SPH resolve the disk with $\lesssim 10^4$ particles (Canup 2008). And because SPH calculates inter-particle forces using a spherically symmetric kernel, objects with a sharp interface are subject to surface-tension forces which can artificially distort their shape (Hess & Springel 2009).

SPH is also difficult to use for this problem from a practical standpoint. As SPH codes require that each particle is aware of the properties of its nearest neighbors, the algorithms used by SPH typically require shared memory, or intensive message passing, and are difficult to parallelize effectively. This problem is compounded by the fact that each particle's list of neighbors changes as a function of time. This have placed a practical limit on the scale of SPH simulations to $\sim 10^6$ particles (Marcus et al. 2009). Grid-based hydrodynamics simulations have the virtue of a fixed topology, and can spread the memory burden to each child node's local memory, which allows for practical simulations that can be performed using as many as 10^9 resolution elements.

A full understanding of the giant impact that led to the formation of the Earth's moon requires that our computational simulations are as realistic as possible. This is because we are seeking to evolve to late time the most energetic few percent of a problem's total mass. These simulations would have to possess all of the following qualities:

- Would include detailed equations of state that properly treat multi-phase fluids and solids over a wide range temperatures and densities.
- Can resolve regions of both high and low density simultaneously.
- Use physically consistent initial models for both the target and the impactor.

- Have flexibility of scale, which allows for both the exploration of the initial condition parameter space and for high resolution realizations of the problem.
- An accurate treatment of self-gravitational forces and radiative transfer.
- A modular architecture that is extendable to the general problem of planetary collisions.

The simulations performed to date are lacking in many (if not most) of these areas. With all the shortcomings of SPH, why haven't grid-based methods been used more widely to address lunar formation? Partially, the issue is related to the history of how the problem was studied, with most groups progressively improving their pre-existing SPH codes. Additionally, the only group attempting to address the problem using a grid-based method (Wada et al. 2006, hereafter W06) did so with a fixed-resolution grid and a simplified two-component equation of state. Using a fixed-resolution grid for a simulation that requires simultaneous resolution of both the target/impactor ($\sim 1R_{\oplus}$) and the accretion disk ($\sim 10R_{\oplus}$) results in a very under-resolved representation of the impact itself. Grid methods also have their own set of problems, including no explicit conservation of angular momentum and issues related to numerical diffusion.

3. Approach

Modern grid-based methods utilize adaptive mesh refinement (AMR), a method that can arbitrarily increase the resolution anywhere within the simulation volume while leaving other regions unresolved. One of the best-developed codes built on an AMR framework is FLASH, a mature and well-tested code (Fryxell et al. 2000; Tasker et al. 2008). A key advantage of FLASH is its modularity, which makes it adaptable to a wide range of astrophysical problems. We have used FLASH for projects with very different requirements, including the tidal disruption of stars by black holes (Guillochon et al. 2009), collisions between white dwarfs (Rosswog et al. 2009), and rapid accretion between two white dwarfs in a close binary (Guillochon et al. 2010). All of the physics routines included with FLASH are interchangeable, which allows us to easily compare and contrast between various methods. In W06, the target is only ~ 50 grid cells wide, and the impactor is only ~ 30 grid cells wide. Our toy model in FLASH (Fig. 1) has a target that is ~ 250 grid cells wide, resulting in an impact that is resolved by 125 times as many cells per volume as W06, while simultaneously being able to represent the accretion disk region with similar resolution.

With sufficient resolution, hydrodynamic mixing between the Earth and the core of the impactor can be directly observed and quantified. Our simulations in FLASH are the first to have enough resolution to resolve the large-scale instabilities that provide the driving necessary to mix the Earth's interior. Mixing between the Earth's mantle and the impactor may be a crucial process that explains why the lunar composition is so similar to that of the Earth's mantle, especially if that mixing can affect the composition of the ejecta. Another aspect of GI that can be studied at this

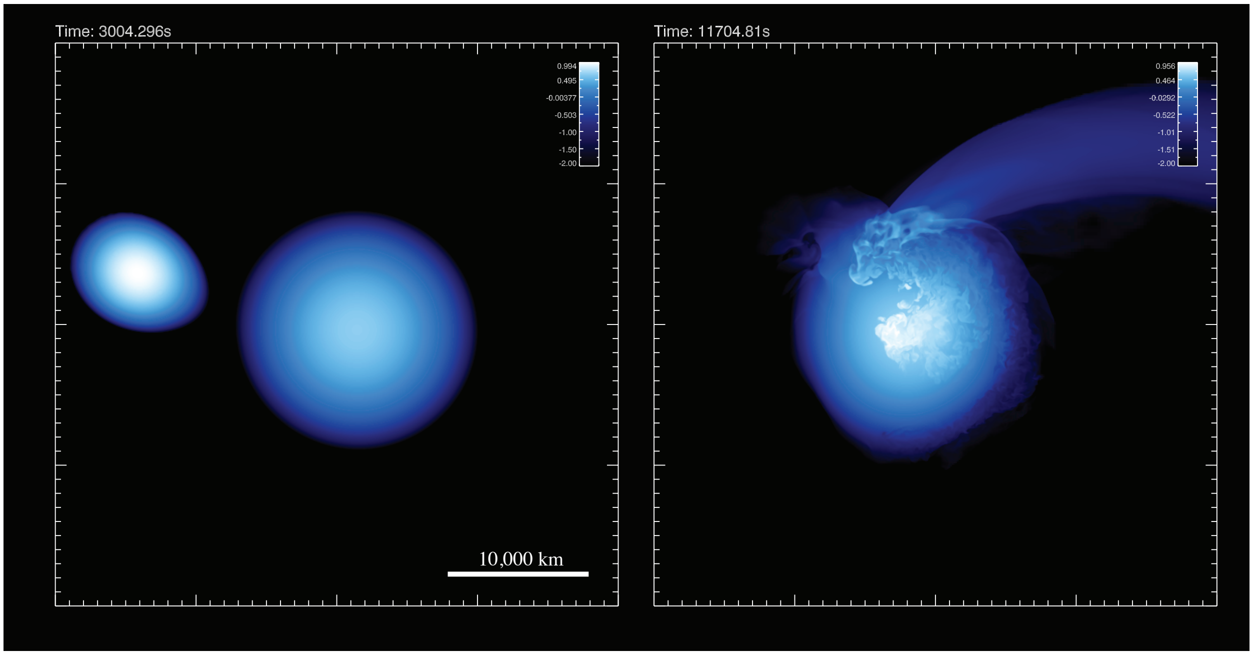

Fig. 1.— A toy model of a collision between a 1.0 $M_\oplus$ target and a 0.1 $M_\oplus$ impactor in FLASH. The fluid obeys a simple $\gamma = 5/3$ equation of state. Color scales with the density of iron, with the left panel showing the system just prior to impact, and the right panel showing the impact's aftermath. Hydrodynamical instabilities are clearly present in the target's mantle, which act to mix the material accreted from the impactor.

high resolution is the evolution of atmospheres and volatile reservoirs (e.g. oceans) during giant impacts.

FLASH includes both a split (Colella & Woodward 1984) and unsplit Godunov-type solver to calculate the transport of matter and energy across grid cell boundaries. The split solver is 3rd-order accurate for smooth flows and minimizes artificial diffusion which plague lower-order flux calculation methods. FLASH includes an FFT-based method for calculating the self-gravity of a system with isolated boundary conditions, which allows for the self-consistent evolution of both the target and the impactor during the encounter and the gravitational instabilities present in the lunar-forming disk.

Another limitation of previous simulations is the equation of state (EOS), with many using the Tillotson EOS, variants of ANEOS, or a combination of polytropic relations. These EOSes are often good enough to determine how planetary collisions qualitatively behave, but the next stage will require the use of state-of-the-art EOSes. We propose to use SESAME, a well-tested suite of material properties which includes equations of state for most of the materials that comprise planetary bodies in the solar system. The tables describing each substance in SESAME are built based on the best-available empirical and/or theoretical data, and the description extends from the regime of cold degenerate fluids to fully-ionized gases. Additionally, each material includes the locations of phase changes and how the material behaves in the presence of shear, both of which are critical to include when simulating GI.

Much progress has already been made by our group towards achieving the above objectives, but there's still significant work to be done before we can meet all of the requirements. As Figure 1 shows, we already have the apparatus for colliding two self-gravitating bodies together within the FLASH framework. We have also already written a SESAME EOS module for FLASH. However, there are still issues in tracking the evolution of the volatiles that are violently decompressed and ejected from the Earth during the collision. Because the FLASH flux solver uses a method that assumes the EOS is always convex ($\partial^2 P/\partial\rho^2|_s > 0$, e.g. the sound speed must increase with increasing density), the solver may return unphysical results when this assumption is violated near phase transitions. Additionally, grid-based methods require the background to have non-zero density, which can cause issues if an object is moving supersonically relative to the background (Springel 2009). Special care must also be taken to ensure that angular momentum is adequately conserved, although Krumholz et al. (2004) have shown that angular momentum loss is minimal when using sufficient resolution. We must address these issues before we can realistically simulate the impact and disk evolution.

The collaborative environment at UCSC enables us to work with experts such as Earth & Marine Sciences Professor Erik Asphaug who is intimately familiar with the practical and theoretical issues associated with GI simulations. We will work closely with Erik and colleagues at UCSC in both the development of this project and in comparing with their SPH results (Canup & Asphaug 2001; Asphaug et al. 2006). We will publish these results in two or more papers, one of which will review our method, and one that will present the simulation we think best represents the actual sequence of events that led to the formation of the Moon. The successful completion of this project will not only shed light on the lunar formation problem, but will provide a set of tools that can easily be used to study collisions between planetary bodies of all kinds.

REFERENCES

Asphaug, E. 2010, Chemie Der Erde, In press

Asphaug, E., Agnor, C., & Williams, Q. 2006, Nature, 439, 155

Benz, W. 1986, The Use of Supercomputers in Stellar Dynamics, 267, 117

Canup, R. M. 2004, Icarus, 168, 433

—. 2005, Science, 307, 546

—. 2008, Icarus, 196, 518

Canup, R. M. & Asphaug, E. 2001, Nature, 412, 708

Clayton, R. N. & Mayeda, T. K. 1996, Geochimica et Cosmochimica Acta, 60, 1999

Colella, P. & Woodward, P. R. 1984, Journal of Computational Physics, 54, 174

Fryxell, B., Olson, K., Ricker, P., Timmes, F. X., Zingale, M., Lamb, D. Q., MacNeice, P., Rosner, R., Truran, J. W., & Tufo, H. 2000, ApJ, 131, 273

Guillochon, J., Dan, M., Ramirez-Ruiz, E., & Rosswog, S. 2010, ApJ, 709, L64

Guillochon, J., Ramirez-Ruiz, E., Rosswog, S., & Kasen, D. 2009, ApJ, 705, 844

Hess, S. & Springel, V. 2009, eprint arXiv:0912.0629

Humayun, M. & Clayton, R. N. 1995, Geochimica et Cosmochimica Acta, 59, 2131

Krumholz, M. R., McKee, C. F., & Klein, R. I. 2004, ApJ, 611, 399

Marcus, R., Stewart, S., Sasselov, D., & Hernquist, L. 2009, ApJ, 700, L118

O'Neill, H. S. C. 1991, Geochimica et Cosmochimica Acta, 55, 1135

Rosswog, S., Kasen, D., Guillochon, J., & Ramirez-Ruiz, E. 2009, ApJ, 705, L128

Shukolyukov, A. & Lugmair, G. W. 2000, Space Science Reviews, 92, 225

Springel, V. 2009, arXiv, astro-ph.CO

Stevenson, D. J. 1989, Geophysical Research Letters, 16, 1067

Tasker, E. J., Brunino, R., Mitchell, N. L., Michielsen, D., Hopton, S., Pearce, F. R., Bryan, G. L., & Theuns, T. 2008, MNRAS, 390, 1267

Wada, K., Kokubo, E., & Makino, J. 2006, ApJ, 638, 1180