CS 5004: Lecture 6

Northeastern University, Spring 2021

At the start of every lecture

- 1. Pull the latest code from the lecture-code repo
- 2. Open the Evening_lectures folder
- 3. Copy this week's folder somewhere else
 - So you can edit it without causing GitHub conflicts
- 4. Open the code:
 - 1. Find the build.gradle file in the folder called LectureX
 - 2. Double click it to open the project

Agenda

- Polymorphism recap
- Recursive data structures
- Recursive linked list
- Stack implementation using a recursive linked list
- Immutable stack using a recursive linked list

Subtype polymorphism recap

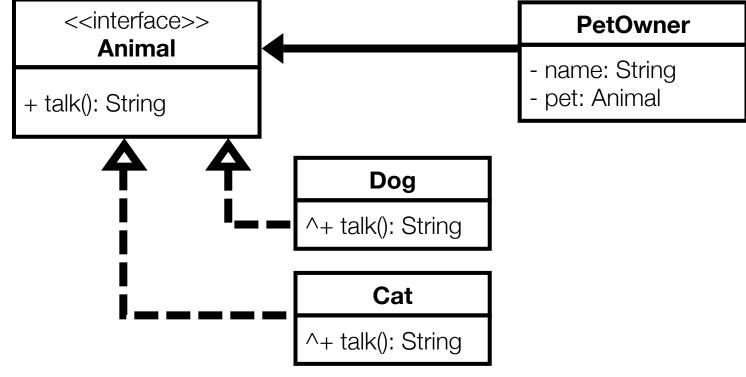
Polymorphism definition

The ability of one **object** to be viewed/used as different **types**.

- Object = an instance of a class (i.e. a variable)
- Type = a data type
 - A class name
 - An abstract class name
 - An interface name

Made possible by inheritance.

- Every object will have multiple types
- An object is an instanceof its runtime type
- An object is an instanceof every type its runtime type inherits from



Cat cat; Dog dog;
dog instanceof Dog
cat instanceof Cat
dog instanceof Animal
cat instanceof Animal

```
public PetOwner(String name, Animal pet) {
   this.name = name;
   this.pet = pet;
}

PetOwner owner = new PetOwner("Darth Vader", new Cat("Mittens"));

owner.getPet().talk();
   An example of dynamic dispatch.
```

 Won't know which implementation of talk() until runtime.

Equals method takes any Object as the parameter.

• All Java classes inherit Object therefore, all are instanceof Object

```
@Override
public boolean equals(Object o) {
   if (this == o) return true;
   if (o == null || getClass() != o.getClass()) return false;
   Node node = (Node) o;
   return Objects.equals(getItem(), node.getItem()) &&
        Objects.equals(getNextNode(), node.getNextNode());
}
```

While an object is being viewed as a base/super class, can't access subclass functionality.

Cast to get access to that functionality

```
@Override
public boolean equals(Object o) {
   if (this == o) return true;
   if (o == null || getClass() != o.getClass()) return false;
   Node node = (Node) o;
   return Objects.equals(getItem(), node.getItem()) &&
        Objects.equals(getNextNode(), node.getNextNode());
}
```

Without the cast:

- compile time error
- class Object has no methods getItem or getNextNode.

```
@Override
public boolean equals(Object o) {
   if (this == o) return true;
   if (o == null || getClass() != o.getClass()) return false;
   Node node = (Node) o
   return Objects.equals(getItem(), node.getItem()) &&
        Objects.equals(getNextNode(), node.getNextNode());
}
```

Recursive data structures

Review: Recursion

- An operation defined in terms of itself.
- Solving a problem recursively means solving smaller occurrences of the same problem.
- Recursive programming functions/methods/objects that call themselves to solve some problem.

Review: Recursive algorithms

Every recursive algorithm consists of:

- **Base case** at least one occurrence of the problem that can be solved directly the simplest case(s).
- Recursive case more complex occurrences that can't be solved directly but can be described in terms of smaller occurrences of the same problem.

Recursion – (Python) example from 5001

```
def gcd(num1, num2):
    min_num = min(num1, num2)
    max_num = max(num1, num2)
    if max_num % min_num == 0:
        return min_num
    else:
        return gcd(min_num, max_num % min_num)
```

Recursion – (Python) example from 5001

```
def gcd(num1, num2):
    min_num = min(num1, num2)
    max num = max(num1, num2)
    if max_num % min_num == 0:
        return min_num
    else:
        return gcd(min_num, max_num % min_num)
Base case - smallest subproblem
```

Recursion – (Python) example from 5001

```
def gcd(num1, num2):
    min_num = min(num1, num2)
    max_num = max(num1, num2)
    if max_num % min_num == 0:
        return min_num
    else:
        return gcd(min_num, max_num % min_num)
```

Recursive case – call self with next smallest problem

Recursive data structures

A data structure partially composed of smaller or simpler instances of the same data structure.

Just like recursive functions, recursive structures have:

- Base case
- Recursive cases

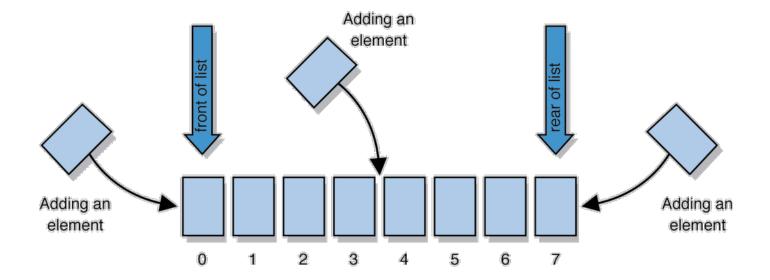
Recursive Linked List

List ADT and a linked list are not the same...

A List ADT may be implemented using a linked list

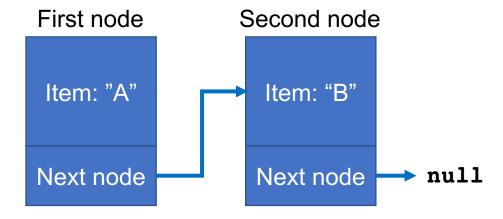
List ADT

Order



Linked List

Sequential* version



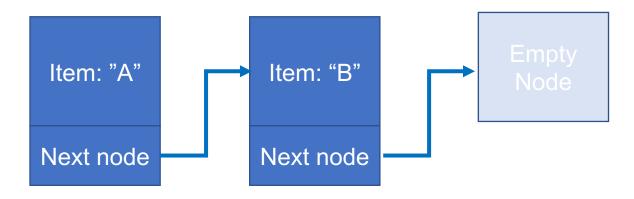
```
public class Node {
   private DataType item;
   private Node next;

   public Node(DataType item, Node next)
{
     this.item = item;
     this.nextNode = nextNode;
   }
   // getters, setters, etc
}
```

^{*}Linked list is always a recursive structure but methods may/may not use recursion

Linked List

Recursive version



Replace null at end of list with special type of Node

Sometimes known as a Cons list

Recursive data structure

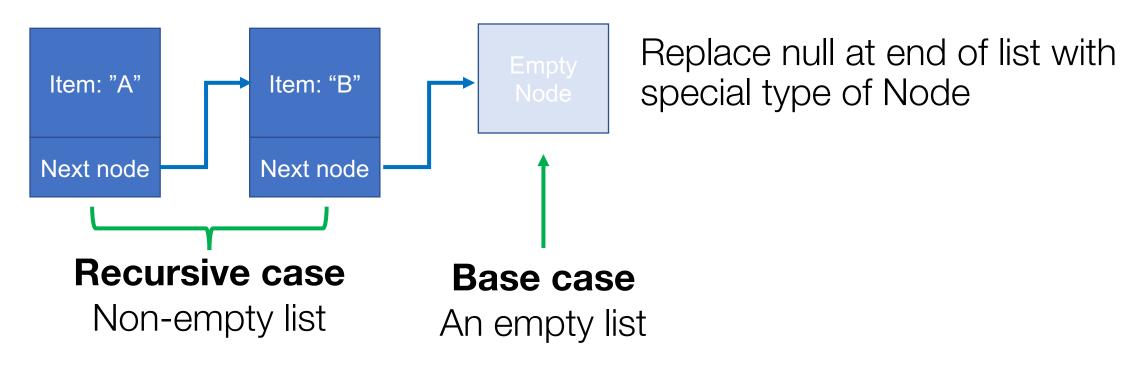
A data structure partially composed of smaller or simpler instances of the same data structure.

Just like recursive functions, recursive structures have:

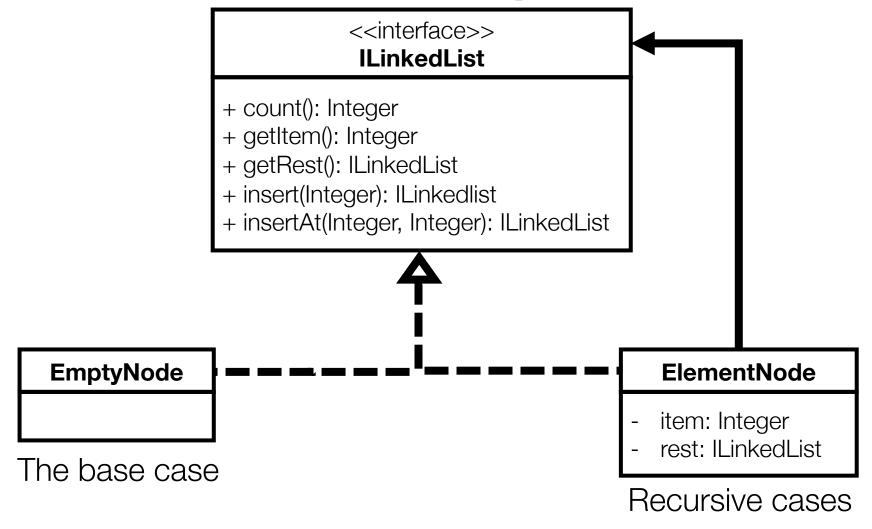
- Base case
- Recursive cases

Linked List

Recursive version



Recursive linked list implementation



Recursive linked list implementation

<<interface>> From now on, **ILinkedList** always + count(): Integer provide an + getItem(): Integer interface for an + getRest(): ILinkedList **ADT** (unless one is + insert(Integer): ILinkedlist + insertAt(Integer, Integer): ILinkedList provided) **EmptyNode ElementNode** item: Integer rest: ILinkedList The base case Recursive cases

Note the subtype polymorphism

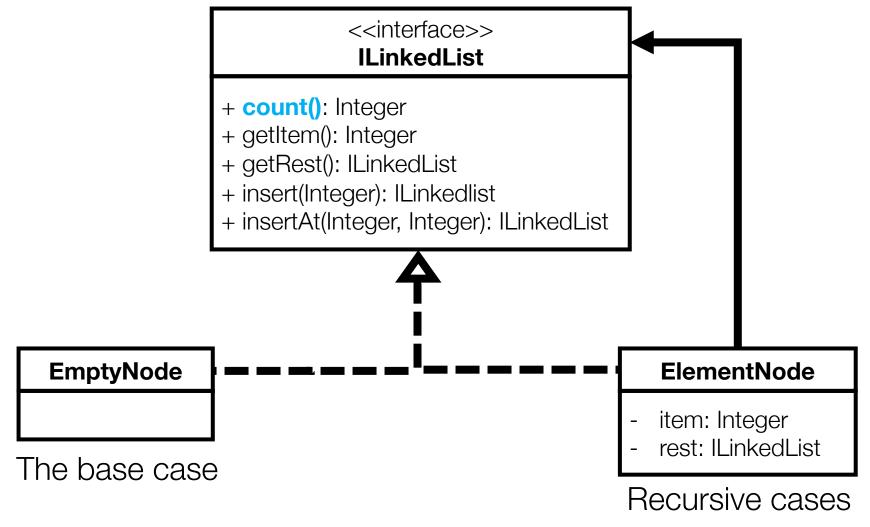
Every **ILinkedList** <<interface>> **ILinkedList** returned will be one of two + count(): Integer + getItem(): Integer implementations: + getRest(): ILinkedList **EmptyNode** + insert(Integer): ILinkedlist **ElementNode** + insertAt(Integer, Integer): ILinkedList **EmptyNode ElementNode** item: Integer rest: |LinkedList The base case Recursive cases

Walkthrough

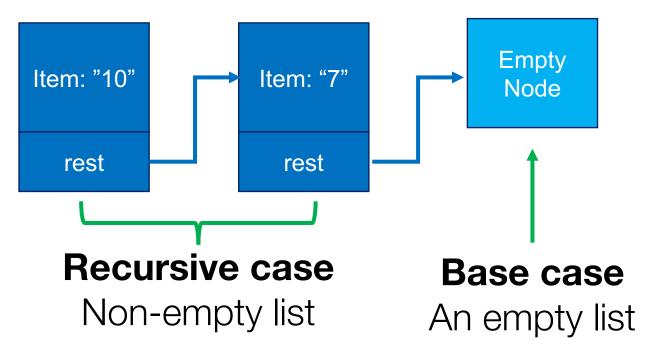
Today's sample code > linkedlist

- The interface
- The test class
- The node classes
 - FYI: ElementNode can be "Cons", EmptyNode can be "Empty"

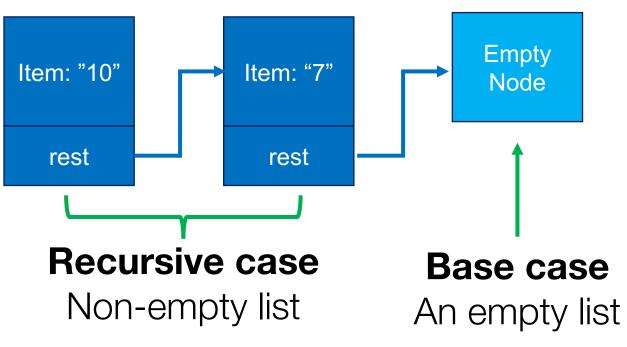
Recursive linked list implementation



Where to start?

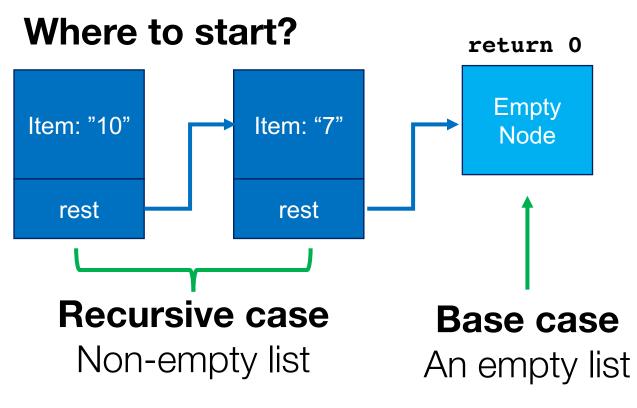


Where to start?



Just like a recursive function, start with the base case

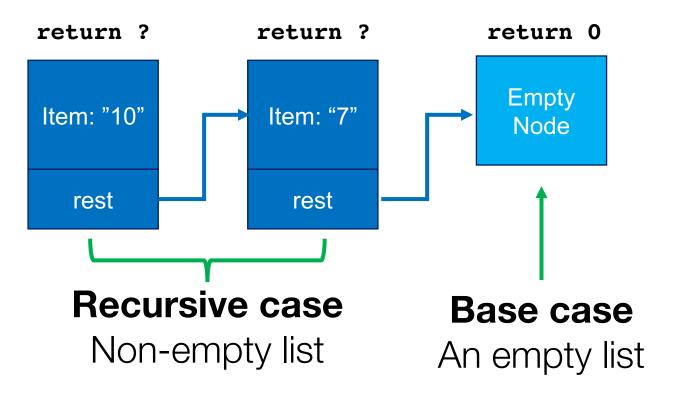
 What should count() do if the list is empty?



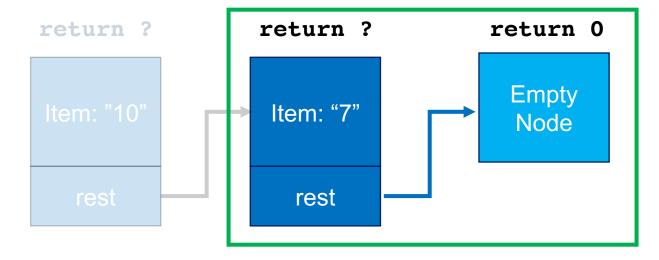
Just like a recursive function, start with the base case

- What should count() do if the list is empty?
 - An empty list has no items
 - > return 0

What about the recursive case?



What about the recursive case?



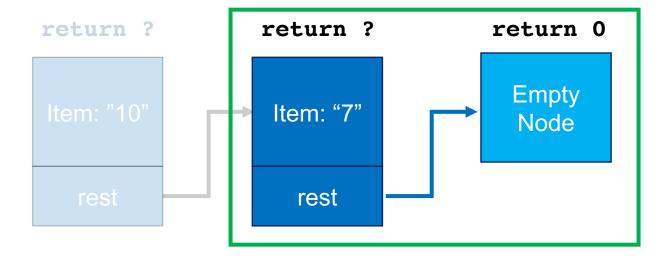
Think about the next simplest case, a list of 1.

What we know:

• this.rest.count() is 0

The size of the list is 1 + the size of the rest of the list

What about the recursive case?



Think about the next simplest case, a list of 1.

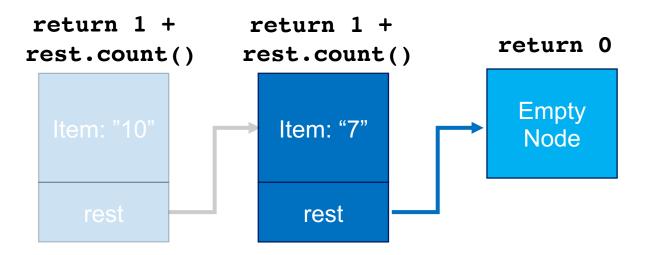
What we know:

• this.rest.count() is 0

So, this.count() should return...

1 + this.rest.count()

What about the recursive case?



Think about the next simplest case, a list of 1.

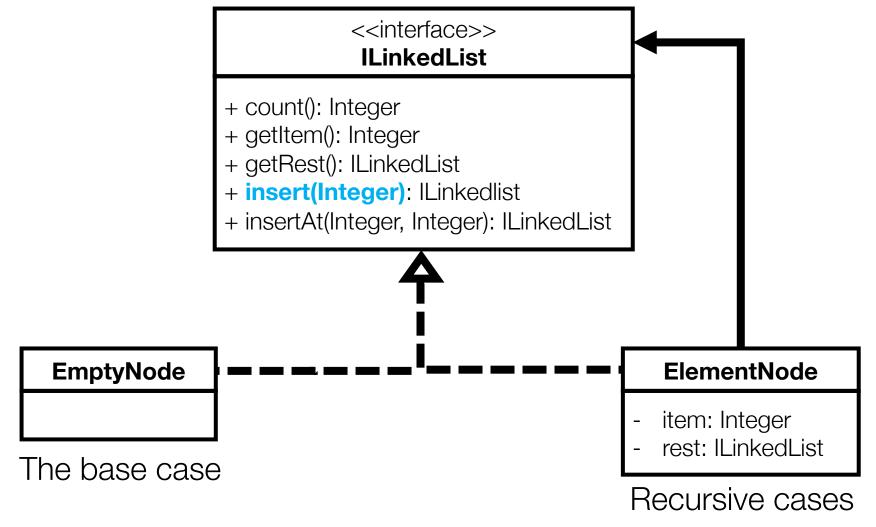
What we know:

• this.rest.count() is 0

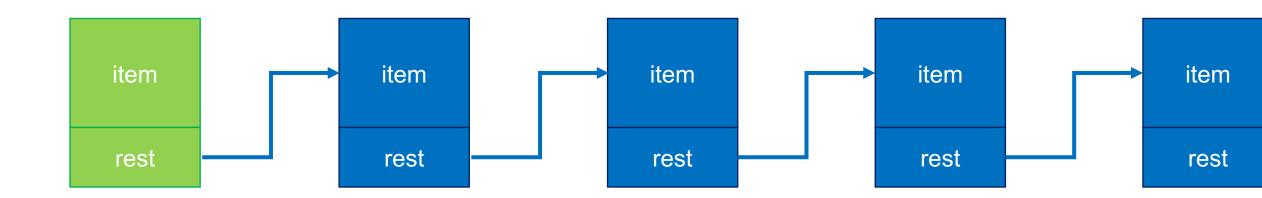
So, this.count() should return...

1 + this.rest.count()

Recursive linked list implementation



Create a new ElementNode containing the Integer, put it at the beginning



- Insert at the head of the list so doesn't need to be recursive
- BUT, still need to tackle insert for both node types
 - Head is list with contents > ElementNode
 - Head is empty list > Empty Node

Exercise 1: implementing insert

Consider how you could implement the insert method for the recursive linked list

- What would the EmptyNode implementation look like?
- What would the ElementNode implementation look like?

Discuss in breakout rooms (actually implementing is optional)

I will call on one or two groups to share their conclusions / questions

EmptyNode.java

```
public ILinkedList insert(Integer item)
{
   return new ElementNode(item, this);
}
```

EmptyNode.java

```
public ILinkedList insert(Integer item)
{
   return new ElementNode(item, this);
}
```

ElementNode.java

```
public ILinkedList insert(Integer item)
{
   return new ElementNode(item, this);
}
```

EmptyNode.java

```
public ILinkedList insert(Integer item)
{
   return new ElementNode(item, this);
}
```

ElementNode.java

```
public ILinkedList insert(Integer item)
{
   return new ElementNode(item, this);
}
```

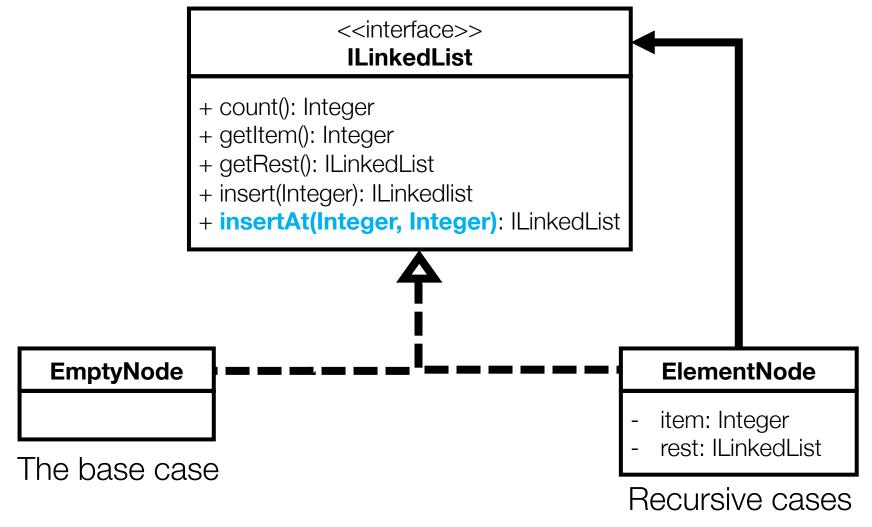
this represents the current "head" of the List

How does Java know which version to call?

Dynamic dispatch

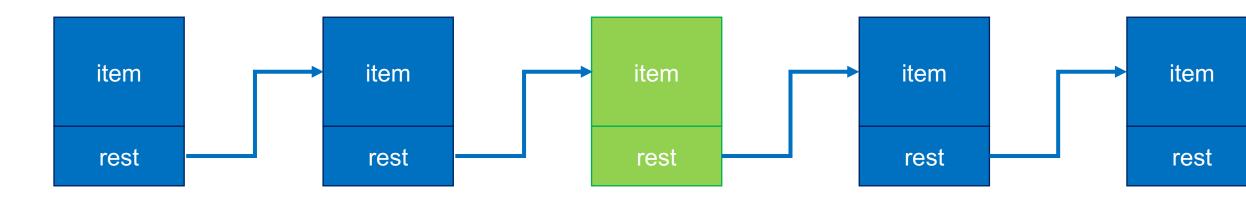
- If the list that calls insert is an **ElementNode**, Java will call the **ElementNode insert** implementation
- If the list that calls insert is an **EmptyNode**, Java will call the **EmptyNode insert** implementation

Recursive linked list implementation



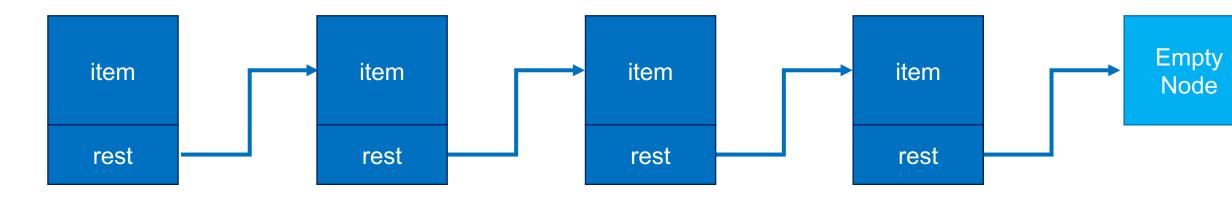
insertAt(Integer, Integer): ILinkedList

Create a new ElementNode containing the Integer, put it at index e.g. 2



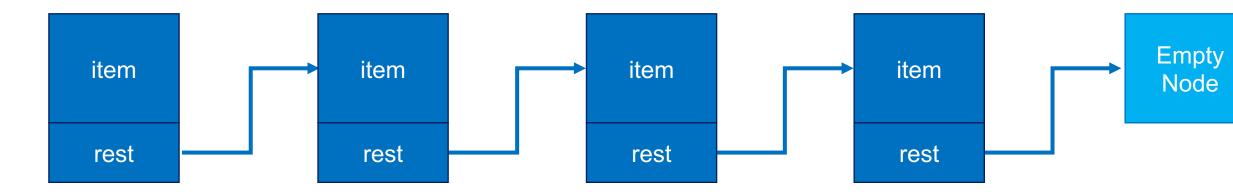
insertAt(Integer, Integer): ILinkedList

- Insert at given index
- Will need to recursively check nodes to find the right index
- Also need to check index is in bounds

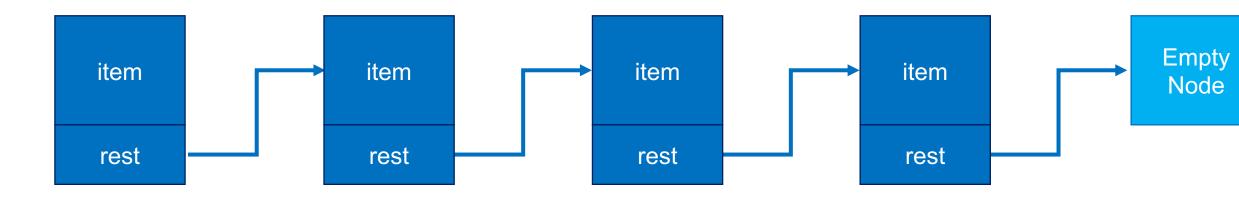


Start at node 0

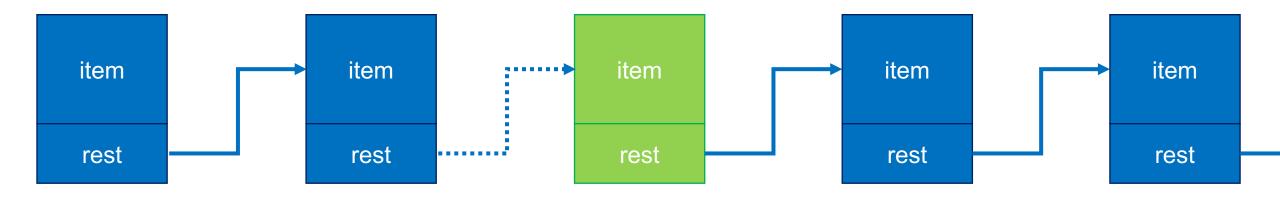
index = 2



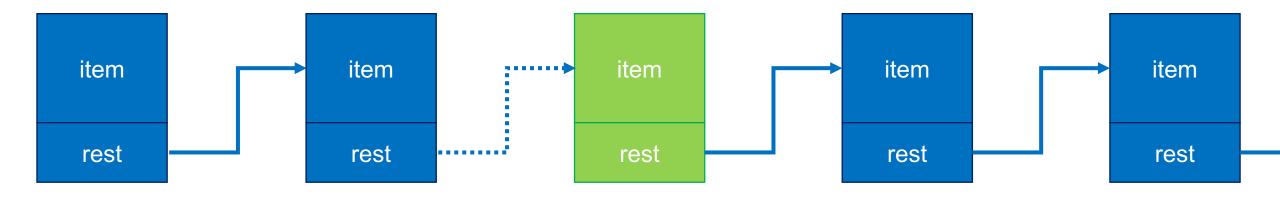
Go to node 1, subtract 1 from index index = 1



Go to node 2, subtract 1 from index Index = 0



Create new node, set its next node to point to node already at node 2...



Create new node, set its next node to point to node already at node 2...

Connect to previous node (index 1)

- Start with the base case existing list is empty
- Two cases:
 - index = 0, same as insert()
 - index is out of range > throw exception

- Three cases:
 - Index is out of range → throw exception
 - This is the index we want to insert at → insert here
 - This is NOT the index we want to insert at → check next node

```
public ILinkedList insertAt(Integer item, Integer index)
                                 throws IndexOutOfBoundsException {
  if (index > this.count() | index < 0) {</pre>
    throw new IndexOutOfBoundsException();
  } else if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
  } else {
    return new ElementNode(this.item,
                                this.rest.insertAt(item, index-1));
```

```
public ILinkedList insertAt(Integer item, Integer index)
                                 throws IndexOutOfBoundsException {
  if (index > this.count() || index < 0) {</pre>
                                                Index out of range
    throw new IndexOutOfBoundsException();
    else if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
  } else {
    return new ElementNode(this.item,
                                this.rest.insertAt(item, index-1));
```

```
public ILinkedList insertAt(Integer item, Integer index)
                                 throws IndexOutOfBoundsException {
  if (index > this.count() | index < 0) {</pre>
    throw new IndexOutOfBoundsException();
  } else if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
    else {
    return new ElementNode(this.item,
                                this.rest.insertAt(item, index-1));
```

This is NOT the index we want to insert at

```
public ILinkedList insertAt(Integer item, Integer index)
                                 throws IndexOutOfBoundsException {
  if (index > this.count() | index < 0) {</pre>
    throw new IndexOutOfBoundsException();
  } else if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
                                                         Recursive call
  } else {
                                                         Reduce index
    return new ElementNode(this.item,
                                this.rest.insertAt(item, index-1));
```

```
public ILinkedList insertAt(Integer item, Integer index)
                                 throws IndexOutOfBoundsException {
                                                   This is the index we
  if (index > this.count() | index < 0) {</pre>
                                                   want to insert at
    throw new IndexOutOfBoundsException();
    else if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
    return new ElementNode(this.item,
                                this.rest.insertAt(item, index-1));
```

```
public ILinkedList insertAt(Integer item, Integer index)
                                  throws IndexOutOfBoundsException {
  if (index > this.count() | index < 0) {</pre>
    throw new IndexOutOfBoundsException();
  } else if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
                                               Copy the contents of this node
  } else {
                                               to maintain link from previous node
    return new ElementNode(this.item,
                                this.rest.insertAt(item, index-1));
```

```
public ILinkedList insertAt(Integer item, Integer index)
                                 throws IndexOutOfBoundsException {
  if (index > this.count() | index < 0) {</pre>
    throw new IndexOutOfBoundsException();
  } else if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
                                                Return new node with
    return new ElementNode(item, thisCopy);
                                                new item to the
  } else {
                                                previous recursive call
    return new ElementNode(this.item,
                                this.rest.insertAt(item, index-1));
```

```
public ILinkedList insertAt(Integer item, Integer index)
                                 throws IndexOutOfBoundsException {
  if (index > this.count() | index < 0) {</pre>
    throw new IndexOutOfBoundsException();
  } else if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
                                                Point "rest" to
    return new ElementNode(item, thisCopy);
                                                the copy node
  } else {
    return new ElementNode(this.item,
                                this.rest.insertAt(item, index-1));
```

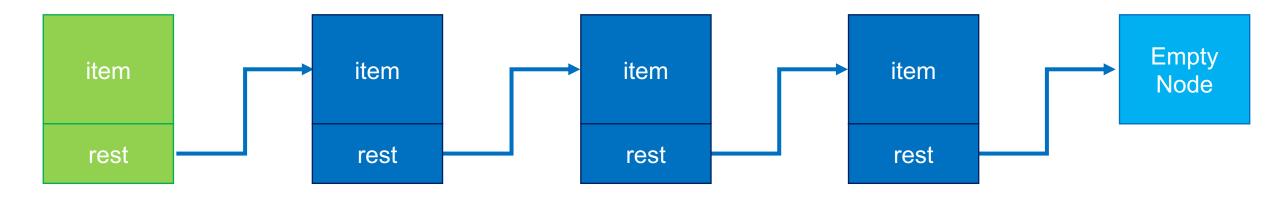
Break (10 mins)

10 8 6 4 2 0

UI Development

Efficiency

Linked List is very efficient when you're only adding/removing from the front



Accessing/inserting/removing anywhere else is less efficient

Have to traverse the list each time

insertAt efficiency - recursive case

```
public ILinkedList insertAt(Integer item, Integer index)
                                 throws IndexOutOfBoundsException {
  if (index > this.count() | index < 0) {</pre>
                                              Do we really
    throw new IndexOutOfBoundsException();
                                              need to do this?
   else if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
  } else {
    return new ElementNode(this.item,
                                this.rest.insertAt(item, index-1));
```

insertAt efficiency - recursive case

Pro

- Prevents list traversal if the index is invalid
- list.insertAt(100, -1);

Con

- If the index is valid, the check is repeated every node
- list.insertAt(100, list.count());

```
if (index > this.count() | | index < 0) {
   throw new IndexOutOfBoundsException();
}</pre>
```

insertAt efficiency - recursive case

Pro

- Prevents list traversal if the index is invalid
- list.insertAt(100, -1);
- Design choice: if you think invalid inserts will be more common, keep the check

Con

- If the index is valid, the check is repeated every node
- list.insertAt(100, list.count());

```
if (index > this.count() | | index < 0) {
   throw new IndexOutOfBoundsException();
}</pre>
```

insertAt efficiency – recursive case

Pro

- Prevents list traversal if the index is invalid
- list.insertAt(100, -1);
- Design choice: if you think invalid inserts will be more common, keep the check

Con

- If the index is valid, the check is repeated every node
- list.insertAt(100, list.count());
- Design choice: if you think valid inserts will be more common, can we remove the check?

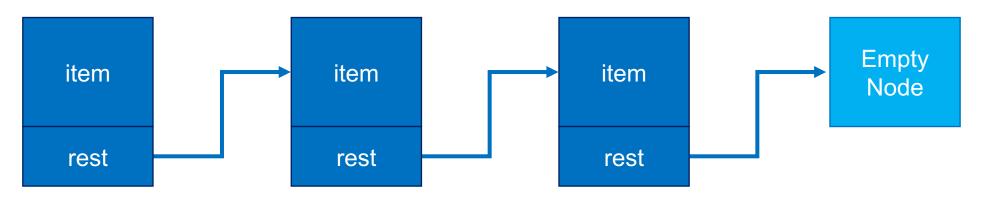
```
if (index > this.count() | | index < 0) {
   throw new IndexOutOfBoundsException();
}</pre>
```

insertAt recursive case – another approach

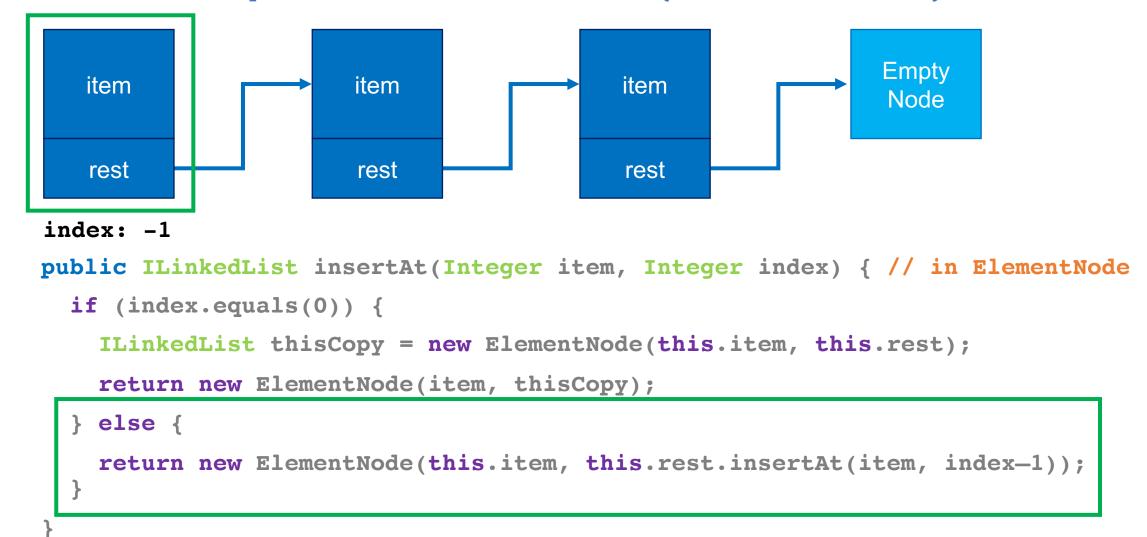
```
public ILinkedList insertAt(Integer item, Integer index)
                                throws IndexOutOfBoundsException {
  if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
  } else {
    return new ElementNode(this.item,
                               this.rest.insertAt(item, index -
1));
```

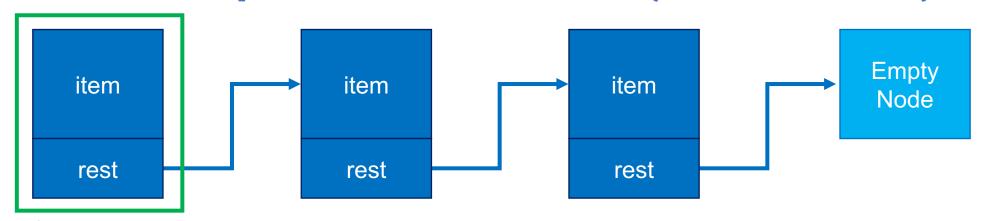
We can remove the index check

EmptyNode will catch an invalid index

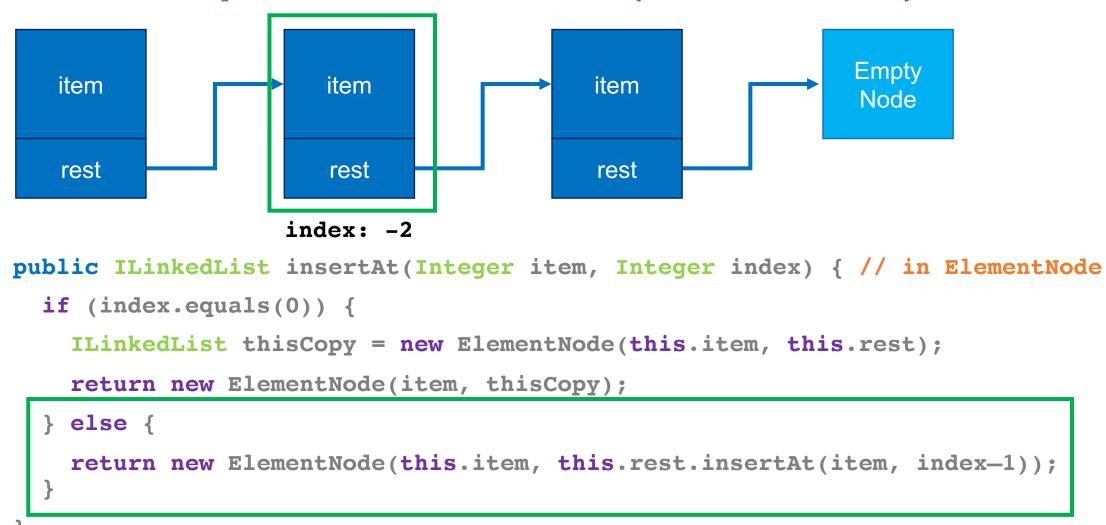


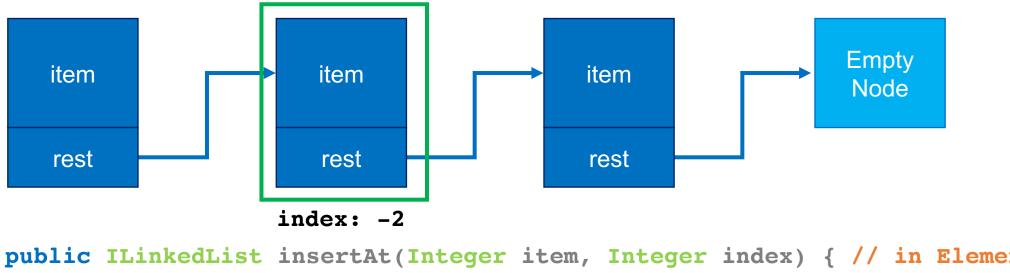
```
public ILinkedList insertAt(Integer item, Integer index) { // in ElementNode
   if (index.equals(0)) {
        ILinkedList thisCopy = new ElementNode(this.item, this.rest);
        return new ElementNode(item, thisCopy);
   } else {
        return new ElementNode(this.item, this.rest.insertAt(item, index-1));
   }
}
```



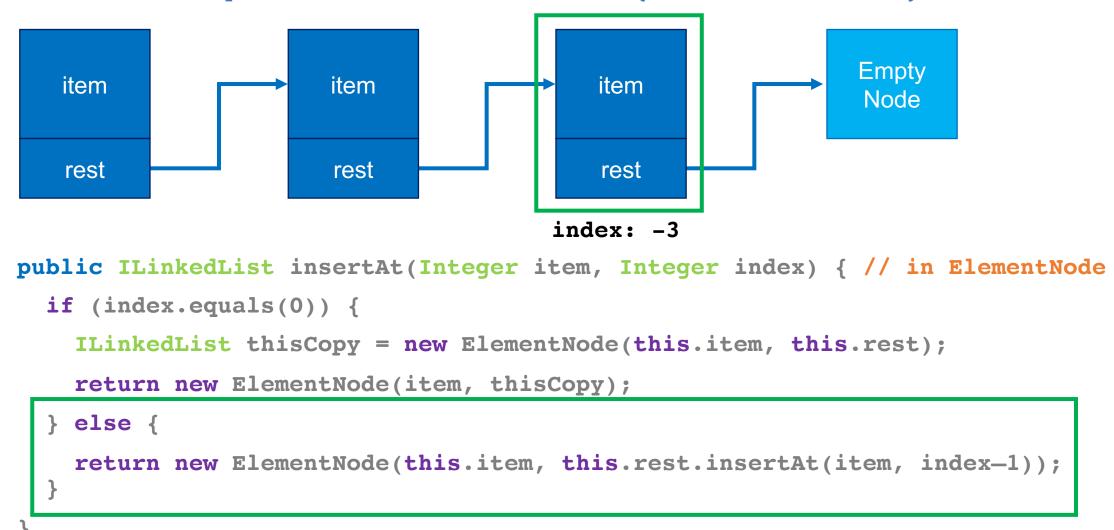


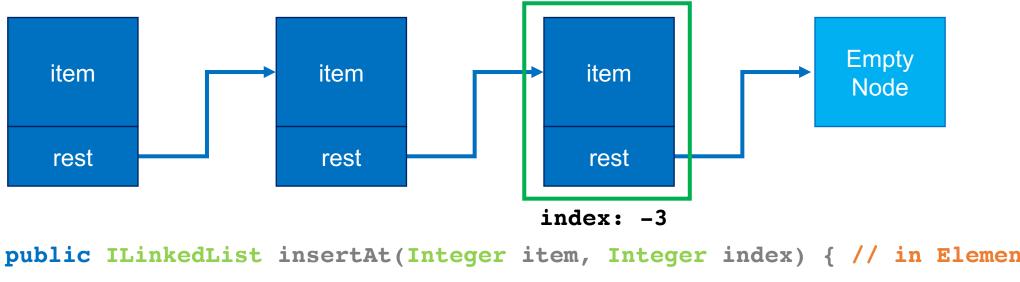
```
index: -1
public ILinkedList insertAt(Integer item, Integer index) { // in ElementNode
  if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
} else {
    return new ElementNode(this.item, this.rest.insertAt(item, index-1));
}
```



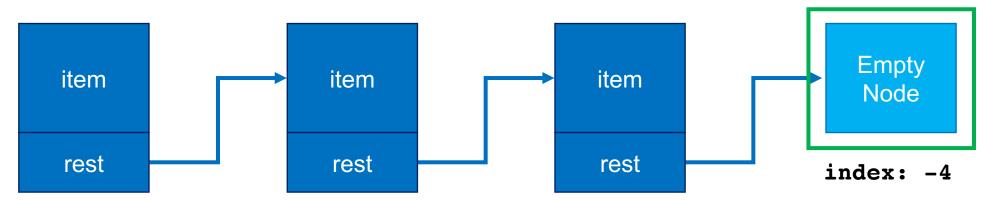


```
public ILinkedList insertAt(Integer item, Integer index) { // in ElementNode
  if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
} else {
    return new ElementNode(this.item, this.rest.insertAt(item, index-1));
}
```

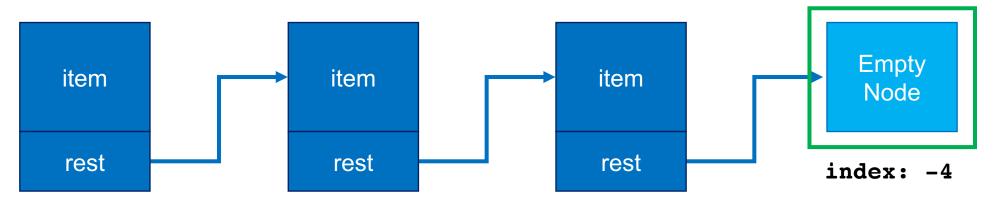




```
public ILinkedList insertAt(Integer item, Integer index) { // in ElementNode
  if (index.equals(0)) {
    ILinkedList thisCopy = new ElementNode(this.item, this.rest);
    return new ElementNode(item, thisCopy);
} else {
    return new ElementNode(this.item, this.rest.insertAt(item, index-1));
}
```



```
public ILinkedList insertAt(Integer item, Integer index) { // in EmptyNode
  if (!index.equals(0)) {
    throw new IndexOutOfBoundsException();
  } else {
    return new ElementNode(item, this);
  }
}
```



```
public ILinkedList insertAt(Integer item, Integer index) { // in EmptyNode
    if (!index.equals(0)) {
        throw new IndexOutOfBoundsException();
    } else {
        return new ElementNode(item, this);
    }
}
```

Walkthrough

Completed Linked List implementation

Stack implementation

Using a recursive linked list

A mutable Stack ADT

- void push(Integer item) push an Integer on to the Stack
- Integer pop() throws EmptyStackException returns and removes the most recently-added item.
- Integer top() throws EmptyStackException returns the most recently-added item
- boolean is Empty() checks if the Stack is empty.

```
public class Stack implements IStack {
 private ILinkedList top;
 private Stack() {
    this.top = new EmptyNode();
  public static Stack createEmpty() {
    return new Stack();
```

```
public class Stack implements IStack {
 private Stack() {
  this.top = new EmptyNode();
 public static Stack createEmpty() {
  return new Stack();
```

```
public class Stack implements IStack {
   private ILinkedList top;
```

```
private Stack() {
  this.top = new EmptyNode();
}
```

Why private?

- Sometimes want to prevent direct access to constructors
- Most useful for immutable
- Not necessary here (but fine)

```
public static Stack createEmpty()
  return new Stack();
}
```

```
public class Stack implements IStack {
   private ILinkedList top;

   private Stack() {
     this.top = new EmptyNode();
   }
```

```
public static Stack createEmpty() {
  return new Stack();
}
```

Convenience method creates a Stack without "new"

Creating a Stack

```
Stack aStack = Stack.createEmpty();
```

```
Inside aStack:
this.top = Empty Node
```

A mutable Stack ADT

- void push(Integer item) push an Integer on to the Stack
- Integer pop() throws EmptyStackException returns and removes the most recently-added item.
- Integer top() throws EmptyStackException returns the most recently-added item
- boolean is Empty() checks if the Stack is empty.

push(Integer): void

```
public void push(Integer item) {
   this.top = this.top.insert(item);
}
```

push(Integer): void

The linked list is immutable so reassign this.top

• Stack is mutable

Pushing items on to the Stack

```
aStack.push(27);
```

```
Inside aStack:
this.top =
item = 27

rest

Empty
Node
```

Pushing items on to the Stack

```
aStack.push(3);

Inside aStack:
this.top = 

item = 3

rest

item = 27

Node
```

A mutable Stack ADT

- void push(Integer item) push an Integer on to the Stack
- Integer pop() throws EmptyStackException returns and removes the most recently-added item.
- Integer top() throws EmptyStackException returns the most recently-added item
- boolean is Empty() checks if the Stack is empty.

isEmpty(): boolean

```
public boolean isEmpty() {
   return this.top.count().equals(0);
}
```

isEmpty(): boolean

```
public boolean isEmpty() {
   return this.top.count().equals(0);

   Dynamic dispatch. One of:
        EmptyNode's count()
        ElementNode count()
```

A mutable Stack ADT

- void push(Integer item) push an Integer on to the Stack
- Integer pop() throws EmptyStackException returns and removes the most recently-added item.
- Integer top() throws EmptyStackException returns the most recently-added item
- boolean is Empty() checks if the Stack is empty.

```
public Integer top() throws EmptyStackException
{
   if (this.isEmpty())
     throw new EmptyStackException();
   return this.top.getItem();
}
```

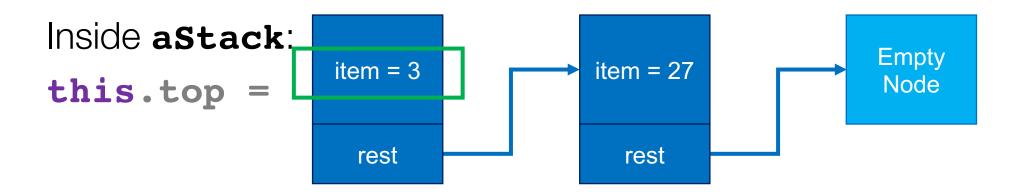
```
public Integer top() throws EmptyStackException
{
    if (this.isEmpty())
        throw new EmptyStackException();
}    Ensures the specification is met
    return this.top.getItem();
}
```

```
public Integer top() throws EmptyStackException
  if (this.isEmpty())
    throw new EmptyStackException();
  return this.top.getItem();
        Dynamic dispatch. One of:
        EmptyNode's getItem()
        ElementNode getItem()
```

(We know it can't be an EmptyNode but the compiler does not)

Getting the top item

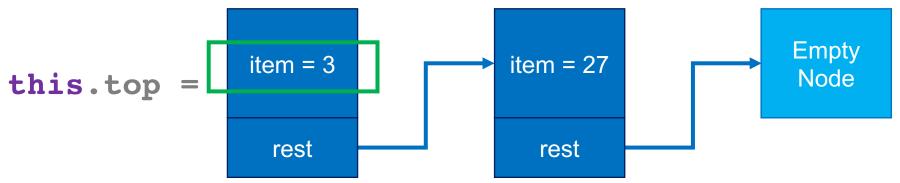
```
aStack.top(); \rightarrow 3
```



A mutable Stack ADT

- void push(Integer item) push an Integer on to the Stack
- Integer pop() throws EmptyStackException returns and removes the most recently-added item.
- Integer top() throws EmptyStackException returns the most recently-added item
- boolean is Empty() checks if the Stack is empty.

```
public Integer pop() throws EmptyStackException
{
   Integer poppedItem = this.top();
   this.top = this.top.getRest();
   return poppedItem;
}
```



```
public Integer pop() throws EmptyStackException
  Integer poppedItem = this.top();
                                     Removes the top
  this.top = this.top.getRest();
                                      element
  return poppedItem;
                         Empty
           item = 27
                          Node
this.top =
            rest
```

Walkthrough

- Completed Stack
- Tests

Exercise 2

- Implement Queue with the recursive linked list as the underlying data structure
- Use the **IQueue** interface & tests from this week's sample code

Immutable Stack

Using a recursive linked list

An immutable Stack ADT

- IImmutableStack createEmpty() creates a new empty stack.
- IImmutableStack push(Integer item) returns a new stack with item at the top.
- IImmutableStack pop() throws EmptyStackException returns and removes the most recently-added item.
- Integer top() throws EmptyStackException returns the most recently-added item
- boolean is Empty() checks if the Stack is empty.

An immutable Stack ADT

- IImmutableStack createEmpty() creates a new empty stack.
- IImmutableStack push(Integer item) returns a new stack with item at the top.
- IImmutableStack pop() throws EmptyStackException removes the most recently-added item.
- Integer top() throws EmptyStackException returns the most recently-added item
- boolean is Empty() checks if the Stack is empty.

```
public class ImmutableStack implements IImmutableStack {
  private final ILinkedList top;
  private ImmutableStack() {
    this.top = new EmptyNode();
  private ImmutableStack(ILinkedList elements) {
    this.top = elements;
```

```
public class ImmutableStack implements IImmutableStack {
  private final ILinkedList top;

    Underlying data structure

    final ensures immutability

  private ImmutableStack() {
    this.top = new EmptyNode();
  private ImmutableStack(ILinkedList elements) {
    this.top = elements;
```

```
public class ImmutableStack implements IImmutableStack {
   private final ILinkedList top;
```

```
private ImmutableStack() {
   this.top = new EmptyNode();
}
```

Doesn't *need* to be private

 emptyStack will serve as constructor

```
private ImmutableStack(ILinkedList elements) {
  this.top = elements;
}
```

```
public class ImmutableStack implements IImmutableStack {
  private final ILinkedList top;
  private ImmutableStack() {
    this.top = new EmptyNode();
  private ImmutableStack(ILinkedList elements) {
    this.top = elements;
```

Definitely private \rightarrow Don't want clients to know about the underlying

Need for immutable methods

createEmpty(): IImmutableStack

Typically static, calls private/public constructor

```
public static ImmutableStack createEmpty() {
   return new ImmutableStack();
}
```

Aside: choosing array vs linked list for underlying data structure

Rules of thumb

- Use an array when random access is important
 - i.e. access by index
 - will be faster for insert at index as well
- Use a linked list when random access/order is not important
 - faster for add/remove (doesn't involve resizing)

Assignment 6

Recursive ADTs

- No choice of underlying data structure
 - i.e. no arrays
- Don't assume today's Linked List is the best ADT for the job!
 - Use two nodes BUT
 - Linked list ADT methods may support different operations
 - Nodes may need different fields

Exercise 3: implement the remaining ImmutableStack methods

- IImmutableStack createEmpty() creates a new empty stack.
- IImmutableStack push(Integer item) returns a new stack with item at the top.
- IImmutableStack pop() throws EmptyStackException removes the most recently-added item.
- Integer top() throws EmptyStackException returns the most recently-added item
- boolean is Empty() checks if the Stack is empty.