Business Analytics Individual Assignment

Problem & Model

Problem

'Whole Food Earth' produces 3 different packs of 'mixed nuts'. These are differentiated by price and proportion of nuts contained in them. 'Regular' contains a high proportion of cheaper peanuts, 'deluxe' packs contain a greater variety with fewer peanuts, and 'super-deluxe' packs contain no peanuts and a higher proportion of the more expensive nuts.

This exercise focuses on optimising the allocation of a given stock of nuts into production of mixed nut packs to maximise potential revenue. In other words, 'How many packs of mixed nuts of each type should be produced to maximise revenue', given certain constraints and assumptions.

Data

The stock for each nut is derived from how much can be bought at wholesale prices with a budget of £10,000 per type of nut. The tables below provide the calculated stock of each type of nut, along with the prices and mass of each nut included in every 1kg pack of mixed nuts.

Nut	Brazil	cashew	almond	peanut	walnut	hazel
Price (per kg)	11	5.96	7.5	3.7	11	9.4
stock bought	909.09	1677.85	1333.33	2702.70	909.09	1063.83

Table 1: Wholesale price (per kg) of each nut and stock (kg) that can be bought with £10,000 for each nut

Nuts	Brazil	cashew	almond	peanut	walnut	hazel	price
Regular	0	0	0.15	0.7	0.15	0	£22.31
Deluxe	0.167	0.167	0.167	0.167	0.167	0.167	£24.42
Super Deluxe	0.2	0.2	0.2	0	0.2	0.2	£27.90

Table 2: Prices (£) and mass of each nut used (kg) for each product

Parameters

- X_n: $Number\ of\ product\ n\ produced$
- P_n: $Price\ of\ product\ n$
- S_m: $Stock\ of\ nut\ m$
- $U(m,n)$: $Usage\ of\ nut\ m\ in\ product\ n$
- R: $Revenue$

Decision Variables

The independent variable is the number of 'regular' (X_1), 'deluxe' (X_2) and 'super-deluxe' (X_3) packs of nuts being produced.

Objective:

The variable to maximise is potential revenue, the projected revenue produced if all goods are sold at their given per-item price. This is given as the sum total of the quantities of each product sold multiplied by their prices or as follows.

$$R = X_1P_1 + X_2P_2 + X_3P_3 = \sum_{n=1}^{3} X_nP_n$$

After substituting in the price values from table 2, we reach the following equation.

$$R = 22.31X_1 + 24.42X_2 + 27.90X_3$$

Constraints:

(1) Each type of nut has a stock constraint. A budget of £10,000 is split evenly among each type of nut, resulting in the following stock. Only this much stock of any nut is available to be used.

$$\sum_{n=1}^{3} X_nU(m,n) \leq S_m$$

(2) Brazil and cashew nuts have especially high fat content. This means that they spoil faster. I have adapted this into the constraint: at least half of the stock of high-fat nuts must be used.

$$\sum_{n=1}^{3} X_nU(cashew,n) \geq \frac{S_{cashews}}{2}$$

$$\sum_{n=1}^{3} X_nU(Brazil,n) \geq \frac{S_{Brazil}}{2}$$

(3) To keep the deluxe packs meaningfully differentiated from regular packs, they can be made to more exclusive through a logical constraint. For every pack of super-deluxe mixed nuts, at least 1.5 packs of regular mixed nuts must be produced. Similarly, the number of regular packs produced must be at least 25% more than that of the deluxe packs produced.

$$X_1 \geq 1.25X_2$$

$$X_1 \geq 1.5X_3$$

(4) Packs of mixed nuts produced is an integer value.

$$X_1, X_2, X_3 \in Z$$

$$Where\ Z\ is\ the\ set\ of\ all\ integers.$$

(5) Packs of mixed nuts are non-negative. In practice, this constraint can be ignored. Through inspection it can be seen that this optimisation would never prescribe producing negative packs of mixed nuts.

$$X_1, X_2, X_3 \geq 0$$

Limitations

Real-world systems are not fully captured in this optimisation as they do not adhere to the following inexhaustive list of assumptions we have made.

- **Infinite demand:** Produced packs are always sold. In reality, if prices exceed equilibrium prices, demand for mixed nuts will not match supply. Furthermore, the market for mixed nuts is limited. In a competitive market, Whole Food Earth has no guarantee of selling its stock before consumer demand is fulfilled.
- **Additivity:** This model assumes that the revenue for each mixed nuts product may be calculated independently. This model would not capture any interaction that may exist between them such as the production of one product acting as a substitute for the other, creating a dynamic whereby the production of one product lowers the demand, price and revenue of another.
- **Proportionality:** The model assumes that the revenue from selling mixed nut packs is proportional to the number produced. In reality, this linear relationship may not exist. For example. prices may fluctuate due to changing market conditions as products are being sold.
- **Divisibility:** This model assumes that packs are not divisible and are sold in 1kg increments only. Although this is more realistic than assuming items are perfectly divisible, this does not fully capture nut sales in the real world. Products are sold in packs of various sizes and these sizes may be varied and adapted to suit the customer. Although they are not perfectly divisible, packs of nuts have more frequent increments than this model assumes.

Solve

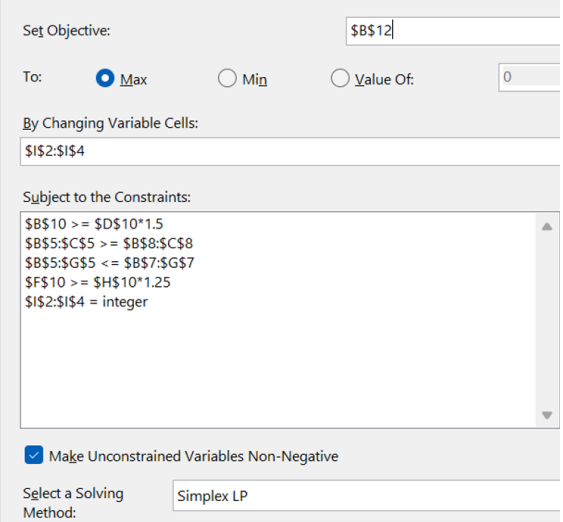
The table used for solver and the results are shown below.

Nuts	Brazil	cashew	almond	peanut	walnut	hazel	price	production
Regular	0	0	0.15	0.7	0.15	0	£22.31	3243
Deluxe	0.167	0.167	0.167	0.167	0.167	0.167	£24.42	2594
Super Deluxe	0.2	0.2	0.2	0	0.2	0.2	£27.90	2072
amount used	847	847	1333	2702	1333	847		

Stock (grams)	909	1678	1333	2703	2004	1064	Constraints
High-Fat	455	839					

Production Differentiation	3243	>= 1.5*	2072		3243	>=1.25*	2594
Integers	The number of packs for each product must be an integer						

Revenue	£193,505.61	Objective



Test

The optimal solution from solver was to produce 3243 packs, 2594 packs and 2072 packs of regular, deluxe and super-deluxe mixed nuts respectively. This allows for a potential £193,505.61 of revenue. To conduct a sensitivity analysis, the integer constraint was ignored resulting in 3242.2, 2594.6 and 2072.1 packs of each product being optimal. This amounts to a potential revenue of £193,527.27. Ignoring the integer constraint at this point makes sense as this produces a very similar result and so our sensitivity analysis and recommendations for the continuous results would be very apt in resolving the integer problem.

This solution is limited as it only takes into account the supply-side of the market and current pricing. Customer preferences and demands would need to be considered as well as changes in prices. A safer approach would take into account whether customers would buy the products, should they be produced, to mitigate the risk of having excess stock.

Furthermore, the initial stock of nuts could be improved through allocating a budget for buying wholesale nuts more effectively. Our knowledge of where there is slack constraints would help here. For example, allocating a greater budget towards buying almonds would improve revenue potential.

Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
I2	Regular to produce	3243.24324	0	22.31	3.529	2.321
I3	Deluxe to produce	2594.59459	0	24.42	1E+30	0.8402381
I4	Super Deluxe to produce	2072.07207	0	27.9	1.28327273	27.9

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B5	amount used Brazil	846.846847	0	454.545455	392.301392	1E+30
C5	amount used cashew	846.846847	0	838.926174	7.92067235	1E+30
B5	amount used Brazil	846.846847	0	909.090909	1E+30	62.2440622
C5	amount used cashew	846.846847	0	1677.85235	1E+30	831.005502
D5	amount used almond	1333.33333	139.5	1333.33333	18.018018	7.92067235
E5	amount used peanut	2702.7027	2.7852	2702.7027	44.0037353	36.036036
F5	amount used walnut	1333.33333	0	2004.00802	1E+30	670.674683
G5	amount used hazel	846.846847	0	1063.82979	1E+30	216.98294

The sensitivity report shows a shadow price of £139.50 and £2.79 for almonds and peanuts. These are the factors limiting Whole Food Earth from achieving higher revenues. By having one more kilogram of almonds and/or peanuts in stock, we could be achieving additional revenues values at the shadow price. In fact, revenue would increase by the shadow price for almonds for an additional 18kg of extra stock. This would improve the potential revenue by £2511. I recommend that budgets are reallocated to have at least this many more almonds in stock.

For cashews, Brazils, walnuts and hazels, the allowable increase reaches ‘1E+30’ (the output for infinity). This means that an increase in stock to these nuts would never change the optimal solution. In fact, these nuts seem to be overstocked. For example, cashews and walnuts have an allowable decrease of 831 and 670 respectively at a shadow price of 0. This means that potential revenue would not change even if stocks were reduced at these values. I recommend that, to reduce costs, these nuts are bought at much lower rates than for this exercise.

The shadow price and degree of slack on the allowable increase for Brazils and cashews with regards to their ‘high-fat’ constraint suggests that this isn’t a necessary constraint that affects the solution. I recommend that this constraint might be revised to be tightened such that even less stock of high-fat, fast-spoiling, nuts are left in stock.

The very high (1E+30) allowable increase and low allowable decrease for the coefficient of deluxe mixed nuts suggests that increases in the price for this product would not affect the optimal solution, whereas decreases would affect the optimal solution very quickly. This demonstrates a flaw in the optimisation where any increase in the price of deluxe nuts would not push Whole Food Earth to produce more. However, given this current sensitivity report, I recommend that the ingredient proportions for deluxe mixed nuts are changed to be more resource efficient.

References

Data for wholesale nut prices obtained from:

1. https://www.buywholefoodsonline.co.uk/brazil-nuts-whole.html
2. https://www.grapetree.co.uk/whole-cashews-raw-bulk-box-22-68kg
3. https://www.buywholefoodsonline.co.uk/almonds-whole.html
4. https://www.buywholefoodsonline.co.uk/peanuts-redskin.html
5. https://www.buywholefoodsonline.co.uk/organic-walnuts-light-halves.html
6. https://www.buywholefoodsonline.co.uk/hazelnuts-whole-raw.html

Data for mixed nut products obtained from:

1. https://wholefoodearth.com/p/wholefood-earth-mixed-chopped-nuts-gmo-free-vegan-dairy-free-no-added-sugar?variant=31877581242465
2. https://wholefoodearth.com/p/wholefood-earth-deluxe-mixed-nuts-gmo-free-vegan-dairy-free-no-added-sugar
3. https://wholefoodearth.com/p/wholefood-earth-super-deluxe-mixed-nuts-gmo-free-vegan-dairy-free-no-added-sugar