Introduction

Definition : Computer Graphics is a field related to generate graphics by using computer. It includes creation, storage, and manipulation of image of an object by using computer. It uses mathematical or Geometrical model to generate required image or graphics but it is quite different than image processing.

Main task of computer Graphics :

(i) Imaging (Creation of 2-D Image)
(ii) Modelling (" " 3-D ")
(iii) Rendering (Conversion of 3-D image into 2-D image)
(iv) Animation (Stimulate image over time)

Terminology on Computer Graphics :

(I) Picture Element / Pels / Pixel
(II) Resolution
(III) PPI (Pixel Per Inch)
(IV) Aspect Ratio
(V) Refresh Rate
(VI) Response time
(VII) Latency (Input lag)
(VIII) Fluorescence / Phosphorescence
(IX) Persistence

CRT Monitor :

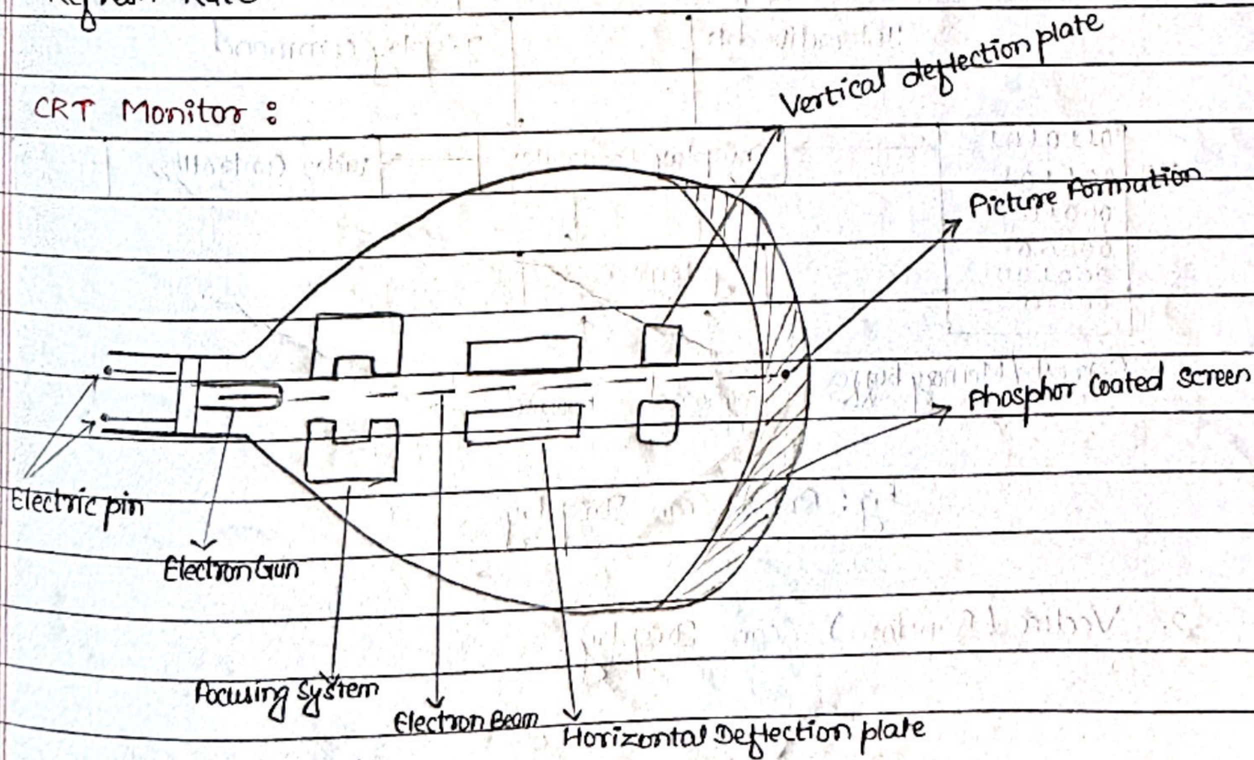

Fig: CRT Monitor

Assignment 1 : Difference between Computer Graphics and Image Processing.

	Computer Graphics	Image Processing
1.	the field related to generation of pictures.	Techniques to modify or interpret existing pictures.
2.	Synthesizes pictures from mathematical or geometrical model.	Analyze picture to derive description in mathematical or geometrical forms.
3.	Includes creation, storage and manipulation of images of objects.	It is part of computer Graphics that handles image manipulation.
4.	e.g. Drawing a picture	e.g. Making blurred image visible.

2080/05/24

Imp: **Display Architecture / Technology, scan line display, scan line display technology**

1. Raster Scan Display

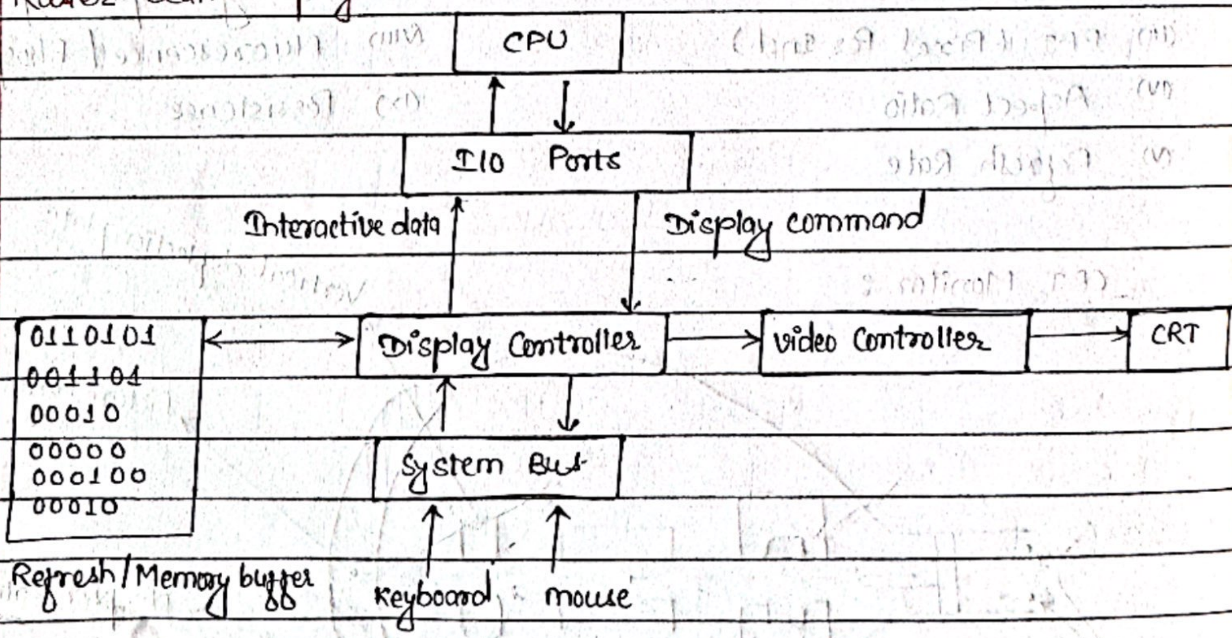

Fig : Raster Scan Display

2. Vector (Random) Scan Display

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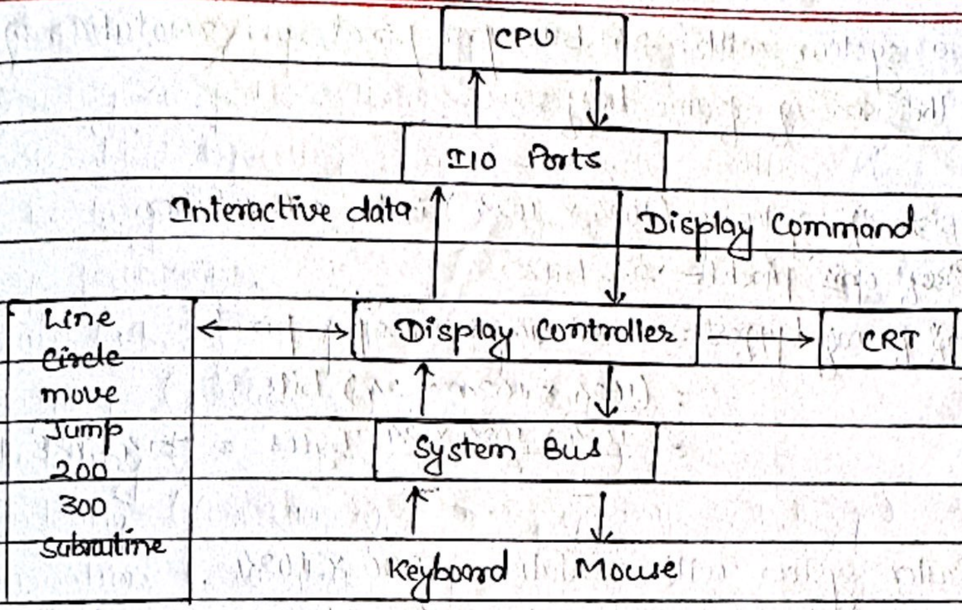

Refresh/ Memory buffer

Fig: Vector Scan Display

Applications of CG

- Computer Aided Design (CAD)
- Education
- Entertainment
- V.R.
- Animations
- Research
- E-Tab
- AnSys
- ABACUS
- CAM

Q.no.1. Consider a RGB Raster System is to be designed using 8 inch by 10 inch screen with a resolution of 100 pixels per inch in each direction. If we want to store 8 bits per pixel in the frame buffer, how much storage do we need for the frame buffer?

Sol^n →

Size of Screen = 8 inch * 10 inch

Pixel per inch (Resolution) = 100

Total no. of pixel = (8 * 100) * (10 * 100) = 800000 pixels

Total Storage required in frame buffer is given by :

= 800000 * 8 bits

= 6400000 bits

= $\frac{6400000}{8}$ bytes

= 800000 bytes

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Q.no.2. There is a system with 24 bits per pixel and resolution of 1024 * 1024. Calculate the size of frame buffer.

Sol^n:

Size of system = (1024 * 1024) = Resolution

Size of one pixel = 24 bits

∴ Size of frame buffer = Resolution * bits per pixel

$= (1024 * 1024 * 24)$ bits

$= \frac{1024 * 1024 * 24}{8}$ bytes $= 3145728$ bytes

Q.no.3. Consider Raster system with resolution 1280 * 1024.

(a) How much pixel could be accessed per second by the video controller that refreshes the screen at the rate of 60 frames per second?

(b) What is the access time for per pixel?

Sol^n:

(a) No. of pixel accessed per second = 1280 * 1024 * 60 = 78643200 pixels

(b) Since, 78643200 pixels are accessed in one second.

Access time per pixel $= \frac{1}{78643200} = 12.7$ nsec

Q.no.4 Consider a raster scan system having 12 inch by 12 inch with a resolution of 100 pixel per inch in each direction. If display controller of this system refresh the screen at the rate of 50 frames per second. How many pixel could be accessed per second and what is the access time per pixel of the system?

Sol^n:

Size of system = 12 * 12 inch

Resolution = 100 pixel per inch

Total no of pixel = (12 * 100) * (12 * 100)

= 1440000

Now,

Pixel accessed per second = 1440000 * 50 = 72000000 pixels

Access time per pixel $= \frac{1}{72000000} = 13.88$ nsec

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Q.no.5 What is the time required to display a pixel on the monitor of size 1024 * 768 with refresh rate of 60 Hz?

Sol:

Size of monitor = 1024 * 768

Refresh rate = 60 Hz i.e. 60 frames per second

Total no. of pixel in one frame = 1024 * 768 pixels = 786432 pixel

60 frame need 1 sec.

1 " " 1/60 sec.

786432 " " 1/60 sec.

$\therefore$ 1 pixel need $\frac{1}{(786432 * 60)} = 21.19$ nsec

Q.no.6 How much time is spent scanning across each row of pixel during screen refresh on a raster system with resolution 1280 * 1024 and refresh rate of 60 frames per second. (Ans: 0.016 sec)

Q.no.7 If a pixel is accessed from the frame buffer with an average access time of 300 nsec then ~~with produce rate and~~ will this rate produce an unflicking effect for the screen size of 640 * 480. (Ans: 10.86 cycle per second)

Q.no.8 Calculate the total memory required to store a 10 min video in a VGA system with 24 bit true caller and 25 FPS (frame per second). (Ans: 32.95 MB)
(Hint: VGA Resolution = 800 * 600)

Sol:

The VGA system allows resolution = 800 * 600

Refresh rate = 25 fps

i.e 25 frames take 1 second.

$\Rightarrow$ 1 frame take $\frac{1}{25}$ seconds = 0.04 secs

Size of video = 10 min

= 10 * 60

= 600 sec

$\therefore$ Total Memory Required $= 800 \times 600 \times 600 \times 0.04 \times 24$ bits

$= 276480000$ bits

$= \frac{276480000}{8}$ bytes

$= \frac{276480000}{8 \times 1024}$ Kbytes

$= \frac{276480000}{8 \times 1024 \times 1024}$ Mb

$= 32.95$ Mega bytes

Q.no.7 Soln, Size of screen $= 640 \times 480$

Total no. of pixels $= 640 \times 480 = 307200$

Average acess time of one pixel $= 300$ ns

Total time required to access entire pixels of image in the screen $= 307200 \times 300$ns

$= 92160000$ nsec $= \frac{92160000}{10^9}$ sec $= 0.09216$ sec

i.e one cycle takes 0.09216 sec.

Now, no. of cycles per second i.e. refresh rate = ?

0.09216 sec $= 1$ cycle

1 sec $= \frac{1}{0.09216} = 10.8506$ cycles per second ≈ 10.86

Since the minimum refresh rate for unflicker image is 60 frames per second, here we can say the monitor produces flickering effect.

Q.no.6 Soln Resolution $= 1280 \times 1024$

Refresh rate = 60fps i.e. 60 frames = 1 second

$\Rightarrow 1$ " $= \frac{1}{60} = 0.016$ sec

$\Rightarrow 1$ frame i.e. (1280×1024) pixel take 0.016 sec.

The scan rate for each pixel row is $= 60 \times 1024 = 61440$ lines/sec

Since one frame consist of 1024 rows, so 1024 rows takes 0.016 sec.

$\therefore$ Time spent across scanning each row of pixel $= 0.016$ sec.

Scan Conversion

- It is a way of representing continuous geographical object as a discrete data pixel available in frame buffer.
- This is a way of digitize or rasterise data pixel (picture element or pels)
- It also quantize data pixel according to the pixel definition and associated rule.
- In field of C.G., e.g. line should have two ends points (x_1, y_1, x_2, y_2).
- In C-program line can be drawn using putpixel function i.e. putpixel (x, y, color).

Scan line conversion algorithm

1. Digital Differential Analyzer (DDA) Algorithm / Incremental Algorithm
2. Bresenham's Algorithm

DDA Algorithm

- DDA is also called as Incremental Algorithm.
- This algorithm depends on differential value along x and y-direction.
- Using this algorithm line can be drawn using slope (m) of line.
- Slope(m) represents the nature and characteristics of line.

The equation of straight line can be given and for two end points (x_1, y_1) and (x_2, y_2),

$$y = mx + b \quad \text{—— (i)}$$

$$\text{Slope}(m) = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{—— (ii)}$$

$$m = \frac{\Delta y}{\Delta x}$$

for interval Δx, $\Delta y = m\Delta x$

" " Δy, $\Delta x = \dfrac{\Delta y}{m}$

To draw line, we should compare the value of Δx and Δy i.e. value of slope(m).

Cases :-

1. If $m < 1$ i.e. $\Delta x > \Delta y$ for Δx-interval,

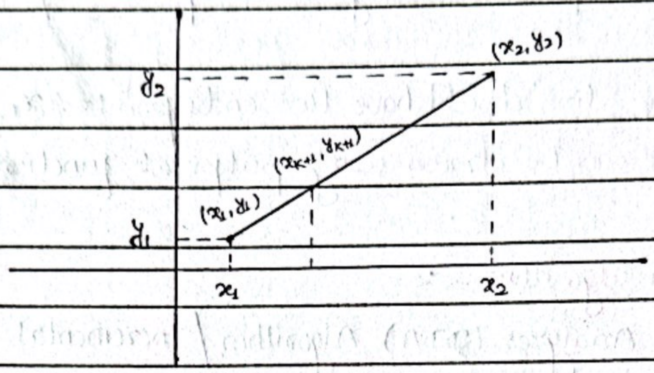
(a) $m < 1$ and starting point is left end i.e. $x_{K+1} = x_K + \Delta x = x_K + 1$

$y_{K+1} = y_K + \Delta y = y_K + m\Delta x = y_K + m \cdot 1 = y_K + m$

(b) $m<1$ and starting point is right end i.e.

$$x_{k+1} = x_k - \Delta x = x_k - 1$$

$$y_{k+1} = y_k - \Delta y = y_k - m\Delta x = y_k - m \cdot 1 = y_k - m$$



2. If $m>1$ i.e $\Delta y > \Delta x$ for Δy-interval,

(a) $m>1$ and starting point is left end, i.e. $y_{k+1} = y_k + \Delta y = y_k + 1$

$$x_{k+1} = x_k + \Delta x = x_k + \frac{\Delta y}{m} = x_k + \frac{1}{m}$$

(b) $m>1$ and starting point is right end, i.e.

$$y_{k+1} = y_k - \Delta y = y_k - 1$$

$$x_{k+1} = x_k - \Delta x = x_k - \frac{\Delta y}{m} = x_k - \frac{1}{m}$$

3. If $m=1$ i.e. $\Delta x = \Delta y$,

$\Delta x = 1$ and $\Delta y = 1$

(a) $m=1$ and starting point is left end i.e.

$$x_{k+1} = x_k + \Delta x = x_k + 1$$

$$y_{k+1} = y_k + \Delta y = y_k + 1$$

(b) $m=1$ and starting point is right end i.e

$$x_{k+1} = x_k - \Delta x = x_k - 1$$

$$y_{k+1} = y_k - \Delta y = y_k - 1$$

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Q. Digitize the line with end points (0,0) and (4,5), and starting point is left end. Draw line using DDA.

Sol^n →

Given that, starting point = (0,0) = (x_1, y_1)

End point = (4,5) = (x_2, y_2)

The slope (m) = $\frac{5-0}{4-0} = \frac{5}{4} = 1.25 > 1$

$y_{K+1} = y_K + \Delta y = y_K + 1$

$x_{K+1} = x_K + \Delta x = x_K + \frac{\Delta y}{m} = x_K + \frac{1}{m}$

x_K	y_K	x_{K+1} plot	y_{K+1} plot	(x, y)
0	0	$0 + \frac{1}{1.25} = 0.8$	0+1 = 1	(0.8, 1)
0.8	1	$0.8 + \frac{1}{1.25} = 1.6$	1+1 = 2	(1.6, 2)
1.6	2	2.4	3	(2.4, 3)
2.4	3	3.2	4	(3.2, 4)
3.2	4	4	5	(4, 5)

Assignment: Digitize the line with end point (3,7) and (8,3) and starting point is left end. Draw line using DDA.

Sol^n →

starting point = (3,7) = (x_1, y_1)

End point = (8,3) = (x_2, y_2)

∴ Slope (m) = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-7}{8-3} = -0.8 < 1$

Since m < 1 and starting point is left end, so

$x_{K+1} = x_K + \Delta x = x_K + 1$

$y_{K+1} = y_K + \Delta y = y_K + m\Delta x = y_K - 0.8 \times 1 = y_K - 0.8$

x_K	y_K	x_{K+1} plot	y_{K+1} plot	(x, y)
3	7	3+1 = 4	7 − 0.8 = 6.2	(4, 6.2)
4	6.2	5	5.4	(5, 5.4)
5	5.4	6	4.6	(6, 4.6)
6	4.6	7	3.8	(7, 3.8)
7	3.8	8	3	(8, 3)

2. Bresenham's Scan Line Algorithm

This technique is also used to draw line or conversion of scan line by eliminating limitation of DDA.

The straight line can be found by,

$$y = mx + b \quad \text{--- (1)}$$

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \quad (\text{for two ends } (x_1, y_1) \text{ and } (x_2, y_2))$$

In Bresenham's algorithm, two cases arises as,

Case 1 : $m < 1$

If slope $m < 1$, then the next two points and decision parameters as,

(a) If $m < 1$, $p < 0$

$$x_{K+1} = x_K + 1$$

$$y_{K+1} = y_K$$

$$P_{K+1} = P_K + 2\Delta y$$

(b) If $m < 1$, $p \geq 0$

$$x_{K+1} = x_K + 1$$

$$y_{K+1} = y_K + 1$$

$$P_{K+1} = P_K + 2\Delta y - 2\Delta x$$

Case 2 : $m \geq 1$

If slope $m \geq 1$, then the next two points and decision parameter as,

(a) If $m \geq 1$, $p < 0$

$$x_{K+1} = x_K$$

$$y_{K+1} = y_K + 1$$

$$P_{K+1} = P_K + 2\Delta x$$

Decision parameter $(P) = 2\Delta y - \Delta x$

(b) If $m \geq 1$, $p \geq 0$

$$x_{K+1} = x_K + 1$$

$$y_{K+1} = y_K + 1$$

$$P_{K+1} = P_K + 2\Delta x - 2\Delta y$$

Q. Draw line with end points (1,1) and (5,5) using Bresenham's algorithm.

Sol→

Given, $x_1 = 1,\ y_1 = 1,\ x_2 = 5,\ y_2 = 5$

$\Delta x = x_2 - x_1 = 5 - 1 = 4$

$\Delta y = y_2 - y_1 = 5 - 1 = 4$

Slope (m) $= \frac{\Delta y}{\Delta x} = \frac{4}{4} = 1$, $P = 2\Delta y - \Delta x = 2\times4-4 = 4$

Now,

Decision parameter (P)	x_K	y_K	x_{K+1}	y_{K+1}	(x,y)
4	1	1	2	2	(2,2)
8	2	2	3	3	(3,3)
12	3	3	4	4	(4,4)
16	4	4	5	5	(5,5)

Q. Draw line with end points (1,1) and (5,3) using Bresenham's algorithm.

Sol→

Slope (m) $= \frac{3-1}{5-1} = \frac{2}{4} = \frac{1}{2} = 0.5 < 1$

$P = 2\Delta y - \Delta x = 2*2-4 = 0$

Now,

P	x_K	y_K	x_{K+1}	y_{K+1}	(x,y)
0	1	1	2	2	(2,2)
-4	2	2	3	2	(3,2)
4	3	2	4	3	(4,3)
-4	4	3	5	3	(5,3)

Mid-point Algorithm

1. Mid-point Circle Scan Conversion Algorithm
2. Mid-point Ellipse Scan Conversion Algorithm

Mid-point Circle Scan Conversion Algorithm

Circle is defined as a set of points that are all at a given distance 'r' from the

Center position (x_c, y_c). Equation of circle at (x_c, y_c) with radius 'r' is

$$(x - x_c)^2 + (y - y_c)^2 = r^2 \quad \text{--- (I)}$$

Assume that the position of x_c, y_c set to $(0,0)$ then eq^n of circle is calculated by,

$$x^2 + y^2 = r^2 \quad \text{--- (II)}$$

In mid-point circle algorithm, we have just plotted (x_K, y_K). The next point is choice between (x_K+1, y_K) and (x_K+1, y_K-1). We would like to choose the point that is nearest to the actual circle.

Let us define a circle function as:

$$f(x,y) = x^2 + y^2 - r^2 = \begin{cases} 0 & \text{if } (x,y) \text{ is on the circle} \\ >0 & \text{if } (x,y) \text{ is outside of circle} \\ <0 & \text{if } (x,y) \text{ is inside the circle} \end{cases}$$

By evaluating this function at the mid-point between pixels, we can make our decision.

Our decision variable can be defined as,

$$P_K = f(x_K+1, y_K - \tfrac{1}{2}) = (x_K+1)^2 + (y_K - \tfrac{1}{2})^2 - r^2$$

If $P_K < 0$, the mid-point is inside the circle and the pixel at y_K is closer to the circle otherwise $y_K - 1$ is closer.

The decision parameter for next position is at $x_{K+1}+1$ i.e. $x_K + 2$.

$$P_{K+1} = f(x_{K+1}+1, y_{K+1} - \tfrac{1}{2}) = (x_K+2)^2 + (y_{K+1} - \tfrac{1}{2})^2 - r^2$$

$x_{K+1} = x_K + 1$
$y_{K+1} = y_K$
$= y_K - 1$

Now,

$$P_{K+1} - P_K = (x_K+2)^2 + (y_{K+1} - \tfrac{1}{2})^2 - r^2 - (x_K+1)^2 - (y_K - \tfrac{1}{2})^2 + r^2$$

$$\therefore P_{K+1} = P_K + x_K^2 + 4x_K + 4 + y_{K+1}^2 - y_{K+1} + \tfrac{1}{4} - x_K^2 - 2x_K - 1 - y_K^2 + y_K - \tfrac{1}{4}$$

$$P_{K+1} = P_K + 2x_K + 3 + (y_{K+1}^2 - y_K^2) - (y_{K+1} - y_K)$$

$$P_{K+1} = P_K + 2x_{K+1} + (y_{K+1}^2 - y_K^2) - (y_{K+1} - y_K) + 1$$

If $P_K < 0$: $y_{K+1} = y_K$, $P_{K+1} = P_K + 2x_{K+1} + 1$

If $P_K \geq 0$: $y_{K+1} = y_K - 1$, $P_{K+1} = P_K + 2x_{K+1} + 1 - 2y_{K+1}$

For Initial Decision Parameter, $(x_0, y_0) = (0, r)$

$$P_0 = f(1, r - \tfrac{1}{2}) = 1 + (r - \tfrac{1}{2})^2 - r^2 = \tfrac{5}{4} - r^2$$

Hence, all integers are integers, rounding $\frac{5}{4}$, will give 1 so,

$$\boxed{P_0 = 1 - r}$$

e.g. Q. Digitize the circle $x^2+y^2=100$ in first octant.

Sol^n →

We have, $x^2+y^2 = 100$

$(x-0)^2 + (y-0)^2 = (10)^2$

$\therefore \quad x^2+y^2 = r^2$

So,

Centre = (0, 0)

Radius (r) = 10

Initial Point = (0, r) = (0, 10)

Initial decision parameter, $P_0 = 1 - r = 1 - 10 = -9$

From mid-point circle algorithm, we have

If $P_K < 0$:

Plot (x_K+1, y_K) and $P_{K+1} = P_K + 2x_{K+1} + 1$

If $P_K \geq 0$:

Plot (x_K+1, y_K-1) and $P_{K+1} = P_K + 2x_{K+1} + 1 - 2y_{K+1}$

Now,

Iteration goes until $x \geq y$ i.e. $x_{K+1} \geq y_{K+1}$

K	P_K	(x_{K+1}, y_{K+1})	$2x_{K+1}$	$2y_{K+1}$
0	-9	(1, 10)	2	20
1	-6	(2, 10)	4	20
2	-1	(3, 10)	6	20
3	6	(4, 9)	8	18
4	-3	(5, 9)	10	18
5	8	(6, 8)	12	16
6	5	(7, 7)	14	14

Q. Digitize the circle with radius r = 10 centered (3, 4) in first octant.

Sol^n →

Here, Centre = (3, 4)

radius (r) = 10

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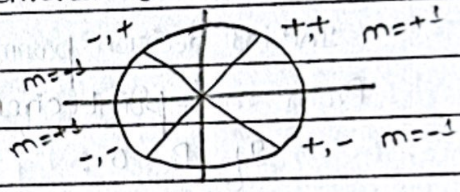
Initial decision parameter, $P_0 = 1 - r = 1 - 10 = 9$

Now,

K	P_K	(x_{K+1}, y_{K+1}) at (0,0)	$2x_{K+1}$	$2y_{K+1}$	(x_{K+1}, y_{K+1}) at (3,4)
0	-9	(1,10)	2	20	(1+3, 10+4) = (4,14)
1	-6	(2,10)	4	20	(5,14)
2	-1	(3,10)	6	20	(6,14)
3	6	(4,9)	8	18	(7,13)
4	-3	(5,9)	10	18	(8,13)
5	8	(6,8)	12	16	(9,12)
6	5	(7,7)	14	14	(10,11)

Q.3. Digitize the circle with radius $r=5$ centered (2,3).

Sol^n →



Centre = (2, 3)

Radius (r) = 5

Initial decision parameter, $P_0 = 1 - 5 = -4$

		1st octant			Other pixel in other octant at (0,0)						
K	P_K	(x_{K+1}, y_{K+1}) at (0,0)	$2x_{K+1}$	$2y_{K+1}$	2nd	3rd	4th	5th	6th	7th	8th
0	-4	(1,5)	2	10	(5,1)	(-5,1)	(-1,5)	(-1,-5)	(-5,-1)	(5,-1)	(1,-5)
1	-1	(2,5)	4	10	(5,2)	(-5,2)	(-2,5)	(-2,-5)	(-5,-2)	(5,-2)	(2,-5)
2	4	(3,4)	6	8	(4,3)	(-4,3)	(-3,4)	(-3,-4)	(-4,-3)	(4,-3)	(3,-4)
3	3	(4,3)	8	6	(3,4)	(-3,4)	(-4,3)	(-4,-3)	(-3,-4)	(3,-4)	(4,-3)

1st octant	Other pixel in other octant at (2,3)						
(x_{K+1}, y_{K+1}) at (2,3)	2nd	3rd	4th	5th	6th	7th	8th
(1+2, 5+3) = (3,8)	(8,3)	(-8,3)	(-3,8)	(-3,-8)	(-8,-3)	(8,-3)	(3,-8)
(4,8)	(8,4)	(-4,8)	(-4,8)	(-4,-8)	(-8,-4)	(8,-4)	(4,-8)
(5,7)	(7,5)	(-5,7)	(-5,7)	(-5,-7)	(-7,-5)	(7,-5)	(5,-7)
(6,6)	(6,6)	(-6,6)	(-6,6)	(-6,-6)	(-6,-6)	(6,-6)	(6,-6)

Mid-point ellipse drawing algorithm

Definition:

Ellipse is elongated circle with 4-symmetric quadrant having two different radii i.e. r_x and r_y.

Equation of ellipse centered at (0,0) is, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ — (I)

Major Axis = $2a = 2r_x$

Minor Axis = $2b = 2r_y$

Semi Major Axis = $a = r_x$

Semi Minor Axis = $b = r_y$

eq^n (I) becomes,

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

or, $$\frac{x^2 r_y^2 + y^2 r_x^2}{r_x^2 r_y^2} = 1$$

or, $$x^2 r_y^2 + y^2 r_x^2 = r_x^2 r_y^2$$

or, $$x^2 r_y^2 + y^2 r_x^2 - r_x^2 r_y^2 = 0 \quad \text{— (II)}$$

If we put any point in eq^n (II),

Case I: point = 0, point lies on ellipse

Case II: point < 0, " " inside the ellipse

Case III: point > 0, " " Outside the ellipse

Difference between circle and ellipse:

1. Circle has 8-way symmetry but ellipse is 4-way symmetry.
2. In circle we need to find or plot only 1 octant of any quadrant but in ellipse we need to plot 2 octants i.e. 1 complete quadrant to plot entire ellipse.

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In quadrant 1

1. Region 1
- start point : $(0, r_y)$
- slope of curve < -1
- Take unit steps in positive x-direction till boundary betn the region '2' is reached.

2. Region 2
- slope of curve > -1
- Take unit steps in -ve y direction till the end of quadrant.

Slope of ellipse,

$$\frac{dy}{dx}\left(r_y^2 \cdot x^2 + r_x^2 \cdot y^2 - r_x^2 \cdot r_y^2 \right) = 0$$

$$\frac{dy}{dx}(y^2) = \frac{dy}{dx}\left(\frac{r_x^2 \cdot r_y^2 - r_y^2 \cdot x^2}{r_x^2}\right)$$

$$\frac{dy}{dx}(y^2) = \frac{dy}{dx}\left(\frac{r_x^2 \cdot r_y^2}{r_x^2}\right) - \frac{dy}{dx}\left(\frac{r_y^2 \cdot x^2}{r_x^2}\right)$$

$$\frac{dy}{dx} \cdot 2y = \frac{-2 r_y^2 \cdot x}{r_x^2}$$

$$\left.\frac{dy}{dx} = \frac{-2 r_y^2 \cdot x}{2 r_x^2 \cdot y}\right\} \text{slope of ellipse}$$

$$\therefore -\frac{2 r_y^2 \cdot x}{2 r_x^2 \cdot y} = -1$$

$$\therefore -2 r_y^2 \cdot x = -2 r_x^2 \cdot y$$

i.e. $\boxed{2 r_y^2 \cdot x = 2 r_x^2 \cdot y}$

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Region 1:

$(x_k+1, y_k), (x_k+1, y_k-1) \Rightarrow (x_k+1, y_k-\frac{1}{2})$

we have eqn,

$$r_y^2 x^2 + r_x^2 . y^2 - r_x^2 . r_y^2 = 0$$

or, $r_y^2 (x_k+1)^2 + r_x^2 (y_k - 1/2)^2 - r_x^2 . r_y^2 = P_{1k}$ — ⓐ

or, $r_y^2 (x_{k+1}+1)^2 + r_x^2 (y_{k+1} - 1/2)^2 - r_x^2 . r_y^2 = P_{1k+1}$ — ⓑ

Now, $P_{1k+1} - P_{1k} = r_y^2 (x_{k+1}+1)^2 + r_x^2 (y_{k+1} - 1/2)^2 - \cancel{r_x^2 r_y^2} - r_y^2 (x_k+1)^2 - r_x^2 (y_k - 1/2)^2 + \cancel{r_x^2 r_y^2}$

$= r_y^2 (x_{k+1}+1)^2 + r_x^2 (y_{k+1} - 1/2)^2 - r_y^2 (x_k+1)^2 - r_x^2 (y_k - 1/2)^2$

$= r_y^2 \{(x_k+1)+1\}^2 + r_x^2 (y_{k+1} - 1/2)^2 - r_y^2 (x_k+1)^2 - r_x^2 (y_k - 1/2)^2$

$= r_y^2 \{(x_k+1)^2 + 2(x_k+1) + 1\} + r_x^2 (y_{k+1} - 1/2)^2 - r_y^2 (x_k+1)^2 - r_x^2 (y_k - 1/2)^2$

$= r_y^2 \{2(x_k+1)+1\} + r_x^2 \{y_{k+1}^2 - y_k^2 - y_{k+1} + y_k\}$

$= P_{1k+1} - P_{1k}$

If : $P_{1k} < 0 : y_k$

$P_{1k} > 0 : y_k - 1$

$\therefore P_{1k+1} - P_{1k} = r_y^2 \{2(x_k+1)+1\} + r_x^2 \{y_{k+1}^2 - y_k^2 - y_{k+1} + y_k\}$

Put, $y_{k+1} = y_k$, $P_{1k} < 0$,

$$P_{1k+1} = P_{1k} + r_y^2 (2x_{k+1} + 1)$$

$$\therefore \boxed{P_{1k+1} = P_{1k} + r_y^2 2x_{k+1} + r_y^2}$$

Put $y_{k+1} = y_k - 1$, $P_{1k} > 0$

$P_{1k+1} - P_{1k} = 2x_{k+1} r_y^2 + r_y^2 + r_x^2 \{(y_k-1)^2 - y_k^2 - (y_k-1) + y_k\}$

$$\therefore \boxed{P_{1k+1} = P_{1k} + 2x_{k+1} r_y^2 + r_y^2 - 2y_{k+1} r_x^2}$$

Now, Initial decision Parameter,

Put $(0, r_y)$ in $P_{1k} = r_y^2 (x_k+1)^2 + r_x^2 \left(y_k - \frac{1}{2}\right)^2 - r_x^2 r_y^2$

or, $P_{10} = r_y^2 (0+1)^2 + r_x^2 (y_k - 1/2) - r_x^2 r_y^2$

or, $P_{10} = r_y^2 + r_x^2 \left(r_y^2 + \frac{1}{4} - r_y\right) - r_x^2 r_y^2$

$\therefore P_{10} = \cancel{r_x^2 r_y} \; r_y^2 + \frac{r_x^2}{4} - r_y r_x^2$

For region 2 :

We have initial point $(x_k, y_k - 1)$, $(x_k+1, y_k-1) \rightarrow (x_k + \frac{1}{2}, y_k - 1)$

and equation : $r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2 = 0$

Now, $r_y^2 \left(x_k + \frac{1}{2}\right)^2 + r_x^2 (y_k - 1)^2 - r_x^2 r_y^2 = P_{2k}$ — (i)

$r_y^2 \left(x_{k+1} + \frac{1}{2}\right)^2 + r_x^2 (y_{k+1} - 1)^2 - r_x^2 r_y^2 = P_{2k+1}$

$r_y^2 \left((x_{k+1}) + \frac{1}{2}\right)^2 + r_x^2 ((y_k - 1) - 1)^2 - r_x^2 r_y^2 = P_{2k+1}$ — (ii)

Eqn (ii) − Eqn (i),

$P_{2k+1} - P_{2k} = r_y^2 \left(x_{k+1} + \frac{1}{2}\right)^2 + r_x^2 ((y_k - 1) - 1)^2 - \cancel{r_x^2 r_y^2} - r_y^2 \left(x_k + \frac{1}{2}\right)^2 - r_x^2 (y_k - 1)^2 + \cancel{r_x^2 r_y^2}$

$y_{k+1} = y_k - 1$

$x_{k+1} = x_k + 1$

$= r_y^2 \left(x_{k+1}^2 + \frac{1}{4} + x_{k+1}\right) + r_x^2 \left((y_k - 1)^2 + 1 - 2(y_k - 1)\right) - r_y^2 \left(x_k + \frac{1}{2}\right)^2 - r_x^2 (y_k - 1)^2$

$= r_y^2 \left[x_{k+1}^2 + \cancel{\frac{1}{4}} + x_{k+1} - x_k^2 - \cancel{\frac{1}{4}} - x_k\right] + r_x^2 \left[\cancel{y_k^2} + 1 - 2y_{k+1} - \cancel{2y_k} + 2 - \cancel{y_k^2} - 1 + \cancel{2y_k}\right]$

$1 - 2y_{k+1} + 2$

$1 - 2(y_{k+1} + 1)$

$= r_y^2 \left[x_{k+1}^2 + x_{k+1} - x_k^2 - x_k\right] + r_x^2 \{1 - 2y_{k+1} + 2\}$

$= r_y^2 \left[x_{k+1}^2 + x_{k+1} - x_k^2 - x_k\right] + r_x^2 \{1 - 2y_k\}$

Now, Initial decision Parameter,

Put $(0, r_y)$ in $P_{1k} = r_y^2 (x_k+1)^2 + r_x^2 \left(y_k - \frac{1}{2}\right)^2 - r_x^2 r_y^2$

or, $P_{10} = r_y^2 (0+1)^2 + r_x^2 (y_k - 1/2) - r_x^2 r_y^2$

or, $P_{10} = r_y^2 + r_x^2 \left(r_y^2 + \frac{1}{4} - r_y\right) - r_x^2 r_y^2$

$\therefore P_{10} = \cancel{x^2_y} \cancel{x} r_y^2 + \frac{r_x^2}{4} - r_y r_x^2$

For region 2 :

We have initial point $(x_k, y_k - 1), (x_k+1, y_k-1) \to (x_k + \frac{1}{2}, y_k - 1)$

and equation : $r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2 = 0$

Now, $r_y^2 \left(x_k + \frac{1}{2}\right)^2 + r_x^2 (y_k - 1)^2 - r_x^2 r_y^2 = P_2 k$ — (1)

$r_y^2 \left(x_{k+1} + \frac{1}{2}\right)^2 + r_x^2 (y_{k+1} - 1)^2 - r_x^2 r_y^2 = P_2 k+1$

$r_y^2 \left((x_{k+1}) + \frac{1}{2}\right)^2 + r_x^2 ((y_k - 1) - 1)^2 - r_x^2 r_y^2 = P_2 k+1$ — (11)

Eqn (11) − Eqn (1),

$P_2 k+1 - P_2 k = r_y^2 \left(x_{k+1} + \frac{1}{2}\right)^2 + r_x^2 ((y_k - 1) - 1)^2 - \cancel{r_x^2 r_y^2} - r_y^2 \left(x_k + \frac{1}{2}\right)^2 - r_x^2 (y_k - 1)^2 + \cancel{r_x^2 r_y^2}$

$y_{k+1} = y_k - 1$

$x_{k+1} = x_k + 1$

$x_{k+1} = x_k$

$= r_y^2 \left(x_{k+1}^2 + \frac{1}{4} + x_{k+1}\right) + r_x^2 \left((y_k - 1)^2 + 1 - 2(y_k - 1)\right) - r_y^2 \left(x_k + \frac{1}{2}\right)^2 - r_x^2 (y_k - 1)^2$

$1 - 2y_{k+1} + 2$

$1 - 2(y_{k+1} + 1)$

$= r_y^2 \left[x_{k+1}^2 + \cancel{\frac{1}{4}} + x_{k+1} - x_k^2 - \cancel{\frac{1}{4}} - x_k\right] + r_x^2 \left[\cancel{y_k^2} + 1 - 2y_{k+1} - \cancel{2y_k} + 2 - \cancel{y_k^2} - 1 + \cancel{2y_k}\right]$

$= r_y^2 \left[x_{k+1}^2 + x_{k+1} - x_k^2 - x_k\right] + r_x^2 \{1 - 2y_{k+1} + 2\}$

$= r_y^2 \left[x_{k+1}^2 + x_{k+1} - x_k^2 - x_k\right] + r_x^2 \{1 - 2y_k\}$

$$P_{2K+1} = P_{2K} + r_y^2\left[x_{K+1}^2 + x_{K+1} - x_K^2 - x_K\right] - 2y_{K+1} r_x^2 + r_x^2$$

Case I : If $P_{2K} > 0 \rightarrow x_{K+1} = x_K$

$$P_{2K+1} = P_{2K} - 2y_{K+1} r_x^2 + r_x^2$$

Case II : If $P_{2K} \leq 0 \rightarrow x_{K+1} = x_K + 1$

$$P_{2K+1} = P_{2K} + r_y^2\left[\cancel{x_K^2} + 1 + 2x_K - \cancel{x_K^2} + 1 + \cancel{x_K} + 1 - \cancel{x_K}\right] - 2y_{K+1} r_x^2 + r_x^2$$

$$P_{2K+1} = P_{2K} + r_y^2 (2x_{K+1}) - 2y_{K+1} r_x^2 + r_x^2$$

Now, Initial decision parameter for region 2 :

It can be obtained by putting the last point of region 1 in the equation:

$$P_{2K} = r_y^2 \left(x_K + \tfrac{1}{2}\right)^2 + r_x^2 (y_K - 1)^2 - r_x^2 r_y^2$$

$$\boxed{P_{2K} = r_y^2 \left(x_K + \tfrac{1}{2}\right)^2 + r_z^2 (y_K - 1)^2 - r_x^2 r_y^2}$$

$P_{2K+1} = P_{2K} + r_y^2 [x_{K+1}^2 + x_{K+1} - x_K^2 - x_K] - 2y_{K+1} r_x^2 + r_x^2$

Case I : If $P_{2K} > 0 \rightarrow x_{K+1} = x_K$

$P_{2K+1} = P_{2K} - 2y_{K+1} r_x^2 + r_x^2$

Case II : If $P_{2K} \leq 0 \rightarrow x_{K+1} = x_K + 1$

$P_{2K+1} = P_{2K} + r_y^2 [\cancel{x_K^2} + 1 + 2x_K \cancel{x_K^2} + \cancel{x_K} + 1 - \cancel{x_K}] - 2y_{K+1} r_x^2 + r_x^2$

$P_{2K+1} = P_{2K} + r_y^2 (2x_{K+1}) - 2y_{K+1} r_x^2 + r_x^2$

Now, Initial decision parameter for region 2 :

It can be obtained by putting the last point of region 1 in the equation:

$P_{2K} = r_y^2 (x_K + \frac{1}{2})^2 + r_x^2 (y_K - 1)^2 - r_x^2 r_y^2$

$\boxed{P_{2K} = r_y^2 (x + \frac{1}{2})^2 + r_x^2 (y - 1)^2 - r_x^2 r_y^2}$

Q. Digitize the ellipse with $r_x = 8$, $r_y = 6$ and center (3, 5).

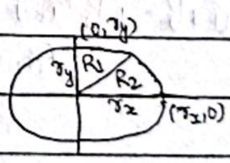
Sol^n →

For region 1

The initial point for the ellipse at origin, $(x_0, y_0) = (0, r_y) = (0, 6)$

The initial decision parameter, $P_{10} = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$

$= 6^2 - 8^2 * 6 + \frac{1}{4} * 8^2$



$= -332$

From mid-point algorithm for region 1, we know,

1. If $P_{1K} < 0$ then,

$x_{K+1} = x_K + 1$, $y_{K+1} = y_K$ and $P_{1K+1} = P_{1K} + 2r_y^2 x_{K+1} + r_y^2$

2. If $P_{1K} \geq 0$ then,

$x_{K+1} = x_K + 1$, $y_{K+1} = y_K - 1$ and $P_{1K+1} = P_{1K} + 2r_y^2 x_{K+1} - 2r_x^2 y_{K+1} + r_y^2$

$P_2K+1 = P_2K + r_y^2\left[x_{K+1}^2 + x_{K+1} - x_K^2 - x_K\right] - 2y_{K+1} r_x^2 + r_x^2$

Case I : If $P_2K > 0 \rightarrow x_{K+1} = x_K$

$P_2K+1 = P_2K - 2y_{K+1} r_x^2 + r_x^2$

Case II : If $P_2K \le 0 \rightarrow x_{K+1} = x_K + 1$

$P_2K+1 = P_2K + r_y^2\left[\cancel{x_K^2} + 1 + 2x_K - \cancel{x_K^2} + 1 + \cancel{x_K} + 1 - \cancel{x_K}\right] - 2y_{K+1} r_x^2 + r_x^2$

$P_2K+1 = P_2K + r_y^2(2x_{K+1}) - 2y_{K+1} r_x^2 + r_x^2$

Now, Initial decision parameter for region 2 :

It can be obtained by putting the last point of region 1 in the equation:

$P_2K = r_y^2(x_K + \frac{1}{2})^2 + r_x^2(y_K - 1)^2 - r_x^2 r_y^2$

$\boxed{P_2K = r_y^2(x_K + \frac{1}{2})^2 + r_x^2(y_K - 1)^2 - r_x^2 r_y^2}$

Q. Digitize the ellipse with $r_x = 8$, $r_y = 6$ and center $(3, 5)$.

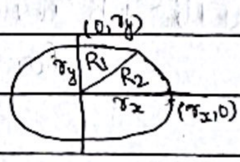
Sol^n →

For region 1

The initial point for the ellipse at origin, $(x_0, y_0) = (0, r_y) = (0, 6)$

The initial decision parameter, $P1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$

$= 6^2 - 8^2 * 6 + \frac{1}{4} * 8^2$



$= -332$

From mid-point algorithm for region 1, we know,

1. If $P_{1K} < 0$ then,

$x_{K+1} = x_K + 1$, $y_{K+1} = y_K$ and $P_{1K+1} = P_{1K} + 2r_y^2 x_{K+1} + r_y^2$

2. If $P_{1K} \geq 0$ then,

$x_{K+1} = x_K + 1$, $y_{K+1} = y_K - 1$ and $P_{1K+1} = P_{1K} + 2r_y^2 x_{K+1} - 2r_x^2 y_{K+1} + r_y^2$

$$P_{2K+1} = P_{2K} + r_y^2\left[x_{K+1}^2 + x_{K+1} - x_K^2 - x_K\right] - 2y_{K+1} r_x^2 + r_x^2$$

Case I : If $P_{2K} > 0 \rightarrow x_{K+1} = x_K$

$$P_{2K+1} = P_{2K} - 2y_{K+1} r_x^2 + r_x^2$$

Case II : If $P_{2K} \le 0 \rightarrow x_{K+1} = x_K + 1$

$$P_{2K+1} = P_{2K} + r_y^2\left[\cancel{x_K^2} + 1 + 2x_K - \cancel{x_K^2} + 1 + \cancel{x_K} + 1 - \cancel{x_K}\right] - 2y_{K+1} r_x^2 + r_x^2$$

$$P_{2K+1} = P_{2K} + r_y^2(2x_{K+1}) - 2y_{K+1} r_x^2 + r_x^2$$

Now, Initial decision parameter for region 2 :

It can be obtained by putting the last point of region 1 in the equation:

$$P_{2K} = r_y^2\left(x_K + \frac{1}{2}\right)^2 + r_x^2(y_K - 1)^2 - r_x^2 r_y^2$$

$$\boxed{P_{2K} = r_y^2\left(x_K + \frac{1}{2}\right)^2 + r_x^2(y_K - 1)^2 - r_x^2 r_y^2}$$

Q. Digitize the ellipse with $r_x = 8$, $r_y = 6$ and center $(3, 5)$.

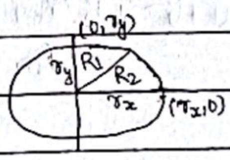
Sol^n:

For region 1

The initial point for the ellipse at origin, $(x_0, y_0) = (0, r_y) = (0, 6)$

The initial decision parameter, $P_{10} = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$

$$= 6^2 - 8^2 * 6 + \frac{1}{4} * 8^2$$



$$= -332$$

From mid-point algorithm for region 1, we know,

1. If $P_{1K} < 0$ then,
$x_{K+1} = x_K + 1$, $y_{K+1} = y_K$ and $P_{1K+1} = P_{1K} + 2r_y^2 x_{K+1} + r_y^2$

2. If $P_{1K} \geq 0$ then,
$x_{K+1} = x_K + 1$, $y_{K+1} = y_K - 1$ and $P_{1K+1} = P_{1K} + 2r_y^2 x_{K+1} - 2r_x^2 y_{K+1} + r_y^2$

$-+ \mid ++$

$3^{rd}\; -- \mid +-\; 4^{th}$

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K	P_K	(x_{K+1}, y_{K+1})	$2r_y^2 x_{K+1}$	$2r_x^2 y_{K+1}$
0	-332	(1, 6)	72	768
1	-224	(2,6)	144	768
2	-44	(3,6)	216	768
3	208	(4,5)	288	640
4	-108	(5,5)	360	640
5	288	(6,4)	432	512
6	244	(7,3)	504	384

Now, we move out of region 1, since $2r_y^2 x_{K+1} > 2r_x^2 y_{K+1}$.
Since the above table is for centre (0,0), but we want to plot ellipse at centre (3,5). Thus,

1^{st} Quadrant	2^{nd} Quadrant	3^{rd} Quadrant	4^{th} Quadrant
(1+3, 6+5) = (4,11)	(-4, 11)	(-4, -11)	(4, -11)
(2+3, 6+5) = (5,11)	(-5, 11)	(-5, -11)	(5, -11)
(3+3, 6+5) = (6,11)	(-6, 11)	(-6, -11)	(6, -11)
(4+3, 5+5) = (7,10)	(-7, 10)	(-7, -10)	(7, -10)
(5+3, 5+5) = (8,10)	(-8, 10)	(-8, -10)	(8, -10)
(6+3, 4+5) = (9,9)	(-9, 9)	(-9, -9)	(9, -9)
(7+3, 3+5) = (10,8)	(-10, 8)	(-10, -8)	(10, -8)

For region 2

Initial point for region 2, $(x_0, y_0) = (7,3)$

The decision parameter is, $P_{20} = r_y^2\left(x_0+\frac{1}{2}\right)^2 + r_x^2(y_0-1)^2 - r_x^2 r_y^2 = -23$

From mid-point algorithm for region 2, we know

1. If $P_{2K} > 0$ then
$x_{K+1} = x_K$, $y_{K+1} = y_K - 1$ and $P_{2K+1} = P_{2K} + 2r_x^2 y_{K+1} + r_x^2$

2. If $P_{2K} \leq 0$ then
$x_{K+1} = x_K + 1$, $y_{K+1} = y_K - 1$ and $P_{2K+1} = P_{2K} + 2r_y^2 x_{K+1} - 2r_x^2 y_{K+1} + r_x^2$

Scanned with CamScanner

K	P_{2k}	(x_{k+1}, y_{k+1})	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	−23	(8, 2)	576	256
1	361	(8, 1)	576	72
2	497	(8, 0)	576	0

We move out of region 2 since we get a point $(r_x, 0)$ i.e. (8,0).

Now

1st Quadrant	2nd Quadrant	3rd Quadrant	4th Quadrant
(8+3, 2+5) = (11, 7)	(−11, 7)	(−11, −7)	(11, −7)
(8+3, 1+5) = (11, 6)	(−11, 6)	(−11, −6)	(11, −6)
(8+3, 0+5) = (11, 5)	(−11, 5)	(−11, −5)	(11, −5)

Note: If centre is given as (−3, −5) then we calculate above as (8−3, 2−5) = (5, −3).

Chapter 03 — 2-D Transformation

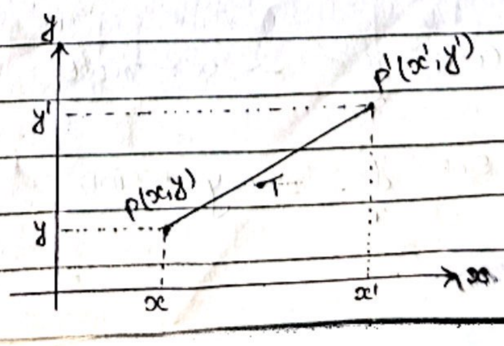
It is the changing of co-ordinate description of an object. Co-ordinate can be changed either rigid body transformation or non-rigid body transformation. When a transformation takes place on a 2-D plane then it is called as 2-D transformation. The basic transformation are as below:

1. Translation
2. Rotation
3. Scaling

Other transformation are:

4. Reflection
5. Shear

Translation



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Repositioning of an object along a straight line path from one co-ordinate location to another is called translation. Translation is performed on a point by adding translation co-ordinate to its original co-ordinate to generate a new co-ordinate position. Let $P(x,y)$ be translated to $P'(x',y')$ using translation co-ordinate (offset) t_x and t_y in x and y direction respectively. Then,

$$T_x = x' - x, \Rightarrow x' = x + T_x$$

$$T_y = y' - y \Rightarrow y' = y + T_y$$

In matrix form, $P' = P + T$

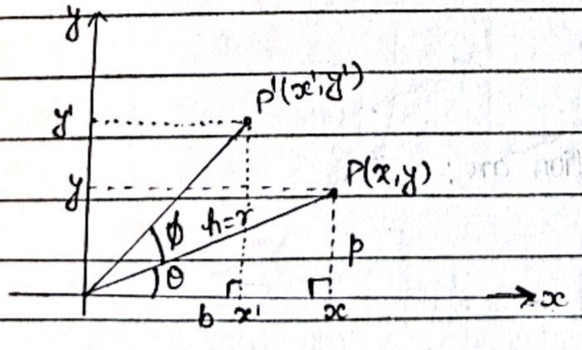
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Rotation

Changing the coordinate position along a circular path is called rotation. 2-D rotation is applied to re-position the object along a circular path in x, y-plane. A rotation is generated by specifying rotation angle (θ) and pivot point (rotation point). The positive θ rotates object in anticlockwise and negative value of θ rotates the object in clockwise direction.

Let $P(x,y)$ be a point rotated by θ about origin to new point $P'(x',y')$.

Here,



b = base = x
p = perpendicular = y
h = hypotenous = radius of circle(r)

$$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$$

Now the object is rotated by angle ϕ.

$$\cos(\theta+\phi) = \frac{x'}{r}$$

$$\Rightarrow x' = r\cos(\theta+\phi)$$
$$= r[\cos\theta\cos\phi - \sin\theta\sin\phi]$$
$$= r\cos\theta\cos\phi - r\sin\theta\sin\phi$$
$$= x\cos\phi - y\sin\phi$$

$$\sin(\theta+\phi) = \frac{y'}{r}$$

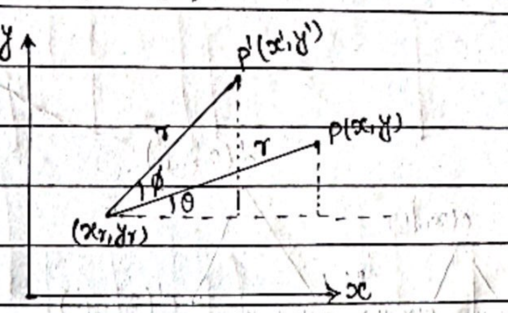
$$\Rightarrow y' = r\sin(\theta+\phi)$$
$$= r[\sin\theta\cos\phi + \cos\theta\sin\phi]$$
$$= r\sin\theta\cos\phi + r\cos\theta\sin\phi$$
$$= y\cos\phi + x\sin\phi$$

In matrix form,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\therefore P' = R \cdot P$$

If pivot point is at (x_r, y_r).



$$\cos\theta = \frac{x-x_r}{r} \Rightarrow x - x_r = r\cos\theta \Rightarrow x = x_r + r\cos\theta$$

$$\sin\theta = \frac{y-y_r}{r} \Rightarrow y - y_r = r\sin\theta \Rightarrow y = y_r + r\sin\theta$$

Now the object is rotated by angle ϕ.

$$\sin(\theta+\phi) = \frac{y'-y_r}{r}$$

$$\Rightarrow y'-y_r = r\sin(\theta+\phi)$$
$$= r\left[\sin\theta\cos\phi + \cos\theta\sin\phi\right]$$
$$= r\sin\theta\cos\phi + r\cos\theta\sin\phi$$
$$\Rightarrow y' = y_r + r\sin\theta\cos\phi + r\cos\theta\sin\phi$$
$$\therefore y' = y_r + (y-y_r)\cos\phi + (x-x_r)\sin\phi$$

And

$$\cos(\theta+\phi) = \frac{x'-x_r}{r}$$

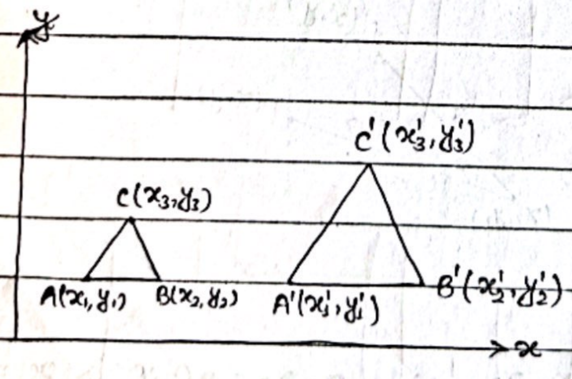
$$\Rightarrow x'-x_r = r\left[\cos\theta\cos\phi - \sin\theta\sin\phi\right]$$
$$\Rightarrow x' = x_r + r\cos\theta\cos\phi - r\sin\theta\sin\phi$$
$$\therefore x' = x_r + (x-x_r)\cos\phi - (y-y_r)\sin\phi$$

In matrix form,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x_r & \cos\phi & -\sin\phi \\ y_r & \cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} 1 \\ (x-x_r) \\ (y-y_r) \end{bmatrix}$$

Hence the value of x', y' gives the rotated co-ordinate point with angle ϕ by taking pivot point (x_r, y_r)

Scaling



Scaling transformation alter the size of object. A simple 2-D scaling operation is performed by multiplying object position (x, y) with scaling factors S_x and S_y along with x and y-direction to produce (x', y').

Now, $x' = x.S_x$ and $y' = y.S_y$

In matrix form, $P' = SP$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

If the scaling factor is less than 1, the size of object is decreased and if it is greater than 1, the size of object is increased. The scaling factor is equal to 1 for both direction doesn't change the size of object.

Matrix Representation And Homogeneous Coordinates

The homogeneous coordinate system provide a uniform framework of for handling different geometric transformation, simply as multiplication of matrices.

To perform more than one transformation at a time, homogeneous coordinates are used. They reduce unwanted calculation, intermediate steps, saves time and memory and produce a sequence of transformation.

We represent each coordinate position (x, y) with the homogeneous coordinate (x_h, y_h, h) where,

$$x = \frac{x_h}{h} \quad \text{and} \quad y = \frac{y_h}{h}$$

'h' is one usually for 2-D transformation. Therefore (x, y) in cartesian system is represented as $(x, y, 1)$ in homogeneous coordinate system.

1. For translation, $T(t_x, t_y)$

$$P' = TP$$

where, $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$, $T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$; $P = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

By which we get

$$x' = x + t_x$$

$$y' = y + t_y$$

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2. For rotation: $R(\theta)$

$$P' = RP$$

where, $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$, $P = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$, $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. For scaling (S_x, y_x)

$$P' = S.P$$

where, $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$, $P = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ and $S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

This gives the eqn,

$$x' = x \cdot S_x$$

and

$$y' = y \cdot S_y$$

Q.1. Find the scaled triangle with vertices A(0,0), B(1,1) and C(5,2) after it has been magnified twice its size.

Soln →

We have coordinate of unscaled triangle, A(0,0), B(1,1) and C(5,2).

$$S_x = 2 \text{ and } S_y = 2$$

Now,

$$x' = x \cdot S_x \quad \text{and} \quad y' = y \cdot S_y$$

For A, $P' = S \cdot P$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = (0,0)$$

For B, $P' = S \cdot P$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = (2,2)$$

For C, $P' = S \cdot P$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \\ 1 \end{bmatrix} = (10,4)$$

Hence, the final co-ordinate points for scaled triangle are A'(0,0), ~~B'(0,2√2)~~ and B'(2,2), C'(10,4).

Q. Rotate a triangle A(0,0), B(2,2), C(4,2) about the origin by the angle of 45°.

Sol^n: The given triangle ABC can be represented by a matrix form homogeneous co-ordinates of vertices, $\begin{bmatrix} 0 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

Also we have, $R_{45°} = \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So the coordinate of the rotated triangle A'B'C' are

$$R_{45}(ABC) = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \sqrt{2} \\ 0 & 2\sqrt{2} & 3\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

Hence, the final coordinate points are A'(0,0), B'(0, 2√2) and C'(√2, 3√2).

Q.2. Rotate a triangle (5,5), (7,3), (3,3) about fixed point (5,4) in counter clockwise by 90°.

Sol^n:

The required steps are:

(i) Translate the fixed point to origin

(ii) Rotate about the origin by 90° degree

(iii) Reverse the translation as performed earlier.

Thus the composite matrix is given by,

$$M = T(x_f, y_f)\, R_\theta\, T(-x_f, -y_f)$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -4 \\ 0 & 0 & +1 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 0 & -1 & 9 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}} \cancel{\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 \times \cos 90 + 0 + 0 & 0 - 1 + 0 \\ & \\ & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & -5 \\ 0 & 0 & +1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -9 \\ 0 & 0 & +1 \end{bmatrix}.$$

Hence the required co-ordinate can be calculated as,

$$P' = M * P$$

$$= \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -9 \\ 0 & 0 & +1 \end{bmatrix} \begin{bmatrix} 5 & 7 & 3 \\ 5 & 3 & 3 \\ 1 & 1 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} -6 & -4 & -4 \\ -4 & -2 & -6 \\ +1 & +1 & +1 \end{bmatrix}$$

Hence the required coordinates are $(-6, -4)$, $(-4, -2)$ and $(-4, -6)$.

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Q. 3. Rotate a triangle A(7, 15), B(5, 8) and C(10, 10) by 45° clockwise about origin and scale it by (2, 3) about origin.

Sol^n →

The required steps are:

(i) Rotate by 45° clockwise

(ii) Scale by $S_x = 2$ and $S_y = 3$

The composite matrix is given by,

$$M = S(2,3) \cdot R_{45^\circ}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(45)^\circ & -\sin(45)^\circ & 0 \\ \sin(45)^\circ & \cos(45)^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} +\frac{\sqrt{2}}{2} & -\left(\frac{-\sqrt{2}}{2}\right) & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -\frac{3\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now the transformation points are,

$$A' = M \cdot A = \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 15 \\ 1 \end{bmatrix} = \begin{bmatrix} 22\sqrt{2} \\ 12\sqrt{2} \\ 1 \end{bmatrix}$$

$$B' = M \cdot B = \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ \frac{-3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 13\sqrt{2} \\ \frac{9\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

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$$C' = M \cdot C = \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ \frac{-3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 20\sqrt{2} \\ 0 \\ 1 \end{bmatrix}$$

Q.1. A square with vertices A(0,0), B(2,0), C(2,2) and D(0,2) is scaled two units in x and y direction about the fixed point (1,1). Find the coordinates of vertices of new square.

Q.2. A triangle having vertices A(3,3), B(8,5) and C(5,8) is first translated by 2 units about fixed point (5,6) and finally rotated by 90° anticlockwise about pivot point (2,5). Find the final position of triangle.

Q.3. Rotate the triangle ABC by 90° anticlockwise direction about (5,8) and scale it by (2,2) about (10,10).

Q.no.1. Soln, The required steps are :

(i) Translate the fixed point to origin

(ii) Scale it by (2,2)

(iii) Reverse the translation as performed earlier i.e. translate again to fixed point

Given, $S_x = 2$, $S_y = 2$

Thus the composite matrix is given by,

$$M = T(1,1)\ S(2,2)\ T(-1,-1)$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence required coordinates are,

$$A' = A \cdot M = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = (-1,-1)$$

$$B' = M \cdot B = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} = (3,-1)$$

$$C' = M \cdot C = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} = (3,3)$$

$$D' = M \cdot D = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} = (-1,3)$$

Q.no.2 Sol^n: The required steps are:

Firstly,

(i) Translate it by 2 units i.e. (2, 2)

Again,

(ii) Translate the pivot point to origin

(iii) Rotate by 90° anticlockwise

(iv) Reverse the translation as performed earlier

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Thus the composite matrix is given by,

$$M = T(2,2)\ T(2,5)\ R_{90^\circ}\ T(-2,-5)$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 9 \\ 1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus the final position of Δ is,

$$P' = M \cdot P$$

$$= \begin{bmatrix} 0 & -1 & 9 \\ 1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 8 & 5 \\ 3 & 5 & 8 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0*3+(-1)(3)+9*1 & 0*8+(-1)(5)+9*1 & 0*5+(-1)(8)+9*1 \\ 1*3+0*3+5*1 & 1*8+0*5+5*1 & 1*5+0*8+5*1 \\ 0*3+0*3+1*1 & 0*8+0*5+1*1 & 0*5+0*8+1*1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 & 1 \\ 8 & 13 & 10 \\ 1 & 1 & 1 \end{bmatrix}$$

Thus, $A' = (6, 8)$

$B' = (3, 13)$

$C' = (1, 10)$

Q no. 3 Sol^n → Required steps are :

(I) Translate the fixed point (5,8) to origin
(II) Rotate by 90° anticlockwise
(III) Reverse the translation as performed earlier
(IV) Translate the fixed point (10, 10) to origin
(V) Scale it by (2, 2)
(VI) Reverse the translation as performed earlier

Thus the composite matrix is given by,

$$M = T(5,8)\, R_{90}\, T(-5,-8)\, T(10,10)\, S(2,2)\, T(-10,-10)$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & 8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -20 \\ 0 & 2 & -20 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & 8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -18 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & 23 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus required co-ordinates of rotated Δ are,

$$P' = M \cdot P$$

$$= \begin{bmatrix} 0 & -2 & 23 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix}$$

Reflection

Reflection is a type of transformation which provides a mirror image about an axis of an object. Following are the different reflection about different axis :

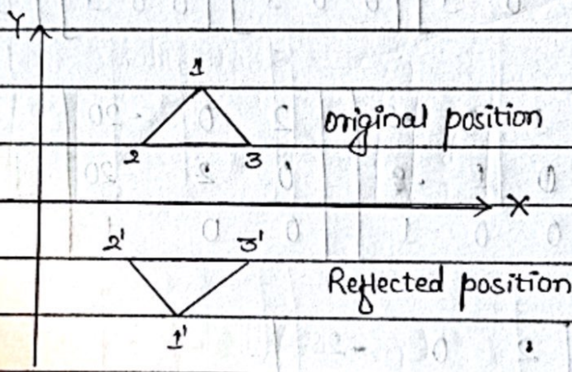
1. Reflection about x-axis ($y=0$)

The reflection of a point $P(x,y)$ on X-axis, changes the y-coordinate sign i.e. $P(x,y)$ changes to $P'(x,-y)$.

In matrix form,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\therefore \boxed{P' = R_{fx} \cdot P}$$



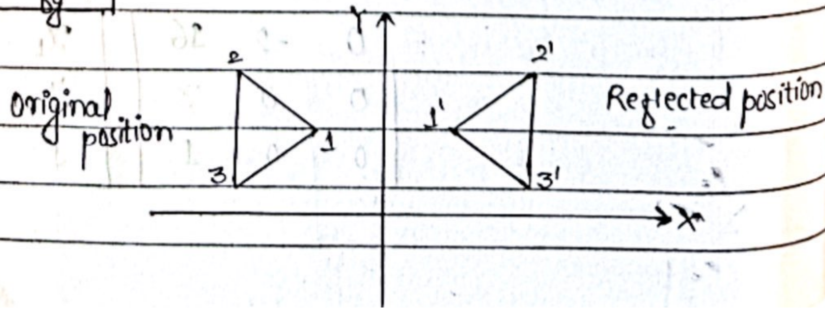
2. Reflection about y-axis ($x=0$)

The reflection of a point $P(x,y)$ on Y-axis, changes the x-coordinate sign i.e. $P(x,y)$ changes to $P'(-x,y)$.

In matrix form,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\therefore \boxed{P' = R_{fy} \cdot P}$$



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3. Reflection about origin
Flip both x and y-coordinate of a point $P(x, y)$ changes to $P'(-x, -y)$.
In matrix form,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\therefore \boxed{P' = R_{f_{x,y}} \cdot P}$$

4. Reflection about $y = mx + c$
Perform the following transformation:
(a) Translate the straight line so that it passes through origin.
(b) Rotate the straight line so that it concides with any coordinate axis
(c) Reflect object about that axis, perform reverse rotation
(d) perform reverse translation, so that straight line is placed to its original position

Shearing

A transformation that distorts the shape of an object such that the transformed shape appears as if the object is composed of internal layers and these layers are caused to ~~slidstigh~~ over is called shearing.

1. X-direction shear
A X-direction shear relative to x-axis is produced with transformation matrix equation,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

which transforms,

$$x' = x + Sh_x \cdot y$$

$$y' = y$$

2. Y-direction shear
A y-direction shear relative to y-axis is produced with transformation matrix eqn,

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$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

which transforms,

$$x' = x$$
$$y' = x \cdot Sh_y + y$$

3 X-direction shear relative to $y = y_{ref}$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & -Sh_x \cdot y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

which transforms,

$$x' = x + Sh_x (y - y_{ref})$$
$$y' = y$$

4. Y-direction shear relative to $x = x_{ref}$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_y & 1 & -Sh_y \cdot x_{ref} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

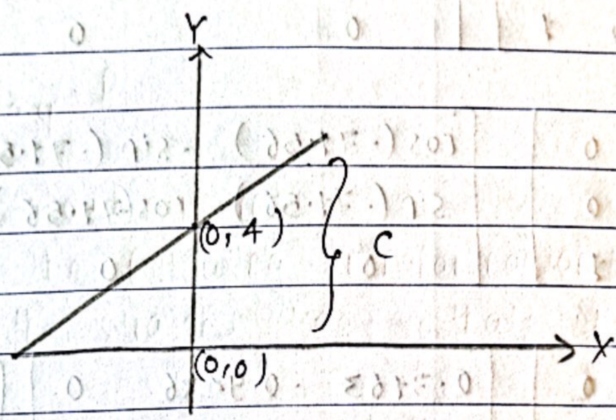
which transforms,

$$x' = x$$
$$y' = y + Sh_y (x - x_{ref})$$

Q. A triangle having vertices A(2,3), B(6,3) and C(4,8) is reflected about $y = 3x+4$ find the final position of a triangle.

⇒ we have straight line $y = 3x+4$ $(y = mx+c)$

$m = 3$, $c = 4$



$(t_x, t_y) = (0, c) = (0, 4)$

$\theta = \tan^{-1}(m) = \tan^{-1}(3) = 71.56$

$$T(t_x, t_y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta) = \begin{bmatrix} \cos(71.56) & -\sin(71.56) & 0 \\ \sin(71.56) & \cos(71.56) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

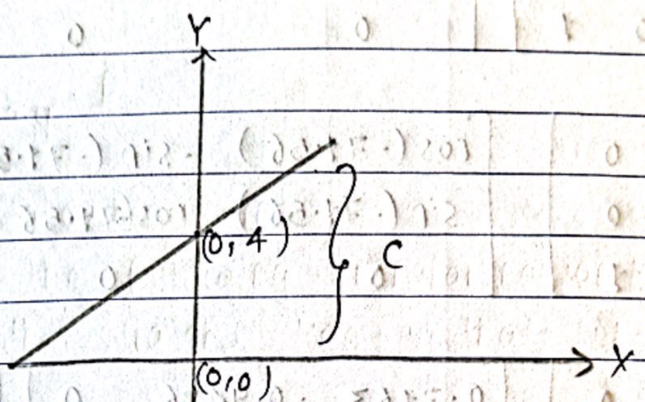
$$T(-t_x, t_y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(-\theta) = \begin{bmatrix} \cos(71.56) & -\sin(71.56) & 0 \\ \sin(-71.56) & \cos(-71.56) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q. A triangle having vertices A(2,3), B(6,3) and C(4,8) is reflected about $y = 3x+4$ find the final position of a triangle.

$\Rightarrow$ we have straight line $y = 3x+4$ ($y = mx+c$)

$m = 3$, $c = 4$



$(t_x, t_y) = (0, c) = (0, 4)$

$\theta = \tan^{-1}(m) = \tan^{-1}(3) = 71.56$

$$T(t_x, t_y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta) = \begin{bmatrix} \cos(71.56) & -\sin(71.56) & 0 \\ \sin(71.56) & \cos(71.56) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(-t_x, -t_y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(-\theta) = \begin{bmatrix} \cos(71.56) & -\sin(71.56) & 0 \\ \sin(-71.56) & \cos(-71.56) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Now, the composite matrix is

$$M = T(t_x, t_y)\ R(\theta)\ R_{fx}\ R(-\theta)\ T(-t_x, -t_y)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(71.56) & -\sin(71.56) & 0 \\ \sin(-71.56) & \cos(-71.56) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-71.56) & -\sin(-71.56) & 0 \\ \sin(-71.56) & \cos(71.56) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.3163 & -0.9486 & 0 \\ 0.9486 & 0.3163 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.3163 & 0.9486 & 0 \\ -0.9486 & 0.3163 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix}$$

Now the transformation point are:

$$A' = M.A = \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.2082 \\ 4.4159 \\ 1 \end{bmatrix}$$

$$= (-2.2082, 4.4159)$$

$$B' = M.B = \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -5.408 \\ 6.8479 \\ 1 \end{bmatrix}$$

$$= (-5.408, 6.8479)$$

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$$c' = M.C = \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.7684 \\ 9.6324 \\ 1 \end{bmatrix}$$

$= (0.7684, 96324)$,,

Q Drive the composite matrix for reflecting an object about any arbitary line $y = mx + c$.

⇒ In order to reflect an object about any line $y = mx + c$ we need to perform composite transformation as below:

$T = T(0, c) \cdot R(\theta) \cdot R_{fx} \cdot R(-\theta)\ T(0, -c)$

and slope $(m) = \tan\theta$

Also we have,

$$\cos^2\theta = \frac{1}{\tan^2\theta + 1} = \frac{1}{m^2 + 1}$$

$$\cos\theta = \frac{1}{\sqrt{m^2+1}}$$

Also, $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \frac{1}{m^2+1}$$

$$\sin\theta = \frac{m}{\sqrt{m^2+1}}$$

So, $T = T(0, c)\ R(\theta)\ R_{fx} \cdot R(-\theta)\ T(0, -c)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$$

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$$c' = M.C = \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.7684 \\ 9.6324 \\ 1 \end{bmatrix}$$

$= (0.7684, 96324)$

Q Drive the composite matrix for reflecting an object about any arbitary line $y = mx + c$.

⇒ In order to reflect an object about any line $y = mx + c$ we need to perform composite transformation as below:

$T = T(0,c) \cdot R(\theta) \cdot R_{fx} \cdot R(-\theta)\; T(0,-c)$

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$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \frac{1}{m^2+1}$$

$$\sin\theta = \frac{m}{\sqrt{m^2+1}}$$

So, $T = T(0,c)\, R(\theta)\, R_{fx} \cdot R(-\theta)\; T(0,-c)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$$

2-D viewing Transformation / 2-D viewing pipeline.

The process of mapping the world coordinate ~~seen~~ sene to device coordinate is called viewing transformation or or windows to view port transformation.

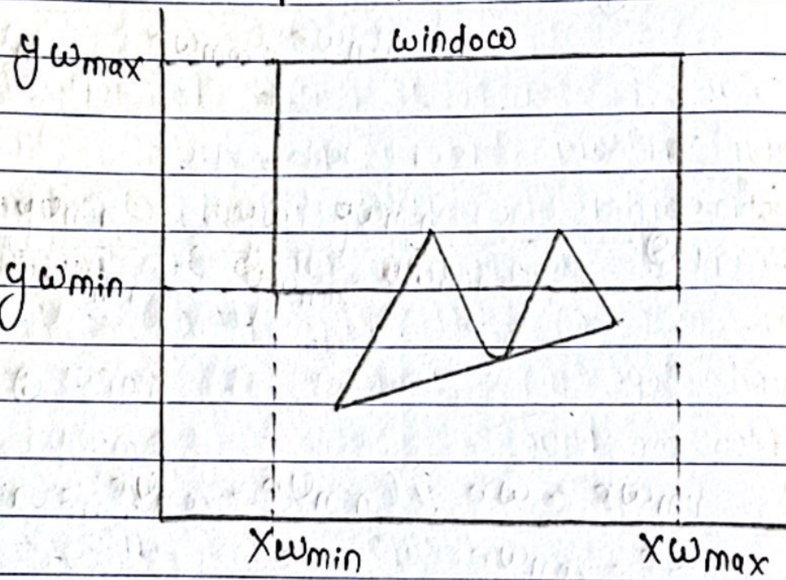


fig ①: World coordinates

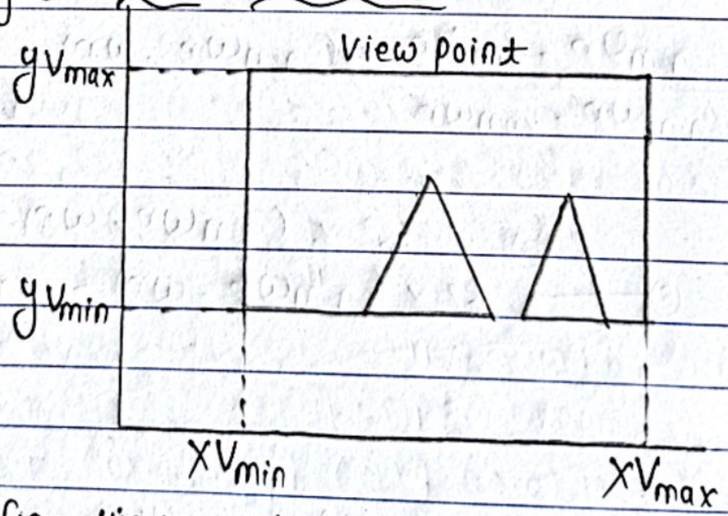


fig: View coordinates

→ World coordinate are selected for displaying (What is to be displays) is called a window and an area on the display devices to which a window is mapped (Where is to be displayed) is called a view port.

→ Window deals with object space where as ~~ima~~ view port deals with image space.

→ Transformation from world coordinates to device coordinate involves specially translation, rotation and scaling operation as well as clipping operation..

The steps for pipeline process are as follows.

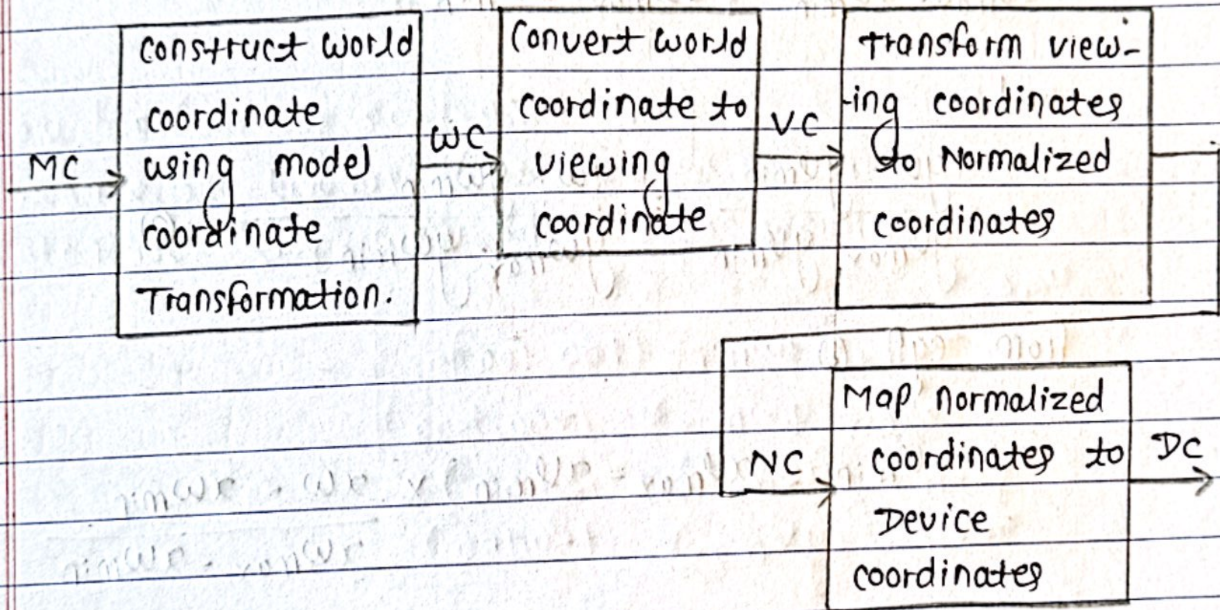
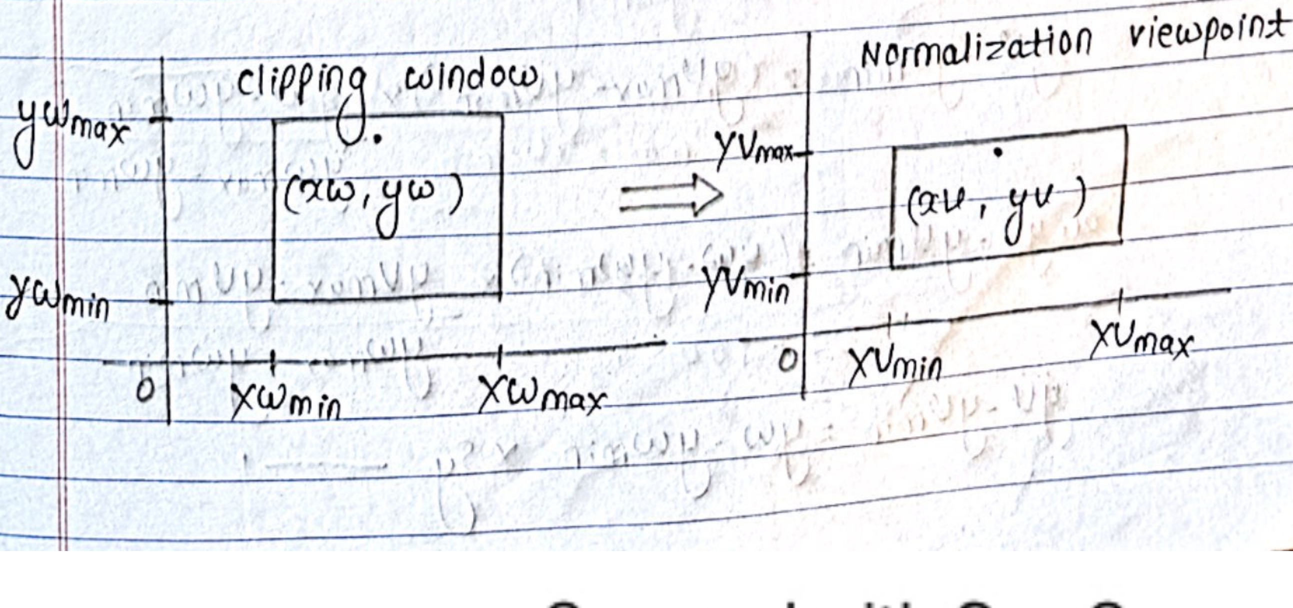


Fig: 2-D viewing Transformation pipeline.

To make the viewing process independent of the requirement of any o/p devices, graphic system convert object description to normalized coordinates.

Windows to Viewport transformation.



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To maintain the same relative placement in the viewport as in the window, we require that

$$\frac{xv - xv_{min}}{xv_{max} - xv_{min}} = \frac{xw - xw_{min}}{xw_{max} - xw_{min}} \quad \text{--- (1)}$$

and,

$$\frac{yv - yv_{min}}{yv_{max} - yv_{min}} = \frac{yw - yw_{min}}{yw_{max} - yw_{min}} \quad \text{--- (2)}$$

from eqⁿ (1)

$$xv - xv_{min} = (xv_{max} - xv_{min}) \times \frac{xw - xw_{min}}{xw_{max} - xw_{min}}$$

$$xv - xv_{min} = (xw - xw_{min}) \times \frac{xv_{max} - xv_{min}}{xw_{max} - xw_{min}}$$

$$xv - xv_{min} = (xw - xw_{min}) \times S_x$$

$$xv = xv_{min} + (xw - xw_{min}) \times S_x \quad \text{--- (3)}$$

from Eqⁿ (2)

$$yv - yv_{min} = (yv_{max} - yv_{min}) \times \frac{yw - yw_{min}}{yw_{max} - yw_{min}}$$

or, $$yv - yv_{min} = (yw - yw_{min}) \times \frac{yv_{max} - yv_{min}}{yw_{max} - yw_{min}}$$

$$yv - yv_{min} = yw - yw_{min} \times S_y \quad \text{---}$$

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$$yv = yv_{min} + (yw - yw_{min}) \times S_y \quad \text{--- (4)}$$

from eqn (3) and (4)

$$xv = xv_{min} + (xw - xw_{min}) \times S_x$$

$$xv = xv_{min} + S_x xw - S_x xw_{min}$$

also

$$yv = yv_{min} + S_y . yw - S_y . yw_{min}$$

for Translation of coordinate to viewpoint

$$xv = S_x . xw_{min} + t_x$$

$$yv = S_y . yw_{min} + t_y$$

And the translation factor (t_x, t_y) would be

$$t_x = \frac{xw_{max} . xv_{min} - xw_{min} . xv_{max}}{xw_{max} - xw_{min}}$$

$$t_y = \frac{yw_{max} . yv_{min} - yw_{min} . yv_{min}}{yw_{max} - yw_{min}}$$

steps for transformation

(1) Scaling
(2) Translation

Here, $S = \begin{bmatrix} S_x & 0 & xw_{min}(1-S_x) \\ 0 & S_y & yw_{min}(1-S_y) \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore T = \begin{bmatrix} 1 & 0 & xv_{min} - xw_{min} \\ 0 & 1 & yv_{min} - yw_{min} \\ 0 & 0 & 1 \end{bmatrix}$

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$$yv = yv_{min} + (yw - yw_{min}) \times S_y \quad \text{--- (4)}$$

from eqn (3) and (4)

$$xv = xv_{min} + (xw - xw_{min}) \times S_x$$

$$xv = xv_{min} + S_x \cdot xw - S_x \cdot xw_{min}$$

also

$$yv = yv_{min} + S_y \cdot yw - S_y \cdot yw_{min}$$

for Translation of coordinate to viewpoint

$$xv = S_x \cdot xw_{min} + t_x$$

$$yv = S_y \cdot yw_{min} + t_y$$

And the translation factor (t_x, t_y) would be

$$t_x = \frac{xw_{max} \cdot xv_{min} - xw_{min} \cdot xv_{max}}{xw_{max} - xw_{min}}$$

$$t_y = \frac{yw_{max} \cdot yv_{min} - yw_{min} \cdot yv_{min}}{yw_{max} - yw_{min}}$$

steps for transformation

1. Scaling
2. Translation

Here, $S = \begin{bmatrix} S_x & 0 & xw_{min}(1-S_x) \\ 0 & S_y & yw_{min}(1-S_y) \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore T = \begin{bmatrix} 1 & 0 & xv_{min} - xw_{min} \\ 0 & 1 & yv_{min} - yw_{min} \\ 0 & 0 & 1 \end{bmatrix}$

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Now window to viewport transformation matrix is

$$T_{wv} = S \cdot T = \begin{bmatrix} S_x & 0 & t_x \\ 0 & S_y & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Q. Window port is given (100, 100, 300, 300) and view port is given by (50, 50, 150, 150). Convert the window port coordinate (200, 200) to the view port coordinate.

⇒ Here

$(xw_{min}, yw_{min}) = (100, 100)$

$(xw_{max}, yw_{max}) = (300, 300)$

$(xv_{min}, yv_{min}) = (50, 50)$

$(xv_{max}, yv_{max}) = (150, 150)$

$(xw, yw) = (200, 200)$

Then, we have

$$S_x = \frac{xv_{max} - xv_{min}}{xw_{max} - yw_{min}} = \left(\frac{150 - 50}{300 - 100}\right) = 0.5$$

$$S_y = \frac{yv_{max} - yv_{min}}{xw_{max} - yw_{min}} = \left(\frac{150 - 50}{300 - 100}\right) = 0.5$$

$$t_x = \frac{xw_{max} \cdot xv_{min} - xw_{min} \cdot xv_{max}}{xw_{max} - xw_{min}}$$

$$= \frac{300 \times 500 - 100 \times 150}{300 - 100}$$

$$= 0$$

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$$t_y = \frac{yw_{max} \cdot yv_{min} - yw_{min} \cdot yv_{max}}{yw_{max} - yw_{min}}$$

$$= 0$$

The eq^n for mapping window coordinate to new coordinate is given by

$$xv = S_x x_w + t_x = 0.5 \times 200 + 0 = 100$$

$$yv = S_y \cdot y_w + t_y = 0.5 \times 200 + 0 = 100$$

∴ The transformed view port coordinate is (100, 100)

Q. Find the normalization transformation matrix for window to view port which uses the rectangle whose lower left corner is at (2,2) and upper right corner is at (6,10) as a window and the view port that has lower left corner at (0,0) and upper right corner at (1,1).

Here,

$$(xw_{min}, yw_{min}) = (2,2)$$

$$(xw_{max}, yw_{max}) = (6,10)$$

$$(xv_{min}, yv_{min}) = (0,0)$$

$$(xv_{max}, yv_{max}) = (1,1)$$

Then we know

$$S_x = \frac{xv_{max} - xv_{min}}{xw_{max} - yw_{min}} = \frac{1-0}{6-2} = \frac{1}{4} = 0.25$$

$$S_y = \frac{yv_{max} - yv_{min}}{xw_{max} - yw_{min}} = \frac{1-0}{10-2} = 1/8 = 0.125$$

Now, $t_x = \frac{6 \times 0 - 2 \times 1}{6-2} = \frac{-2}{4} = -1/2 = -0.5$