Programming Skills – Exercises 2

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All the exercises here are about stochastic processes, i.e. processes that are based on chance. You can usually obtain a solution to such problems by simulating the process many times (say N times, where N is sufficiently large). For each simulation, you measure the same output. The average of these outputs is then the expected value, $E()$. If q_i, $i = 1, \cdots, N$ are measured then

$$E(q) = \frac{1}{N} \sum_{i=1}^{N} q_i.$$

You have to find a 'decent' value for N, and this value can vary for different exercises. There are various ways of determining that value, but I usually start with the value of N for which I do not get bored waiting...

So for all problems we expect more than 'just a code'. You must pay attention to the interpretation of the results that your codes produce.

Of course you will need to generate many random numbers. For that, use `help rand` and/or `help randi`.

1.) **A drunken sailor leaves the bar.**

Consider a drunken sailor that leaves a bar. He makes a step forward, or backward, both with 0.5 (so 50%) chance. As he is completely drunk ("straalbezopen") he does not know what direction he is going, and this process is repeated indefinitely, forward and backboard each with a 0.5 likelihood.

Simulate this process, which means that you let this drunken sailor coming out of the bar and starting his journey many many times.

a.) Measure the probability, P_m, that the sailor returns to the bar for the first time (!) after exactly m steps. Make a plot of this probability P_m for many values of m, with for example $m \leq 50$.
Verify your result with the exact answer:

$$P_m = \begin{cases} \begin{pmatrix} m \\ m/2 \end{pmatrix} \dfrac{2^{-m}}{m-1}, & m \text{ even}, \\ 0, & m \text{ odd}. \end{cases}$$

b.) Also measure the average of the number of steps, $E(m)$, for returning to the bar. Can you understand this result? You are allowed to / should use Mathematica here!

c.) What is the probability of returning to the origin at least once if we wait very (very (very)) long?

2.) **A drunken sailor walking on a plaza.**

Consider the same drunken sailor that leaves a bar which is now situated in the middle of a square. He now every time makes a step forward, OR backward, OR left OR right, each with 0.25 chance.

a.) Measure the probability, P_m, that the sailor returns to the bar for the first time after exactly m steps. Make a plot of this probability P_m for many values of m (for example, with $m \leq 50$).

b.) What is the average distance to the origin after m steps? How does this depend on m? Use a log-log plot for distance and m; fit this result using the standard MATLAB function `polyfit`.

c.) Can you guess the probability of returning to the origin at least once if we wait very (very (very)) long? If you cannot do it because of time (which is possible), can you indicate how long your script should run in order to give an educated guess?