

COMP 7370/7376
Advanced Computer and Network Security
Homework Assignment 2, Mar. 3
Due on Tuesday, Mar. 16 in class (submit your answer sheet)

Instruction: Every student should finish the following questions independently. Give justification for the results (i.e., show the calculation process/steps) to receive full credits.

In all the following encryption/decryption questions, let's assume that "a" corresponds to 0, "b" to 1, and so on.

1. Consider a cryptosystem in which $\mathcal{P}=\{a, b, c\}$, $\mathcal{K}=\{K_1, K_2, K_3\}$, and $\mathcal{C}=\{1, 2, 3, 4\}$. Suppose the encryption matrix is as follows:

	a	b	c
K_1	1	2	3
K_2	2	3	4
K_3	3	4	1

Given that keys are chosen equiprobably, and the plaintext probability distribution is $\Pr[a]=1/2$, $\Pr[b]=1/3$, $\Pr[c]=1/6$, compute the entropies $H(\mathbf{P})$, $H(\mathbf{C})$, $H(\mathbf{K})$, $H(\mathbf{K}|\mathbf{C})$ and $H(\mathbf{P}|\mathbf{C})$.

2. (Chinese Remainder Theorem) Solve the following system of congruences:

$$x \equiv 12 \pmod{25}$$

$$x \equiv 9 \pmod{26}$$

$$x \equiv 23 \pmod{27}.$$

3. (Discrete Logarithm) Let $p = 227$. The element $\alpha=2$ is primitive in \mathbb{Z}_p^* .

(a) Compute α^{32} , α^{40} , α^{59} and α^{156} modulo p , and factor them over the factor base $\{2, 3, 5, 7, 11\}$.

(b) Using the fact that $\log 2 = 1$, compute $\log 3$, $\log 5$, $\log 7$ and $\log 11$ from the factorizations obtained above (all logarithms are discrete logarithms in \mathbb{Z}_p^* to the base α).()

4. Compute $\gcd(57, 93)$, and find integers s and t such that $57s+93t = \gcd(57, 93)$.